A2. Emergence of Mass and Gravity: The Coupled Pressure Model

1. Cell Surface Variables

- Total reflective boundary area:
$$\mathfrak{A}_s \approx \pi l_p^2$$
(the external reflective surface of a single Qaether cell).

- Area blocked per bond:
$$\mathfrak{A}_b \ll \mathfrak{A}_s \quad \Longrightarrow \quad \alpha \equiv \frac{\mathfrak{A}_b}{\mathfrak{A}_s} \ll 1$$

2. Remaining Reflective Area

For a cell i with m_i bonds,

$$\mathfrak{A}_i(m_i) = \mathfrak{A}_s - m_i\mathfrak{A}_b = (1 - \alpha m_i)\,\mathfrak{A}_s.$$

Even for the FCC lattice maximum $m_i = 12$, we have $\alpha m_i \ll 1$, thus $\mathfrak{A}_i > 0$.

3. Amplitude Coefficient $F_\star$: Gauge-Invariant (Plaquette-Based)

The sum below is taken over all minimal plaquettes sharing site i (the smallest loops such as squares or triangles on the FCC lattice):

$$\boxed{F_\star(i) \equiv \frac{\sum_{\Box\ni i} \omega_\Box \big(1 - \frac{1}{2}\mathrm{Re}\,\mathrm{Tr}\,U_\Box\big)}{\sum_{\Box\ni i} \omega_\Box}}.$$

This definition represents a local average of the standard Wilson density $(1 - \frac{1}{2}\mathrm{Re}\,\mathrm{Tr}\,U_\Box)$, hence is gauge invariant.

For small angles,

$$U_\Box \simeq e^{i\Theta_\Box}, \qquad \tfrac{1}{2}\mathrm{Re}\mathrm{Tr}\,U_\Box \simeq 1 - \tfrac{1}{2}\|\Theta_\Box\|^2,$$

which gives

$$\boxed{F_\star(i) \simeq \frac{1}{2N_i}\sum_{\Box\ni i} \|\Theta_\Box\|^2 \;\propto\; \text{(local curvature intensity average).}}$$

4. Internal Phase Oscillation Energy of Qaether Cell i

Assume the Qaether wavelength equals the Planck length l_p. The corresponding angular frequency is

$$\omega_q = \frac{2\pi c}{l_p},$$

and the internal phase oscillation energy is

$$E_q = \tfrac{1}{2}\hbar\omega_q = \hbar\frac{\pi c}{l_p}.$$

Then the phase energy density is

$$u_\phi = \frac{E_q}{V_s} = \frac{\tfrac{1}{2}\hbar\omega_q}{\tfrac{1}{6}\pi l_p^3} = \frac{6\hbar c}{l_p^4}.$$

5. Reference Pressure p_0

Define the pressure corresponding to 100% reflection per unit area:

$$p_0 = 2u_\phi = \frac{12\hbar c}{l_p^4}.$$

The blocked region ($m_i\mathfrak{A}_b$) receives no phase-wave impact, hence zero pressure.

6. Local Effective Pressure

With effective boundary thickness $\delta = \eta l_p$ ($\eta \sim \mathcal{O}(1)$; limit $\eta \to 0$ if needed):

$$\boxed{P_i(m_i) = p_0\frac{\mathfrak{A}_i(m_i)}{\mathfrak{A}_s}F_\star(i) = p_0(1-\alpha m_i)F_\star(i).}$$

7. Pressure–Energy Mapping and Mass (Background Difference Definition)

Stored energy:

$$U_{\text{press}}(i) = P_i\mathfrak{A}_s\delta = p_0\mathfrak{A}_s\eta l_p(1-\alpha m_i)F_\star(i).$$

Relative to the uniform (unobservable) vacuum background:

$$\boxed{\Delta U_{\text{press}}(i) = -\alpha m_i p_0\mathfrak{A}_s\eta l_p F_\star(i) = -\alpha m_i\eta 12\pi\frac{\hbar c}{l_p}F_\star(i) = -\alpha m_i\eta 12\pi E_{\text{Pl}}F_\star(i),}$$

where $E_{\text{Pl}} = \hbar c/l_p$.

Binding energy per bond (uniform per-cell reference):

$$\boxed{\Delta U_{\text{bond}}^{(\text{per cell})} = -\alpha\eta 12\pi E_{\text{Pl}}F_\star(i).}$$

Thus the effective inertial (rest) mass of a cell:

$$\boxed{m_{\text{eff}}(i) = \frac{E_q + U_{\text{link}}(i) + \Delta U_{\text{press}}(i)}{c^2},}$$

where U_{link} represents link/plaquette contributions such as $W_{ij}(\Delta\phi_{ij})$. Since $\Delta U_{\text{press}} < 0 \Rightarrow$ increasing bonds ($m_i \uparrow$) reduces rest energy (inertial mass) $\Rightarrow$ stronger binding.

8. Stress–Energy Tensor (Apparent Fluid Approximation: Isotropic + Anisotropic Corrections)

Isotropic approximation:

$$T_i^{00} \approx u_\phi (1 - \alpha m_i) F_\star(i), \qquad T_i^{aa} \approx p_0 (1 - \alpha m_i) F_\star(i) \quad (a = 1, 2, 3).$$

Anisotropic correction (bond patch normal $\hat{\mathbf{n}}_e$):

$$\Delta T_i^{ab} \approx -p_0 F_\star(i) \sum_{e \in \mathcal{N}(i)} \alpha n_e^a n_e^b.$$

If the bond directions are random/uniform:

$$\sum_e \alpha n_e^a n_e^b \simeq \tfrac{\alpha m_i}{3} \delta^{ab}.$$

9. Local Moment of Inertia

Assume the cell's internal energy is distributed over a sphere of radius $r = l_p/2$:

$$E_q = \frac{\pi \hbar c}{l_p}, \qquad m_q = \frac{E_q}{c^2} = \frac{\pi \hbar}{c l_p}, \qquad r_q = \frac{l_p}{2}.$$

Then the moment of inertia for a uniform solid sphere:

$$I_i = \frac{2}{5} m_q r_q^2 = \frac{2}{5} \left(\frac{\pi \hbar}{c l_p} \right) \left(\frac{l_p}{2} \right)^2.$$

Simplified:

$$\boxed{I_i = \frac{\pi}{10} \frac{\hbar l_p}{c}.}$$