

## *Lecture 31, Canopy Evaporation and Transpiration, part 1, Theory*

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### Topics

#### Concepts, 1

1. Evaporation
2. Potential Evaporation
3. Equilibrium Evaporation
4. Transpiration
5. Dew, Condensation, Distillation

#### Modeling Approaches

- A. Bowen Ratio
- B. Thornthwaite Equation
- C. Penman's Combination Equation
- D. Penman Monteith Equation
- E. Isothermal Energy Balance
- F. Equilibrium Evaporation
- G. Priestly-Taylor Equation

#### Concepts, 2

1. Canopy conductance
2. Coupling Theory, Canopy Scale
3. Sensitivity and Feedbacks

#### Others Concepts

Drought Indices  
Water Balance

#### *L31.1. Introduction*

Evaporation is the “physical process by which a liquid or solid is converted to a gaseous state” (Glossary of Meteorology).

Plant canopies introduce water vapor into the atmosphere via **transpiration** and the **evaporation** of water from the soil and free water on the leaves and stems. Some scientists call the summed rate **evapotranspiration**.

This field has a long and rich history with over 7000 peer-reviewed articles identified on the web of science, circa 2008.

Different and opposing views have been used historically to define evaporation (Jarvis and McNaughton 1986). Meteorologists argue from the thermodynamic viewpoint that **energy** is required to drive the **latent heat of vaporization**. Evaporation also causes the surface and surrounding air to cool, which condensation releases heat. Physiologists counter that evaporation is driven by a **potential difference** between the humidity of the surface and the free atmosphere. The problem these opposing arguments is that they are valid for the scale at which they were defined and applied. We will see later how to derive a more general equation for evaporation.

### *Potential Evaporation*

Potential Evaporation is:

“the evaporation from an extended surface of a short grass that is supplied with water and the canopy covers the ground completely.”

The original assumption is that potential evaporation cannot exceed evaporation from a free water surface. Several problems arise with this definition. In Davis, Potential evaporation may be only 80% of pan evaporation. In Nebraska, PET can **exceed** pan evaporation. Hence, free water evaporation may not represent the maximal rate in a region.

The surface area of transpiring leaf area exceeds the surface area of a water body, as an evaporation pan. Water is partially transparent to sunlight and stores heat energy. So the energy available to evaporate water will be different than that used to evaporate water from the land surface. Evaporation pans are also subject to error due to the **oasis effect** and from animals drinking from it.

### *Actual Evaporation*

Over the years numerous environmental factors have been correlated with evaporation. They include the vapor pressure of the air, which is a function of temperature, wind speed, solar radiation. Plant factors affecting evaporation include stomatal conductance and leaf area index. Surveys on modeling evaporation have been produced by Brutsaert (Brutsaert 1982), Shuttleworth (Shuttleworth 2007), Monteith (Monteith 1981),

Rosenberg et al (1968), Rana and Katerji (Rana and Katerji 2000) Raupach (Raupach 2001), among others.

## 1. Aerodynamic Approach

Theories on evaporation go as far back as Aristotle, who recognized the power that sun and wind have on evaporating water from puddles and ponds. Dalton's (1801) experiments demonstrated that evaporation from warm water was a function of the vapor pressure of the liquid and that of the air, as determined from the dew point temperature. He derived an empirical equation for evaporation

$$E = (e_s - e)f(u) \quad (1)$$

$f(u)$  is an empirical wind speed function.

If we express Eq.1 using an electrical analog, where the aerodynamic resistance  $R_a$  is used to evaluate the wind speed function and the vapor pressure deficits are expressed in terms of vapor concentration then:

$$E = \rho_a \varepsilon \frac{\frac{e_s}{(p - e_s)} - \frac{e}{(p - e)}}{R_w} = \rho_a \varepsilon \frac{(c_{ws} - c_w)}{R_w} \quad (2)$$

## 2. Energy Balance Approach

The surface energy balance contains a term for latent heat exchange. From the view point of simple algebra one can arrive at a simple equation for evaporation from the net radiation budget.

$$R_n = \lambda E + H + S + G \quad (3)$$

$$\lambda E = R_n - H - S - G \quad (4)$$

We can also examine the energy balance between available energy (A) and how it is partitioned into H and  $\lambda E$ .

$$\lambda E + H = R_n - S - G = A$$

Dividing both sides of the equation by  $\lambda E$  produces

$$1 + \frac{H}{\lambda E} = \frac{R_n - S - G}{\lambda E}$$

If the partitioning of energy is conserved, we defined by the Bowen ratio,  $\beta = \frac{H}{\lambda E}$

We can re-express the equation for latent heat exchange

$$\lambda E = \frac{R_n - G - S}{1 + \beta}$$

Similarly, one can use the same logic to define an equation for H.

$$H = \frac{\beta}{1 + \beta} (R_n - G)$$

When this relation was first derived by Bowen, it was assumed that  $\beta$  was constant. Today we know this is not true, but we can still utilize this relationship by measuring the Bowen ratio with temperature and humidity gradients.

$$\beta = \frac{C_p (T_2 - T_1)}{\lambda \frac{M_v}{M_a} (e_2 - e_1)} = \gamma \frac{(T_2 - T_1)}{(e_2 - e_1)}$$

Where the psychrometric constant is defined as:

$$\gamma = \frac{C_p P m_a}{\lambda m_v}$$

But this relation assumes that the sources and sinks of heat and water vapor are identical, as when vapor may originate from vegetation and heat from the hot soil underneath an open and sparse canopy. A second example is when there is advection of heat or moisture, such as at the transition between desert and an irrigated crop or a lake and rough forest.

Under many circumstances, like tall forests or open canopies, the eddy exchange coefficients for momentum, heat and water vapor transfer differ (Thom et al. 1975), as can be shown with the correlation between  $q$  and  $T$  fluctuations differ from one. Hence the Bowen ratio version of the equation for latent heat exchange should be modified

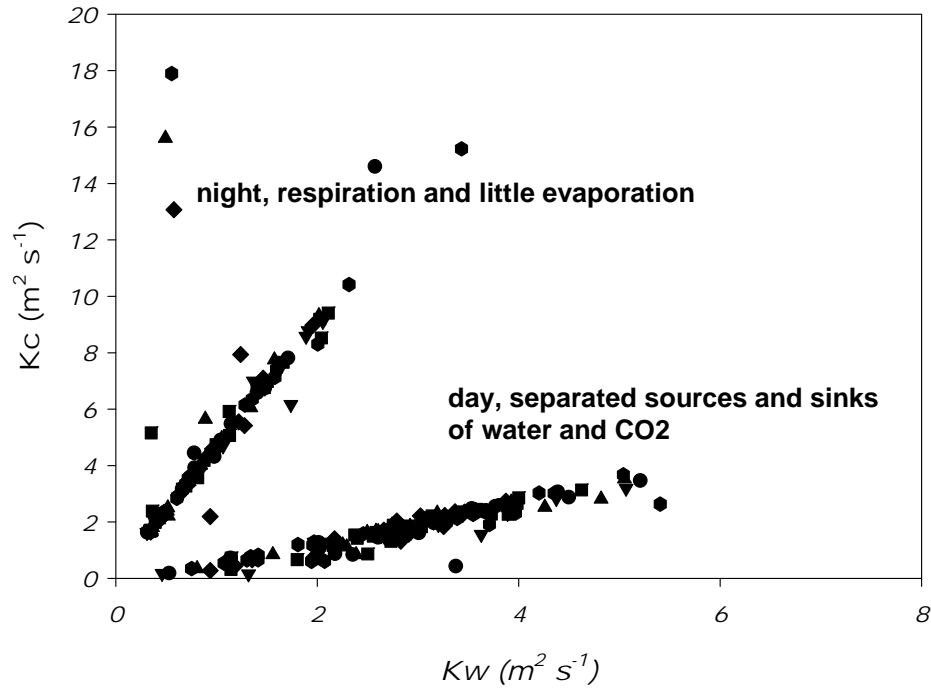
$$\lambda E = \frac{R_n - G}{1 + \frac{K_T}{K_q} \beta}$$

$$H = \frac{\frac{K_T}{K_q} \beta}{1 + \frac{K_T}{K_q} \beta} (R_n - G)$$

If the sources and sinks of heat and vapor are co-located the ratio of the eddy exchange coefficients will approach unity and fluctuations in temperature and humidity will be highly correlated. We see this to be true over a transpiring wheat crop.

