Team notebook

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June 16, 2018

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1.	1 (Convex Hull Trick
	ruct	cht
{		vector <pii> hull;</pii>

```
vector<int> id;
int cur=0;
cht()
{
       hull.clear();
       id.clear();
}
// Might need double here
bool useless(const pii left, const pii middle, const pii right)
       return
       1LL*(middle.second-left.second)*(middle.first-right.first)
       >=1LL*(right.second-middle.second)*(left.first-middle.first);
}
// Inserting line a*x+b with index idx
// Before inserting one by one, all the lines are sorted by slope
void insert(int idx, int a, int b)
       if(hull.empty())
       {
              hull.pb(MP(a, b));
              id.pb(idx);
       }
       else
       {
              if(hull.back().first==a)
                     if(hull.back().second>=b)
                             return;
                     }
                     else
                     {
                            hull.pop_back();
                             id.pop_back();
                     }
              while(hull.size()>=2 &&
                  useless(hull[hull.size()-2], hull.back(), MP(a,
```

```
b)))
                      {
                             hull.pop_back();
                             id.pop_back();
                      hull.pb(MP(a,b));
                      id.pb(idx);
              }
       }
       // returns maximum value and the index of the line
       // Pointer approach: the queries are sorted non-decreasing
       // Otherwise, we will need binary search
       pair<11,int> query(int x)
              11 ret=-INF;
              int idx=-1;
              for(int i=cur ; i < hull.size() ; i++)</pre>
                      ll tmp=1LL*hull[i].first*x + hull[i].second;
                      if(tmp>ret)
                      {
                             ret=tmp;
                             cur=i;
                             idx=id[i]:
                      }
                      else
                      {
                             break;
              return {ret,idx};
       }
};
// Slope decreasing, query minimum - Query point increasing.
// Slope increasing, query maximum - Query point increasing.
// Slope decreasing, query maximum - Query point decreasing.
// Slope increasing, query minimum - Query point decreasing.
```

1.2 Digit DP Sample 2

```
// For each case, output the case number and the number of integers in
    the range [A, B] which are
// divisible by K and the sum of its digits
// is also divisible by K.
int k, cases = 1;
ll dp[11][2][83][83];
int visited[11][2][83][83], flag;
string toString(int x)
       string temp = "";
       if (x == 0) return "0";
       while (x > 0)
              int r = x \% 10;
              temp = char(r + '0') + temp;
              x /= 10;
       }
       return temp;
ll calc(int idx, bool low, int modVal, int sumMod, string s)
{
       if (idx == s.size()) return (!modVal && !sumMod);
       if (visited[idx][low][modVal][sumMod] == flag)
              return dp[idx][low][modVal][sumMod];
       visited[idx][low][modVal][sumMod] = flag;
       int digit = low ? 9 : (s[idx] - '0');
       11 \text{ ret} = 0;
       for (int i = 0; i <= digit; i++)</pre>
              ret += calc(idx + 1, low || i < s[idx] - '0', (modVal * 10
                   + i) % k, (sumMod + i) % k, s);
       return dp[idx][low][modVal][sumMod] = ret;
}
int main()
{
       int test;
       int a, b;
       cin >> test;
       while (test--)
              cin >> a >> b >> k;
              if (k > 90)
```

1.3 Digit DP Sample

```
// Calculate how many numbers in the range from A to B that have digit d
    in only the even positions and
// no digit occurs in the even position and the number is divisible by m.

string A, B; int m, d;
11 dp[2002][2002][2][2];

11 calc(int idx, int Mod, bool s, bool b)
{
    if(idx==B.size()) return Mod==0;
    if(dp[idx][Mod][s][b]!=-1)
        return dp[idx][Mod][s][b];

    ll ret=0;
    int low=s ? 0 : A[idx]-'0';
    int high=b ? 9 : B[idx]-'0';
    for(int i=low; i<=high; i++)
    {
        if(idx%2 && i!=d) continue;
        if(idx%2==0 && i==d) continue;
}</pre>
```

1.4 Divide and Conquer DP

```
// http://codeforces.com/blog/entry/8219
// Divide and conquer optimization:
// Original Recurrence
// dp[i][j] = min(dp[i-1][k] + C[k][j]) for k < j
// Sufficient condition:
// A[i][j] <= A[i][j+1]
// where A[i][j] = smallest k that gives optimal answer
// // compute i-th row of dp from L to R. optL <= A[i][L] <= A[i][R] <=
    optR
// compute(i, L, R, optL, optR)
//
        1. special case L == R
        2. let M = (L + R) / 2. Calculate dp[i][M] and opt[i][M] using
    O(optR - optL + 1)
        3. compute(i, L, M-1, optL, opt[i][M])
        4. compute(i, M+1, R, opt[i][M], optR)
```

```
// Example: http://codeforces.com/contest/321/problem/E
#include "../template.h"
const int MN = 4011;
const int inf = 1000111000;
int n, k;
11 cost[MN][MN], dp[811][MN];
inline ll getCost(int i, int j) {
   return cost[i][i] - cost[i][i-1] - cost[i-1][i] + cost[i-1][i-1];
}
void compute(int i, int L, int R, int optL, int optR) {
   if (L > R) return ;
   int mid = (L + R) >> 1, savek = optL;
   dp[i][mid] = inf;
   FOR(k,optL,min(mid-1, optR)) {
       ll cur = dp[i-1][k] + getCost(k+1, mid);
       if (cur < dp[i][mid]) {</pre>
           dp[i][mid] = cur;
           savek = k;
       }
   compute(i, L, mid-1, optL, savek);
   compute(i, mid+1, R, savek, optR);
}
void solve() {
   cin >> n >> k;
   FOR(i,1,n) FOR(j,1,n) {
       cin >> cost[i][j];
       cost[i][j] = cost[i-1][j] + cost[i][j-1] - cost[i-1][j-1] +
            cost[i][j];
   }
   dp[0][0] = 0;
   FOR(i,1,n) dp[0][i] = inf;
   FOR(i,1,k) {
       compute(i, 1, n, 0, n);
   }
   cout << dp[k][n] / 2 << endl;</pre>
```

1.5 Dynamic Convex Hull Trick

```
// source:
    https://github.com/niklasb/contest-algos/blob/master/convex_hull/dynamic.cpp
// Used in problem CS Squared Ends
const ll is_query = -(1LL<<62);</pre>
struct Line {
   ll m, b;
   mutable function<const Line*()> succ:
   bool operator<(const Line& rhs) const {</pre>
       if (rhs.b != is_query) return m < rhs.m;</pre>
       const Line* s = succ();
       if (!s) return 0;
       11 x = rhs.m:
       return b - s->b < (s->m - m) * x;
   }
}:
struct HullDynamic : public multiset<Line> { // will maintain upper hull
    for maximum
   bool bad(iterator y) {
       auto z = next(y);
       if (y == begin()) {
           if (z == end()) return 0;
           return y->m == z->m && y->b <= z->b;
       }
       auto x = prev(y);
       if (z == end()) return y->m == x->m && y->b <= x->b;
       // **** May need long double typecasting here
       return (long double)(x->b-y->b)*(z->m-y->m) >= (long
            double) (y->b - z->b)*(y->m - x->m);
   void insert_line(ll m, ll b) {
       auto y = insert({ m, b });
       y->succ = [=] { return next(y) == end() ? 0 : &*next(y); };
       if (bad(y)) { erase(y); return; }
       while (next(y) != end() && bad(next(y))) erase(next(y));
       while (y != begin() && bad(prev(y))) erase(prev(y));
   11 eval(ll x) {
       auto 1 = *lower_bound((Line) { x, is_query });
       return 1.m * x + 1.b:
};
```

1.6 Edit Distance Recursive

```
int dp[34][34];
string a, b;
int editDistance(int i, int j)
       if (dp[i][j]!=-1)
              return dp[i][j];
       if (i==0)
              return dp[i][j]=j;
       if (j==0)
              return dp[i][j]=i;
       int cost;
       if (a[i-1]==b[j-1])
              cost=0;
       else
              cost=1;
       return
            dp[i][j]=min(editDistance(i-1,j)+1,min(editDistance(i,j-1)+1,
                                     editDistance(i-1,j-1)+cost));
}
int main()
{
       ms(dp,-1);
       cin>>a>>b;
       prnt(editDistance(a.size(),b.size()));
   return 0;
```

1.7 LCS

```
string a, b;
int dp[100][100];
string 1;
void printLcs(int i, int j)
{
```

```
if (a[i] == '\0' || b[j] == '\0')
               cout << 1 << endl;</pre>
              return;
       }
       if (a[i] == b[j])
              1 += a[i];
               printLcs(i + 1, j + 1);
       }
       else
       {
               if (dp[i + 1][j] > dp[i][j + 1])
                      printLcs(i + 1, j);
               else
                      printLcs(i, j + 1);
       }
}
void printAll(int i, int j)
       if (a[i] == '\0' || b[j] == '\0')
              prnt(1);
               return;
       }
       if (a[i] == b[j])
       {
              1 += a[i];
              printAll(i + 1, j + 1);
              1.erase(1.end() - 1);
       }
       else
       {
               if (dp[i + 1][j] > dp[i][j + 1])
                      printAll(i + 1, j);
               else if (dp[i + 1][j] < dp[i][j + 1])</pre>
                      printAll(i, j + 1);
               else
               {
                      printAll(i + 1, j);
                      printAll(i, j + 1);
              }
       }
int lcslen (int i, int j)
```

```
{
       if (a[i] == '\0' || b[j] == '\0')
               return 0;
       if (dp[i][j] != -1)
               return dp[i][j];
       int ans = 0;
       if (a[i] == b[j])
               ans = 1 + lcslen(i + 1, j + 1);
       }
       else
               int x = lcslen(i, j + 1);
               int y = lcslen(i + 1, j);
               ans = max(x, y);
       return dp[i][j] = ans;
}
int main()
{
       cin >> a >> b;
       ms(dp, -1);
       cout << lcslen(0, 0) << endl;</pre>
       printLcs(0, 0);
       1.clear();
       printAll(0, 0);
       return 0;
```

1.8 LIS nlogk

```
vector<int> d;
int ans, n;

int main() {
    scanf("%d", &n);
    for (int i = 0; i < n; i++) {
        int x;
        scanf("%d", &x);
        vector<int>::iterator it = lower_bound(d.begin(), d.end(), x);
        if (it == d.end()) d.push_back(x);
        else *it = x;
```

```
}
printf("LIS = %d", d.size());
return 0;
}
```

1.9 Matrix Expo Class

Matrix expo(Matrix &M, int n, int sz)

```
struct Matrix
{
       11 mat[MAX][MAX];
       Matrix(){}
       // This initialization is important.
       // Input matrix should be initialized separately
       void init(int sz)
              ms(mat,0);
              for(int i=0; i<sz; i++) mat[i][i]=1;</pre>
} aux;
void matMult(Matrix &m, Matrix &m1, Matrix &m2, int sz)
       ms(m.mat,0);
       // This only works for square matrix
       FOR(i,0,sz)
              FOR(j,0,sz)
                      FOR(k,0,sz)
                             m.mat[i][k]=(m.mat[i][k]+m1.mat
                                     [i][j]*m2.mat[j][k])%mod;
              }
       }
}
```

```
{
       Matrix ret;
       ret.init(sz);
       if(n==0) return ret;
       if(n==1) return M;
       Matrix P=M;
       while(n!=0)
               if (n&1)
               {
                      aux=ret;
                      matMult(ret,aux,P,sz);
              }
               n>>=1;
               aux=P; matMult(P,aux,aux,sz);
       }
       return ret;
```

1.10 Palindrome in a String

2 Data Structures

2.1 Best Partial Sum in a Range

```
struct Node
₹
       11 bestSum, bestPrefix, bestSuffix, segSum;
       Node()
              bestSum=bestPrefix=bestSuffix=segSum=-INF;
       void merge(Node &1, Node &r)
              segSum=1.segSum+r.segSum;
              bestPrefix=max(1.bestPrefix,r.bestPrefix+1.segSum);
              bestSuffix=max(r.bestSuffix,r.segSum+1.bestSuffix);
              bestSum=max(max(1.bestSum,r.bestSum),1.bestSuffix+r.bestPrefix);
}tree[150005];
void init(int node, int start, int end)
       if(start==end)
              tree[node].bestSum=tree[node].segSum=a[start];
              tree[node].bestSuffix=tree[node].bestPrefix=a[start];
              return:
```

```
}
       int left=node<<1;</pre>
       int right=left+1;
       int mid=(start+end)>>1;
       init(left,start,mid);
       init(right, mid+1, end);
       tree[node].merge(tree[left],tree[right]);
}
void update(int node, int start, int end, int i, int val)
{
       if(i<start || i>end)
               return:
       if(start>=i && end<=i)</pre>
               tree[node].bestSum=tree[node].segSum=val;
               tree[node].bestSuffix=tree[node].bestPrefix=val;
               a[start]=val;
               return:
       }
       int left=node<<1;</pre>
       int right=left+1;
       int mid=(start+end)>>1;
       update(left,start,mid,i,val);
       update(right, mid+1, end, i, val);
       tree[node].merge(tree[left],tree[right]);
}
Node query(int node, int start, int end, int i, int j)
{
       if(i>end || j<start)</pre>
               return Node();
       if(start>=i && end<=j)</pre>
       {
               return tree[node];
       int left=node<<1;</pre>
       int right=left+1;
       int mid=(start+end)>>1;
       Node l=query(left,start,mid,i,j);
       Node r=query(right,mid+1,end,i,j);
       Node n;
       n.merge(1,r);
       return n;
```

2.2 Binary Indexed Tree

```
11 Tree[MAX];
void update(int idx, ll x)
       while(idx<=n)</pre>
              Tree[idx]+=x;
              idx+=(idx&-idx):
       }
}
11 query(int idx)
{
       11 sum=0:
       while(idx>0)
              sum+=Tree[idx];
              idx = (idx \& - idx);
       }
       return sum;
}
int main()
{
       // For point update range query:
       // Point update: update(x,val);
       // Range query (a,b): query(b)-query(a-1);
       // For range update point query:
       // Range update (a,b): update(a,v); update(b+1,-v);
       // Point query: query(x);
       // Let's just consider only one update: Add v to [a, b] while the
           rest elements of the array is 0.
       // Now, consider sum(0, x) for all possible x, again three
           situation can arise:
       // 1. 0 x < a : which results in 0
       // 2. a x b : we get v * (x - (a-1))
       // 3. b < x < n : we get v * (b - (a-1))
       // This suggests that, if we can find v*x for any index x, then we
           can get the sum(0, x) by subtracting T from it, where:
       // 1. 0 x < : Sum should be 0, thus, T = 0
       // 2. a x : Sum should be v*x-v*(a-1), thus, T = v*(a-1)
```

2.3 Centroid Decomposition Sample

```
/* You are given a tree consisting of n vertices. A number is written on
    each vertex:
the number on vertex i is equal to a[i].
Let, g(x,y) is the gcd of the numbers written on the vertices belonging
    to the path from
x to y, inclusive. For i in 1 to 200000, count number of pairs (x,y)
    (1 \le x \le y) such
that g(x,y) equals to i.
Note that 1<=x<=y does not really matter.
vi graph[MAX];
int n, a[MAX], sub[MAX], total, cnt[MAX], cent, upto[MAX];
11 ans[MAX];
bool done[MAX];
set<int> take[MAX];
void dfs(int u, int p)
{
       sub[u]=1;
       total++;
       for(auto v: graph[u])
              if(v==p || done[v]) continue;
```

```
dfs(v,u);
               sub[u]+=sub[v];
       }
}
int getCentroid(int u, int p)
       // cout<<u<<" "<<sub[u]<<endl;
       for(auto v: graph[u])
              if(!done[v] && v!=p && sub[v]>total/2)
                      return getCentroid(v,u);
       }
       return u;
}
void go(int u, int p, int val)
       ans[val]++;
       take[cent].insert(val);
       cnt[val]++;
       for(auto v: graph[u])
              if(!done[v] && v!=p)
              {
                      go(v,u,upto[v]);
       }
}
void calc(int u, int p, int val)
       for(auto it: take[cent])
              int g=gcd(val,it);
              ans[g]+=cnt[it];
       }
       for(auto v: graph[u])
              if(!done[v] && v!=p)
                      calc(v,u,upto[v]);
```

```
}
       }
}
void clean(int u, int p, int val)
       cnt[val]=0;
       for(auto v: graph[u])
              if(!done[v] && v!=p)
              {
                      clean(v,u,upto[v]);
              }
       }
}
void calcgcd(int u, int p, int val)
       upto[u]=val;
       for(auto v: graph[u])
              if(!done[v] && v!=p)
                      calcgcd(v,u,gcd(val,a[v]));
       }
}
void solve(int u)
{
       total=0;
       dfs(u,-1);
       cent=getCentroid(u,-1);
       calcgcd(cent,-1,a[cent]);
       // debug("cent",cent);
       done[cent]=true;
       for(auto v: graph[cent])
              if(done[v]) continue;
```

```
// cout<<"from centroid "<<cent<<" going to node:</pre>
                   "<<v<<endl;
              calc(v,cent,upto[v]);
              go(v,cent,upto[v]);
       }
       for(auto v: graph[cent])
              if(!done[v])
                      clean(v,cent,upto[v]);
       }
       for(auto v: graph[cent])
              if(!done[v])
                      solve(v);
       }
}
int main()
{
   // ios_base::sync_with_stdio(0);
   // cin.tie(NULL); cout.tie(NULL);
   // freopen("in.txt","r",stdin);
   int test, cases=1;
   scanf("%d", &n);
   FOR(i,1,n+1)
       scanf("%d", &a[i]);
       ans[a[i]]++;
   int u, v;
   FOR(i,1,n)
       scanf("%d%d", &u, &v);
       graph[u].pb(v);
       graph[v].pb(u);
   solve(1);
   FOR(i,1,MAX) if(ans[i]) printf("%d %lld\n", i, ans[i]);
```

```
return 0;
}
```

2.4 Centroid Decomposition

```
int n, m, a, b, Table[MAX][20];
set<int> Graph[MAX];
int Level[MAX], nodeCnt, Subgraph[MAX], Parent[MAX], Ans[MAX];
void findLevel(int u)
{
       itrALL(Graph[u], it)
              int v = *it;
              if (v != Table[u][0])
                      Table[v][0] = u;
                      Level[v] = Level[u] + 1;
                      findLevel(v);
              }
       }
void Process()
       Level[0] = 0;
       ms(Table, -1);
       Table[0][0] = 0;
       findLevel(0);
// debug;
       for (int j = 1; 1 << j < n; j++)
              for (int i = 0; i < n; i++)
              {
                      if (Table[i][j - 1] != -1)
                             Table[i][j] = Table[Table[i][j-1]][j-1];
              }
// debug;
int findLCA(int p, int q)
       if (Level[p] < Level[q]) swap(p, q);</pre>
       int x = 1;
```

```
while (true)
              if ((1 << (x + 1)) > Level[p]) break;
              x++;
       FORr(i, x, 0)
              if (Level[p] - (1 << i) >= Level[q])
                     p = Table[p][i];
       if (p == q) return p;
       FORr(i, x, 0)
              if (Table[p][i] != -1 && Table[p][i] != Table[q][i])
              {
                     p = Table[p][i];
                     q = Table[q][i];
              }
       return Table[p][0];
}
int Dist(int a, int b)
{
       return Level[a] + Level[b] - 2 * Level[findLCA(a, b)];
void findSubgraph(int u, int parent)
       Subgraph[u] = 1;
       nodeCnt++;
       itrALL(Graph[u], it)
              int v = *it;
              if (v == parent) continue;
              findSubgraph(v, u);
              Subgraph[u] += Subgraph[v];
       }
}
int findCentroid(int u, int p)
       itrALL(Graph[u], it)
              int v = *it;
              if (v == p) continue;
              if (Subgraph[v] > nodeCnt / 2) return findCentroid(v, u);
       }
```

```
return u;
}
void Decompose(int u, int p)
       nodeCnt = 0;
       findSubgraph(u, u);
       int Cent = findCentroid(u, u);
       if (p == -1) p = Cent;
       Parent[Cent] = p;
       itrALL(Graph[Cent], it)
              int v = *it;
              Graph[v].erase(Cent);
              Decompose(v, Cent);
       Graph[Cent].clear();
}
void update(int u)
       int x = u;
       while (true)
              Ans[x] = min(Ans[x], Dist(x, u));
              if (x == Parent[x]) break;
              x = Parent[x];
       }
}
int query(int u)
       int x = u;
       int ret = INF;
       while (true)
              ret = min(ret, Dist(u, x) + Ans[x]);
              if (x == Parent[x]) break;
              x = Parent[x];
       }
       return ret;
}
int main()
// ios_base::sync_with_stdio(0);
// cin.tie(NULL); cout.tie(NULL);
// freopen("in.txt","r",stdin);
// All the nodes are initially blue
```

```
// Then by updating, one node is colored red
// Upon query, return the closest red node of the given node
       scanf("%d%d", &n, &m);
       FOR(i, 0, n-1)
              scanf("%d%d", &a, &b);
              a--, b--;
              Graph[a].insert(b);
              Graph[b].insert(a);
       }
       Process();
// debug;
       Decompose(0, -1);
       FOR(i, 0, n) Ans[i] = INF;
       update(0);
       while (m--)
              int t, x;
              scanf("%d%d", &t, &x);
              x--;
              if (t == 1) update(x);
              else printf("%d\n", query(x));
       }
       return 0;
```

