

# Inha University

## Team : yoojin and dllrary

Coach : Prof. Jeong-seop Sim

Contestant : Min-kyum Kim, Hyeon-min Kim, Seung-hyun Jin

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# 1. Graph

## 1. Dijkstra

```
typedef pair<ll, int> p;
const ll INF = 1e12;
auto dijkstra = [&](int start, vector<p> e[]) -> vector<ll>{
    vector<ll> v(N + 1);
    for (auto &i : v) i = INF;
    priority_queue<p, vector<p>, greater<p>> pq;
    v[start] = 0; pq.push({ 0, start });
    while (!pq.empty()) {
        ll cost, des, next, there;
        tie(cost, des) = pq.top(); pq.pop();
        for (auto k : e[des]) {
            tie(next, there) = k; next += cost;
            if (v[there] > next) {
                v[there] = next;
                pq.push({ next, there });
            }
        }
    }
    return v;
};
```

## 2. Floyd-Warshall

```
const int INF = 1000000000;
int N, M, dist[101][101];
int main() {
    cin >> N >> M;
    for (int i = 1; i <= N; i++) {
        for (int j = 1; j <= N; j++) {
            dist[i][j] = i == j ? 0 : INF;
        }
    }
    for (int i = 1; i <= M; i++) {
        int in1, in2, in3;
        cin >> in1 >> in2 >> in3;
        dist[in1][in2] = min(dist[in1][in2], in3);
    }
}
```

```
for (int k = 1; k <= N; k++) {
    for (int i = 1; i <= N; i++) {
        for (int j = 1; j <= N; j++) {
            dist[i][j] = min(dist[i][j], dist[i][k] + dist[k][j]);
        }
    }
}
for (int i = 1; i <= N; i++) {
    for (int j = 1; j <= N; j++) {
        cout << dist[i][j] << ' ';
    }
    cout << '\n';
}
```

## 3. Minimum Spanning Tree

```
typedef tuple<int, int, int> t;
int uf[10001];
int N, M, in1, in2, in3;
priority_queue<t, vector<t>, greater<t>> pq;
int find(int a) {
    if (uf[a] == a) return a;
    return uf[a] = find(uf[a]);
}
void merge(int a, int b) {
    a = find(a);
    b = find(b);
    if (a != b) uf[b] = a;
}
int main() {
    cin >> N >> M;
    for (int i = 0; i < M; i++) {
        cin >> in1 >> in2 >> in3;
        pq.push({ in3, in1, in2 }); //Distance, Start, End
    }
    for (int i = 1; i <= N; i++) uf[i] = i; //Union-Find Initialization
    ll cnt = 0, ans = 0;
    while (!pq.empty()) {
        if (cnt == N - 1) break; //N - 1 connections are enough for a spanning tree
        tie(cost, st, se) = pq.top();
```

```

    pq.pop();
    if (find(st) == find(se)) continue;
    merge(st, se);
    cnt++; ans += cost;
}
cout << ans;
}

```

#### 4. Topological Sort

```

int main() {
    int N, M;
    int indegree[32001] = { 0 };
    vector<int> edge[32001];
    queue<int> q;
    vector<int> res;
    cin >> N >> M;
    for (int i = 1; i <= M; i++) {
        int in1, in2;
        cin >> in1 >> in2;
        indegree[in2]++;
        edge[in1].push_back(in2);
    }
    for (int i = 1; i <= N; i++) {
        if (indegree[i] == 0) q.push(i);
    }
    for (int i = 1; i <= N; i++) {
        if (q.empty()) {
            cout << "Cannot Sort";
            return 0;
        }
        int cur = q.front();
        q.pop();
        res.push_back(cur);
        for (int next : edge[cur]) {
            if (--indegree[next] == 0) q.push(next);
        }
    }
    for (int i = 0; i < res.size(); i++) {
        cout << res[i] << ' ';
    }
}

```

```

}

```

#### 5. Strongly Connected Components

```

const int MAX = 10001;
int V, E;
vector<int> edge[MAX];
int sccCnt; //How many SCCs?
vector<vector<int>> SCC; //Stores Vertices of each SCCs
int dfsCnt, dfsn[MAX], sccInd[MAX];
bool finished[MAX];
stack<int> s;
int makeSCC(int cur) { //return index cur's SCC number
    dfsn[cur] = ++dfsCnt;
    s.push(cur);
    int res = dfsn[cur];
    for (int next : edge[cur]) {
        if (dfsn[next] == 0) res = min(res, makeSCC(next));
        else if (finished[next] == 0) res = min(res, dfsn[next]);
    }
    if (res == dfsn[cur]) {
        vector<int> curSCC;
        while (1) {
            int t = s.top();
            s.pop();
            curSCC.push_back(t);
            finished[t] = 1;
            sccInd[t] = sccCnt;
            if (t == cur) break;
        }
        //sort(curSCC.begin(), curSCC.end());
        SCC.push_back(curSCC);
        sccCnt++;
    }
    return res;
}
int main() {
    cin >> V >> E;
    for (int i = 0; i < E; i++) {
        int in1, in2;
        cin >> in1 >> in2;
    }
}

