

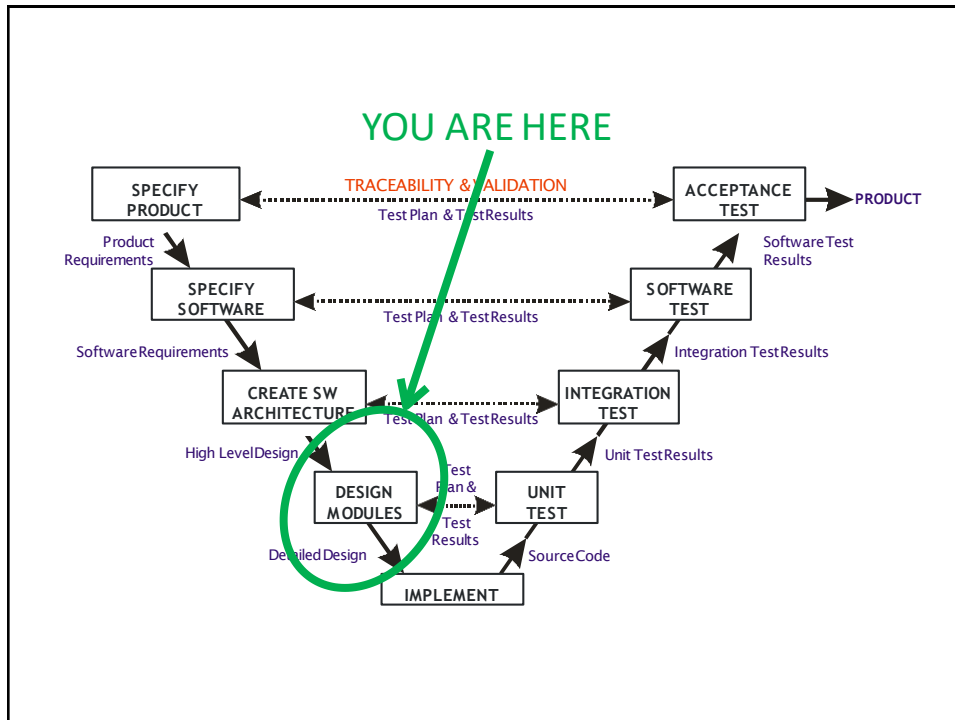
Lecture 6: Modeling Discrete Dynamics

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Based on the slides by Edward Lee and Peter Marwedel

Review

- High Level Design (HLD)
 - Architecture
 - Architecture description languages
 - Refined requirements
 - Sequence Diagrams



Discrete Systems

- **Discrete** = “individually separate / distinct”
- A **discrete system** is one that operates in a sequence of discrete *steps* or has signals taking discrete *values*.
- It is said to have **discrete dynamics**.

Recap: Models of Computation

- **What does it mean, “to compute”?**
- **Models of computation define:**
 - Components and an execution model for computations for each component
 - Communication model for exchange of information between components.



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Models of Computation

(Considered by Marwedel)

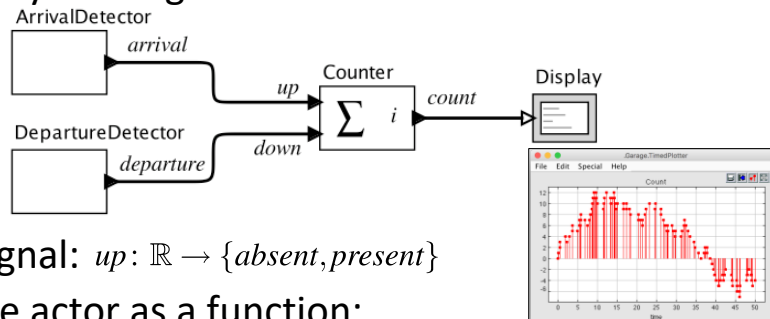
Communication/ local computations	Shared memory	Message passing	
		Synchronous	Asynchronous
Undefined components	Plain text, use cases (Message) sequence charts		
Communicating finite state machines	StateCharts		SDL
Data flow	Scoreboarding + Tomasulo Algorithm (☞ Comp.Archit.)		Kahn networks, SDF
Petri nets		C/E nets, P/T nets, ...	
Discrete event (DE) model	VHDL*, Verilog*, SystemC*, ...	Only experimental systems, e.g. distributed DE in Ptolemy	
Von Neumann model	C, C++, Java	C, C++, Java with libraries CSP, ADA	

