* $u = e^{-2xy} \sin(x^2 - y^2)$

Sol

$u_x = 2xe^{-2xy} \cos(x^2 - y^2) - 2ye^{-2xy} \sin(x^2 - y^2)$

$u_{xx} = 2e^{-2xy} \cos(x^2 - y^2) - 4xye^{-2xy} \cos(x^2 - y^2) - 4x^2 \sin(x^2 - y$

$-2y(2xe^{-2xy} \cos(x^2 - y^2) - 2ye^{-2xy} \sin(x^2 - y^2)$

$\boxed{u_y = -2ye^{-2xy} \cos(x^2 - y^2) - 2xe^{-2xy} \sin(x^2 - y^2)}$

$u_{yy} = -2e^{-2xy} \cos(x^2 - y^2) + 4xye^{-2xy} \cos(x^2 - y^2) + 4y^2 e^{-2xy}$

$-2x(-2ye^{-2xy} \cos(x^2 - y^2) - 2xe^{-2xy} \sin(x^2 - y^2$

$u_{xx} + u_{yy} = 0$ Harmonic

$v_y = u_x$

$= 2xe^{-2xy} \cos(x^2 - y^2) - 2ye^{2xy} \sin(x^2 - y^2)$ Analytic

*

Date Subject

* $u = xe^x \cos y - ye^x \sin y$

Sol

$u_x = xe^x \cos y + e^x \cos y - ye^x \sin y$

$u_{xx} = e^x \cos y + e^x \cos y + e^x \cos y - ye^x \sin y$

$u_y = -xe^x \sin y - e^x \sin y - ye^x \cos y$

$u_{yy} = -xe^x \cos y - e^x \cos y - e^x \cos y + ye^x \sin y$

$u_{xx} + u_{yy} = 0$

Harmonic

$\Downarrow$

Analytic

$v_y = u_x = e^x \cos y + xe^x \cos y - ye^x \sin y$

$v = \int u_x dy = e^x \cancel{\sin y} + xe^x \sin y - e^x[-y\cos y + \cancel{\sin y}] + h(x)$

$= xe^x \sin y + ye^x \cos y + h(x)$

$v_x = -u_y$

$e^x \sin y + xe^x \sin y + ye^x \cos y + h'(x) = xe^x \sin y + e^x \sin y + ye^x \cos$

$h'(x) = 0 \Rightarrow h(x) = c$

$v = xe^x \sin y + ye^x \cos y + c$

ãka

6

Date Subject

* $u = 3x^2y + 2x^2 - y^3 - 2y^2$

Sol

$u_x = 6xy + 4x$ $\qquad$ $u_y = 3x^2 - 3y^2 - 4y$

$u_{xx} = 6y + 4$ $\qquad$ $u_{yy} = -6y - 4$

$u_{xx} + u_{yy} = 0$ $\qquad$ Harmonic

$\downarrow$

analytic

$v_y = u_x$

$v_y = 6xy + 4x \Rightarrow v = 3xy^2 + 4xy + h(x)$

$v_x = -u_y$

$3y^2 + 4y + h'(x) = -3x^2 + 3y^2 + 4y \Rightarrow h'(x) = -3x^2$

$h = -x^3$

$v = 3xy^2 + 4xy - x^3$

* $u = 2xy + 3xy^2 - 2y^3$

Sol

$u_x = 2y + 3y^2$ $\qquad$ $u_y = 2x + 6xy - 6y^2$

$u_{xx} = 0$ $\qquad$ $u_{yy} = 6x - 12y$

$u_{xx} + u_{yy} \neq 0$

Not harmonic

$\downarrow$

Not analytic

$* f_{(z)} = \underset{u}{(x^2+y)} + i\,\underset{v}{(y^2-x)}$

sol

$u_x = 2x$ $\quad$ $v_x = 1$

$u_y = 1$ $\quad$ $v_y = 2y$ $\quad$ not analytic

$* f_{(z)} = \sin(2z)$

sol

$\underset{u}{(\sin 2x\, ch2y)} + \underset{v}{(\cos 2x\, sh2y)}\, i$

$u_x = 2 (\cos 2x\, ch2y$ $\quad$ $v_x = -2\sin 2x\, sh2y$

$u_y = 2 \sin^2 x\, sh2y$ $\quad$ $v_y = 2\cos 2x\, ch2y$

analytic

$* f(z) = sh4z$

sol

$\underset{u}{(sh4x \cos 4y)} + \underset{v}{(ch4x \sin 4y)}\, i$

$u_x = 4\,ch4x \cos 4y$ $\quad$ $v_x = 4\,sh4x \sin 4y$

$u_y = -4\,sh4x \sin 4y$ $\quad$ $v_y = 4\,ch4x \cos 4y$

$u_x = v_y$

$u_y = -v_x$

analytic

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(5)