```

```

    edge[in1].push_back(in2);
}
for (int i = 1; i <= V; i++) if (dfs[i] == 0) makeSCC(i);
//sort(SCC.begin(), SCC.end());
//cout << sccCnt << '\n';
for (auto& curSCC : SCC) {
    for (int cur : curSCC) {
        cout << cur << ' ';
    }
    cout << '\n';
}
}

```

## 6. Maximum Flow

```

const int MAX = 800;
const ll INF = 2100000000;
int c[MAX][MAX], f[MAX][MAX], visited[MAX];
vector<int> edge[MAX];
/*
S에서 T로 가는 증가 경로 구하기(에드몬드 카프)
S : 시작점, T : 도착점
c[a][b] : a에서 b로 흐를 수 있는 최대 양 (Capacity)
f[a][b] : a에서 b로 흐른 실제 양 (Flow)
조건
용량 제한 : f[a][b] <= c[a][b]
유량의 대칭성 : f[a][b] == -f[b][a]
나오는 유량의 합 == 들어오는 유량의 합
*/
int maxFlow(int S, int T);
int bfs(int S, int T);
int maxFlow(int S, int T) {
    int result = 0;
    while (1) {
        int flow = bfs(S, T);
        if (!flow) break;
        result += flow;
    }
    return result;
}
int bfs(int S, int T) {

```

```

memset(visited, -1, sizeof(visited));
queue<int> q;
q.push(S);
while (!q.empty()) {
    int cur = q.front();
    q.pop();
    for (int next : edge[cur]) { //방문했는지, 용량이 남아 있는지 체크
        if (c[cur][next] - f[cur][next] <= 0) continue;
        if (visited[next] != -1) continue;
        q.push(next);
        visited[next] = cur; //cur->next 경로 기억
        if (next == T) break; //도착했을 경우 종료
    }
}
if (visited[T] == -1) return 0;
int flow = INF;
for (int i = T; i != S; i = visited[i]) { //최소 유량 탐색
    flow = min(flow, c[visited[i]][i] - f[visited[i]][i]);
}
for (int i = T; i != S; i = visited[i]) { //최소 유량 추가
    f[visited[i]][i] += flow;
    f[i][visited[i]] -= flow;
}
return flow;
}
int main() {
    int N; cin >> N;
    for (int i = 0; i < N; i++) {
        int in1, in2, in3;
        cin >> in1 >> in2 >> in3;
        c[in1][in2] += in3;
        c[in2][in1] += in3;
        edge[in1].push_back(in2);
        edge[in2].push_back(in1);
    }
    int S, T; cin >> S >> T;
    cout << maxFlow(S, T);
}

```

## 7. Bipartite Matching

```

const int MAX = 1001;
int N, M, A[MAX], B[MAX]; //A[i], B[i] : 각 정점이 매칭된 반대편 정점 인덱스
vector<int> edge[MAX];
bool visited[MAX];
//A그룹의 정점 a를 매칭시키는 데 성공하면 1 반환
//반대편이 매칭되지 않았으면 매칭시키고,
//이미 매칭되어 있다면 원래 매칭된 정점을 다른 정점과 매칭시킨다.
bool dfs(int a) {
    visited[a] = 1;
    for (int b : edge[a]) {
        if (B[b] == -1 || !visited[B[b]] && dfs(B[b])) {
            A[a] = b;
            B[b] = a;
            return 1;
        }
    }
    return 0;
}

int main() {
    cin >> N >> M;
    for (int i = 1; i <= N; i++) {
        int cnt;
        cin >> cnt;
        while (cnt--) {
            int in;
            cin >> in;
            edge[i].push_back(in);
        }
    }
    int match = 0;
    fill(A + 1, A + MAX - 1, -1);
    fill(B + 1, B + MAX - 1, -1);
    for (int i = 1; i <= N; i++) {
        if (A[i] == -1) {
            fill(visited + 1, visited + MAX - 1, 0);
            if (dfs(i)) match++;
        }
    }
    cout << match;
}

```

## 8. Lowest Common Ancestor

```

const int MAX = 100000;
const int MAXDEP = 18; //more than log(2, 100000)
int N, M;
int par[MAX][MAXDEP]; //par[i][k] : i의 2^k번째 부모
int dep[MAX];
vector<int> edge[MAX];
void makeTree(int cur) {
    for (int next : edge[cur]) {
        if (dep[next] == -1) {
            par[next][0] = cur;
            dep[next] = dep[cur] + 1;
            makeTree(next);
        }
    }
}

int getLCA(int in1, int in2) {
    in1--; in2--;
    if (dep[in1] < dep[in2]) swap(in1, in2);
    int dif = dep[in1] - dep[in2];
    for (int j = 0; dif; j++) {
        if (dif % 2) in1 = par[in1][j];
        dif /= 2;
    }
    if (in1 != in2) {
        for (int j = MAXDEP - 1; j >= 0; j--) {
            if (par[in1][j] != -1 && par[in1][j] != par[in2][j]) {
                in1 = par[in1][j];
                in2 = par[in2][j];
            }
        }
        in1 = par[in1][0];
    }
    return in1 + 1;
}

int main() {
    cin >> N;
    for (int i = 0; i < N - 1; i++) {
        int in1, in2;