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Discrete Systems: Example Design Problem

Example: count the number of cars in a parking garage by sensing those that enter and leave:



Pure signal: $up: \mathbb{R} \rightarrow \{absent, present\}$

Discrete actor as a function:

$$Counter: (\mathbb{R} \rightarrow \{absent, present\})^P \rightarrow (\mathbb{R} \rightarrow \{absent\} \cup \mathbb{N})$$

$$P = \{up, down\}$$

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Reflection

- What (mathematical) properties should a discrete signal have?

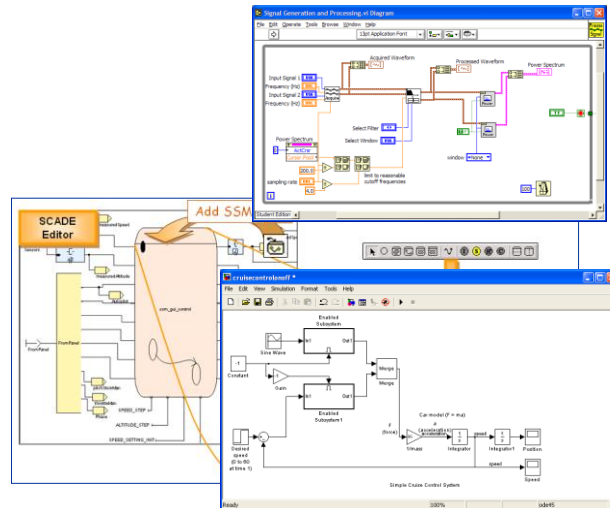
$$e: \mathbb{R} \rightarrow \{absent\} \cup X.$$

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Actor Modeling Languages / Frameworks

- LabVIEW
- Simulink
- Scade
- ...
- Reactors
- StreamIT
- ...



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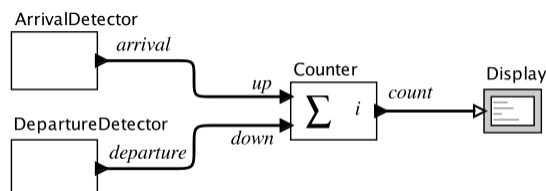
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Reaction / Transition

For any $t \in \mathbb{R}$ where $up(t) \neq absent$ or $down(t) \neq absent$ the Counter **reacts**. It produces an output value in \mathbb{N} and changes its internal **state**.

State: condition of the system at a particular point in time

- Encodes everything about the past that influences the system's reaction to current input



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Reflection

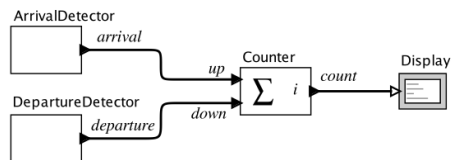
What are some scenarios that the given parking garage (interface) design does not handle well?

For $t \in \mathbb{R}$ the inputs are in a set

$$Inputs = (\{up, down\} \rightarrow \{absent, present\})$$

and the outputs are in a set

$$Outputs = (\{count\} \rightarrow \{absent\} \cup \mathbb{N}) ,$$



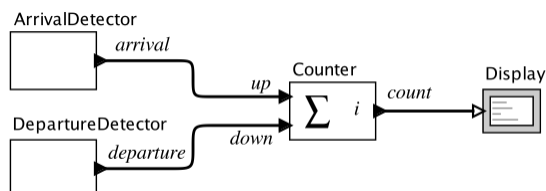
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State Space

A practical parking garage has a finite number M of spaces, so the state space for the counter is

$$States = \{0, 1, 2, \dots, M\} .$$

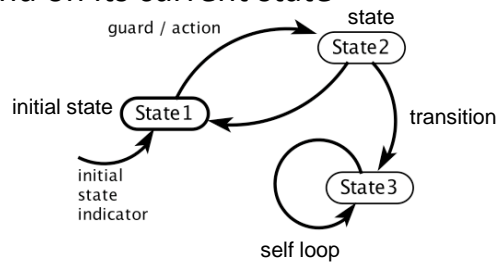


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Finite State Machine (FSM)

