DanceParty

Team Reference Document

Contents

1. Code Templates

1.1. Basic Configuration. Vim and (Caps Lock = Escape) configuration.

```
o.yqtxmal ekrpat  # setxkbmap dvorak for dvorak on qwerty
setxkbmap -option caps:escape
set -o vi
xset r rate 150 100
cat > ~/.vimrc
set nocp et sw=4 ts=4 sts=4 si cindent hi=1000 nu ru noeb showcmd showmode
syn on | colorscheme slate
```

1.2. C++ Header. A C++ header.

```
#include <algorithm>------------------------------------------------------------// 5e
#include <cassert>--------------------------------------------------------------// 65
#include <cmath>----------------------------------------------------------------// 7d
#include <cstdio>---------------------------------------------------------------// 2e
#include <cstdlib>--------------------------------------------------------------// 11
#include <cstring>--------------------------------------------------------------// d0
#include <ctime>----------------------------------------------------------------// 28
#include <iomanip>--------------------------------------------------------------// 29
#include <iostream>-------------------------------------------------------------// ec
#include <map>------------------------------------------------------------------// 02
#include <queue>----------------------------------------------------------------// 75
#include <set>------------------------------------------------------------------// e0
#include <sstream>--------------------------------------------------------------// 18
#include <stack>----------------------------------------------------------------// cf
#include <string>---------------------------------------------------------------// a9
#include <utility>--------------------------------------------------------------// d8
#include <vector>---------------------------------------------------------------// 4f
using namespace std;------------------------------------------------------------// 7b
--------------------------------------------------------------------------------// 7e
#define foreach(u, o) \---------------------------------------------------------// ea
----for (typeof((o).begin()) u = (o).begin(); u != (o).end(); ++u)-------------// 1a
const int INF = 2147483647;-----------------------------------------------------// be
const double EPS = 1e-9;--------------------------------------------------------// 0f
const double pi = acos(-1);-----------------------------------------------------// 49
typedef long long ll;-----------------------------------------------------------// 8f
typedef unsigned long long ull;-------------------------------------------------// 81
typedef pair<int, int> ii;------------------------------------------------------// 56
typedef vector<int> vi;---------------------------------------------------------// f1
typedef vector<ii> vii;---------------------------------------------------------// 29
typedef vector<vi> vvi;---------------------------------------------------------// 31
typedef vector<vii> vvii;-------------------------------------------------------// 4b
template <class T> T mod(T a, T b) { return (a % b + b) % b; }------------------// 70
template <class T> int size(const T &x) { return x.size(); }--------------------// 68
```

1.3. Java Template. A Java template.

```
import java.util.*;-------------------------------------------------------------// 37
import java.math.*;-------------------------------------------------------------// 89
import java.io.*;---------------------------------------------------------------// 28
--------------------------------------------------------------------------------// a3
public class Main {-------------------------------------------------------------// 17
----public static void main(String[] args) throws Exception {-------------------// 02
--------Scanner in = new Scanner(System.in);------------------------------------// ef
--------PrintWriter out = new PrintWriter(System.out, false);-------------------// 62
--------// code-----------------------------------------------------------------// e6
--------out.flush();------------------------------------------------------------// 56
----}---------------------------------------------------------------------------// 79
}-------------------------------------------------------------------------------// 00
```

2. Data Structures

2.1. Union-Find. An implementation of the Union-Find disjoint sets data structure.

```
struct union_find {-------------------------------------------------------------// 42
----vi parent;------------------------------------------------------------------// 1c
----int cnt;--------------------------------------------------------------------// 0d
----union_find(int n) { parent.resize(cnt = n);---------------------------------// 92
--------for (int i = 0; i < cnt; i++) parent[i] = i; }--------------------------// 6f
----int find(int i) {-----------------------------------------------------------// a6
--------return parent[i] == i ? i : (parent[i] = find(parent[i])); }-----------// a9
----bool unite(int i, int j) {--------------------------------------------------// 89
--------int ip = find(i), jp = find(j);-----------------------------------------// 32
--------parent[ip] = jp; return ip != jp; } };----------------------------------// b5
```

2.2. Segment Tree. An implementation of a Segment Tree.

```
// const int ID = INF;----------------------------------------------------------// d2
// int f(int a, int b) { return min(a, b); }------------------------------------// 0c
const int ID = 0;---------------------------------------------------------------// 82
int f(int a, int b) { return a + b; }-------------------------------------------// 6d
struct segment_tree {-----------------------------------------------------------// e5
----int n; vi data, lazy;-------------------------------------------------------// fa
----segment_tree(const vi &arr) : n(size(arr)), data(4*n), lazy(4*n,INF) {-----// 77
--------mk(arr, 0, n-1, 0); }---------------------------------------------------// 3b
----int mk(const vi &arr, int l, int r, int i) {--------------------------------// 02
--------if (l == r) return data[i] = arr[l];------------------------------------// 37
--------int m = (l + r) / 2;----------------------------------------------------// 0f
--------return data[i] = f(mk(arr, l, m, 2*i+1), mk(arr, m+1, r, 2*i+2)); }----// 09
----int query(int a, int b) { return q(a, b, 0, n-1, 0); }----------------------// f5
----int q(int a, int b, int l, int r, int i) {----------------------------------// ad
--------propagate(l, r, i);-----------------------------------------------------// f7
--------if (r < a || b < l) return ID;------------------------------------------// da
--------if (a <= l && r <= b) return data[i];-----------------------------------// 6f
--------int m = (l + r) / 2;----------------------------------------------------// a2
--------return f(q(a, b, l, m, 2*i+1), q(a, b, m+1, r, 2*i+2)); }---------------// 9e
----void update(int i, int v) { u(i, v, 0, n-1, 0); }---------------------------// 65
----int u(int i, int v, int l, int r, int j) {----------------------------------// b5
--------propagate(l, r, j);-----------------------------------------------------// 3c
--------if (r < i || i < l) return data[j];-------------------------------------// 6a
--------if (l == i && r == i) return data[j] = v;-------------------------------// 74
--------int m = (l + r) / 2;----------------------------------------------------// 78
--------return data[j] = f(u(i, v, l, m, 2*j+1), u(i, v, m+1, r, 2*j+2)); }----// 68
----void range_update(int a, int b, int v) { ru(a, b, v, 0, n-1, 0); }---------// a3
----int ru(int a, int b, int v, int l, int r, int i) {--------------------------// d7
--------propagate(l, r, i);-----------------------------------------------------// 82
```

```
--------if (l > r) return ID;---------------------------------------------------// 72
--------if (r < a || b < l) return data[i];-------------------------------------// bb
--------if (l == r) return data[i] += v;----------------------------------------// e2
--------if (a <= l && r <= b) return (lazy[i] = v) * (r - l + 1) + data[i];----// 89
--------int m = (l + r) / 2;----------------------------------------------------// 9d
--------return data[i] = f(ru(a, b, v, l, m, 2*i+1), ru(a, b, v, m+1, r, 2*i+2));
----}---------------------------------------------------------------------------// ff
----void propagate(int l, int r, int i) {---------------------------------------// 4c
--------if (l > r || lazy[i] == INF) return;------------------------------------// 42
--------data[i] += lazy[i] * (r - l + 1);---------------------------------------// 7c
--------if (l < r) {------------------------------------------------------------// 6e
------------if (lazy[2*i+1] == INF) lazy[2*i+1] = lazy[i];----------------------// 1d
------------else lazy[2*i+1] += lazy[i];----------------------------------------// 12
------------if (lazy[2*i+2] == INF) lazy[2*i+2] = lazy[i];----------------------// f9
------------else lazy[2*i+2] += lazy[i];----------------------------------------// d1
--------}-----------------------------------------------------------------------// 0f
--------lazy[i] = INF;----------------------------------------------------------// 7e
----}---------------------------------------------------------------------------// b2
};------------------------------------------------------------------------------// c6
```

2.3. **Fenwick Tree.** A Fenwick Tree is a data structure that represents an array of n numbers. It supports adjusting the i-th element in $O(\log n)$ time, and computing the sum of numbers in the range $i..j$ in $O(\log n)$ time. It only needs $O(n)$ space.

```
struct fenwick_tree {-----------------------------------------------------------// 98
----int n; vi data;-------------------------------------------------------------// d3
----fenwick_tree(int _n) : n(_n), data(vi(n)) { }-------------------------------// db
----void update(int at, int by) {-----------------------------------------------// 76
--------while (at < n) data[at] += by, at |= at + 1; }--------------------------// fb
----int query(int at) {---------------------------------------------------------// 71
--------int res = 0;------------------------------------------------------------// c3
--------while (at >= 0) res += data[at], at = (at & (at + 1)) - 1;--------------// 37
--------return res; }-----------------------------------------------------------// e4
----int rsq(int a, int b) { return query(b) - query(a - 1); }-------------------// be
};------------------------------------------------------------------------------// 57
struct fenwick_tree_sq {--------------------------------------------------------// d4
----int n; fenwick_tree x1, x0;-------------------------------------------------// 18
----fenwick_tree_sq(int _n) : n(_n), x1(fenwick_tree(n)),-----------------------// 2e
--------x0(fenwick_tree(n)) { }-------------------------------------------------// 7c
----// insert f(y) = my + c if x <= y-------------------------------------------// 17
----void update(int x, int m, int c) { x1.update(x, m); x0.update(x, c); }------// 45
----int query(int x) { return x*x1.query(x) + x0.query(x); }--------------------// 73
};------------------------------------------------------------------------------// 13
void range_update(fenwick_tree_sq &s, int a, int b, int k) {--------------------// 89
----s.update(a, k, k * (1 - a)); s.update(b+1, -k, k * b); }--------------------// 7f
int range_query(fenwick_tree_sq &s, int a, int b) {-----------------------------// 15
----return s.query(b) - s.query(a-1); }-----------------------------------------// f3
```

