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Project 2: OFDM Reception

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Chapter 1

Introduction

1.1 Background and Up-to-date development

Orthogonal Frequency Division Multiplexing can be considered as one of the most newest modulations of Frequency selective technology for combating transmission channels, which can reach the goal of transmitting data with high rate but without inter-symbol interference (Oltean, 2004); The idea of Orthogonal Frequency Division Multiplexing (OFDM) has been established for a long time. With the development of software and electronic technologies, OFDM had also became e a practical reality (LaSorte et al., 2008). In the 1870s, because of the stimulated by telegraphy companies, some capitalists want to increase their profit by add multiply non-interfering information channels to enhance the capacity of a telegraph transmission line, Baudot and others had invented Time-division multiplexing(TDM) to reach this purposeBeauchamp (2001). Later on, Alexander Graham Bell (Bell, 1876) has worked on the aspect of ” “harmonic telegraphy”, which is also known as FDM transmission of multiple telegraph channels. However, the technique of FDM also have some drawbacks. Change (Chang, 1966) has make a viable OFDM system “without interchannel and intersymbol interferences.” Then Saltzberg (Saltzberg, 1967) has extended Chang’s work into the scope of complex data which is QAM. OFDM then has been enhanced by Hirosaki(Hirosaki, 1981), who made the data transmitted faster by replacing the an N-point DFT with an N/2-point DFT, at the same period, Cyclic-prefix has beed invented by Peled (Peled and Ruiz, 1980)

Later on, this technique grown rapidly, this technique is widely used in the aspect of DVB-T, The DVB-T broadcaster uses a 2048 (“2K”) or 8192 (“8K”) DFT and employs coded OFDM (COFDM) (Le Floch et al., 1995). Within the past few years, OFDM has been applied to long-haul-optical communication networks, by using it, it can help to reduce the degradations of chromatic dispersion (Shieh et al., 2008).In addition, the implementation of using OFDM in the Digital Video Broadcasting-Second Generation Terrestrial (DVB-T2) is quite important which is one of the most important part of this semester.

1.2 Scope, approach and contribution

In this project, the main work can be divided into three parts, the OFDM symbol generation, channel generation and also OFDM Synchronization. The simulation works were done by using Matlab.

To discuss these three part in details, in the first part of OFDM symbol generation, Edge pilots and scattered pilots are used to generate the whole series of symbols, the rest of the symbols are data.

For the part of channel generation, There are mainly three channel type were designed, which are AWGN channel, multi-path Rayleigh fading channel (MRFC), multi-path Ricean fading channel(MRIFC) The last part of this project is OFDM synchronization, there are three techniques were used at this section, which involves cyclic-prex correlation (CPC) technique, frequency domain pilot time domain correlation (FDPC) technique, Integer Frequency Synchronization (IFS) which used both 1 symbol and 2 symbol. The aim of using these techniques is to find the estimated timing $\hat{n}$ and also the estimated frequency offset $\hat{v}$

Chapter 2

System Model

2.1 Terminology

Notation	Description
D_x	FD pilot distance
$\boldsymbol{h}$	TD channel impulse response (CIR)
$\boldsymbol{H}$	FD channel transform function (CTF)
L_d	The length of $\boldsymbol{h}$, a.k.a., channel delay spread
$\hat{n}$	estimated timing
N	FD OFDM symbol size, also the FFT/IFFT size
N_g	the length of cyclic-prefix (CP)
T_s	TD sampling interval with a value of $7/64\mu s$
v	normalized carrier frequency offset
v_i	integer frequency offset
v_f	fractional frequency offset
w	TD AWGN noise
W	FD AWGN noise
$\boldsymbol{x}$	TD OFDM symbol without CP
$\boldsymbol{x}_{cp}$	TD OFDM symbol with CP
$\boldsymbol{x}_p$	TD OFDM symbol generated using the pilot carriers
$\boldsymbol{x}_d$	TD OFDM symbol generated using the data carriers
X	FD OFDM symbol
$\boldsymbol{y}$	received TD OFDM symbol
$\boldsymbol{Y}$	received FD OFDM symbol

Figure 2.1: Terminology

2.2 Determinations

In this project, some determinations are being specified including:

- The symbol size of Frequency domain is $N = 2048$ (Group K).
- The cyclic prefix length $N_g = 512$.
- The FD pilot distance $D_x = 12$.
- QPSK modulation is used for OFDM modulation.
- Consider Edge pilots and Scattered pilots only.
- The amplitude and position of pilots are exactly known to the receiver.
- The signal experiences Gaussian distribution for OFDM synchronization.
- No ISI for integer frequency synchronization and v_f is able to be correctly estimated and compensated.

