
LADR 2C

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Dimension

2.35 Basis length doesn't depend on basis

Proof. Let B_1, B_2 be bases for V . Then, $\text{len}(B_1) \geq \text{len}(B_2) \wedge \text{len}(B_2) \geq \text{len}(B_1) \implies \text{len}(B_1) = \text{len}(B_2)$. \square

2.36 Definition: dimension ($\dim V$)

Length of any basis of V .

2.38 Dimension of a subspace \leq the space

Proof. Trivial. \square

2.39 Linearly independent list of right length is basis

2.42 Spanning list of right length is basis

2.43 Dimension of a sum

$$\dim(U_1 + U_2) = \dim U_1 + \dim U_2 - \dim(U_1 \cap U_2)$$

Proof. Intersection is a subspace w/ dimension m . We can extend it by j vectors to create a basis for U_1 . Similarly, we can extend it by k vectors to create a basis for U_2 . Finally, $\dim(U_1 + U_2) = (m + j) + (m + k) - m$. \square