2.5 Counting Inversions with BIT

```
while(idx>0)
              sum+=tree[idx];
              idx-=(idx&-idx);
       return sum;
}
int main()
{
       // ios_base::sync_with_stdio(0);
       // cin.tie(NULL); cout.tie(NULL); // No 'endl'
       // freopen("in.txt","r",stdin);
       int test:
       // cin>>test;
       scanf("%d", &test);
       while(test--)
              ms(tree,0);
              scanf("%d", &n);
              FOR(i,1,n+1)
              {
                      scanf("%d", &a[i]);
                      b[i]=a[i];
              }
              sort(b+1,b+n+1);
              // Compressing the array
              FOR(i,1,n+1)
              {
                      int rank=int(lower_bound(b+1,b+1+n,a[i])-b-1);
                      a[i]=rank+1;
              }
              // FOR(i,1,n+1) cout<<a[i]<<" "; cout<<endl;
              11 ans=0;
              FORr(i,n,1)
              {
                      ans+=query(a[i]-1);
                      update(a[i],1);
              }
              // prnt(ans);
              printf("%lld\n",ans);
```

```
return 0;
}
```

2.6 How Many Values Less than a Given Value

```
// How many values in a range are less than or equal to the given value?
// The key idea is to sort the values under a node in the segment tree
    and use binary search to find
// the required count
// Complexity is O(nlog^2n) for building
// The actual problem needed the number of such values and the cumulative
    sum of them
// Tree[node].All has all the values and Tree[node].Pref has the prefix
// Remember: upper_bound gives the number of values less than or equal to
    given value in a sorted range
struct info
       vector<ll> All, Pref;
} Tree[MAX * 4];
11 T[MAX], Prefix[MAX];
void build(int node, int 1, int r)
       if (1 == r)
              Tree[node].All.pb(T[1]);
              Tree[node].Pref.pb(T[1]);
              return;
       }
       int mid = (1 + r) / 2;
       build(lc, 1, mid);
       build(rc, mid + 1, r);
       for (auto it : Tree[lc].All)
              Tree[node].All.pb(it);
       for (auto it : Tree[rc].All)
              Tree[node].All.pb(it);
       SORT(Tree[node].All);
       11 \text{ now = 0};
       for (auto it : Tree[node].All)
```

```
Tree[node].Pref.pb(now + it);
              now += it;
       }
}
pair<11, 11> query(int node, int 1, int r, int x, int y, int val)
       if (x > r || y < 1) return MP(OLL, OLL);</pre>
       if (x <= 1 && r <= y)</pre>
       {
              int idx = upper_bound(Tree[node].All.begin(),
                   Tree[node].All.end(), val) - Tree[node].All.begin();
              if (idx > 0) return MP(Tree[node].Pref[idx - 1], idx);
              return MP(OLL, OLL);
       }
       int mid = (1 + r) / 2;
       pair<ll, ll> ret, left, right;
       left = query(lc, l, mid, x, y, val);
       right = query(rc, mid + 1, r, x, y, val);
       ret.first += left.first; ret.second += left.second;
       ret.first += right.first; ret.second += right.second;
       return ret;
```

2.7 Mo Algorithm Example

```
struct info
{
        int l, r, id;
        info(){}
        info(int l, int r, int id) : l(l), r(r), id(id){}
};

int n, t, a[2*MAX];
info Q[2*MAX];
int Block, cnt[1000004];
ll ans=0;
ll Ans[2*MAX];

inline bool comp(info a, info b)
{
        if(a.l/Block==b.l/Block) return a.r<b.r;
        return a.l<b.l;
}</pre>
```

```
inline void Add(int idx)
{
       ans+=(2*cnt[a[idx]]+1)*a[idx];
       cnt[a[idx]]++;
       /* Actual meaning of the above code
       ans-=cnt[a[idx]]*cnt[a[idx]]*a[idx];
       cnt[a[idx]]++;
       ans+=cnt[a[idx]]*cnt[a[idx]]*a[idx];
}
inline void Remove(int idx)
       ans-=(2*cnt[a[idx]]-1)*a[idx];
       cnt[a[idx]]--;
       /* Actual meaning of the above code
       ans-=cnt[a[idx]]*cnt[a[idx]]*a[idx];
       cnt[a[idx]]--;
       ans+=cnt[a[idx]]*cnt[a[idx]]*a[idx];
       */
int main()
{
       // ios_base::sync_with_stdio(0);
       // cin.tie(NULL); cout.tie(NULL);
       // freopen("in.txt","r",stdin);
       // Problem: For each query, find the value cnt[a[i]]*cnt[a[i]]*a[i]
       scanf("%d%d", &n, &t);
       Block=sqrt(n);
       FOR(i,1,n+1) a[i]=getnum();
       FOR(i,0,t)
```

```
Q[i].l=getnum();
       Q[i].r=getnum();
       Q[i].id=i;
}
sort(Q,Q+t,comp);
int Left=0, Right=-1;
FOR(i,0,t)
       while(Left<Q[i].1)</pre>
       {
               Remove(Left);
               Left++;
       }
       while(Left>Q[i].1)
       {
               Left--;
               Add(Left);
       }
       while(Right<Q[i].r)</pre>
       {
               Right++;
               Add(Right);
       }
       while(Right>Q[i].r)
       {
               Remove(Right);
               Right--;
       }
       Ans[Q[i].id]=ans;
}
FOR(i,0,t) printf("%lld\n", Ans[i]);
return 0;
```

2.8 Mo on Tree Path

}

```
int aux[MAX], b[MAX], n, m, weight[MAX], u, v;
```

```
vi graph[MAX];
int parent[MAX][17], st[MAX], en[MAX], tag = 0, dist[MAX], blocSZ;
int go[100005], lca[100005], cnt[MAX], t[MAX];
bool seen[MAX];
struct info
{
       int u, v, id;
       bool fl;
       info() {}
       info(int u, int v, int id, bool fl) : u(u), v(v), id(id), fl(fl) {
};
vector<info> Q;
// "Unordered"
void compress(int n, int *in, int *out)
       unordered_map <int, int> mp;
       for (int i = 1; i <= n; i++) out[i] = mp.emplace(in[i],</pre>
           mp.size()).first->second;
}
void dfs(int u, int p, int d)
       parent[u][0] = p;
       st[u] = ++tag;
       dist[u] = d;
       for (auto v : graph[u])
              if (v != p) dfs(v, u, d + 1);
       en[u] = ++tag;
       aux[st[u]] = u;
       aux[en[u]] = u;
}
void sparse()
{
       for (int j = 1; 1 << j < n; j++)
       {
              for (int i = 1; i <= n; i++)</pre>
                      if (parent[i][j - 1] != -1)
                             parent[i][j] = parent[parent[i][j - 1]][j -
              }
       }
}
```

```
int query(int p, int q)
{
       if (dist[p] < dist[q]) swap(p, q);</pre>
       int x = 1;
       while (true)
              if ((1 << (x + 1)) > dist[p]) break;
       }
       FORr(i, x, 0) if (dist[p] - (1 << i) >= dist[q]) p = parent[p][i];
       if (p == q) return p;
       FORr(i, x, 0)
              if (parent[p][i] != -1 && parent[p][i] != parent[q][i])
              {
                      p = parent[p][i];
                      q = parent[q][i];
              }
       return parent[p][0];
}
int ans = 0;
void doit(int idx)
       if (!seen[aux[idx]])
       {
              cnt[b[idx]]++;
              if (cnt[b[idx]] == 1) ans++;
       }
       else
       {
              cnt[b[idx]]--;
              if (cnt[b[idx]] == 0) ans--;
       seen[aux[idx]] ^= 1;
}
int main()
       // Each node has some weight associated with it
       // u v : ask for how many different integers that represent the
            weight of
       // nodes there are on the path from u to v.
       ms(parent, -1);
       scanf("%d%d", &n, &m);
       blocSZ = sqrt(n);
```

```
FOR(i, 1, n + 1)
{
       scanf("%d", &weight[i]);
FOR(i, 1, n)
       scanf("%d%d", &u, &v);
       graph[u].pb(v);
       graph[v].pb(u);
}
dfs(1, 0, 0);
sparse();
compress(n, weight, t);
(1, 1) << endl;
FOR(i, 1, 2 * n + 1) b[i] = t[aux[i]];
FOR(i, 0, m)
{
       scanf("%d%d", &u, &v);
       lca[i] = query(u, v);
       if (st[u] > st[v]) swap(u, v);
       if (lca[i] == u) Q.pb(info(st[u], st[v], i, 0));
       else Q.pb(info(en[u], st[v], i, 1));
}
sort(Q.begin(), Q.end(), [](const info & a, const info & b)->bool
       if (a.u / blocSZ == b.u / blocSZ) return a.v < b.v;</pre>
       return a.u < b.u:
});
int L = 1, R = 0;
FOR(i, 0, Q.size())
       int l = Q[i].u, r = Q[i].v, anc = lca[Q[i].id];
       while (R < r) \{ R++; doit(R); \}
       while (R > r) { doit(R); R--; }
       while (L > 1) { L--; doit(L); }
       while (L < 1) { doit(L); L++; }</pre>
       if (Q[i].fl)
       {
               if (!cnt[b[st[anc]]])
                      go[Q[i].id] = ans + 1;
               else go[Q[i].id] = ans;
       else go[Q[i].id] = ans;
```

```
}
FOR(i, 0, m) printf("%d\n", go[i]);
return 0;
}
```

2.9 Persistent Segment Tree 1

```
// Calculate how many distinct values are there in a given range
// Persistent Segment Tree implementation
// Actually used in Codeforces - The Bakery
int n, k, a[MAX], last[MAX], nxt[MAX];
int idx=1;
int Tree[64*MAX], L[64*MAX], R[64*MAX], root[2*MAX], rt[MAX];
int pos[MAX];
void build(int node, int 1, int r)
{
       if(1==r)
       {
              Tree[node]=0;
              return:
       }
       L[node]=++idx;
       R[node] = ++idx;
       // cout<<node<<" "<<L[node]<<" "<<R[node]<<endl;
       int mid=(1+r)/2;
       build(L[node],1,mid);
       build(R[node],mid+1,r);
       Tree[node]=0;
}
int update(int node, int 1, int r, int pos, int val)
{
       int x;
       x=++idx;
       if(l==r)
```

```
{
               Tree[x]=val;
               return x;
       }
       L[x]=L[node]; R[x]=R[node];
       int mid=(1+r)/2;
       if(pos<=mid) L[x]=update(L[x],1,mid,pos,val);</pre>
       else R[x]=update(R[x],mid+1,r,pos,val);
       Tree[x]=Tree[L[x]]+Tree[R[x]];
       return x;
}
int query(int node, int 1, int r, int x, int y)
{
       if(x>r || y<1) return 0;</pre>
       if(x<=1 && r<=y) return Tree[node];</pre>
       int mid=(1+r)/2;
       int q1=query(L[node],1,mid,x,y);
       int q2=query(R[node],mid+1,r,x,y);
       return q1+q2;
}
int getCost(int 1, int mid)
{
       return query(root[rt[mid]],1,n,1,mid);
}
int main()
    int test, cases=1;
    scanf("%d%d", &n, &k);
    build(1,1,n);
    root[0]=1;
    int t=1;
```

```
FOR(i,1,n+1)
   scanf("%d", &a[i]);
   int k=pos[a[i]];
   if(!k)
   {
           root[t]=update(root[t-1],1,n,i,1);
   }
   else
   {
           root[t] = update(root[t-1],1,n,k,0);
          root[t]=update(root[t-1],1,n,i,1);
   }
   rt[i]=t-1;
   pos[a[i]]=i;
}
return 0;
```

2.10 Persistent Segment Tree 2

```
const int MAXN = (1 << 20);
struct node
{
    int sum;
    node *1, *r;
    node() { 1 = nullptr; r = nullptr; sum = 0; }
    node(int x) { sum = x; 1 = nullptr; r = nullptr; }
};
typedef node* pnode;
pnode merge(pnode 1, pnode r)
{</pre>
```

```
pnode ret = new node(0);
       ret->sum = 1->sum + r->sum;
       ret->1 = 1:
       ret->r = r;
       return ret;
}
pnode init(int 1, int r)
{
       if(1 == r) { return (new node(0));}
       int mid = (1 + r) >> 1;
       return merge(init(l, mid), init(mid + 1, r));
}
pnode update(int pos, int val, int 1, int r, pnode nd)
       if(pos < 1 || pos > r) return nd;
       if(l == r) { return (new node(val)): }
       int mid = (1 + r) >> 1;
       return merge(update(pos, val, 1, mid, nd->1), update(pos, val, mid
           + 1, r, nd->r));
}
int query(int qL, int qR, int 1, int r, pnode nd)
       if(qL <= 1 && r <= qR) return nd->sum;
       if(qL > r || qR < 1) return 0;</pre>
       int mid = (1 + r) >> 1;
   return query(qL, qR, 1, mid, nd->1) + query(qL, qR, mid + 1, r,
       nd->r);
}
int get_kth(int k, int l, int r, pnode nd)
{
       if(l == r) return 1;
       int mid = (1 + r) >> 1;
       if(nd->l->sum < k) return get_kth(k - nd->l->sum, mid + 1, r,
           nd->r):
       else return get_kth(k, 1, mid, nd->1);
```

2.11 Persistent Trie

```
#include <bits/stdc++.h>
using namespace std;
// Problem: find maximum value (x^a[j]) in the range (1,r) where l<=j<=r
const int N = 1e5 + 100;
const int K = 15:
struct node_t;
typedef node_t * pnode;
struct node_t {
 int time:
 pnode to[2];
 node t() : time(0) {
   to[0] = to[1] = 0;
 bool go(int 1) const {
   if (!this) return false;
   return time >= 1;
 pnode clone() {
   pnode cur = new node_t();
   if (this) {
     cur->time = time;
     cur->to[0] = to[0];
     cur->to[1] = to[1];
   }
   return cur;
 }
};
pnode last;
pnode version[N];
void insert(int a, int time) {
 pnode v = version[time] = last = last->clone();
 for (int i = K - 1; i \ge 0; --i) {
   int bit = (a >> i) & 1;
   pnode &child = v->to[bit];
   child = child->clone();
   v = child;
```

```
v->time = time:
int query(pnode v, int x, int 1) {
  int ans = 0;
  for (int i = K - 1; i >= 0; --i) {
   int bit = (x >> i) & 1;
    if (v->to[bit]->go(1)) { // checking if this bit was inserted before
        the range
     ans |= 1 << i:
     v = v \rightarrow to[bit];
   } else {
     v = v \rightarrow to[bit ^ 1];
  }
  return ans;
void solve() {
 int n, q;
  scanf("%d %d", &n, &q);
 last = 0;
  for (int i = 0; i < n; ++i) {</pre>
   int a;
   scanf("%d", &a);
   insert(a, i);
  while (q--) {
   int x, 1, r;
    scanf("%d %d %d", &x, &l, &r);
    --1, --r;
   printf("%d\n", query(version[r], ~x, 1));
   // Trie version[r] contains the trie for [0...r] elements
 }
}
```

2.12 Range Sum Query by Lazy Propagation

```
int a[MAX + 7], tree[4 * MAX + 7], lazy[4 * MAX + 7];
void build(int node, int 1, int r)
{
    if (1 == r)
```

```
{
              tree[node] = a[1];
              return;
       }
       if (1 >= r) return;
       int mid = (1 + r) / 2;
       build(node * 2, 1, mid);
       build(node * 2 + 1, mid + 1, r);
       tree[node] = tree[node * 2] + tree[node * 2 + 1];
}
void upd(int node, int 1, int r, int v)
{
       lazy[node] += v;
       tree[node] += (r - 1 + 1) * x;
void pushDown(int node, int 1, int r) //passing update information to the
    children
{
       int mid = (1 + r) / 2;
       upd(node * 2, 1, mid, lazy[node]);
       upd(node * 2 + 1, mid + 1, r, lazy[node]);
       lazy[node] = 0;
void update(int node, int 1, int r, int x, int y, int v)
{
       if (x > r \mid | y < 1) return;
       if (x >= 1 && r <= y)
       {
              upd(node, 1, r, v);
              return:
       pushDown(node, 1, r);
       int mid = (1 + r) / 2;
       update(node * 2, 1, mid, x, y, v);
       update(node * 2 + 1, mid + 1, r, x, y, v);
       tree[node] = tree[node * 2] + tree[node * 2 + 1];
}
```

2.13 Splay Tree

```
/**
Splay Tree :
Node:
```

```
void addIt(int ad) : adding an integer in a range
       void revIt() : reversing flag
       void upd() : push_up( gather from child)
       void pushdown() : pass values to the child( like lazy propagation)
       Node* newNode(int v,Node* f) :Returns Pointer of a node whose
           parent is f, and value v
       Node* build(int 1,int r,Node* f) : building [L,R] which parent is f
       void rotate(Node* t,int d) : Rotation of Splay Tree
       void splay(Node* t, Node* f) : Splaying , t resides just below the f
       void select(int k,Node *f) : Select k th element in the tree
           ,splay it to the just below f
       Node*&get(int 1, int r): Getting The node for segment [L,R]
       void reverse(int 1,int r) : Reverse a segment
       void del(int p) : deletes entry a[p]
       void split(int l,int r,Node*&s1) : Split the array and s1 stores
           the [L,R] segment
       void cut(int 1,int r): Cut the segment [L,R] and insert in at the
       void insert(int p,int v): Insert after p,( 0 means before the
           array) an element whose value is v
       void insertRange(int pos, Node *s): Insert after pos, an segment
           denoted by s
       int query(int 1,int r): Output desired result for [L,R]
       void addRange(int 1,int r,int v): Add v to all the element in
           segment [L,R]
       void output(int 1,int r) : Output the segment [L,R]
**/
The following code answers the following queries
1 L R Output Maximum value in range [L,R]
2 L R Reverse the array [L,R]
3 L R v add v in range [L,R]
4 pos removes entry from pos
5 pos v - insert an element after position v
We assumes the initial array stored in ar[]=\{1,2,3,4...n\}
*/
typedef int T;
const int N = 2e5+50; // >= Node + Query
T ar[N];
                  // Initial Array
struct Node{
```

```
Node *ch[2],*pre; // child and parent
   T val; // Value stored in each node
   int size: //size of the subtree rooted at this node
   T mx; // additional info stored to solve problems, here maximum value
   T add;//lazy updates
   bool rev;// reverse flag
   Node(){size=0; val=mx=-1e9; add=0;}
   void addIt(T ad){
       add+=ad;
       mx+=ad:
       sum += size*ad;
       val+=ad;
   }
   void revIt(){
       rev^=1:
   }
   void upd(){
       size=ch[0]->size+ch[1]->size+1;
       mx=max(val,max(ch[0]->mx,ch[1]->mx));
       sum = ch[0] -> sum + ch[1] -> sum + val;
   }
   void pushdown();
}Tnull,*null=&Tnull;
void Node::pushdown(){
   if (add!=0){
       for (int i=0;i<2;++i)</pre>
           if (ch[i]!=null) ch[i]->addIt(add);
       add = 0;
   }
   if (rev){
       swap(ch[0],ch[1]);
       for (int i=0;i<2;i++)</pre>
           if (ch[i]!=null) ch[i]->revIt();
       rev = 0;
   }
}
struct Splay{
   Node nodePool[N],*cur; // Static Memory and cur pointer
   Node* root; // root of the splay tree
   Splay(){
       cur=nodePool;
       root=null;
   }
```

```
void clear(){
   cur=nodePool;
   root=null;
Node* newNode(T v,Node* f){
   cur->ch[0]=cur->ch[1]=null;
   cur->size=1;
   cur->val=v:
   cur->mx=v; cur->sum = 0;
   cur->add=0;
   cur->rev=0:
   cur->pre=f;
   return cur++;
Node* build(int 1,int r,Node* f){
   if(l>r) return null;
   int m=(1+r)>>1:
   Node* t=newNode(ar[m],f);
   t \rightarrow ch[0] = build(1, m-1, t);
   t->ch[1]=build(m+1,r,t);
   t->upd();
   return t;
}
void rotate(Node* x,int c){
   Node* y=x->pre;
   y->pushdown();
   x->pushdown();
   y->ch[!c]=x->ch[c];
   if (x->ch[c]!=null) x->ch[c]->pre=y;
   x->pre=y->pre;
   if (y->pre!=null)
       if (y->pre->ch[0]==y) y->pre->ch[0]=x;
       else y->pre->ch[1]=x;
   x->ch[c]=y;
   y->pre=x;
   y->upd();
   if (y==root) root=x;
}
void splay(Node* x,Node* f){
```

```
x->pushdown();
   while (x->pre!=f){
       if (x->pre->pre==f){
          if (x->pre->ch[0]==x) rotate(x,1);
          else rotate(x,0);
       }else{
          Node *y=x->pre,*z=y->pre;
          if (z->ch[0]==y){
              if (y->ch[0]==x) rotate(y,1),rotate(x,1);
              else rotate(x,0),rotate(x,1);
          }else{
              if (y->ch[1]==x) rotate(y,0),rotate(x,0);
              else rotate(x,1),rotate(x,0);
          }
       }
   }
   x->upd();
}
void select(int k,Node* f){
   int tmp;
   Node* x=root;
   x->pushdown();
   k++;
   for(::){
       x->pushdown();
       tmp=x->ch[0]->size;
       if (k==tmp+1) break;
       if (k<=tmp) x=x->ch[0];
       else{
          k-=tmp+1;
          x=x->ch[1];
       }
   }
   splay(x,f);
}
Node*&get(int 1, int r){
   select(1-1,null);
   select(r+1,root);
   return root->ch[1]->ch[0];
}
void reverse(int l,int r){
   Node* o=get(1,r);
   o->rev^=1;
```

```
splay(o,null);
void del(int p)
       select(p-1,null);
       select(p+1,root);
       root->ch[1]->ch[0] = null;
       splay(root->ch[1],null);
void split(int l,int r,Node*&s1)
   Node* tmp=get(1,r);
   root->ch[1]->ch[0]=null;
   root->ch[1]->upd();
   root->upd();
   s1=tmp;
void cut(int 1,int r)
   Node* tmp;
   split(l,r,tmp);
   select(root->size-2,null);
   root->ch[1]->ch[0]=tmp;
   tmp->pre=root->ch[1];
   root->ch[1]->upd();
   root->upd();
}
void init(int n){
   clear();
   root=newNode(0,null);
   root->ch[1]=newNode(n+1,root);
   root->ch[1]->ch[0]=build(1,n,root->ch[1]);
   splay(root->ch[1]->ch[0],null);
}
void insertPos(int pos,T v)
       select(pos,null);
       select(pos+1,root);
       root \rightarrow ch[1] \rightarrow ch[0] = newNode(v,root \rightarrow ch[1]);
       splay(root->ch[1]->ch[0],null);
void insertRange(int pos,Node *s)
```

```
{
       select(pos,null);
       select(pos+1,root);
       root->ch[1]->ch[0] = s;
       s->pre = root->ch[1];
       root->ch[1]->upd();
       root->upd();
    }
    T query(int 1,int r)
           Node *o = get(1,r);
           return o->mx;
    }
    void addRange(int 1,int r,T v)
           Node *o = get(1,r);
           o->add += v;
           o \rightarrow val += v;
           o->sum += o->size * v;
           splay(o,null);
   }
    void output(int 1,int r){
       for (int i=1;i<=r;i++){</pre>
           select(i,null);
           cout<<root->val<<endl;</pre>
       };
    }
}St;
int main()
{
    int n,m,a,b,c;
    scanf("%d%d", &n, &m);
    for(int i= 1;i <= n;i ++ ) ar[i] = i;</pre>
    St.init(n);
    FOR(i,1,m+1)
       scanf("%d%d", &a, &b);
```

```
St.cut(a,b);
}
St.output(1,n);
return 0;
}
```

