Problems, problems, problems...

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University of Connecticut

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- The function assigning each person their biological mother.
- The function assigning each whole number its square ($F(n) = n^2$).

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- The range of the function assigning each 9-digit phone number its owner will include me, but not my friend Denis who doesn't have a phone.
- The range of the function assigning each whole number its square will include 1 (because $1 = 1^2$), and it will include 9 (because $9 = 3^2$), but it won't include 5 (because there is no whole number n such that $n^2 = 5$).

Ranges (continued).

Ranges are super **useful!**

...but only if we know the range, i.e., if we know which things are in the range and which things aren't.

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In all our examples, figuring out the range seems **doable**. But for some functions, it is clearly easier to do than for others.

How hard is it to know the range of a function, in general?

The Range Problem.

Problem.

Suppose we are given:

- A function F assigning a whole number to each whole number.
- A whole number k.

Figure out whether or not k is in the range of the function F.

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Million $$$ iPhone App Idea.

- Design an app that, when you enter your computer's current configuration after you get a spinning beachball, tells you whether to reboot or wait it out.

How hard would it be to write an app like this?

The Spinning Beachball Problem.

Problem.

Suppose we are given:

- A computer, running a program.
- A spinning beachball on our screen.

Figure out whether the beachball will stop spinning or not.

Algorithms.

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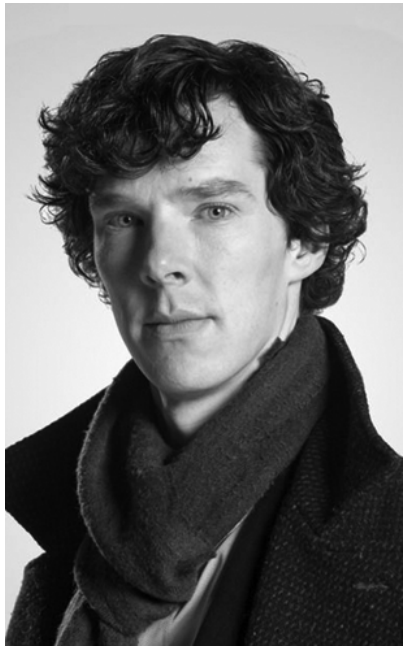
- You can easily write this in your favorite language (C, Python, etc.).

Alan Turing.



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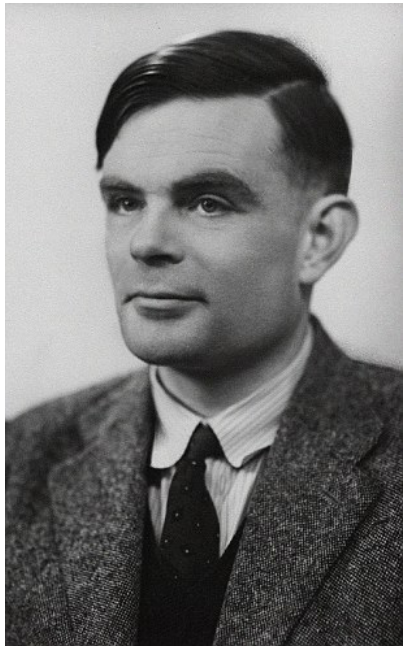

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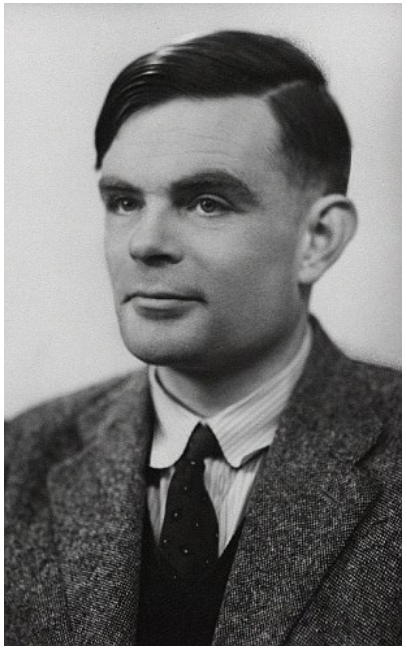

