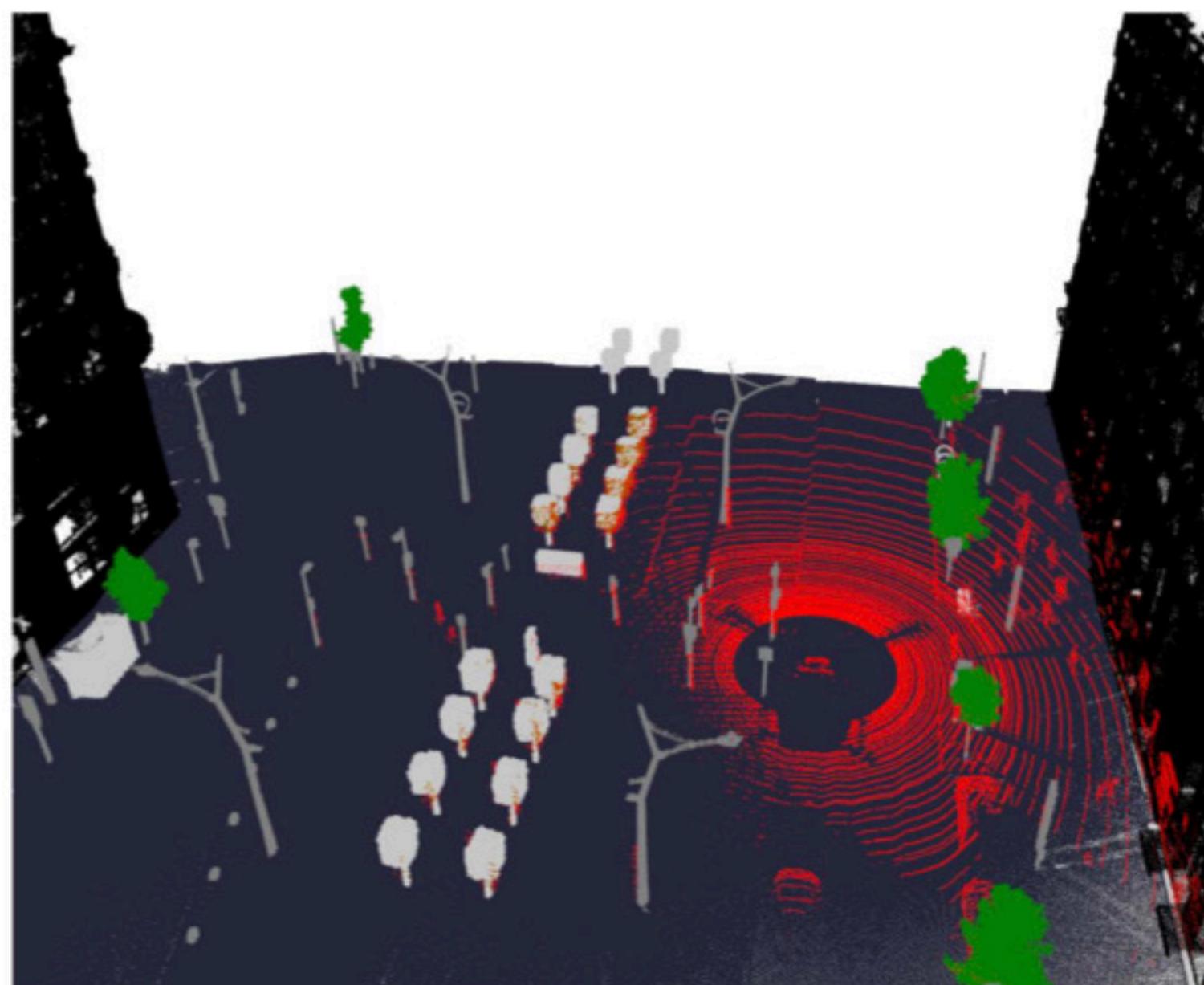


nerf2nerf: Pairwise Registration of Neural Radiance Fields

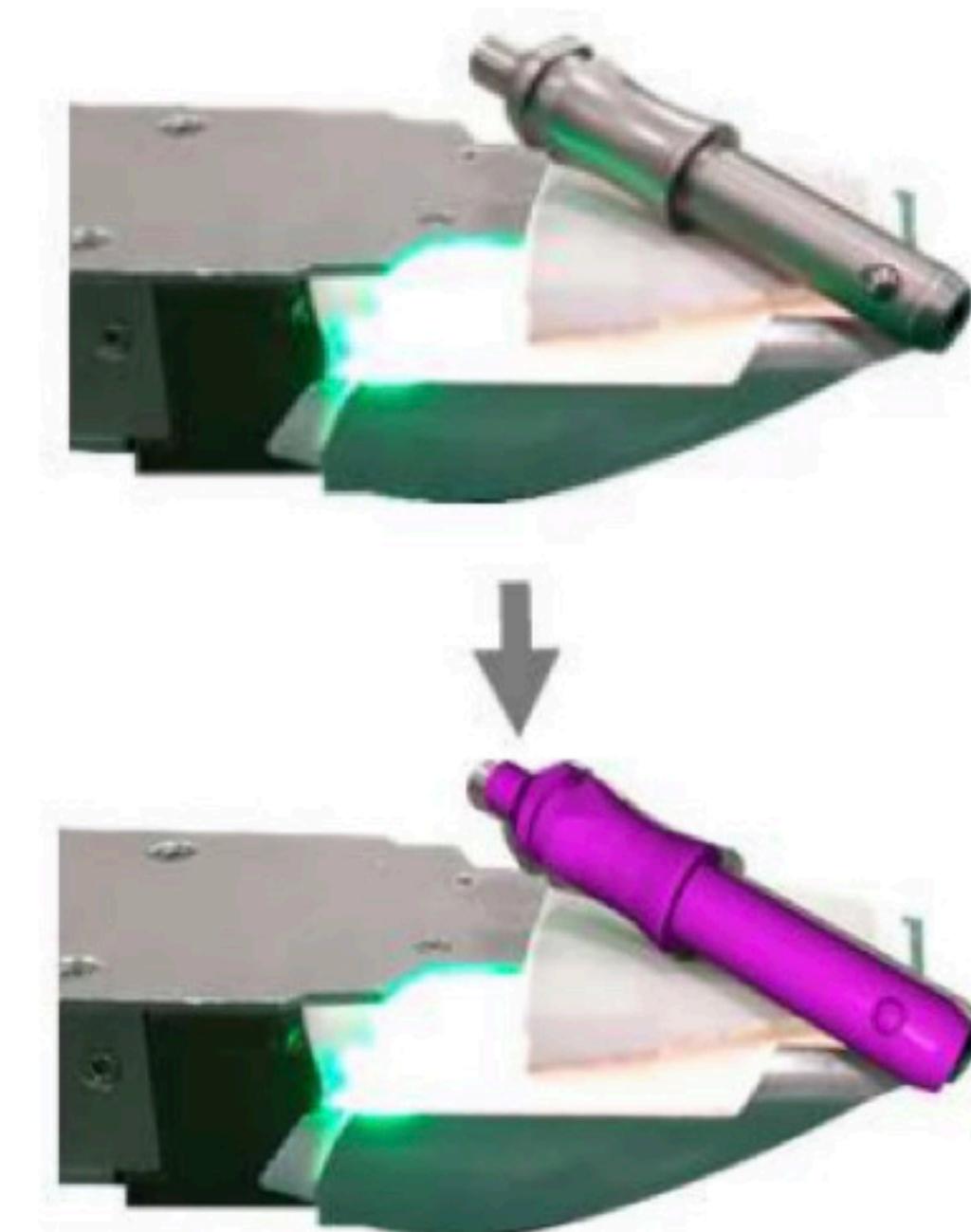
Lily Goli^{1,2}, Daniel Rebain⁴, Sara Sabour^{1,2,6}, Animesh Garg^{1,2,5}, Andrea Tagliasacchi^{1,3,6}

¹University of Toronto, ²Vector Institute, ³SFU, ⁴UBC, ⁵NVIDIA, ⁶Google Research

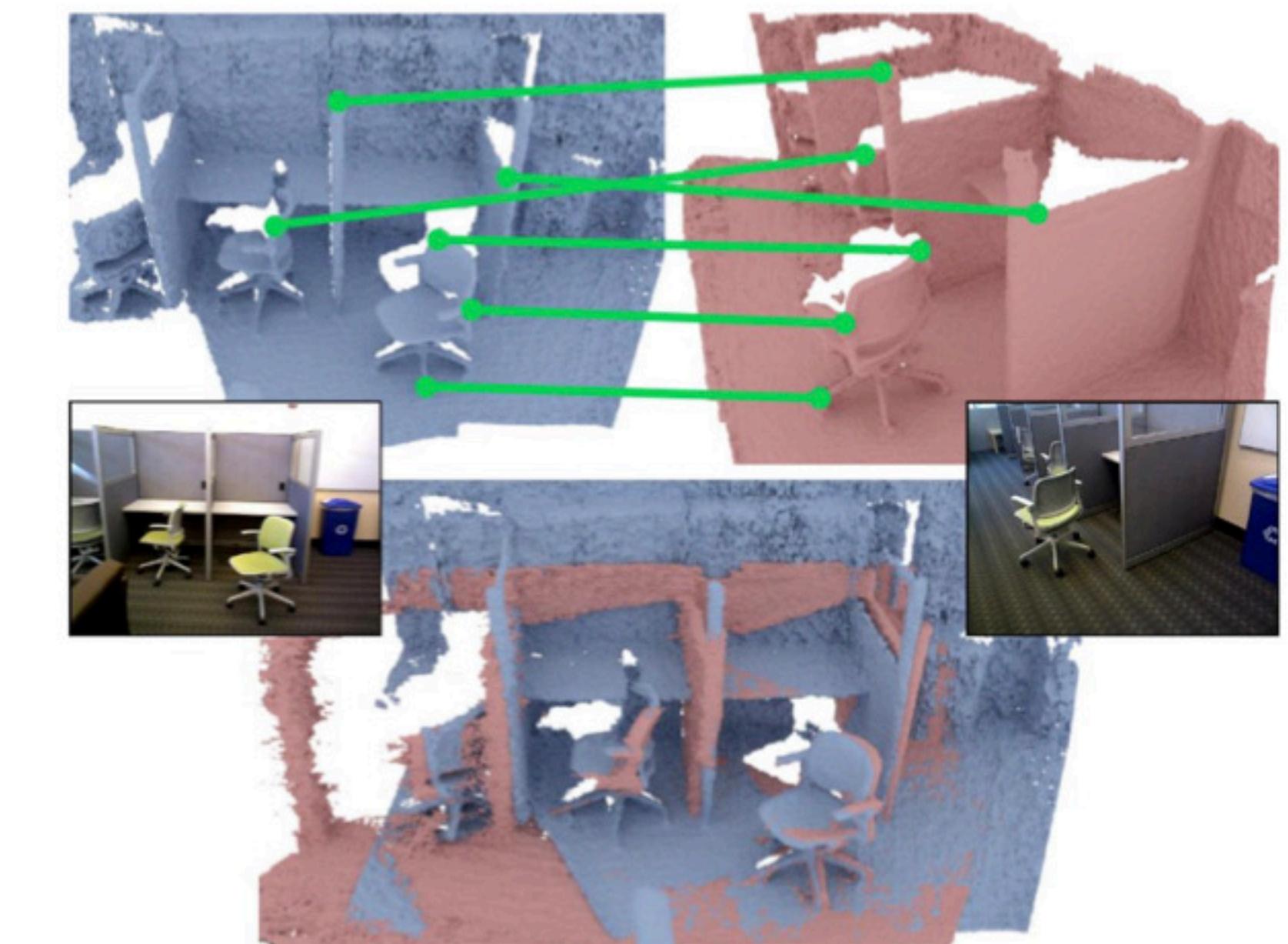
Registration Applications



Nagy et al. ECCV'18
“localization”

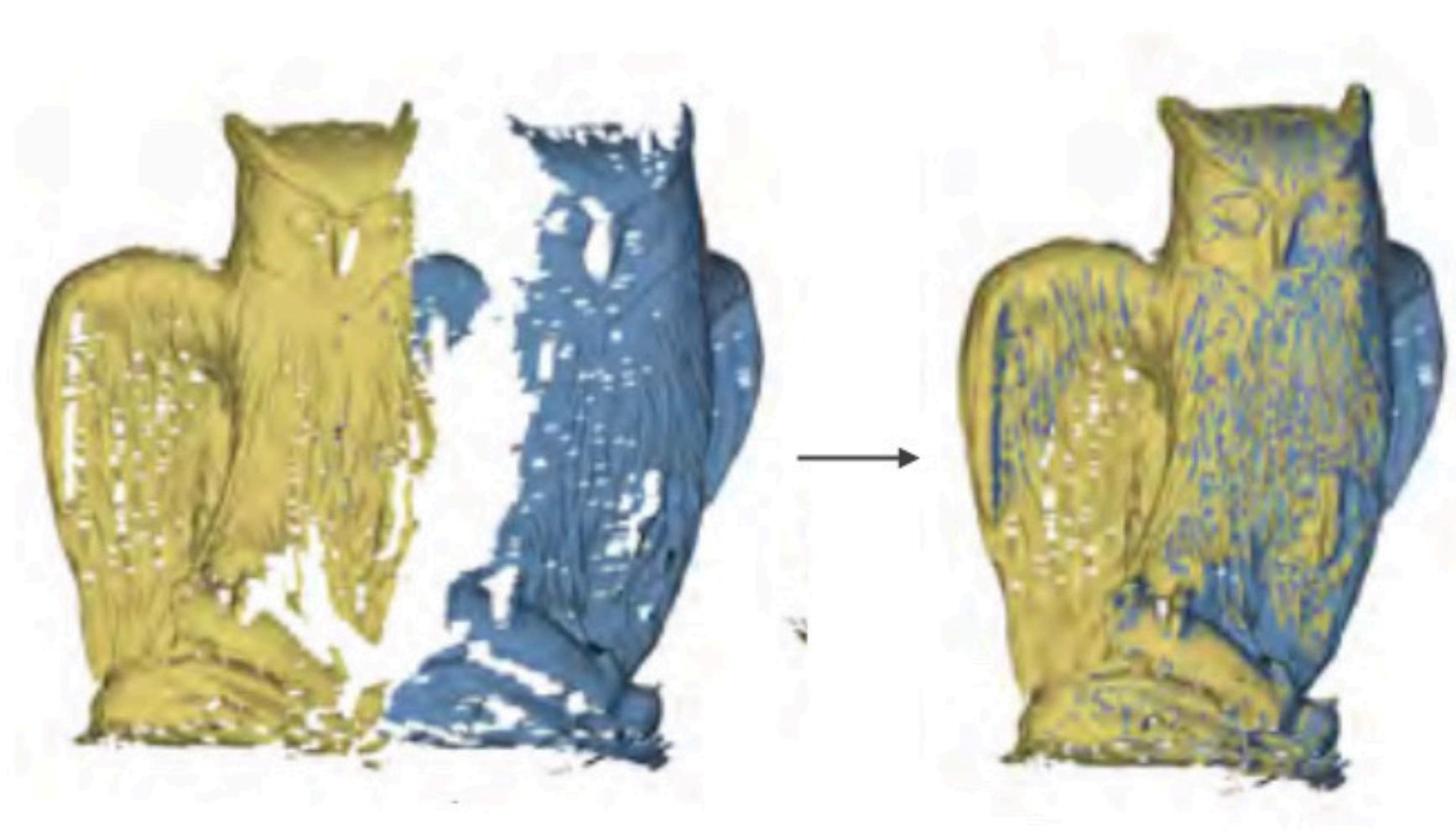


Bauzá et al. CoRL'20
“Pose Estimation”