2.4. **Matrix.** A Matrix class.

```
template <class K> bool eq(K a, K b) { return a == b; }-------------------------// 2a
template <> bool eq<double>(double a, double b) { return abs(a - b) < EPS; }---// a7
template <class T>--------------------------------------------------------------// 53
class matrix {------------------------------------------------------------------// 85
public:-------------------------------------------------------------------------// be
----int rows, cols;-------------------------------------------------------------// d3
----matrix(int r, int c) : rows(r), cols(c), cnt(r * c) {-----------------------// 34
--------data.assign(cnt, T(0)); }-----------------------------------------------// d0
----matrix(const matrix& other) : rows(other.rows), cols(other.cols),-----------// fe
--------cnt(other.cnt), data(other.data) { }------------------------------------// ed
----T& operator()(int i, int j) { return at(i, j); }----------------------------// e0
----void operator +=(const matrix& other) {-------------------------------------// c9
--------for (int i = 0; i < cnt; i++) data[i] += other.data[i]; }---------------// e5
----void operator -=(const matrix& other) {-------------------------------------// 68
--------for (int i = 0; i < cnt; i++) data[i] -= other.data[i]; }---------------// 88
----void operator *=(T other) {-------------------------------------------------// ba
--------for (int i = 0; i < cnt; i++) data[i] *= other; }-----------------------// 40
----matrix<T> operator +(const matrix& other) {---------------------------------// ee
--------matrix<T> res(*this); res += other; return res; }-----------------------// 5d
----matrix<T> operator -(const matrix& other) {---------------------------------// 8f
--------matrix<T> res(*this); res -= other; return res; }-----------------------// cf
----matrix<T> operator *(T other) {---------------------------------------------// be
--------matrix<T> res(*this); res *= other; return res; }-----------------------// 37
----matrix<T> operator *(const matrix& other) {---------------------------------// 95
--------matrix<T> res(rows, other.cols);----------------------------------------// 57
--------for (int i = 0; i < rows; i++) for (int j = 0; j < other.cols; j++)----// 7a
------------for (int k = 0; k < cols; k++)--------------------------------------// fc
----------------res(i, j) += at(i, k) * other.data[k * other.cols + j];--------// eb
--------return res; }-----------------------------------------------------------// 70
----matrix<T> transpose() {-----------------------------------------------------// dd
--------matrix<T> res(cols, rows);----------------------------------------------// b5
--------for (int i = 0; i < rows; i++)------------------------------------------// 9c
------------for (int j = 0; j < cols; j++) res(j, i) = at(i, j);----------------// a3
--------return res; }-----------------------------------------------------------// c3
----matrix<T> pow(int p) {------------------------------------------------------// 68
--------matrix<T> res(rows, cols), sq(*this);-----------------------------------// 4d
--------for (int i = 0; i < rows; i++) res(i, i) = T(1);------------------------// bf
--------while (p) {-------------------------------------------------------------// cb
------------if (p & 1) res = res * sq;------------------------------------------// c1
------------p >>= 1;------------------------------------------------------------// 68
------------if (p) sq = sq * sq;------------------------------------------------// 9c
--------} return res; }---------------------------------------------------------// 50
----matrix<T> rref(T &det) {----------------------------------------------------// 89
--------matrix<T> mat(*this); det = T(1);---------------------------------------// 21
--------for (int r = 0, c = 0; c < cols; c++) {---------------------------------// c4
------------int k = r;----------------------------------------------------------// e5
------------while (k < rows && eq<T>(mat(k, c), T(0))) k++;---------------------// f9
------------if (k >= rows) continue;--------------------------------------------// 3f
------------if (k != r) {-------------------------------------------------------// a3
----------------det *= T(-1);---------------------------------------------------// 7a
----------------for (int i = 0; i < cols; i++)---------------------------------// ab
--------------------swap(mat.at(k, i), mat.at(r, i));---------------------------// 8d
------------} det *= mat(r, r);-------------------------------------------------// 19
------------if (!eq<T>(mat(r, c), T(1)))----------------------------------------// 2c
----------------for (int i = cols-1; i >= c; i--) mat(r, i) /= mat(r, c);------// 5d
```

```
------------for (int i = 0; i < rows; i++) {-------------------------------------// 3d
----------------T m = mat(i, c);-------------------------------------------------// e8
----------------if (i != r && !eq<T>(m, T(0)))-----------------------------------// 33
--------------------for (int j = 0; j < cols; j++) mat(i, j) -= m * mat(r, j);-// f6
------------} r++;---------------------------------------------------------------// 3a
--------} return mat; }----------------------------------------------------------// 8f
private:-------------------------------------------------------------------------// e0
----int cnt;---------------------------------------------------------------------// 6a
----vector<T> data;--------------------------------------------------------------// 41
----inline T& at(int i, int j) { return data[i * cols + j]; }--------------------// 74
};-------------------------------------------------------------------------------// b8
```

