Graph Concepts that I Forget Frequently

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1 Definitions

Chromatic Number: Minimum number of colors required to color graph G such that no two adjacent vertex gets same color.

Brooks' Therorem: For any connected undirected graph G with maximum degree x, the chromatic number of G is at most x unless G is a complete graph or an odd cycle, in which case the chromatic number is $x + 1$.

Independent Vertex Set: Set of vertices in a graph G , no two of which are adjacent that means no two vertices in this set is connected by an edge.
Maximum Independence Set is the such set with maximum size.

Vertex Cover: Set of Vertices in a graph G, such that each edge in G incident on at least one vertex in the Set.
Minimum Vertex Cover is such a cover of minimum size.

Note that,

- A set of vertices is a vertex cover if and only if its complement is an independent set.
- The number of vertices of graph G is equal to sum of minimum vertex cover and maximum independent set.

Dominating Set: A dominating set for a graph $G = (V, E)$ is a subset D of V such that every vertex not in D is adjacent to at least one member of D. The domination number is the number of vertices in a smallest dominating set for G.

Any *maximal independent set* in a graph is necessarily also a minimal dominating set. Note that, *maximal independent set* is any independent set such that adding any other vertex to this set is not possible as an edge is created. But *maximum independent set* is such set of maximum size.

Clique: A clique of a graph G is a complete subgraph of G, and the clique of largest possible size is referred to as a maximum clique.

Suppose we are asked to find the *maximum clique* of a bipartite graph. We take the compliment graph. Then find the *maximum independent set* by finding the *maximum matching*. We know *maximum independent set cardinality* is equal to the difference between the number of vertices of the graph and the maximum bipartite matching. This value is the *maximum clique* of our given bipartite graph.