Many workers, however, show cases where  $K_T > K_q$ . Others, including Verma et al at Mead, NE found  $K_q > K_T$  for numerous studies using independent measurement methods. When  $K_T < K_q$ , as within an advective inversion, large scale eddies bring in air from above with different temperature and humidity profiles (Lang et al. 1983).

Sometimes  $CO_2$  is used to evaluate eddy exchange coefficients. The following case shows how the role of separated sources and sinks can cause Reynold's analogy to fail.



**Figure 1** Calculations of eddy exchange coefficients for CO<sub>2</sub> and water with equal and separated sources and sinks

The Bowen ratio has some biophysical relevance. Stewart and Thom (Stewart and Thom 1973) defined a relation for Bowen ratio as a function of the surface's resistances.

$$\beta = \frac{1 + R_c / R_{av} - R_i / R_{av}}{s / \gamma + R_i / R_{av}}$$

$R_i$  is a climatological resistance

$$R_i = \frac{\rho_a C_p D}{\gamma A}$$

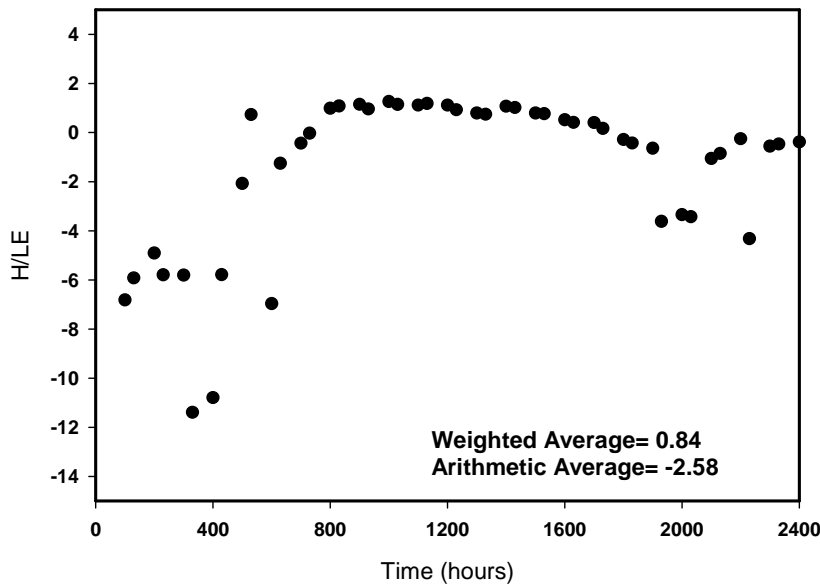
An **oceanic** climate will have a smaller climatological resistance than a **continental** climate. Hence we should expect the Bowen ratio of England to differ than that of Iowa (Jarvis and McNaughton 1986).

### *New Concepts*

Often scientists want to compute daily average evaporation fluxes from mean daily meteorological variables. But a mathematically-averaged Bowen ratio produces a biased and nonsensical value, since it is weighted towards high values when the sun is low and evaporation rates are low. To get a representative value to apply to daily average available energy one should flux weighted the Bowen Ratio

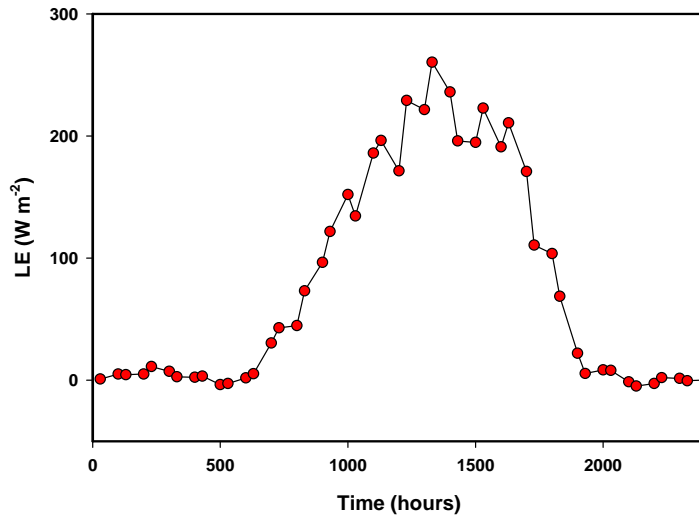
$$\langle \beta \rangle = \frac{\int_0^T R_g(t) \beta(t) dt}{\int_0^T R_g(t) dt}$$

Consider the data in the following figure. This is a daily course of  $\beta$  for a grassland. The arithmetic average is  $-2.58$ , while the flux weighted average is  $0.84$ .



**Figure 2 Grassland, D129, 2001**

As we see in the following figure little evaporation occurs at night, so we don't want to weight this information



### Thornthwaite Equation

At mid Century, numerous investigators were trying to define a way to compute evaporation. Thornthwaite (Thornthwaite 1948) developed a relation that evaluated monthly **potential** evaporation as a function of temperature (cm per month) and day length.

$$E = 1.6 \frac{L}{12} \frac{N}{30} \left(10 \frac{T}{I}\right)^a$$

L is daylength in hours

N is number of days in a month

T is mean monthly air temperature

I is a heat index, computed as a function of the sum of 12 monthly temperature indices,  $i$

$$i = (T/5)^{1.514}$$

The exponent,  $a$ , is a function of  $I$ .

$$a = 6.75 \cdot 10^{-7} I^3 - 7.71 \cdot 10^{-5} I^2 + 1.79 \cdot 10^{-2} I + 0.49$$

The method has many shortcomings. Evaporation and temperature are out of phase with one another. It has no physiological feedback and could not be applied to short term studies, as temperature is not a suitable proxy for radiation on such time scales.

Nevertheless, the method has found much favor by biogeographers (Rosenzweig 1968) trying to forge correlations between plant distributions and water balances. Yet, this is



where one should be cautious and where one could use modern evaporation theory to refine some of the biogeography maps on plant distribution based on climate.

### Penman Equation

In the late 1940's H.L. Penman (Penman 1948) recognized the weakness of the Thornthwaite approach. He consequently developed a relation that had a physical basis. In 1948 he derived the famous Penman equation:

$$E_o = \frac{sR_{no} + \gamma E_a}{s + \gamma}$$

$$E_a = f(u)(e_s - e_a)$$

$$f(u) = 0.27(1 + u/100)$$

The rate of evaporation for free open water ( $E_o$ ) is:

$$E_o = \frac{sR_{no} + 0.27(0.35(e_s(T_a) - e_s(T_d))(1 + 0.01u))}{s + 0.27} \quad (\text{mm d}^{-1})$$

which is a function of the radiation balance over open water. In this version the humidity deficit is in terms of millibars and wind is in terms of wind run, kilometers per day (measured at 2 m).

Penman's equation gained early popularity as it relied on simple meteorological variables. It also produced accurate estimates of evaporation across England. Unfortunately,  $R_{net}$  was rarely measured in early days, so it was evaluated by empirical relation to solar radiation. To apply the relation over vegetation one computed potential evaporation:

$$E_p = fE_o$$

Developed in England,  $f$  was about 0.8 for summer and 0.6 for winter. The relation had a 15% error in temperate climates (see Rosenberg et al.)

This equation ignores heat storage and advection and crop water status.

In applications like agriculture and irrigation management, practitioners compute a reference and potential evaporation rates and then adjust them with a crop coefficient to evaluate actual evaporation (Allen et al. 1989).

$$E_a = k_{crop} E_p$$

## Penman-Monteith Equation

A major and revolutionary breakthrough on quantifying evaporation was made by J.L. Monteith, a colleague of Penman at the Rothamsted Research station in England. Monteith modified the Penman equation by introducing a canopy resistance. This is the basis of the now-famous Penman-Monteith equation (Monteith 1981). We will discuss this equation below.