2.5. **AVL Tree.** A fast, easily augmentable, balanced binary search tree.

```
#define AVL_MULTISET 0-----------------------------------------------------------// b5
---------------------------------------------------------------------------------// 61
template <class T>---------------------------------------------------------------// 22
class avl_tree {-----------------------------------------------------------------// ff
public:--------------------------------------------------------------------------// f6
----struct node {----------------------------------------------------------------// 45
--------T item; node *p, *l, *r;-------------------------------------------------// a6
--------int size, height;--------------------------------------------------------// 33
--------node(const T &item, node *p = NULL) : item(item), p(p),------------------// c5
--------l(NULL), r(NULL), size(1), height(0) { } };------------------------------// e1
----avl_tree() : root(NULL) { }--------------------------------------------------// dc
----node *root;------------------------------------------------------------------// c1
----node* find(const T &item) const {--------------------------------------------// d2
--------node *cur = root;--------------------------------------------------------// cf
--------while (cur) {------------------------------------------------------------// ad
------------if (cur->item < item) cur = cur->r;----------------------------------// eb
------------else if (item < cur->item) cur = cur->l;-----------------------------// de
------------else break; }--------------------------------------------------------// 05
--------return cur; }------------------------------------------------------------// e7
----node* insert(const T &item) {------------------------------------------------// 89
--------node *prev = NULL, **cur = &root;----------------------------------------// 60
--------while (*cur) {-----------------------------------------------------------// b0
------------prev = *cur;---------------------------------------------------------// 31
------------if ((*cur)->item < item) cur = &((*cur)->r);-------------------------// 39
#if AVL_MULTISET-----------------------------------------------------------------// d1
------------else cur = &((*cur)->l);---------------------------------------------// 3b
#else----------------------------------------------------------------------------// dc
------------else if (item < (*cur)->item) cur = &((*cur)->l);--------------------// e5
------------else return *cur;----------------------------------------------------// 19
#endif---------------------------------------------------------------------------// c6
--------}------------------------------------------------------------------------// d8
--------node *n = new node(item, prev);------------------------------------------// 5b
--------*cur = n, fix(n); return n; }--------------------------------------------// 86
----void erase(const T &item) { erase(find(item)); }-----------------------------// c0
----void erase(node *n, bool free = true) {--------------------------------------// 89
--------if (!n) return;----------------------------------------------------------// 4d
--------if (!n->l && n->r) parent_leg(n) = n->r, n->r->p = n->p;-----------------// f5
--------else if (n->l && !n->r) parent_leg(n) = n->l, n->l->p = n->p;------------// 3d
--------else if (n->l && n->r) {-------------------------------------------------// 1a
------------node *s = successor(n);----------------------------------------------// 16
------------erase(s, false);-----------------------------------------------------// 17
------------s->p = n->p, s->l = n->l, s->r = n->r;-------------------------------// 37
------------if (n->l) n->l->p = s;-----------------------------------------------// 88
------------if (n->r) n->r->p = s;-----------------------------------------------// 42
------------parent_leg(n) = s, fix(s);-------------------------------------------// 87
------------return;--------------------------------------------------------------// 32
--------} else parent_leg(n) = NULL;---------------------------------------------// 58
--------fix(n->p), n->p = n->l = n->r = NULL;------------------------------------// 70
--------if (free) delete n; }----------------------------------------------------// 99
----node* successor(node *n) const {---------------------------------------------// 1b
--------if (!n) return NULL;-----------------------------------------------------// b3
--------if (n->r) return nth(0, n->r);-------------------------------------------// 5b
--------node *p = n->p;----------------------------------------------------------// 7c
--------while (p && p->r == n) n = p, p = p->p;----------------------------------// 04
--------return p; }--------------------------------------------------------------// 03
----node* predecessor(node *n) const {-------------------------------------------// e6
--------if (!n) return NULL;-----------------------------------------------------// 96
--------if (n->l) return nth(n->l->size-1, n->l);--------------------------------// 15
--------node *p = n->p;----------------------------------------------------------// 33
--------while (p && p->l == n) n = p, p = p->p;----------------------------------// 03
--------return p; }--------------------------------------------------------------// 83
----inline int size() const { return sz(root); }---------------------------------// e2
----void clear() { delete_tree(root), root = NULL; }-----------------------------// d4
----node* nth(int n, node *cur = NULL) const {-----------------------------------// f4
--------if (!cur) cur = root;----------------------------------------------------// 0a
--------while (cur) {------------------------------------------------------------// 55
------------if (n < sz(cur->l)) cur = cur->l;------------------------------------// 25
------------else if (n > sz(cur->l)) n -= sz(cur->l) + 1, cur = cur->r;--------// 8b
------------else break;----------------------------------------------------------// 4c
--------} return cur; }----------------------------------------------------------// 8f
private:-------------------------------------------------------------------------// 49
----inline int sz(node *n) const { return n ? n->size : 0; }---------------------// 69
----inline int height(node *n) const { return n ? n->height : -1; }--------------// e4
----inline bool left_heavy(node *n) const {--------------------------------------// d7
--------return n && height(n->l) > height(n->r); }-------------------------------// 9d
----inline bool right_heavy(node *n) const {-------------------------------------// 91
--------return n && height(n->r) > height(n->l); }-------------------------------// 77
----inline bool too_heavy(node *n) const {---------------------------------------// 18
--------return n && abs(height(n->l) - height(n->r)) > 1; }----------------------// fb
----void delete_tree(node *n) {--------------------------------------------------// 48
--------if (n) { delete_tree(n->l), delete_tree(n->r); delete n; } }-------------// ea
----node*& parent_leg(node *n) {-------------------------------------------------// 0d
--------if (!n->p) return root;--------------------------------------------------// af
--------if (n->p->l == n) return n->p->l;----------------------------------------// 95
--------if (n->p->r == n) return n->p->r;----------------------------------------// 0e
--------assert(false); }---------------------------------------------------------// f4
----void augment(node *n) {------------------------------------------------------// 2c
--------if (!n) return;----------------------------------------------------------// 46
--------n->size = 1 + sz(n->l) + sz(n->r);---------------------------------------// 14
--------n->height = 1 + max(height(n->l), height(n->r)); }-----------------------// a1
```

```
    #define rotate(l, r) \ // b7
        node *l = n->l; \ // 40
        l->p = n->p; \ // 66
        parent_leg(n) = l; \ // 02
        n->l = l->r; \ // 08
        if (l->r) l->r->p = n; \ // eb
        l->r = n, n->p = l; \ // c3
        augment(n), augment(l) // 2e
    void left_rotate(node *n) { rotate(r, l); } // 43
    void right_rotate(node *n) { rotate(l, r); } // ac
    void fix(node *n) { // 42
        while (n) { augment(n); // c9
            if (too_heavy(n)) { // a9
                if (left_heavy(n) && right_heavy(n->l)) left_rotate(n->l); // 05
                else if (right_heavy(n) && left_heavy(n->r)) // 09
                    right_rotate(n->r); // 7c
                if (left_heavy(n)) right_rotate(n); // 44
                else left_rotate(n); // 02
                n = n->p; } // af
            n = n->p; } } }; // 85
```

Also a very simple wrapper over the AVL tree that implements a map interface.

```
#include "avl_tree.cpp" // 01
 // ba
template <class K, class V> // da
class avl_map { // 3f
public: // 5d
    struct node { // 2f
        K key; V value; // 32
        node(K k, V v) : key(k), value(v) { } // 29
        bool operator <(const node &other) const { return key < other.key; } }; // 92
    avl_tree<node> tree; // b1
    V& operator [](K key) { // 7c
        typename avl_tree<node>::node *n = tree.find(node(key, V(0))); // ba
        if (!n) n = tree.insert(node(key, V(0))); // cb
        return n->item.value; // ec
    } // 2e
}; // af
```

2.6. **Heap.** An implementation of a binary heap.

```
#define RESIZE // d0
#define SWP(x,y) tmp = x, x = y, y = tmp // fb
struct default_int_cmp { // 8d
    default_int_cmp() { } // 35
    bool operator ()(const int &a, const int &b) { return a < b; } }; // e9
template <class Compare = default_int_cmp> // 30
class heap { // 05
private: // 39
    int len, count, *q, *loc, tmp; // 0a
    Compare _cmp; // 98
    inline bool cmp(int i, int j) { return _cmp(q[i], q[j]); } // a0
    inline void swp(int i, int j) { // 1c
        SWP(q[i], q[j]), SWP(loc[q[i]], loc[q[j]]); } // 67
    void swim(int i) { // 33
        while (i > 0) { // 1a
            int p = (i - 1) / 2; // 77
            if (!cmp(i, p)) break; // a9
            swp(i, p), i = p; } } // 93
    void sink(int i) { // ce
        while (true) { // 3c
            int l = 2*i + 1, r = l + 1; // b4
            if (l >= count) break; // d5
            int m = r >= count || cmp(l, r) ? l : r; // cc
            if (!cmp(m, i)) break; // 42
            swp(m, i), i = m; } } // 1d
public: // cd
    heap(int init_len = 128) : count(0), len(init_len), _cmp(Compare()) { // 17
        q = new int[len], loc = new int[len]; // f8
        memset(loc, 255, len << 2); } // f7
    ~heap() { delete[] q; delete[] loc; } // 09
    void push(int n, bool fix = true) { // b7
        if (len == count || n >= len) { // 0f
#ifdef RESIZE // a9
            int newlen = 2 * len; // 22
            while (n >= newlen) newlen *= 2; // 2f
            int *newq = new int[newlen], *newloc = new int[newlen]; // e3
            for (int i = 0; i < len; i++) newq[i] = q[i], newloc[i] = loc[i]; // 94
            memset(newloc + len, 255, (newlen - len) << 2); // 18
            delete[] q, delete[] loc; // 74
            loc = newloc, q = newq, len = newlen; // 61
#else // 54
            assert(false); // 84
#endif // 64
        } // 4b
        assert(loc[n] == -1); // 8f
        loc[n] = count, q[count++] = n; // 6b
        if (fix) swim(count-1); } // bf
    void pop(bool fix = true) { // 43
        assert(count > 0); // eb
        loc[q[0]] = -1, q[0] = q[--count], loc[q[0]] = 0; // 50
        if (fix) sink(0); // 80
    } // 16
    int top() { assert(count > 0); return q[0]; } // ab
    void heapify() { for (int i = count - 1; i > 0; i--) // 39
        if (cmp(i, (i - 1) / 2)) swp(i, (i - 1) / 2); } // 0b
    void update_key(int n) { // 26
        assert(loc[n] != -1), swim(loc[n]), sink(loc[n]); } // 7d
    bool empty() { return count == 0; } // f8
    int size() { return count; } // 86
    void clear() { count = 0, memset(loc, 255, len << 2); } }; // 58
```