2.3 System model used

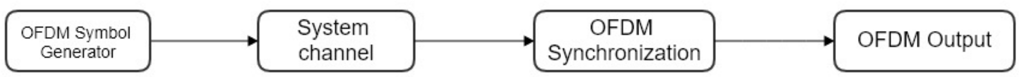

Figure 2.2: System model

The system model of this project consists of four sections, including OFDM symbol generator, system channel that signals are propagating through, OFDM synchronization and output eventually.

In OFDM symbol generator, edge pilots and scattered pilots are taken into account, which means that the first and the last carriers as well as evenly distributed carriers are included in all carriers.

In the channel model, there are three types of possible channels that are about to be considered including AWGN channel($h = 1$), Multi-path Rayleigh fading channel(MRFC) with baseband sampling interval $T_s = 7/64\mu s$ and Multi-path Ricean fading channel (MRIFC) with one extra path h_ric of 10 times of the total power of h_ray.

In OFDM synchronizer, the cyclic-prefix correlation and FPTC are applied to TD synchronization while 1-symbol and 2-symbol are applied to the integer frequency synchronization.

Chapter 3

OFDM Synchronization-Cyclic-prefix Correlation

3.1 Cyclic-prefix Correlation

The aim of doing cyclic prefix is to avoid Inter box interference, and cyclic prefix correlation is a unique for OFDM Synchronization. The principle of this technique is that, when a series of symbol is applied, copy the last N_g symbols and paste it into the beginning of this series of symbol as cyclic prefix, then do the correlation check with it self. The highest correlation appears at the correct timing.

3.2 Denote the correct timing by n0. What is the value of n0?

Because we are required to generate three samples, therefore, the total length of this symbol array is $3(N_g + N)$ when cyclic prefix is applied. Look at the figure below, it showed the three continuous OFDM samples with cyclic prefix.

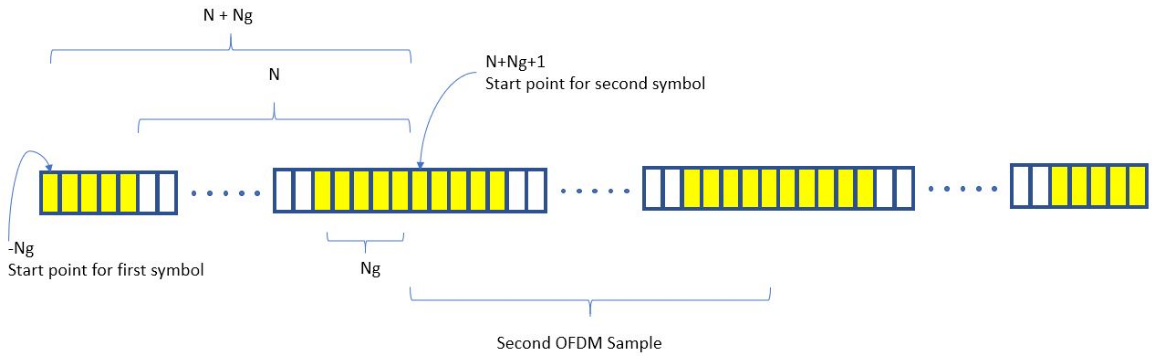

Figure 3.1: three continuous OFDM sample with cyclic prefix

Because for each symbol whose length is N, N_g samples are copied from it tail and paste it to its beginning, the index of these repeated samples are from -N_g, therefore, for a single sample, the correct time n_0 is located at -N_g, but in this case of three continuous sample, the start point of n_0 is located at $N + N_g + 1$.