### 3. Physiological Approach

Physiologists define evaporation using the network between the leaf stomatal cavity and the surface of a leaf:

$$\lambda E = \frac{\rho_a C_p}{\gamma} \frac{(e_s(T_s) - e_0)}{R_s}$$

### 2. Concepts/Theory Penman Monteith Equation

From a leaf or physiological perspective, the flux density of water vapor is proportional to the total conductance exerted across the pathway and the chemical potential (as quantified by the concentration difference between the surface and atmosphere)

$$E = g_w \Delta c_w$$

E: mmol m<sup>-2</sup> s<sup>-1</sup>

g<sub>w</sub>: mol m<sup>-2</sup> s<sup>-1</sup>

Δc: mmol mol<sup>-1</sup>

Humidity is typically measured in terms of vapor pressure. In this case

$$\Delta c = \rho_a \frac{M_w}{P \cdot M_a} \Delta e \quad (2)$$

P: pressure (kPa)

M<sub>w</sub>: molecular weight of water vapor (18.0 g mole<sup>-1</sup>)

M<sub>a</sub>: molecular weight of dry air (28.97 g mole<sup>-1</sup>)

Δe: (e<sub>s</sub>(T<sub>s</sub>) – e<sub>a</sub>) (kPa)

Substituting Equation 2 into Equation 1 yields:

$$E = G_w \rho_a \frac{M_w}{P \cdot M_a} (e_s(T_s) - e_a)$$

The clever attribute of the Penman Monteith equation revolves around how it eliminates a need to assess the surface temperature. The humidity difference can be assessed through application of Taylor's expansion series.

$$e_s(T_s) - e_a = (e_s(T_a) - e_a) - s(T_s - T_a)$$

s is the slope of the saturation vapor pressure relation:

$$s = \frac{de_s}{dT} = \frac{17.502 \cdot 240.97 \cdot e_s(T)}{(240.97 + T)^2}$$

s equals 145 Pa °C<sup>-1</sup> at 20 °C and ranges between 32 to 473 as air temperature increases from -5 to 44 °C.

The energy balance of the land surface is different from that of a leaf. First there are new terms introduced due to storage of heat in the air, ground and plants. Second the ground is only one sided:

$$R_n = (1 - \alpha)R_g + \varepsilon L \downarrow - \varepsilon \sigma T_s^4 = \lambda E + H + G + S$$

Remember, the net longwave balance is computed as the difference between incoming and reflected incoming long wave.

$$L \downarrow - (1 - \varepsilon) L \downarrow = \varepsilon L \downarrow$$

Like our derivation of linearized forms of evaporation for leaves we can manipulate the energy balance, resistance equations for latent heat transfer and linearized forms of equations defining surface temperature to arrive at the Penman-Monteith Equation in terms of a canopy conductance for water,  $G_w$ , that includes a surface and boundary layer effect:

$$\lambda E = \frac{s(R_n - S) + \rho \cdot C_p \cdot G_H \cdot D}{s + \frac{\gamma \cdot G_H}{G_w}}$$

or a canopy conductance for the stomata, that is denoted separately from the boundary layer conductance:

$$\lambda E = \frac{s(R_n - S) + \rho \cdot C_p \cdot G_H \cdot D}{s + \gamma + \gamma \frac{G_H}{G_s}}$$

In the terminology used here, capital letters assigned to conductances represent canopy scale. Lower case letters represent leaf scale.

### *Turbulence and Diffusion.*

The canopy-scale aerodynamic conductance is much different than the leaf boundary layer conductance. Turbulent eddies are responsible for transporting material through the surface boundary layer. The aerodynamic resistance determines the rate that momentum, and other scalars, are transported between a given level in the atmosphere and the vegetation's effective surface sink. The aerodynamic resistance to momentum transfer is expressed as:

$$R_a = \frac{u}{u_*^2} = \frac{1}{ku^*} \ln \frac{z-d}{z_o} - \psi_c(z/L)$$

where  $k$  is von Karman's constant (0.4),  $u^*$  is friction velocity,  $z$  is height,  $d$  is the zero-plane displacement,  $z_o$  is the roughness parameter and  $\psi_c$  is a diabatic correction function which is a function of  $z/L$ .

Momentum transfer is associated with pressure forces, which have no analogue in association with heat and mass transfer (Garratt and Hicks 1973, Stewart and Thom 1973, Massman 1999). Consequently, heat and mass transfer encounter an additional resistance as they are transferred through the well-mixed and turbulent surface layer and the laminar boundary layer of the amalgam of leaves, in the 'big-leaf' canopy. In principles,

$$R_{a,h} = R_{a,v} = R_{a,m} + R_b$$

$R_b$  is the quasi-laminar boundary layer resistance. Fluid dynamicists evaluate this term as a function of the inverse Stanton number,  $B^{-1}$  (a dimensionless resistance,  $B^{-1} = R_b u^*$ ), from a reference view-point). This point, in my opinion obfuscates its meaning, as I have trouble with the concept of an inverse dimensionless number. A better way to explain this resistance as there is an additional layer that must be crossed, from  $z_o$  to  $z_c$ , the scalar roughness length.

$$R_b = \frac{1}{ku^*} \ln \frac{z_o}{z_c} = \frac{(kB^{-1})}{ku^*} (Sc / Pr)^{2/3}$$

We also note it can be parameterized as a function of  $Sc$  is the Schmidt number and  $Pr$  is the Prandtl number, so  $R_b$  is a function of the mass or heat entity being transferred.

Another approach to evaluating  $R_b$  comes from the atmospheric chemistry community. Kramm et al. present the total resistance as:

$$R_{at} = \int_0^{z_r} \frac{dz}{D_x + K_x} + \int_{z_r}^z \frac{dz}{K_x}$$

$$R_b = \int_0^z \frac{dz}{D_x + K_x}$$

The first term on the RHS is the combined turbulent and molecular diffusion term across the viscous sublayer. The second term is the resistance to turbulent transfer across the boundary layer.

$$R_{at} = \frac{l}{D_x} \int_0^{z_r} \frac{dz}{1 + \frac{K_x}{D_x}} + \int_{z_r}^z \frac{dz}{K_x}$$

$$K_m = u_* k z$$

$$Re = \frac{u_* z}{\nu}$$

$$R_{at} = \frac{l}{D_x} \int_0^{z_r} \frac{dz}{1 + Sc \frac{K_m}{\nu}} + \frac{u_1 - u_2}{u_*}$$

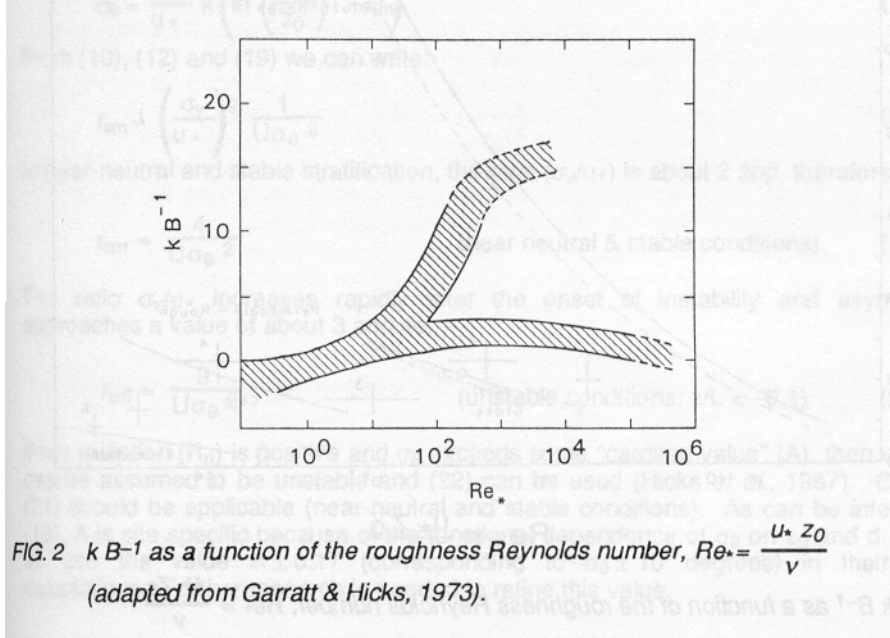
$$R_{at} = R_a + R_b = \frac{u}{u_*^2} + \frac{Sc}{u_*} \int_0^{Re(z_r)} \frac{d Re}{1 + Sc \frac{K_m}{\nu}}$$

Stanton number equals B

$$B^{-1} = Sc \int_0^{Re(z_r)} \frac{d Re}{1 + Sc \frac{K_m}{\nu}}$$

$$R_b = kB^{-1} / u_*$$

The product  $kB^{-1}$  can be treated as a constant, but one that varies with canopy roughness and roughness Reynolds number ( $u^* z_0/v$ ).