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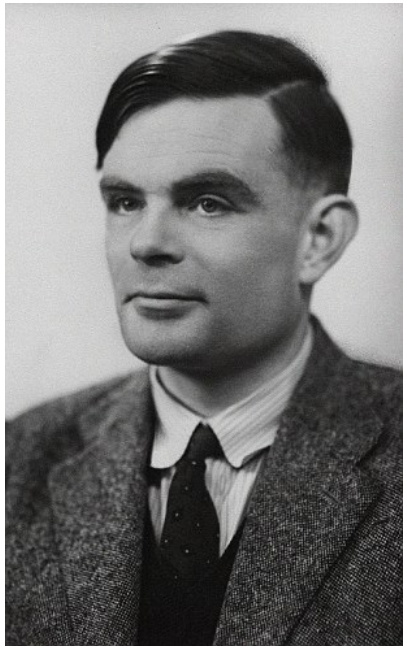

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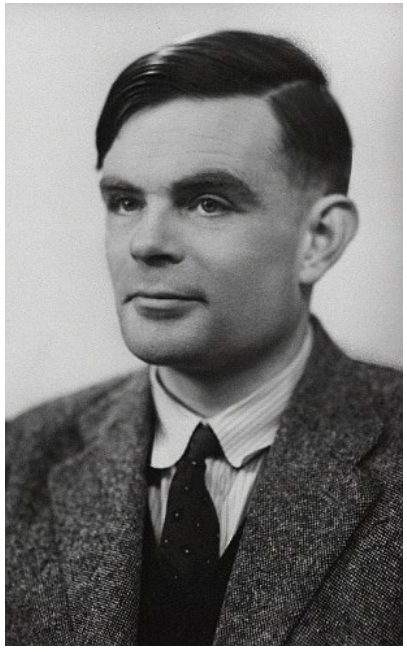

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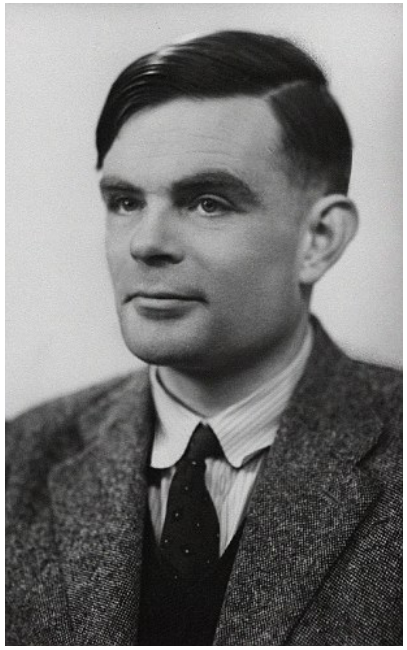

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Arrested for being a homosexual, and forced to accept agonizing hormone “therapy”, which led to his suicide in 1954.

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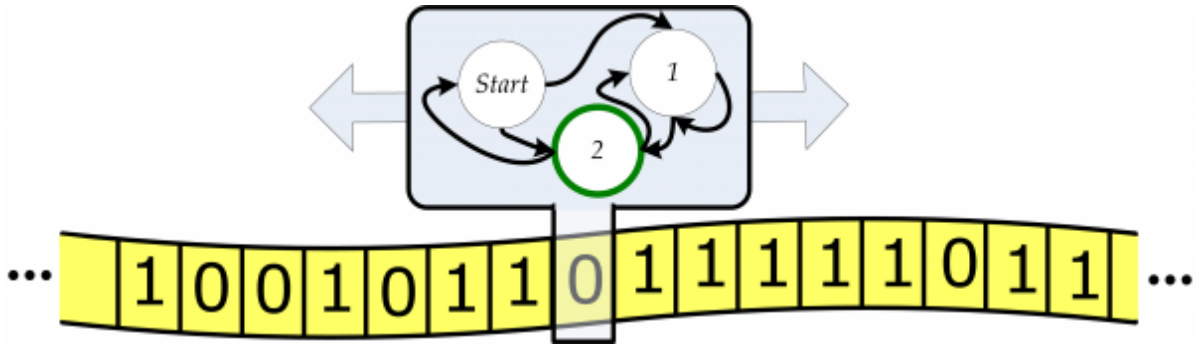
- A system of internal states.
- Instructions on what to do when in a given state, and reading a given symbol.

Turing machines (continued).