3.3 What is the maximum possible value of $C(\hat{n})$?

From the lecture 24, we can easily compute the received signal for OFDM is

$$y[n] = x_{cp}[n]e^{j2\pi(v)n/N} + \omega[n] \tag{3.1}$$

Where x_{cp} is the TD sample with Cyclic-prefix, let v = $v_i andv_f$ are Integer frequency offset (IFO) and Fractional frequency offset(FFO) and $\omega[n]$ is the AWGN.// Let's pick a starting point $y[\hat{n}]$, The correlation between $y[\hat{n}] \sim y[\hat{n} + N_g - 1]$ with $y[\hat{n} + N] \sim y[\hat{n} + N + N_g - 1]$

$$C(\hat{n}) = \sum_{n=\hat{n}}^{\hat{n}+N_g-1} y[n]. * y[n+N] \tag{3.2}$$

from previous question, we know that, when there are three continuous OFDM samples, the start point is at $N + N_g + 1$, therefore, $\hat{n} = N + N_g + 1$, then we have

$$\begin{aligned}
C(\hat{n}) &= \sum_{n=N+N_g+1}^{n+2N_g} y[n]. * y[n+N] \\
&= \sum_{n=N+N_g+1}^{n+2N_g} (x_{cp}[n]e^{j2\pi(v)n/N} + \omega[n])(x_{cp} * [n+N]e^{j2\pi(v)(n+N)/N} + \omega[n]) \\
&= e^{j2\pi(v)N/N} \sum_{n=N+N_g+1}^{n+2N_g} |x[n]|^2 + \omega \\
&= e^{j2\pi(v_i+v_f)} \sum_{n=N+N_g+1}^{n+2N_g} |x[n]|^2 + \omega
\end{aligned} \tag{3.3}$$

Because after Cyclic-prefix, the effect of v_i will be removed, only v_f left, then we can have

$$C(\hat{n}) = e^{j2\pi v_f} \sum_{n=N+N_g+1}^{n+2N_g} |x[n]|^2 + \omega \tag{3.4}$$

Therefore, the maximum value of $C(\hat{n})$ is $e^{j2\pi v_f} \sum_{n=N+N_g+1}^{n+2N_g} |x[n]|^2 + \omega$ and when the SNR is large enough, the effect of noise will be removed, then the maximum value of it will become

$$e^{j2\pi v_f} \sum_{n=N+N_g+1}^{n+2N_g} |x[n]|^2 \tag{3.5}$$

3.4 Will the value of v affect the timing and fractional frequency synchronization performance

3.4.1 Timing synchronization

From previous question, we know that, after Cyclic-prefix, the effect of integer frequency has been removed, therefore, only fractional frequency offset is take into account. However, in the timing synchronization process, only the symbol's real timing point was estimated, which means, the fractional frequency offset will not give an effect, therefore, both v_i and v_f will have no effect on timing synchronization.

3.4.2 Frequency synchronization

As for the frequency synchronization part, the range of v_f is [-0.5, 0.5], which is based on its definition which can be shown below

$$v_f = -\frac{1}{2\pi}\angle C(\hat{n}) \tag{3.6}$$

and due to the range of $C(\hat{n})$ is $[-\pi, \pi]$, then follows the equation 3.4, v_f is a factor within $e^{j2\pi v_f}$, therefore, it will affect the frequency synchronization performance.

3.5 What is the average timing offset $\Delta n = |\hat{n} - n_0|$ v.s. SNR? Among the estimated timings, how many of them incur ISI?

To get the relationship between SNR and average timing offset, multiply times of simulation are used to find the average value for a more accurate results. then the relationship between timing offset and SNR can be shown below. The value of SNR is set from 0 to 30 with gab 3.