The constant is often assumed to equal 2 over closed canopies, but can be much greater over rough incomplete canopies (Wesely and Hicks, 1977).

In recent years the idea of  $R_b$  has been refined to account of canopies of different structure, as it can also vary with leaf area and the source sink distribution (Massman 1999). One model by Choudhury and Monteith (1988) yields:

$$kB^{-1} = \frac{kC_d}{4C_t u_* / u(h) (1 - \exp(-\alpha / 2))}$$

Another by McNaughton and van den Hurk (McNaughton and van den Hurk 1995)

$$kB^{-1} = \frac{ku_* / u(h)}{C_t LAI} \frac{\int_0^h S(z) ((u(z) / u(h))^{-1/2} dz}{\int_0^h S(z) dz}$$

Massman (1999) recently evaluated these algorithms, with his own new one, and shows how  $kB^{-1}$  can vary with canopy structure, yielding a more modern and sophisticated view of this otherwise ill-defined parameter.

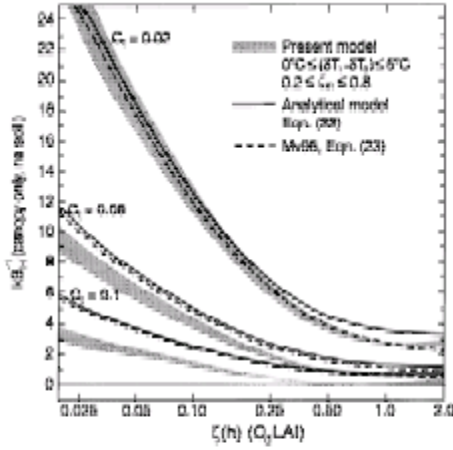


Fig. 3. Comparison of three models of  $kH_m^{mi}$  as functions of  $\zeta(h)$  and  $C_d$  for the canopy leaf surfaces only, i.e. no soil or sheltering effects are explicitly included in the model. The shaded area shows the range of variation of the present LNF model, Eqs. (1)–(19), for different canopy profiles of the temperature difference,  $\Delta T_g(z)$  of Eq. (8), and for different types of vertical foliage distribution. See Fig. 2 for more details concerning the foliage distribution profiles. The line  $kH_m^{mi} = 0$  is highlighted and Mv95 is McNaughton and van den Hurk (1995).

**Figure 3 After Massman 1999**

One point to be stressed with this portion of the discussion is that there is much value and potential to use detailed process level models, such as multi-layer Eulerian higher order closure and Lagrangian random walk models to develop big-leaf parameterization that incorporate aspects of varying canopy structure. These features have generally been ignored by many practitioners. The next section provides another example of using detailed models to understand big-leaf parameterizations.

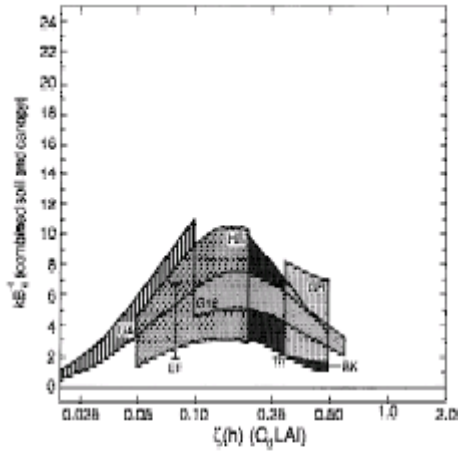


Fig. 6. Simulation of observed values of  $kH_R^{\text{canopy}}$  with the simplified model, Eq. (24), for all ecosystems shown in Fig. 4. Model simulations are based on the observed values of  $C_h$  and canopy height  $h$ , and (where possible) the observed foliage distribution. If the foliage distribution was unknown, then simulations of it were included as a possible source of variation. Otherwise the predicted range of variation is associated with uncertainties or natural variations in  $C_h$  and the foliage distribution. Note here that, primarily for the sake of clarity and simplicity in the visual presentation, only the lower portion of the range of variation in BK is shown. The simplified model is quite sensitive to  $C_h$ , which for BK may be very small due to the relatively tall evergreen trees. See Figs. 4 and 5 for a discussion of the data sources and symbols associated with the different ecosystems. The line  $kH_R^{\text{canopy}} = 0$  is highlighted.

Figure 4 after Massman 1999

### L33.2 Aerodynamic Temperature and Infrared Temperature

There have been many efforts to assess canopy sensible heat transfer with a measure of the canopy infrared temperature. However, it is the aerodynamic temperature that is the temperature of the canopy that drives sensible heat transfer. This problem is of concern because numerous studies show that the aerodynamic canopy temperature of agricultural crops and forests, determined with ground based radiometers (Choudhury et al, 1986; Huband and Monteith, 1986; Stewart et al., 1989; Dunin et al. 1989) and aircraft-mounted thermal scanners (Heilman et al., 1976), does not equal the canopy radiative temperature.

The aerodynamic temperature is computed by manipulating the Resistance Equation for sensible heat transfer

$$H = \rho C_p \frac{(T_{aero} - T_{air}(z))}{R_a + R_b}$$

$$T_{aero} = \frac{H(R_a + R_b)}{\rho C_p} + T_{air}$$



From boundary layer theory we evaluate,  $R_a$  is the aerodynamic resistance as:

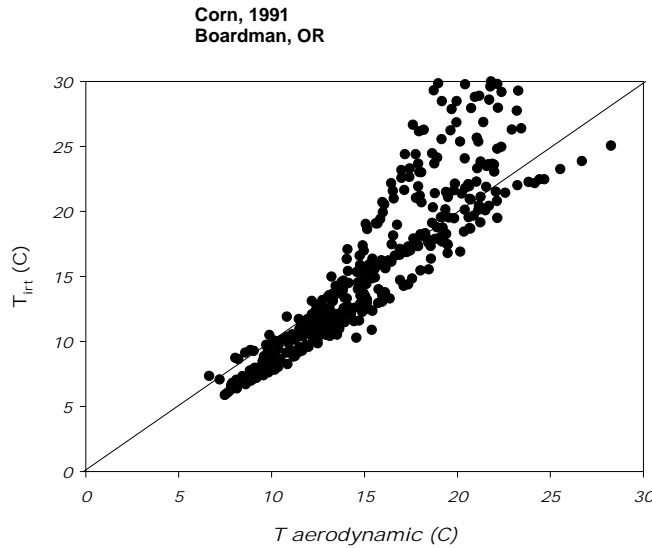
$$R_a = \frac{1}{ku^*} \ln \frac{z-d}{z_o} - \psi_c$$

and  $R_b$  is the quasi laminar boundary layer resistance

$$R_b = \frac{1}{ku^*} \ln \frac{z_o}{z_c} = \frac{const}{ku^*} (Sc / Pr)^{2/3}$$

An infrared thermometer measures the thermal radiance of a scene.

Errors in evaluating the aerodynamic temperature with the infrared temperature will have many consequences with regards with using remote sensing to evaluate canopy latent and sensible heat exchange and canopy surface conductance. The differences between the radiative and aerodynamic temperatures are typically on the order of 2 to 6 C.



**Figure 5** The relation between aerodynamic and radiative temperature at a corn site, Boardman, OR. (data of D. Baldocchi).