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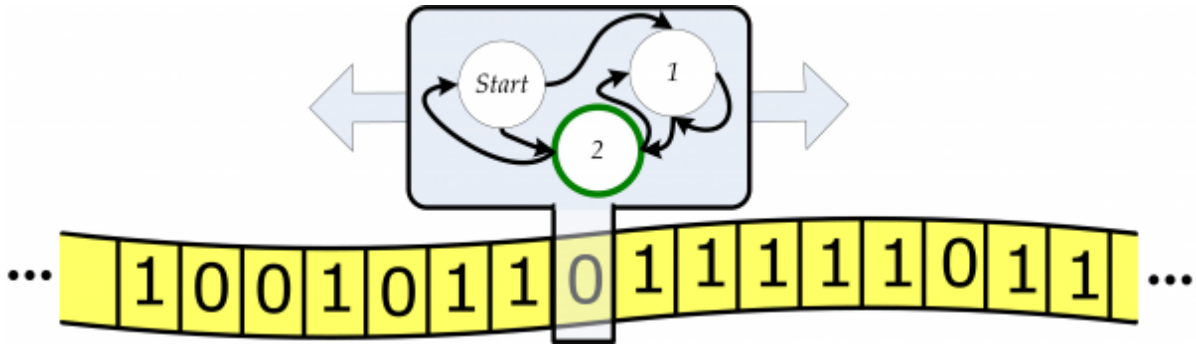

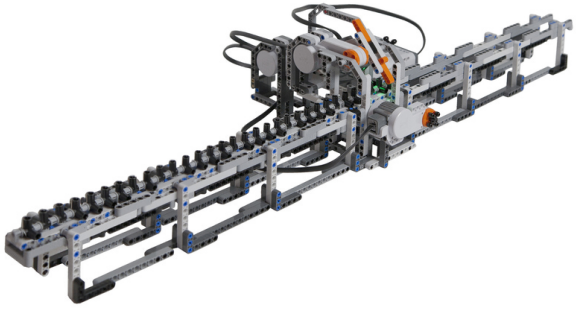

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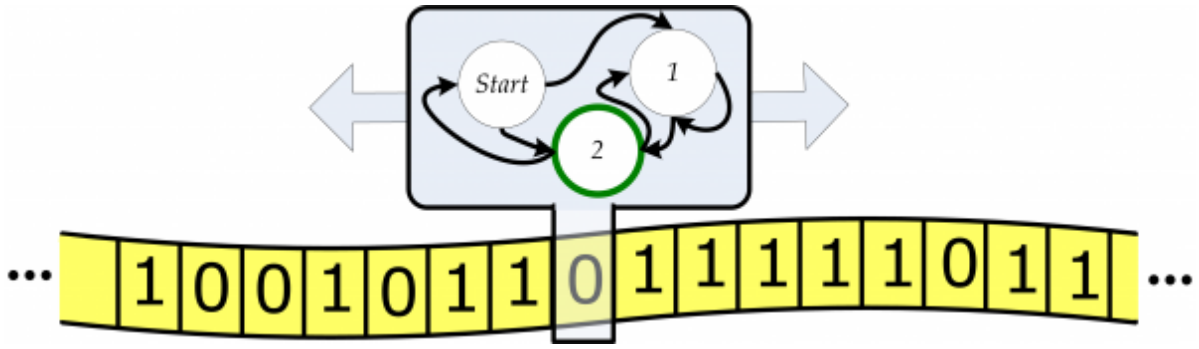