```

```

cin >> in1 >> in2;
in1--; in2--;
edge[in1].push_back(in2);
edge[in2].push_back(in1);
}
memset(par, -1, sizeof(par));
fill(dep, dep + N, -1);
dep[0] = 0;
makeTree(0);
for (int j = 0; j < MAXDEP - 1; j++) {
    for (int i = 1; i < N; i++) {
        if (par[i][j] != -1) {
            par[i][j + 1] = par[par[i][j]][j];
        }
    }
}
cin >> M;
for (int i = 0; i < M; i++) {
    int in1, in2;
    cin >> in1 >> in2;
    cout << getLCA(in1, in2) << '\n';
}
}

```

## 2. Data Structure

### 1. Segment Tree

```

struct SegTree {
    ll sz, st, dep;
    vector<ll> pw2, v;
    SegTree(ll size) {
        ll k = 1;
        for (int i = 0; i < 30; i++) pw2.push_back(k), k <<= 1;
        sz = size; v.resize(sz * 4, 0); dep = 1;
        while(1) {
            if (sz <= pw2[dep - 1]) break;
            dep++;
        }
        st = pw2[dep - 1] - 1;
    }
}

```

```

}
void init(ll val) {
    for (int i = 1; i < 4 * sz; i++) v[i] = val;
}
ll val(ll ind) {
    return v[st + ind];
}
void update(ll ind, ll val) {
    v[st + ind] = val; ind = (ind + 1) / 2;
    for (int i = dep - 1; i >= 1; i--) {
        ll cur = pw2[i - 1] - 1 + ind;
        v[cur] = v[cur * 2] + v[cur * 2 + 1];
        ind = (ind + 1) / 2;
    }
}
ll query(ll start, ll end) {
    ll ret = 0;
    start += st; end += st;
    while (start <= end) {
        if (start % 2 == 1) ret += v[start];
        if (end % 2 == 0) ret += v[end];
        start = (start + 1) / 2;
        end = (end - 1) / 2;
    }
    return ret;
}
};

```

### 2. Fenwick Tree with Lazy Propagation

```

struct lazy {
    struct fenwick {
        ll sz;
        ll *arr;
        fenwick(int size) {
            sz = size + 1;
            arr = new ll[sz];
            fill(arr, arr + sz, 0);
        }
        void update(int i, ll x) {
            while (i <= sz) {

```

```

        arr[i] += x;
        i += i & -i;
    }
}

ll sum(int i) const {
    ll x = 0;
    while (i) {
        x += arr[i];
        i -= i & -i;
    }
    return x;
}

};

fenwick *suma, *sumb;
lazy(int size) {
    suma = new fenwick(size);
    sumb = new fenwick(size);
}

void add(int L, int R, ll val) {
    suma->update(L, val);
    suma->update(R + 1, -val);
    sumb->update(L, (1LL - L)*val);
    sumb->update(R + 1, 1LL * R*val);
}

ll query(int L, int R) {
    ll ans = 0;
    ans += suma->sum(R)*R + sumb->sum(R);
    ans -= suma->sum(L - 1)*(L - 1) + sumb->sum(L - 1);
    return ans;
}

};

```

# 3. String

## 1. KMP

```

string src, tar;
vector<int> fail, KMP;
void getFail(string t) {
    fail.resize(t.length());

```

```

    int j = 0;
    for (int i = 1; i < t.length(); ++i) {
        while (j > 0 && t[i] != t[j]) j = fail[j - 1];
        if (t[i] == t[j]) fail[i] = ++j;
    }
}

void getKMP(string s, string t) {
    int sLen = s.length(), tLen = t.length(), j = 0;
    for (int i = 0; i < sLen; ++i) {
        while (j > 0 && s[i] != t[j]) j = fail[j - 1];
        if (s[i] == t[j]) {
            if (j == tLen - 1) {
                KMP.push_back(i - tLen + 1 + 1);
                j = fail[j];
            }
            else j++;
        }
    }
}

int main() {
    getline(cin, src); getline(cin, tar);
    getFail(tar);
    getKMP(src, tar);
    cout << KMP.size() << "\n";
    for (int i : KMP) {
        cout << i << " ";
    }
}

```

## 2. Rabin-Karp

```

const int MOD = 1000000007;
ll mod(ll n) { if (n >= 0) return n % MOD; return ((-n / MOD + 1) * MOD + n) % MOD; }
int L;
string src;
int rabinKarp(int len) {
    ll H = 0, power = 1;
    unordered_map<int, vector<int>> hashTable;
    for (int i = 0; i <= src.length() - len; ++i) {
        if (i == 0) {
            for (int j = 0; j < len; ++j) {