Zeng et al. CVPR'17
“Large Scene Reconstruction”

Classic 3D Registration



Bouaziz et al. SGP'13
Sparse Iterative Closest Point
(Sparse ICP)



Zhou et al. ECCV'16
Fast Global Registration (FGR)

Neural Radiance Field (NeRF)

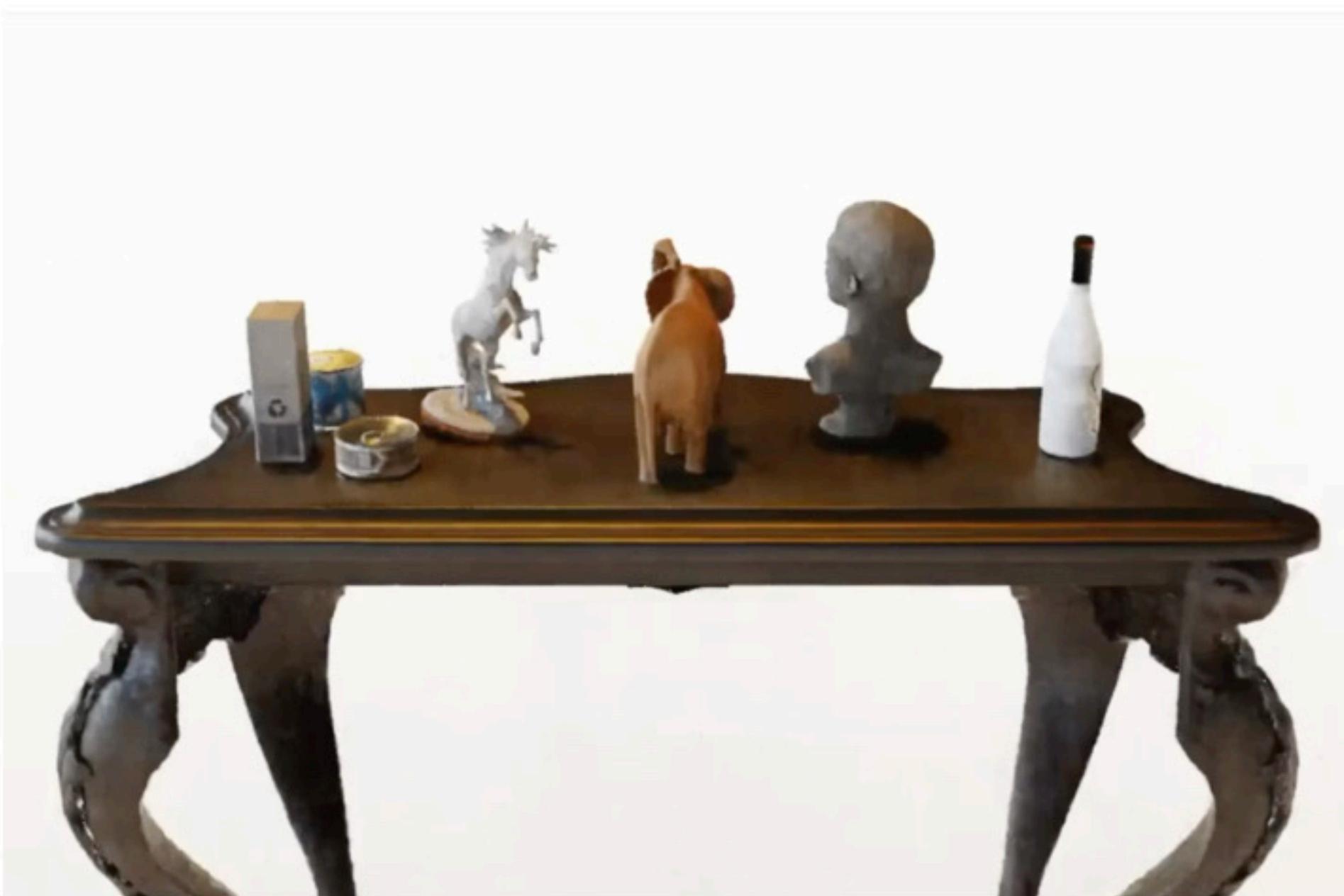


Mildenhall, Srinivasan, Tanick et al. ECCV'20
Neural Radiance Field (NeRF)

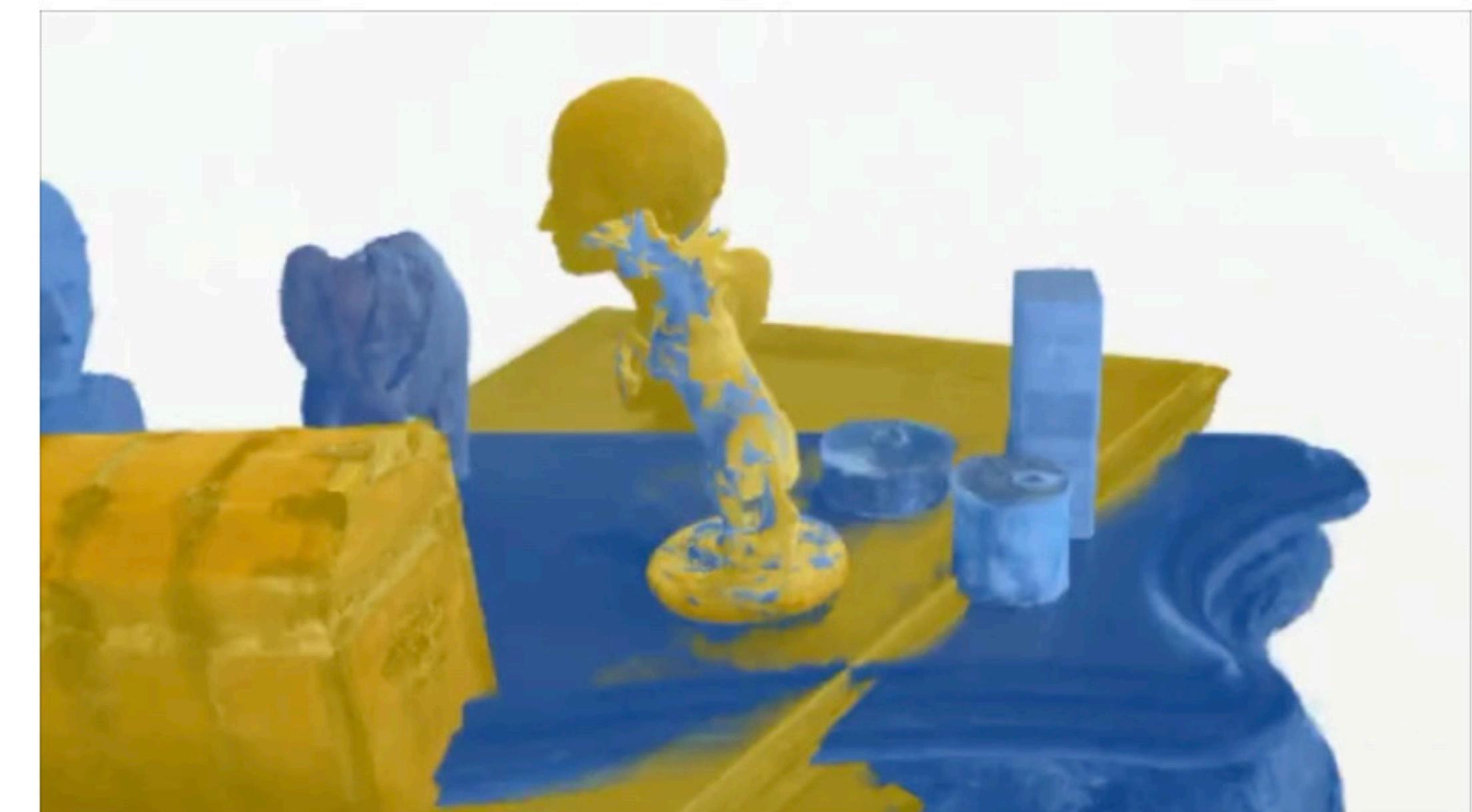
Yen-Chen et al. IROS'21
Inverting Neural Radiance Fields for Pose Estimation

nerf2nerf

Scene A



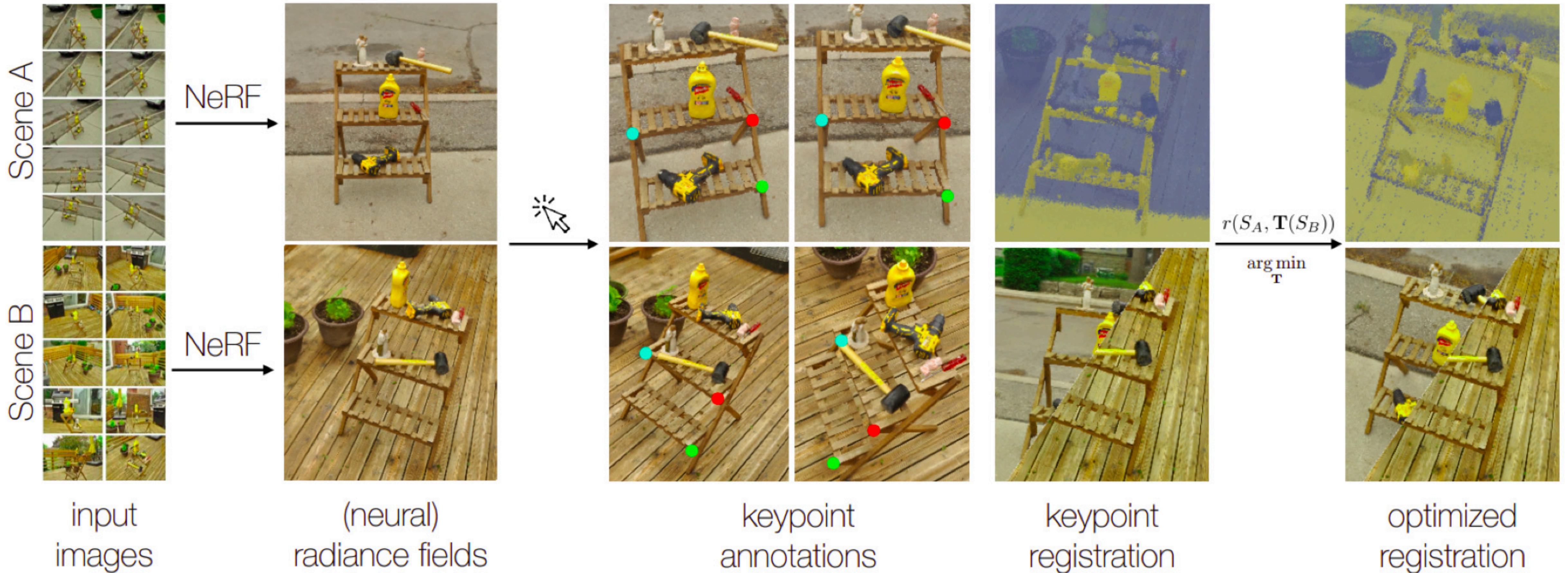
Scene B



nerf2nerf

to register horse figurine

Problem Setting



Talk Outline

How to extract a geometric representation from NeRF that can be used for the registration task?

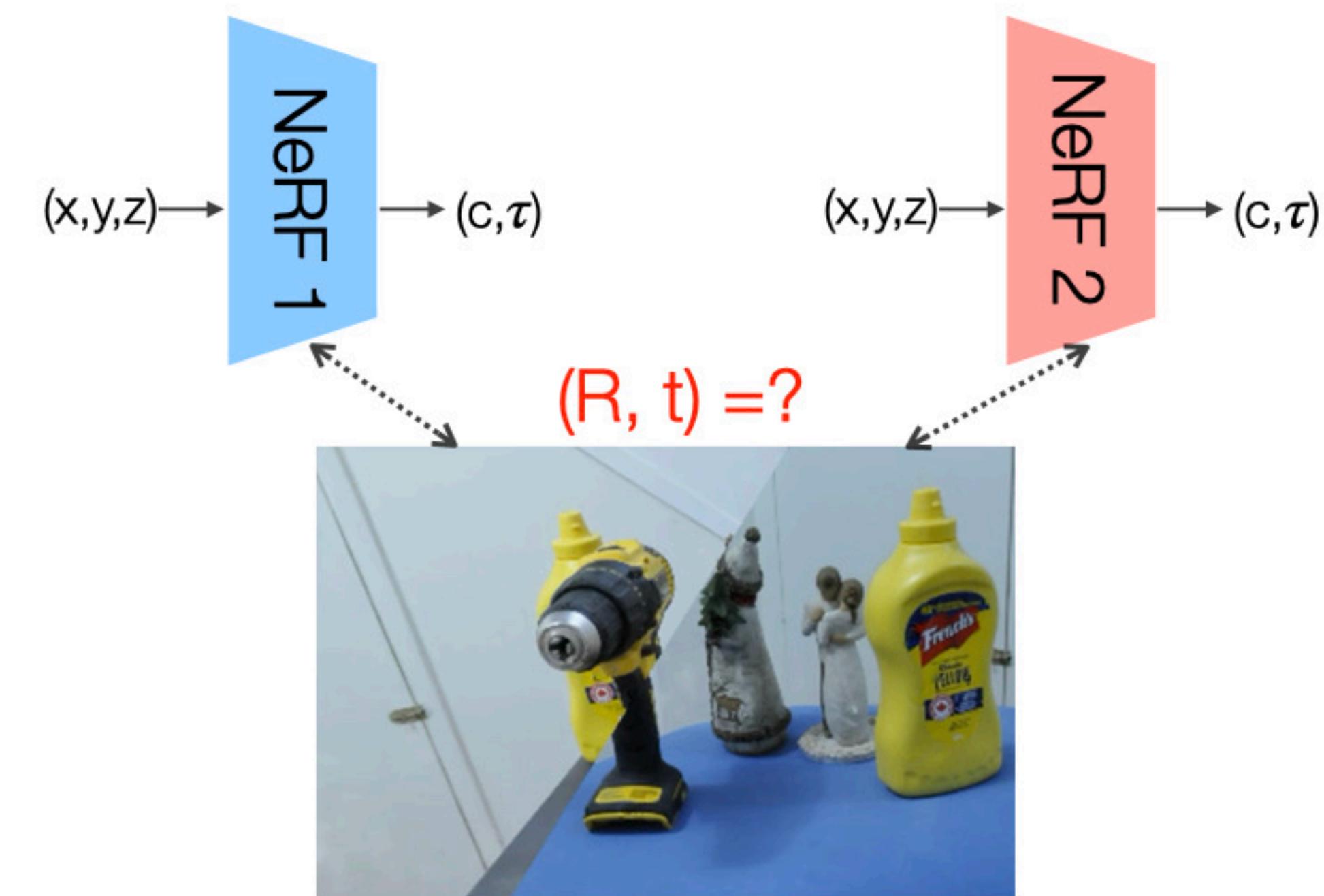
- Should accurately capture object geometry
- Should be invariant to scene illumination
- Should be invariant to view direction



Talk Outline

How to register two neural fields?

