CS70 Note 0: Mathematical Foundations

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1 Sets

Math is built on the foundation of the theory of sets. In sets, order and count do not matter.
Set builder notation: $\mathbb{Q} = \{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\}$
Cardinality: the size of a set $|S|$
Subset / superset / proper subset: $A \subseteq B$ / $A \supseteq B$ / $A \subset B$
Natural numbers: $\mathbb{N} = \{0, 1, 2, 3, \dots\}$
Integer numbers: $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$
Rational numbers: $\mathbb{Q} = \{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\}$
Real numbers: $\mathbb{R}$
Intersection: $A \cap B$
Union: $A \cup B$
Relative complement: $B \setminus A := \{x \in B : x \notin A\}$
Cartesian product: $A \times B = \{(a, b) : a \in A, b \in B\}$
Power set: $2^A = \{S : S \subseteq A\}$

2 Sums and Products

$\sum, \prod$
Geometric series (infinite), taylor series expansion of the exponential

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$