LADR 3C

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Matrices

3.30 Definition: matrix, $A_{j,k}$

An m-by-n matrix A is a rectangular array of elements of F

$$A = \begin{pmatrix} A_{1,1} & \dots & A_{1,n} \\ \vdots & & \vdots \\ A_{m,1} & \dots & A_{m,n} \end{pmatrix}$$

$A_{j,k} = A[j][k]$ like a software 2D array.

3.32 Definition: matrix of a linear map, $\mathcal{M}(T)$

Suppose $T \in \mathcal{L}(V, W)$ and $v_1, \dots, v_n$ is a basis of V and $w_1, \dots, w_m$ is a basis of W. $A = \mathcal{M}(T)$ where $Tv_k = A_{1,k}w_1 + \dots + A_{m,k}w_m$.

$$\mathcal{M}(T) = \begin{array}{c} \\ w_1 \\ \vdots \\ w_m \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & \dots & v_k & \dots & v_m \end{array} \\ \begin{pmatrix} A_{1,1} & \dots & A_{1,k} & \dots & A_{1,n} \\ \vdots & & \vdots & & \vdots \\ A_{m,1} & \dots & A_{m,k} & \dots & A_{m,n} \end{pmatrix} \end{array}$$

Unless explicitly stated, assume the bases used are the standard ones. You can imagine elements of $\mathbb{F}^m$ as columns of m numbers. Then, k^{th} column of $\mathcal{M}(T)$ is T applied to the k^{th} standard basis vector.

3.35 Definition: matrix addition

Matrix of sum of linear maps. A, C must be same size. $(A+C)_{j,k} = A_{j,k} + C_{j,k}$

3.37 Definition: scalar multiplication of a matrix

Matrix of a scalar times a linear map. $(\lambda A)_{j,k} = \lambda A_{j,k}$

3.39 Notation: $\mathbb{F}^{m,n}$

Set of all m by n matrices with entries in $\mathbb{F}$.

3.40 $\mathbb{F}^{m,n}$ is a vector space

Closed under addition and multiplication (3.35, 3.37). It is clearly a non-empty set, the 0 element is the matrix with all 0 entries.
Property: $\dim \mathbb{F}^{m,n} = mn$

3.41 Definition: matrix multiplication

We want an operator so that the equality holds:

$$\mathcal{M}(ST) = \mathcal{M}(S)\mathcal{M}(T).$$

Let A be m by n, C be n by p. Then, AC is m by p.

$$(AC)_{j,k} = \sum_{r=1}^{n} A_{j,r}C_{r,k}$$

Another way:

$$(AC)_{j,k} = A_{j,\cdot}C_{\cdot,k}$$

3.44 Notation: $A_{j,\cdot}, A_{\cdot,k}$

$A_{j,\cdot}$ is 1 by n matrix for row j of A.
$A_{\cdot,k}$ is m by 1 matrix for col k of A.

3.49 Column of matrix product equals matrix times column

$$(AC)_{\cdot,k} = AC_{\cdot,k}$$

3.52 Linear combination of columns

$$Ac = c_1 A_{\cdot,1} + \cdots + c_n A_{\cdot,n}$$