- work on Eulerian representation
- robust to partial overlap

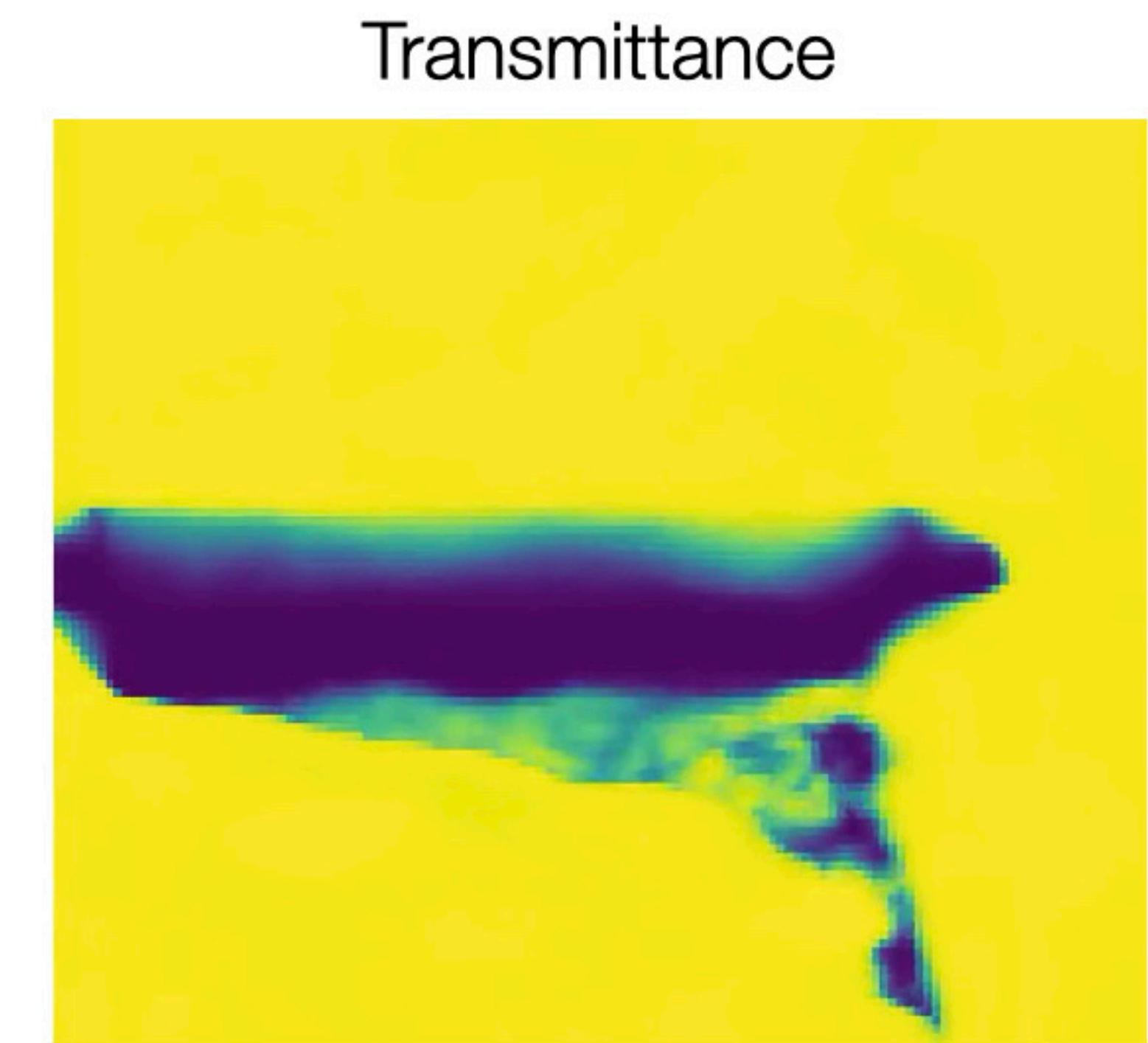
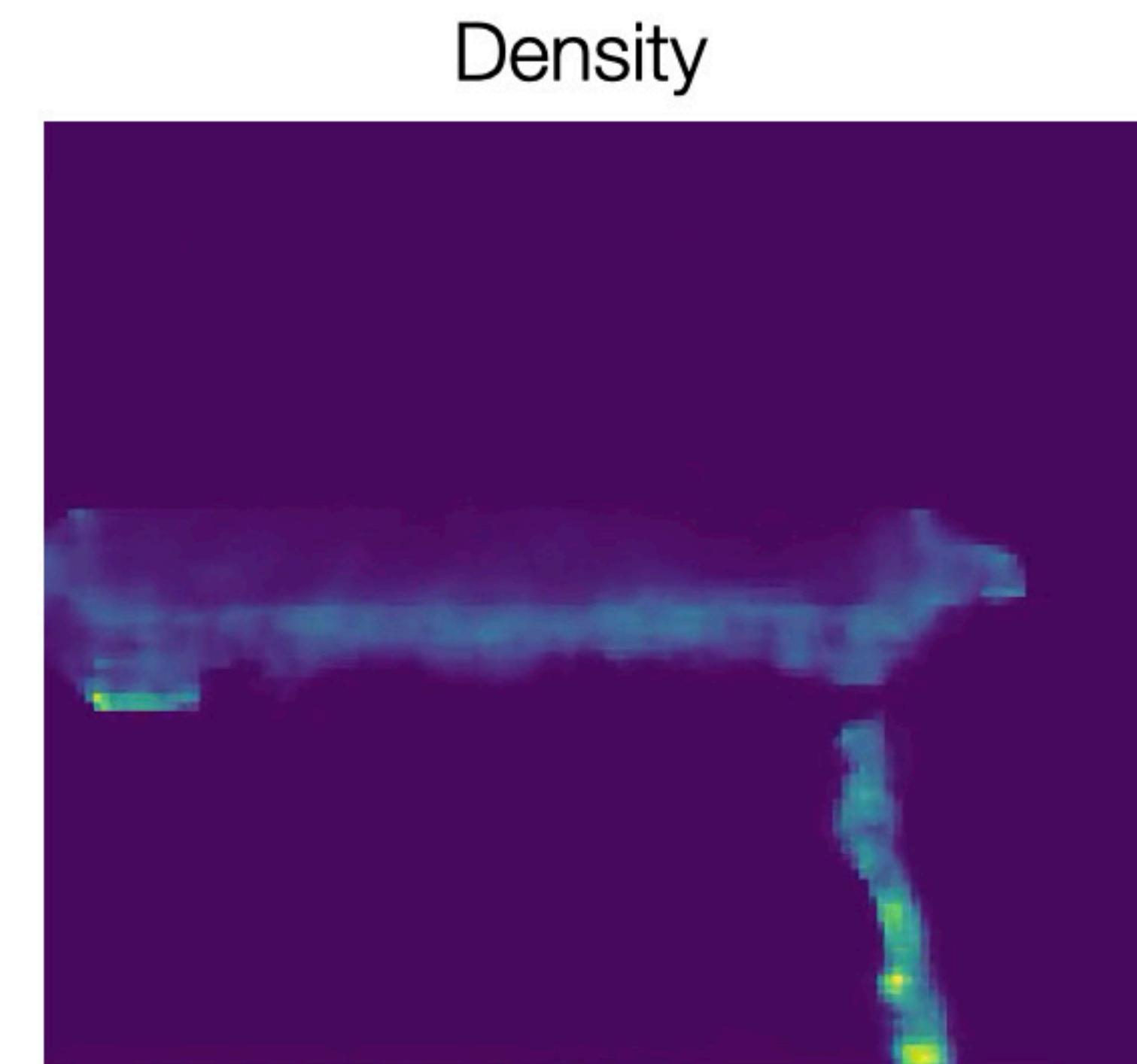
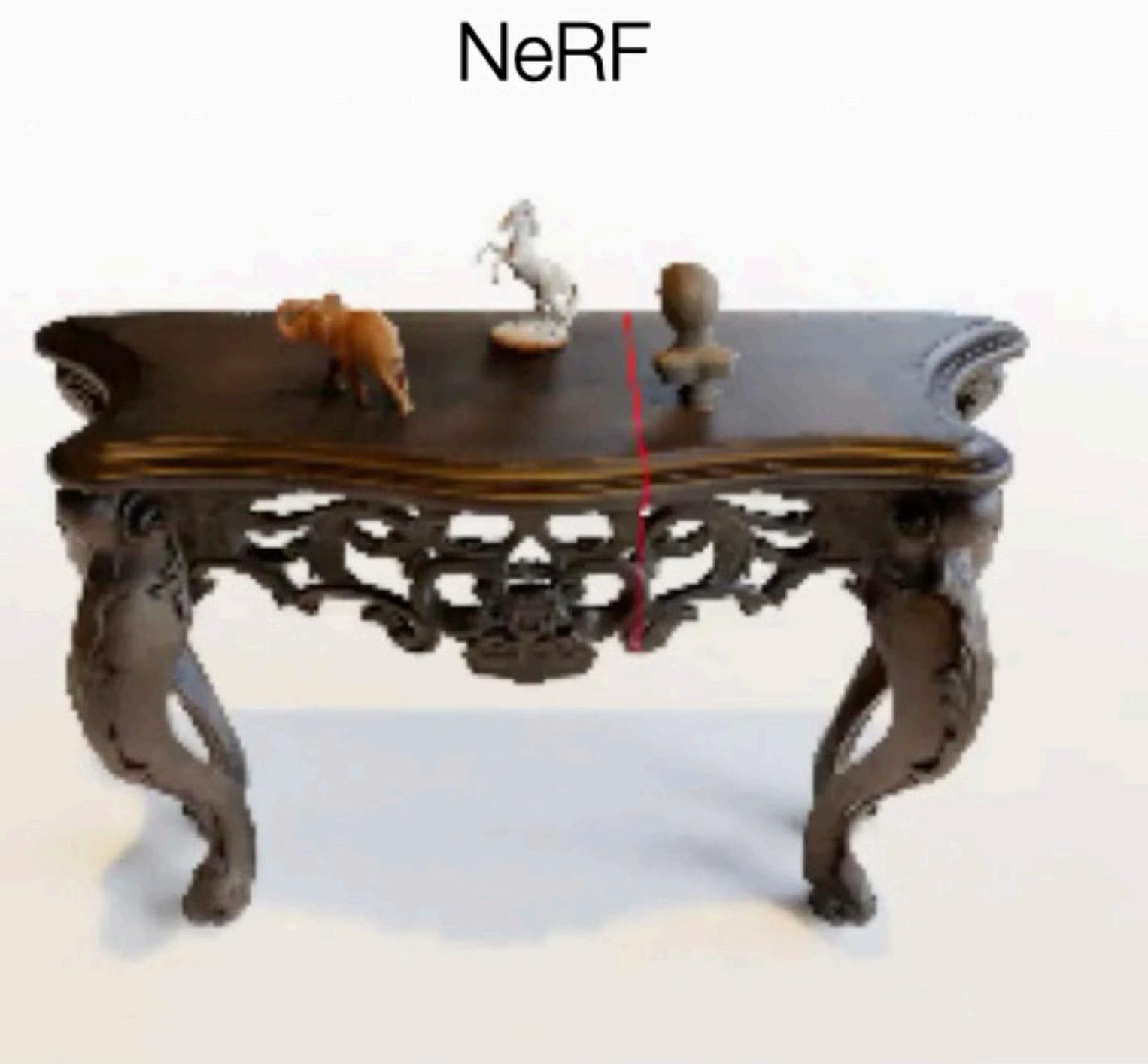


Surface Fields

Two products of NeRF:

Density $\tau(s)$: differential probability of hitting a solid particle at a point s .

Transmittance $\mathcal{T}(0 \rightarrow s | r)$: the probability that ray r hits no solid particles on its way to point s



