Assignment 1

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Problem 1

Given Problem: To find condition number of a function f: R2 $\rightarrow$ R
Methodology used: Took 1000 random points near the given (x,y) and found the average of all the obtained condition numbers.
The condition number for the function is obtained by

$$k = \frac{(|\Delta f|/|f|)}{(|\Delta x|/|x|)}$$

Two functions are used, one (x-y) and another $\sin(x^2+y^3)$. The condition number for $\sin(x^2+y^3)$ was varying very much, because the function has quadratic dependence on x and cubic dependence on y inside the sin() function. So, a little change causes a big trouble. Reliability of the Calculation:
In the regions where condition number is high, it means that a small change in x and y will cause a big change in the function output, thereby decreasing the reliability of our approximation of f

Problem 2

Here we formulate two ways of finding roots of a Quadratic equation.

Take the equation to be $ax^2 + bx + c = 0$

To find the roots, we first find Discriminant = b^2–$4ac$. The roots are:

$$x = \frac{-b\pm\sqrt{D}}{2a}$$

Multiplying numerator and denomination with -b$\mp\sqrt{D}$ gives a second form

$$x = \frac{2c}{-b\pm\sqrt{D}}$$

Now we use the value which avoids division with small numbers as it can affect the accuracy of the result.

Problem 3

We have
$f(x+h) = f(x) + f'(x)h + f''(x)\frac{h^2}{2} + f'''(x)\frac{h^3}{6} + f^{(4)}(x)\frac{h^4}{24} + f^{(5)}(x_1)\frac{h^5}{120}$

$f(x-h) = f(x) - f'(x)h + f''(x)\frac{h^2}{2} - f'''(x)\frac{h^3}{6} + f^{(4)}(x)\frac{h^4}{24} - f^{(5)}(x_2)\frac{h^5}{120}$

$f(x+2h) = f(x) + f'(x)2h + f''(x)2h^2 + f'''(x)\frac{4h^3}{3} + f^{(4)}(x)\frac{2h^4}{3} + f^{(5)}(x_3)\frac{4h^5}{15}$

$f(x-2h) = f(x) - f'(x)2h + f''(x)2h^2 - f'''(x)\frac{4h^3}{3} + f^{(4)}(x)\frac{2h^4}{3} - f^{(5)}(x_4)\frac{4h^5}{15}$

From above equations we can get:
$f(x+2h) - f(x-2h) - 2[f(x+h) - f(x-h)] = f'''(x)2h^3 + \xi h^5$
or
$\frac{1}{2h^3}\left(f(x_0+2h) - f(x_0-2h) - 2[f(x_0+h) - f(x_0-h)]\right) + kh^2 = f'''(x_0)$

Here k is a finite quantity and x_1, x_2, x_3, x_4 are some numbers between x-2h to x+2h
Hence the order of error is Quadratic.

Again, from the above four equations, we can get
$-f(x+2h) + 8[f(x+h) - f(x-h)] + f(x-2h) = 8(2hf'(x) + \frac{f'''(x)h^3}{3}) - 4hf'(x) - 8\frac{h^3}{3} + \xi h^5$
or

$$f'(x) = \frac{8(f(x+h)-f(x-h)) - (f(x+2h)-f(x-2h))}{12h}$$

Here the order of error is bi-quadratic

For getting the second derivative, we proceed as follows:

$f(x+2h) + f(x-2h) - f(x+h) - f(x-h) = 3h^2 f''(x) + \frac{5h^4 f^{(4)}x}{4}$
or

$$f''(x) = \frac{f(x+2h)+f(x-2h)-f(x+h)-f(x-h)}{3h^2}$$

The order of error is quadratic

Problem 4

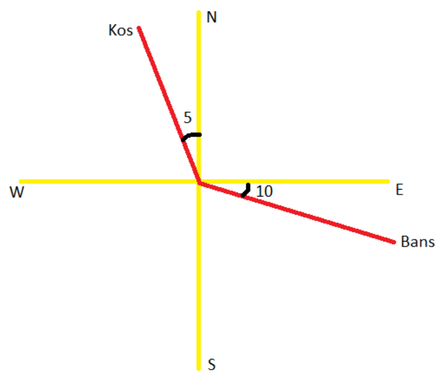

Figure 1: Valley and Standard System

We are given two systems of measurement, to transform one system to another(see figure 1), we simply take the corresponding components, keeping in mind the magnitudes.
For eg: $NorthData = 15 * Kos * cos(5°) + 0.2 * Bans * cos(100°)$
Similarly,
$EastData = 0.2 * bans * cos(pi/18) + 15 * kos * cos(19 * pi/36)$
$bans = 5 * (EastData * cos(pi/36) + NorthData * cos(5 * pi/9))$
$kos = 1/15.0 * (EastData * cos(19 * pi/36) + NorthData * cos(pi/36))$

Problem 5

From figure 2, its clear that the patter followed will be 4Ω, 2Ω, 1Ω,0.5Ω and so on for the resistance of BC, CD, DE branch(call it branch 1) and IH, HG, GF branch.
So let's assume we are in some branch where the branch 1 resistors are 2^n where $n < 3, nisaninteger$, and current value of resistance is x_k
For the next step, the resistance will $(x_k + 2^{n+1})||1$, or equivalently,

$$x_{k+1} = \frac{(x_k+2^{n+1})}{(x_k+2^{n+1})+1}$$

This is the underlying algorithm by which the Overall Resistance for different loops will be calculated

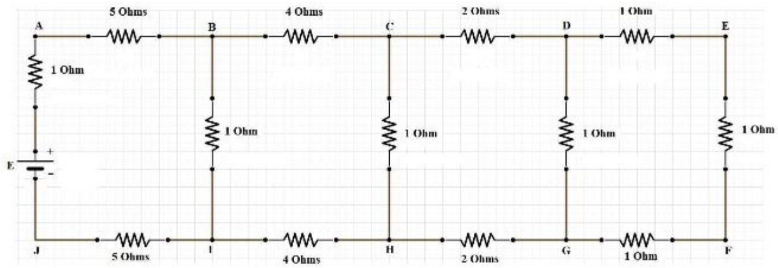

Figure 2: Electrical Circuit

Problem 6

Given System to solve: Ax = b, where A and b are known and x is to be determined.

Methodology used: $A^T Ax = A^T b$

Now, $A^T A$ is made stiff using a parameter μ, which can be made small depending on accuracy we need.

So the rigged system is:

$$(A^T A + \mu^2 I)x = A^T b$$

Let $M = (A^T A + \mu^2 I)$. Since $A^T A$ is symmetric and at least Positive Semi-Definite, therefore introducing μ will make the system positive definite.

So, we can find the LU decomposition of M. Since, M is symmetric positive definite, we can find L and U such that $L = U^T$ using Cholesky Decomposition, i.e. $M = LL^T$

So, the system now becomes: $LL^T x = A^T b$

Put $y = L^T x$ to get Ly = b. This can be solved for y easily as L is a lower triangular matrix. After finding y we find x using y = $L^T x$, which is also easily solvable as L^T is an upper triangular matrix.