```

```

        H = mod(H + 1LL * src[len - 1 - j] * power);
        if (j < len - 1) power = mod(power * 127);
    }
}
else {
    H = mod(127 * (H - 1LL * src[i - 1] * power) + src[i + len - 1]);
}
if (hashTable[H].size() == 0) hashTable[H].push_back(i);
else {
    for (int pos : hashTable[H]) {
        for (int p = 0; p < len; ++p) {
            if (src[pos + p] != src[i + p]) break;
            if (p == len - 1) return true;
        }
    }
    hashTable[H].push_back(i);
}
}
return false;
}

```

### 3. Trie

```

struct Trie;
typedef pair<char, Trie*> pct;
struct Trie {
    vector<pct> child;
    bool isRet;
    Trie() {
        isRet = false;
    }
    ~Trie() {
        for (pct c : child) delete c.second;
    }
    void insert(const char* key) {
        int k = *key;
        if (k == '\0') {
            isRet = true;
            return;
        }
        for (pct c : child) {

```

```

            if (c.first == k) {
                c.second->insert(key + 1);
                return;
            }
        }
        child.push_back(pct(*key, new Trie));
        child.back().second->insert(key + 1);
    }
    bool demoFunc(const char* key) {
        int k = *key;
        if (isRet && k == '\0') return true;
        for (pct c : child) {
            if (c.first == k) return c.second->demoFunc(key + 1);
        }
        return false;
    }
};

int main() {
    Trie* root = new Trie;
    int n; cin >> n;
    for (int i = 0; i < n; ++i) {
        string input; cin >> input;
        root->insert(input.c_str());
    }
    int m; cin >> m;
    for (int i = 0; i < m; ++i) {
        string input; cin >> input;
        cout << (root->demoFunc(input.c_str()) ? "YES" : "NO") << "\n";
    }
    delete root;
}

```

### 4. Aho-Corasick

```

struct Trie {
    unordered_map<char, Trie*> child;
    Trie* fail;
    bool isRet = false;
    void push(string& s) {
        Trie* cur = this;
        for (char const& c : s) {

```



```

    if (cur->child.find(c) == cur->child.end())
        cur->child[c] = new Trie;
    cur = cur->child[c];
    cur->fail = this;
}
cur->isRet = true;
}

void build() {
    queue<Trie*> Q;
    for (auto const& p : child)
        if (p.second) Q.push(p.second);
    while (!Q.empty()) {
        Trie* cur = Q.front(); Q.pop();
        for (auto const& c : cur->child) {
            Trie* p = cur->fail;
            while (p != this && p->child.find(c.first) == p->child.end()) p = p->fail;
            p = p->child[c.first];
            if (!p) p = this;
            c.second->fail = p;
            if (p->isRet) c.second->isRet = true;
            Q.push(c.second);
        }
    }
}

bool ask(string& s) {
    Trie* p = this;
    for (char& c : s) {
        while (p != this && p->child.find(c) == p->child.end()) p = p->fail;
        p = p->child[c];
        if (!p) p = this;
        if (p->isRet) return 1;
    }
    return 0;
}

int main() {
    Trie* root = new Trie;
    int N, Q;
    string input;
    cin >> N; while (N--) {

```

```

        cin >> input; root->push(input);
    }
    root->build();
    cin >> Q; while (Q--) {
        cin >> input; cout << (root->ask(input) ? "YES" : "NO") << "\n";
    }
}

```

## 5. Suffix Array and Longest Common Prefix Array

```

const char baseChar = '\0'; // @ for uppercase ` for lowercase
const int baseSize = 27; // 27 : 59
vector<int> suffixArray, LCPArray;
void getLCP(vector<int>& sa, vector<int>& lcpa, string& s) {
    int i, j, k, u = 0, m = baseSize, sLen = s.length();
    sa.resize(sLen, 0); lcpa.resize(sLen, 0);
    vector<int> cnt(max(sLen, m), 0), x(sLen, 0), y(sLen, 0);
    for (i = 0; i < sLen; ++i) cnt[x[i] = s[i] - baseChar]++;
    for (i = 0; i < m; ++i) cnt[i] += (i == 0 ? 0 : cnt[i - 1]);
    for (i = sLen - 1; i >= 0; --i) sa[--cnt[x[i]]] = i;
    for (int len = 1, p = 1; p < sLen; len <= 1, m = p + 1) {
        for (i = sLen - len - 1, p = 0; ++i < sLen;) y[p++] = i;
        for (i = 0; i < sLen; ++i) if (sa[i] >= len) y[p++] = sa[i] - len;
        for (i = 0; i < m; ++i) cnt[i] = 0;
        for (i = 0; i < sLen; ++i) cnt[x[y[i]]]++;
        for (i = 0; i < m; ++i) cnt[i] += (i == 0 ? 0 : cnt[i - 1]);
        for (i = sLen - 1; i >= 0; --i) sa[--cnt[x[y[i]]]] = y[i];
        swap(x, y); p = 1; x[sa[0]] = 1;
        for (i = 0; i < sLen - 1; ++i) x[sa[i + 1]] = sa[i] + len < sLen && sa[i + 1] + len < sLen &&
y[sa[i]] == y[sa[i + 1]] && y[sa[i] + len] == y[sa[i + 1] + len] ? p : ++p;
    }
    vector<int> rank(sLen, 0);
    for (i = 0; i < sLen; ++i) rank[sa[i]] = i;
    for (i = 0; i < sLen; ++i) if (k = rank[i]) {
        j = sa[k - 1];
        while (i + u < sLen && j + u < sLen && s[i + u] == s[j + u]) u++;
        lcpa[k] = u;
        u = u ? u - 1 : 0;
    }
}

int main() {