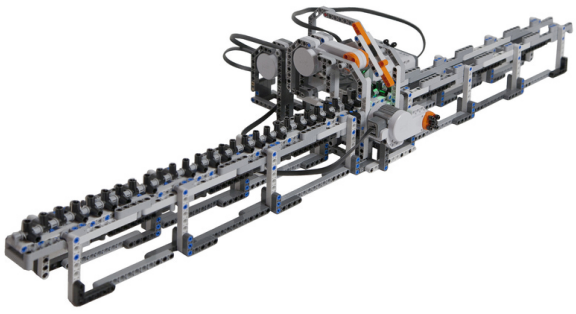

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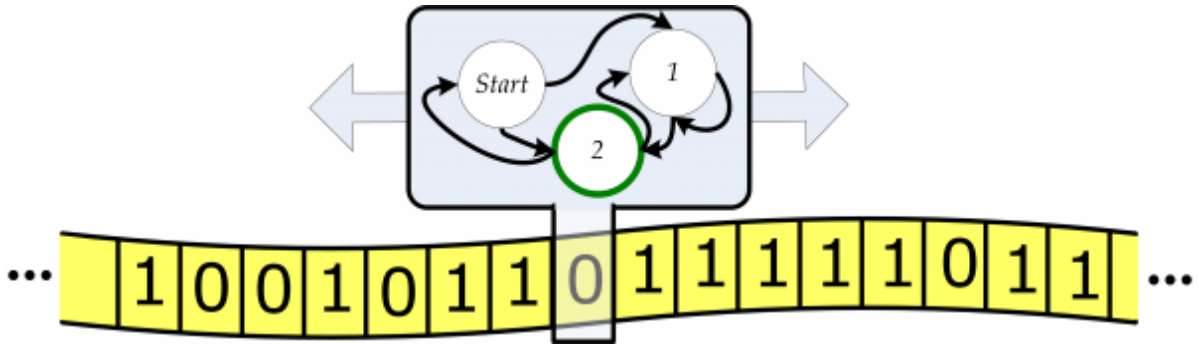

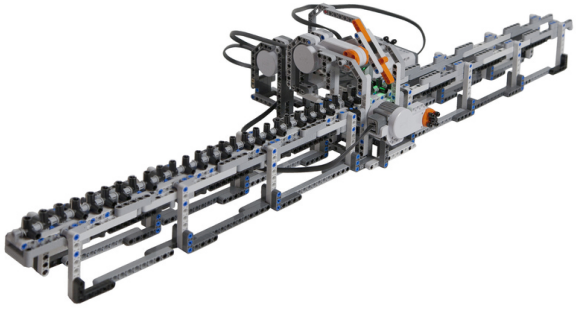

Universal TMs.

Key fact: there exist **universal Turing machines**. These can solve any problem any other Turing machine can solve (more on this later).

Not all Turing machines are universal Turing machines.

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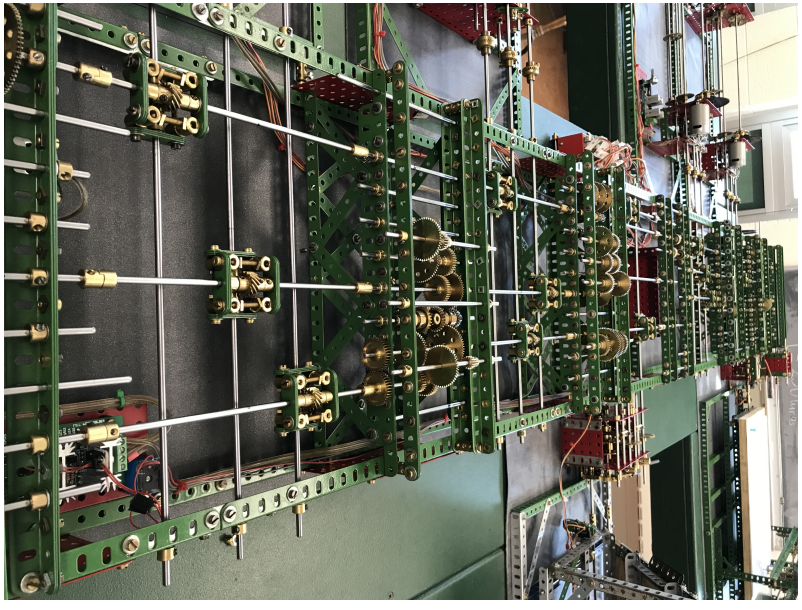

