Assignment 5

Name:__________________; UID:__________________

1 Question 1 20 pt

Suppose $f(x) = \sin(x), x \in [-1, 1]$. Let $x_i, i = 0, ..., n, n \geq 1$ be equadistributed grid points in $[-1, 1]$. That is, $x_i = -1 + \frac{2i}{n}$. Suppose $p_n(x)$ is the Lagragian polynomial approximating $f(x)$. That is, $p_n(x_i) = f(x_i)$. Give an upper error bound for the approximation error

$$|f(x) - p_n(x)|, x \in [-1, 1] \tag{1}$$

w.r.t. n.

2 Question 2 20 pt

Write code to implement the Hermite interpolation algorithm and approximate the function $f(x) = \tanh(x)$ at $x_i = i, i = 0, ..., 20$ using $p_{41}(x)$ such that $p_{41}(x_i) = f(x_i), p'_{41}(x_i) = f'(x_i)$. Make a plot.

3 Question 3 20 pt

Suppose that $n \geq 2$. The function f and its derivatives of order up to and including $2n$ are continuous on $[a, b]$. The points x_i, $i = 0, 1, \ldots, n$, are distinct and lie in $[a, b]$. Explain how to construct polynomials $l_0(x)$, $l_n(x)$, $h_i(x)$, $k_i(x)$, $i = 1, \ldots, n-1$, of degree $2n - 1$ such that the polynomial

$$p_{2n-1}(x) = l_0(x)f(x_0) + l_n(x)f(x_n) + \sum_{i=1}^{n-1}[h_i(x)f(x_i) + k_i(x)f'(x_i)]$$

satisfies the conditions $p_{2n-1}(x_i) = f(x_i)$, $i = 0, 1, \ldots, n$, and $p'_{2n-1}(x_i) = f'(x_i)$, $i = 1, \ldots, n-1$. It is not necessary to give explicit expressions for these polynomials.

4 Question 4 20pt

Write code to implement the trapezium formula and the Simpson's formula. Try to numerically calculate the following integrations. (h should be viewed as an input variable) Take $h = 1, 1/2, 1/4, 1/8, 1/16, 1/32, 1/64$.

$$1. \int_{-h}^{h} \exp(x^3) + \sin(x^2) + x\underset{\cdot}{\mathrm{x}} \tag{2}$$

$$2. \int_{0}^{h} \cos(x^5 + 4x^4 + 3x^3 + 2x^2 + x + 1)\underset{\cdot}{\mathrm{x}} \tag{3}$$

For both, plot the values w.r.t. different h for the trapezium formula and the Simpson's formula.

5 Question 5 20pt

Give the points $x_0 = -1, x_1 = 0, x_2 = \frac{1}{2}, x_3 = 1$.

(a) Determine the Lagragian interpolation polynomial such that

$$f(x_i) = p_3(x_i), i = 0, 1, 2, 3 \tag{4}$$

(b) Using the numerical integration approximation

$$\int_0^1 f(x)\underset{\cdot}{\mathrm{x}} \approx \int_0^1 p_3(x)\mathrm{d}x = \sum_{i=0}^{3} w_i p_3(x_i), \tag{5}$$

determine the parameters w_i.