```

```

string s; cin >> s;
getLCP(suffixArray, LCPArray, s);
cout << "SA: "; for (int i = 0; i < s.length(); ++i) cout << setw(2) << suffixArray[i] << " "; cout
<< "\n";
cout << "LA: "; for (int i = 0; i < s.length(); ++i) cout << setw(2) << LCPArray[i] << " "; cout
<< "\n";
}

```

## 6. Manacher

```

vector<int> manacher(string& src) {
    int srcLen = src.size(); src.resize(srcLen * 2 + 1, '#');
    for (int i = srcLen - 1; i >= 0; --i) src[i * 2 + 1] = src[i], src[i] = '#';
    int c = 0, r = 0, len = src.size();
    vector<int> ret(len, 0);
    for (int i = 0; i < len; ++i) {
        int sym = 2 * c - i;
        if (i < r) ret[i] = min(r - i, ret[sym]);
        while (i - ret[i] > 0 && i + ret[i] - 1 < len && src[i - ret[i] - 1] == src[i + ret[i] + 1])
            ret[i]++;
        if (ret[i] + i > r) r = ret[i] + i, c = i;
    }
    return ret;
}

int main() {
    string src; cin >> src;
    vector<int> pal = manacher(src);
    for (int i : pal) cout << i << ' ';
}

```

## 7. Z

```

vector<int> Z;
void getZ(string& src) {
    int l = 0, r = 0, len = src.length();
    Z.resize(len);
    for (int i = 1; i < len; i++) {
        Z[i] = max(0, min(Z[i - 1], r - i));
        while (src[i + Z[i]] && src[Z[i]] == src[i + Z[i]]) Z[i]++;
        if (i + Z[i] > r) l = i, r = i + Z[i];
    }
    Z[0] = len; // Z[0] = 0;
}

```

```

int main() {
    string src; cin >> src;
    getZ(src);
    for (auto i : Z) cout << i << " ";
}

```

# 4. Math

## 1. Greatest Common Divisor, Least Common Multiple

```

ll gcd(ll a, ll b) { for (; b; a %= b, swap(a, b)); return a; }
ll lcm(ll a, ll b) { return a * b / gcd(a, b); }

```

## 2. Sieve of Eratosthenes

```

const ll MAX = 100001;
bool isprime[MAX];
void sieve(){
    fill(isprime, isprime + MAX, 1);
    isprime[1] = 0;
    for (int i = 2; i*i <= MAX; i++) {
        if (isprime[i]) {
            for (int j = i*i; j < MAX; j += i) {
                isprime[j] = 0;
            }
        }
    }
}

```

## 3. Binomial Coefficient

```

const ll MOD = 1000000007;
const ll MAX_NUM = 4000001;
ll f[MAX_NUM];
ll mypow(ll base, ll exp, ll MOD) {
    ll ans = 1;
    while (exp > 0) {
        if (exp % 2 != 0) {
            ans *= base;
            ans %= MOD;
        }
        base *= base;
        base %= MOD;
        exp /= 2;
    }
}

```

```

    }
    return ans;
}

void nCrInit() {
    f[0] = 1; f[1] = 1;
    for (int i = 2; i <= MAX_NUM - 1; i++) {
        f[i] = f[i - 1] * i;
        f[i] %= MOD;
    }
}

ll nCr(int n, int r) {
    return (f[n] * mypow((f[r] * f[n - r]) % MOD, MOD - 2, MOD)) % MOD;
}

int main() {
    ll n, r;
    nCrInit();
    cin >> n >> r;
    cout << nCr(n, r) << "\n";
}

```

#### 4. Matrix Exponential

```
ll a[3][3], a2[3][3], ans[3][3], temp[3][3], N;
```

```

void mult(ll a[3][3], ll b[3][3]) {
    for (int i = 0; i < 3; i++) {
        for (int j = 0; j < 3; j++) {
            for (int k = 0; k < 3; k++) {
                temp[i][j] += a[i][k] * b[k][j];
                temp[i][j] %= 1000000007;
            }
        }
    }

    for (int i = 0; i < 3; i++) {
        for (int j = 0; j < 3; j++) {
            a[i][j] = temp[i][j];
            temp[i][j] = 0;
        }
    }
}

int main() {
    cin >> N; N--;