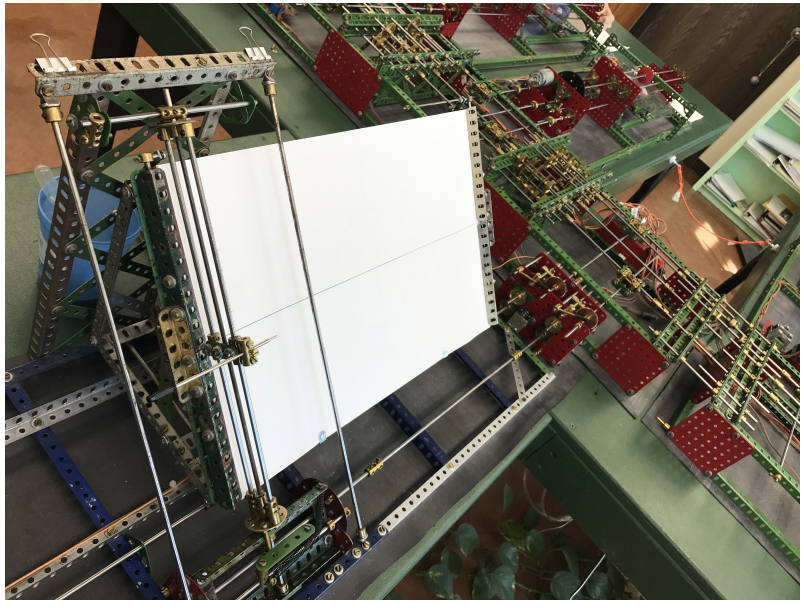

Mechanical Integrator/Differential Analyzer Lab at Marshall University.

A list of computer programs.

Imagine infinitely many monkeys, each typing randomly on a keyboard...

- Eventually, one of them write all of Shakespeare.

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- And...one of them will also write all the Harry Potter books.

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- Angry Birds might be P_{208}, for example.
- P_{637} might be " bv hjjhhjbiuillmoik8k,bj cfy".

The Halting Problem.

If we run P_{637}, the computer will be confused, and show us 

The Halting Problem.

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If we run P_{208}, it will parse and compile the code, and play Angry Birds.

The Halting Problem.

If we run P_{637}, the computer will be confused, and show us 

If we run P_{208}, it will parse and compile the code, and play Angry Birds.

The technical term for this is that P_{208} halts, while P_{637} does not halt.

Problem.

Suppose we are given:

- A positive whole number k.

Figure out whether P_k halts or not.

The Halting Problem (continued).

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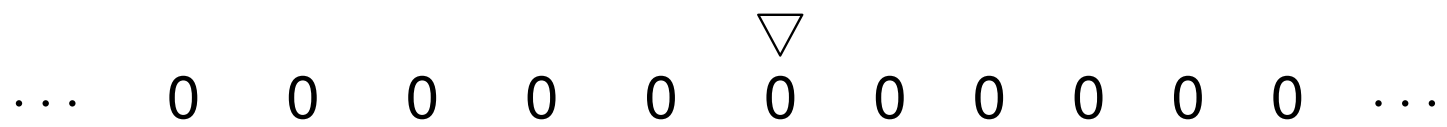

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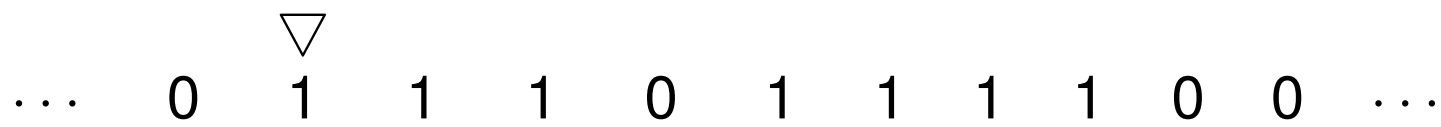

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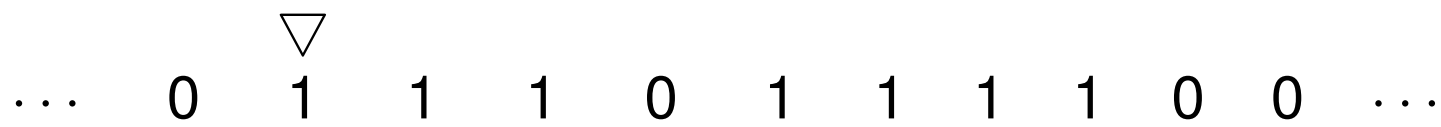

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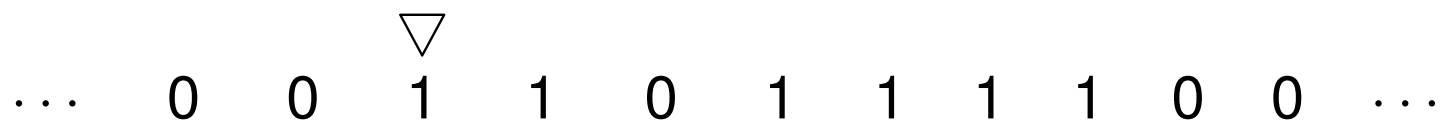