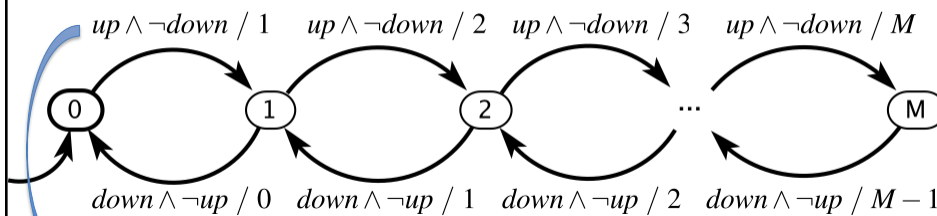
- FSM
 - A model of a system with discrete dynamics
 - At each reaction maps valuations of the inputs to valuations of the outputs
 - The map may depend on its current state
- Transitions
 - Guards
 - Actions



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Garage Counter Finite State Machine (FSM) in Pictures



Guard $g \subseteq Inputs$ is specified using the shorthand

$$up \wedge \neg down$$

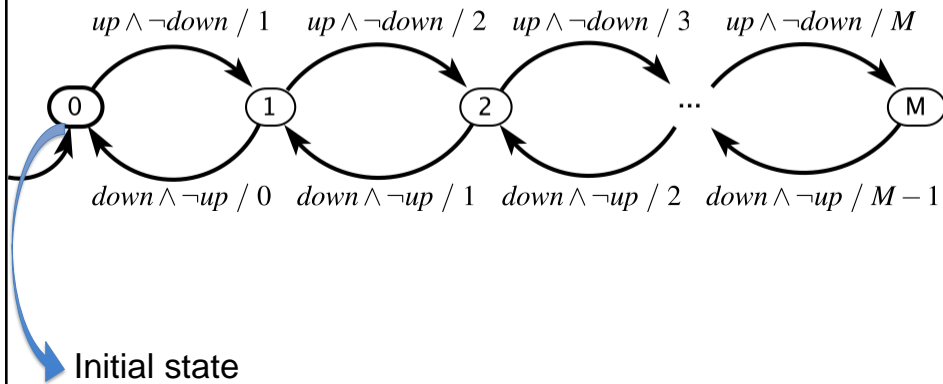
which means

$$g = \{\{up\}\}.$$

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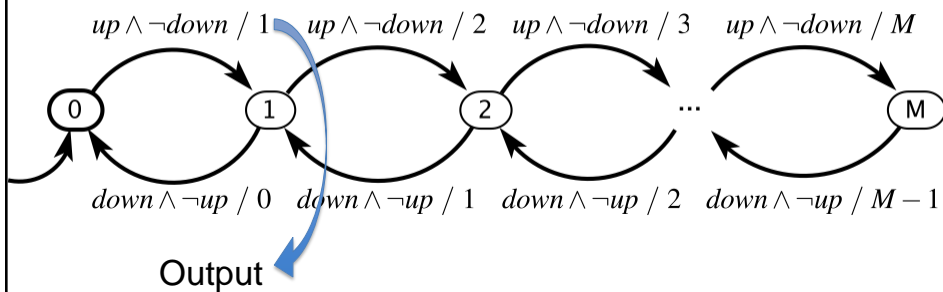
Garage Counter Finite State Machine (FSM) in Pictures



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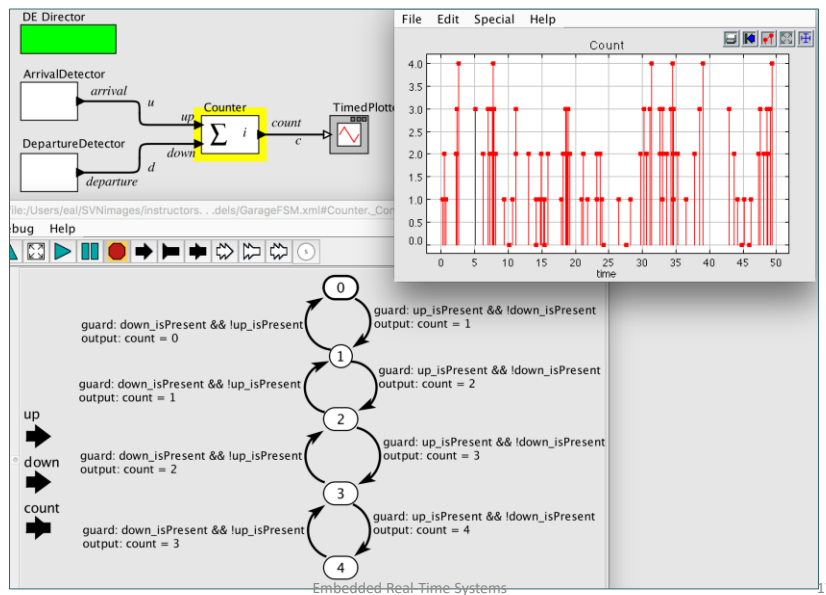
Garage Counter Finite State Machine (FSM) in Pictures