```

```

/*
행렬 설명
a : 가운데 거듭제곱되는 행렬
    초기화는 점화식으로 유도된 행렬로 한다.
a2 : 거듭제곱된 결과가 저장되는 행렬
    초기화는 영행렬로 한다.
temp : mult함수에서 사용되는 임시 행렬
ans : 답이 저장되어 있는 행렬.
    초기화를 할 때는 ans[0][i]의 콜로 베이스 값을 저장해놓는다.
mult(ans, a2)를 하면 ans[0][i]에 원하는 값이 들어간다.
*/
for (int i = 0; i < 3; i++) {
    for (int j = 0; j < 3; j++) {
        a[i][j] = 1;
        if (i == j)a2[i][i] = 1;
    }
}

a[0][0] = a[1][1] = 0;
ans[0][0] = ans[0][1] = ans[0][2] = 1;
while (N > 0) {
    if (N % 2 == 1) {
        mult(a2, a);
    }
    mult(a, a);
    N /= 2;
}

mult(ans, a2);
cout << (ans[0][0] + ans[0][1] + ans[0][2]) % 1000000007;
}

```

#### 5. Fast Fourier Transform

```

using ld = double;
using base = complex<ld>;
const ld pi = acos(-1);
void fft(vector<base> &A, bool f) {
    int k = A.size(), i, j, l, t;
    base w, x, y; ld th;
    for (i = 1, j = 0; i < k; i++) {
        for (l = k >> 1; j >= l; l >>= 1) j -= l;
        j += l; if (i < j) swap(A[i], A[j]);
    }
}

```

```

}
for (i = 1; i < k; i <= 1, t--) {
    th = (f ? -pi : pi) / i;
    w = base(cos(th), sin(th));
    for (j = 0; j < k; j += i + 1) {
        for (l = 0; l < i; l++) {
            if (l & 2047) x *= w;
            else x = l ? base(cos(th * l), sin(th * l)) : 1;
            y = x * A[l | i | j];
            A[l | i | j] = A[l | j] - y;
            A[l | j] += y;
        }
    }
}
if (f) for (i = 0; i < k; i++) {
    A[i] /= k;
}
}

vector<ll> mult(vector<ll> &X, vector<ll> &Y) { //return vector's size is |X| + |Y| - 1
    ll s, i, j;
    for (s = 1; s < X.size() + Y.size(); s <= 1);
    vector<base> P(s), Q(s);
    vector<ll> Z(X.size() + Y.size() - 1);
    for (i = 0; i < X.size(); i++)
        P[i] = base(X[i] >> 12, X[i] & 4095);
    for (i = 0; i < Y.size(); i++)
        Q[i] = base(Y[i] >> 12, Y[i] & 4095);
    fft(P, 0); fft(Q, 0);
    for (i = 0; i + i <= s; i++) {
        j = i ? s - i : 0;
        base v1 = P[i] + conj(P[j]), v2 = conj(P[i]) - P[j];
        tie(P[i], Q[i], P[j], Q[j]) = make_tuple(
            v1 * Q[i], conj(v2) * conj(Q[j]),
            conj(v1) * Q[j], -v2 * conj(Q[i]));
    }
    fft(P, 1); fft(Q, 1);
    for (i = 0; i < Z.size(); i++) {
        Z[i] = (((ll)round(P[i].real()) << 23) + ((ll)round(Q[i].real()) >> 1)
            + ((ll)round(P[i].imag() + Q[i].imag()) << 11));
    }
}

```

```

return Z;
}

6. Berlekamp-Massey
ll ipow(ll x, ll p) {
    ll ret = 1, piv = x;
    while(p){
        if(p & 1) ret = ret * piv % MOD;
        piv = piv * piv % MOD;
        p >>= 1;
    }
    return ret;
}

vector<ll> berlekamp_massey(vector<ll> x) {
    vector<ll> ls, cur;
    ll lf, ld;
    for(int i = 0; i < x.size(); i++) {
        ll t = 0;
        for(int j = 0; j < cur.size(); j++) {
            t = (t + 1ll * x[i - j - 1] * cur[j]) % MOD;
        }
        if((t - x[i]) % MOD == 0) continue;
        if(cur.empty()) {
            cur.resize(i + 1);
            lf = i;
            ld = (t - x[i]) % MOD;
            continue;
        }
        ll k = -(x[i] - t) * ipow(ld, MOD - 2) % MOD;
        vector<ll> c(i - lf - 1);
        c.push_back(k);
        for(auto &j : ls) c.push_back(-j * k % MOD);
        if(c.size() < cur.size()) c.resize(cur.size());
        for(int j = 0; j < cur.size(); j++) {
            c[j] = (c[j] + cur[j]) % MOD;
        }
        if(i - lf + (ll)ls.size() >= (ll)cur.size()) {
            tie(ls, lf, ld) = make_tuple(cur, i, (t - x[i]) % MOD);
        }
        cur = c;
    }
}