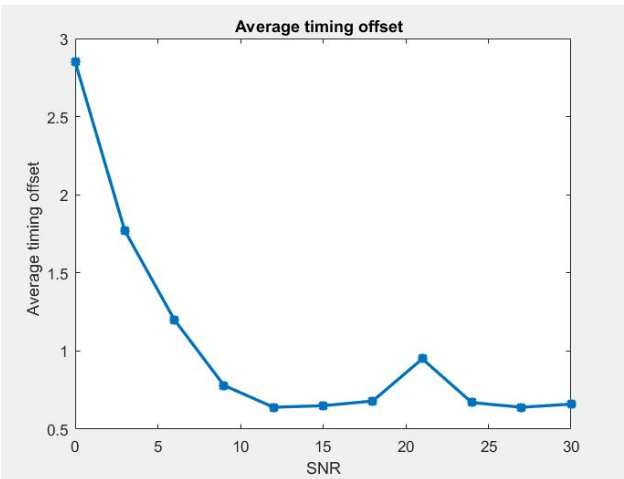

Figure 3.2: "Average timing offset against SNR"

From the figure, it is obvious that, with SNR increased, the amplitude of average timing offset decrease and will fluctuate around some value, which can be considered

as that when the SNR is big enough, the value of average timing offset can be considered as a constant value.
To find the number of how many estimated timing incurs with ISI, multiply times of simulation are also used. the relationship between them can be found in the figure below.

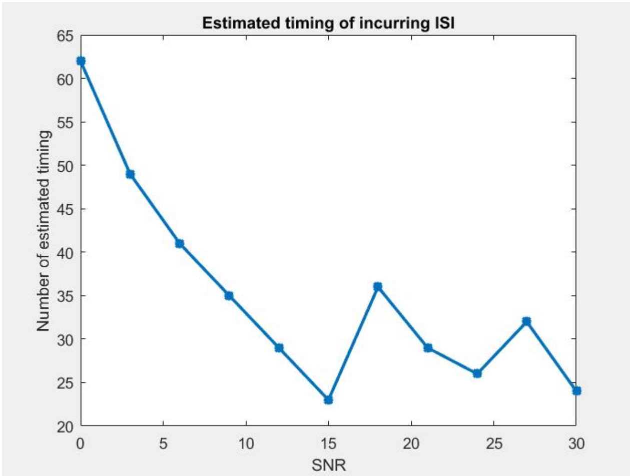

Figure 3.3: Estimated timing of incurring ISI against SNR

It has the same trend with the relationship between SNR and average timing offset, that is when the SNR is small, there are more number of estimated timing with ISI and it will reduce when the value of SNR is increased. In conclusion, the increase of SNR will enhance the performance of Cyclic-prefix correlation.

3.6 What is the the mean-square-error (MSE) of the estimated v_f v.s. SNR?

To get the relationship between SNR and mean-square-error (MSE) of the estimated v_f, multiply times of simulation are used to find the average value for a more accurate results.hen the relationship between them can be shown below. The value of SNR is set from 0 to 30 with gab 3.

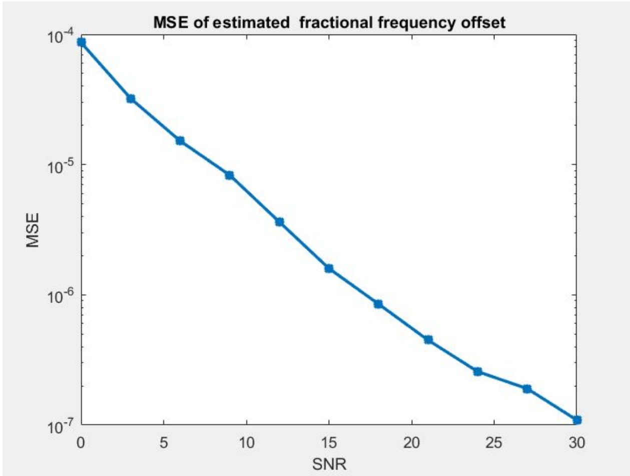

Figure 3.4: MSE of estimated fractional frequency offset

It is obvious that, with the increase of SNR, the MSE decrease respectively.

FD pilot TD Correlation (FPTC)