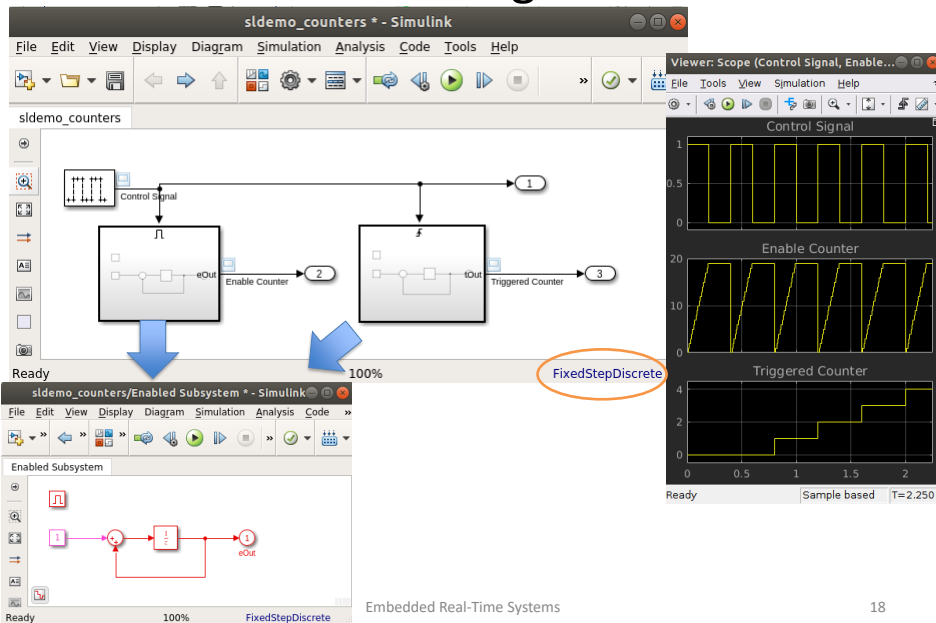
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Ptolemy II Model

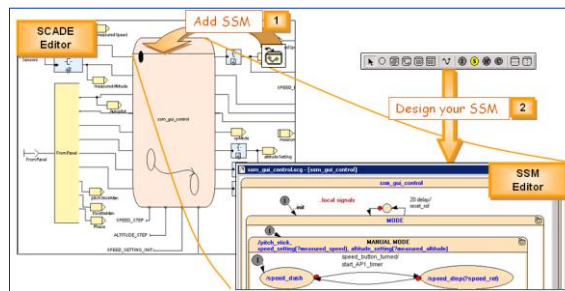


Discrete Modeling in Simulink



FSM Modeling Languages / Frameworks

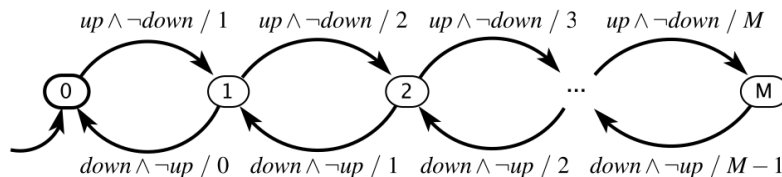
- LabVIEW Statecharts
- Simulink Stateflow
- Scade
- ...



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Garage Counter Mathematical Model



Formally: $(States, Inputs, Outputs, update, initialState)$, where

- $States = \{0, 1, \dots, M\}$
- $Inputs = (\{up, down\} \rightarrow \{absent, present\})$
- $Outputs = (\{count\} \rightarrow \{absent\} \cup \mathbb{N})$
- $update : States \times Inputs \rightarrow States \times Outputs$
- $initialState = 0$

The picture above defines the update function.