```

```

}
for(auto &i : cur) i = (i % MOD + MOD) % MOD;
return cur;
}

ll get_nth(vector<ll> rec, vector<ll> dp, ll n) {
    ll m = rec.size();
    vector<ll> s(m), t(m);
    s[0] = 1;
    if(m != 1) t[1] = 1;
    else t[0] = rec[0];
    auto mul = [&rec](vector<ll> v, vector<ll> w) {
        int m = v.size();
        vector<ll> t(2 * m);
        for(int j = 0; j < m; j++) {
            for(int k = 0; k < m; k++) {
                t[j + k] += 1ll * v[j] * w[k] % MOD;
                if(t[j + k] >= MOD) t[j + k] -= MOD;
            }
        }
        for(int j = 2 * m - 1; j >= m; j--) {
            for(int k = 1; k <= m; k++) {
                t[j - k] += 1ll * t[j] * rec[k - 1] % MOD;
                if(t[j - k] >= MOD) t[j - k] -= MOD;
            }
        }
        t.resize(m);
        return t;
    };
    while(n) {
        if(n & 1) s = mul(s, t);
        t = mul(t, t);
        n >>= 1;
    }
    ll ret = 0;
    for(int i = 0; i < m; i++) ret += 1ll * s[i] * dp[i] % MOD;
    return ret % MOD;
}

ll guess_nth_term(vector<ll> x, ll n){
    if(n < x.size()) return x[n];
    vector<ll> v = berlekamp_massey(x);

```

```

    if(v.empty()) return 0;
    return get_nth(v, x, n);
}

7. Extended Euclidean Algorithm
tuple<ll, ll, ll> euclid(ll x, ll y) {
    if (x < y) swap(x, y);
    if (y == 0) return { x, 1, 0 };
    ll g, x1, y1; tie(g, x1, y1) = euclid(y, x%y);
    return { g, y1, x1 - (x / y) * y1 };
}
ll inv = (get<2>(euclid(base, MOD)) + MOD) % MOD;

```

## 5. Geometry

### 1. CCW

```

//determines direction of 3 points
ll ccw(const point& A, const point& B, const point& C) {
    ll t = 1LL * (B.x - A.x)*(C.y - A.y) - 1LL * (B.y - A.y)*(C.x - A.x);
    if (t > 0) return 1; //Counter-Clockwise
    if (t == 0) return 0; //Line
    return -1; //Clockwise
}

```

### 2. Convex Hull

```

const int MAX = 100000;
struct point {
    int x, y; //실제 위치
    int p, q; //기준점으로부터 상대 위치
    point() : point(0, 0, 1, 0) {}
    point(int x1, int y1) : point(x1, y1, 1, 0) {}
    point(int x1, int y1, int p1, int q1) : x(x1), y(y1), p(p1), q(q1) {}
    bool operator<(const point& O) {
        if (1LL * q*O.p != 1LL * p*O.q) return 1LL * q*O.p < 1LL * p*O.q;
        if (y != O.y) return y < O.y;
        return x < O.x;
    }
};

point p[MAX];
int main() {
    int N;

```

```

cin >> N;
for (int i = 0; i < N; i++) {
    int in1, in2;
    cin >> in1 >> in2;
    p[i] = point(in1, in2);
}
sort(p, p + N);
for (int i = 1; i < N; i++) {
    p[i].p = p[i].x - p[0].x;
    p[i].q = p[i].y - p[0].y;
}
sort(p + 1, p + N); //반시계 방향 정렬
stack<int> cvh;
cvh.push(0); cvh.push(1);
int next = 2;
while (next < N) {
    while (cvh.size() >= 2) {
        int fst, scd;
        fst = cvh.top();
        cvh.pop();
        scd = cvh.top();
        if (ccw(p[scd], p[fst], p[next]) > 0) {
            cvh.push(fst);
            break;
        }
    }
    cvh.push(next++);
}
cout << cvh.size();
}

```

### 3. Line Intersection

```

struct point {
    int x, y;
    point(int x, int y) {
        this->x = x;
        this->y = y;
    }
    bool operator< (point a) {
        if (a.x != this->x) {

```

```

            return a.x < this->x;
        }
        else {
            return a.y < this->y;
        }
    }
    bool operator<= (point a) {
        if (a.x == this->x && a.y == this->y) return 1;
        if (a.x != this->x) {
            return a.x < this->x;
        }
        else {
            return a.y < this->y;
        }
    }
};
bool isIntersect(pair<point, point> line1, pair<point, point> line2) {
    point p1 = line1.first;
    point p2 = line1.second;
    point p3 = line2.first;
    point p4 = line2.second;
    int l1 = ccw(p1, p2, p3) * ccw(p1, p2, p4);
    int l2 = ccw(p3, p4, p1) * ccw(p3, p4, p2);
    if (l1 == 0 && l2 == 0) {
        if (p2 < p1) swap(p1, p2);
        if (p4 < p3) swap(p3, p4);
        return (p3 <= p2 && p1 <= p4);
    }
    return (l1 <= 0 && l2 <= 0);
}