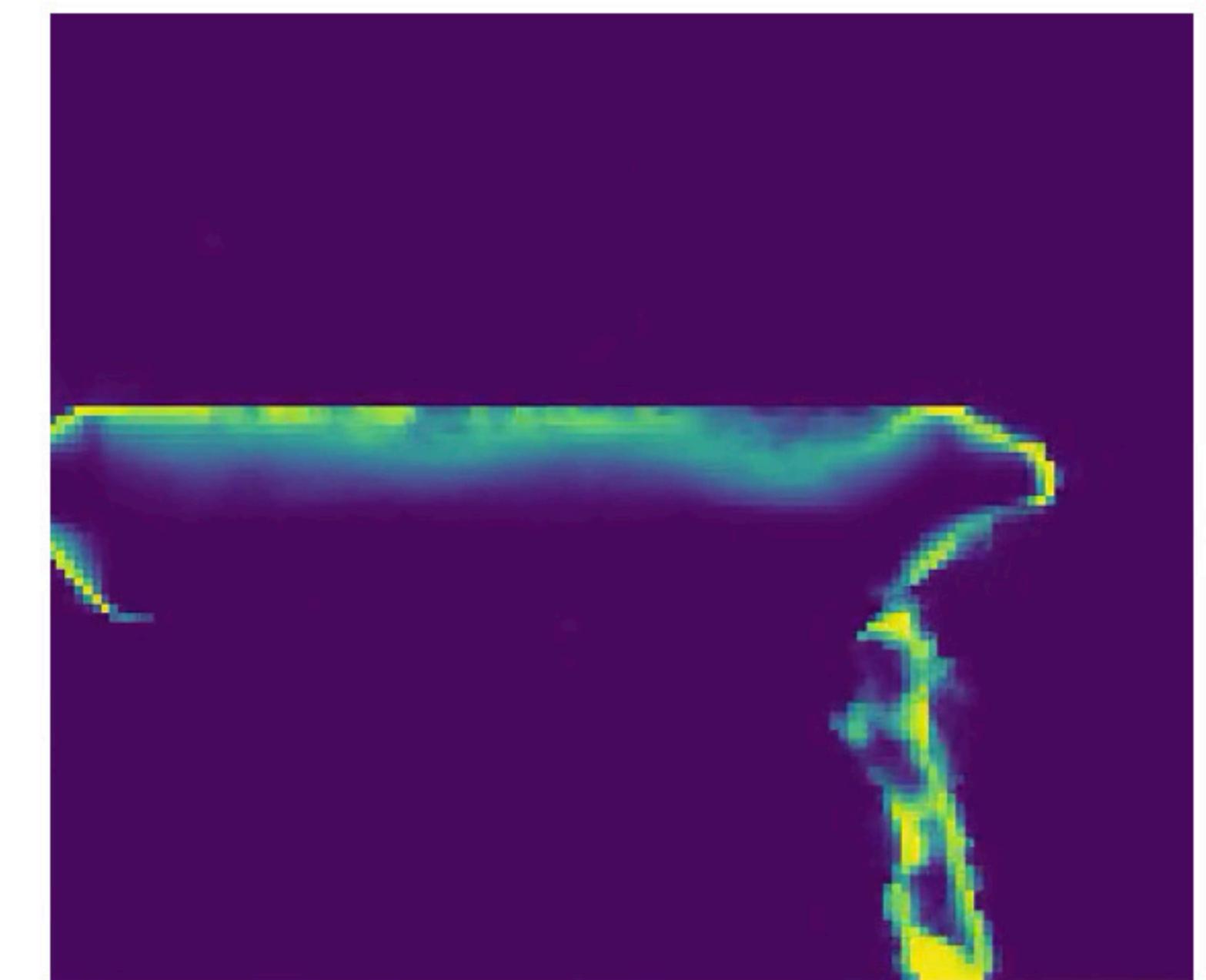
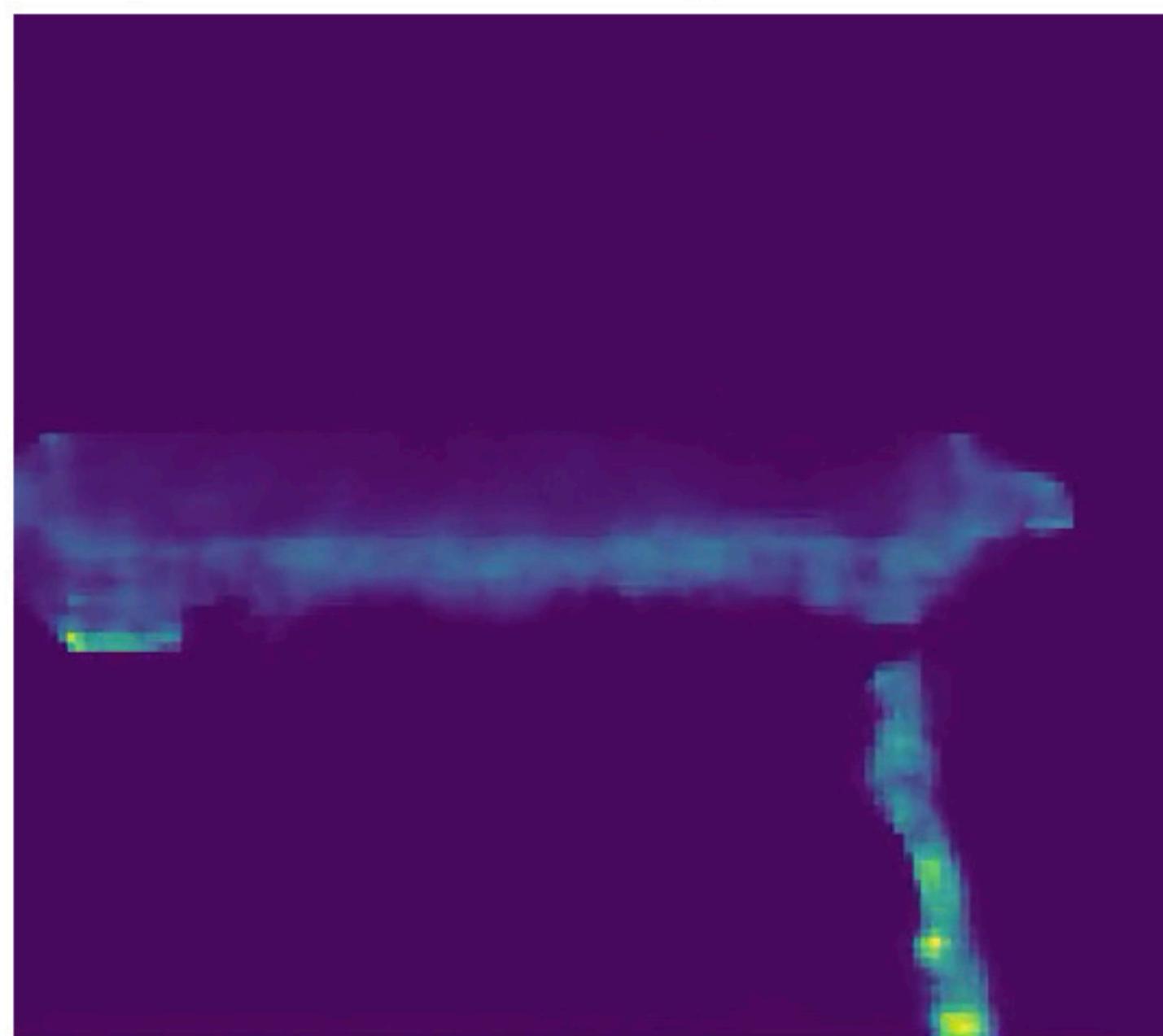
Surface Fields

Density $\tau(s)$
Transmittance $\mathcal{T}(0 \rightarrow s | r)$

} Surface

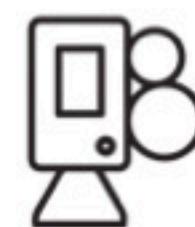
\rightarrow

$$\mathcal{S}(s | r) = \int_{s-\delta}^{s+\delta} \mathcal{T}(0 \rightarrow t | r) \cdot \tau(t | r) dt$$

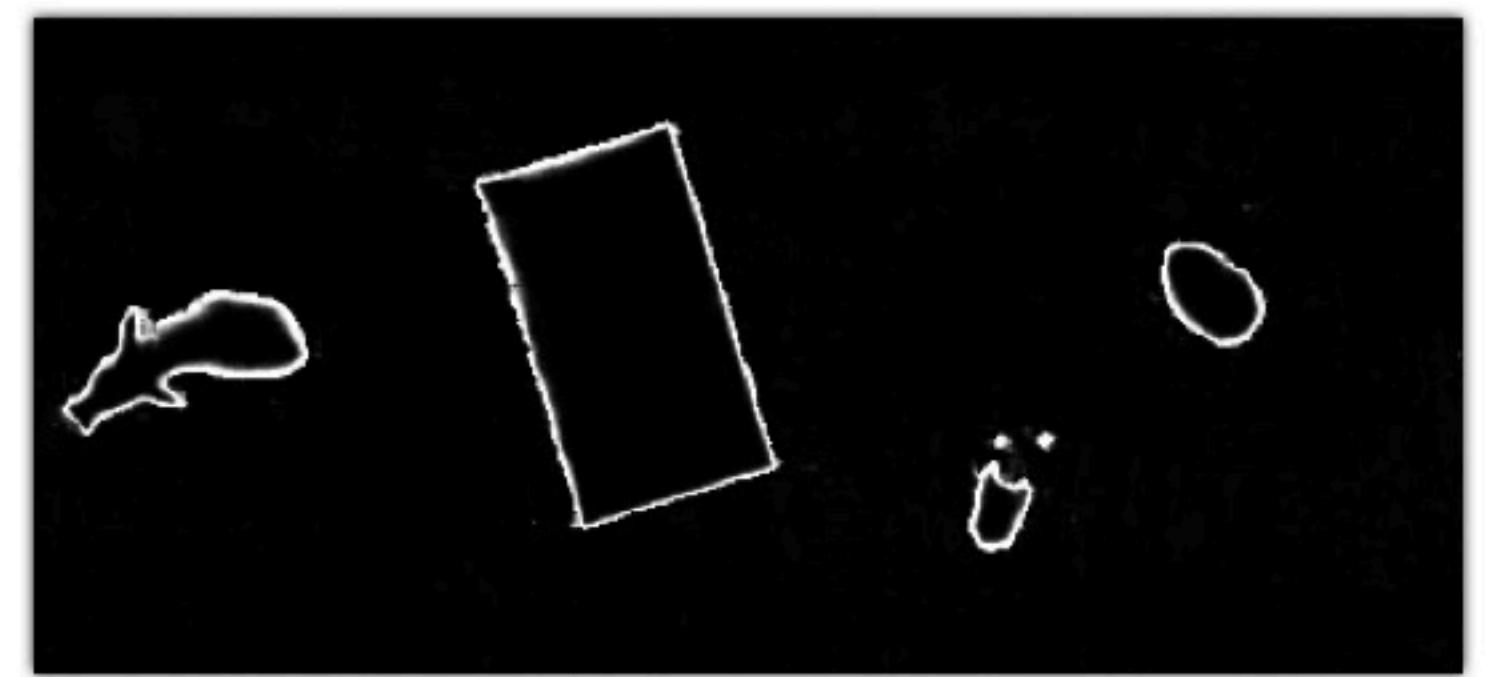
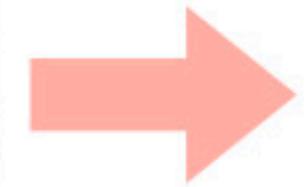
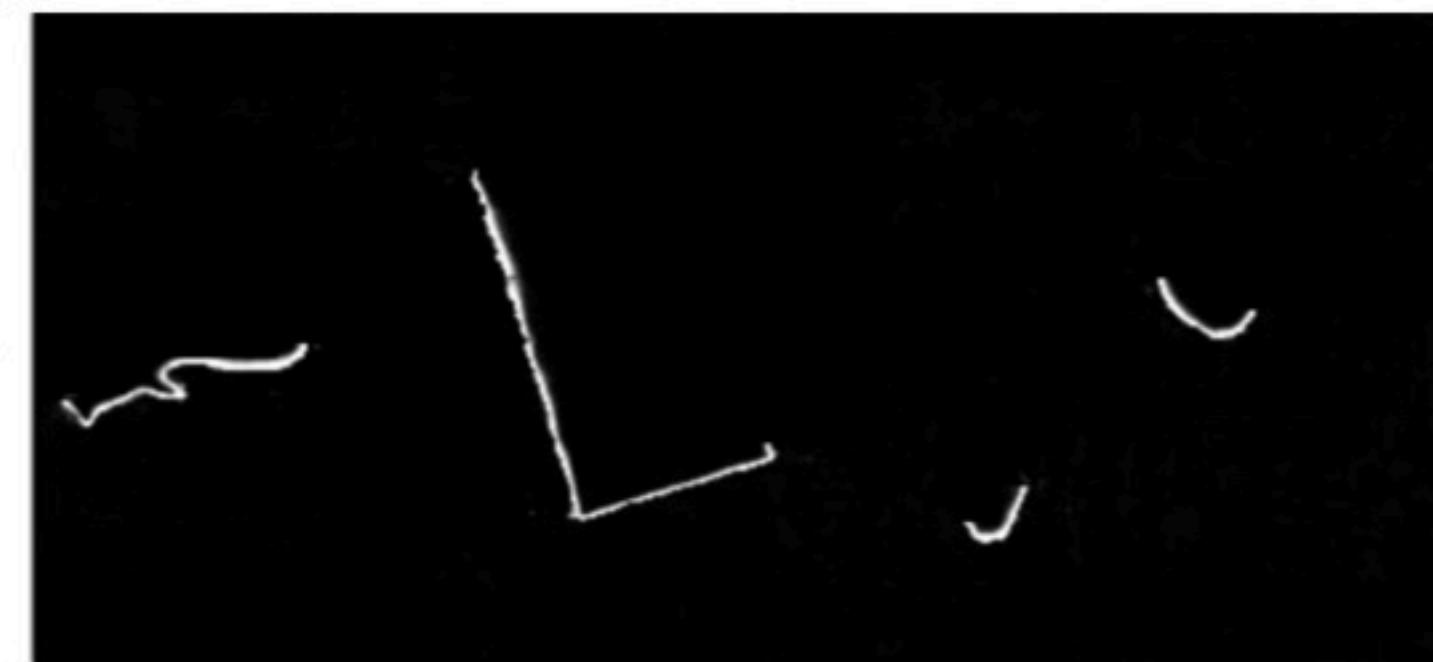
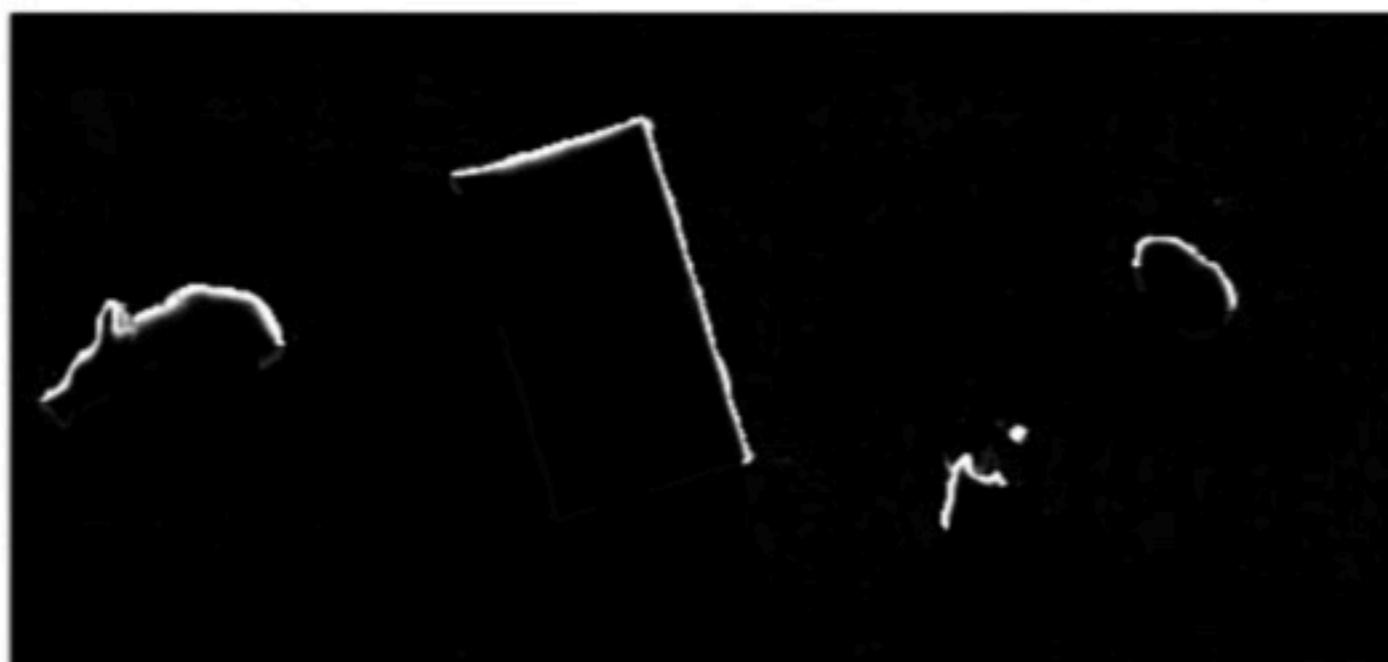


Surface Fields

$$\left. \begin{array}{l} \text{Density } \tau(s) \\ \text{Transmittance } \mathcal{T}(0 \rightarrow s | r) \end{array} \right\} \xrightarrow{\text{Surface}} \mathcal{S}(s | r) = \int_{s-\delta}^{s+\delta} \mathcal{T}(0 \rightarrow t | r) \cdot \tau(t | r) dt$$