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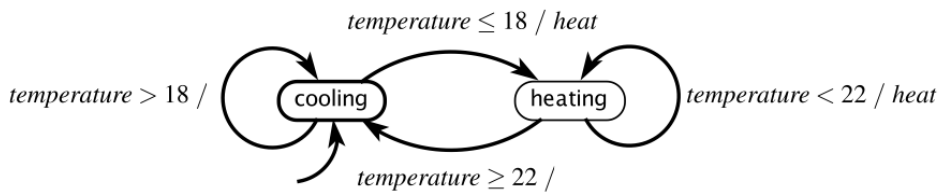
Examples of Guards for Pure Signals

<i>true</i>	Transition is always enabled.
p_1	Transition is enabled if p_1 is <i>present</i> .
$\neg p_1$	Transition is enabled if p_1 is <i>absent</i> .
$p_1 \wedge p_2$	Transition is enabled if both p_1 and p_2 are <i>present</i> .
$p_1 \vee p_2$	Transition is enabled if either p_1 or p_2 is <i>present</i> .
$p_1 \wedge \neg p_2$	Transition is enabled if p_1 is <i>present</i> and p_2 is <i>absent</i> .

Examples of Guards for Signals with Numerical Values

p_3	Transition is enabled if p_3 is <i>present</i> (not <i>absent</i>).
$p_3 = 1$	Transition is enabled if p_3 is <i>present</i> and has value 1.
$p_3 = 1 \wedge p_1$	Transition is enabled if p_3 has value 1 and p_1 is <i>present</i> .
$p_3 > 5$	Transition is enabled if p_3 is <i>present</i> with value greater than 5.

Example of *Modal* Model: Thermostat



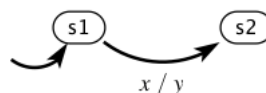
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When does a reaction occur?

- The FSM definition does NOT specify when it reacts.

input: $x \in \{present, absent\}$
 output: $y \in \{present, absent\}$



Suppose all inputs are discrete and a reaction occurs *when any input is present*. Then the above transition will be taken whenever the current state is s1 and x is present.

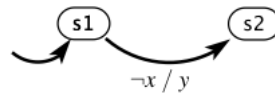
This is an *event-triggered model*.

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When does a reaction occur?

input: $x \in \{present, absent\}$
output: $y \in \{present, absent\}$



Suppose x and y are discrete and pure signals.
When does the transition occur?

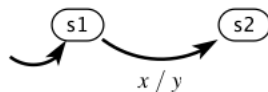
Answer: when the *environment* triggers a reaction and x is absent.
If this is a (complete) event-triggered model, then the transition will never be taken because the reaction will only occur when x is present!

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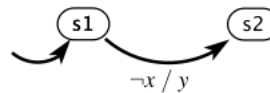
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When does a reaction occur?

input: $x \in \{present, absent\}$
output: $y \in \{present, absent\}$



input: $x \in \{present, absent\}$
output: $y \in \{present, absent\}$



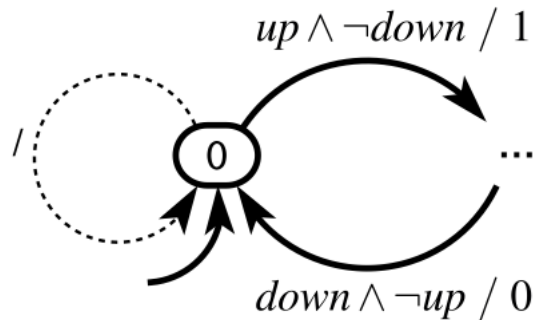
Suppose all inputs are discrete and a reaction occurs
on the tick of an external clock.

This is a *time-triggered model*.