Why does radiative temperature not equal aerodynamic temperature? Let's consider case where fraction of soil is 0.25 ( $f_{soil}$ ) and that of vegetation is 0.75 ( $f_{veg}$ ), soil temperature is 50C and vegetation temperature is 25C and the soil resistance is 500 s/m and the vegetation resistance is 50 s/m; we are considering a case where the soil gets very warm because the

flow regime is decoupled and turbulent mixing is weak deep in the canopy. We can examine the difference between the radiative and aerodynamic temperatures using heat transfer and radiative transfer equations. The aerodynamic temperature is computed from a ‘big-leaf’ model that represents the combined fluxes from the soil and vegetation

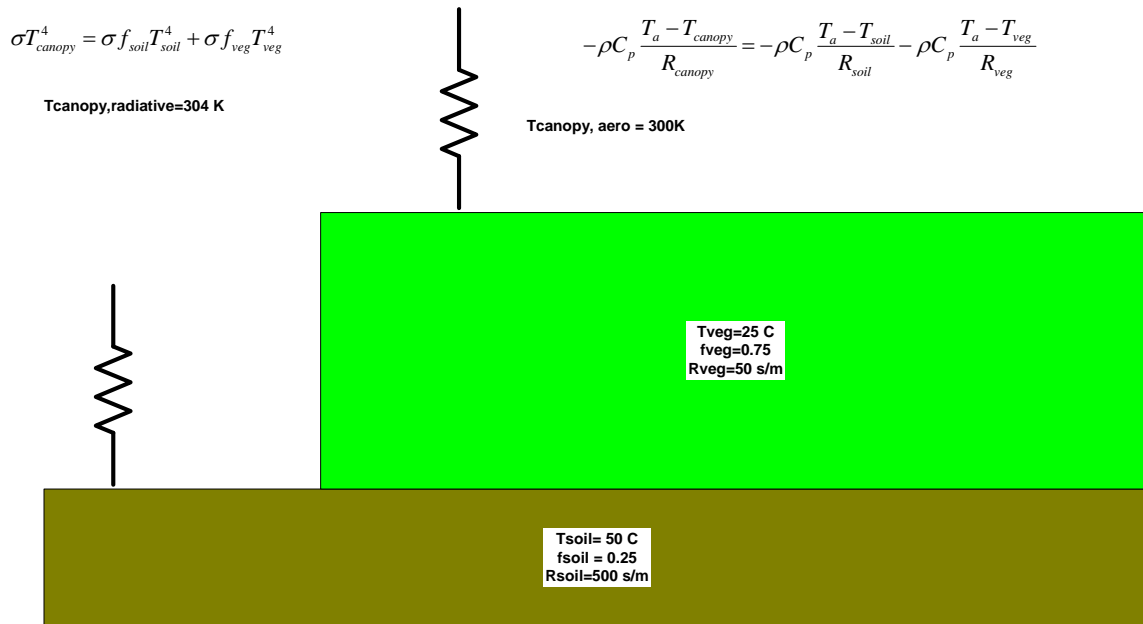
$$-\rho C_p \frac{T_a - T_{canopy}}{R_{canopy}} = -\rho C_p \frac{T_a - T_{soil}}{R_{soil}} - \rho C_p \frac{T_a - T_{veg}}{R_{veg}}$$

For our case the mean temperature is 300 K, assuming air temperature is 293K.

The other situation computes the radiative temperature based on the relative viewing of the soil and vegetation

$$\sigma T_{canopy}^4 = \sigma f_{soil} T_{soil}^4 + \sigma f_{veg} T_{veg}^4$$

This temperature is 304 K. So even though the vegetation and soil components are the same temperature for both cases, the aerodynamic and radiative canopy temperatures differ by 4 K!



A demonstration is presented on the basis of measurements made over a corn site had partial cover. The soil was exposed to the sun and very warm, so the radiative temperature was quite high. In contrast, the heat exchange between the soil and air was decoupled, relative to the vegetation, so the aerodynamic temperature was much less than the infrared temperature.

Atmospheric stability determines whether radiative canopy temperature measurements will over- or underestimate aerodynamic canopy temperature (Verma et al., 1976; Heilman et al., 1976; Choudhury et al., 1986). Representative estimates of the canopy aerodynamic temperature measured with radiometers are difficult to attain because radiative temperature measurements depend on view and sun angles, degree of crop cover, soil-canopy temperature differences and—for airplane and satellite mounted sensors—atmospheric attenuation of radiation and the spatial variability in canopy emissivity (Heilman et al., 1976; Kimes et al., 1980; Kimes, 1983; Huband and Monteith, 1986). Nadir viewed radiometric measurements of canopy temperatures are the least representative of aerodynamic temperatures since radiative measurements are highly biased by the emission of thermal radiation from the soil (Kimes et al., 1980). More representative estimates of radiative temperature of the foliage are possible by measuring canopy radiative temperature from multiple viewing angles or using the model inversion technique of Kimes (1983).

The impact of errors in  $T_{\text{ir}}$  for computing sensible heat exchange can be examined with the aid of sensitivity tests. For the surface energy balance a 1 C error in the surface to air temperature difference can yield a  $40 \text{ W m}^{-2}$  error in the estimate of  $\mathbf{H}$ .

$$\frac{dH}{dT} = \frac{\rho C_p}{R_a + R_b}$$

We assume a rough surface,  $R_a=20$ ,  $R_b=10 \text{ s/m}$ .

The radiative temperature may also imply the wrong direction of  $\mathbf{H}$  for there are instances where counter-gradient flux can occur. One can have instances where the radiative temperature is warmer than the air temperature as a radiometer may be weighted by patches of hot soil. But this portion of the canopy may be uncoupled from the atmosphere. In reality upper transpiring leaves can be cooler than the atmosphere and more closely coupled and may actually draw sensible heat from the atmosphere. Here would be an instance of counter gradient transfer.

### *Canopy Conductance vs Canopy Stomatal Conductance*

Though the Penman-Monteith equation attempts to incorporate physiology, its canopy conductance is ill-defined. In the field of evaporation it is common practice to make a

first order estimate of  $G_c$  on the basis of a leaf area weighted integral of stomatal conductance.

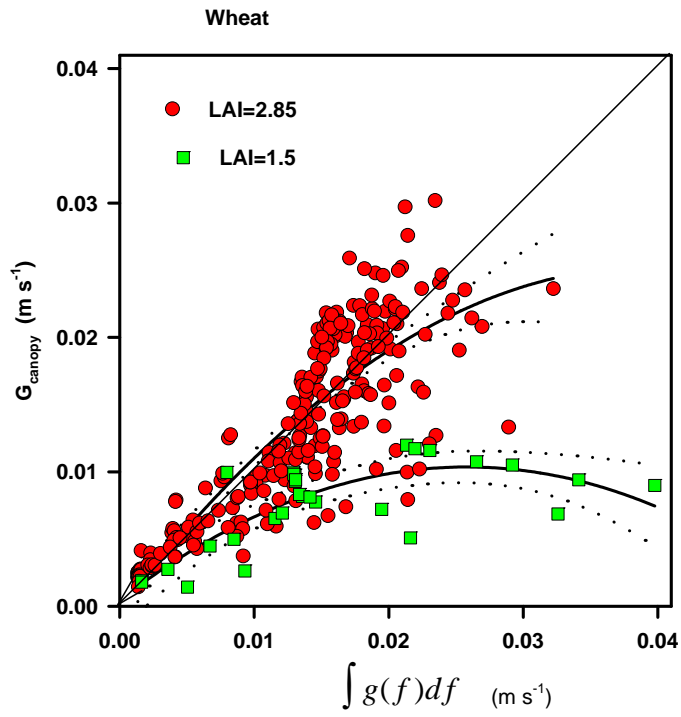
$$G_{sfc} \approx G_{stom} = \int_0^L g_s(l) dl$$

Or in the field of hydrology, biogeochemistry the integrated canopy stomatal conductance is approximated by inverting the Penman-Monteith evaporation model. The relationship between these two metrics, however, has merit only when there is little or no exchange coming from the soil. Because of the wide interest in determining bulk canopy conductances, several teams of scientists (Baldocchi et al. 1987, Paw U and Meyers 1989, Kelliher et al. 1995, Baldocchi and Meyers 1998)(Finnigan and Raupach, 1985) have attempted to examine how the bulk canopy conductance responds to physiological and biophysical factors. One can compute  $G_{canopy}$  by inverting the total Penman-Monteith equation for whole canopy, yielding:

$$G_{sfc}^{-1} = \frac{(\varepsilon + 1)}{G_a} \left( \frac{\varepsilon A}{(\varepsilon + 1) \lambda E} - 1 \right) + \frac{\rho D}{E}$$

Now this relation does not yield any new information, because we need latent heat flux a priori to make this inversion. On the other hand, it gives us a variable with which to work with and explore its dependency upon independent variables that can be measured. Furthermore, use of the inverted Penman Monteith conductance does not yield a satisfactory estimate of the leaf area weight stomatal conductance.

Several empirical studies show how  $G_c$  and  $G_s$  relate to one another. Theoretical studies (Raupach and Finnigan, 1986)(Paw U and Meyers 1989, Baldocchi and Meyers 1998) report conditions where  $G_c > G_s$  and when  $G_s > G_c$ . The relations are a factor of the ratio  $G_h/G_s$ , canopy leaf area, soil wetness.