4.1 What is the value of the correct timing n_0

Assume that n_0 is the last value of cyclic-prefix, the correct timing n_0 is able to be obtained by

$$n_0 = N + N_g * 2 + 1 = 2048 + 2 * 512 + 1 = 3073 \tag{4.1}$$

4.2 Given q, what is the range of v that FPTC can estimate? Provide mathematical derivations.

Since the correlation is divided into q segments which have $M = N/q =$ samples, the segment correlation is able to be obtained by

$$C(i) = \sum_{(i-1)M}^{iM-1} y_{\hat{n}}[n]x_p^*[n] = e^{j\theta_i} \sum_{(i-1)M}^{iM-1} |x_p[n]|^2 + w_n \tag{4.2}$$

where θ_i represents the i^{th} angular segment and w_n stands for the noise. Assume that $\sum_{(i-1)M}^{iM-1} |x_p|^2 = \sum_{iM}^{(i+1)M-1} |x_p|^2$ and obtain that $C(i+1) = C(i)e^{j2\pi Mv/N}$. Therefore,

$$C(i)C^*(i+1) = c * e^{-j2\pi Mv/N} + w_n \tag{4.3}$$

Average the $q-1$ estimations to derive the differential correlation as

$$Cor_{diff} = \frac{1}{q-1}\sum_{1}^{q-1} C(i)C^*(i+1) = c * e^{-j2\pi Mv/N} + w_n \tag{4.4}$$

Then, the estimated offset frequency is calculated by

$$v_{est} = -\frac{N}{2\pi M}\angle Cor_{diff} = -\frac{q}{2\pi}\angle Cor_{diff} \tag{4.5}$$

Since the differential correlation is ranged between $-\pi$ and π, the range of v is estimated between $-q/2$ and $q/2$.

4.3 Can we freely increase q to extend the estimation range on v?

No, we can't. When q increases, M decreases; however, M can not achieve a much small value. While $M = 1$, $N = 4$ and $x_p = \{j, j, -j, -j\}$, incorrect $y_{\hat{n}} = \{1, 1, 1, 1\}$ would be achieved. The correlation between $y_{\hat{n}}$ with x_p is calculated as $Cor = \sum_{i=1}^{4} |C(i)| = 4$. Hence, the correlation has the maximum available value of 4, but with incorrect timing estimation.

4.4 What is the average timing offset $\Delta n = |\hat{n} - n_0|$ v.s. SNR?

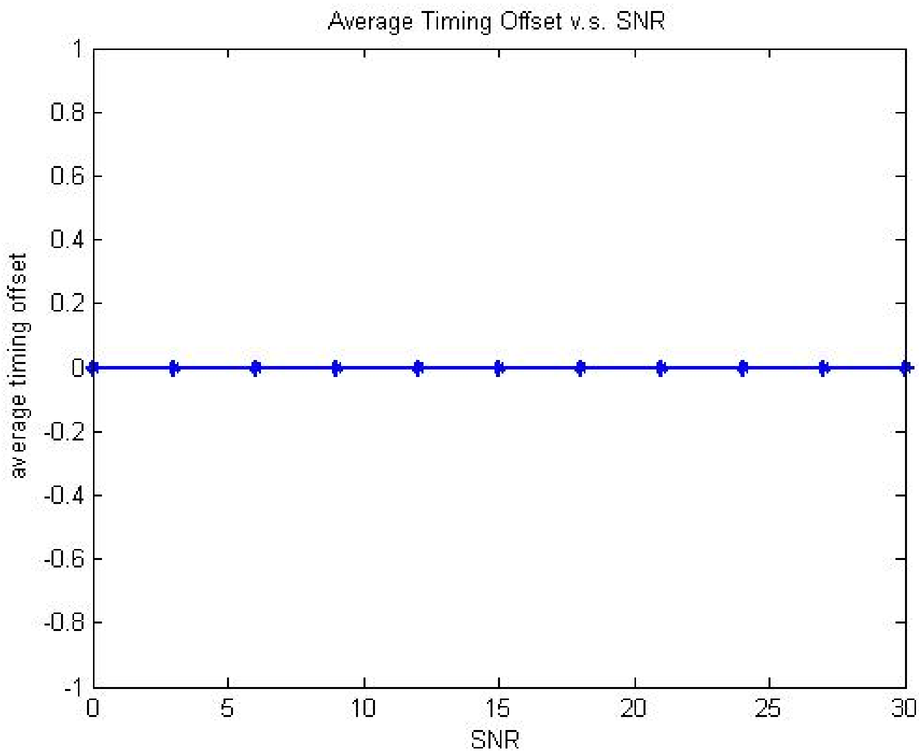

Figure 4.1: Average Timing Offset v.s. SNR

Based on the figure above showing the time offset v.s. SNR, the average timing offset Δn appears to be zero at all time.