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More Notation: Default Transitions

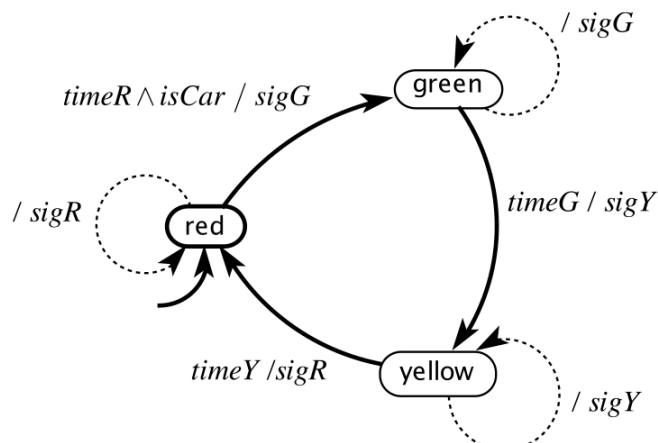


- What happens if state=0 and a car departs?
- A default transition is enabled if no non-default transition is enabled and it either has no guard or the guard evaluates to true. When is the above default transition enabled?

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Example: Traffic Light Controller



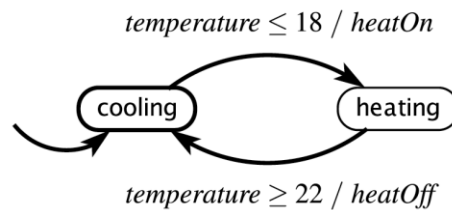
Only show default transitions if they are guarded or produce outputs (or go to other states).

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Where Default Transitions Need Not Be shown

input: $temperature : \mathbb{R}$
outputs: $heatOn, heatOff : \text{pure}$



Exercise: From this picture, construct the formal mathematical model.

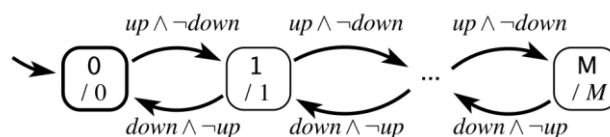
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Mealy and Moore Machines

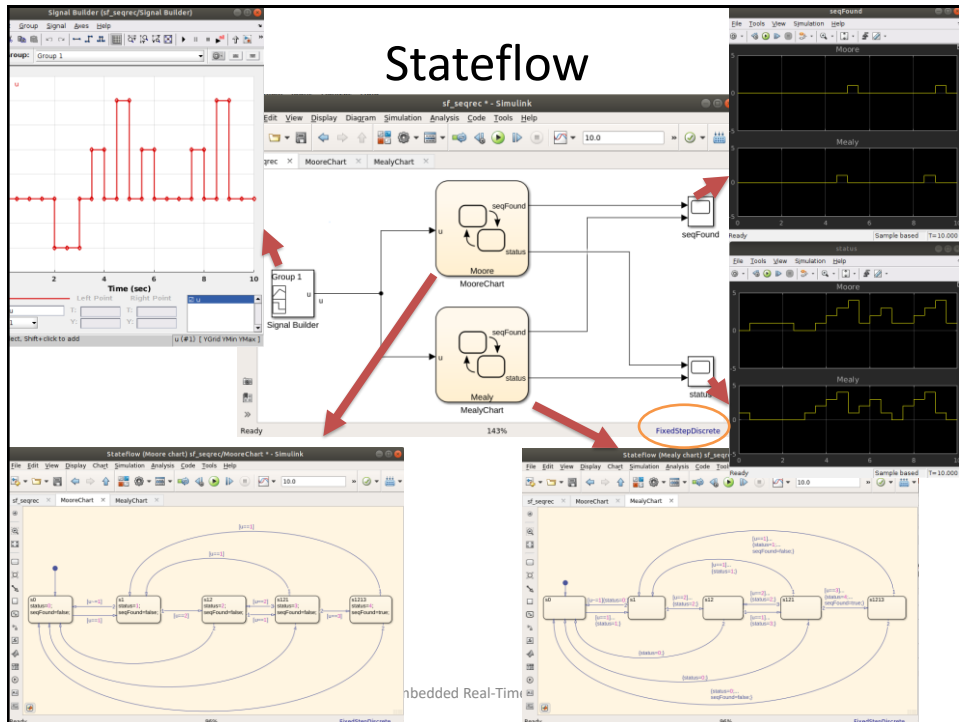
- The state machines we describe in this lecture are known as Mealy machines.
- Moore machine produces outputs when the machine is in a state, rather than when a transition is taken.

inputs: $up, down : \text{pure}$
output: $count : \{0, \dots, M\}$



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Some Definitions

- **Stuttering transition:** (possibly implicit) default transition that is enabled when inputs are absent, that does not change state, and that produces absent outputs.
- **Receptiveness:** For any input values, some transition is enabled. Our structure together with the implicit default transition ensures that our FSMs are receptive.
- **Determinism:** In every state, for all input values, exactly one (possibly implicit) transition is enabled.