3 Game

3.1 Green Hacenbush

```
// Green Hackenbush
vi graph[505];
int go(int u, int p)
{
       int ret = 0;
       for (auto &v : graph[u])
              if (v == p) continue;
              ret = (go(v, u) + 1);
       }
       return ret;
int u, v, n;
int main()
{
// ios_base::sync_with_stdio(0);
// cin.tie(NULL); cout.tie(NULL);
// freopen("in.txt","r",stdin);
       int test, cases = 1;
       cin >> test:
       while (test--)
       {
              cin >> n;
              FOR(i, 0, n - 1)
              {
                     cin >> u >> v;
                     graph[u].pb(v);
                     graph[v].pb(u);
              }
```

```
if (go(1, 0)) puts("Alice");
    else puts("Bob");
    FOR(i, 1, n + 1) graph[i].clear();
}
return 0;
}
```

3.2 Green Hackenbush 2

```
//
// Green Hackenbush
//
// Description:
    Consider a two player game on a graph with a specified vertex (root).
    In each turn, a player eliminates one edge.
    Then, if a subgraph that is disconnected from the root, it is
    removed.
// If a player cannot select an edge (i.e., the graph is singleton),
    he will lose.
//
    Compute the Grundy number of the given graph.
// Algorithm:
    We use two principles:
      1. Colon Principle: Grundy number of a tree is the xor of
//
         Grundy number of child subtrees.
//
         (Proof: easy).
      2. Fusion Principle: Consider a pair of adjacent vertices u, v
         that has another path (i.e., they are in a cycle). Then,
//
         we can contract u and v without changing Grundy number.
//
         (Proof: difficult)
//
// We first decompose graph into two-edge connected components.
    Then, by contracting each components by using Fusion Principle,
    we obtain a tree (and many self loops) that has the same Grundy
    number to the original graph. By using Colon Principle, we can
    compute the Grundy number.
//
// Complexity:
// O(m + n).
//
// Verified:
```

```
// SPOJ 1477: Play with a Tree
    IPSC 2003 G: Got Root?
//
#include <iostream>
#include <vector>
#include <cstdio>
#include <algorithm>
#include <functional>
using namespace std;
#define fst first
#define snd second
#define all(c) ((c).begin()), ((c).end())
#define TEST(s) if (!(s)) { cout << __LINE__ << " " << #s << endl;</pre>
    exit(-1); }
struct hackenbush {
       int n;
       vector<vector<int>> adj;
       hackenbush(int n) : n(n), adj(n) { }
       void add_edge(int u, int v) {
              adj[u].push_back(v);
              if (u != v) adj[v].push_back(u);
       }
       // r is the only root connecting to the ground
       int grundy(int r) {
              vector<int> num(n), low(n);
              int t = 0;
              function<int(int, int)> dfs = [&](int p, int u) {
                     num[u] = low[u] = ++t;
                      int ans = 0;
                      for (int v : adj[u]) {
                             if (v == p) { p += 2 * n; continue; }
                             if (num[v] == 0) {
                                    int res = dfs(u, v);
                                    low[u] = min(low[u], low[v]);
                                    if (low[v] > num[u]) ans ^= (1 + res)
                                         ^ 1; // bridge
                                     else
                                                        ans ^= res;
                                                 // non bridge
                             } else low[u] = min(low[u], num[v]);
```

```
}
                      if (p > n) p = 2 * n;
                      for (int v : adj[u])
                              if (v != p && num[u] <= num[v]) ans ^= 1;</pre>
                      return ans;
               };
               return dfs(-1, r);
       }
};
int main() {
       int cases; scanf("%d", &cases);
       for (int icase = 0; icase < cases; ++icase) {</pre>
               int n; scanf("%d", &n);
               vector<int> ground(n);
               int r;
               for (int i = 0; i < n; ++i) {</pre>
                      scanf("%d", &ground[i]);
                      if (ground[i] == 1) r = i;
               }
               int ans = 0;
               hackenbush g(n);
               for (int i = 0; i < n - 1; ++i) {</pre>
                      int u, v;
                      scanf("%d %d", &u, &v);
                      --u: --v:
                      if (ground[u]) u = r;
                      if (ground[v]) v = r;
                      if (u == v) ans ^= 1;
                      else g.add_edge(u, v);
               int res = ans ^ g.grundy(r);
               printf("%d\n", res != 0);
       }
```

4 Geometry

4.1 Convex Hull

```
struct PT
{
```

```
int x, y;
       PT(){}
       PT(int x, int y) : x(x), y(y) {}
       bool operator < (const PT &P) const</pre>
              return x<P.x || (x==P.x && y<P.y);
       }
};
11 cross(const PT p, const PT q, const PT r)
       return (11)(q.x-p.x)*(11)(r.y-p.y)-(11)(q.y-p.y)*(11)(r.x-p.x);
}
vector<PT> Points, Hull;
void findConvexHull()
       int n=Points.size(), k=0;
       SORT(Points);
       // Build lower hull
       FOR(i,0,n)
               while(Hull.size()>=2 &&
                   cross(Hull[Hull.size()-2],Hull.back(),Points[i])<=0)</pre>
               {
                      Hull.pop_back();
                      k--;
               Hull.pb(Points[i]);
               k++;
       }
       // Build upper hull
       for(int i=n-2, t=k+1; i>=0; i--)
               while(Hull.size()>=t &&
                   cross(Hull[Hull.size()-2],Hull.back(),Points[i])<=0)</pre>
```

4.2 Counting Closest Pair of Points

```
int n;
struct Points
       double x, y;
       Points() {}
       Points(double x, double y) : x(x), y(y) { }
       bool operator<(const Points &a) const</pre>
               return x < a.x;</pre>
};
bool comp1(const Points &a, const Points &b)
       return a.x < b.x;</pre>
bool comp2(const Points &a, const Points &b)
       return a.y < b.y;</pre>
void printPoint(Points a)
       cout << a.x << " " << a.y << endl;
Points P[10005];
typedef set<Points, bool(*)(const Points&, const Points&)> setType;
typedef setType::iterator setIT;
setType s(&comp2);
double euclideanDistance(const Points &a, const Points &b)
// prnt((double)(a.x-b.x)*(a.x-b.x)+(a.y-b.y)*(a.y-b.y));
       return (a.x - b.x) * (a.x - b.x) + (a.y - b.y) * (a.y - b.y);
```

```
map<double, map<double, int> > CNT;
int main()
// ios_base::sync_with_stdio(0);
// cin.tie(NULL); cout.tie(NULL);
// freopen("in.txt","r",stdin);
       while ((cin >> n) && n)
              FOR(i, 0, n) cin >> P[i].x >> P[i].y;
              sort(P, P + n, comp1);
              FOR(i, 0, n)
// printPoint(P[i]);
                      s.insert(P[i]);
                      CNT[P[i].x][P[i].y]++;
// To check repeated points :/
// for(auto it: s) printPoint(it);
              double ans = 10000;
              int idx = 0;
              FOR(j, 0, n)
// cout<<"Point now: "; printPoint(P[j]);</pre>
                      if (CNT[P[j].x][P[j].y] > 1) ans = 0;
                      Points it = P[j];
                      while (it.x - P[idx].x > ans)
                             s.erase(P[idx]);
                             idx++;
                      Points low = Points(it.x, it.y - ans);
                      Points high = Points(it.x, it.y + ans);
                      setIT lowest = s.lower_bound(low);
                      if (lowest != s.end())
                              setIT highest = s.upper_bound(high);
                             for (setIT now = lowest; now != highest;
                                  now++)
                              {
                                     double cur = sqrt(euclideanDistance
                                                      (*now, it));
// prnt(cur);
                                     if (cur == 0) continue;
// cout<<"Here:"<<endl;</pre>
```

4.3 Maximum Points to Enclose in a Circle of Given Radius with Angular Sweep

```
typedef pair<double,bool> pdb;

#define START 0
#define END 1

struct PT
{
          double x, y;
          PT() {}
          PT(double x, double y) : x(x), y(y) {}
          PT(const PT &p) : x(p.x), y(p.y) {}
          PT operator + (const PT &p) const { return PT(x+p.x, y+p.y); }
          PT operator - (const PT &p) const { return PT(x-p.x, y-p.y); }
          PT operator * (double c) const { return PT(x*c, y*c); }
          PT operator / (double c) const { return PT(x/c, y/c); }
};

PT p[505];
double dist[505][505];
```

```
int n, m;
void calcDist()
       FOR(i,0,n)
              FOR(j,i+1,n)
                     dist[i][j]=dist[j][i]=sqrt((p[i].x-p[j].x)*(p[i].x-p[j].x)
                             +(p[i].y-p[j].y)*(p[i].y-p[j].y));
       }
}
// Returns maximum number of points enclosed by a circle of radius
// where the circle is pivoted on point 'point'
// 'point' is on the circumfurence of the circle
int intelInside(int point, double radius)
{
       vector<pdb> ranges;
       FOR(j,0,n)
              if(j==point || dist[j][point]>2*radius) continue;
              double a1=atan2(p[point].y-p[j].y,p[point].x-p[j].x);
              double a2=acos(dist[point][j]/(2*radius));
              ranges.pb({a1-a2,START});
              ranges.pb({a1+a2,END});
       }
       sort(ALL(ranges));
       int cnt=1, ret=cnt;
       for(auto it: ranges)
              if(it.second) cnt--;
              else cnt++;
              ret=max(ret,cnt);
       }
       return ret;
```

```
// returns maximum amount of points enclosed by the circle of radius r
// Complexity: O(n^2*log(n))

int go(double r)
{
    int cnt=0;
    FOR(i,0,n)
    {
        cnt=max(cnt,intelInside(i,r));
    }
    return cnt;
}
```

4.4 Point in Polygon Binary Search

```
int sideOf(const PT &s, const PT &e, const PT &p)
{
       ll a = cross(e-s,p-s);
       return (a > 0) - (a < 0);
}
bool onSegment(const PT &s, const PT &e, const PT &p)
{
       PT ds = p-s, de = p-e;
       return cross(ds,de) == 0 && dot(ds,de) <= 0;
}
Main routine
Description: Determine whether a point t lies inside a given polygon
    (counter-clockwise order).
The polygon must be such that every point on the circumference is visible
    from the first point in the vector.
It returns 0 for points outside, 1 for points on the circumference, and 2
    for points inside.
*/
int insideHull2(const vector<PT> &H, int L, int R, const PT &p) {
       int len = R - L:
       if (len == 2) {
```

```
int sa = sideOf(H[0], H[L], p);
              int sb = sideOf(H[L], H[L+1], p);
              int sc = sideOf(H[L+1], H[0], p);
              if (sa < 0 || sb < 0 || sc < 0) return 0;</pre>
              if (sb==0 || (sa==0 && L == 1) || (sc == 0 && R ==
                   (int)H.size()))
                      return 1;
              return 2:
       }
       int mid = L + len / 2;
       if (sideOf(H[0], H[mid], p) >= 0)
              return insideHull2(H, mid, R, p);
       return insideHull2(H, L, mid+1, p);
}
int insideHull(const vector<PT> &hull, const PT &p) {
       if ((int)hull.size() < 3) return onSegment(hull[0], hull.back(),</pre>
       else return insideHull2(hull, 1, (int)hull.size(), p);
```

4.5 Rectangle Union

```
struct info
{
    int x, ymin, ymax, type;
    info(){}
    info(int x, int ymin, int ymax, int type) :
        x(x), ymin(ymin), ymax(ymax), type(type) { }

    bool operator < (const info &p) const
    {
        return x<p.x;
    }
};

vector<info> in;
int n, x, y, p, q, m;
vi take;
int Lazy[4*MAX], Tree[4*MAX];

void update(int node, int l, int r, int ymin, int ymax, int val)
{
```

```
if(take[1]>ymax || take[r]<ymin) return;</pre>
       if(ymin<=take[1] && take[r]<=ymax)</pre>
               Lazy[node] +=val;
               if(Lazy[node]) Tree[node]=take[r]-take[1];
               else Tree[node] = Tree[lc] + Tree[rc];
               return:
       }
       if(l+1>=r) return;
       int mid=(1+r)/2;
       update(lc,1,mid,ymin,ymax,val);
       update(rc,mid,r,ymin,ymax,val);
       if(Lazy[node]) Tree[node]=take[r]-take[l];
       else Tree[node] = Tree[lc] + Tree[rc];
}
ll solve()
{
       take.clear(); ms(Tree,0); ms(Lazy,0);
       take.pb(-1);
       FOR(i,0,in.size())
               take.pb(in[i].ymin);
               take.pb(in[i].ymax);
       }
       SORT(take);
       take.erase(unique(ALL(take)),take.end());
       m=take.size()-1;
       // VecPrnt(take);
       update(1,1,m,in[0].ymin,in[0].ymax,in[0].type);
       int prv=in[0].x; ll ret=0;
       FOR(i,1,in.size())
```

```
{
              ret+=(11)(in[i].x-prv)*Tree[1];
              prv=in[i].x;
              update(1,1,m,in[i].ymin,in[i].ymax,in[i].type);
       }
       return ret;
}
int main()
   // ios_base::sync_with_stdio(0);
   // cin.tie(NULL); cout.tie(NULL);
   // freopen("in.txt","r",stdin);
   int test, cases=1;
   scanf("%d", &test);
   while(test--)
       scanf("%d", &n);
       in.clear();
       FOR(i,0,n)
       {
              scanf("%d%d%d%d", &x, &y, &p, &q);
              in.pb(info(x,y,q,1));
              in.pb(info(p,y,q,-1));
           }
       SORT(in);
       11 ans=solve();
       printf("Case %d: %lld\n", cases++, ans);
   return 0;
```

4.6 Stanford ACM Team Geometry

```
double INF = 1e100;
double EPS = 1e-12;
struct PT {
 double x, y;
 PT() {}
 PT(double x, double y) : x(x), y(y) {}
 PT(const PT &p) : x(p.x), y(p.y) {}
 PT operator + (const PT &p) const { return PT(x+p.x, y+p.y); }
 PT operator - (const PT &p) const { return PT(x-p.x, y-p.y); }
 PT operator * (double c) const { return PT(x*c, y*c ); }
 PT operator / (double c) const { return PT(x/c, y/c ); }
}:
double dot(PT p, PT q) { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q) { return dot(p-q,p-q); }
double cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {
 return os << "(" << p.x << "," << p.y << ")";
}
// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p) { return PT(-p.y,p.x); }
PT RotateCW90(PT p) { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
 return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
}
// project point c onto line through a and b
// assuming a != b
PT ProjectPointLine(PT a, PT b, PT c) {
 return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
}
// project point c onto line segment through a and b
PT ProjectPointSegment(PT a, PT b, PT c) {
 double r = dot(b-a,b-a);
 if (fabs(r) < EPS) return a;</pre>
 r = dot(c-a, b-a)/r;
 if (r < 0) return a;</pre>
 if (r > 1) return b:
 return a + (b-a)*r:
}
```

```
// compute distance from c to segment between a and b
double DistancePointSegment(PT a, PT b, PT c) {
 return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
// compute distance between point (x,y,z) and plane ax+by+cz=d
double DistancePointPlane(double x, double y, double z,
                       double a, double b, double c, double d)
 return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
// determine if lines from a to b and c to d are parallel or collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
 return fabs(cross(b-a, c-d)) < EPS:</pre>
bool LinesCollinear(PT a, PT b, PT c, PT d) {
 return LinesParallel(a, b, c, d)
     && fabs(cross(a-b, a-c)) < EPS
     && fabs(cross(c-d, c-a)) < EPS;
}
// determine if line segment from a to b intersects with
// line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
 if (LinesCollinear(a, b, c, d)) {
   if (dist2(a, c) < EPS || dist2(a, d) < EPS ||</pre>
     dist2(b, c) < EPS || dist2(b, d) < EPS) return true;</pre>
   if (dot(c-a, c-b) > 0 \&\& dot(d-a, d-b) > 0 \&\& dot(c-b, d-b) > 0)
     return false:
   return true;
 if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return false;
 if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return false;
 return true:
// compute intersection of line passing through a and b
// with line passing through c and d, assuming that unique
// intersection exists; for segment intersection, check if
// segments intersect first
PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
  b=b-a; d=c-d; c=c-a;
```

```
assert(dot(b, b) > EPS && dot(d, d) > EPS);
 return a + b*cross(c, d)/cross(b, d);
}
// compute center of circle given three points
PT ComputeCircleCenter(PT a, PT b, PT c) {
 b=(a+b)/2;
 c=(a+c)/2:
 return ComputeLineIntersection(b, b+RotateCW90(a-b), c,
      c+RotateCW90(a-c));
}
// determine if point is in a possibly non-convex polygon (by William
// Randolph Franklin); returns 1 for strictly interior points, 0 for
// strictly exterior points, and 0 or 1 for the remaining points.
// Note that it is possible to convert this into an *exact* test using
// integer arithmetic by taking care of the division appropriately
// (making sure to deal with signs properly) and then by writing exact
// tests for checking point on polygon boundary
bool PointInPolygon(const vector<PT> &p, PT q) {
 bool c = 0;
 for (int i = 0; i < p.size(); i++){</pre>
   int j = (i+1)%p.size();
   if ((p[i].y <= q.y && q.y < p[j].y ||</pre>
     p[j].y \le q.y && q.y \le p[i].y) &&
     q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y -
         p[i].y))
     c = !c;
 }
 return c;
}
// determine if point is on the boundary of a polygon
bool PointOnPolygon(const vector<PT> &p, PT q) {
 for (int i = 0; i < p.size(); i++)</pre>
   if (dist2(ProjectPointSegment(p[i], p[(i+1)%p.size()], q), q) < EPS)</pre>
     return true;
   return false;
}
// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<PT> CircleLineIntersection(PT a, PT b, PT c, double r) {
 vector<PT> ret:
 b = b-a;
```

```
a = a-c:
  double A = dot(b, b);
  double B = dot(a, b);
  double C = dot(a, a) - r*r;
  double D = B*B - A*C;
  if (D < -EPS) return ret;</pre>
 ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
  if (D > EPS)
   ret.push_back(c+a+b*(-B-sqrt(D))/A);
 return ret;
// compute intersection of circle centered at a with radius r
// with circle centered at b with radius R
vector<PT> CircleCircleIntersection(PT a, PT b, double r, double R) {
  vector<PT> ret:
  double d = sqrt(dist2(a, b));
 if (d > r+R || d+min(r, R) < max(r, R)) return ret;</pre>
  double x = (d*d-R*R+r*r)/(2*d);
  double y = sqrt(r*r-x*x);
  PT v = (b-a)/d;
 ret.push_back(a+v*x + RotateCCW90(v)*y);
 if (y > 0)
   ret.push_back(a+v*x - RotateCCW90(v)*y);
 return ret;
// This code computes the area or centroid of a (possibly nonconvex)
// polygon, assuming that the coordinates are listed in a clockwise or
// counterclockwise fashion. Note that the centroid is often known as
// the "center of gravity" or "center of mass".
double ComputeSignedArea(const vector<PT> &p) {
  double area = 0;
 for(int i = 0; i < p.size(); i++) {</pre>
   int j = (i+1) % p.size();
   area += p[i].x*p[j].y - p[j].x*p[i].y;
 return area / 2.0;
double ComputeArea(const vector<PT> &p) {
 return fabs(ComputeSignedArea(p));
}
PT ComputeCentroid(const vector<PT> &p) {
```

```
PT c(0,0);
 double scale = 6.0 * ComputeSignedArea(p);
 for (int i = 0; i < p.size(); i++){</pre>
   int j = (i+1) % p.size();
   c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
 }
 return c / scale;
}
// tests whether or not a given polygon (in CW or CCW order) is simple
bool IsSimple(const vector<PT> &p) {
 for (int i = 0; i < p.size(); i++) {</pre>
   for (int k = i+1; k < p.size(); k++) {</pre>
     int j = (i+1) % p.size();
     int 1 = (k+1) % p.size();
     if (i == 1 || j == k) continue;
     if (SegmentsIntersect(p[i], p[j], p[k], p[1]))
       return false;
   }
 }
 return true;
```