$|z_1 \; z_2| = |z_1| \, |z_2|$ Prove

sol

$|z_1 z_2| = x_1 x_2 + x_1 y_2 i + x_2 y_1 i - y_1 y_2$

$|z_1| \, |z_2| = x_1 x_2 + x_1 y_2 i + x_2 y_2 i - y_1 y_2$

ex (6)

(1) $* f(z) = z^2$

sol

$= (x+iy)^2 = \underbrace{(x^2-y^2)}_{u} + \underbrace{(2xy)}_{v} i$

$u_x = 2x \qquad v_x = 2y$

$u_y = -2y \qquad v_y = 2x$

$u_x = v_y \quad , \; u_y = -v_x$ analytic

$* f(z) = z e^{-z}$

sol

$(x+iy)(e^{-x} e^{-iy}) = (x+iy)(e^{-x}\cos y - ie^{-x}\sin y)$

$\underbrace{(xe^{-x}\cos y + ye^{-x}\sin y)}_{u} - \underbrace{(xe^{-x}\sin y + ye^{-x}\cos y)}_{v} i$

$u_x = -xe^{-x}\cos y + e^{-x}\cos y - ye^{-x}\sin y$

$u_y = -xe^{-x}\sin y + ye^{-x}\cos y + e^{-x}\sin y$

$v_x = xe^{-x}\sin y - ye^{-x}\cos y - e^{-x}\sin y$

$v_y = -xe^{-x}\cos y + e^{-x}\cos y - ye^{-x}\sin y$

$u_x = v_y \quad , \; u_y = v_x$ analytic

* $z^{10}+1=0$

sol

$x=-1$, $y=0$, $r=1$

$\theta=0$, $\theta=\pi$

for $K=0:9$

$$Z_0=\left[\cos\frac{\pi}{10}+i\sin\frac{\pi}{10}\right]$$

$$Z_1=\left[\cos\frac{3\pi}{10}+i\sin\frac{3\pi}{10}\right]$$

$$Z_2=\left[\cos\frac{5\pi}{10}+i\sin\frac{5\pi}{10}\right]$$

$$Z_3=\left[\cos\frac{7\pi}{10}+i\sin\frac{7\pi}{10}\right]$$

$$Z_4=\left[\cos\frac{9\pi}{10}+i\sin\frac{9\pi}{10}\right]$$

$$Z_5=\left[\cos\frac{11\pi}{10}+i\sin\frac{11\pi}{10}\right]$$

$$Z_6=\left[\cos\frac{13\pi}{10}+i\sin\frac{13\pi}{10}\right]$$

$$Z_7=\left[\cos\frac{15\pi}{10}+i\sin\frac{15\pi}{10}\right]$$

$$Z_8=\left[\cos\frac{17\pi}{10}+i\sin\frac{17\pi}{10}\right]$$

$$Z_9=\left[\cos\frac{19\pi}{10}+i\sin\frac{19\pi}{10}\right]$$

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* $z^3 = i$

sol

$z = (i)^{\frac{1}{3}}$

$x = 0$, $y = 1$, $r = 1$

$\theta = \frac{\pi}{2}$

$z = 1^{\frac{1}{3}} \left[\cos \frac{\theta + 2\pi k}{3} + i \sin \frac{\theta + 2\pi k}{3} \right]$

for $k = 0, 1, 2$

$z_0 = \cos 30 + i \sin 30$

$z_1 = \cos 150 + i \sin 150$

$z_2 = \cos 270 + i \sin 270$

* $z^4 - 1 = i\sqrt{3}$

sol

$z = (1 + i\sqrt{3})^{\frac{1}{4}}$

$x = 1$, $y = \sqrt{3}$, $r = 2$

$\theta = 60°$

for $k = 0, 1, 2, 3$

$z_0 = 2^{0.25} [\cos 15 + i \sin 15]$

$z_1 = 2^{0.25} [\cos 105 + i \sin 105]$

$z_2 = 2^{0.25} [\cos 195 + i \sin 195]$

$z_3 = 2^{0.25} [\cos 285 + i \sin 285]$

barak

Date Subject

3

* $(-1+i)^3$

sol

$x=-1$, $y=1$, $r=\sqrt{2}$

$\theta' = \tan^{-1}\left(\frac{y}{x}\right) = 45^\circ$, $\theta = \pi - \theta' = 135^\circ$