**Figure 6 Comparisons between canopy resistance and integrated stomatal resistance**

Using a biophysical model we can explore in more detail the interrelations between  $G_c$  and  $G_s$ .

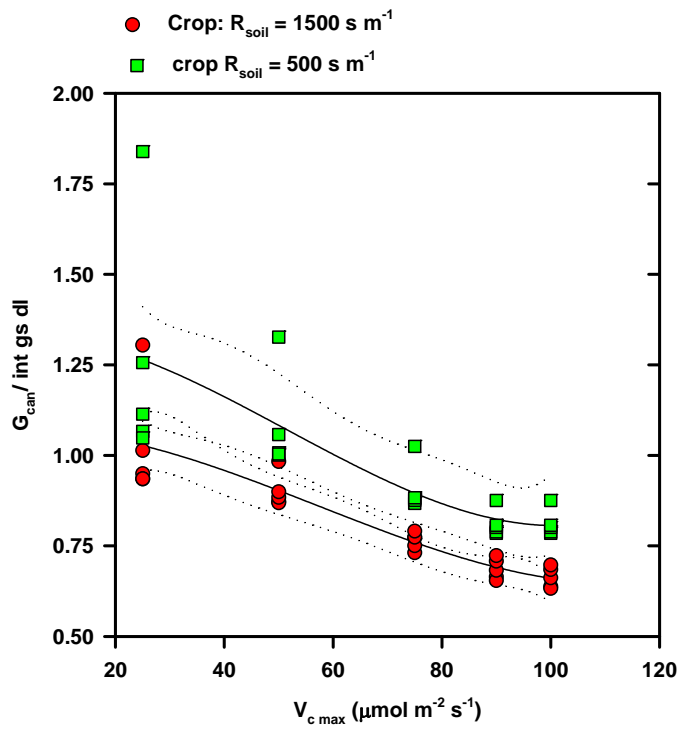


Figure 7 Theoretical relation between the ratio of canopy and stomatal conductance as a function of the underlying soil resistance and photosynthetic capacity of the canopy

*F.M. Kelliher et al. / Agricultural and Forest Meteorology 73 (1995) 1–16*

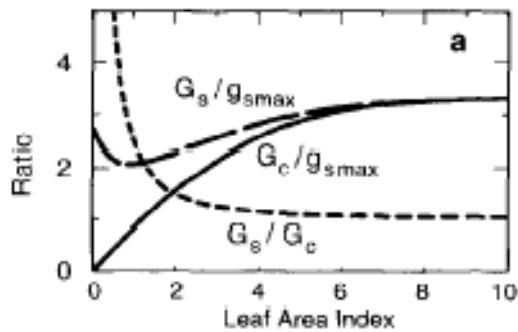


Figure 8 After Kelliher et al. 1995

*Isothermal Penman Monteith Equation*

In a manner similar to our derivation of the energy balance of a leaf we can derive the isothermal net radiation balance to compute evaporation from a canopy, to remove the complication from not explicitly considering surface temperature when examining the sensitivity between latent heat exchange and incoming energy:

$$\lambda E = \frac{s(R_{iso} - G) + D\rho_a C_p (G_r + G_h)}{(s + \frac{\gamma G_r}{G_w} + \frac{\gamma G_h}{G_w})}$$

Details for this derivation are given in Appendix A33.2.

### *Omega Theory*

Jarvis and McNaughton (Jarvis and McNaughton 1986, Mcnaughton and Jarvis 1991) developed omega theory to examine the paradox of scales. Meteorologists contended that evaporation was driven by available energy and was independent of plant and surface control (e.g. Thornthwaite, 1944). Physiologists contend that the opening and closing of stomata control plant water loss.

They therefore address to what extent do stomata control evaporation. At the canopy scale, they arrive at

$$\frac{dE_c}{E_c} = (1 - \Omega_c) \frac{dG_c}{G_c}$$

$$E_c = \Omega_c E_{eq} + (1 - \Omega_c) E_{imp}$$

$$\Omega_c = \frac{\varepsilon + 1}{\varepsilon + 1 + \frac{G_a}{G_c}}$$

$$\lambda E_{eq} = \frac{s}{s + \gamma} (R_n - G_{soil})$$

$$\lambda E_{imp} = \frac{C_p}{\gamma} G_c D_m$$

The first case (1) is the limit when the vegetation is short and poorly efficient from an areodynamic point of view. Evapotranspiration depend largely from the available energy, irrespectively from the stomatal control. In the second case (2) evapotranspiration is more directly controlled by VPD and the stomata.

$$\lim_{Ra \rightarrow 0} LE = \frac{\rho c_p VPD}{\gamma R_s}$$

Broadly speaking short vegetation, like crops, is more dependent on the available energy (they are decoupled from the atmosphere, according to McNaughton and Jarvis terminology) while tall vegetation like forests is affected more by VPD and can control through stomata the water fluxes (they are coupled).

### *Equilibrium Evaporation*

Equilibrium evaporation is one limit of Penman-Monteith evaporation theory:

$$\lambda E_{eq} = \frac{s}{s + \gamma} (R_n - S)$$

Equilibrium Equation can be derived via three routes of logic. Mathematically, it describes the case when the aerodynamic conductance goes to zero, as when wind is calm.

$$\lim_{Ra \rightarrow \infty} LE = \frac{sA}{s + \gamma}$$

Theoretically, one can arrive at the present definition of equilibrium evaporation by examining the time dependence of evaporation into a closed volume and its feedback with the humidity deficit of the volume (McNaughton and Spriggs 1986).

$$\frac{dE(D)}{dt} \rightarrow 0$$

From inspection of Equilibrium evaporation one can conclude that cooler zones in northern portion of the boreal climates have a lower potential to evaporate than locales in the southern zone (see Table 1; Nijssen et al. 1997). One explanation for this

observations stems from the fact that the factor,  $\frac{s}{s + \gamma}$ , is a strong function of

temperature. For example,  $\frac{s}{s + \gamma}$  is 0.32 at  $-5^\circ\text{C}$  and 0.47 at  $5^\circ\text{C}$ , a 47% difference with a  $5^\circ\text{C}$  increase in temperature.



$$s = \frac{de_s}{dT} = \frac{17.502 \cdot 240.97 \cdot e_s(T)}{(240.97 + T)^2}$$

s equals 145 Pa °C<sup>-1</sup> at 20 °C and ranges between 32 to 473 as air temperature increases from -5 to 44 °C.

γ is the psychrometric constant. Its value is 66.1 Pa K<sup>-1</sup> at 20 °C and ranges between 64.6 to 67.8 as air temperature varies between -5 and 45 °C.

### *Priestly-Taylor Equation*

Simple models can be used for calculating evaporation flux densities under restricted conditions (see McNaughton and Jarvis, 1986; Jarvis and McNaughton, 1986). Over well-watered and aerodynamically-smooth vegetation, evaporation flux densities are proportional to equilibrium evaporation flux densities ( $\lambda E_{eq}$  is a function of available energy and air temperature) and are relatively independent of surface control (Jarvis and McNaughton, 1986). An assortment of studies report that the proportionality constant between actual and equilibrium evaporation flux densities ranges between 1.2 and 1.3 (Priestley and Taylor 1972, Debruin 1983). An appeal of using an equilibrium-based evaporation model is its dependence on variables that are readily computed by global and mesoscale meteorological models.

The ratio,  $LE/LE_{eq}$ , deviates from the cited narrow range under non-ideal conditions and over heterogeneous canopies. Shuttleworth and Calder (1979) report that evaporation rates from wet forests exceed 1.3 times  $LE_{eq}$  and evaporation rates from dry forests are much less than 1.2 times  $LE_{eq}$ . Other experimental and theoretical studies (over crops and forests) show that the ratio,  $LE/LE_{eq}$ , decreases drastically from 1.2 as soil moisture deficits (Priestley and Taylor, 1972; Davies and Allen, 1973; Flint and Childs, 1991; Bailey and Davies, 1981; Spittlehouse, 1987) and surface conductance (de Bruin, 1983; McNaughton and Spriggs, 1986, 1989) cross some threshold. Furthermore, an energy-dependent evaporation model will break down at night if significant evaporation occurs when available energy is being lost to the environment.