5 Graph

5.1 Articulation Points and Bridges

```
vi graph[100];
int dfs_num[100], dfs_low[100], parent[100], cnt;
int dfsroot, rootchild;
int art_v[100];

void articulate(int u)
{
         dfs_low[u]=dfs_num[u]=cnt++;
         for (ul j=0; j<graph[u].size(); j++)
         {
             int v=graph[u][j];
            if (dfs_num[v]==-1)
            {
                  parent[v]=u;
                  if (u==dfsroot)</pre>
```

```
rootchild++;
                       articulate(v);
                       if (dfs_low[v]>=dfs_num[u])
                               art_v[u]=true;
                       if (dfs_low[v]>dfs_num[u])
                               cout<<"Edge "<<u<<" & "<<v<<" is a
                                   bridge."<<endl;</pre>
                       dfs_low[u]=min(dfs_low[u],dfs_low[v]);
               else if (v!=parent[u])
                       dfs_low[u]=min(dfs_low[u],dfs_num[v]);
       }
}
int main()
{
       int n, m, u, v;
       cin>>n>>m;
       for (int i=0; i<m; i++)</pre>
               cin>>u>>v;
               graph[u].pb(v);
               graph[v].pb(u);
       }
       cnt=0;
       ms(dfs_num,-1);
       for (int i=0; i<n; i++)</pre>
               if (dfs_num[i] ==-1)
               {
                       dfsroot=i;
                       rootchild=0:
                       articulate(i);
                       art_v[dfsroot] = (rootchild>1);
               }
       }
       prnt("Articulation points:");
       for (int i=0; i<n; i++)</pre>
               if (art_v[i])
                       cout<<"Vertex: "<<i<<endl;</pre>
       }
       return 0;
```

5.2 BCC

```
struct MagicComponents {
       struct edge {
              ll u, v, id;
       };
       ll num, n, edges;
       vector<ll> dfs_num, low, vis;
       vector<ll> cuts; // art-vertices
       vector<edge> bridges; // bridge-edges
       vector<vector<edge>> adj; // graph
       vector<vector<edge>> bccs; // all the bccs where bcc[i] has all
           the edges inside it
       deque<edge> e_stack;
       // Nodes are numberd from 0
       MagicComponents(const ll& _n) : n(_n) {
              adj.assign(n, vector<edge>());
              edges = 0;
       }
       void add_edge(const ll& u, const ll& v) {
              adj[u].push_back({u,v,edges});
              adj[v].push_back({v,u,edges++});
       }
       void run(void) {
              vis.assign(n, 0);
              dfs_num.assign(n, 0);
              low.assign(n, 0);
              bridges.clear();
              cuts.clear():
              bccs.clear();
              e_stack = deque<edge>();
              num = 0;
              for (ll i = 0; i < n; ++i) {</pre>
                     if (vis[i]) continue;
                     dfs(i, -1);
              }
       }
```

```
void dfs(const ll% node, const ll% par) {
       dfs num[node] = low[node] = num++:
       vis[node] = 1;
       ll n_child = 0;
       for (edge& e : adj[node]) {
              if (e.v == par) continue;
              if (vis[e.v] == 0) {
                     ++n_child;
                      e_stack.push_back(e);
                      dfs(e.v, node);
                      low[node] = min(low[node], low[e.v]);
                      if (low[e.v] >= dfs_num[node]) {
                             if (dfs_num[node] > 0 || n_child > 1)
                                    cuts.push_back(node);
                             if (low[e.v] > dfs_num[node]) {
                                    bridges.push_back(e);
                                    pop(node);
                             } else pop(node);
                     }
              } else if (vis[e.v] == 1) {
                     low[node] = min(low[node], dfs_num[e.v]);
                      e_stack.push_back(e);
              }
       }
       vis[node] = 2;
}
void pop(const ll& u) {
       vector<edge> list;
       for (;;) {
              edge e = e_stack.back();
              e_stack.pop_back();
              list.push_back(e);
              if (e.u == u) break;
       bccs.push_back(list);
}
//# Make sure to call run before calling this function.
// Function returns a new graph such that all two connected
// components are compressed into one node and all bridges
// in the previous graph are the only edges connecting the
```

```
// components in the new tree.
// map is an integer array that will store the mapping
// for each node in the old graph into the new graph. //$
MagicComponents component_tree(vector<11>& map) {
       vector<char> vis(edges);
       for (const edge& e : bridges)
              vis[e.id] = true;
       11 \text{ num\_comp} = 0;
       map.assign(map.size(), -1);
       for (ll i = 0; i < n; ++i) {</pre>
              if (map[i] == -1) {
                      deque<11> q;
                      q.push_back(i);
                      map[i] = num_comp;
                      while (!q.empty()) {
                             11 node = q.front();
                              q.pop_front();
                              for (const edge& e : adj[node]) {
                                     if (!vis[e.id] && map[e.v] ==
                                          -1) {
                                             vis[e.id] = true;
                                             map[e.v] = num_comp;
                                             q.push_back(e.v);
                                     }
                             }
                      }
               ++num_comp;
       }
       MagicComponents g(num_comp);
       vis.assign(vis.size(), false);
       for (ll i = 0; i < n; ++i) {</pre>
               for (const edge& e : adj[i]) {
                      if (!vis[e.id] && map[e.v] < map[e.u]) {</pre>
                              vis[e.id] = true;
                              g.add_edge(map[e.v], map[e.u]);
                      }
              }
       return g;
}
//# Make sure to call run before calling this function.
```

```
// Function returns a new graph such that all biconnected
       // components are compressed into one node. Cut nodes will
       // be in multiple components, so these nodes will also have
       // their own component by themselves. Edges in the graph
       // represent components to articulation points
       // map is an integer array that will store the mapping
       // for each node in the old graph into the new graph.
       // Cut points to their special component, and every other node
       // to their specific component. //$
       MagicComponents bcc_tree(vector<11>& map) {
              vector<ll> cut(n, -1):
              11 size = bccs.size();
              for (const auto& i : cuts)
                     map[i] = cut[i] = size++;
              MagicComponents g(size);
              vector<ll> used(n);
              for (ll i = 0; i < bccs.size(); ++i) {</pre>
                     for (const edge& e : bccs[i]) {
                             vector<ll> tmp = {e.u,e.v};
                             for (const ll& node : tmp) {
                                    if (used[node] != i+1) {
                                           used[node] = i+1;
                                           if (cut[node] != -1)
                                                   g.add_edge(i,
                                                       cut[node]);
                                           else map[node] = i;
                                    }
                             }
                     }
              return g;
       }
};
```

5.3 Bridges and Arts

```
struct MagicComponents {
    struct edge {
         ll u, v, id;
    };
```

```
ll num, n, edges;
vector<ll> dfs_num, low, vis;
vector<ll> cuts; // art-vertices
vector<edge> bridges; // bridges
vector<vector<edge>> adj; // graph
vector<vector<edge>> bccs; // contains the bccs, each bccs[i]
    contiains all the edges in a bcc
deque<edge> e_stack;
MagicComponents(const ll& _n) : n(_n) {
       adj.assign(n, vector<edge>());
       edges = 0;
}
void add_edge(const ll& u, const ll& v) {
       adj[u].push_back({u,v,edges});
       adj[v].push_back({v,u,edges++});
}
void run(void) {
       vis.assign(n, 0);
       dfs_num.assign(n, 0);
       low.assign(n, 0);
       bridges.clear();
       cuts.clear();
       bccs.clear():
       e_stack = deque<edge>();
       num = 0;
       for (ll i = 0; i < n; ++i) {</pre>
              if (vis[i]) continue;
              dfs(i, -1);
       }
}
void dfs(const ll& node, const ll& par) {
       dfs_num[node] = low[node] = num++;
       vis[node] = 1;
       ll n_child = 0;
       for (edge& e : adj[node]) {
              if (e.v == par) continue;
              if (vis[e.v] == 0) {
                      ++n_child;
                      e_stack.push_back(e);
```

```
dfs(e.v, node);
                             low[node] = min(low[node], low[e.v]);
                             if (low[e.v] >= dfs_num[node]) {
                                    if (dfs_num[node] > 0 || n_child > 1)
                                            cuts.push_back(node);
                                    if (low[e.v] > dfs_num[node]) {
                                            bridges.push_back(e);
                                            pop(node);
                                    } else pop(node);
                             }
                      } else if (vis[e.v] == 1) {
                             low[node] = min(low[node], dfs_num[e.v]);
                             e_stack.push_back(e);
                      }
              }
              vis[node] = 2;
       }
       void pop(const ll& u) {
              vector<edge> list;
              for (;;) {
                      edge e = e_stack.back();
                      e_stack.pop_back();
                      list.push_back(e);
                      if (e.u == u) break;
              }
              bccs.push_back(list);
       }
};
```

5.4 Dijkstra!

```
struct road
{
    int u, w;
    road (int a, int b)
    {
        u=a; w=b;
    }
    bool operator < (const road & p) const
    {
        return w>p.w;
}
```

```
}
};
int d[100], parent[100], start, end;
mvii g, cost;
void dijkstra (int n)
       ms(d,INF);
       ms(parent,-1);
       priority_queue <road> Q;
       Q.push(road(start,0));
       d[start]=0;
       while (!Q.empty())
       {
               road t=Q.top();
               Q.pop();
               int u=t.u;
               for (ul i=0; i<g[u].size(); i++)</pre>
                       int v=g[u][i];
                       if (d[u]+cost[u][i]<d[v])</pre>
                       {
                               d[v]=d[u]+cost[u][i];
                              parent[v]=u;
                               Q.push(road(v,d[v]));
                       }
               }
       return;
}
int main()
{
       int n, m, road_out, road_cost, cases=1;
       while (scanf("%d", &n) && n)
               for (int i=1; i<=n; i++)</pre>
                       cin>>m;
                       for (int j=1; j<=m; j++)</pre>
                               cin>>road_out>>road_cost;
                              g[i].pb(road_out);
                               cost[i].pb(road_cost);
```

```
}
       }
       scanf("%d%d", &start, &end);
       dijkstra(n);
       //cout<<d[end]<<endl;</pre>
       g.clear(); cost.clear();
       int current=end;
       vi path;
       while (current!=start)
              path.pb(parent[current]);
              current=parent[current];
       printf("Case %d: Path = ", cases++);
       for (int j=(int)path.size()-1; j>-1; j--)
              cout<<path[j]<<" ";
       printf("%d; %d second delay\n", end, d[end]);
}
return 0;
```

5.5 Dominator Tree

```
// Problem: LightOJ Sabotaging Contest
// n - number of cities, m - number of edges, (u,v,t) - edge and cost
// Each of the q lines gives a query of k cities n[1], n[2], \ldots, n[k];
// We have to find the number of nodes where if any one of them is
    removed, the
// shortest path to 0 from n[1]...n[k] will be increased. We also have to
// the number of nodes which will be affected by such removal.
/* Solution
       Run Dijkstra, build shortest path dag, take topsort order and
       according to the reversed order add one edge at a time to build
           dominator tree
       Finally, run dfs to find the level of each node and subtree size.
           Answer is the
       (level of the lca of the nodes n[1]...n[k] + 1) and subtree size
           of this ancestor
*/
```

```
vi graph[MAX], cost[MAX], dag[MAX], parent[MAX], Tree[MAX];
int u, v, t, n, m;
int dist[MAX];
vector<int> all;
int L[MAX], table[MAX][18], sub[MAX];
bool visited[MAX];
void clear()
{
       FOR(i,0,n)
               graph[i].clear();
               cost[i].clear();
               dag[i].clear();
               parent[i].clear();
              Tree[i].clear();
               sub[i]=0;
       all.clear();
       ms(table,-1);
       ms(visited, false);
}
void dfs(int u)
{
       sub[u]++;
       FOR(j,0,Tree[u].size())
               int v=Tree[u][j];
               dfs(v);
               sub[u]+=sub[v];
       }
}
int query(int p, int q)
{
       if(L[p]<L[q]) swap(p,q);</pre>
       int x=1;
       while(true)
               if((1<<(x+1))>L[p])
                      break;
```

```
x++;
       }
       FORr(i,x,0)
       {
              if(L[p]-(1<<i) >= L[q])
                      p=table[p][i];
       }
       if(p==q) return p;
       FORr(i,x,0)
              if(table[p][i]!=-1 && table[p][i]!=table[q][i])
              {
                      p=table[p][i];
                      q=table[q][i];
              }
       }
       return table[p][0];
}
void build(int curr)
       for(int j=1; (1<<j) < n; j++)</pre>
       {
              if(table[curr][j-1]!=-1)
                      table[curr][j]=table[table[curr][j-1]][j-1];
       }
}
void dijkstra()
{
       priority_queue<pii,vpii,greater<pii> > PQ;
       PQ.push(pii(0,0));
       FOR(i,0,n) dist[i]=INF;
       dist[0]=0;
       while(!PQ.empty())
              pii t=PQ.top();
              PQ.pop();
              int u=t.second;
```

```
FOR(j,0,graph[u].size())
                      int v=graph[u][j];
                      if(dist[u]+cost[u][j]<dist[v])</pre>
                      {
                              dist[v]=dist[u]+cost[u][j];
                              PQ.push(pii(dist[v],v));
                      }
               }
       }
}
void buildDag()
{
       FOR(i,0,n)
       {
              FOR(j,0,graph[i].size())
                      int v=graph[i][j];
                      if(dist[i]!=INF && dist[v]!=INF &&
                           dist[v] == dist[i] + cost[i][j])
                      {
                              dag[i].pb(v);
                              parent[v].pb(i);
                      }
               }
       }
}
void topsort(int u)
{
       visited[u]=true;
       FOR(j,0,dag[u].size())
               if(!visited[dag[u][j]]) topsort(dag[u][j]);
       }
       all.pb(u);
}
void buildTree()
```

```
{
       L[0]=0;
       REVERSE(all);
       FOR(i,0,all.size())
              int now=all[i];
              if(parent[now].size())
                      int anc=parent[now][0];
                      FOR(j,1,parent[now].size())
                      {
                             anc=query(anc,parent[now][j]);
                      }
                      L[now]=L[anc]+1;
                      table[now][0]=anc;
                     Tree[anc].pb(now);
                      build(now);
              }
       }
}
int main()
   int test, cases=1;
   scanf("%d", &test);
   while(test--)
       scanf("%d%d", &n, &m);
       FOR(i,0,m)
              scanf("%d%d%d", &u, &v, &t);
              graph[u].pb(v);
              graph[v].pb(u);
              cost[u].pb(t);
              cost[v].pb(t);
       }
```

```
dijkstra();
   buildDag();
   topsort(0);
   buildTree();
   dfs(0);
   int q; scanf("%d", &q);
   printf("Case %d:\n", cases++);
   while(q--)
           int x, u;
           scanf("%d", &x);
           int anc=-1;
           FOR(i,0,x)
           {
                  scanf("%d", &u);
                  if(dist[u] == INF) continue;
                  if(anc==-1) anc=u;
                  else anc=query(anc,u);
          }
           if (anc==-1) printf("0\n");
           else printf("%d %d\n", L[anc]+1, sub[anc]);
   }
   clear();
}
return 0;
```

5.6 Edmonds Matching

}

```
/*
 * Algorithm: Edmonds Blossom Maximum Matching in Generel Graph
 * Order : O( N^4 )
```

```
* Note : vertx must be indexing based
 */
#include<stdio.h>
#include<string.h>
using namespace std;
#define MAX_V 103
#define MAX_E MAX_V*MAX_V
long nV,nE,Match[MAX_V];
long Last[MAX_V], Next[MAX_E], To[MAX_E];
long eI;
long q[MAX_V], Pre[MAX_V], Base[MAX_V];
bool Hash[MAX_V], Blossom[MAX_V], Path[MAX_V];
void Insert(long u, long v) {
   To[eI] = v, Next[eI] = Last[u], Last[u] = eI++;
   To[eI] = u, Next[eI] = Last[v], Last[v] = eI++;
}
long Find_Base(long u, long v) {
   memset( Path,0,sizeof(Path));
   for (;;) {
       Path[u] = 1:
       if (Match[u] == -1) break;
       u = Base[Pre[Match[u]]];
   while (Path[v] == 0) v = Base[Pre[Match[v]]];
   return v;
}
void Change_Blossom(long b, long u) {
   while (Base[u] != b) {
       long v = Match[u];
       Blossom[Base[u]] = Blossom[Base[v]] = 1;
       u = Pre[v];
       if (Base[u] != b) Pre[u] = v;
}
long Contract(long u, long v) {
   memset( Blossom, 0, size of (Blossom));
   long b = Find_Base(Base[u], Base[v]);
   Change_Blossom(b, u);
   Change_Blossom(b, v);
```

```
if (Base[u] != b) Pre[u] = v;
   if (Base[v] != b) Pre[v] = u;
   return b;
}
void Augment(long u) {
   while (u != -1) {
       long v = Pre[u];
       long k = Match[v];
       Match[u] = v;
       Match[v] = u;
       u = k;
   }
}
long Bfs( long p ){
   memset( Pre,-1,sizeof(Pre));
   memset( Hash,0,sizeof(Hash));
   long i;
   for( i=1;i<=nV;i++ ) Base[i] = i;</pre>
   q[1] = p, Hash[p] = 1;
   for (long head=1, rear=1; head<=rear; head++) {</pre>
       long u = q[head];
       for (long e=Last[u]; e!=-1; e=Next[e]) {
           long v = To[e];
           if (Base[u]!=Base[v] and v!=Match[u]) {
               if (v==p or (Match[v]!=-1 and Pre[Match[v]]!=-1)) {
                  long b = Contract(u, v);
                  for( i=1;i<=nV;i++ ) if (Blossom[Base[i]]==1) {</pre>
                      Base[i] = b;
                      if (!Hash[i]) {
                          Hash[i] = 1;
                          q[++rear] = i;
                      }
              } else if (Pre[v]==-1) {
                  Pre[v] = u;
                  if (Match[v] == -1) {
                      Augment(v);
                      return 1;
                  }
                  else {
                      g[++rear] = Match[v];
                      Hash[Match[v]] = 1;
                  }
```

```
}
}
}
return 0;
}
long Edmonds_Blossom( void ){
  long i,Ans = 0;
  memset( Match,-1,sizeof(Match));
  for( i=1;i<=nV;i++ ) if (Match[i] == -1) Ans += Bfs(i);
  return Ans;
}

int main( void ){
  eI = 0;
  memset( Last,-1,sizeof(Last));
}</pre>
```

