CS70 Note 3: Induction

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1 Induction

Powerful tool to prove a statement holds for all natural numbers.
Prove $\forall n \in \mathbb{N}, \sum_{i=0}^{n} i = \frac{n(n+1)}{2}$

Proof. We do induction on n.
Base case (n=0): can easily see LHS = RHS = 0 and it works.
Induction hypothesis: assume we have proved the statement works for an arbitrary $n = k \geq 0$.
Inductive step: prove the statement for $n = k + 1$.

$$\sum_{i=0}^{k+1} = \sum_{i=0}^{k} i + (k+1) = \frac{k(k+1)}{2} + (k+1) = \frac{(k+1)(k+2)}{2}$$

□

2 Strengthening the Induction Hypothesis

Ex: "Sum of the first n odd numbers is a perfect square" vs "sum of the first n odd numbers is n^2"
The introduction of more constraints/structure was necessary to create the proof.

3 Simple vs Strong Induction

Simple: assume $P(k)$ is true.
Strong: assume $P(0), P(1), \ldots, P(k)$ are all true.