CS70 Note 0: Mathematical Foundations

Lucas Zheng

1 Propositions

A statement that's either true or false is called a **proposition**.
Ex: $\sqrt{3}$ is rational, $1+1=5$, CS70 is cool
Propositional variables P, Q, R represent arbitrary propositions.
Connectives like and$(P \wedge Q)$, or$(P \vee Q)$, not$(\neg P)$ join propositions together to form more complex ones.
Ex: $\sqrt{3}$ is rational $\wedge$ $1+1=5$
Propositional formulas are created by combining propositional variables with connectives.
Ex: $P \wedge Q \vee \neg R$

2 Propositional Logic

Tautology is a propositional formula that is always true regardless of the truth values of the variables.
Contradiction always false regardless of the truth values of the variables (tautologically false).
Truth table is an algorithm to verify if a propositional formula is a tautology.

P	P	$\wedge$	$\neg P$
T	T	F	F
F	T	T	T

Implication $P \implies Q$ means "if P, then Q". Only false if P is T and Q is F. Equivalent to $\neg P \vee Q$.
2 propositional formulas are **tautologically equivalent** if they have the same truth table, written as $P \equiv Q$