LADR 2B

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Bases

2.27 Definition: basis

A basis of V is a list of vectors of V both linearly independent and spans V. The standard basis is $(1, 0, \ldots, 0), ..., (0, \ldots, 0, 1)$.

Problem 1

Find all vector spaces that have exactly 1 basis.

Proof. Only the trivial vector space: $\{0\}$. If the vector space has a non-zero basis, all scalar multiples (except 0) of the non-zero basis is also a basis. $\square$

Problem 5

Prove or disprove: there exists a basis p_0, p_1, p_2, p_3 of $\mathcal{P}_3(F)$ such that none of the polynomials p_0, p_1, p_2, p_3 has degree 2.

Proof. Consider $[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 1], [0, 0, 0, 1]$. Constants a_1, a_2, a_3, a_4 are unique for any linear combination. $\square$

Problem 7

Prove or disprove: If v_1, v_2, v_3, v_4 is a basis of V and U is a subspace of V such that $v_1, v_2 \in U$ and $v3, v4 \notin U$, then v_1, v_2 is a basis of U.

Proof. Disprove: $U = \text{span}([1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1])$. $V = \text{span}([1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 1])$. $\square$