### *Evaporation over a hill*

As air passes over a hill numerous perturbations occur. The wind accelerates and decelerates, the radiation balance varies as a function of slope, azimuth and sun angle. Elevation changes affect temperature and can cause condensation and rain, with orographic lifting. Different slopes have different amounts of available soil moisture, which together with environmental forcings affect leaf area index, stomatal conductance and evaporation rates. Studies on evaporation from hills is rather rare, but it is an important topic when studying the biometeorology of California. Blyth et al and Huntingford et al have developed simple algorithms for assessing evaporation on hills, by modifying the radiation geometry that affects the Penman-Monteith equation.

$$\lambda E = \int_x^x \frac{sA(f_{diffuse} + f_{beam}|\sin(\alpha + \beta(x))| + \rho C_p (q_{sat}(T) - q) / R_a}{(s + \gamma + \gamma \frac{R_s}{R_a}) \cos \beta(x)}$$

alpha is sun elevation angle, beta is hill angle, X is distance of the hill.

They compute that potential evaporation can decrease by 20% over a 300 m hill. It can increase by 10% over a hill with 30 degrees slope because of more surface area.

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## APPENDIX FOR GRADUATE STUDENTS

### *Assessing Average Parameters for a Non-Linear Equation*

One issue relates to the fact that the Penman-Monteith equation can be envisioned as the average of latent heat fluxes. On inspection we see that these components are a non-linear function of the local surface conductance. The important point to stress and conclude is that a simple average of multi-layer canopy conductances do not yield the appropriate metric (Raupach 1991, McNaughton 1994, Baldocchi et al. 2005).

For example, let's consider the Penman-Monteith equation as a function of its mean components:

$$\lambda < E > = \frac{s(< R_n >) + \rho \cdot C_p \cdot < G_H > \cdot < D >}{s + \frac{\gamma \cdot < G_H >}{< G_w >}}$$

Raupach (1991) and McNaughton (1994) show that the mean conductances that are needed to evaluate the proper evaporation rate are not the algebraic average of all the leaves in the canopy. They must be weighted by the isothermal radiation balance, or in other words, by their potential contribution to canopy evaporation:

$$\frac{1}{< G_w >} = < R_w > = \frac{\sum a_i \frac{R_{n,i} R_{w,i}}{< R_n > (R_{w,i} + \epsilon R_{h,i})}}{\sum \frac{a_i}{(R_{w,i} + \epsilon R_{h,i})}}$$

$$\frac{1}{< G_h >} = < R_h > = \frac{\sum a_i \frac{R_{n,i} R_{h,i}}{< R_n > (R_{w,i} + \epsilon R_{h,i})}}{\sum \frac{a_i}{(R_{w,i} + \epsilon R_{h,i})}}$$

This may sound counter-intuitive, but the correct resistance to use is weighted by leaf area and net radiation! We stress that not all leaves contribute equally or proportionally, as some deep in the canopy are decoupled from controlling evaporation, may have a huge resistance and bias the canopy average. Consequently, the parameters should be flux-weighted.

We can draw upon a simple numerical experiment to understand the merits and demerits of using strict arithmetic and flux weighted resistances. Lets examine the simple case of

$$F = \sum f = \sum \frac{a_i}{R_i}$$

Leaf area density, a	Resistance	a/R
2	25	0.08
1	50	0.02
1	100	0.01
1	200	0.005

--	--	--

The Average resistance is 94 s/m (which produces a mean flux of  $A/R=5/94=0.0533$ . The layer integrated flux is 0.115, a factor of two difference. To produce a conserved flux, the flux-weighted resistance is 43 s/m.

## Appendix

### *Penman Monteith Derivation*

For this analysis we start with the surface energy budget, which defines that the net radiative flux density is partitioned into sensible and latent heat flux density:

$$R_n = H + \lambda E + G$$

Solving for latent heat flux density yields:

$$\lambda E = R_n - H - G$$

Next we invoke the conductance/resistance relations for sensible and latent heat exchange. The conductance equation for sensible heat exchange is:

$$H = \rho_a C_p (T_s - T_a) G_{a,h}$$

H is positive if heat enters the atmosphere, and negative if heat is lost from the atmosphere.

The relation for latent heat exchange is:

$$\begin{aligned} \lambda E &= \frac{(m_v / m_a) \lambda \rho_a G_w (e_s(T_s) - e_a)}{P} = \\ &= \frac{(m_v / m_a) \lambda \rho_a (e_s(T_s) - e_a)}{P} \frac{G_s G_{a,v}}{G_{a,v} + G_s} = : \\ &= \frac{(m_v / m_a) \lambda \rho_a (e_s(T_s) - e_a)}{P(R_s + R_{a,v})} \end{aligned}$$

We simplify the equation for  $\lambda E$  by applying the linearized form of the equation for  $e_s(T_s) - e_a$ :

$$\lambda E = \frac{\rho_a (m_v / m_a) \lambda G_w (D + e_s'(T_s - T_a))}{P}$$

$$e_s(T_s) - e_a = D + e_s'(T_s - T_a) = \frac{\lambda E \cdot P}{\rho_a \lambda (m_v / m_a) G_w}$$

As an aside, it should be noted that many derivations include the psychrometric constant, defined in terms of air pressure, the specific heat of dry air at constant pressure and the latent heat of evaporation.

$$\gamma = \frac{PC_p}{0.622\lambda}$$

The next key step is to eliminate the leaf-air temperature difference. This is accomplished by solving for it in terms of latent heat exchange:

$$T_s - T_a = \Delta T = \frac{\lambda E \gamma}{s \rho_a C_p G_w} - \frac{D}{s}$$

$$\lambda E = R_n - G - \left( \frac{\lambda E \gamma}{s \rho_a C_p G_w} - \frac{D}{s} \right) \rho_a C_p G_{a,h}$$

$$\lambda E = R_n - G - \left( \frac{\lambda E \gamma G_{a,h}}{s G_w} - \frac{D \rho_a C_p G_{a,h}}{s} \right)$$

Next we re-arrange the equation so we can solve for  $\lambda E$ :

$$\lambda E + \frac{\lambda E \gamma G_{a,h}}{s G_w} = R_n - G + \frac{D \rho_a C_p G_{a,h}}{s}$$

$$\lambda E \left( 1 + \frac{\gamma G_{a,h}}{s G_w} \right) = R_n - G + \frac{D \rho_a C_p G_{a,h}}{s}$$

A final simplification is made by multiplying the equation by  $s$  to remove it from the denominator:

$$\lambda E \left( s + \frac{\gamma G_{a,h}}{G_w} \right) = s(R_n - G) + D \rho_a C_p G_{a,h}$$

And Voila', we now have an equation for the evaporation from the canopy, knowing its net radiation balance, without needing to know its surface temperature''

$$\lambda E = \frac{s(R_n - G) + D\rho_a C_p G_{a,h}}{(s + \gamma \frac{G_{a,h}}{G_w})}$$

Breaking  $G_w$  into  $G_s$  and  $G_{av}$ , while retaining  $R_n$  yields:

$$\lambda E = \frac{s(R_n - G) + D\rho_a C_p G_{a,h}}{(s + \gamma(1 + \frac{G_{a,h}}{G_s}))}$$

### A33.2 Quadratic Solutions for canopy Temperature and canopy Latent Heat Exchange

The leaf energy balance can also be used to derive a quadratic equation for latent heat exchange ( $\lambda E$ ;  $W\ m^{-2}$ ):

$$a\ \lambda E^2 + b\ \lambda E + c = 0$$

$$Q = R_g(1 - \alpha) + \varepsilon L \downarrow$$

$$R_n = R_g(1 - \alpha) + \varepsilon L \downarrow - \varepsilon \sigma T_s^4$$

$$R_n = H + \lambda E + G_{soil}$$

We can solve directly for latent heat flux density

$$\lambda E = \frac{0.622\lambda\rho_a(e_s(T_l) - e_a)}{P} \frac{G_s G_a}{G_a + G_s} = \frac{0.622\lambda\rho_a(e_s(T_l) - e_a)}{P(R_s + R_a)}$$

$$\lambda E = \frac{0.622\lambda\rho_a G_w}{P} ((e_s(T_a) - e_a) + s(T_a)(T_l - T_a) + \frac{e_s(T_a)''}{2}(T_l - T_a)^2)$$

Substituting

$$T_l - T_a = \frac{Q - \lambda E - \sigma \varepsilon T_a^4 - G_{soil}}{\rho_a C_p G_h + 4\varepsilon \sigma T_a^3}$$

One derives a quadratic equation for  $\lambda E$  ( $a\lambda E^2 + b\lambda E + c = 0$ ).