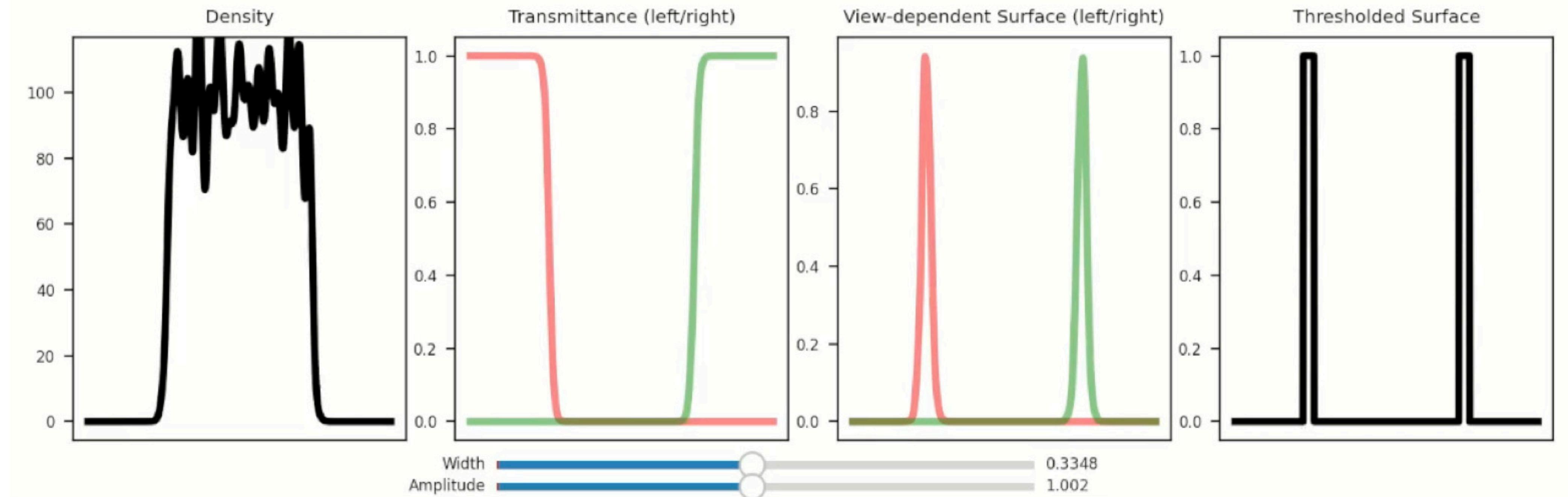
$$\mathcal{S}(s) = \max_{\mathbf{o} \in \mathcal{O}} \mathcal{S}\left(\|\mathbf{o} - s\| \mid \left(\mathbf{o}, \frac{\mathbf{o} - s}{\|\mathbf{o} - s\|}\right)\right) \in [0, 1]$$



Surface Field

To have a conservative estimate of the field we can threshold it:

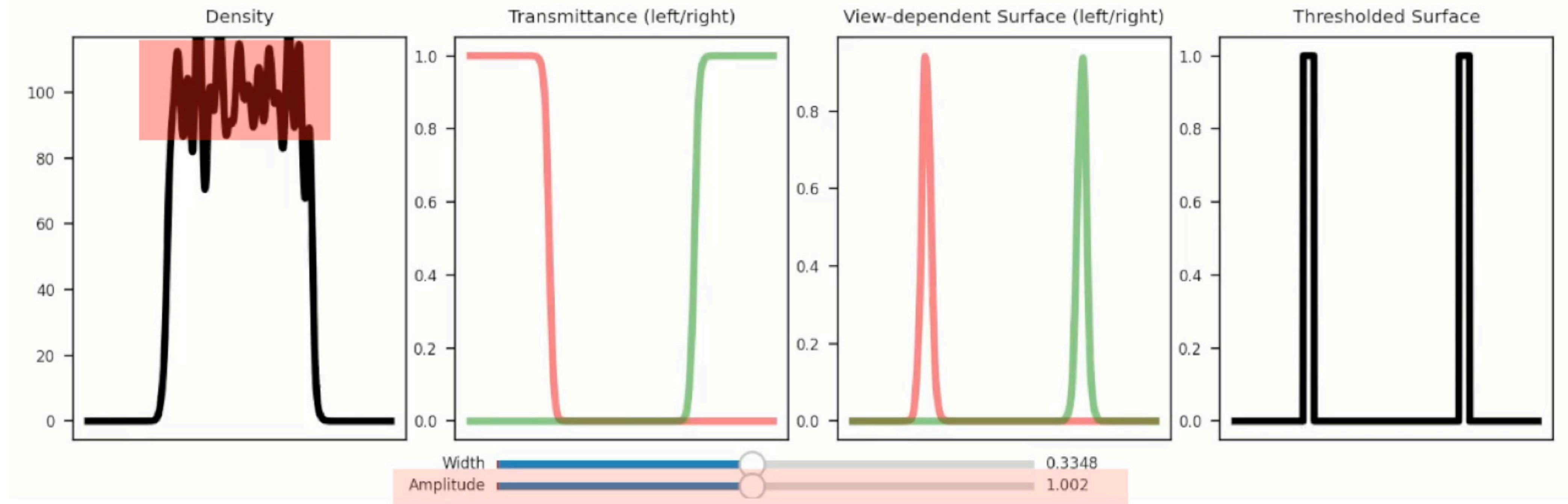
$$\mathcal{S}^\epsilon(\mathbf{x}) = \mathbf{1}(\mathcal{S}(\mathbf{x}) > \epsilon) \in \{0, 1\}$$



Surface Field

To have a conservative estimate of the field we can threshold it:

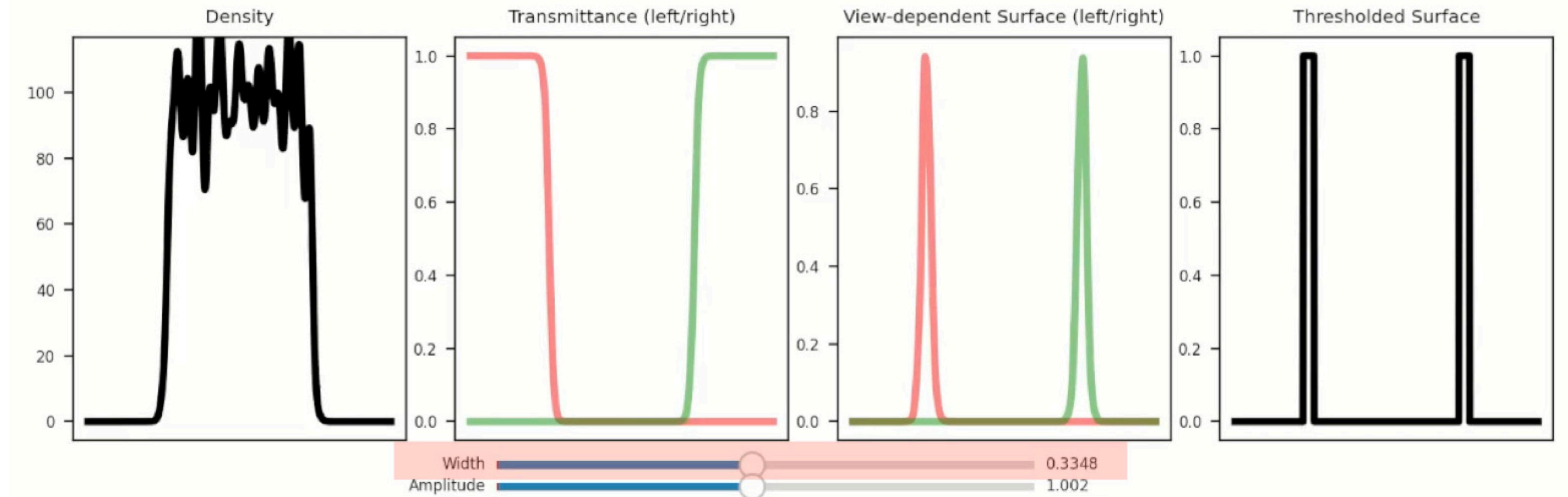
$$\mathcal{S}^\epsilon(\mathbf{x}) = \mathbb{1}(\mathcal{S}(\mathbf{x}) > \epsilon) \in \{0, 1\}$$



Surface Field

To have a conservative estimate of the field we can threshold it:

$$\mathcal{S}^\epsilon(\mathbf{x}) = \mathbb{1}(\mathcal{S}(\mathbf{x}) > \epsilon) \in \{0, 1\}$$



Distilling Surface Fields

The thresholded surface has a categorical co-domain and is not suitable for differentiation.

We can smooth it with a Gaussian convolution:

$$\mathcal{S}^\sigma(\mathbf{x}) = \mathcal{S}^\epsilon(\mathbf{x}) \circledast \mathcal{N}(0, \sigma^2) \approx \mathbb{E}_{\mathbf{z} \sim \mathcal{N}(\mathbf{x}, \sigma^2)} [\mathcal{S}^\epsilon(\mathbf{z})]$$

To have faster query time on surface field we can distill it into another network.

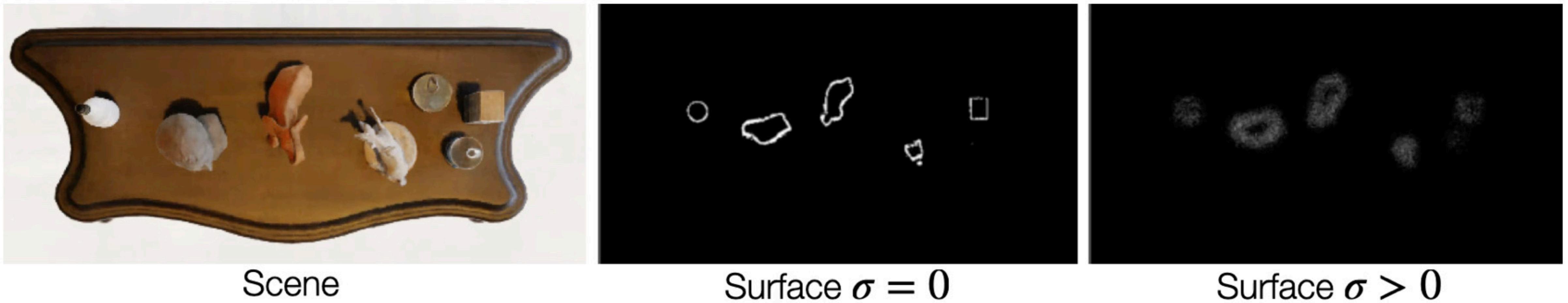
We use integrated positional encoding¹ to encode different levels of smoothness:

$$\gamma_\sigma(\mathbf{x}) = \mathbb{E}_{\mathbf{z} \in \mathcal{N}(\mathbf{x}, \sigma^2)} [\gamma(\mathbf{z})] = \left\{ \left(\frac{\sin(2^l \mathbf{z})}{\sqrt{\exp(4^l \sigma^2)}}, \frac{\cos(2^l \mathbf{z})}{\sqrt{\exp(4^l \sigma^2)}} \right) \right\}_{l=0}^L$$

$$\arg \min_{\theta} \mathbb{E}_{\mathbf{x} \in \mathcal{B}(\mathbf{0}, r)} [\mathcal{L}_{\text{poisson}}(\mathcal{S}^\sigma(\gamma_\sigma(\mathbf{x}); \theta)), \mathcal{S}^\sigma(\mathbf{x})]]$$
$$\mathcal{L}_{\text{poisson}}(x, y) = x - y \log x$$

1. Barron et al “Mip-nerf: A multiscale representation for anti-aliasing neural radiance fields,” in ICCV, 2021.

Distilling Surface Field



Scene

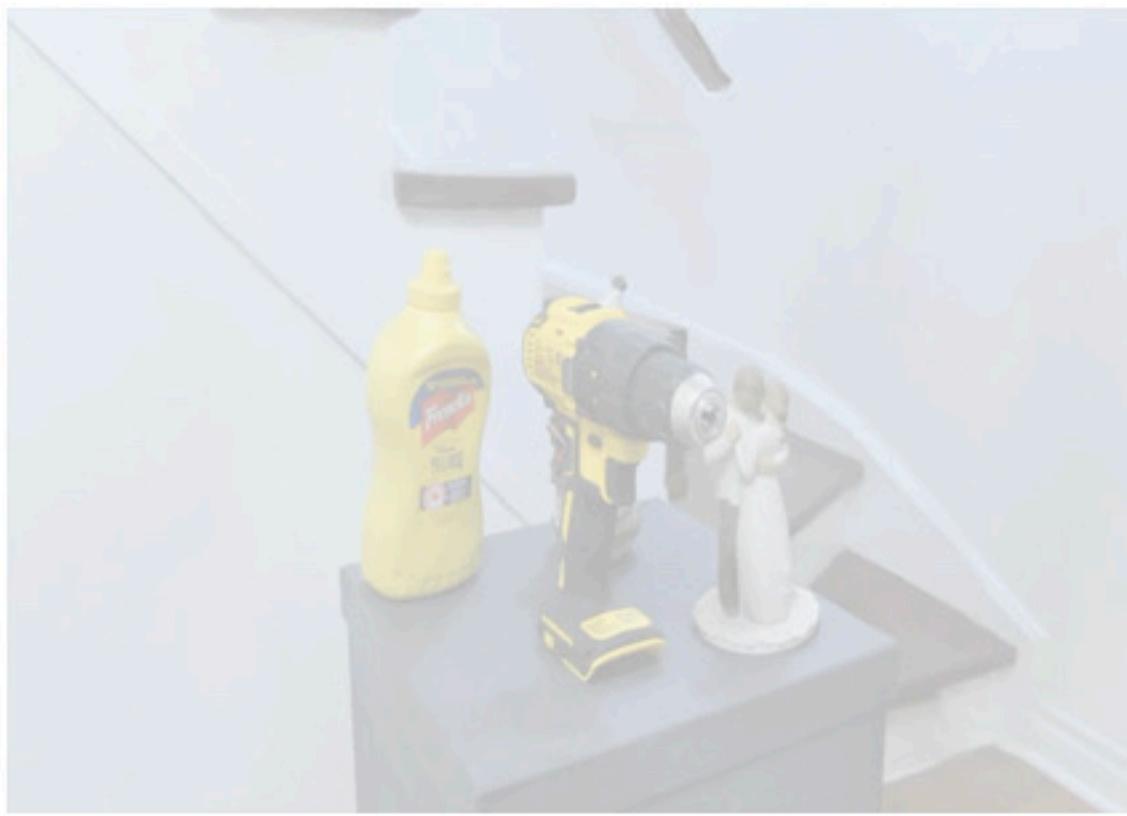
Surface $\sigma = 0$

Surface $\sigma > 0$

Talk Outline

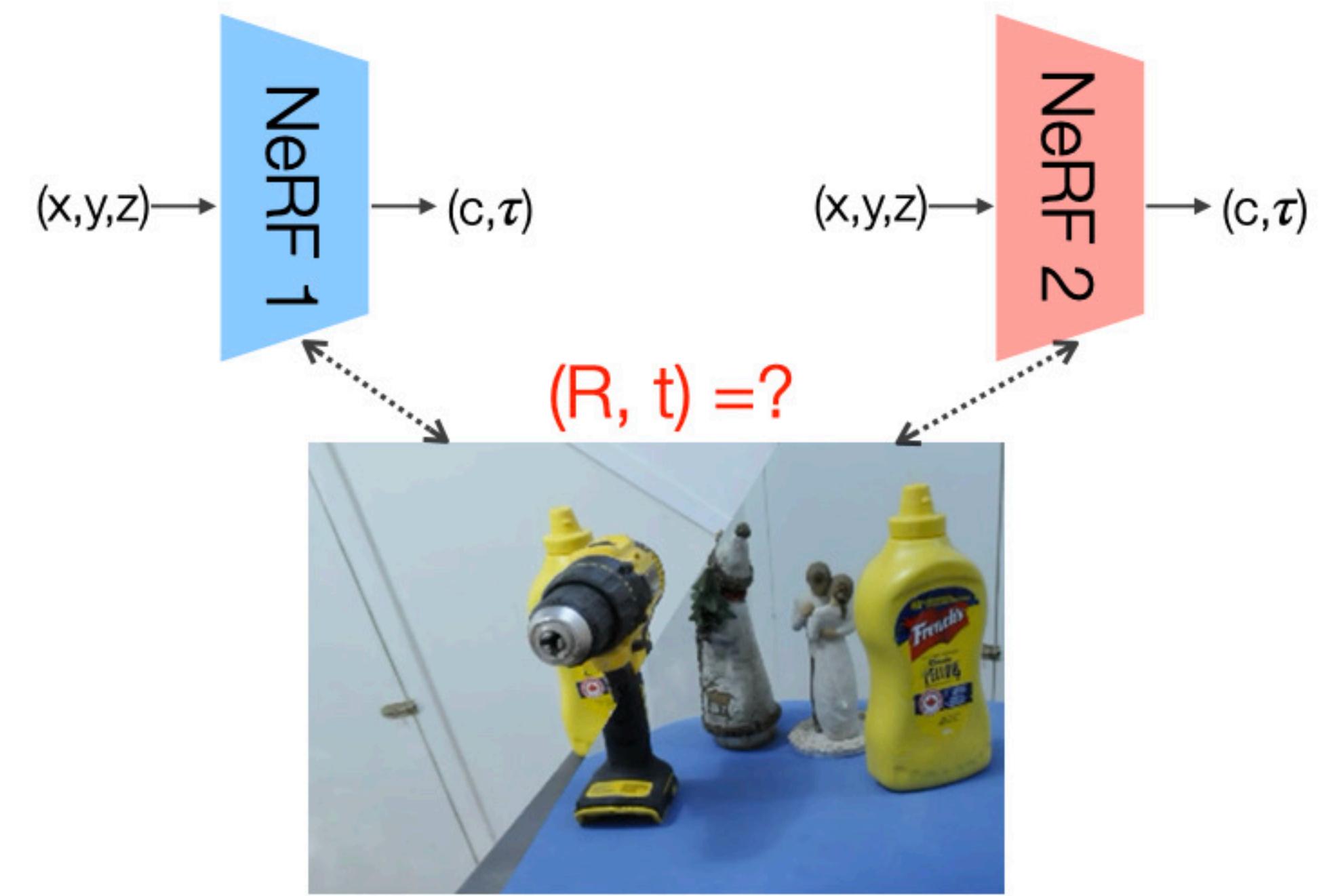
How to extract a geometric representation from NeRF that can be used for the registration task?

