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# Introduction



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# 1

## Logic

## 1.1 Formal languages



## 1.2 Propositional logic

## 1.3 First order logic

## 1.4 Modal logic



# 2

## Introduction to set theory

## 2.1 Elements of set theory

## 2.2 Relations and functions

## 2.3 Natural numbers



## 2.4 Integers and rationals

## 2.5 Real numbers

## 2.6 Ordinals

## 2.7 Cardinality

## 2.8 Choice axiom

## 2.9 Well foundedness

# 3

## More set theory

## 3.1 Classes and sets



## 3.2 General infinity

## 3.3 Universes

## 3.4 Further topics on set theory



# 4

## Structures

## 4.1 Historical note

## 4.2 Sets as the foundation

## 4.3 Universal algebra



## 4.4 Introduction to categories



# 5

## Beyond sets

## 5.1 Category theory

## 5.2 Metamathematics

## 5.3 Mathematical logic

## 5.4 Type theory

## 5.5 Philosophical concerns