2.7. **Skiplist.** An implementation of a skiplist.

```
#define BP 0.20-------------------------------------------------------// aa
#define MAX_LEVEL 10--------------------------------------------------// 56
unsigned int bernoulli(unsigned int MAX) {----------------------------// 7b
----unsigned int cnt = 0;---------------------------------------------// 28
----while(((float) rand() / RAND_MAX) < BP && cnt < MAX) cnt++;-------// d1
----return cnt; }-----------------------------------------------------// a1
template<class T> struct skiplist {-----------------------------------// 34
----struct node {-----------------------------------------------------// 53
--------T item;-------------------------------------------------------// e3
--------int *lens;----------------------------------------------------// 07
--------node **next;--------------------------------------------------// 0c
--------#define CA(v, t) v((t*)calloc(level+1, sizeof(t)))-------------// 25
--------node(int level, T i) : item(i), CA(lens, int), CA(next, node*) {}-----// 7c
--------~node() { free(lens); free(next); }; };-----------------------// aa
----int current_level, _size;-----------------------------------------// 61
----node *head;-------------------------------------------------------// b7
----skiplist() : current_level(0), _size(0), head(new node(MAX_LEVEL, 0)) { };-// 7a
----~skiplist() { clear(); delete head; head = NULL; }----------------// aa
----#define FIND_UPDATE(cmp, target) \---------------------------------// c3
--------int pos[MAX_LEVEL + 2]; \-------------------------------------// 18
--------memset(pos, 0, sizeof(pos)); \--------------------------------// f2
--------node *x = head; \---------------------------------------------// 0f
--------node *update[MAX_LEVEL + 1]; \--------------------------------// 01
--------memset(update, 0, MAX_LEVEL + 1); \---------------------------// 38
--------for(int i = MAX_LEVEL; i >= 0; i--) { \-----------------------// 87
------------pos[i] = pos[i + 1]; \------------------------------------// 68
------------while(x->next[i] != NULL && cmp < target) { \-------------// 93
----------------pos[i] += x->lens[i]; x = x->next[i]; } \-------------// 10
------------update[i] = x; \------------------------------------------// dd
--------} x = x->next[0];---------------------------------------------// fc
----int size() const { return _size; }--------------------------------// 9a
----void clear() { while(head->next && head->next[0])-----------------// 91
--------erase(head->next[0]->item); }---------------------------------// e6
----node *find(T target) { FIND_UPDATE(x->next[i]->item, target);-----// 36
--------return x && x->item == target ? x : NULL; }-------------------// 50
----node *nth(int k) { FIND_UPDATE(pos[i] + x->lens[i], k+1); return x; }-----// b8
----int count_less(T target) { FIND_UPDATE(x->next[i]->item, target);-// 80
--------return pos[0]; }----------------------------------------------// 19
----node* insert(T target) {------------------------------------------// 80
--------FIND_UPDATE(x->next[i]->item, target);------------------------// 3a
--------if(x && x->item == target) return x; // SET-------------------// 07
--------int lvl = bernoulli(MAX_LEVEL);-------------------------------// 7a
--------if(lvl > current_level) current_level = lvl;------------------// 8a
--------x = new node(lvl, target);------------------------------------// 36
--------for(int i = 0; i <= lvl; i++) {-------------------------------// 49
------------x->next[i] = update[i]->next[i];--------------------------// 46
------------x->lens[i] = pos[i] + update[i]->lens[i] - pos[0];--------// bc
------------update[i]->next[i] = x;-----------------------------------// 20
------------update[i]->lens[i] = pos[0] + 1 - pos[i];-----------------// 42
--------}-------------------------------------------------------------// fc
--------for(int i = lvl + 1; i <= MAX_LEVEL; i++) update[i]->lens[i]++;--------// 07
--------_size++;------------------------------------------------------// 19
--------return x; }---------------------------------------------------// c9
----void erase(T target) {--------------------------------------------// 4d
--------FIND_UPDATE(x->next[i]->item, target);------------------------// 6b
--------if(x && x->item == target) {----------------------------------// 76
------------for(int i = 0; i <= current_level; i++) {-----------------// 97
----------------if(update[i]->next[i] == x) {-------------------------// b1
--------------------update[i]->next[i] = x->next[i];------------------// 59
--------------------update[i]->lens[i] = update[i]->lens[i] + x->lens[i] - 1;--// b1
----------------} else update[i]->lens[i] = update[i]->lens[i] - 1;---// 88
------------}---------------------------------------------------------// dd
------------delete x; _size--;----------------------------------------// 81
------------while(current_level > 0 && head->next[current_level] == NULL)------// 7f
----------------current_level--; } } };-------------------------------// 59
```

2.8. **Dancing Links.** An implementation of Donald Knuth's Dancing Links data structure. A linked list supporting deletion and restoration of elements.

```
template <class T>----------------------------------------------------// 82
struct dancing_links {------------------------------------------------// 9e
----struct node {-----------------------------------------------------// 62
--------T item;-------------------------------------------------------// dd
--------node *l, *r;--------------------------------------------------// 32
--------node(const T &item, node *l = NULL, node *r = NULL)-----------// 88
------------: item(item), l(l), r(r) {--------------------------------// 04
------------if (l) l->r = this;---------------------------------------// 1c
------------if (r) r->l = this;---------------------------------------// 0b
--------}-------------------------------------------------------------// 61
----};----------------------------------------------------------------// 97
----node *front, *back;-----------------------------------------------// 23
----dancing_links() { front = back = NULL; }--------------------------// 8c
----node *push_back(const T &item) {----------------------------------// d7
--------back = new node(item, back, NULL);----------------------------// 5d
--------if (!front) front = back;-------------------------------------// a2
--------return back;--------------------------------------------------// b4
----}-----------------------------------------------------------------// b1
----node *push_front(const T &item) {---------------------------------// ea
--------front = new node(item, NULL, front);--------------------------// 75
--------if (!back) back = front;--------------------------------------// d6
--------return front;-------------------------------------------------// ef
----}-----------------------------------------------------------------// 30
----void erase(node *n) {---------------------------------------------// 88
--------if (!n->l) front = n->r; else n->l->r = n->r;-----------------// d5
--------if (!n->r) back = n->l; else n->r->l = n->l;------------------// 96
----}-----------------------------------------------------------------// ae
----void restore(node *n) {-------------------------------------------// 6d
--------if (!n->l) front = n; else n->l->r = n;-----------------------// ab
--------if (!n->r) back = n; else n->r->l = n;------------------------// 8d
----}-----------------------------------------------------------------// 02
};--------------------------------------------------------------------// 4f
```

2.9. **Misof Tree.** A simple tree data structure for inserting, erasing, and querying the nth largest element.

```
#define BITS 15----------------------------------------------------------------// 7b
struct misof_tree {------------------------------------------------------------// fe
----int cnt[BITS][1<<BITS];----------------------------------------------------// aa
----misof_tree() { memset(cnt, 0, sizeof(cnt)); }------------------------------// b0
----void insert(int x) { for (int i = 0; i < BITS; cnt[i++][x]++, x >>= 1); }--// 5a
----void erase(int x) { for (int i = 0; i < BITS; cnt[i++][x]--, x >>= 1); }---// 49
----int nth(int n) {-----------------------------------------------------------// 8a
--------int res = 0;-----------------------------------------------------------// a4
--------for (int i = BITS-1; i >= 0; i--)--------------------------------------// 99
------------if (cnt[i][res <<= 1] <= n) n -= cnt[i][res], res |= 1;------------// f4
--------return res;------------------------------------------------------------// 3a
----}--------------------------------------------------------------------------// b5
};-----------------------------------------------------------------------------// 0a
```

3. Graphs

3.1. **Breadth-First Search.** An implementation of a breadth-first search that counts the number of edges on the shortest path from the starting vertex to the ending vertex in the specified unweighted graph (which is represented with an adjacency list). Note that it assumes that the two vertices are connected. It runs in $O(|V| + |E|)$ time.

```
int bfs(int start, int end, vvi& adj_list) {-----------------------------------// d7
----queue<ii> Q;---------------------------------------------------------------// 75
----Q.push(ii(start, 0));------------------------------------------------------// 49
-------------------------------------------------------------------------------// 0b
----while (true) {-------------------------------------------------------------// 0a
--------ii cur = Q.front(); Q.pop();-------------------------------------------// e8
-------------------------------------------------------------------------------// 06
--------if (cur.first == end)--------------------------------------------------// 6f
------------return cur.second;-------------------------------------------------// 8a
-------------------------------------------------------------------------------// 3c
--------vi& adj = adj_list[cur.first];-----------------------------------------// 3f
--------for (vi::iterator it = adj.begin(); it != adj.end(); it++)-------------// bb
------------Q.push(ii(*it, cur.second + 1));-----------------------------------// b7
----}--------------------------------------------------------------------------// 93
}------------------------------------------------------------------------------// 7d
```

Another implementation that doesn't assume the two vertices are connected. If there is no path from the starting vertex to the ending vertex, a -1 is returned.

```
int bfs(int start, int end, vvi& adj_list) {-----------------------------------// d7
----set<int> visited;----------------------------------------------------------// b3
----queue<ii> Q;---------------------------------------------------------------// bb
----Q.push(ii(start, 0));------------------------------------------------------// 3a
----visited.insert(start);-----------------------------------------------------// b2
-------------------------------------------------------------------------------// db
----while (!Q.empty()) {-------------------------------------------------------// f7
--------ii cur = Q.front(); Q.pop();-------------------------------------------// 03
-------------------------------------------------------------------------------// 9c
--------if (cur.first == end)--------------------------------------------------// 22
------------return cur.second;-------------------------------------------------// b9
-------------------------------------------------------------------------------// ba
--------vi& adj = adj_list[cur.first];-----------------------------------------// f9
--------for (vi::iterator it = adj.begin(); it != adj.end(); it++)-------------// 44
------------if (visited.find(*it) == visited.end()) {--------------------------// 8d
----------------Q.push(ii(*it, cur.second + 1));-------------------------------// ab
----------------visited.insert(*it);-------------------------------------------// cb
------------}------------------------------------------------------------------// a1
----}--------------------------------------------------------------------------// 0b
-------------------------------------------------------------------------------// 63
----return -1;-----------------------------------------------------------------// f5
}------------------------------------------------------------------------------// 03
```