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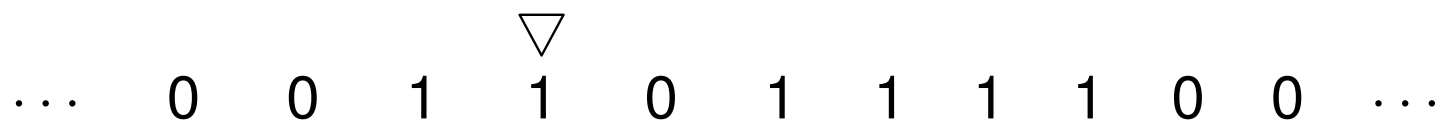

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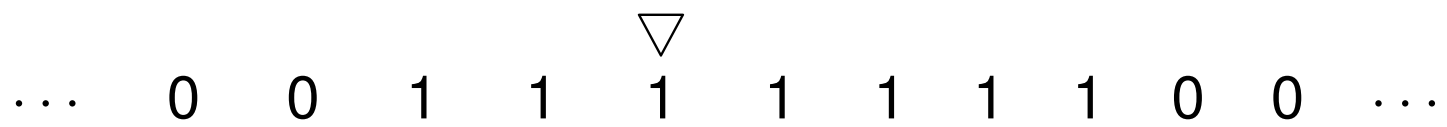

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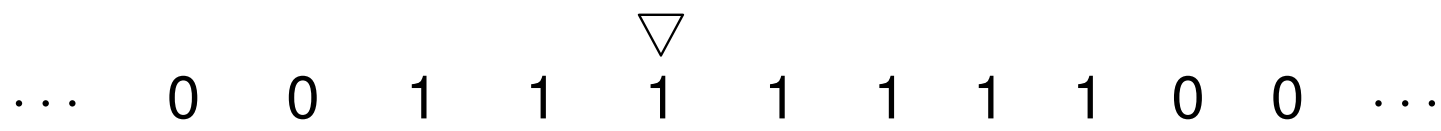

Theorem (Turing, 1936).

There is no Turing machine that can solve the halting problem.

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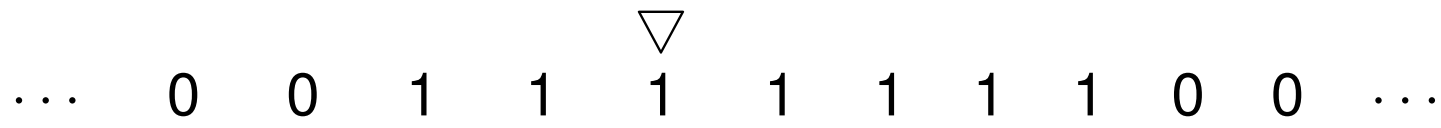

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(Sorry, app developers...)

The Range Problem as a computer program.

Recall the range problem: We are given a function F and a number, say 6, and we need to know if 12 is in the range of F.

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Write a program to check if $F(1) = 6$, $F(2) = 6$, $F(3) = 6$, $F(4) = 6$, ..., which keeps going forever unless it finds some n with $F(n) = 6$.

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This program is somewhere on our list, say $P_{3817834}$.

Now $P_{3817834}$ halts if and only if 6 is in the range of F.

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Let's call a number a coding number if it is equal to some number of **powers of** 2 times some number of **powers of** 3.

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Non-examples.

- 10 is a not coding number, since it is equal to $2 \cdot 5$.
- Neither is $15 = 3 \cdot 5$, or $28 = 2^2 \cdot 7$, etc.

The Halting Problem as the range of a function.

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Define a function F as follows.

- Given a number n, first check if n is a coding number.
- If n is not a coding number, define $F(n) = 0$.
- If n is a coding number, say $n = 2^k 3^t$. Run P_k for t minutes.
- If P_k halts in t minutes, let $F(2^k 3^t) = k$.
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So 1 is in the range of F if and only if the Goldbach Conjecture is false.

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So 1 is in the range of F if and only if the Goldbach Conjecture is false.

This means that if we could solve the range problem (or equivalently, the halting problem), then we could figure out whether or not the Goldbach Conjecture is true—and I'd be out of a job!

Thanks for your attention!