- Should be invariant to scene illumination
- Should be invariant to view direction
- Should accurately capture object geometry



How to register two neural fields?

- robust to partial overlap **Optimization**
- work on Eulerian representation **Sampling**



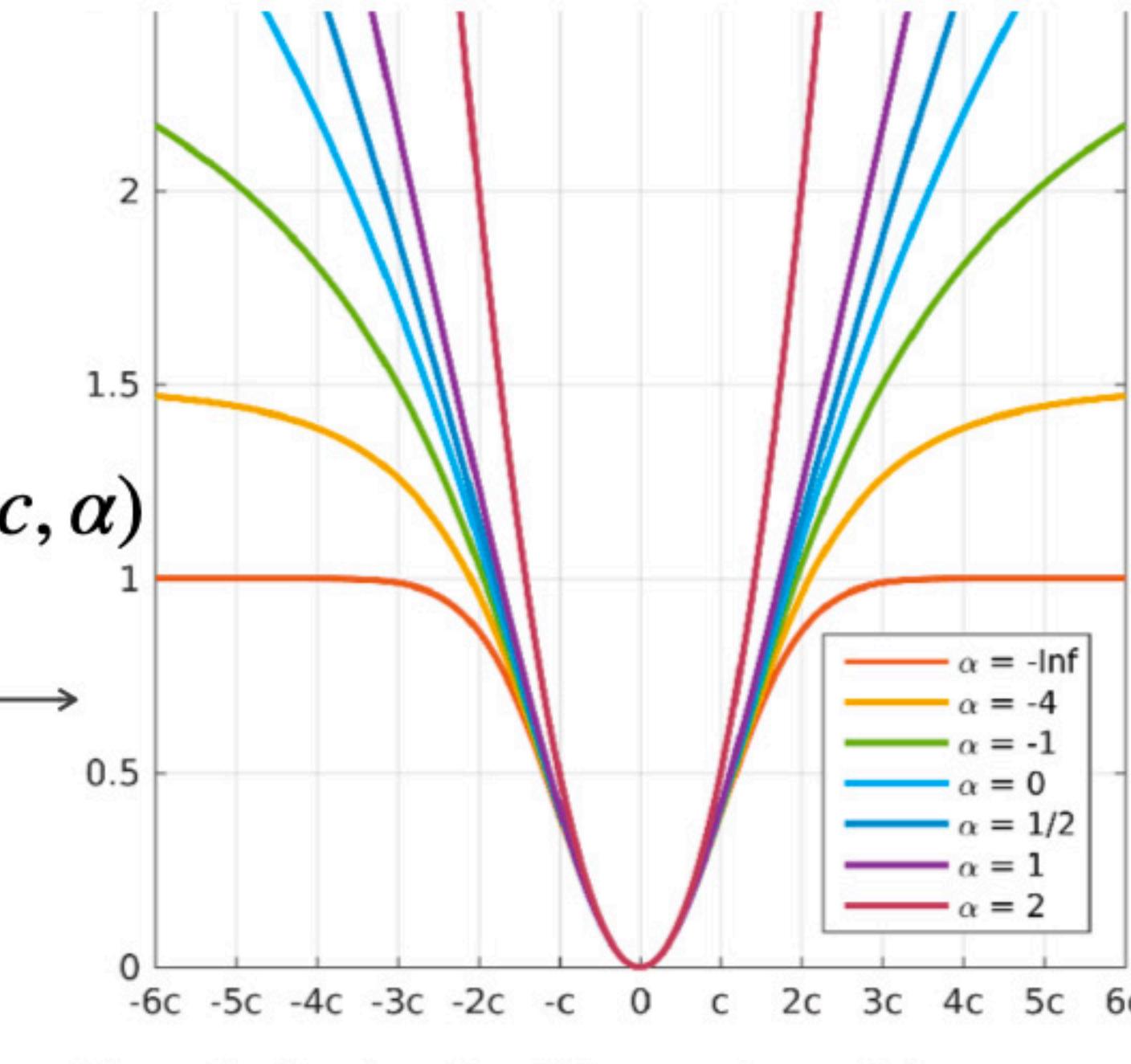
Energy-based Optimization

$$\operatorname{argmin}_{R,t} \mathcal{L}_{match} + \lambda \mathcal{L}_{key}$$

$$r(x; \mathcal{S}_a^\sigma, \mathcal{S}_b^\sigma, R, t) = \| \mathcal{S}_a^\sigma(x) - \mathcal{S}_b^\sigma(Rx + t) \|_2$$

$$\mathcal{L}_{match}(\mathcal{S}_a^\sigma, \mathcal{S}_b^\sigma, R, t) = \mathbb{E}_{x \in \mathcal{A}} k(\| \mathcal{S}_a^\sigma(x) - \mathcal{S}_b^\sigma(Rx + t) \|_2; c, \alpha)$$

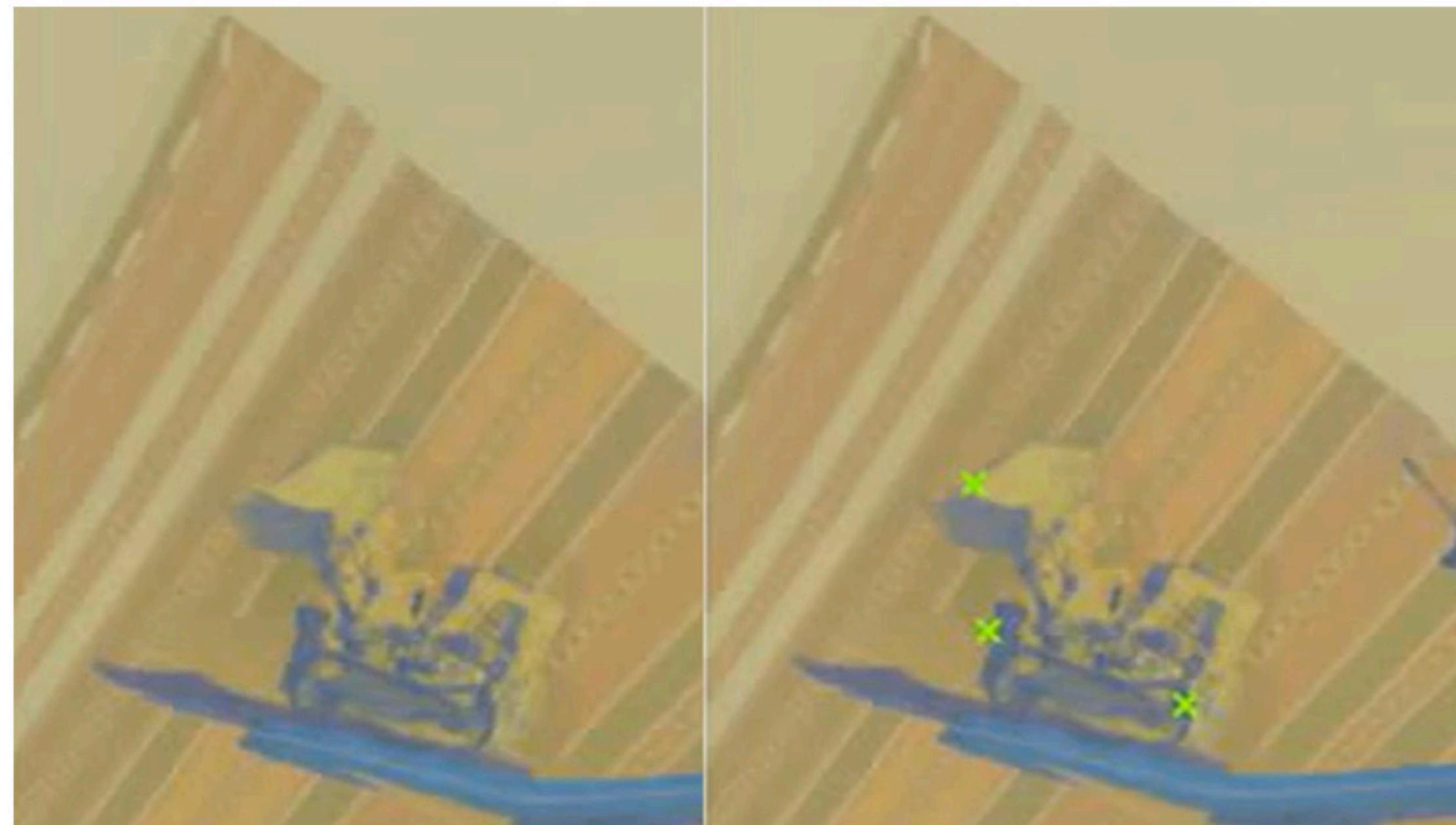
$$\mathcal{L}_{key}(R, t; \mathcal{Q}_a, \mathcal{Q}_b) = \sum_{q_a, q_b} \| q_a - (Rq_b + t) \|_2^2$$



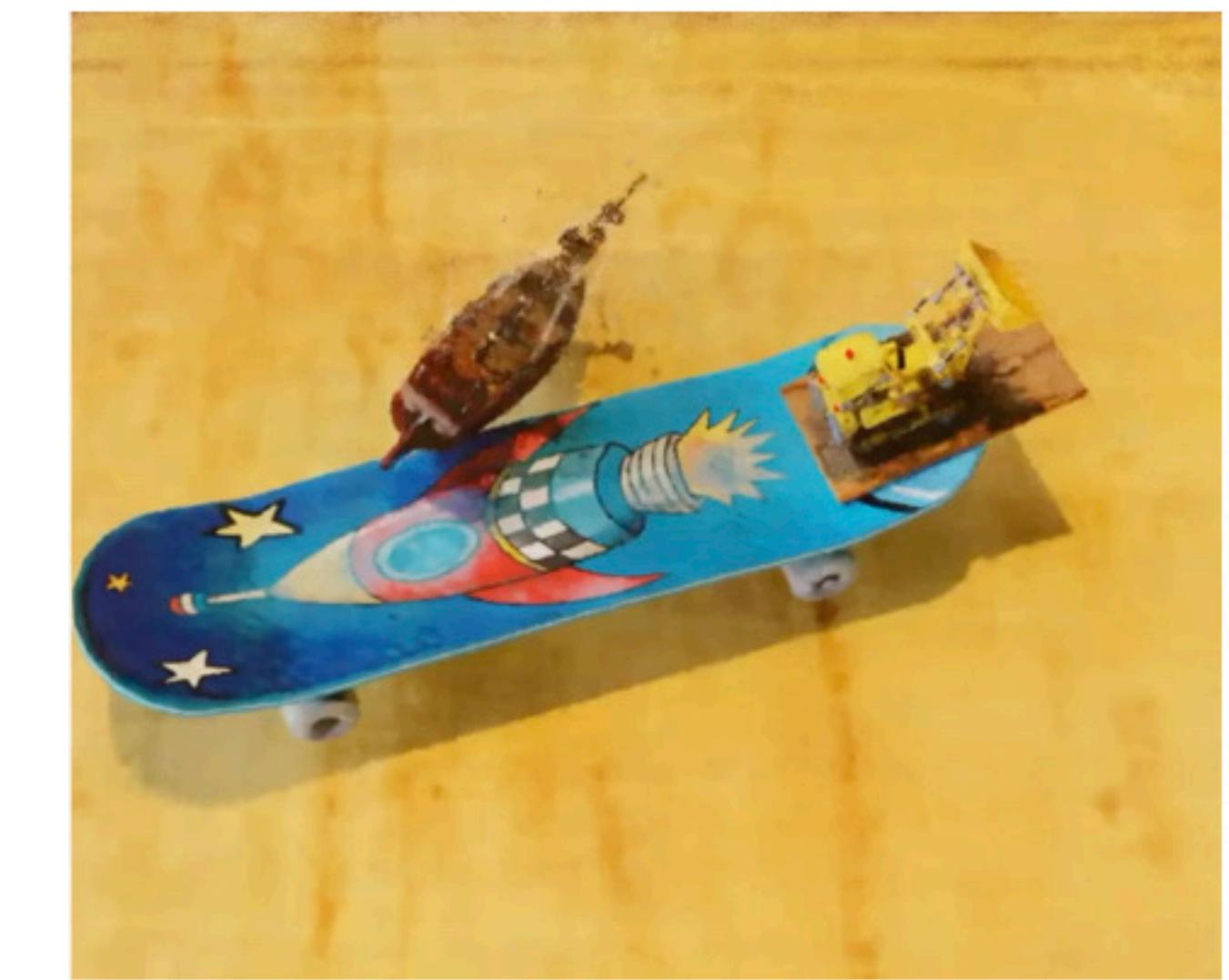
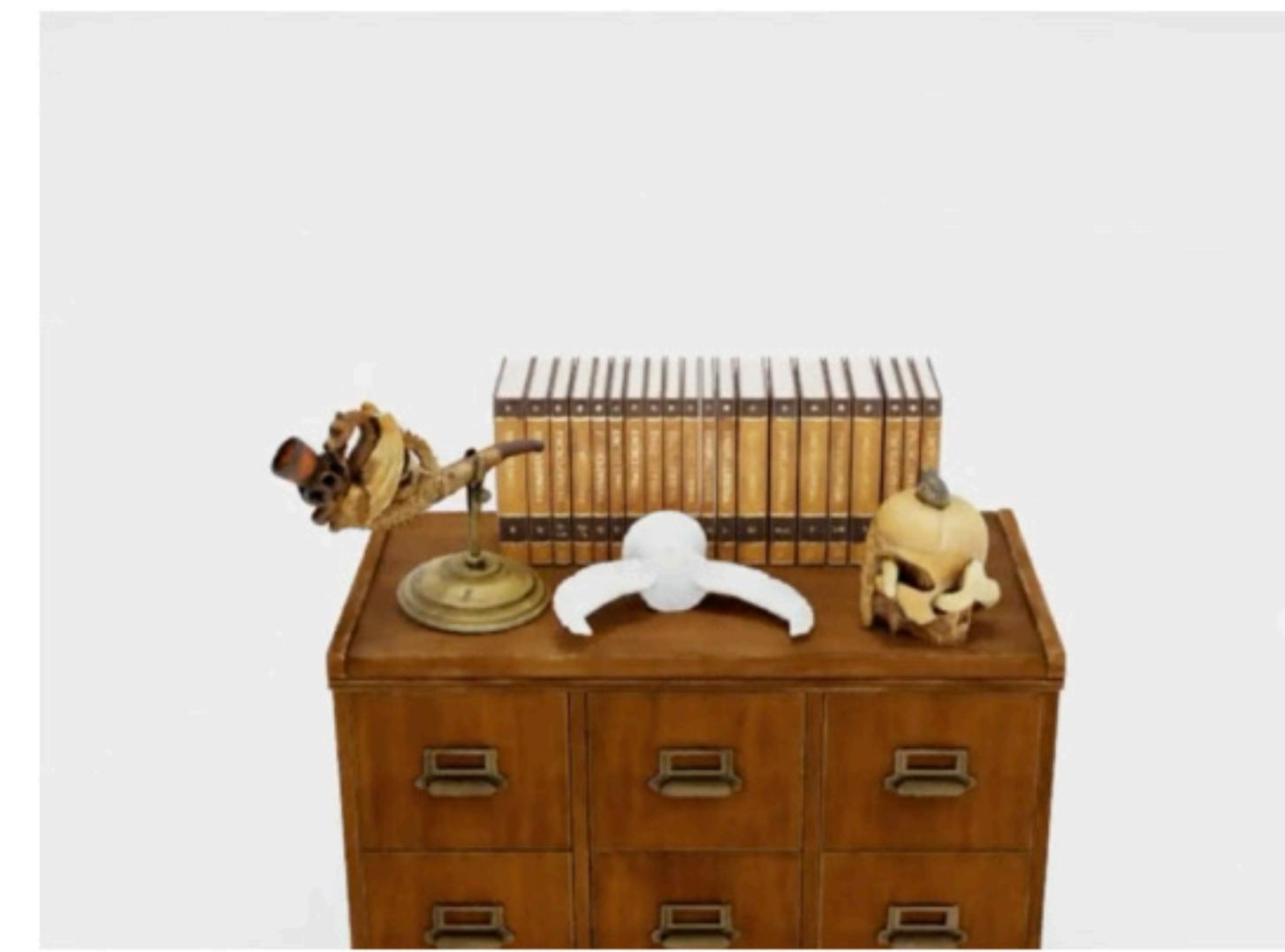
Sampling