3.2. Single-Source Shortest Paths.

3.2.1. *Dijkstra's algorithm.* An implementation of Dijkstra's algorithm. It runs in $\Theta(|E| \log |V|)$ time.

```
int *dist, *dad;---------------------------------------------------------------// 46
struct cmp {-------------------------------------------------------------------// a5
----bool operator()(int a, int b) {--------------------------------------------// bb
--------return dist[a] != dist[b] ? dist[a] < dist[b] : a < b; }---------------// e6
};-----------------------------------------------------------------------------// 41
pair<int*, int*> dijkstra(int n, int s, vii *adj) {----------------------------// 53
----dist = new int[n];---------------------------------------------------------// 84
----dad = new int[n];----------------------------------------------------------// 05
----for (int i = 0; i < n; i++) dist[i] = INF, dad[i] = -1;--------------------// d6
----set<int, cmp> pq;----------------------------------------------------------// 04
----dist[s] = 0, pq.insert(s);-------------------------------------------------// 1b
----while (!pq.empty()) {------------------------------------------------------// 57
--------int cur = *pq.begin(); pq.erase(pq.begin());---------------------------// 7d
--------for (int i = 0; i < size(adj[cur]); i++) {-----------------------------// 9e
------------int nxt = adj[cur][i].first,---------------------------------------// b8
----------------ndist = dist[cur] + adj[cur][i].second;------------------------// 0c
------------if (ndist < dist[nxt]) pq.erase(nxt),------------------------------// e4
----------------dist[nxt] = ndist, dad[nxt] = cur, pq.insert(nxt);-------------// 0f
--------}----------------------------------------------------------------------// 75
----}--------------------------------------------------------------------------// e8
----return pair<int*, int*>(dist, dad);----------------------------------------// cc
}------------------------------------------------------------------------------// af
```

3.2.2. *Bellman-Ford algorithm.* The Bellman-Ford algorithm solves the single-source shortest paths problem in $O(|V||E|)$ time. It is slower than Dijkstra's algorithm, but it works on graphs with negative edges and has the ability to detect negative cycles, neither of which Dijkstra's algorithm can do.

```
int* bellman_ford(int n, int s, vii* adj, bool& has_negative_cycle) {----------// cf
----has_negative_cycle = false;------------------------------------------------// 47
----int* dist = new int[n];----------------------------------------------------// 7f
----for (int i = 0; i < n; i++) dist[i] = i == s ? 0 : INF;--------------------// 10
----for (int i = 0; i < n - 1; i++)--------------------------------------------// a1
--------for (int j = 0; j < n; j++)--------------------------------------------// c4
------------if (dist[j] != INF)------------------------------------------------// 4e
----------------for (int k = 0; k < size(adj[j]); k++)------------------------// 3f
--------------------dist[adj[j][k].first] = min(dist[adj[j][k].first],---------// 61
------------------------------dist[j] + adj[j][k].second);--------------------// 47
```

```
----for (int j = 0; j < n; j++)-------------------------------------------------// 13
--------for (int k = 0; k < size(adj[j]); k++)---------------------------------// a0
------------if (dist[j] + adj[j][k].second < dist[adj[j][k].first])------------// ef
----------------has_negative_cycle = true;-------------------------------------// 2a
----return dist;---------------------------------------------------------------// 2e
}------------------------------------------------------------------------------// c2
```

3.3. All-Pairs Shortest Paths.

3.3.1. *Floyd-Warshall algorithm.* The Floyd-Warshall algorithm solves the all-pairs shortest paths problem in $O(|V|^3)$ time.

```
void floyd_warshall(int** arr, int n) {----------------------------------------// 21
----for (int k = 0; k < n; k++)------------------------------------------------// 49
--------for (int i = 0; i < n; i++)--------------------------------------------// 21
------------for (int j = 0; j < n; j++)----------------------------------------// 77
----------------if (arr[i][k] != INF && arr[k][j] != INF)----------------------// b1
--------------------arr[i][j] = min(arr[i][j], arr[i][k] + arr[k][j]);---------// e1
}------------------------------------------------------------------------------// 86
```

3.4. Strongly Connected Components.

3.4.1. *Kosaraju's algorithm.* Kosarajus's algorithm finds strongly connected components of a directed graph in $O(|V| + |E|)$ time.

```
#include "../data-structures/union_find.cpp"-----------------------------------// 5e
-------------------------------------------------------------------------------// 11
vector<bool> visited;----------------------------------------------------------// 66
vi order;----------------------------------------------------------------------// 9b
-------------------------------------------------------------------------------// a5
void scc_dfs(const vvi &adj, int u) {------------------------------------------// a1
----int v; visited[u] = true;--------------------------------------------------// e3
----for (int i = 0; i < size(adj[u]); i++)-------------------------------------// c5
--------if (!visited[v = adj[u][i]]) scc_dfs(adj, v);--------------------------// 6e
----order.push_back(u);--------------------------------------------------------// 19
}------------------------------------------------------------------------------// dc
-------------------------------------------------------------------------------// 96
pair<union_find, vi> scc(const vvi &adj) {-------------------------------------// 3e
----int n = size(adj), u, v;---------------------------------------------------// bd
----order.clear();-------------------------------------------------------------// 22
----union_find uf(n);----------------------------------------------------------// 6d
----vi dag;--------------------------------------------------------------------// ae
----vvi rev(n);----------------------------------------------------------------// 20
----for (int i = 0; i < n; i++) for (int j = 0; j < size(adj[i]); j++)---------// b9
--------rev[adj[i][j]].push_back(i);-------------------------------------------// 77
----visited.resize(n), fill(visited.begin(), visited.end(), false);------------// 04
----for (int i = 0; i < n; i++) if (!visited[i]) scc_dfs(rev, i);--------------// e4
----fill(visited.begin(), visited.end(), false);-------------------------------// c2
----stack<int> S;--------------------------------------------------------------// 04
----for (int i = n-1; i >= 0; i--) {-------------------------------------------// 3f
--------if (visited[order[i]]) continue;---------------------------------------// 94
--------S.push(order[i]), dag.push_back(order[i]);-----------------------------// 40
--------while (!S.empty()) {---------------------------------------------------// 03
------------visited[u = S.top()] = true, S.pop(), uf.unite(u, order[i]);-------// 1b
------------for (int i = 0; i < size(adj[u]); i++)-----------------------------// 90
----------------if (!visited[v = adj[u][i]]) S.push(v);------------------------// 43
--------}----------------------------------------------------------------------// da
----}--------------------------------------------------------------------------// 7c
----return pair<union_find, vi>(uf, dag);--------------------------------------// 94
}------------------------------------------------------------------------------// 97
```

3.5. Minimum Spanning Tree.

3.5.1. *Kruskal's algorithm.*

```
#include "../data-structures/union_find.cpp"-----------------------------------// 5e
-------------------------------------------------------------------------------// 11
// n is the number of vertices-------------------------------------------------// 18
// edges is a list of edges of the form (weight, (a, b))-----------------------// c6
// the edges in the minimum spanning tree are returned on the same form--------// 4d
vector<pair<int, ii> > mst(int n, vector<pair<int, ii> > edges) {--------------// a7
----union_find uf(n);----------------------------------------------------------// 04
----sort(edges.begin(), edges.end());------------------------------------------// 51
----vector<pair<int, ii> > res;------------------------------------------------// 71
----for (int i = 0; i < size(edges); i++)--------------------------------------// ce
--------if (uf.find(edges[i].second.first) !=----------------------------------// d5
----------------uf.find(edges[i].second.second)) {-----------------------------// 8c
------------res.push_back(edges[i]);-------------------------------------------// d1
------------uf.unite(edges[i].second.first, edges[i].second.second);-----------// a2
--------}----------------------------------------------------------------------// 5b
----return res;----------------------------------------------------------------// 46
}------------------------------------------------------------------------------// 88
```

3.6. Topological Sort.

3.6.1. *Modified Depth-First Search.*

```
void tsort_dfs(int cur, char* color, const vvi& adj, stack<int>& res,----------// ca
--------bool& has_cycle) {-----------------------------------------------------// a8
----color[cur] = 1;------------------------------------------------------------// 5b
----for (int i = 0; i < size(adj[cur]); i++) {---------------------------------// 96
--------int nxt = adj[cur][i];-------------------------------------------------// 53
--------if (color[nxt] == 0)---------------------------------------------------// 00
------------tsort_dfs(nxt, color, adj, res, has_cycle);------------------------// 5b
--------else if (color[nxt] == 1)----------------------------------------------// 53
------------has_cycle = true;--------------------------------------------------// c8
--------if (has_cycle) return;-------------------------------------------------// 7e
----}--------------------------------------------------------------------------// 3d
----color[cur] = 2;------------------------------------------------------------// 16
----res.push(cur);-------------------------------------------------------------// cb
}------------------------------------------------------------------------------// 9e
-------------------------------------------------------------------------------// ae
vi tsort(int n, vvi adj, bool& has_cycle) {------------------------------------// 37
----has_cycle = false;---------------------------------------------------------// 37
----stack<int> S;--------------------------------------------------------------// 54
----vi res;--------------------------------------------------------------------// d1
----char* color = new char[n];-------------------------------------------------// b1
----memset(color, 0, n);-------------------------------------------------------// ce
----for (int i = 0; i < n; i++) {----------------------------------------------// 96
--------if (!color[i]) {-------------------------------------------------------// d5
```

```
------------tsort_dfs(i, color, adj, S, has_cycle);---------------------------// 40
------------if (has_cycle) return res;----------------------------------------// 6c
--------}---------------------------------------------------------------------// 70
----}-------------------------------------------------------------------------// df
----while (!S.empty()) res.push_back(S.top()), S.pop();-----------------------// 94
----return res;---------------------------------------------------------------// 07
}-----------------------------------------------------------------------------// 1f
```

