Assignment 3

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Problem 1

Given Problem: To bracket a minimum for a function
Methodology used: Let the function be 'f', if $x_1 < x_2 < x_3$ are three points such that $f(x_1), f(x_3) > f(x_2)$, then there will be a minimum of f in the interval (x_1, x_3).
Then we use bisection to find two points $a = \frac{x_1+x_2}{2}$ and $b = \frac{x_2+x_3}{2}$.
If $f(a) > f(x_2)$ *and* $f(b) > f(x_2)$, we will take the next (x_1, x_2, x_3) as (a, x_2, b).
If $f(a) \leq f(x_2)$, then we take (x_1, a, x_2) in the next iteration
If none of the above is the case, then we take (x_2, b, x_3) for the next iteration
Iteration is done till convergence of $x_i s$.
In this particular problem, the function given is $x^4 - 14x^3 + 60x^2 - 70x$, the initial points were taken to be $x_1 = -20, x_2 = 0, x_3 = 20$, and the convergence was obtained at $x = 0.78088$, where the function value is -24.36960

Problem 2

In this problem, we will fit the function using Levenberg-Marquardt model (non-linear least squares), from the given data, the results obtained were:

$$x = [12.570364, 105.197341, -55.912090, 12.609959, 0.056693]$$

Also, when x_5 is taken zero, the new result obtained is:

$$x = [12.590757, 105.903848, -55.886924, 12.590759, 0]$$

Then minimizing the error with respect to x_5 gives $x_5 = O(10^{-7})$, taken as zero

Problem 3

Here we assume that Power method can be applied(i.e. there is a single eigenvalue of maximum magnitude). Now we apply power method to get the largest eigenvalue and the corresponding eigenvector, and then apply deflation to make the magnitude of largest eigenvalue zero in the new matrix. And thus find the second largest eigenvalue and the corresponding eigenvector.
In the suggested case, the matrix is:

$$\begin{bmatrix} 2 & 3 & 2 & 4 \\ 3 & 3 & 4 & 1 \\ 2 & 6 & 1 & 2 \\ 4 & 1 & 2 & 0 \end{bmatrix}$$

The following results were obtained:
Largest eigenalue: 10.284877
Corresponding eigenvector:

$$\begin{bmatrix} 0.50799458 \\ 0.55939319 \\ 0.54814402 \\ 0.35855162 \end{bmatrix}$$

Second largest eigenvalue: -4.01285551365
Corresponding eigenvector:

$$\begin{bmatrix} 0.49005442 \\ -0.29494233 \\ 0.65129033 \\ -0.49867571 \end{bmatrix}$$

Problem 4

First we use the polynomial to obtain the matrix whose eigenvalues are the roots of the polynomial(as described in the Problem).
Now we do qr decomposition using gram-schmidt algorithm to obtain Q and R matrices.
Now we do QR iterations to reduce the matrix into Quasi upper triangular form.
After obtaining the Quasi upper triangular form, we look for block matrices(which contains eigenvalues of equal/close magnitude) and 2x2 matrices(which contains complex eigenvalues).
After that, we find the eigenvalues of the block matrix and 2x2 matrix using some other method.
Remaining Diagonal Entries are also the eigenvalues of the given Matrix.
All the eigenvalues put together gives the solution set of the given polynomial equation.

In suggested data, the polynomial is:

$$p(x) = x^4 - 13x^3 + 35x^2 - 40x + 24$$

Here the matrix will become:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -24 & 40 & -35 & 13 \end{bmatrix}$$

After QR decomposition:

$$Q = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \end{bmatrix}, R = \begin{bmatrix} 24 & -40 & 35 & -13 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

After QR iterations, the Quasi Upper triangular form of the matrix is:

$$\begin{bmatrix} 9.82742 & 14.9798 & -29.23471 & 48.57353 \\ 0.0 & 1.80476 & -2.38158 & 4.24500 \\ 0.0 & 0 & 0 & 1.76754 \\ 0.0 & 0 & -1.67577 & 2.12493 \end{bmatrix}$$

Here, two eigenvalues are straightforward i.e. 9.87242 and 1.80476.
Remaining two are obtained by taking the last 2x2 block of the matrix and finding it eigenvalues which comes out to be $0.683904 \pm 0.940977\imath$

Problem 5

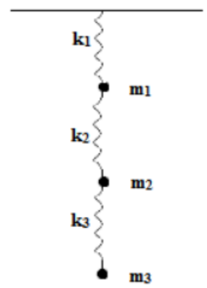

Figure 1: Mass Spring System

Taking each mass m_i to be displaced by a distance d_i, We have the following equations of motion:

$$m_1\ddot{x_1} + k_1x_1 = 0$$
$$m_2\ddot{x_2} + k_2(x_2 - x_1) = 0$$
$$m_3\ddot{x_3} + k_3(x_3 - x_2) = 0$$

which can be put in a matrix form as

$$M\ddot{X} + KX = 0$$

where:

$$M = \begin{bmatrix} \mathrm{m}_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}, K = \begin{bmatrix} \mathrm{k}_1 & 0 & 0 \\ -k_2 & k_2 & 0 \\ 0 & -k_3 & k_3 \end{bmatrix}, X = \begin{bmatrix} \mathrm{x}_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Now,

$$M\ddot{X} + KX = 0$$
$$\text{put } X = X_0 sin(\omega t) \text{ to get}$$
$$-\omega^2 MX + KX = 0$$
$$\implies (-\omega^2 M + K)X = 0$$
$$\implies det(-\omega^2 M + K) = 0 \text{ (since } X \neq 0 \rightarrow \text{For non-trivial Solution)}$$
$$\implies det(M^{-1}K + \omega^2 I) = 0 \text{ (since } det(M) \neq 0)$$

So, we'll find eigenvalues and take the square roots which will give the natural frequencies of the system.
The eigenvalues are found out by power method with continuous use of deflation to remove the determined eigenvalues.
The natural frequencies come out to be: 0.70710, 0.57736, 0.50000