5.7 Hopcroft Karp

```
vector< int > graph[MAX];
int n, m, match[MAX], dist[MAX];
int NIL=0;

bool bfs()
{
    int i, u, v, len;
    queue< int > Q;
    for(i=1; i<=n; i++)
    {
        if(match[i]==NIL)
        {
            dist[i] = 0;
            Q.push(i);
        }
        else dist[i] = INF;
    }
    dist[NIL] = INF;
    while(!Q.empty())
    {</pre>
```

```
u = Q.front(); Q.pop();
       if(u!=NIL)
       {
           len = graph[u].size();
           for(i=0; i<len; i++)</pre>
               v = graph[u][i];
               if(dist[match[v]] == INF)
                   dist[match[v]] = dist[u] + 1;
                   Q.push(match[v]);
           }
       }
    }
    return (dist[NIL]!=INF);
bool dfs(int u)
    int i, v, len;
    if(u!=NIL)
    {
       len = graph[u].size();
       for(i=0; i<len; i++)</pre>
           v = graph[u][i];
           if(dist[match[v]] == dist[u] + 1)
               if(dfs(match[v]))
                   match[v] = u;
                   match[u] = v;
                   return true;
               }
           }
       dist[u] = INF;
       return false;
    }
    return true;
}
int hopcroft_karp()
```

```
int matching = 0, i;
   // match[] is assumed NIL for all vertex in graph
   // All nodes on left and right should be distinct
   while(bfs())
       for(i=1; i<=n; i++)</pre>
           if(match[i]==NIL && dfs(i))
              matching++;
   return matching;
}
void clear()
   FOR(j,0,MAX) graph[j].clear();
   ms(match,NIL);
}
int main()
{
       // ios_base::sync_with_stdio(0);
       // cin.tie(NULL); cout.tie(NULL);
       // freopen("in.txt","r",stdin);
   // SPOJ - Fast Maximum Matching
       int p, x, y;
       scanf("%d%d%d", &n, &m, &p);
       FOR(i,0,p)
              scanf("%d%d", &x, &y);
              graph[x].pb(n+y);
              graph[n+y].pb(x);
       }
       printf("%d\n", hopcroft_karp());
       return 0;
```

5.8 Hungarian Weighted Matching

```
// hungarian weighted matching algo
```

```
// finds the max cost of max matching, to find mincost, add edges as
    negatives
template<typename T>
struct KuhnMunkras { // n for left, m for right
 int n, m, match[maxM];
 T g[maxN] [maxM], lx[maxN], ly[maxM], slack[maxM];
 bool vx[maxN], vy[maxM];
 void init(int n_, int m_) {
   MEM(g,0); n = n_{,} m = m_{,};
 void add(int u, int v, T w) {
   g[u][v] = w;
 }
 bool find(int x) {
   vx[x] = true:
   for (int y = 1; y \le m; ++y) {
     if (!vy[y]) {
       T delta = lx[x] + ly[y] - g[x][y];
       if (equalT(delta, T(0))) {
         vy[y] = true;
         if (match[y] == 0 || find(match[y])) {
           match[y] = x;
          return true:
       } else slack[y] = min(slack[y], delta);
   }
   return false;
 }
 T matching() { // maximum weight matching
   fill(lx + 1, lx + 1 + n, numeric_limits<T>::lowest());
   MEM(ly,0);
   MEM(match,0);
   for (int i = 1; i <= n; ++i) {</pre>
     for (int j = 1; j \le m; ++j) lx[i] = max(lx[i], g[i][j]);
   for (int k = 1; k \le n; ++k) {
     fill(slack + 1, slack + 1 + m, numeric_limits<T>::max());
     while (true) {
       MEM(vx,0);
```

```
MEM(vy,0);
       if (find(k)) break;
       else {
         T delta = numeric_limits<T>::max();
         for (int i = 1; i <= m; ++i) {</pre>
           if (!vy[i]) delta = min(delta, slack[i]);
         for (int i = 1; i <= n; ++i) {</pre>
           if (vx[i]) lx[i] -= delta:
         for (int i = 1; i <= m; ++i) {</pre>
           if (vy[i]) ly[i] += delta;
           if (!vy[i]) slack[i] -= delta;
       }
     }
   T result = 0;
   for (int i = 1; i <= n; ++i) result += lx[i];</pre>
   for (int i = 1; i <= m; ++i) result += ly[i];</pre>
   return result;
 }
};
```

5.9 Kruskal

```
struct edge
{
    int u, v, w;
    bool operator < (const edge & p) const
    {
        return w < p.w;
    }
};
edge get;
int parent[100];
vector <edge> e;
int find(int r)
{
    if (parent[r] == r)
        return r;
    return parent[r] = find(parent[r]);
}
```

```
int mst(int n)
{
       sort(e.begin(), e.end());
       for (int i = 1; i <= n; i++)</pre>
               parent[i] = i;
       int cnt = 0, s = 0;
       for (int i = 0; i < (int)e.size(); i++)</pre>
               int u = find(e[i].u);
               int v = find(e[i].v);
               if (u != v)
               {
                      parent[u] = v;
                      cnt++;
                      s += e[i].w;
                      if (cnt == n - 1)
                              break;
               }
       }
}
```

5.10 LCA

```
int x=1;
       while(true)
               if((1<<(x+1))>L[p])
                      break;
               x++;
       }
       FORr(i,x,0)
               if(L[p]-(1<<i) >= L[q])
                      p=table[p][i];
       }
       if(p==q) return p;
       FORr(i,x,0)
              if(table[p][i]!=-1 && table[p][i]!=table[q][i])
              {
                      p=table[p][i];
                      q=table[q][i];
              }
       }
       return P[p];
}
void build(int n)
{
       ms(table,-1);
       FOR(i,0,n)
               table[i][0]=P[i];
       for(int j=1; 1<<j < n; j++)</pre>
               for(int i=0; i<n; i++)</pre>
                      if(table[i][j-1]!=-1)
                              table[i][j]=table[table[i][j-1]][j-1];
              }
       }
```

}

5.11 Max Flow Dinic 2

```
11
// Dinic's maximum flow
//
// Description:
// Given a directed network G = (V, E) with edge capacity c: E \rightarrow R.
    The algorithm finds a maximum flow.
//
// Algorithm:
    Dinic's blocking flow algorithm.
//
// Complexity:
// O(n^2 m), but very fast in practice.
// In particular, for a unit capacity graph,
// it runs in O(m \min\{m^{1/2}, n^{2/3}\}).
// Verified:
    SPOJ FASTFLOW
// Reference:
    E. A. Dinic (1970):
// Algorithm for solution of a problem of maximum flow in networks with
    power estimation.
// Soviet Mathematics Doklady, vol. 11, pp. 1277-1280.
//
// B. H. Korte and J. Vygen (2008):
// Combinatorial Optimization: Theory and Algorithms.
    Springer Berlin Heidelberg.
//
#include <iostream>
#include <vector>
#include <cstdio>
#include <queue>
#include <algorithm>
#include <functional>
using namespace std;
#define fst first
```

```
#define snd second
#define all(c) ((c).begin()), ((c).end())
const long long INF = (111 << 50);</pre>
struct graph {
       typedef long long flow_type;
       struct edge {
              int src, dst;
              flow_type capacity, flow;
              size_t rev;
       };
       int n;
       vector<vector<edge>> adj;
       graph(int n) : n(n), adj(n) { }
       void add_edge(int src, int dst, flow_type capacity) {
              adj[src].push_back({src, dst, capacity, 0,
                   adj[dst].size()});
              adj[dst].push_back({dst, src, 0, 0, adj[src].size() - 1});
       flow_type max_flow(int s, int t) {
              vector<int> level(n), iter(n);
              function<int(void)> levelize = [&]() { // foward levelize
                      level.assign(n, -1); level[s] = 0;
                      queue<int> Q; Q.push(s);
                      while (!Q.empty()) {
                             int u = Q.front(); Q.pop();
                             if (u == t) break;
                             for (auto &e : adj[u]) {
                                     if (e.capacity > e.flow &&
                                         level[e.dst] < 0) {</pre>
                                            Q.push(e.dst);
                                            level[e.dst] = level[u] + 1;
                                    }
                             }
                      return level[t];
              };
              function<flow_type(int, flow_type)> augment = [&](int u,
                   flow_type cur) {
                      if (u == t) return cur;
                      for (int &i = iter[u]; i < adj[u].size(); ++i) {</pre>
                             edge &e = adj[u][i], &r = adj[e.dst][e.rev];
                             if (e.capacity > e.flow && level[u] <</pre>
                                 level[e.dst]) {
```

```
flow_type f = augment(e.dst, min(cur,
                                         e.capacity - e.flow));
                                     if (f > 0) {
                                            e.flow += f;
                                            r.flow -= f;
                                            return f;
                                    }
                             }
                      }
                      return flow_type(0);
              }:
              for (int u = 0; u < n; ++u) // initialize
                      for (auto &e : adj[u]) e.flow = 0;
              flow_type flow = 0;
              while (levelize() >= 0) {
                      fill(all(iter), 0);
                      for (flow_type f; (f = augment(s, INF)) > 0; )
                             flow += f;
              }
              return flow;
       }
};
int main() {
       for (int n, m; scanf("%d %d", &n, &m) == 2; ) {
              graph g(n);
              for (int i = 0; i < m; ++i) {</pre>
                      int u, v, w;
                      scanf("%d %d %d", &u, &v, &w);
                      //g.add_edge(u, v, w);
                      g.add\_edge(u - 1, v - 1, w);
              printf("\frac{1}{n}, g.max_flow(0, n - 1));
       }
}
```

5.12 Max Flow Dinic

```
/*
Feasible flow in network with upper + lower constraint, no source, no
    sink:
cap' = upper bound - lower bound.
```

```
Add source s, sink t.
Let M[v] = (sum of lowerbounds of ingoing edges to v) - (sum of lower
    bounds of outgoing edges from v).
For all v, if M[v] > 0, add (s, v, M), else add (v, t, -M).
If all outgoing edges from S are full --> feasible flow exists, it is
    flow + lower bounds.
Max flow with both upper + lower constraints, source s, sink t: add edge
    (t, s, +INF).
Binary search lower bound, check whether feasible flow exists WITHOUT
    source / sink
*/
struct Edge
       int to, rev, f, cap;
};
class Dinic
public:
       int dist[MAX], q[MAX], work[MAX], src, dest;
       vector<Edge> graph[MAX];
       // MAX equals to node_number
       void init(int sz)
              FOR(i,0,sz+1) graph[i].clear();
       void clearFlow(int sz)
              FOR(i,0,sz+1)
              {
                     FOR(j,0,graph[i].size())
                             graph[i][j].f=0;
              }
       }
       void addEdge(int s, int t, int cap)
              Edge a={t,(int)graph[t].size(),0,cap};
```

```
Edge b={s,(int)graph[s].size(),0,0};
       // If our graph has bidirectional edges
       // Capacity for the Edge b will equal to cap
       // For directed, it is 0
       graph[s].emplace_back(a);
       graph[t].emplace_back(b);
}
bool bfs()
       ms(dist,-1);
       dist[src]=0;
       int qt=0;
       q[qt++]=src;
       for(int qh=0; qh<qt; qh++)</pre>
               int u=q[qh];
               for(auto &e: graph[u])
                      int v=e.to;
                      if(dist[v]<0 && e.f<e.cap)</pre>
                      {
                              dist[v]=dist[u]+1;
                              q[qt++]=v;
                      }
               }
       }
       return dist[dest]>=0;
}
int dfs(int u, int f)
       if(u==dest) return f;
       for(int &i=work[u]; i<(int)graph[u].size(); i++)</pre>
               Edge &e=graph[u][i];
               if(e.cap<=e.f) continue;</pre>
```

```
int v=e.to;
                      if(dist[v]==dist[u]+1)
                             int df=dfs(v,min(f,e.cap-e.f));
                             if(df>0)
                             {
                                    e.f+=df;
                                    graph[v][e.rev].f-=df;
                                    return df;
                             }
                      }
              }
              return 0;
       }
       int maxFlow(int _src, int _dest)
              src=_src;
              dest=_dest;
              int result=0;
              while(bfs())
                      // debug;
                      fill(work,work+MAX,0);
                      while(int delta=dfs(src,INF))
                             result+=delta;
              }
              return result;
       }
};
```

5.13 Max Flow Edmond Karp

```
//
// Maximum Flow (Edmonds-Karp)
```

```
//
// Description:
    Given a directed network G = (V, E) with edge capacity c: E->R.
    The algorithm finds a maximum flow.
//
// Algorithm:
    Edmonds-Karp shortest augmenting path algorithm.
// Complexity:
// O(n m<sup>2</sup>)
// Verified:
// AOJ GRL_6_A: Maximum Flow
//
// Reference:
// B. H. Korte and J. Vygen (2008):
// Combinatorial Optimization: Theory and Algorithms.
    Springer Berlin Heidelberg.
//
#include <iostream>
#include <vector>
#include <queue>
#include <cstdio>
#include <algorithm>
#include <functional>
using namespace std;
#define fst first
#define snd second
#define all(c) ((c).begin()), ((c).end())
const int INF = 1 << 30;</pre>
struct graph {
       int n;
       struct edge {
              int src, dst;
              int capacity, residue;
              size_t rev;
       edge &rev(edge e) { return adj[e.dst][e.rev]; };
       vector<vector<edge>> adj;
       graph(int n) : n(n), adj(n) { }
```

```
void add_edge(int src, int dst, int capacity) {
              adj[src].push_back({src, dst, capacity, 0,
                   adj[dst].size()});
              adj[dst].push_back({dst, src, 0, 0, adj[src].size() - 1});
       int max_flow(int s, int t) {
              for (int u = 0; u < n; ++u)
                      for (auto &e : adj[u]) e.residue = e.capacity;
              int total = 0:
              while (1) {
                      vector<int> prev(n, -1); prev[s] = -2;
                      queue<int> que; que.push(s);
                      while (!que.empty() && prev[t] == -1) {
                             int u = que.front(); que.pop();
                             for (edge &e : adj[u]) {
                                    if (prev[e.dst] == -1 && e.residue >
                                            prev[e.dst] = e.rev;
                                            que.push(e.dst);
                                    }
                             }
                      if (prev[t] == -1) break;
                      int inc = INF;
                      for (int u = t; u != s; u = adj[u][prev[u]].dst)
                             inc = min(inc, rev(adj[u][prev[u]]).residue);
                      for (int u = t; u != s; u = adj[u][prev[u]].dst) {
                             adj[u][prev[u]].residue += inc;
                             rev(adj[u][prev[u]]).residue -= inc;
                      }
                      total += inc;
              } // { u : visited[u] == true } is s-side
              return total:
       }
};
int main() {
       for (int n, m; scanf("%d %d", &n, &m) == 2; ) {
              graph g(n);
              for (int i = 0; i < m; ++i) {</pre>
                      int u, v, w;
                      scanf("%d %d %d", &u, &v, &w);
                      g.add_edge(u, v, w);
              printf("d\n", g.max_flow(0, n - 1));
```

```
}
```

5.14 Max Flow Ford Fulkerson

```
// Ford-Fulkerson's maximum flow
// Description:
    Given a directed network G = (V, E) with edge capacity c: E \rightarrow R.
    The algorithm finds a maximum flow.
//
// Algorithm:
    Ford-Fulkerson's augmenting path algorithm
//
// Complexity:
    O(m F), where F is the maximum flow value.
//
// Verified:
    AOJ GRL_6_A: Maximum Flow
//
// Reference:
// B. H. Korte and J. Vygen (2008):
// Combinatorial Optimization: Theory and Algorithms.
    Springer Berlin Heidelberg.
//
#include <iostream>
#include <vector>
#include <cstdio>
#include <algorithm>
#include <functional>
using namespace std;
#define fst first
#define snd second
#define all(c) ((c).begin()), ((c).end())
const int INF = 1 << 30;</pre>
struct graph {
       typedef long long flow_type;
       struct edge {
```

```
int src, dst;
              flow_type capacity, flow;
              size_t rev;
       };
       int n;
       vector<vector<edge>> adj;
       graph(int n) : n(n), adj(n) { }
       void add_edge(int src, int dst, flow_type capacity) {
              adj[src].push_back({src, dst, capacity, 0,
                   adj[dst].size()});
              adj[dst].push_back({dst, src, 0, 0, adj[src].size() - 1});
       }
       int max_flow(int s, int t) {
              vector<bool> visited(n);
              function<flow_type(int, flow_type)> augment = [&](int u,
                   flow_type cur) {
                     if (u == t) return cur;
                      visited[u] = true;
                      for (auto &e : adj[u]) {
                             if (!visited[e.dst] && e.capacity > e.flow) {
                                    flow_type f = augment(e.dst,
                                        min(e.capacity - e.flow, cur));
                                    if (f > 0) {
                                           e.flow += f;
                                           adj[e.dst][e.rev].flow -= f;
                                           return f;
                                    }
                             }
                      return flow_type(0);
              };
              for (int u = 0; u < n; ++u)
                     for (auto &e : adj[u]) e.flow = 0;
              flow_type flow = 0;
              while (1) {
                     fill(all(visited), false);
                     flow_type f = augment(s, INF);
                     if (f == 0) break;
                     flow += f;
              return flow;
       }
};
```

5.15 Max Flow Goldberg Tarjan

```
//
// Maximum Flow (Goldberg-Tarjan, aka. Push-Relabel, Preflow-Push)
// Description:
// Given a directed network G = (V, E) with edge capacity c: E \rightarrow R.
// The algorithm finds a maximum flow.
//
// Algorithm:
    Goldberg-Tarjan's push-relabel algorithm with gap-heuristics.
//
// Complexity:
// O(n^3)
// Verified:
    SPOJ FASTFLOW
//
// Reference:
// B. H. Korte and Jens Vygen (2008):
// Combinatorial Optimization: Theory and Algorithms.
    Springer Berlin Heidelberg.
#include <iostream>
#include <vector>
#include <cstdio>
#include <queue>
#include <algorithm>
#include <functional>
```

```
using namespace std;
#define fst first
#define snd second
#define all(c) ((c).begin()), ((c).end())
const long long INF = (111 << 50);</pre>
struct graph {
       typedef long long flow_type;
       struct edge {
              int src, dst;
              flow_type capacity, flow;
              size_t rev;
       };
       int n;
       vector<vector<edge>> adj;
       graph(int n) : n(n), adj(n) { }
       void add_edge(int src, int dst, int capacity) {
              adj[src].push_back({src, dst, capacity, 0,
                   adj[dst].size()});
              adj[dst].push_back({dst, src, 0, 0, adj[src].size() - 1});
       }
       flow_type max_flow(int s, int t) {
              vector<flow_type> excess(n);
              vector<int> dist(n), active(n), count(2 * n);
              queue<int> Q;
              auto enqueue = [&](int v) {
                      if (!active[v] && excess[v] > 0) { active[v] =
                          true; Q.push(v); }
              auto push = [&](edge & e) {
                     flow_type f = min(excess[e.src], e.capacity -
                          e.flow);
                     if (dist[e.src] <= dist[e.dst] || f == 0) return;</pre>
                      e.flow += f;
                      adj[e.dst][e.rev].flow -= f;
                      excess[e.dst] += f;
                      excess[e.src] -= f;
                      enqueue(e.dst);
              };
              dist[s] = n; active[s] = active[t] = true;
              count[0] = n - 1; count[n] = 1;
```

```
for (int u = 0; u < n; ++u)
                      for (auto &e : adj[u]) e.flow = 0;
              for (auto &e : adj[s]) {
                      excess[s] += e.capacity;
                      push(e);
              }
              while (!Q.empty()) {
                      int u = Q.front(); Q.pop();
                      active[u] = false:
                      for (auto &e : adj[u]) push(e);
                      if (excess[u] > 0) {
                             if (count[dist[u]] == 1) {
                                     int k = dist[u]; // Gap Heuristics
                                     for (int v = 0; v < n; v++) {
                                            if (dist[v] < k) continue;</pre>
                                            count[dist[v]]--;
                                            dist[v] = max(dist[v], n + 1);
                                            count[dist[v]]++;
                                            enqueue(v);
                                    }
                             } else {
                                     count[dist[u]]--; // Relabel
                                     dist[u] = 2 * n:
                                     for (auto &e : adj[u])
                                            if (e.capacity > e.flow)
                                                   dist[u] = min(dist[u].
                                                        dist[e.dst] + 1);
                                     count[dist[u]]++;
                                     enqueue(u);
                             }
                      }
              }
              flow_type flow = 0;
              for (auto e : adj[s]) flow += e.flow;
              return flow;
       }
};
int main() {
       for (int n, m; scanf("%d %d", &n, &m) == 2; ) {
              graph g(n);
              for (int i = 0; i < m; ++i) {</pre>
                      int u, v, w;
```