3.7. **Euler Path.** Finds an euler path (or circuit) in a directed graph, or reports that none exist.

```
#define MAXV 1000---------------------------------------------------------------// 2f
#define MAXE 5000---------------------------------------------------------------// 87
vi adj[MAXV];-----------------------------------------------------------------// ff
int n, m, indeg[MAXV], outdeg[MAXV], res[MAXE + 1];---------------------------// 49
ii start_end() {--------------------------------------------------------------// 30
----int start = -1, end = -1, any = 0, c = 0;---------------------------------// 74
----for (int i = 0; i < n; i++) {---------------------------------------------// 96
--------if (outdeg[i] > 0) any = i;-------------------------------------------// f2
--------if (indeg[i] + 1 == outdeg[i]) start = i, c++;------------------------// 98
--------else if (indeg[i] == outdeg[i] + 1) end = i, c++;---------------------// 4f
--------else if (indeg[i] != outdeg[i]) return ii(-1,-1);---------------------// fa
----}-------------------------------------------------------------------------// ef
----if ((start == -1) != (end == -1) || (c != 2 && c != 0)) return ii(-1,-1);--// 6e
----if (start == -1) start = end = any;---------------------------------------// db
----return ii(start, end);----------------------------------------------------// 9e
}-----------------------------------------------------------------------------// 35
bool euler_path() {-----------------------------------------------------------// d7
----ii se = start_end();------------------------------------------------------// 45
----int cur = se.first, at = m + 1;-------------------------------------------// 8c
----if (cur == -1) return false;----------------------------------------------// 45
----stack<int> s;-------------------------------------------------------------// f6
----while (true) {------------------------------------------------------------// 04
--------if (outdeg[cur] == 0) {-----------------------------------------------// 32
------------res[--at] = cur;--------------------------------------------------// a6
------------if (s.empty()) break;---------------------------------------------// ee
------------cur = s.top(); s.pop();-------------------------------------------// d7
--------} else s.push(cur), cur = adj[cur][--outdeg[cur]];--------------------// d8
----}-------------------------------------------------------------------------// ba
----return at == 0;-----------------------------------------------------------// c8
}-----------------------------------------------------------------------------// aa
```

3.8. **Bipartite Matching.**

3.8.1. *Alternating Paths algorithm.* The alternating paths algorithm solves bipartite matching in $O(mn^2)$ time, where m, n are the number of vertices on the left and right side of the bipartite graph, respectively.

```
vi* adj;----------------------------------------------------------------------// cc
bool* done;-------------------------------------------------------------------// b1
int* owner;-------------------------------------------------------------------// 26
int alternating_path(int left) {----------------------------------------------// da
----if (done[left]) return 0;-------------------------------------------------// 08
----done[left] = true;--------------------------------------------------------// f2
----for (int i = 0; i < size(adj[left]); i++) {-------------------------------// 34
--------int right = adj[left][i];---------------------------------------------// b6
--------if (owner[right] == -1 || alternating_path(owner[right])) {-----------// d2
------------owner[right] = left; return 1;------------------------------------// 26
--------} }-------------------------------------------------------------------// 7a
----return 0; }---------------------------------------------------------------// 83
```

3.8.2. *Hopcroft-Karp algorithm.* An implementation of Hopcroft-Karp algorithm for bipartite matching. Running time is $O(|E|\sqrt{|V|})$.

```
#define MAXN 5000---------------------------------------------------------------// f7
int dist[MAXN+1], q[MAXN+1];--------------------------------------------------// b8
#define dist(v) dist[v == -1 ? MAXN : v]----------------------------------------// 0f
struct bipartite_graph {------------------------------------------------------// 2b
----int N, M, *L, *R; vi *adj;------------------------------------------------// fc
----bipartite_graph(int N, int M) : N(N), M(M),-------------------------------// e7
--------L(new int[N]), R(new int[M]), adj(new vi[N]) {}-----------------------// 46
----~bipartite_graph() { delete[] adj; delete[] L; delete[] R; }--------------// b9
----bool bfs() {--------------------------------------------------------------// 3e
--------int l = 0, r = 0;-----------------------------------------------------// a4
--------for(int v = 0; v < N; ++v) if(L[v] == -1) dist(v) = 0, q[r++] = v;-----// 31
------------else dist(v) = INF;-----------------------------------------------// c4
--------dist(-1) = INF;-------------------------------------------------------// f3
--------while(l < r) {--------------------------------------------------------// 3f
------------int v = q[l++];---------------------------------------------------// 69
------------if(dist(v) < dist(-1)) {------------------------------------------// b2
----------------foreach(u, adj[v]) if(dist(R[*u]) == INF)---------------------// 63
--------------------dist(R[*u]) = dist(v) + 1, q[r++] = R[*u];----------------// f8
------------}-----------------------------------------------------------------// 9d
--------}---------------------------------------------------------------------// 21
--------return dist(-1) != INF;-----------------------------------------------// e4
----}-------------------------------------------------------------------------// b5
----bool dfs(int v) {---------------------------------------------------------// f6
--------if(v != -1) {---------------------------------------------------------// 6c
------------foreach(u, adj[v])------------------------------------------------// 19
----------------if(dist(R[*u]) == dist(v) + 1)--------------------------------// d9
--------------------if(dfs(R[*u])) {------------------------------------------// c7
------------------------R[*u] = v, L[v] = *u;---------------------------------// 2e
------------------------return true;------------------------------------------// 56
--------------------}---------------------------------------------------------// 8c
------------dist(v) = INF;----------------------------------------------------// d4
------------return false;-----------------------------------------------------// de
--------}---------------------------------------------------------------------// 67
--------return true;----------------------------------------------------------// 7b
----}-------------------------------------------------------------------------// 61
----void add_edge(int i, int j) { adj[i].push_back(j); }----------------------// 87
----int maximum_matching() {--------------------------------------------------// ae
--------int matching = 0;-----------------------------------------------------// 7d
--------memset(L, -1, sizeof(int) * N);---------------------------------------// 16
--------memset(R, -1, sizeof(int) * M);---------------------------------------// e4
--------while(bfs()) for(int i = 0; i < N; ++i)-------------------------------// f6
------------matching += L[i] == -1 && dfs(i);---------------------------------// c9
--------return matching;------------------------------------------------------// 82
```

```
----}----------------------------------------------// 86
};-------------------------------------------------// dd
```

3.9. Maximum Flow.

3.9.1. *Dinic's algorithm.* An implementation of Dinic's algorithm that runs in $O(|V|^2|E|)$. It computes the maximum flow of a flow network.

```
#define MAXV 2000--------------------------------------// ba
int q[MAXV], d[MAXV];----------------------------------// e6
struct flow_network {----------------------------------// 12
----struct edge {--------------------------------------// 1e
--------int v, cap, nxt;-------------------------------// ab
--------edge() { }-------------------------------------// 38
--------edge(int v, int cap, int nxt) : v(v), cap(cap), nxt(nxt) { }----// f7
----};-------------------------------------------------// 7a
----int n, ecnt, *head, *curh;-------------------------// 77
----vector<edge> e, e_store;---------------------------// d0
----flow_network(int n, int m = -1) : n(n), ecnt(0) {--// 80
--------e.reserve(2 * (m == -1 ? n : m));--------------// 5d
--------head = new int[n], curh = new int[n];----------// 6d
--------memset(head, -1, n * sizeof(int));-------------// f6
----}--------------------------------------------------// 73
----void destroy() { delete[] head; delete[] curh; }---// 21
----void reset() { e = e_store; }----------------------// 60
----void add_edge(int u, int v, int uv, int vu = 0) {--// dd
--------e.push_back(edge(v, uv, head[u])); head[u] = ecnt++;----// 7a
--------e.push_back(edge(u, vu, head[v])); head[v] = ecnt++;----// b2
----}--------------------------------------------------// 35
----int augment(int v, int t, int f) {-----------------// a1
--------if (v == t) return f;--------------------------// 84
--------for (int &i = curh[v], ret; i != -1; i = e[i].nxt)------// f4
------------if (e[i].cap > 0 && d[e[i].v] + 1 == d[v])----------// 8f
----------------if ((ret = augment(e[i].v, t, min(f, e[i].cap))) > 0)----// a3
--------------------return (e[i].cap -= ret, e[i^1].cap += ret, ret);----// ed
--------return 0;--------------------------------------// d3
----}--------------------------------------------------// 48
----int max_flow(int s, int t, bool res = true) {------// 87
--------if(s == t) return 0;---------------------------// 2e
--------e_store = e;-----------------------------------// 59
--------int f = 0, x, l, r;----------------------------// 82
--------while (true) {---------------------------------// 36
------------memset(d, -1, n * sizeof(int));------------// 6c
------------l = r = 0, d[q[r++] = t] = 0;--------------// 47
------------while (l < r)------------------------------// 87
----------------for (int v = q[l++], i = head[v]; i != -1; i = e[i].nxt)----// cd
--------------------if (e[i^1].cap > 0 && d[e[i].v] == -1)----------------// 23
------------------------d[q[r++] = e[i].v] = d[v]+1;-------------------// c2
------------if (d[s] == -1) break;---------------------// a3
------------memcpy(curh, head, n * sizeof(int));-------// 0e
------------while ((x = augment(s, t, INF)) != 0) f += x;----// 5f
--------}----------------------------------------------// ea
--------if (res) reset();------------------------------// 71
--------return f;--------------------------------------// 72
----}--------------------------------------------------// 75
};-------------------------------------------------// 97
```