```

## 6. Others

### 1. Decimal in python

```

import decimal: D = decimal.Decimal
# 정밀도 설정
decimal.getcontext().prec = 28000
# Decimal 객체에 인자 넘겨주기
g = D(1/(10**200))

```

## 2. Fraction in python

```
Import fractions: F = fractions.Fraction
F(f.numerator, f.denominator)
```

## 3. Bit Operations

```
k & (1 << N) // return N-th bit of k
k | (1 << N) // make N-th bit of k to 1
k & ~(1 << N) // make N-th bit of k to 0
k & ~k // return least index of bit which has value 1
```

## 4. Coordinate Compression

```
ll getind(ll in){
    return lower_bound(a.begin(), a.end(), in) - a.begin();
}

sort(a.begin(), a.end());
a.erase(unique(a.begin(), a.end()), a.end());
```

## 5. Knuth Optimization

Optimization of  $O(N^3)$  DP to  $O(N^2)$

Condition 0 :  $dp[i][j]$  is defined under  $1 \leq i \leq j \leq n$

Condition 1 :  $dp[i][j] = \min(dp[i][k] + dp[k+1][j]) + cost[i][j]$

Condition 2 : under  $a \leq b \leq c \leq d$ ,  $cost[a][c] + cost[b][d] \leq cost[a][d] + cost[b][c]$

Condition 3 : under  $a \leq b \leq c \leq d$ ,  $cost[b][c] \leq cost[a][d]$

Original  $O(N^3)$  DP :

```
for(i = 1; i <= n; i++){
    for(j = i; j < n; j++){
        for(k = j + 1; k < i; k++){
            dp[i][j] = min(dp[i][j], dp[i][k] + dp[k + 1][j]);
        }
        dp[i][j] += C[i][j];
    }
}
```

Optimized  $O(N^2)$  DP :

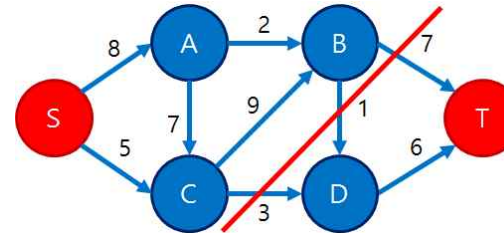
```
for(i=1; i<=n; i++){
    for(j=i; j<n; j++){
        dp[i][j] = MAX;
        for(k=A[i][j - 1]; k<=A[i + 1][j]; k++){ //상수 번 반복
            if(dp[i][j] > dp[i][k] + dp[k + 1][j]){
                dp[i][j] = dp[i][k] + dp[k + 1][j];
                A[i][j] = k;
            }
        }
    }
}
```

```
dp[i][j] += C[i][j];
} //A[i][j] = minimum k to minimize dp[i][j]
} //A[i][j - 1] <= A[i][j] <= A[i + 1][j]
```

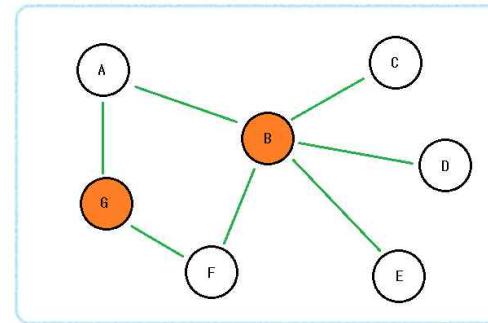
## 6. Some Prime Numbers

127, 131, 137, 139, 10007, 100003, 998244353, 1000000007, 1000000009

# 7. Tips



1. Minimum Cut of vertices is same as Maximum Flow of Vertices.



2. Minimum Vertex Cover is same as Maximum Matching.

3. Maximum Independent Set is Number of Vertices - Minimum Vertex Cover.

4. If node C is between node A and node B in a tree,

$\{LCA(A, C) == C \ \&\& \ LCA(A, B) == LCA(C, B)\} \ || \ \{LCA(B, C) == C \ \&\& \ LCA(A, B) == LCA(C, A)\}$  is true.

5. Derangement :  $dp[i] = (i - 1) * (dp[i - 1] + dp[i - 2])$ ;

6. Erasing particular value in a vector

```
void erase(vector<int> &v, int val){
    for(auto it = v.begin(); it != v.end(); ){
        if(*it == val) it = v.erase(it);
        else it++;
    }
}
```

## 7. Partition of Number

Let  $dp[n][k]$  be the number of cases dividing  $n$  to  $k$  numbers.

a.  $dp[n][1] = dp[n][n] = 1$

b.  $dp[n][k] = dp[n - k][1] + dp[n - k][2] + \dots + dp[n - k][k]$

c.  $dp[n][k] = dp[n - 1][k - 1] + dp[n - k][k]$

## 8. Fermat's Little Theorem

If  $p$  is a prime number and  $a$  is not divisible by  $p$ ,  $a^{(p-1)} \% p == 1$  is true.