4.5 What is the MSE of the estimated $\hat{v}$ v.s. SNR?

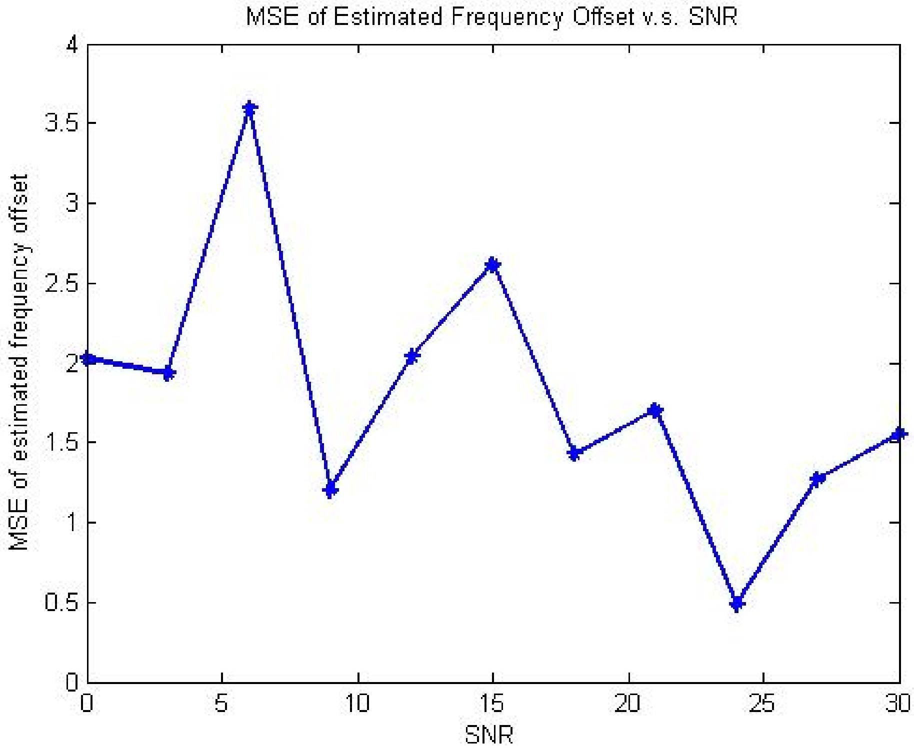

Figure 4.2: Average Frequency Offset v.s. SNR

We can clearly see that a fluctuation of MSE appears under the value of 4 over entire SNRs, which are relatively low enough. The curve slightly decreases when SNR increases.

4.6 Will this x_p provide better synchronization performance?

No, it will not. Considering timing synchronization, the average time offset is always zero and hence the performance is already very good. Considering frequency synchronization, the CP attached x_p does not influence the segments q and will not provide better performance as well.

Integer Frequency Synchronization (IFS)

5.1 IFS using one symbol

5.1.1 What is the maximum possible value of $C(\hat{v}_i)$

The maximum possible value of $C(\hat{v}_i)$ is 305.778, which is calculated as shown below:

$$\begin{aligned} C(\hat{v}_i) &= (\frac{4}{3})^2 \cdot ([\frac{N}{D_x}] + min\{2, remainder(N\%D_x)\} \\ &= \frac{16}{9} \cdot (170+2) \\ &= 305.778 \end{aligned} \tag{5.1}$$

5.1.2 Can this technique distinguish between an IFO of v_i and an IFO of $N - v_i$?Answer analytically

Yes, it can be distinguished,We can see that for the IFO if v_i,we can have:

$$Y_{\hat{n}}(k') = e^{j2\pi k' \Delta n/N} \cdot X([k' - v_i]_N) \tag{5.2}$$

And for the IFO of $N - v_i$, we can have:

$$\begin{aligned} Y'_{\hat{n}}(k') &= e^{j2\pi k' \Delta n/N} \times X([k' - (N - v_i)]_N) \\ &= e^{j2\pi k' \Delta n/N} \cdot X([k' + v_i - N]_N) \\ &= e^{j2\pi k' \Delta n/N} \cdot X([k' + v_i]_N) \end{aligned} \tag{5.3}$$

Therefore, compare equation 5.2 and equation 5.3, we can tell that the technique is able to distinguish between an IFO of v_i and an IFO of $N - v_i$.