Metropolis-Hastings Sampling: to sample points near key-point locations that

- are near surface
- are far enough and informative
- are in correspondence according to residual



Synthetic Dataset



Scene 1

Scene 2

Scene 3

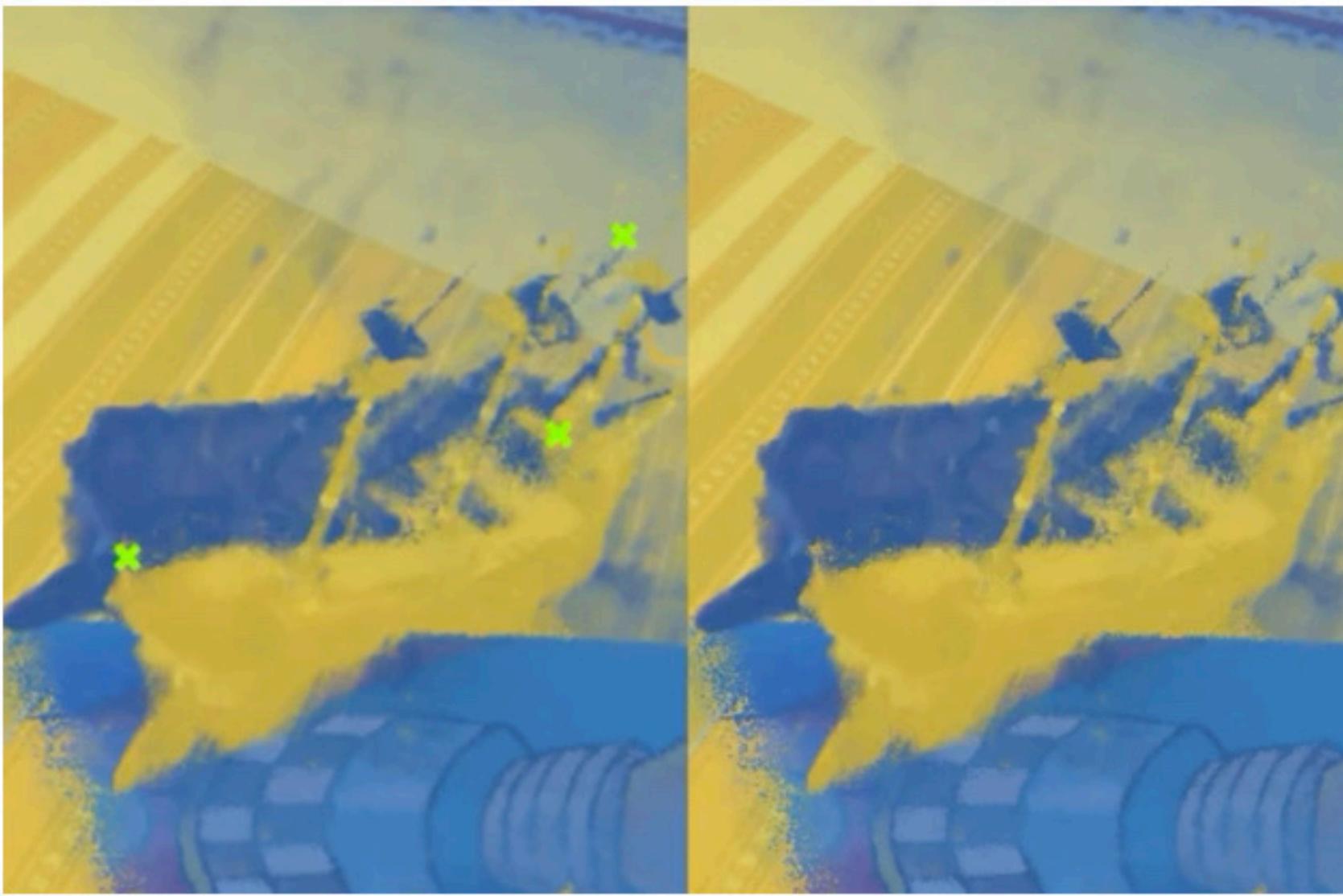
Real Dataset



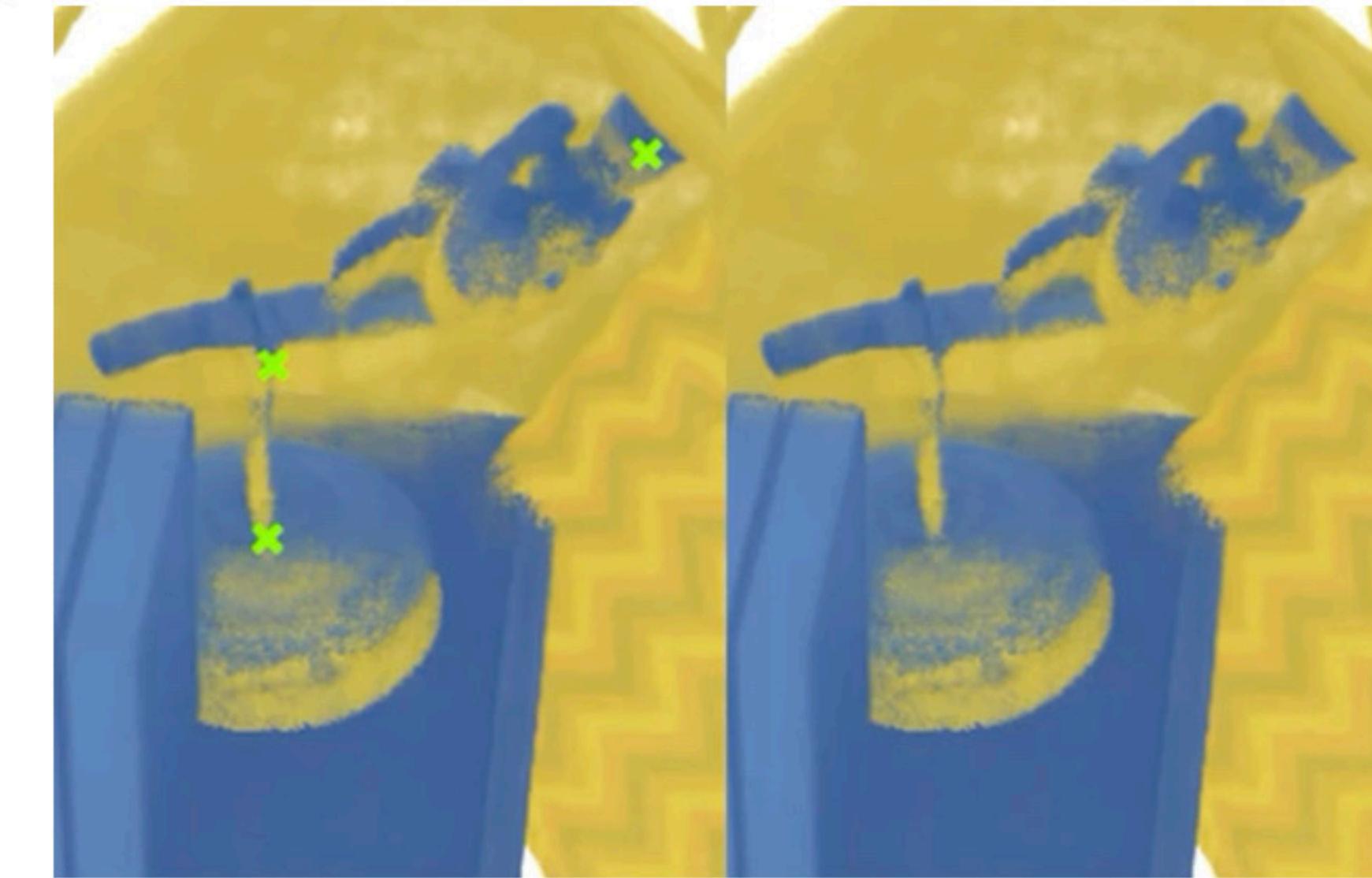
Outdoor

Indoor

Results (Synthetic)



“Ship” $T = 0$



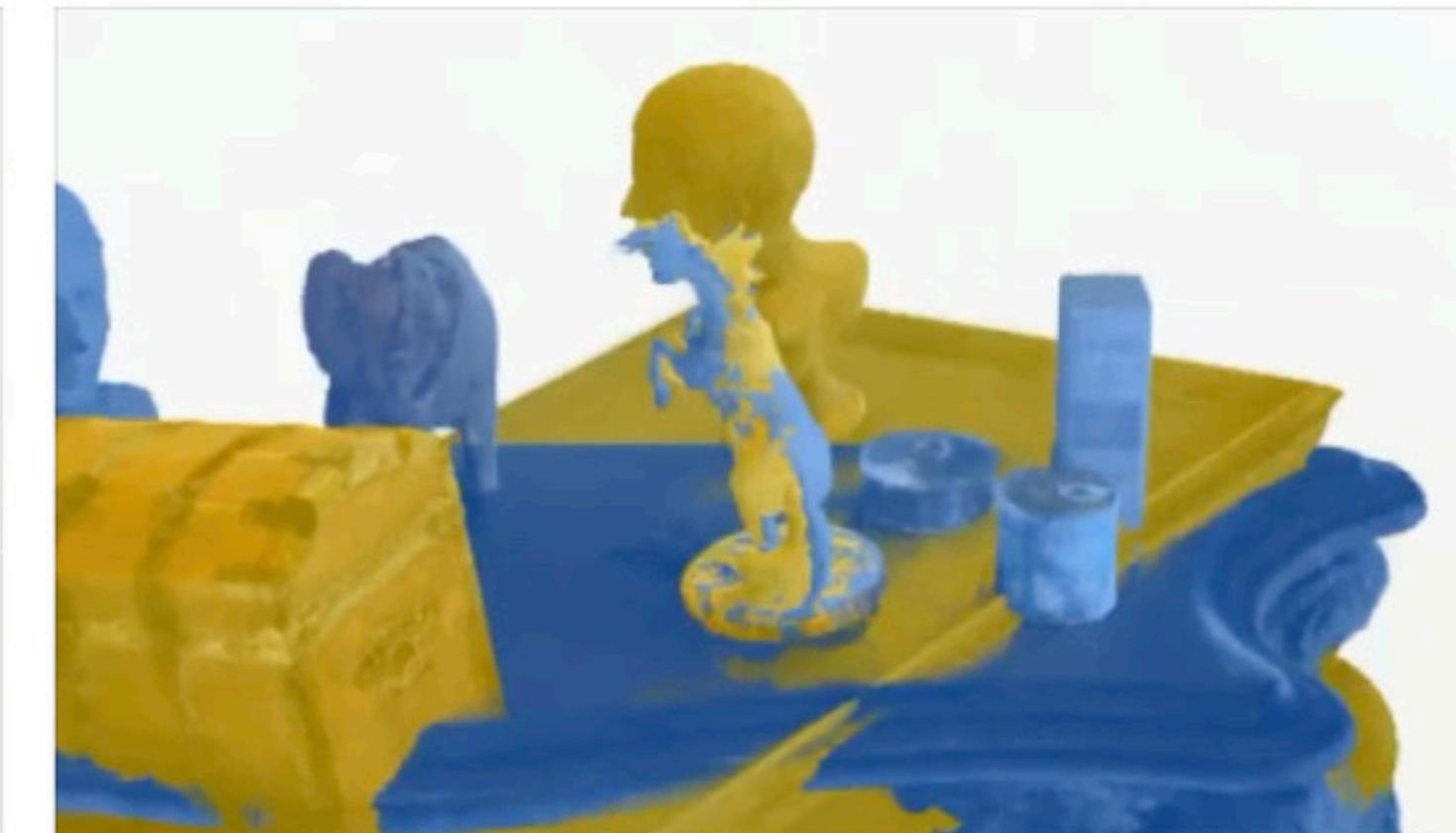
“Pip” $T = 0$

Results (Synthetic)

