



Adiel González



# Introduction



# Contents

<b>Introduction</b>	<b>i</b>
<b>1 Logic</b>	<b>1</b>
1.1 Formal languages . . . . .	2
1.2 Propositional logic . . . . .	3
1.3 First order logic . . . . .	4
1.4 Modal logic . . . . .	5
<b>2 Introduction to set theory</b>	<b>7</b>
2.1 Elements of set theory . . . . .	8
2.2 Relations and functions . . . . .	9
2.3 Natural numbers . . . . .	10
2.4 Integers and rationals . . . . .	11
2.5 Real numbers . . . . .	12
2.6 Ordinals . . . . .	13
2.7 Cardinality . . . . .	14
2.8 Choice axiom . . . . .	15
2.9 Well foundedness . . . . .	16
<b>3 More set theory</b>	<b>17</b>
3.1 Classes and sets . . . . .	18
3.2 Limits of infinity . . . . .	19
3.3 Universes . . . . .	20
3.4 Constructibility . . . . .	21
3.5 Further topics on set theory . . . . .	22
<b>4 Structures</b>	<b>23</b>
4.1 Historical note . . . . .	24
4.2 Sets as the foundation . . . . .	25
4.3 Universal algebra . . . . .	26
4.4 Introduction to categories . . . . .	27
<b>5 Beyond sets</b>	<b>29</b>
5.1 Category theory . . . . .	30
5.2 Metamathematics . . . . .	31
5.3 Mathematical logic . . . . .	32
5.4 Type theory . . . . .	33
5.5 Philosophical concerns . . . . .	34



# 1

## Logic

## 1.1 Formal languages



## 1.2 Propositional logic

## 1.3 First order logic

## 1.4 Modal logic



# 2

## Introduction to set theory

## 2.1 Elements of set theory

## 2.2 Relations and functions

## 2.3 Natural numbers



## 2.4 Integers and rationals

## 2.5 Real numbers

## 2.6 Ordinals

## 2.7 Cardinality

## 2.8 Choice axiom

## 2.9 Well foundedness

# 3

## More set theory

## 3.1 Classes and sets



## 3.2 Limits of infinity

## 3.3 Universes

## 3.4 Constructibility

## 3.5 Further topics on set theory

# 4

## Structures

## 4.1 Historical note

## 4.2 Sets as the foundation

## 4.3 Universal algebra



## 4.4 Introduction to categories



# 5

## Beyond sets

## 5.1 Category theory

## 5.2 Metamathematics

## 5.3 Mathematical logic

## 5.4 Type theory

## 5.5 Philosophical concerns