3.9.2. *Edmonds Karp's algorithm.* An implementation of Edmonds Karp's algorithm that runs in $O(|V||E|^2)$. It computes the maximum flow of a flow network.

```
#define MAXV 2000--------------------------------------// ba
int q[MAXV], d[MAXV], p[MAXV];-------------------------// 7b
struct flow_network {----------------------------------// 5e
----struct edge {--------------------------------------// fc
--------int v, cap, nxt;-------------------------------// cb
--------edge(int v, int cap, int nxt) : v(v), cap(cap), nxt(nxt) {  }----// a1
----};-------------------------------------------------// f9
----int n, ecnt, *head;--------------------------------// 00
----vector<edge> e, e_store;---------------------------// 5f
----flow_network(int n, int m = -1) : n(n), ecnt(0) {--// 0d
--------e.reserve(2 * (m == -1 ? n : m));--------------// 0d
--------memset(head = new int[n], -1, n << 2);---------// bc
----}--------------------------------------------------// ec
----void destroy() { delete[] head; }------------------// f1
----void reset() { e = e_store; }----------------------// 1c
----void add_edge(int u, int v, int uv, int vu=0) {----// ba
--------e.push_back(edge(v, uv, head[u])); head[u] = ecnt++;----// 85
--------e.push_back(edge(u, vu, head[v])); head[v] = ecnt++;----// 9b
----}--------------------------------------------------// ce
----int max_flow(int s, int t, bool res = true) {------// b1
--------if (s == t) return 0;--------------------------// bb
--------e_store = e;-----------------------------------// f8
--------int f = 0, l, r, v;----------------------------// 62
--------while (true) {---------------------------------// 3a
------------memset(d, -1, n << 2);---------------------// 73
------------memset(p, -1, n << 2);---------------------// 0d
------------l = r = 0, d[q[r++] = s] = 0;--------------// ff
------------while (l < r)------------------------------// af
----------------for (int u = q[l++], i = head[u]; i != -1; i = e[i].nxt)----// 39
--------------------if (e[i].cap > 0 && (d[v = e[i].v] == -1 || d[u] + 1 < d[v]))
------------------------d[v] = d[u] + 1, p[q[r++] = v] = i;----------// 14
------------if (p[t] == -1) break;---------------------// 84
------------int x = INF, at = p[t];--------------------// 43
------------while (at != -1) x = min(x, e[at].cap), at = p[e[at^1].v];----// 70
------------at = p[t], f += x;-------------------------// 3c
------------while (at != -1)---------------------------// 58
----------------e[at].cap -= x, e[at^1].cap += x, at = p[e[at^1].v];----// 59
--------}----------------------------------------------// b8
--------if (res) reset();------------------------------// e7
--------return f;--------------------------------------// 62
----}--------------------------------------------------// 65
};-------------------------------------------------// 83
```

3.10. **Minimum Cost Maximum Flow.** An implementation of Edmonds Karp's algorithm, modified to find shortest path to augment each time (instead of just any path). It computes the maximum flow of a flow network, and when there are multiple maximum flows, finds the maximum flow with minimum cost. Running time is $O(|V|^2|E|\log|V|)$.

```
#define MAXV 2000--------------------------------------------------------------// ba
int d[MAXV], p[MAXV], pot[MAXV];-----------------------------------------------// 80
struct cmp {-------------------------------------------------------------------// d1
----bool operator ()(int i, int j) {-------------------------------------------// 8a
--------return d[i] == d[j] ? i < j : d[i] < d[j];-----------------------------// 89
----}--------------------------------------------------------------------------// df
};-----------------------------------------------------------------------------// cf
struct flow_network {----------------------------------------------------------// eb
----struct edge {--------------------------------------------------------------// 9a
--------int v, cap, cost, nxt;-------------------------------------------------// ad
--------edge(int v, int cap, int cost, int nxt)--------------------------------// 01
------------: v(v), cap(cap), cost(cost), nxt(nxt) {  }------------------------// 5d
----};-------------------------------------------------------------------------// ae
----int n, ecnt, *head;--------------------------------------------------------// 57
----vector<edge> e, e_store;---------------------------------------------------// d7
----flow_network(int n, int m = -1) : n(n), ecnt(0) {--------------------------// 83
--------e.reserve(2 * (m == -1 ? n : m));--------------------------------------// 2c
--------memset(head = new int[n], -1, n << 2);---------------------------------// e1
----}--------------------------------------------------------------------------// 64
----void destroy() { delete[] head; }------------------------------------------// 89
----void reset() { e = e_store; }----------------------------------------------// 64
----void add_edge(int u, int v, int cost, int uv, int vu=0) {------------------// 30
--------e.push_back(edge(v, uv, cost, head[u])); head[u] = ecnt++;-------------// 04
--------e.push_back(edge(u, vu, -cost, head[v])); head[v] = ecnt++;------------// 70
----}--------------------------------------------------------------------------// a2
----ii min_cost_max_flow(int s, int t, bool res = true) {----------------------// 5e
--------if (s == t) return ii(0, 0);-------------------------------------------// c4
--------e_store = e;-----------------------------------------------------------// e8
--------memset(pot, 0, n << 2);------------------------------------------------// 32
--------int f = 0, c = 0, v;---------------------------------------------------// 9c
--------while (true) {---------------------------------------------------------// a7
------------memset(d, -1, n << 2);---------------------------------------------// a2
------------memset(p, -1, n << 2);---------------------------------------------// 71
------------set<int, cmp> q;---------------------------------------------------// a8
------------q.insert(s); d[s] = 0;---------------------------------------------// 1d
------------while (!q.empty()) {-----------------------------------------------// 6f
----------------int u = *q.begin();--------------------------------------------// e6
----------------q.erase(q.begin());--------------------------------------------// 0f
----------------for (int i = head[u]; i != -1; i = e[i].nxt) {-----------------// 9c
--------------------if (e[i].cap == 0) continue;-------------------------------// b8
--------------------int cd = d[u] + e[i].cost + pot[u] - pot[v = e[i].v];-----// f5
--------------------if (d[v] == -1 || cd < d[v]) {-----------------------------// d2
------------------------if (q.find(v) != q.end()) q.erase(q.find(v));----------// 47
------------------------d[v] = cd; p[v] = i;-----------------------------------// 52
------------------------q.insert(v);-------------------------------------------// 0f
--------------------}----------------------------------------------------------// f1
----------------}--------------------------------------------------------------// d1
------------}------------------------------------------------------------------// d7
------------if (p[t] == -1) break;---------------------------------------------// 51
------------int x = INF, at = p[t];--------------------------------------------// fe
------------while (at != -1) x = min(x, e[at].cap), at = p[e[at^1].v];---------// 82
------------at = p[t], f += x;-------------------------------------------------// 30
------------while (at != -1)---------------------------------------------------// 84
----------------e[at].cap -= x, e[at^1].cap += x, at = p[e[at^1].v];-----------// a5
------------c += x * (d[t] + pot[t] - pot[s]);---------------------------------// 97
------------for (int i = 0; i < n; i++) if (p[i] != -1) pot[i] += d[i];--------// 66
--------}----------------------------------------------------------------------// 9f
--------if (res) reset();------------------------------------------------------// da
--------return ii(f, c);-------------------------------------------------------// f4
----}--------------------------------------------------------------------------// d1
};-----------------------------------------------------------------------------// 80
```