5.16 Maximum Bipartite Matching and Min Vertex Cover

```
int n, m, p; // n = # of nodes on left, m = # of nodes on right
vi bp[N]; // bipartite graph
int matched[N], revmatch[N];
bool seen[N], visited[2][N];
bool trymatch(int u)
{
       FOR(j,0,bp[u].size())
              int v=bp[u][j];
              if(seen[v]) continue;
              seen[v]=true;
              // v is on right, u on left
              if(matched[v]<0 || trymatch(matched[v]))</pre>
              {
                      matched[v]=u;
                      revmatch[u]=v;
                      return true;
              }
       }
       return false;
}
// O based
int maxbpm(int sz)
       ms(matched,-1);
       ms(revmatch,-1); // for min-vertex-cover
       int ret=0:
```

```
FOR(i,0,sz)
       {
               ms(seen,false);
              if(trymatch(i)) ret++;
       }
       return ret;
}
void dfsLast(int u, bool side)
       if(visited[side][u]) return;
       visited[side][u]=true;
       if(!side)
               for(int i=0; i<n; i++)</pre>
               {
                      if(graph[u][i] && matched[u]!=i)
                             dfsLast(i,1-side);
              }
       else dfsLast(matched[u],1-side);
}
void findMinVertexCover()
{
       FOR(i,0,n)
               if(revmatch[i]==-1)
                      dfsLast(i,0);
              }
       // Assuming both sides have n nodes
       vi mvc, mis; // min vertex cover, max independent set
       FOR(i,0,n)
       {
               if(!visited[0][i] || visited[1][i]) mvc.pb(i);
              if(!(!visited[0][i] || visited[1][i])) mis.pb(i);
       }
```

5.17 Min Cost Arborescence

```
// Min Cost Arboroscense class in C++
// Directed MST
// dir_mst returns the cost O(EV)?
struct Edge {
    int u, v;
   ll dist;
    int kbps;
};
struct MinCostArborescence{
    int n, m;
    Edge allEdges[MAX];
    int done[62], prev[62], id[62];
    ll in[62];
    void init(int n)
       this -> n = n;
       m = 0;
    void add_Edge(int u, int v, ll dist)
       allEdges[m++] = {u,v,dist,0};
    void add_Edge(Edge e)
       allEdges[m++] = e;
    11 dir_mst(int root) {
       11 \text{ ans} = 0;
       while (true) {
           for (int i = 0; i < n; i++) in[i] = INF;</pre>
           for (int i = 0; i < m; i++) {</pre>
               int u = allEdges[i].u;
               int v = allEdges[i].v;
               if (allEdges[i].dist < in[v] && u != v) {</pre>
                   in[v] = allEdges[i].dist;
                   prev[v] = u;
```

```
for (int i = 0; i < n; i++) {</pre>
              if (i == root) continue;
              if (in[i] == INF) return -1;
           }
           int cnt = 0:
           memset(id, -1, sizeof(id));
           memset(done, -1, sizeof(done));
           in[root] = 0:
           for (int i = 0; i < n; i++)</pre>
              ans += in[i];
              int v = i:
              while (done[v] != i && id[v] == -1 && v != root) {
                  done[v] = i;
                  v = prev[v];
              }
              if (v != root && id[v] == -1) {
                  for (int u = prev[v]; u != v; u = prev[u])
                      id[u] = cnt;
                  id[v] = cnt++;
           }
           if (cnt == 0) break;
           for (int i = 0; i < n; i++)</pre>
              if (id[i] == -1) id[i] = cnt++;
           for (int i = 0; i < m; i++) {</pre>
              int v = allEdges[i].v;
              allEdges[i].u = id[allEdges[i].u];
              allEdges[i].v = id[allEdges[i].v];
              if (allEdges[i].u != allEdges[i].v)
                  allEdges[i].dist -= in[v];
          }
           n = cnt;
           root = id[root];
       return ans;
   }
} Arboroscense;
```

}

5.18 Min Cost Max Flow 1

```
11
// Minimum Cost Maximum Flow (Tomizawa, Edmonds-Karp's successive
    shortest path)
//
// Description:
// Given a directed graph G = (V,E) with nonnegative capacity c and
    The algorithm find a maximum s-t flow of G with minimum cost.
//
// Algorithm:
// Tomizawa (1971), and Edmonds and Karp (1972)'s
// successive shortest path algorithm,
// which is also known as the primal-dual method.
//
// Complexity:
// O(F m log n), where F is the amount of maximum flow.
// Caution: Probably does not support Negative Costs
// Negative cost is supported in an implementation named:
    mincostmaxflow2.cpp
#define fst first
#define snd second
#define all(c) ((c).begin()), ((c).end())
#define TEST(s) if (!(s)) { cout << __LINE__ << " " << #s << endl;</pre>
    exit(-1); }
const long long INF = 1e9;
struct graph {
  typedef int flow_type;
  typedef int cost_type;
  struct edge {
   int src, dst;
   flow_type capacity, flow;
   cost_type cost;
   size_t rev;
  vector<edge> edges;
  void add_edge(int src, int dst, flow_type cap, cost_type cost) {
   adj[src].push_back({src, dst, cap, 0, cost, adj[dst].size()});
```

```
adj[dst].push_back({dst, src, 0, 0, -cost, adj[src].size()-1});
}
int n;
vector<vector<edge>> adj;
graph(int n) : n(n), adj(n) { }
pair<flow_type, cost_type> min_cost_max_flow(int s, int t) {
  flow_type flow = 0;
  cost_type cost = 0;
  for (int u = 0: u < n: ++u) // initialize
   for (auto &e: adj[u]) e.flow = 0;
  vector<cost_type> p(n, 0);
  auto rcost = [&](edge e) { return e.cost + p[e.src] - p[e.dst]; };
  for (int iter = 0; ; ++iter) {
    vector < int > prev(n, -1); prev[s] = 0;
    vector<cost_type> dist(n, INF); dist[s] = 0;
    if (iter == 0) { // use Bellman-Ford to remove negative cost edges
     vector<int> count(n); count[s] = 1;
     queue<int> que;
     for (que.push(s); !que.empty(); ) {
       int u = que.front(); que.pop();
       count[u] = -count[u];
       for (auto &e: adj[u]) {
         if (e.capacity > e.flow && dist[e.dst] > dist[e.src] +
             rcost(e)) {
           dist[e.dst] = dist[e.src] + rcost(e);
           prev[e.dst] = e.rev;
           if (count[e.dst] <= 0) {</pre>
             count[e.dst] = -count[e.dst] + 1;
             que.push(e.dst);
       }
    } else { // use Dijkstra
     typedef pair<cost_type, int> node;
     priority_queue<node, vector<node>, greater<node>> que;
     que.push(\{0, s\});
     while (!que.empty()) {
       node a = que.top(); que.pop();
       if (a.snd == t) break;
       if (dist[a.snd] > a.fst) continue;
```

```
for (auto e: adj[a.snd]) {
           if (e.capacity > e.flow && dist[e.dst] > a.fst + rcost(e)) {
             dist[e.dst] = dist[e.src] + rcost(e);
            prev[e.dst] = e.rev;
             que.push({dist[e.dst], e.dst});
        }
       }
     if (prev[t] == -1) break;
     for (int u = 0; u < n; ++u)
       if (dist[u] < dist[t]) p[u] += dist[u] - dist[t];</pre>
     function<flow_type(int,flow_type)> augment = [&](int u, flow_type
         cur) {
       if (u == s) return cur;
       edge &r = adj[u][prev[u]], &e = adj[r.dst][r.rev];
       flow_type f = augment(e.src, min(e.capacity - e.flow, cur));
       e.flow += f; r.flow -= f;
       return f;
     };
     flow_type f = augment(t, INF);
     flow += f:
     cost += f * (p[t] - p[s]);
   return {flow, cost};
};
```

5.19 Min Cost Max Flow 2

```
// By zscoder
// From problem: CF Anti Palindromize - 884F
// Thank you ZS.
// Works as max-cost-max-flow if the costs are considered negative
// Slower due to SPFA in some cases?

struct Edge{
   int u, v;
   long long cap, cost;

Edge(int _u, int _v, long long _cap, long long _cost){
```

```
u = _u; v = _v; cap = _cap; cost = _cost;
   }
};
struct MinCostFlow{
   int n, s, t;
   long long flow, cost;
   vector<vector<int> > graph;
   vector<Edge> e;
   vector<long long> dist;
   vector<int> parent;
   MinCostFlow(int _n){
       // 0-based indexing
       n = _n;
       graph.assign(n, vector<int> ());
   }
   void addEdge(int u, int v, long long cap, long long cost, bool
        directed = true){
       graph[u].push_back(e.size());
       e.push_back(Edge(u, v, cap, cost));
       graph[v].push_back(e.size());
       e.push_back(Edge(v, u, 0, -cost));
       if(!directed)
           addEdge(v, u, cap, cost, true);
   }
   pair<long long, long long> getMinCostFlow(int _s, int _t){
       s = _s; t = _t;
       flow = 0, cost = 0;
       while(SPFA()){
           flow += sendFlow(t, 1LL<<62);</pre>
       }
       return make_pair(flow, cost);
   }
   // not sure about negative cycle
   bool SPFA(){
       parent.assign(n, -1);
       dist.assign(n, 1LL<<62);</pre>
                                     dist[s] = 0;
```

```
vector<int> queuetime(n, 0); queuetime[s] = 1;
   vector<bool> inqueue(n, 0); inqueue[s] = true;
   queue<int> q;
                                 q.push(s);
   bool negativecycle = false;
   while(!q.empty() && !negativecycle){
       int u = q.front(); q.pop(); inqueue[u] = false;
       for(int i = 0; i < graph[u].size(); i++){</pre>
          int eIdx = graph[u][i];
          int v = e[eIdx].v; ll w = e[eIdx].cost, cap = e[eIdx].cap;
          if(dist[u] + w < dist[v] && cap > 0){
              dist[v] = dist[u] + w;
              parent[v] = eIdx;
              if(!inqueue[v]){
                  q.push(v);
                  queuetime[v]++;
                  inqueue[v] = true;
                  if(queuetime[v] == n+2){
                      negativecycle = true;
                      break:
                  }
              }
          }
       }
   }
   return dist[t] != (1LL<<62);</pre>
long long sendFlow(int v, long long curFlow){
   if(parent[v] == -1)
       return curFlow;
   int eIdx = parent[v];
   int u = e[eIdx].u; ll w = e[eIdx].cost;
   long long f = sendFlow(u, min(curFlow, e[eIdx].cap));
   cost += f*w;
   e[eIdx].cap -= f;
   e[eIdx^1].cap += f;
```

```
return f;
};
```

5.20 Min Cost Max Flow 3

```
// This gave AC for CF 813D Two Melodies but the other one was TLE
// By sgtlaugh
namespace mcmf{
   const int MAX = 1000010;
   const int INF = 1 << 25;</pre>
   int cap[MAX], flow[MAX], cost[MAX], dis[MAX];
   int n, m, s, t, Q[10000010], adj[MAX], link[MAX], last[MAX],
        from[MAX], visited[MAX];
   void init(int nodes, int source, int sink){
       m = 0, n = nodes, s = source, t = sink;
       for (int i = 0; i <= n; i++) last[i] = -1;
   }
   void addEdge(int u, int v, int c, int w){
       adj[m] = v, cap[m] = c, flow[m] = 0, cost[m] = +w, link[m] =
           last[u], last[u] = m++;
       adj[m] = u, cap[m] = 0, flow[m] = 0, cost[m] = -w, link[m] =
           last[v], last[v] = m++;
   }
   bool spfa(){
       int i, j, x, f = 0, l = 0;
       for (i = 0; i <= n; i++) visited[i] = 0, dis[i] = INF;</pre>
       dis[s] = 0, Q[1++] = s;
       while (f < 1)
          i = Q[f++];
           for (j = last[i]; j != -1; j = link[j]){
              if (flow[j] < cap[j]){</pre>
                  x = adi[i];
                  if (dis[x] > dis[i] + cost[j]){
                      dis[x] = dis[i] + cost[j], from[x] = j;
                      if (!visited[x]){
```

```
visited[x] = 1;
                      if (f \&\& rand() \& 7) Q[--f] = x;
                      else Q[1++] = x;
              }
           }
       }
       visited[i] = 0;
   }
   return (dis[t] != INF);
pair <int, int> solve(){
   int i, j;
   int mincost = 0, maxflow = 0;
   while (spfa()){
       int aug = INF;
       for (i = t, j = from[i]; i != s; i = adj[j ^ 1], j = from[i]){
           aug = min(aug, cap[j] - flow[j]);
       for (i = t, j = from[i]; i != s; i = adj[j ^ 1], j = from[i]){
           flow[j] += aug, flow[j ^ 1] -= aug;
       maxflow += aug, mincost += aug * dis[t];
   }
   return make_pair(mincost, maxflow);
}
```

5.21 Prim MST

```
if (!visited[i] && d[i] <mini)</pre>
                       mini=d[i], minidx=i;
       }
       return minidx;
}
11 Prim()
{
       FOR(i,0,10003)
               d[i]=INF:
               visited[i]=false;
       }
       d[1]=0:
       for (int i=1; i<=n-1; i++)</pre>
               int u=minKey();
               visited[u]=true;
               FOR(j,0,graph[u].size())
               {
                       int v=graph[u][j];
                       if(!visited[v] && cost[u][j]<d[v])</pre>
                               d[v]=cost[u][j];
               }
       }
       ll ret=0:
       FOR(j,1,n+1)
       {
               // cout<<d[j]<<endl;</pre>
               if(d[j]!=INF)
                       ret+=d[j];
       }
       return ret;
}
int main()
       int a, b, c;
       scanf("%d%d", &n, &m);
       FOR(i,0,m)
       {
               scanf("%d%d%d", &a, &b, &c);
               graph[a].pb(b);
               graph[b].pb(a);
```

```
cost[a].pb(c);
    cost[b].pb(c);
}
cout<<Prim()<<endl;
return 0;
}</pre>
```

5.22 Push Relabel

```
#define sz(x) (int)(x).size()
struct Edge {
   int v;
   ll flow, C;
   int rev:
};
template <int SZ> struct PushRelabel {
   vector<Edge> adj[SZ];
   11 excess[SZ];
   int dist[SZ], count[SZ+1], b = 0;
   bool active[SZ];
   vi B[SZ]:
   void addEdge(int u, int v, 11 C) {
       Edge a{v, 0, C, sz(adj[v])};
       Edge b{u, 0, 0, sz(adj[u])};
       adj[u].pb(a), adj[v].pb(b);
   }
   void enqueue (int v) {
       if (!active[v] && excess[v] > 0 && dist[v] < SZ) {</pre>
           active[v] = 1;
           B[dist[v]].pb(v);
           b = max(b, dist[v]);
       }
   }
   void push (int v, Edge &e) {
       11 amt = min(excess[v], e.C-e.flow);
       if (dist[v] == dist[e.v]+1 && amt > 0) {
           e.flow += amt, adj[e.v][e.rev].flow -= amt;
```

```
excess[e.v] += amt, excess[v] -= amt;
       enqueue(e.v);
   }
}
void gap (int k) {
   FOR(v,1,SZ+1) if (dist[v] >= k) {
       count[dist[v]] --;
       dist[v] = SZ;
       count[dist[v]] ++;
       enqueue(v);
}
void relabel (int v) {
   count[dist[v]] --; dist[v] = SZ;
   for (auto e: adj[v]) if (e.C > e.flow) dist[v] = min(dist[v],
       dist[e.v] + 1);
   count[dist[v]] ++;
   enqueue(v);
}
void discharge(int v) {
   for (auto &e: adj[v]) {
       if (excess[v] > 0) push(v,e);
       else break;
   }
   if (excess[v] > 0) {
       if (count[dist[v]] == 1) gap(dist[v]);
       else relabel(v);
   }
}
11 maxFlow (int s, int t) {
   for (auto &e: adj[s]) excess[s] += e.C;
   count[0] = SZ;
   enqueue(s); active[t] = 1;
   while (b >= 0) {
       if (sz(B[b])) {
          int v = B[b].back(); B[b].pop_back();
          active[v] = 0; discharge(v);
       } else b--;
   }
```

```
return excess[t];
};
PushRelabel<50000> network;
```

5.23 SCC Kosaraju

```
// Kosaraju's strongly connected component
// Description:
    For a graph G = (V, E), u and v are strongly connected if
    there are paths u \rightarrow v and v \rightarrow u. This defines an equivalent
    relation, and its equivalent class is called a strongly
    connected component.
//
// Algorithm:
// Kosaraju's algorithm performs DFS on G and rev(G).
// First DFS finds topological ordering of SCCs, and
    the second DFS extracts components.
11
// Complexity:
    O(n + m)
11
// Verified:
    SPOJ 6818
// References:
   A. V. Aho, J. E. Hopcroft, and J. D. Ullman (1983):
    Data Structures and Algorithms,
    Addison-Wesley.
11
#include <iostream>
#include <vector>
#include <cstdio>
#include <cstdlib>
#include <map>
#include <set>
#include <cmath>
#include <cstring>
#include <functional>
#include <algorithm>
```

```
#include <unordered_map>
#include <unordered_set>
using namespace std;
#define fst first
#define snd second
#define all(c) ((c).begin()), ((c).end())
struct graph {
 int n;
 vector<vector<int>> adj, rdj;
  graph(int n) : n(n), adj(n), rdj(n) { }
 void add_edge(int src, int dst) {
   adj[src].push_back(dst);
   rdj[dst].push_back(src);
 }
  vector<vector<int>> strongly_connected_components() { // kosaraju
   vector<int> ord, visited(n);
   vector<vector<int>> scc;
   function<void(int, vector<vector<int>>&, vector<int>&)> dfs
   = [&](int u, vector<vector<int>> &adj, vector<int> &out) {
     visited[u] = true;
     for (int v : adj[u])
       if (!visited[v]) dfs(v, adj, out);
     out.push_back(u);
   };
   for (int u = 0; u < n; ++u)
     if (!visited[u]) dfs(u, adj, ord);
   fill(all(visited), false);
   for (int i = n - 1; i \ge 0; --i)
     if (!visited[ord[i]])
       scc.push_back({}), dfs(ord[i], rdj, scc.back());
   return scc;
 }
};
int main() {
 int n, m;
 scanf("%d %d", &n, &m);
 graph g(n);
 for (int k = 0; k < m; ++k) {
   int i, j;
```

```
scanf("%d %d", &i, &j);
   g.add_edge(i - 1, j - 1);
 vector<vector<int>> scc = g.strongly_connected_components();
 vector<int> outdeg(scc.size());
 vector<int> id(n);
 for (int i = 0; i < scc.size(); ++i)</pre>
   for (int u : scc[i]) id[u] = i;
 for (int u = 0; u < n; ++u)
   for (int v : g.adj[u])
     if (id[u] != id[v]) ++outdeg[id[u]];
 if (count(all(outdeg), 0) != 1) {
   printf("0\n");
 } else {
   int i = find(all(outdeg), 0) - outdeg.begin();
   sort(all(scc[i]));
   printf("%d\n%d", scc[i].size(), scc[i][0] + 1);
   for (int j = 1; j < scc[i].size(); ++j)</pre>
     printf(" %d", scc[i][j] + 1);
   printf("\n");
 }
}
```

5.24 SCC Tarjan

```
stack<int> st;
vector<vector<int> > scc;
int low[MAX], disc[MAX], comp[MAX];
int dfs_time;
bool in_stack[MAX];
vi graph[MAX];
int n; // node count indexed from 1

void dfs(int u)
{
    low[u] = dfs_time;
    disc[u] = dfs_time;
    dfs_time++;
    in_stack[u] = true;
```

```
st.push(u);
       int sz = graph[u].size(), v;
       for(int i = 0; i < sz; i++)</pre>
              v = graph[u][i];
              if(disc[v] == -1)
              {
                      dfs(v);
                      low[u] = min(low[u], low[v]);
              else if(in_stack[v] == true)
                      low[u] = min(low[u], disc[v]);
       }
       if(low[u] == disc[u])
              scc.push_back(vector<int>());
              while(st.top() != u)
              {
                      scc[scc.size() - 1].push_back(st.top());
                      in_stack[st.top()] = false;
                      st.pop();
              }
              scc[scc.size() - 1].push_back(u);
              in_stack[u] = false;
              st.pop();
       }
}
int tarjan()
{
       memset(comp, -1, sizeof(comp));
       memset(disc, -1, sizeof(disc));
       memset(low, -1, sizeof(low));
       memset(in_stack, 0, sizeof(in_stack));
       dfs\_time = 0;
       while(!st.empty())
              st.pop();
       for(int i = 1; i <= n; i++)</pre>
              if(disc[i] == -1)
```