The coefficients for LE:

$$a = \frac{\rho_a \lambda G_w m_v}{2m_a P(\rho_a C_p G_h + 4\sigma \varepsilon T_k^3)} \frac{d^2 e_s(T)}{dT^2}$$

$$b = -4 \cdot \varepsilon \cdot \sigma \cdot T_k^3 - \frac{\rho_a \lambda G_w m_v}{m_a P} \frac{de_s(T)}{dT} - \rho C_p G_h + \frac{\rho_a \lambda G_w m_v}{2m_a P(\rho_a C_p G_h + 4\sigma \varepsilon T_k^3)} \frac{d^2 e_s(T)}{dT^2} [-2Q + 2\sigma \varepsilon T_k^4 + 2G_{soil}]$$

$$c = (\rho_a C_p G_h + 4\varepsilon \cdot \sigma \cdot T_k^3) \frac{\rho_a \lambda G_w m_v (e_s(T) - e_a)}{m_a P} + \frac{\rho_a \lambda G_w m_v}{m_a P} \frac{de_s(T)}{dT} (Q - \varepsilon \cdot \sigma \cdot T_k^4 - G_{soil}) + \frac{\rho_a \lambda G_w m_v}{2m_a P(\rho_a C_p G_h + 4\sigma \varepsilon T_k^3)} \frac{d^2 e_s(T)}{dT^2} [Q^2 + (\sigma \varepsilon T_k^4)^2 + G_{soil}^2 - 2Q \cdot \sigma \varepsilon T_k^4 - 2Q G_{soil} + 2G_{soil} \sigma \varepsilon T_k^4]$$

A quadratic equation, defining the difference between leaf and air temperature ( $\Delta T$ ), was derived from the leaf energy balance relationship so an analytical solution could be used to compute leaf temperature (Paw U, 1987):

$$a \Delta T^2 + b \Delta T + c = 0 \quad (A18)$$

The coefficients are defined as:

$$a = 6 \cdot \varepsilon \cdot \sigma \cdot T_k^2 + \frac{\rho_a \lambda G_w m_v}{2m_a P} \frac{d^2 e_s(T)}{dT^2}$$

$$b = 4 \cdot \varepsilon \cdot \sigma \cdot T_k^3 + \frac{\rho_a \lambda G_w m_v}{m_a P} \frac{de_s(T)}{dT} + \rho C_p G_h$$

$$c = \varepsilon \cdot \sigma \cdot T_k^4 + \frac{\rho_a \lambda G_w m_v (e_s(T) - e_a)}{m_a P} + -Q + G_{soil}$$

where  $\varepsilon$  is emissivity,  $\sigma$  is the Stefan-Boltzman constant,  $\rho_a$  is air density,  $\lambda$  is the latent heat of vaporization,  $T_k$  is absolute temperature (K),  $g_s$  is the stomatal conductance ( $\text{m s}^{-1}$ ),  $g_h$  is the aerodynamic conductance for sensible heat transfer ( $\text{m s}^{-1}$ ),  $C_p$  is the specific heat of air,  $Q$  is absorbed energy (incoming short and long wave radiation, minus reflected shortwave radiation;  $\text{W m}^{-2}$ ),  $m_v$  and  $m_a$  are the molecular weights of vapor and dry air ( $\text{g mol}^{-1}$ ),  $P$  is pressure (kPa),  $e_s$  is saturated vapor pressure (kPa) and  $e_a$  is the ambient vapor pressure (kPa).

### ***A33.3Evaporation and Isothermal net radiation.***

It is the radiation flux density that occurs when the leaf and air temperature are equal.

$$R_{ni} = Q - \sigma \varepsilon T_a^4$$

$$\begin{aligned} R_n &= Q - \sigma \varepsilon T_l^4 = \\ Q &- (\sigma \varepsilon T_a^4 + 4\sigma \varepsilon T_a^3 (T_l - T_a)) = \\ R_{ni} &- 4\sigma \varepsilon T_a^3 (T_l - T_a) \end{aligned}$$

$$R_n = H + \lambda E + G_{soil}$$

Solving for latent heat flux density yields:

$$\lambda E = R_n - H - G_{soil}$$

Substituting the isothermal radiation balance produces:

$$\lambda E = R_{ni} - 4\sigma \varepsilon T_a^3 (T_l - T_a) - H - G_{soil}$$

Which allows us to produce an equation for latent heat exchange that is a function of the leaf-air temperature difference:

$$\lambda E = R_{iso} - 4\sigma \varepsilon T_a^3 (T_l - T_a) - \rho C_p G_h (T_l - T_a) - G_{soil}$$

$$\Delta T = (T_l - T_a)$$

$$\lambda E = R_{iso} - \Delta T (4\sigma \varepsilon T_a^3 - \rho_a C_p g_h) - G_{soil}$$

$$\Delta T = \frac{\lambda E \cdot P}{\rho_a \lambda (m_v / m_a) s G_w} - \frac{D}{s}$$



or

$$\Delta T = \frac{\lambda E \gamma}{s \rho_a C_p G_w} - \frac{D}{s}$$

Substituting the leaf-air temperature difference equation into the relation for  $\lambda E$  produces:

$$\lambda E = R_{iso} - \left( \frac{\lambda E \gamma}{s \rho_a C_p G_w} - \frac{D}{s} \right) (4 \sigma \epsilon T_a^3 - \rho_a C_p G_h) - G_{soil}$$

$$\lambda E = R_{iso} - \left( \frac{\lambda E \gamma}{s \rho_a C_p G_w} \right) (4 \sigma \epsilon T_a^3 - \rho_a C_p G_h) + \frac{D}{s} (4 \sigma \epsilon T_a^3 - \rho_a C_p G_h) - G_{soil}$$

$$\lambda E \left( 1 + \frac{\gamma 4 \sigma \epsilon T_a^3}{s \rho_a C_p G_w} + \frac{\gamma G_h}{s G_w} \right) = R_{iso} - G_{soil} + \frac{D 4 \sigma \epsilon T_a^3}{s} + \frac{\rho_a C_p D G_h}{s}$$

$$\lambda E \left( s + \frac{\gamma 4 \sigma \epsilon T_a^3}{\rho_a C_p G_w} + \frac{\gamma G_h}{G_w} \right) = s (R_{iso} - G_{soil}) + D 4 \sigma \epsilon T_a^3 + \rho_a C_p D G_h$$

An algebraic simplification can be made using the radiative conductance:

$$G_r = \frac{4 \sigma \epsilon T_a^3}{\rho C_p}$$

This term can be thought of as the resistance defining the conditions where sensible heat and emissive energy flux densities are in balance:

$$H = L \uparrow = 4 \sigma \epsilon T_a^3 (T_l - T_a) = \rho C_p (T_l - T_a) G_h$$

Substitution of  $g_r$  produces:

$$\lambda E \left( s + \frac{\gamma G_r}{G_w} + \frac{\gamma G_h}{G_w} \right) = s (R_{iso} - G_{soil}) + D \rho_a C_p (G_r + G_h)$$

$$\lambda E = \frac{s (R_{iso} - G_{soil}) + D \rho_a C_p (G_r + G_h)}{\left( s + \frac{\gamma G_r}{G_w} + \frac{\gamma G_h}{G_w} \right)}$$

Next, we can refine the derivation by breaking the water vapor conductance into its components.

$$\frac{1}{G_w} = \frac{1}{G_s} + \frac{1}{G_{av}} = \frac{G_s + G_{av}}{G_s G_{av}}$$

Assuming, that the conductances for heat and water vapor are identical,  $g_h = g_{av}$ , leads to:

$$\lambda E = \frac{s(R_{iso} - G_{soil}) + D\rho_a C_p (G_r + G_h)}{(s + \gamma(\frac{G_s + G_{av}}{G_s G_{av}}))(G_r + G_h)}$$

or:

$$\lambda E = \frac{s(R_{iso} - G_{soil}) + D\rho_a C_p (G_r + G_h)}{(s + \gamma(1 + \frac{G_r}{G_{av}})(1 + \frac{G_{av}}{G_s}))}$$

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