$(-1+i)^3 = (\sqrt{2})^3 \ (\cos(3\times135) + i\sin(3\times135))$

$= 2\sqrt{2}\,(\cos 405 + i\sin 405)$

* $(1+\sqrt{3}i)^8$

sol

$x=1$, $y=\sqrt{3}$, $r=2$

$\theta' = \theta = 60^\circ$

$(1+\sqrt{3}i)^8 = 2^8\,(\cos(8*60) + i\sin(8*60))$

$= 256\,(\cos 480 + i\sin 480) = -128 + 221{,}7i$

* $(1-\sqrt{3}i)^8$

$x=1$, $y=-\sqrt{3}$, $r=r$

$\theta' = 60$ $\theta = -60^\circ$

$(1-\sqrt{3}i)^8 = 256\,(\cos -480 + i\sin -480) = -128 - 221{,}7$

$(1+\sqrt{3}i)^8 + (1-\sqrt{3}i)^8 = -256 + 0i$

baraka

$P(z) = 2(-1+i)(-1-i)$

$Z = 2$ $\quad y = 0, \; r = 2$

$x = 2$

$\theta = 0$

$Z = 2$

$Z = -1+i$

$x = -1, \; y = 1 \; r = \sqrt{2}$

$\theta' = \tan^{-1}(1) = \frac{\pi}{4}$

$\theta = \frac{3\pi}{4}$

$\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

$Z = -1-i$

$x = -1, \; y = 1, \; r = \sqrt{2}$

$\theta' = \tan^{-1}(1) \; \frac{\pi}{4}$

$\theta = \pi + \theta' = \frac{5\pi}{4}$

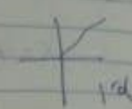
$Z = \sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

* $(3-2i)(1+3i)$

sol

$(3-2i)(1+3i) = 9+7i$

$x = 9 \quad , y = 7 \quad , r = \sqrt{130}$



$\theta = \tan^{-1}\left(\frac{7}{9}\right) = 37.8°$ $\quad , r = \sqrt{130}$

$\theta = \theta' = 37.8°$

$\boxed{2}$

$Z^3 - 2Z - 4 = 0$

sol

Long division by $(Z-2)$

$$
\begin{array}{r}
Z^2 + 2Z + 2 \\
Z-2 \,\big)\, \overline{Z^3 \qquad - 2Z - 4} \\
-\,(Z^3 - 2Z^2) \qquad\quad \\
\hline
2Z^2 - 2Z - 4 \\
2Z^2 - 4Z \qquad \\
\hline
2Z - 4 \\
-\,(2Z - 4) \\
\hline
0
\end{array}
$$

$P_{(z)} = (Z-2)(Z^2 + 2Z + 2)$

$Z = \frac{-b \pm \sqrt{b^2 + 4ac}}{2a} = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm i$

ex (4)

[1] * $4-3i$

sol

$x=4$, $y=3$, $r=\sqrt{4^2+3^2}=5$

4rd

* $-2+i$

sol

$x=-2$, $y=1$, $r=\sqrt{5}$

$\theta' = \tan^{-2}\left(\frac{1}{2}\right) = 26,6$ $\theta = \pi - \theta' = 153,4°$

2nd

* $\frac{(1+i)(3-i)}{3+i}$

sol

$$\frac{(1+i)(3-i)}{3+i} \times \frac{3-i}{3-i} = \frac{14+2i}{10} = \frac{7}{5} + \frac{1}{5} i$$

$x = \frac{7}{5}$, $y = \frac{1}{5}$, $r = \sqrt{2}$

$\theta' = \tan^{-1}\left(\frac{1/5}{7/5}\right) = 8,13°$ $\theta = \theta' = 8,13°$