5.25 kth Shortest Path Length

```
int n, m, x, y, k, a, b, c;
vi Graph[103], Cost[103];
vector<priority_queue<int> > d(103);
priority_queue < pii > Q;
void goDijkstra()
{
       // Here, elements are sorted in decreasing order of the first
       // of the pairs and then the second elements if equal first
           element.
       // d[i] is the priority_queue of the node i where the best k path
       // will be stored in decreasing order. So, d[i].top() has the
           longest of the
       // first k shortest path.
       d[x].push(0);
       Q.push(MP(x,0));
       // Q contains the nodes in the increasing order of their cost
       // Since the priority_queue sorts the pairs in decreasing order of
           their
       // first element and then second element, to sort it in increasing
       // we will negate the cost and push it.
       while(!Q.empty())
       {
              pii t=Q.top(); Q.pop();
```

```
int u=t.first, costU=-t.second;
              // Since the actual cost was negated.
              FOR(j,0,Graph[u].size())
                      int v=Graph[u][j];
                      // prnt(v); prnt(d[v].size());
                      // Have we already got k shortest paths? Or is the
                          longest path can be made better?
                      if(d[v].size()<k || d[v].top()>costU+Cost[u][j])
                             int temp=costU+Cost[u][j];
                             d[v].push(temp);
                             Q.push(MP(v,-temp));
                      if(d[v].size()>k) d[v].pop();
       // If we have more than k shortest path for the current node, we
            can pop
       // the worst ones.
       }
       if(d[y].size()<k) prnt(-1);</pre>
       // We have not found k shortest path for our destination.
       else prnt(d[y].top());
}
int main()
{
       // ios_base::sync_with_stdio(0);
       // cin.tie(NULL); cout.tie(NULL);
       // freopen("in.txt","r",stdin);
       while(scanf("%d%d", &n, &m) && n+m)
              scanf("%d%d%d", &x, &y, &k);
              FOR(i,0,m)
                      scanf("%d%d%d", &a, &b, &c);
                      Graph[a].pb(b);
                      Cost[a].pb(c);
```

6 Math

6.1 CRT Diophantine

```
// Chinese remainder theorem (special case): find z such that
// z % x = a, z % y = b. Here, z is unique modulo M = lcm(x,y).
// Return (z,M). On failure, M = -1.
PII chinese_remainder_theorem(int x, int a, int y, int b) {
       int s, t;
       int d = extended_euclid(x, y, s, t);
       if (a % d != b % d) return make_pair(0, -1);
       return make_pair(mod(s * b * x + t * a * y, x * y) / d, x * y / d);
}
// Chinese remainder theorem: find z such that
// z % x[i] = a[i] for all i. Note that the solution is
// unique modulo M = lcm_i (x[i]). Return (z,M). On
// failure, M = -1. Note that we do not require the a[i]'s
// to be relatively prime.
PII chinese_remainder_theorem(const VI &x, const VI &a) {
       PII ret = make_pair(a[0], x[0]);
       for (int i = 1; i < x.size(); i++) {</pre>
              ret = chinese_remainder_theorem(ret.second, ret.first,
                   x[i], a[i]);
```

```
if (ret.second == -1) break;
}
return ret;
}

// computes x and y such that ax + by = c; on failure, x = y =-1
void linear_diophantine(int a, int b, int c, int &x, int &y) {
   int d = gcd(a, b);
   if (c % d) {
        x = y = -1;
   } else {
        x = c / d * mod_inverse(a / d, b / d);
        y = (c - a * x) / b;
}
```

6.2 Euler Phi

6.3 FFT 1

```
/*
FFT is used for fast multiplication.
Main Functionality : mult(a,b)
```

```
Parameter : vector<int>a, vector<int> b (Representing a polynomial where
contains co-efficient of the polynomial of degree 3)
Output : The polynomial a*b
typedef complex<double> Complex;
void fft(vector<Complex> & a, bool inv)
   int n = (int)a.size();
   for (int i = 1, j = 0; i < n; ++i)
       int bit = n >> 1;
       for (; j >= bit; bit >>= 1)
           j -= bit;
       j += bit;
       if (i < j)
           swap(a[i], a[j]);
   }
   for (int len = 2; len <= n; len <<= 1)</pre>
       double ang = 2 * PI / len * (inv ? -1 : 1);
       Complex wlen(cos(ang), sin(ang));
       for (int i = 0; i < n; i += len)</pre>
           Complex w(1);
           for (int j = 0; j < len / 2; ++j)
               Complex u = a[i + j], v = a[i + j + len / 2] * w;
               a[i + j] = u + v;
               a[i + j + len / 2] = u - v;
               w *= wlen;
           }
       }
   if(inv)
   for (int i = 0; i<n; ++i)</pre>
       a[i] /= n;
}
vector<int> mult(vector<int>& a, vector<int>& b)
```

```
{
    vector<Complex> fa(a.begin(), a.end()), fb(b.begin(), b.end());
    size_t n = 1;
    while (n < max(a.size(), b.size())) n <<= 1;</pre>
    n <<= 1:
    fa.resize(n), fb.resize(n);
    fft(fa, false), fft(fb, false);
    for (size_t i = 0; i<n; ++i)</pre>
       fa[i] *= fb[i]:
    fft(fa, true);
    vector<int> res;
    res.resize(n);
    for (size_t i = 0; i<n; ++i)</pre>
       res[i] = int(fa[i].real() + 0.5);
    return res;
}
vector<int> squ(vector<int>& a)
    vector<Complex> fa(a.begin(), a.end()), fb(a.begin(), a.end());
    size_t n = 1;
    while (n < a.size()) n <<= 1;</pre>
    n <<= 1:
    fa.resize(n), fb.resize(n);
    fft(fa, false); fb = fa;
    for (size_t i = 0; i < n; ++i)</pre>
       fa[i] *= fb[i]:
    fft(fa, true);
    vector<int> res;
    res.resize(n);
    for (size_t i = 0; i < n; ++i)</pre>
       res[i] = int(fa[i].real() + 0.5);
    return res;
}
```

6.4 FFT 2

```
typedef vector<long long> VL;
typedef complex<long double> CN;
void FFT( vector<CN> &a, bool invert ){
    long i,j,n = a.size();
    for( i=1, j=0; i<n; i++ ){</pre>
               long bit = n >> 1;
               for( ; j>=bit; bit>>=1 ) j -= bit ;
               j += bit ;
               if( i < j ) swap( a[i],a[j] );</pre>
       }
       long len;
       for( len=2;len<=n;len<<=1 ){</pre>
           double ang = 2*PI / len * ( invert ? -1:1 );
           CN wlen(cos(ang), sin(ang));
               for( i=0;i<n;i+=len ){</pre>
                      CN w(1);
                      for( j=0; j<len/2; j++ ){</pre>
                              CN u = a[i+j], v = a[i+j+len/2]*w;
                              a[i+j] = u+v; a[i+j+len/2] = u-v; w *= wlen;
                      }
               }
       }
       if( invert ){
           for( i=0;i<n;i++ ) a[i] /= n;</pre>
       }
}
void Multiply( const VL &a, const VL &b, VL &res ){
    vector<CN> fa( a.begin(),a.end() ), fb( b.begin(),b.end() );
       long i,n = 1;
       while( n < max( a.size(),b.size()) ) n <<= 1;</pre>
       n <<= 1;
       fa.resize( n ),fb.resize( n );
       FFT( fa,false ) , FFT( fb,false );
       for( i=0;i<n;i++ ) fa[i] *= fb[i];</pre>
       FFT( fa,true );
       res.resize( n );
       for( i=0;i<n;i++ ) res[i] = long( fa[i].real() + 0.5 );</pre>
       /* if multiplication between 2 number then res need to be mod by
            10 */
```

6.5 Gauss Elimination Equations Mod Number Solutions

```
11 pow(ll base, ll p, ll MOD)
{
       if(p == 0) return 1;
       if(p % 2 == 0) { 11 d = pow(base, p / 2, MOD); return (d * d) %
            MOD: }
       return (pow(base, p - 1, MOD) * base) % MOD;
}
11 inv(11 x, 11 MOD) { return pow(x, MOD - 2, MOD); }
// If MOD equals 2, it becomes XOR operation and we can use vector of
    bitsets to build equation
// Complexity becomes 1/32
11 gauss(vector<vector<1l> > &a, 11 MOD)
       int n = a.size(), m = a[0].size() - 1;
       for(int i = 0; i < n; i++)</pre>
               for(int j = 0; j <= m; j++)</pre>
                      a[i][j] = (a[i][j] \% MOD + MOD) \% MOD;
       vector<int> where(m, -1);
       for(int col = 0, row = 0; col < m && row < n; col++)
       int sel = row;
       for(int i = row; i < n; i++)</pre>
               if(a[i][col] > a[sel][col])
                      sel = i:
               if(a[sel][col] == 0) { where[col] = -1; continue; }
       for(int i = col; i <= m; i++)</pre>
                      swap(a[sel][i], a[row][i]);
               where[col] = row;
               11 c_inv = inv(a[row][col], MOD);
               for(int i = 0; i < n; i++)</pre>
                      if(i != row)
                              if(a[i][col] == 0) continue;
               11 c = (a[i][col] * c_inv) % MOD;
               for(int j = 0; j <= m; j++)</pre>
```

```
a[i][j] = (a[i][j] - c * a[row][j] % MOD + MOD) % MOD;
          row++;
}
vector<ll> ans(m, 0);
11 result = 1:
// for counting rank, take the count of where[i]==-1
for(int i = 0; i < m; i++)</pre>
   if(where[i] != -1) ans[i] = (a[where[i]][m] * inv(a[where[i]][i],
        MOD)) % MOD;
          else result = (result * MOD) % mod;
   // This is validity check probably.
   // May not be needed
for(int i = 0: i < n: i++)
          11 sum = a[i][m] % MOD;
          for(int j = 0; j < m; j++)
                  sum = (sum + MOD - (ans[j] * a[i][j]) % MOD) % MOD;
          if(sum != 0) return 0;
   }
   return result;
```

6.6 Gauss Jordan Elimination

```
// Gauss-Jordan elimination with full pivoting.
//
// Uses:
// (1) solving systems of linear equations (AX=B)
// (2) inverting matrices (AX=I)
// (3) computing determinants of square matrices
//
// Running time: O(n^3)
//
// INPUT: a[][] = an nxn matrix
// b[][] = an nxm matrix
```

```
//
                  = an nxm matrix (stored in b[][])
// OUTPUT: X
            A^{-1} = an nxn matrix (stored in a[][])
//
            returns determinant of a[][]
// Example used: LightOJ Snakes and Ladders
#include <iostream>
#include <vector>
#include <cmath>
using namespace std;
const double EPS = 1e-10;
typedef vector<int> VI;
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;
T GaussJordan(VVT &a, VVT &b) {
  const int n = a.size();
 const int m = b[0].size();
 VI irow(n), icol(n), ipiv(n);
 T \det = 1;
 for (int i = 0: i < n: i++) {</pre>
   int pj = -1, pk = -1;
   for (int j = 0; j < n; j++) if (!ipiv[j])</pre>
     for (int k = 0; k < n; k++) if (!ipiv[k])</pre>
       if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk = k;}
   if (fabs(a[pj][pk]) < EPS) { cerr << "Matrix is singular." << endl;</pre>
        exit(0): }
   ipiv[pk]++;
   swap(a[pj], a[pk]);
   swap(b[pj], b[pk]);
   if (pj != pk) det *= -1;
   irow[i] = pj;
   icol[i] = pk;
   T c = 1.0 / a[pk][pk];
   det *= a[pk][pk];
   a[pk][pk] = 1.0;
   for (int p = 0; p < n; p++) a[pk][p] *= c;
```

```
for (int p = 0; p < m; p++) b[pk][p] *= c;
   for (int p = 0; p < n; p++) if (p != pk) {</pre>
     c = a[p][pk];
     a[p][pk] = 0;
     for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
     for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
   }
 }
 for (int p = n-1; p >= 0; p--) if (irow[p] != icol[p]) {
   for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);</pre>
 return det;
int main() {
 const int n = 4;
 const int m = 2:
 double A[n][n] = \{ \{1,2,3,4\}, \{1,0,1,0\}, \{5,3,2,4\}, \{6,1,4,6\} \};
 double B[n][m] = \{ \{1,2\}, \{4,3\}, \{5,6\}, \{8,7\} \};
 VVT a(n), b(n);
 for (int i = 0; i < n; i++) {</pre>
   a[i] = VT(A[i], A[i] + n);
   b[i] = VT(B[i], B[i] + m);
 double det = GaussJordan(a, b);
 // expected: 60
 cout << "Determinant: " << det << endl;</pre>
 // expected: -0.233333 0.166667 0.133333 0.0666667
 //
             0.166667 0.166667 0.333333 -0.333333
 //
             0.05 -0.75 -0.1 0.2
 cout << "Inverse: " << endl;</pre>
 for (int i = 0; i < n; i++) {</pre>
  for (int j = 0; j < n; j++)
     cout << a[i][j] << ' ';
   cout << endl;</pre>
 // expected: 1.63333 1.3
             -0.166667 0.5
```

```
// 2.36667 1.7
// -1.85 -1.35
cout << "Solution: " << endl;
for (int i = 0; i < n; i++) {
  for (int j = 0; j < m; j++)
     cout << b[i][j] << ' ';
  cout << endl;
}
</pre>
```

6.7 Gauss Xor

```
const int MAXN = (1 \ll 20);
const int MAXLOG = 64;
struct basis
   int64_t base[MAXLOG];
   void clear()
              for(int i = MAXLOG - 1; i >= 0; i--)
                      base[i] = 0;
   }
   void add(int64_t val)
       for(int i = MAXLOG - 1; i >= 0; i--)
                      if((val >> i) & 1)
                             if(!base[i]) { base[i] = val; return; }
                             else val ^= base[i];
                      }
       }
   inline int size()
       int sz = 0;
       for(int i = 0; i < MAXLOG; i++)</pre>
                      sz += (bool)(base[i]);
              return sz:
   }
```

```
int64_t max_xor()
{
    int64_t res = 0;
    for(int i = MAXLOG - 1; i >= 0; i--)
        if(!((res >> i) & 1) && base[i])
            res ^= base[i];

    return res;
}

bool can_create(int64_t val)
{
    for(int i = MAXLOG - 1; i >= 0; i--)
        if(((val >> i) & 1) && base[i])
            val ^= base[i];

    return (val == 0);
}
;
```

6.8 Gaussian 1

```
void gauss(vector< vector<double> > &A) {
   int n = A.size();
       for(int i = 0; i < n; i++){</pre>
       int r = i;
       for(int j = i+1; j < n; j++)</pre>
           if(fabs(A[j][i]) > fabs(A[r][i]))
               r = j;
       if(fabs(A[r][i]) < EPS) continue;</pre>
       if(r != i)
           for(int j = 0; j <= n; j++)</pre>
               swap(A[r][j], A[i][j]);
       for(int k = 0; k < n; k++){
           if(k != i){
               for(int j = n; j >= i; j--)
                  A[k][j] -= A[k][i]/A[i][i]*A[i][j];
           }
       }
```

```
// solve: A[x][n]/A[x][x] for each x }
```

6.9 Gaussian 2

```
const double eps = 1e-9;
// *****may return empty vector
vector<double> gauss(vector<vector<double>> &a)
{
       int n = a.size(), m = a[0].size() - 1;
       vector<int> where(m, -1);
       for(int col = 0, row = 0; col < m && row < n; col++)</pre>
       int sel = row;
       for(int i = row; i < n; i++)</pre>
               if(abs(a[i][col]) > abs(a[sel][col]))
                       sel = i;
               if(abs(a[sel][col]) < eps) { where[col] = -1; continue; }</pre>
       for(int i = col; i <= m; i++)</pre>
                       swap(a[sel][i], a[row][i]);
               where[col] = row;
               for(int i = 0; i < n; i++)</pre>
                       if(i != row)
                               if(abs(a[i][col]) < eps) continue;</pre>
               double c = a[i][col] / a[row][col];
               for(int j = 0; j \le m; j++)
                   a[i][j] -= c * a[row][j];
               row++;
    }
    vector<double> ans(m, 0);
    for(int i = 0; i < m; i++)</pre>
       if(where[i] != -1)
                       ans[i] = a[where[i]][m] / a[where[i]][i];
```

7 Miscellaneous

7.1 Header

```
#pragma comment(linker, "/stack:200000000")
#pragma GCC optimize("unroll-loops")
#include <bits/stdc++.h>
// #include <ext/pb_ds/assoc_container.hpp>
// #include <ext/pb_ds/tree_policy.hpp>
// #include <ext/pb_ds/detail/standard_policies.hpp>
using namespace std;
// using namespace __gnu_pbds;
// using namespace __gnu_cxx;
typedef long long 11;
typedef unsigned int ul;
typedef unsigned long long ull;
typedef vector <int> vi;
typedef map<int, vector<int> > mvii;
typedef map<int, int> mii;
typedef queue <int> qi;
typedef vector <string> vs;
typedef pair <int, int> pii;
typedef vector<pii > vpii;
```

```
// Order Statistic Tree
/* Special functions:
               find_by_order(k) --> returns iterator to the kth largest
                    element counting from 0
               order_of_key(val) --> returns the number of items in a set
                    that are strictly smaller than our item
*/
// typedef tree<
// int,
// null_type,
// less<int>,
// rb_tree_tag,
// tree_order_statistics_node_update>
// ordered set:
#define MP make_pair
#define SORT(a) sort (a.begin(), a.end())
#define REVERSE(a) reverse (a.begin(), a.end())
#define ALL(a) a.begin(), a.end()
#define PI acos(-1)
#define ms(x,y) memset (x, y, sizeof (x))
#define inf 1e9
#define INF 1e16
#define pb push_back
#define MAX 100005
#define debug(a,b) cout<<a<<": "<<b<<endl</pre>
#define Debug cout<<"Reached here"<<endl</pre>
#define prnt(a) cout<<a<<"\n"</pre>
#define mod 100000007LL
#define FOR(i,a,b) for (int i=(a); i<(b); i++)</pre>
#define FORr(i,a,b) for (int i=(a); i>=(b); i--)
#define itrALL(c,itr) for(__typeof((c).begin())
    itr=(c).begin();itr!=(c).end();itr++)
#define lc ((node)<<1)</pre>
#define rc ((node)<<1|1)
#define VecPrnt(v) FOR(J,0,v.size()) cout<<v[J]<<" "; cout<<endl</pre>
#define endl "\n"
#define PrintPair(x) cout<<x.first<<" "<<x.second<<endl</pre>
#define EPS 1e-9
#define ArrPrint(a,st,en) for(int J=st; J<=en; J++) cout<<a[J]<<" ";</pre>
    cout << endl;
```

```
/* Direction Array */
// int fx[]=\{1,-1,0,0\};
// int fy[]=\{0,0,1,-1\};
// int fx[]=\{0,0,1,-1,-1,1,-1,1\};
// int fy[]=\{-1,1,0,0,1,1,-1,-1\};
template <class T> inline T bigmod(T p,T e,T M)
   ll ret = 1:
   for(; e > 0; e >>= 1)
       if(e & 1) ret = (ret * p) % M;
       p = (p * p) % M;
   } return (T)ret:
template <class T> inline T gcd(T a,T b){if(b==0)return a;return
    gcd(b,a%b);}
template <class T> inline T modinverse(T a,T M){return bigmod(a,M-2,M);}
template <class T> inline T lcm(T a,T b) {a=abs(a);b=abs(b); return
    (a/gcd(a,b))*b;}
template <class T, class X> inline bool getbit(T a, X i) { T t=1; return
    ((a&(t<<i))>0);}
template <class T, class X> inline T setbit(T a, X i) { T t=1;return
    (al(t<<i)): }
template <class T, class X> inline T resetbit(T a, X i) { T t=1; return
    (a&(~(t<<i)));}
inline 11 getnum()
   char c = getchar();
   11 num,sign=1;
   for(;c<'0'||c>'9';c=getchar())if(c=='-')sign=-1;
   for(num=0;c>='0'&&c<='9';)</pre>
       c-='0':
       num = num*10+c;
       c=getchar();
   return num*sign;
inline ll power(ll a, ll b)
```

7.2 Russian Peasant Multiplication

```
if(a>=m) a-=m;
b>>=1;
}
    return ret;
}
```

8 String

8.1 Aho Corasick 2

```
int n;
string s, p[MAX];
map<char, int> node[MAX];
int root, nnode, link[MAX], occ[MAX];
vi ending[MAX], exist[MAX];
// exist[i] has all the ending occurrences of the input strings
void insertword(int idx)
{
       int len = p[idx].size();
       int now = root;
       FOR(i, 0, len)
              if (!node[now][p[idx][i]])
              {
                      node[now][p[idx][i]] = ++nnode;
                     node[nnode].clear();
              now = node[now][p[idx][i]];
       }
// which strings end in node number 'now'?
       ending[now].pb(idx);
void populate(int curr)
// Because 'suffix-links'. It links a node with the longest proper suffix
       for (auto it : ending[link[curr]])
              ending[curr].pb(it);
void populate(vi &curr, int idx)
```

```
// So word number it ends in idx-th character of the text
       for (auto it : curr)
       {
              exist[it].pb(idx);
}
void push_links()
       queue<int>q;
       link[0] = -1;
       q.push(0);
       while (!q.empty())
              int u = q.front();
              q.pop();
              itrALL(node[u], it)
              {
                      char ch = it->first;
                      int v = it->second:
                      int j = link[u];
                      while (j != -1 && !node[j][ch])
                             j = link[j];
                      if (j != -1)link[v] = node[j][ch];
                      else link[v] = 0;
                      q.push(v);
                      populate(v);
              }
}
void traverse()
       int len = s.size();
       int now = root:
       FOR(i, 0, len)
              while (now != -1 && !node[now][s[i]])
                      now = link[now];
              if (now != -1) now = node[now][s[i]];
              else now = 0;
              populate(ending[now], i);
       }
}
```

