

System of Difference Constraints

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1 Problem

Given some inequality on some variable (x_i, x_j, \dots) in form $x_j - x_i \leq w$, we need to determine whether we can assign values to the variables so that all the given inequalities are satisfiable or not. If satisfiable, then output a solution.

2 Solution

- For each variable we create a vertex.
- For each inequality, $x_j - x_i \leq w$, we give a directed edge (v_i, v_j) with cost w .
- Create a source vertex S and give an edge (S, v_i) for all vertices with cost 0. Can be solved without source vertex.

If the constraint graph contains a negative cycle, then the system of differences is unsatisfiable.

3 Determining a Possible Solution

- If there is no negative cycle in the constraint graph, then there is a solution for the system.
- For each variable x_i , $x_i =$ shortest path distance of v_i from the source vertex in constraint graph.
- Let $x = x_1, x_2, \dots, x_n$ be a solution to a system of difference constraints and let d be any constant. Then $x + d = x_1 + d, x_2 + d, \dots, x_n + d$ is a solution as well.
- Shortest Path can be calculated from Bellman-Ford algorithm.
- Bellman-Ford maximizes $x_1 + x_2 + \dots + x_n$ subject to the constraints $x_j - x_i \leq w_{ij}$ and $x_i \leq 0$
- Bellman-Ford also minimizes $\max(x_i) - \min(x_i)$