Reflection: Three Kinds of Transitions

- Self-Loop
 - Default Transition
 - Stuttering Transition
1. Is a default transition always a self-loop?
 2. Is a stuttering transition always a self-loop?
 3. Is a self-loop always stuttering?

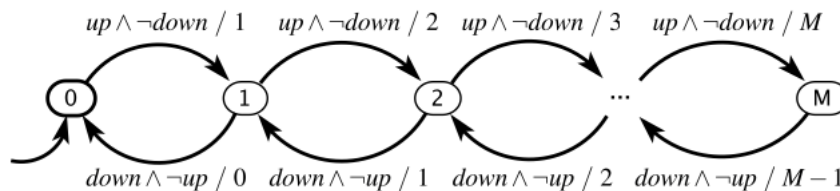
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Garage Counter Example

Is the following model manageable with large Ms?

inputs: $up, down \in \{present, absent\}$
 output $\in \{0, \dots, M\}$



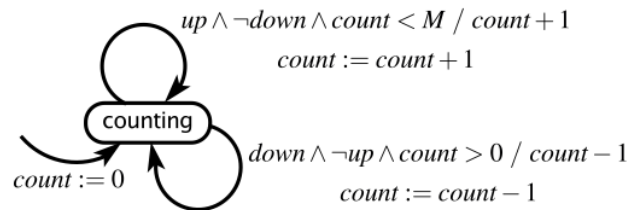
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Extended State Machines

Extended state machines augment the FSM model with *variables* that may be read or written. E.g.:

variable: $count \in \{0, \dots, M\}$
 inputs: $up, down \in \{present, absent\}$
 output $\in \{0, \dots, M\}$



Question: What is the size of the state space?

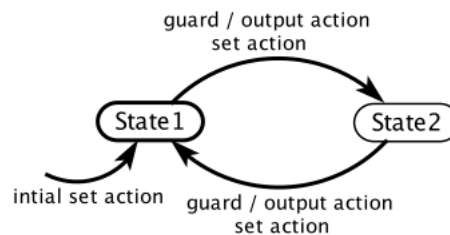
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General Notation for Extended State Machines

We make explicit declarations of variables, inputs, and outputs to help distinguish the three.

variable declaration(s)
 input declaration(s)
 output declaration(s)



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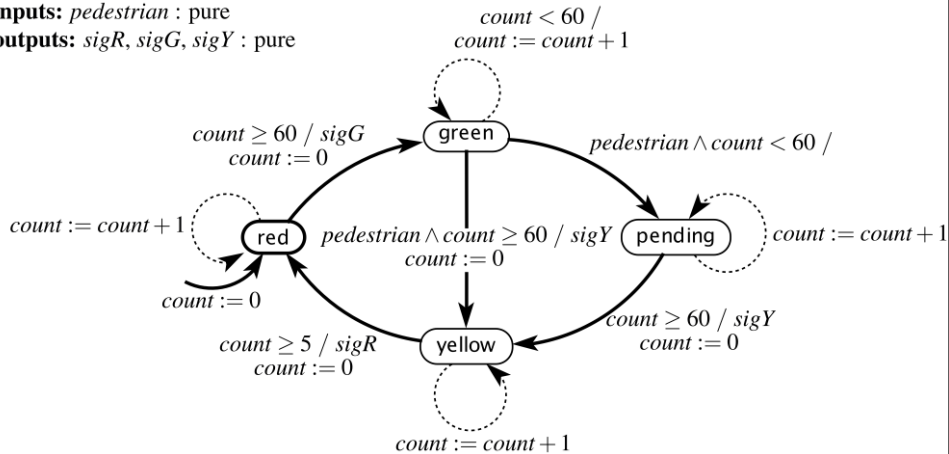
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Extended state machine model of a traffic light controller at a pedestrian crossing

variable: *count*: $\{0, \dots, 60\}$

inputs: *pedestrian*: pure

outputs: *sigR*, *sigG*, *sigY*: pure



This model assumes one reaction per second
(a time-triggered model)