8.2 Aho Corasick Occurrence Relation

```
// Suppose we have n<=1000 strings. Total summation of the length of
    these strings
// can be 1e7. Now we are given queries. In each query, we are given
// two strings and asked if one of them occurs in another as a substring.
// We need to find this relation efficiently. We will use Aho-Corasick.
// The solution is to build a graph where vertices denote indices of
    strings and an edge
// from u to v denotes that string[u] occurs in string[v].
#define ALPHABET_SIZE 26
#define MAX_NODE 1e6
int n; // number of strings
string in[N], p;
int node[MAX_NODE] [ALPHABET_SIZE];
int root, nnode, link[MAX_NODE], termlink[MAX_NODE], terminal[MAX_NODE];
bool graph[N][N];
// termlink[u] = a link from node u to a node which is a terminal node
// terminal node is a node where an ending of an input string occurs
// terminal[node] = the index of the string which ends in node
/* Solution:
// For every node of the Aho-Corasick structure find and remember the
nearest terminal node (termlink[u]) in the suffix-link path; Once again
all strings through Aho-Corasick. Every time new symbol is added, add an
    arc from the node
corresponding to the current string (in the graph we build, not
    Aho-Corasick) to
the node of the graph corresponding to the nearest terminal in the
    suffix-link path;
The previous step will build all essential arcs plus
some other arcs, but they do not affect the next step in any way;
Find the transitive closure of the graph.
void init()
   root=0:
   nnode=0;
```

```
ms(terminal,-1);
    ms(termlink,-1);
}
void insertword(int idx)
{
       p=in[idx];
    int len=p.size();
    int now=root;
    FOR(i,0,len)
       int x=p[i]-'a';
       if(!node[now][x])
           node[now][x]=++nnode;
       now=node[now][x];
    }
    terminal[now]=idx; // string with index idx ends in now
}
void push_links()
{
    queue<int>q;
    link[0]=-1;
    q.push(0);
    while(!q.empty())
       int u=q.front();
       q.pop();
       for(int i=0; i<ALPHABET_SIZE; i++)</pre>
               if(!node[u][i]) continue;
           int v=node[u][i];
           int j=link[u];
           while(j!=-1 && !node[j][i])
               j=link[j];
```

```
if(j!=-1) link[v]=node[j][i];
           else link[v]=0;
                     // Finding nearest terminal nodes
           if(terminal[link[v]]!=-1)
              termlink[v]=link[v];
           else termlink[v]=termlink[link[v]];
          q.push(v);
}
void buildgraph()
{
       FOR(i,0,n)
              int curr=root:
              FOR(j,0,in[i].size())
                      char ch=in[i][j];
                      curr=node[curr][(int)ch-'a'];
                      int st=curr:
                      if(terminal[st]==-1) st=termlink[st]:
                      for(int k=st; k>=0; k=termlink[k])
                             if(terminal[k]==i) continue;
                             if(graph[i][terminal[k]]) break;
                             graph[i][terminal[k]]=true;
                             // cout<<"edge: "<<i<" "<<terminal[k]<<endl;
                     }
              }
       }
}
// Finally, find transitive closure of the graph. If O(n^3) is possible,
// Floyd-Warshall. Otherwise, run dfs from each node and add an edge from
    current starting
```

// node to each reachable node. An edge in this transitive closure
 denotes the occurrence relation.

8.3 Aho Corasick

```
int n; // n is the number of dictionary word
string s,p; // dictionary words are inputted in p, s is the traversed text
#define MAX_NODE 250004
map<char,int> node[MAX_NODE];
int root, nnode, link[MAX_NODE], endof[504], travis[MAX_NODE];
pii level[MAX_NODE];
void init()
   root=0;
   nnode=0;
   travis[root] = 0; // number of time a node is traversed by s
   level[root] = MP(0, root); // level, node
   node[root].clear();
}
void insertword(int idx)
   int len=p.size();
   int now=root;
   FOR(i,0,len)
       if(!node[now][p[i]])
           node[now][p[i]]=++nnode;
           node[nnode].clear();
           travis[nnode]=0;
           level[nnode] = MP(level[now].first+1,nnode);
       }
       now=node[now][p[i]];
   }
   endof[idx]=now; // end of dictionary word idx
```

```
}
void push_links()
   queue<int>q;
   link[0]=-1;
   q.push(0);
   while(!q.empty())
       int u=q.front();
       q.pop();
       itrALL(node[u],it)
           char ch=it->first;
           int v=it->second;
           int j=link[u];
           while(j!=-1 && !node[j][ch])
              j=link[j];
           if(j!=-1) link[v]=node[j][ch];
           else link[v]=0;
           q.push(v);
       }
}
void traverse()
   int len=s.size();
   int now=root;
   travis[root]++;
   FOR(i,0,len)
       while(now!=-1 && !node[now][s[i]])
           now=link[now];
       if(now!=-1) now=node[now][s[i]];
       else now=0;
```

```
travis[now]++;
   }
   sort(level,level+nnode+1,greater<pii>());
   FOR(i,0,nnode+1)
       now=level[i].second;
       travis[link[now]]+=travis[now];
}
void driver()
   init();
   FOR(i,0,n)
   {
       // input p
       insertword(i);
   }
   // input s
   push_links();
   traverse();
   // number of occurrence of word i in s is travis[endof[i]]
```

8.4 KMP 2

```
}
void Search()
       int now=-1;
       for(int i=0; i<n; i++)</pre>
               while(now!=-1 && patt[now+1]!=text[i])
                      now=pi[now];
              if(patt[now+1] == text[i]) ++now;
               else now=-1;
               if(now==m-1)
               {
                      cout<<"match at "<<i-now<<endl;</pre>
                      now=pi[now]; // match again
              }
       }
}
int main()
       // ios_base::sync_with_stdio(0);
       // cin.tie(NULL); cout.tie(NULL);
       // freopen("in.txt","r",stdin);
       cin>>text>>patt;
       n=strlen(text); m=strlen(patt);
       Process();
       Search();
       // FOR(i,0,m) cout<<pi[i]<<" "; cout<<endl;
       return 0;
}
```

8.5 KMP 3

```
string p, t;
int pi[MAX], cnt[MAX];
```

```
void prefixFun()
       int now;
       pi[0]=now=-1;
       for(int i=1; i<p.size(); i++)</pre>
               while(now!=-1 && p[now+1]!=p[i])
                      now=pi[now];
               if(p[now+1]==p[i]) pi[i]=++now;
               else pi[i]=now=-1;
       }
}
int kmpMatch()
       int now=-1;
       FOR(i,0,t.size())
               cout<<"now: "<<i<" "<<now<<endl;
               while(now!=-1 && p[now+1]!=t[i])
                      now=pi[now];
               if(p[now+1] ==t[i])
               {
                      ++now;
                      cnt[now]++;
               }
               else now=-1;
               if(now+1==p.size())
               {
                      // match found
                      // cout<<"match and setting "<<now<<" to
                           "<<pi[now] << endl;</pre>
                      now=pi[now]; // match again
              }
       }
}
int main()
       // ios_base::sync_with_stdio(0);
       // cin.tie(NULL); cout.tie(NULL);
       // freopen("in.txt","r",stdin);
```

```
cin>>t>>p;

prefixFun();
FOR(i,0,p.size()) cout<<pi[i]<<" "; cout<<endl;
prnt(kmpMatch());
FOR(i,0,p.size()) cout<<cnt[i]<<" "; cout<<endl;
FORr(i,p.size()-1,0)
{
        if(pi[i]==-1) continue;
        cnt[pi[i]]+=cnt[i];
}
FOR(i,0,p.size()) cout<<cnt[i]<<" "; cout<<endl;
return 0;
}</pre>
```

8.6 Palindromic Tree

```
class PalindromicTree
public:
       int s[MAX], Link[MAX], Len[MAX], Edge[MAX][26];
       int node, lastPal, n;
       11 cnt[MAX]:
       void init()
              s[n++]=-1;
              Link[0]=1; Len[0]=0;
              Link[1]=1; Len[1]=-1;
              node=2;
       }
       int getLink(int v)
              while(s[n-Len[v]-2]!=s[n-1]) v=Link[v];
              return v;
       }
       void addLetter(int c)
              // cout<<char(c+'a')<<" "<<n<<endl;</pre>
```

```
s[n++]=c;
               lastPal=getLink(lastPal);
               if(!Edge[lastPal][c])
                      Len[node] = Len[lastPal] + 2;
                      Link[node] = Edge [getLink(Link[lastPal])][c];
                      cnt[node]++:
                      Edge[lastPal][c]=node++;
               }
               else
               {
                      cnt[Edge[lastPal][c]]++;
               }
               lastPal=Edge[lastPal][c];
       }
       void clear()
               FOR(i,0,node+1)
                      cnt[i]=0;
                      ms(Edge[i],0);
               }
               n=0:
               lastPal=0:
       }
} PTA;
```

8.7 String Split by Delimiter

```
template<typename Out>
void split(const std::string &s, char delim, Out result) {
   std::stringstream ss(s);
   std::string item;
   while (std::getline(ss, item, delim)) {
       *(result++) = item;
   }
}
std::vector<std::string> split(const std::string &s, char delim) {
   std::vector<std::string> elems;
```

```
split(s, delim, std::back_inserter(elems));
   return elems;
}
```

8.8 Suffix Array 2

```
// You are given two strings A and B, consisting only of lowercase
    letters from the English alphabet.
// Count the number of distinct strings S, which are substrings of A, but
    not substrings of B
LL substr_count(int n,char *s)
{
        VI cnt(128);
        for(int i=0;i<n;i++)</pre>
               cnt[s[i]]++:
        for(int i=1;i<128;i++)</pre>
               cnt[i]+=cnt[i-1];
        VI p(n);
        for(int i=0;i<n;i++)</pre>
               p[--cnt[s[i]]]=i;
        VVI c(1,VI(n));
        int w=0:
        for(int i=0;i<n;i++)</pre>
               if(i==0 || s[p[i]]!=s[p[i-1]]) w++;
               c[0][p[i]] = w-1;
        }
        for(int k=0,h=1;h<n;k++,h*=2)</pre>
               VI pn(n);
               for(int i=0;i<n;i++) {</pre>
                       pn[i] = p[i] - h;
                       if(pn[i]<0) pn[i] += n;</pre>
               }
               VI cnt(w,0);
               for(int i=0;i<n;i++)</pre>
                       cnt[c[k][pn[i]]]++;
               for(int i=1;i<w;i++)</pre>
                       cnt[i]+=cnt[i-1];
               for(int i=n;i--;)
                       p[--cnt[c[k][pn[i]]]=pn[i];
```

```
w=0;
               c.push_back(VI(n));
               for(int i=0;i<n;i++)</pre>
                      if(i==0 || c[k][p[i]] != c[k][p[i-1]]) {
                              w++;
                      } else {
                              int i1 = p[i] + h; if(i1>=n) i1-=n;
                              int i2 = p[i-1] + h; if(i2>=n) i2-=n;
                              if(c[k][i1]!=c[k][i2]) w++;
                      c[k+1][p[i]] = w-1;
              }
       }
       LL ans = LL(n)*(n-1)/2;
       for(int k=1;k<n;k++)</pre>
       {
              int i=p[k];
              int j=p[k-1];
               int cur = 0;
               for (int h=c.size(); h--;)
                      if (c[h][i] == c[h][j]) {
                              cur += 1<<h;
                              i += 1<<h;
                              j += 1<<h;
                      }
               ans-=cur;
       }
       return ans;
}
char s[200005];
int n, m;
void input()
{
       scanf("%s", s);
       n=strlen(s)+1;
       s[n-1]='a'-1;
       scanf("%s", s+n);
       m=strlen(s+n)+1;
```

```
s[n+m-1]='a'-2;
       s[n+m]=0;
}
void solve()
{
       11 p=substr_count(n,s);
       11 r=substr_count(m,s+n);
       11 q=substr_count(n+m,s)-(11)n*m;
       // cout<<p<<" "<<r<" "<<q<<endl;
       11 t=q-p-r;
       t=abs(t);
       prnt(p-t);
}
int main()
{
   // ios_base::sync_with_stdio(0);
   // cin.tie(NULL); cout.tie(NULL);
   // freopen("in.txt","r",stdin);
   int test, cases=1;
   input();
   solve();
   return 0;
```

8.9 Suffix Array

```
// rak[i] -> The position of i th index of the main string in height
    array.
// 0 or 1 based ranking (check)
const int N = 2e6+5;
int wa[N],wb[N],wv[N],wc[N];
int r[N],sa[N],rak[N], height[N];
int cmp(int *r,int a,int b,int 1)
      return r[a] == r[b] && r[a+1] == r[b+1]:
}
void da(int *r,int *sa,int n,int m)
{
      int i,j,p,*x=wa,*y=wb,*t;
      for( i=0;i<m;i++) wc[i]=0;</pre>
      for( i=0;i<n;i++) wc[x[i]=r[i]] ++;</pre>
      for( i=1;i<m;i++) wc[i] += wc[i-1];</pre>
      for( i = n-1; i > 0; i--)sa[--wc[x[i]]] = i;
      for( j= 1,p=1;p<n;j*=2,m=p){</pre>
           for(p=0,i=n-j;i< n;i++)y[p++] = i;
           for(i=0;i< n;i++)if(sa[i] >= j) y[p++] = sa[i] - j;
           for(i=0;i<n;i++)wv[i] = x[y[i]];</pre>
           for(i=0;i<m;i++) wc[i] = 0;</pre>
           for(i=0;i<n;i++) wc[wv[i]] ++;</pre>
           for(i=1;i<m;i++) wc[i] += wc[i-1];</pre>
           for(i=n-1;i>=0;i--) sa[--wc[wv[i]]] = y[i];
           for(t=x,x=y,y=t,p=1,x[sa[0]] = 0,i=1;i < n;i++) x[sa[i]] =
                cmp(y,sa[i-1],sa[i],j) ? p-1:p++;
}
void calheight(int *r,int *sa,int n)
      int i,j,k=0;
      for(i=1;i<=n;i++) rak[sa[i]] = i;</pre>
      for(i=0;i<n;height[rak[i++]] = k ) {</pre>
           for(k?k--:0, j=sa[rak[i]-1] ; r[i+k] == r[j+k] ; k ++) ;
     }
}
```

```
int dp[N][22];
void initRMQ(int n)
      for(int i= 1;i<=n;i++) dp[i][0] = height[i];</pre>
      for(int j= 1; (1<<j) <= n; j ++ ){</pre>
           for(int i = 1; i + (1 << j) - 1 <= n; i ++ ) {
                 dp[i][j] = min(dp[i][j-1], dp[i + (1 << (j-1))][j-1]);
           }
     }
int askRMQ(int L,int R)
{
      int k = 0;
      while((1<<(k+1)) <= R-L+1) k++;</pre>
      return min(dp[L][k], dp[R - (1<<k) + 1][k]);</pre>
}
int main()
{
       return 0;
}
```

8.10 Suffix Automata

```
// Counts number of distinct substrings

struct suffix_automaton
{
    map<char, int> to[MAX];
    int len[MAX], link[MAX];
    int last, psz = 0;

    void add_letter(char c)
    {
        int p = last, cl, q;
        if(to[p].count(c))
        {
            q = to[p][c];
            if(len[q] == len[p] + 1)
```

```
{
       last = q;
       return;
   }
   cl = psz++;
   len[cl] = len[p] + 1;
   to[c1] = to[q];
   link[cl] = link[q];
   link[q] = cl;
   last = cl;
   for(; to[p][c] == q; p = link[p])
       to[p][c] = c1;
   return;
}
last = psz++;
len[last] = len[p] + 1;
for(; to[p][c] == 0; p = link[p])
   to[p][c] = last;
if(to[p][c] == last)
   link[last] = p;
   return;
}
q = to[p][c];
if(len[q] == len[p] + 1)
   link[last] = q;
   return;
}
cl = psz++;
len[cl] = len[p] + 1;
to[c1] = to[q];
link[cl] = link[q];
link[q] = cl;
link[last] = cl;
for(; to[p][c] == q; p = link[p])
```

```
to[p][c] = c1;
   void clear()
       for(int i = 0; i < psz; i++)</pre>
           len[i] = 0, link[i] = 0, to[i].clear();
       psz = 1;
       last = 0;
   void init(string s)
       clear();
       for(int i = 0; i < s.size(); i++)</pre>
           add_letter(s[i]);
   }
   suffix_automaton() {psz = 0; clear();}
};
string s;
suffix_automaton SA;
11 cnt[MAX];
vi endpos[MAX];
int main()
   // ios_base::sync_with_stdio(0);
   // cin.tie(NULL); cout.tie(NULL);
   // freopen("in.txt","r",stdin);
   int test, cases=1;
   cin>>s;
   SA.clear();
   FOR(i,0,s.size())
       SA.add_letter(s[i]), cnt[SA.last]++;
   FOR(i,0,SA.psz)
       endpos[SA.len[i]].pb(i);
```

```
fORr(i,SA.psz-1,1)
{
    for(auto it: endpos[i])
    {
        cnt[SA.link[it]]+=cnt[it];
        ans+=(SA.len[it]-SA.len[SA.link[it]]); // distinct
        occurrences
        // cnt[it] has occurrence of substring ending at node it
    }
}

// cnt[x] has occurrences of state x
// To calculate occurrence of an input string, we visit the automata using the letters
// of the input string and find the last_state where it finishes
// The cnt[last_state] should be the occurrence of this string
prnt(ans);
return 0;
```

8.11 Trie 1

```
struct Node
       int cntL, cntR, lIdx, rIdx;
       Node()
       {
              cntL = cntR = 0;
              lIdx = rIdx = -1;
       }
};
Node Tree[MAX];
int globalIdx = 0;
class Trie
{
public:
       void insert(int val, int idx, int depth)
       {
              for (int i = depth - 1; i >= 0; i--)
```

```
{
               bool bit = val & (1 << i);
               // cout<<"bit now: "<<bit<<endl;</pre>
               if (bit)
               {
                      Tree[idx].cntR++;
                      if (Tree[idx].rIdx == -1)
                              Tree[idx].rIdx = ++globalIdx;
                              idx = globalIdx;
                      else idx = Tree[idx].rIdx;
               }
               else
               {
                      Tree[idx].cntL++;
                      if (Tree[idx].lIdx == -1)
                              Tree[idx].lIdx = ++globalIdx;
                              idx = globalIdx;
                      }
                      else idx = Tree[idx].lIdx;
              }
       }
int query(int val, int compVal, int idx, int depth)
       int ans = 0;
       for (int i = depth - 1; i >= 0; i--)
               bool valBit = val & (1 << i);</pre>
               bool compBit = compVal & (1 << i);</pre>
               if (compBit)
               {
                      if (valBit)
                              ans += Tree[idx].cntR;
                              idx = Tree[idx].lIdx;
                      }
                      else
                              ans += Tree[idx].cntL;
                              idx = Tree[idx].rIdx;
                      }
               }
```

```
else
                              if (valBit)
                              {
                                     idx = Tree[idx].rIdx;
                              }
                              else
                              {
                                     idx = Tree[idx].lIdx;
                             }
                      }
                      if (idx == -1) break;
              }
              return ans;
       void clear()
               for (int i = 0; i <= globalIdx; i++)</pre>
                      Tree[i].cntL = 0;
                      Tree[i].cntR = 0;
                      Tree[i].rIdx = -1;
                      Tree[i].lIdx = -1;
              }
               globalIdx = 0;
       }
};
int main()
{
       // ios_base::sync_with_stdio(0);
       // cin.tie(NULL); cout.tie(NULL);
       // freopen("in.txt","r",stdin);
       // Given an array of positive integers you have to print the
            number of
       // subarrays whose XOR is less than K.
       int test, n, k, x;
       Trie T;
       scanf("%d", &test);
       while (test--)
       {
               scanf("%d%d", &n, &k);
              T.insert(0, 0, 20);
              int pre = 0;
              11 \text{ ans} = 0;
              FOR(i, 0, n)
```

8.12 Trie 2

```
const int MaxN = 100005;
int sz;
int nxt[MaxN][55];
int en[MaxN];
bool isSmall(char ch)
{
       return ch>='a' && ch<='z';</pre>
}
int getId(char ch)
{
       if(isSmall(ch)) return ch-'a';
       else return ch-'A'+26;
}
void insert (char *s, int 1)
{
       int v = 0;
       for (int i = 0; i < 1; ++i) {</pre>
               int c=getId(s[i]);
               if (nxt[v][c]==-1)
                      ms(nxt[sz],-1);
```

```
nxt[v][c]=sz++;
                     en[sz]=0;
                     // created[sz] = true;
              }
              v = nxt[v][c];
       }
       ++en[v];
}
int search (char *tmp, int 1) {
       int v = 0;
       for (int i = 0; i < 1; ++i) {</pre>
              int c=getId(tmp[i]);
              if (nxt[v][c]==-1)
                     return 0;
              v = nxt[v][c];
       }
       return en[v];
}
void init()
{
       sz=1;
       en[0]=0;
       ms(nxt[0],-1);
```