Object (Pair No.)	$10^2 \cdot \Delta t \downarrow$			$\Delta R \downarrow$			$10^2 \cdot \text{3D-ADD} \downarrow$		
	KP	FGR	Ours	KP	FGR	Ours	KP	FGR	Ours
bust ①	7.78	7.94	0.73	9.93	10.08	1.39	7.16	7.02	0.77
elephant ①	14.01	12.96	0.73	17.16	15.59	1.00	13.97	12.87	0.76
horse ①	7.88	2.10	0.99	14.56	7.32	1.77	6.67	2.09	0.88
pip ②	4.01	18.93	0.25	6.40	30.01	1.18	4.61	20.36	0.46
jar ②	10.40	7.17	0.18	20.46	17.03	2.84	11.31	8.13	0.97
pedestal ②	5.62	8.13	0.69	15.83	11.69	2.42	8.42	9.51	1.23
lego ③	14.41	12.97	2.09	38.08	86.29	3.89	18.75	18.40	2.97
ship ③	13.48	4.80	0.75	20.52	10.74	1.35	15.89	5.36	0.91



nerf2nerf

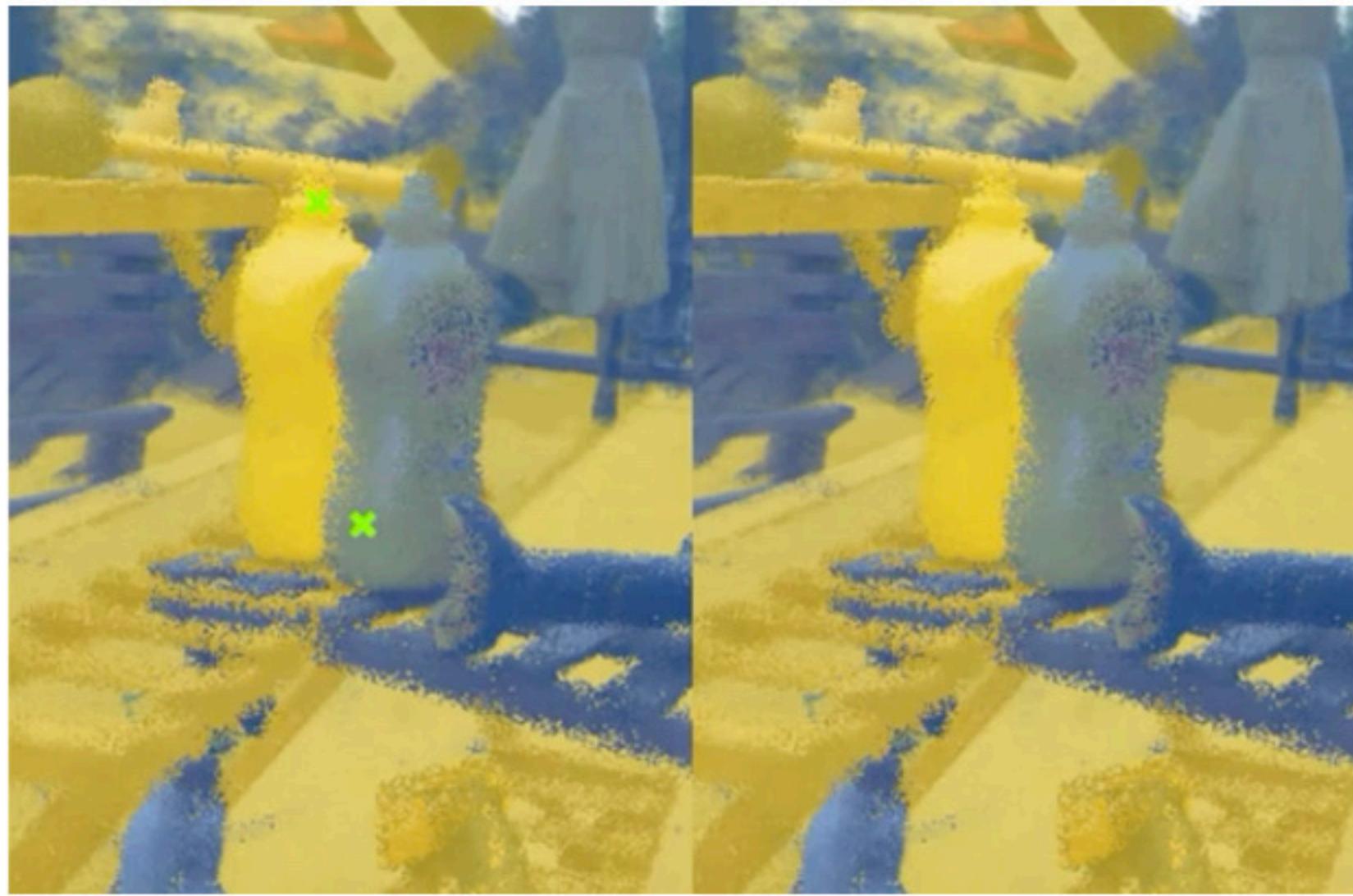


FGR

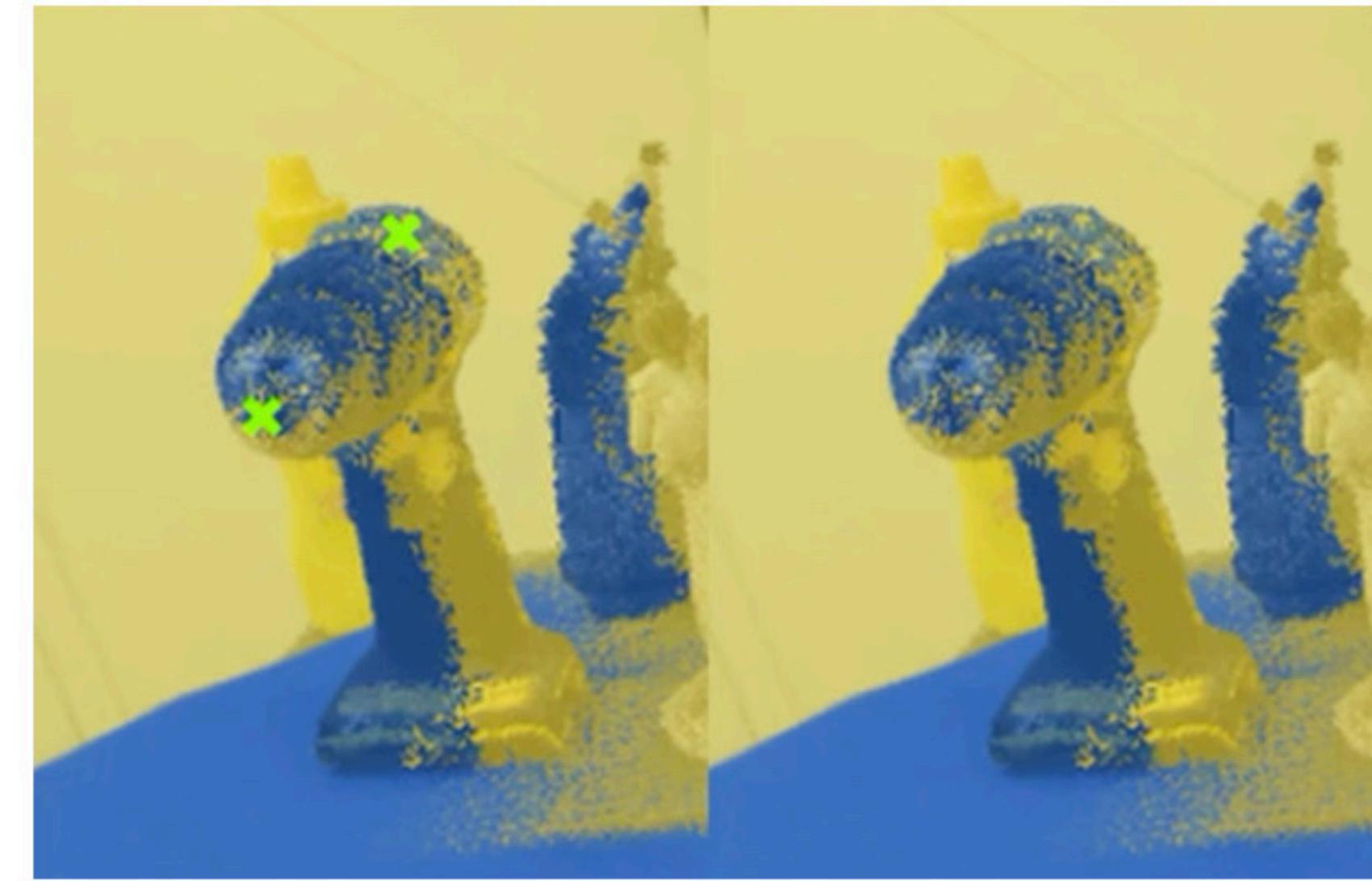


NeRF Point Cloud

Results (Real)

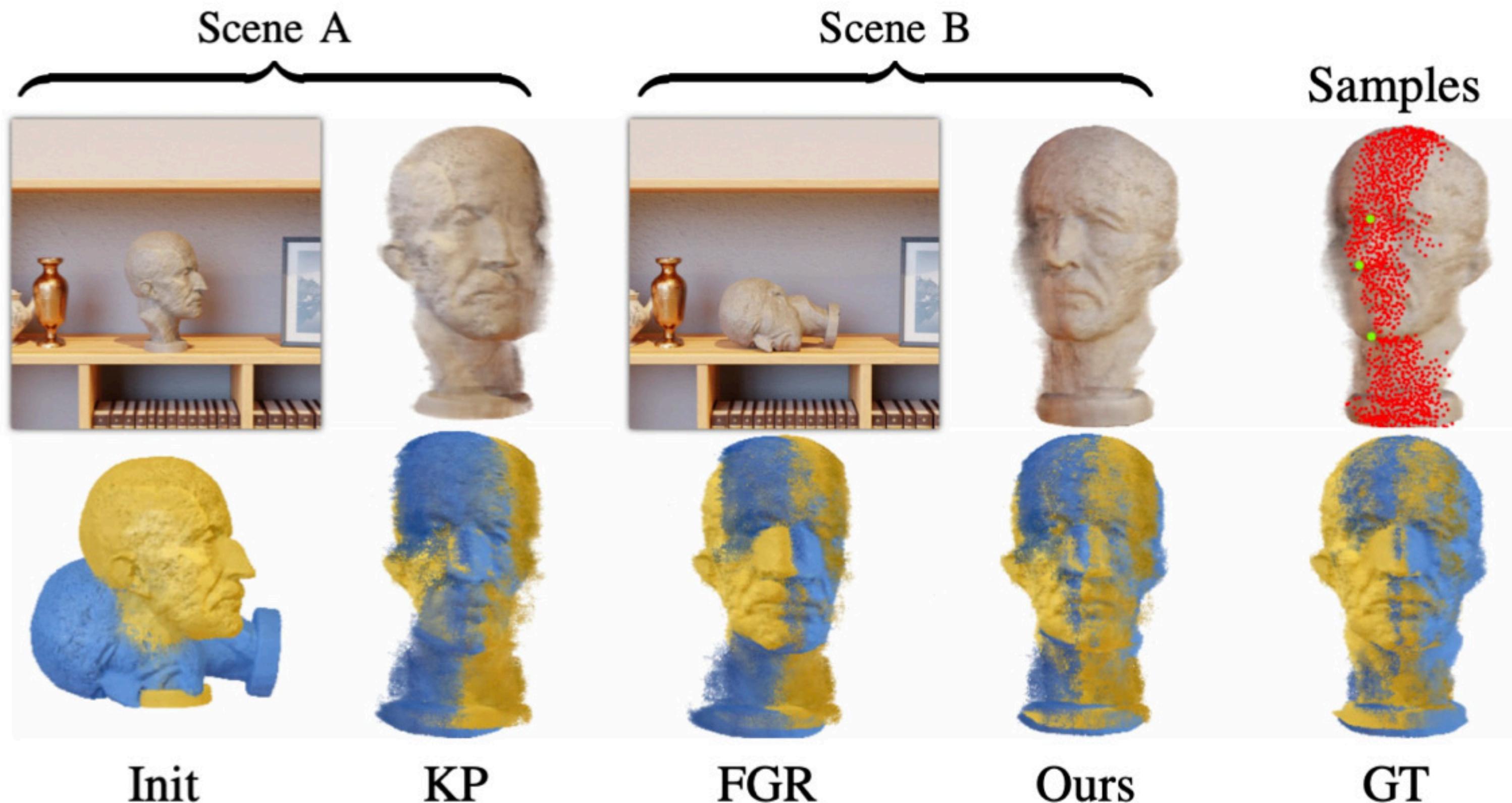


“Mustard” $T = 0$



“Drill” $T = 0$

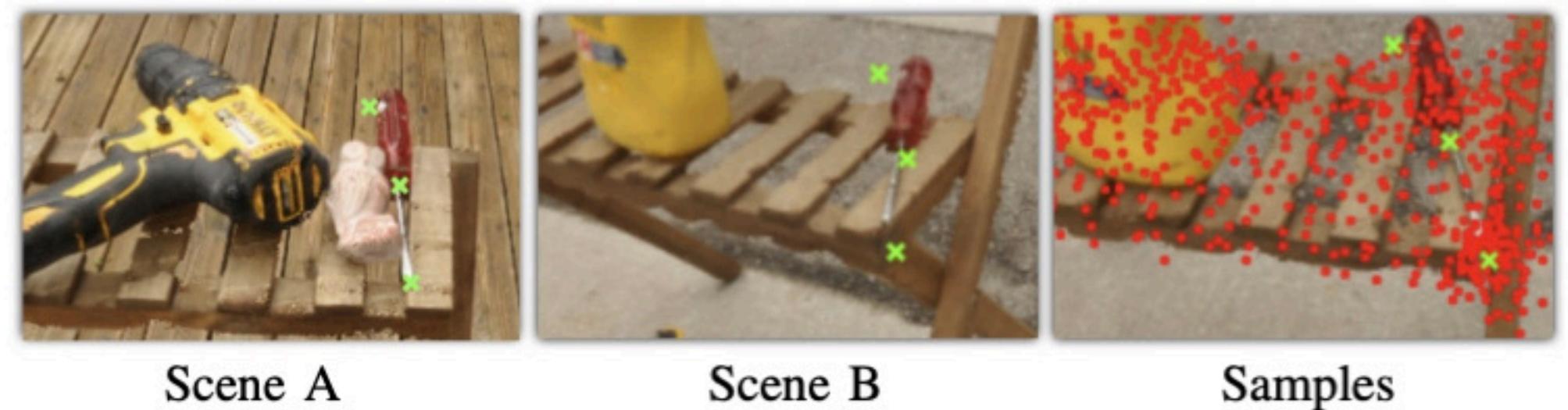
An Application



Future work: How to get rid of those *uncertain* parts for a high quality blend of two NeRFs?
(Coming soon!!!)

Conclusion

- We introduce:
 - a novel geometric representation extracted from NeRF called “Surface Fields”
 - a rigid registration method between two neural radiance fields (NeRFs)
 - a dataset of paired NeRFs of real and synthetic scenes with differently posed multiple objects
- Limitations:
 - Sampling can fail on thin or small objects
 - Registering geometrically symmetric objects with asymmetric texture



nerf2nerf: Pairwise Registration of Neural Radiance Fields

Lily Goli^{1,2}, Daniel Rebain⁴, Sara Sabour^{1,2,6}, Animesh Garg^{1,2,5}, Andrea Tagliasacchi^{1,3,6}

¹University of Toronto, ²Vector Institute, ³SFU, ⁴UBC, ⁵NVIDIA, ⁶Google Research

<https://nerf2nerf.github.io>