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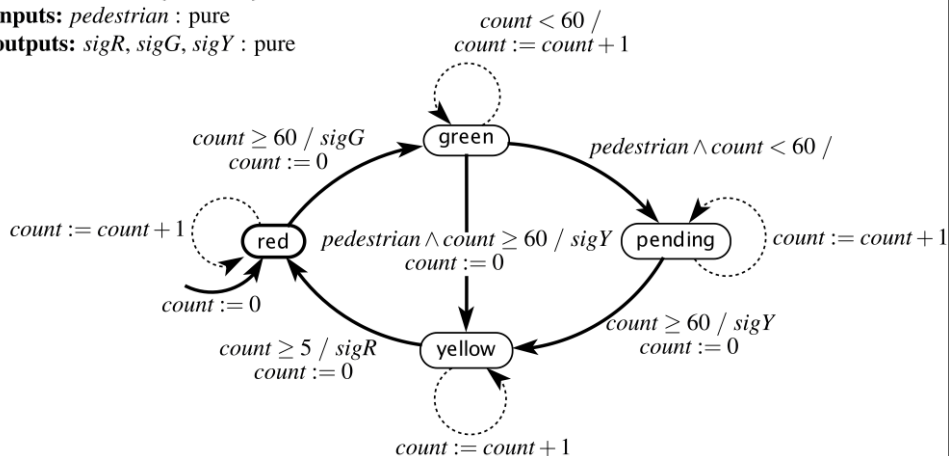
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Quiz: What is the Size of the State Space for the Traffic Light Controller?

variable: *count*: $\{0, \dots, 60\}$

inputs: *pedestrian*: pure

outputs: *sigR*, *sigG*, *sigY*: pure

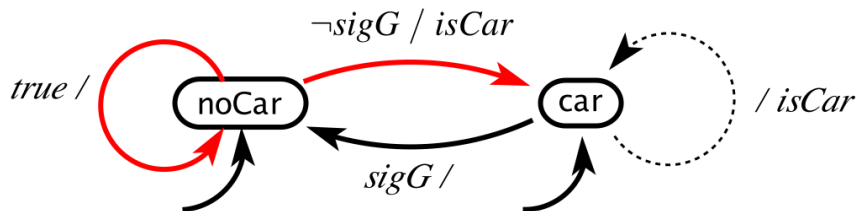


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Example: Nondeterministic FSM

Model of the environment for a traffic light, abstracted using nondeterminism:



Formally, the update function is replaced by such a function:

$$\text{possibleUpdates} : \text{States} \times \text{Inputs} \rightarrow 2^{\text{States} \times \text{Outputs}}$$

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Uses of Nondeterminism

- Modeling **unknown aspects** of the **environment** or **system**
 - Such as: how the environment changes a robot's orientation
- **Hiding detail** in a *specification* of the system
 - See the text

Any other reasons why nondeterministic FSMs might be preferred over deterministic FSMs?

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Size Matters

Non-deterministic FSMs are **more compact** than deterministic FSMs

- A classic result in automata theory shows that a nondeterministic FSM has a related deterministic FSM that is equivalent in a technical sense.
- But the deterministic machine has, in the worst case, many more states (exponential in the number of states of the nondeterministic machine, Appendix B).

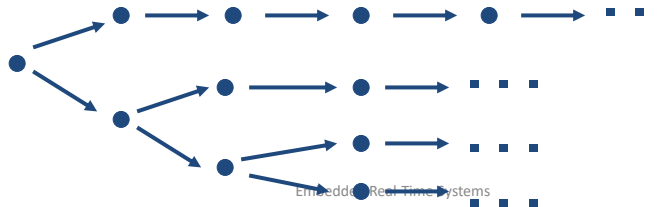
Non-deterministic Behavior: Tree of Computations

- For a fixed input sequence:
 - A deterministic system exhibits a single behavior
 - A non-deterministic system exhibits a **set of behaviors**
 - visualized as a *computation tree*

Deterministic FSM behavior:



Non-deterministic FSM behavior:



Non-deterministic \neq Probabilistic (Stochastic)

In a probabilistic FSM, each transition has an associated probability with which it is taken.

In a non-deterministic FSM, no such probability is known. We just know that any of the enabled transitions from a state can be taken.

Next Lecture

- Timed automata
- Hybrid systems
- Read chapter 4 of LeeSeshia