5.1.3 Will the value of $\hat{n}$ affect the estimation performance? Answer analytically

Yes, the value of $\hat{n}$ can affect the estimation performance, let $\Delta n = |\hat{n} - n_0|$, which is the value of mistiming from the actual time, and we assume the timing offset exists, and then, we can have:

$$Y_{\hat{n}}(k') = e^{j2\pi k' \Delta n/N} \times X([k' + v_i]_N) \tag{5.4}$$

Therefore, the correlation between $Y_{\hat{n}}(k)$ and $X_p(k)$, and the correlation between $X(k)$ and $X_p(k)$ are not the same, which we can see from the equation below:

$$Corr = \sum_{k=0}^{N-1} Y(k) \cdot X_p^*(k) = \sum_{k=0} N - 1X(k) \cdot X_p^*(k) e^{j2\pi k \Delta n/N} \tag{5.5}$$

We can determine that if Δn not equal to 0, because pilot carrier position cannot match to each other due to the time offset, the correlation is very low even for v_i equals to 0 , therefore, the estimation performance will not be accurate when Δn not equals to 0. Hence, the value of $\hat{n}$ will have the impact on the estimation performance.

5.1.4 What is the MSE of the estimated $\hat{n}_i$ v.s. SNR when $\hat{n} = n_0$? Answer numerically

We can determine that when $\hat{n} = n_0$, the time difference $\Delta n = 0$. Therefore, by equation 5.4 and 5.5, $Y_{\hat{n}}(k') = X([k' - v_i]_N)$ and $Corr = \sum_{k=0}^{N-1} X(k) \cdot X_p^*(k)$. The highest correlation appears when the correct frequency offset encountered. The MSE of the estimated $\hat{v}_i$ against SNR from 0 db to 30 db is shown below.

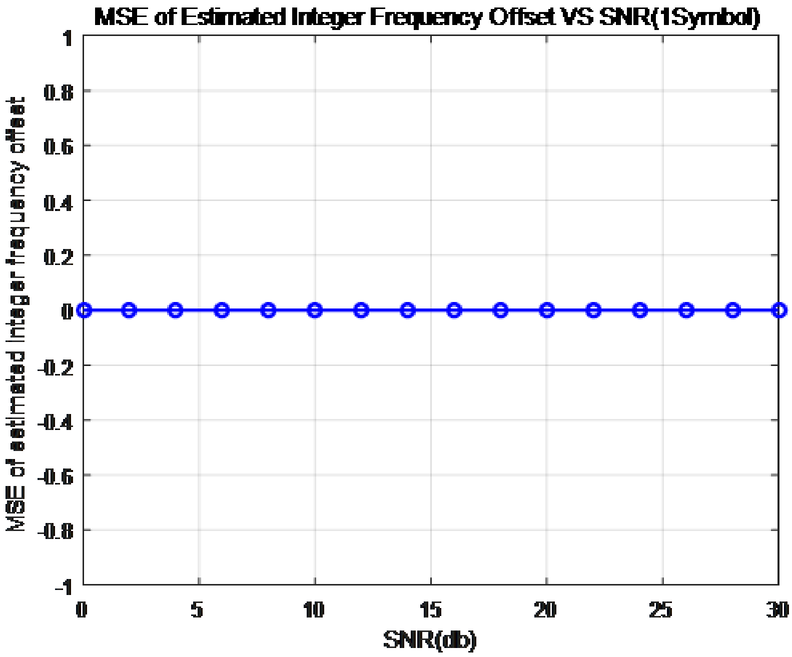

Figure 5.1: MSE of Estimated Integer Frequency Offset VS SNR (1 symbol)

When $\hat{n} = n_0$, the MSE of $\hat{v}_i$ shows on the figure indicate 0 no matter what value SNR is, which means the estimated $\hat{v}_i$ is always correct when $\hat{n} = n_0$ and 1 symbol IFS is applied.

5.2 IFS using two symbols

5.2.1 Prove that the effect of timing offset is removed from Z

For the OFDM symbols with TO and IFO, we can have:

$$Y_{1,\hat{n}}(k') = e^{j2\pi k' \Delta n/N} \cdot X([k' + v_i]_N) \tag{5.6}$$

$$Y_{2,\hat{n}}(k') = e^{j2\pi k' \Delta n/N} \cdot X([k' + v_i]_N) \tag{5.7}$$

and Z can be calculated as:

$$\begin{aligned} Z(k') &= Y_{1,\hat{n}}(k') \cdot Y_{2,\hat{n}}(k') \\ &= X_1([k' - v_i]_N) \cdot X_2^*([k' - v_i]_N) \end{aligned} \tag{5.8}$$

Hence, the effect of timing offset Δn is removed from Z.

5.2.2 What is the MSE of the estimated $\hat{v}_i$ v.s. SNR? Answer numerically

From the proof in 5.2.1, we can tell that the Z has no timing offset. The highest correlation appears when the correct frequency offset encountered. The figure below shows the MSE of the estimated $\hat{v}_i$ against SNR from 0 db to 30 db is shown below, using two symbols.

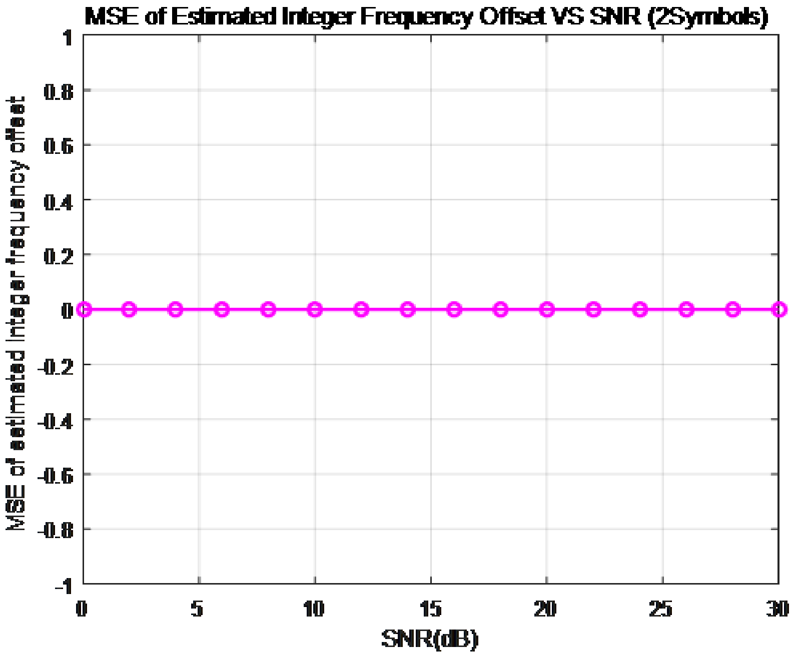

Figure 5.2: MSE of Estimated Integer Frequency Offset VS SNR (2 symbol)

Similarly, the MSE of $\hat{v}_i$ shows on the figure indicate 0 no matter what value SNR is, which means the estimated $\hat{v}_i$ is always correct when two symbol IFS is applied.

Conclusion

In this project, we use Matlab to implement the OFDM reception process, including OFDM symbol generation, channel simulation, and OFDM synchronization. By simulating OFDM signal processing and research on the questions, the knowledge of fundamental principles of OFDM are acquired.

In OFDM synchronization, we realized that, although the cyclic-prefix correlation has a acceptable performance on frequency synchronization, it cannot accurately work on timing synchronization due to the frequency offset. To amendment this situation, the FPTC technique is applied, which can perform a good timing estimation once a suitable segment length (M) is selected. The project has been simplified to make us quickly understand and practice the OFDM technique.

For IFS, the one symbol synchronization technique is favorable when work with the correct timing. For two symbol synchronization, the timing offset can be ignored if the timing offset is within the range of $[n_0 - N_g, n_0]$. In the end, the last step, channel estimation can be used to w estimate the channel for one time interval,and recover the data more accurately, then, the OFDM receiving is complete.

In conclusion, the project covers several main part of OFDM reception. The transmission, channel estimation and many other techniques, channels or issues can be discussed in the future.

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