3.11. **All Pairs Maximum Flow.**

3.11.1. *Gomory-Hu Tree.* An implementation of the Gomory-Hu Tree. The spanning tree is constructed using Gusfield's algorithm in $O(|V|^2)$ plus $|V|-1$ times the time it takes to calculate the maximum flow. If Dinic's algorithm is used to calculate the max flow, the running time is $O(|V|^3|E|)$.

```
#include "dinic.cpp"-----------------------------------------------------------// 58
-------------------------------------------------------------------------------// 25
bool same[MAXV];---------------------------------------------------------------// 59
pair<vii, vvi> construct_gh_tree(flow_network &g) {----------------------------// 77
----int n = g.n, v;------------------------------------------------------------// 5d
----vii par(n, ii(0, 0)); vvi cap(n, vi(n, -1));-------------------------------// 49
----for (int s = 1; s < n; s++) {----------------------------------------------// 9e
--------int l = 0, r = 0;------------------------------------------------------// 9d
--------par[s].second = g.max_flow(s, par[s].first, false);--------------------// 38
--------memset(d, 0, n * sizeof(int));-----------------------------------------// 79
--------memset(same, 0, n * sizeof(int));--------------------------------------// b0
--------d[q[r++] = s] = 1;-----------------------------------------------------// 8c
--------while (l < r) {--------------------------------------------------------// 45
------------same[v = q[l++]] = true;-------------------------------------------// c8
------------for (int i = g.head[v]; i != -1; i = g.e[i].nxt)-------------------// 33
----------------if (g.e[i].cap > 0 && d[g.e[i].v] == 0)------------------------// 3f
--------------------d[q[r++] = g.e[i].v] = 1;----------------------------------// f8
--------}----------------------------------------------------------------------// b5
--------for (int i = s + 1; i < n; i++)----------------------------------------// 68
------------if (par[i].first == par[s].first && same[i]) par[i].first = s;-----// ea
--------g.reset();-------------------------------------------------------------// 9a
----}--------------------------------------------------------------------------// 1e
----for (int i = 0; i < n; i++) {----------------------------------------------// 2a
--------int mn = INF, cur = i;-------------------------------------------------// 19
--------while (true) {---------------------------------------------------------// 3a
------------cap[cur][i] = mn;--------------------------------------------------// 63
------------if (cur == 0) break;-----------------------------------------------// 35
------------mn = min(mn, par[cur].second), cur = par[cur].first;---------------// 28
--------}----------------------------------------------------------------------// 91
----}--------------------------------------------------------------------------// 4a
----return make_pair(par, cap);------------------------------------------------// 6b
}------------------------------------------------------------------------------// 99
int compute_max_flow(int s, int t, const pair<vii, vvi> &gh) {-----------------// 16
----if (s == t) return 0;------------------------------------------------------// d4
```

```
----int cur = INF, at = s;------------------------------------------------------// 65
----while (gh.second[at][t] == -1)----------------------------------------------// ef
--------cur = min(cur, gh.first[at].second), at = gh.first[at].first;-----------// bd
----return min(cur, gh.second[at][t]);------------------------------------------// 6d
}-------------------------------------------------------------------------------// a2
```

4. Strings

4.1. Trie. A Trie class.

```
template <class T>--------------------------------------------------------------// 82
class trie {--------------------------------------------------------------------// 9a
private:------------------------------------------------------------------------// f4
----struct node {---------------------------------------------------------------// ae
--------map<T, node*> children;-------------------------------------------------// a0
--------int prefixes, words;----------------------------------------------------// e2
--------node() { prefixes = words = 0; } };-------------------------------------// 42
public:-------------------------------------------------------------------------// 88
----node* root;-----------------------------------------------------------------// a9
----trie() : root(new node()) {  }----------------------------------------------// 8f
----template <class I>----------------------------------------------------------// 89
----void insert(I begin, I end) {-----------------------------------------------// 3c
--------node* cur = root;-------------------------------------------------------// 82
--------while (true) {----------------------------------------------------------// 67
------------cur->prefixes++;----------------------------------------------------// f1
------------if (begin == end) { cur->words++; break; }--------------------------// db
------------else {--------------------------------------------------------------// 3e
----------------T head = *begin;------------------------------------------------// fb
----------------typename map<T, node*>::const_iterator it;----------------------// 01
----------------it = cur->children.find(head);----------------------------------// 77
----------------if (it == cur->children.end()) {--------------------------------// 95
--------------------pair<T, node*> nw(head, new node());------------------------// cd
--------------------it = cur->children.insert(nw).first;------------------------// ae
----------------} begin++, cur = it->second; } } }-----------------------------// 64
----template<class I>-----------------------------------------------------------// b9
----int countMatches(I begin, I end) {------------------------------------------// 7f
--------node* cur = root;-------------------------------------------------------// 32
--------while (true) {----------------------------------------------------------// bb
------------if (begin == end) return cur->words;--------------------------------// a4
------------else {--------------------------------------------------------------// 1e
----------------T head = *begin;------------------------------------------------// 5c
----------------typename map<T, node*>::const_iterator it;----------------------// 25
----------------it = cur->children.find(head);----------------------------------// d9
----------------if (it == cur->children.end()) return 0;------------------------// 14
----------------begin++, cur = it->second; } } }-------------------------------// 7c
----template<class I>-----------------------------------------------------------// 9c
----int countPrefixes(I begin, I end) {-----------------------------------------// 85
--------node* cur = root;-------------------------------------------------------// 95
--------while (true) {----------------------------------------------------------// 3e
------------if (begin == end) return cur->prefixes;-----------------------------// f5
------------else {--------------------------------------------------------------// 66
----------------T head = *begin;------------------------------------------------// 43
----------------typename map<T, node*>::const_iterator it;----------------------// 7a
----------------it = cur->children.find(head);----------------------------------// 43
----------------if (it == cur->children.end()) return 0;------------------------// 71
----------------begin++, cur = it->second; } } } };-----------------------------// 26
```

4.2. Suffix Array. An $O(n \log n)$ construction of a Suffix Tree.

```
struct entry { ii nr; int p; };-------------------------------------------------// f9
bool operator <(const entry &a, const entry &b) { return a.nr < b.nr; }---------// 77
struct suffix_array {-----------------------------------------------------------// 87
----string s; int n; vvi P; vector<entry> L; vi idx;----------------------------// b6
----// REMINDER: Append a large character ('\x7F') to s-------------------------// 70
----suffix_array(string s) : s(s), n(size(s)) {---------------------------------// fc
--------L = vector<entry>(n), P.push_back(vi(n)), idx = vi(n);------------------// 96
--------for (int i = 0; i < n; i++) P[0][i] = s[i] - 'a';-----------------------// 69
--------for (int stp = 1, cnt = 1; cnt >> 1 < n; stp++, cnt <<= 1) {------------// bb
------------P.push_back(vi(n));-------------------------------------------------// e9
------------for (int i = 0; i < n; i++)-----------------------------------------// 50
----------------L[L[i].p = i].nr = ii(P[stp - 1][i],----------------------------// 0e
------------------------i + cnt < n ? P[stp - 1][i + cnt] : -1);----------------// 18
------------sort(L.begin(), L.end());-------------------------------------------// 29
------------for (int i = 0; i < n; i++)-----------------------------------------// 38
----------------P[stp][L[i].p] = i > 0 &&---------------------------------------// 36
--------------------L[i].nr == L[i - 1].nr ? P[stp][L[i - 1].p] : i;------------// 61
--------}-----------------------------------------------------------------------// 39
--------for (int i = 0; i < n; i++) idx[P[size(P) - 1][i]] = i;-----------------// 28
----}---------------------------------------------------------------------------// 0e
----int lcp(int x, int y) {-----------------------------------------------------// 10
--------int res = 0;------------------------------------------------------------// 62
--------if (x == y) return n - x;-----------------------------------------------// f4
--------for (int k = size(P) - 1; k >= 0 && x < n && y < n; k--)----------------// 3e
------------if (P[k][x] == P[k][y]) x += 1 << k, y += 1 << k, res += 1 << k;---// 05
--------return res;-------------------------------------------------------------// 1e
----}---------------------------------------------------------------------------// 89
};------------------------------------------------------------------------------// 66
```

4.3. Aho-Corasick Algorithm. An implementation of the Aho-Corasick algorithm. Constructs a state machine from a set of keywords which can be used to search a string for any of the keywords.

```
struct aho_corasick {-----------------------------------------------------------// 78
----struct out_node {-----------------------------------------------------------// 3e
--------string keyword; out_node *next;-----------------------------------------// f0
--------out_node(string k, out_node *n) : keyword(k), next(n) { }---------------// 26
----};--------------------------------------------------------------------------// b9
----struct go_node {------------------------------------------------------------// 40
--------map<char, go_node*> next;-----------------------------------------------// 6b
--------out_node *out; go_node *fail;-------------------------------------------// 3e
--------go_node() { out = NULL; fail = NULL; }----------------------------------// 0f
----};--------------------------------------------------------------------------// c0
----go_node *go;----------------------------------------------------------------// b8
----aho_corasick(vector<string> keywords) {-------------------------------------// 4b
--------go = new go_node();-----------------------------------------------------// 77
--------foreach(k, keywords) {--------------------------------------------------// e4
------------go_node *cur = go;--------------------------------------------------// 9d
------------foreach(c, *k)------------------------------------------------------// 38
```

```
----------------cur = cur->next.find(*c) != cur->next.end() ? cur->next[*c] :--// 3d
--------------------(cur->next[*c] = new go_node());----------------------------// 75
------------cur->out = new out_node(*k, cur->out);------------------------------// 6e
--------}-----------------------------------------------------------------------// 96
--------queue<go_node*> q;------------------------------------------------------// 8a
--------foreach(a, go->next) q.push(a->second);---------------------------------// a3
--------while (!q.empty()) {----------------------------------------------------// 43
------------go_node *r = q.front(); q.pop();------------------------------------// 2e
------------foreach(a, r->next) {-----------------------------------------------// 25
----------------go_node *s = a->second;-----------------------------------------// cb
----------------q.push(s);------------------------------------------------------// 76
----------------go_node *st = r->fail;------------------------------------------// fa
----------------while (st && st->next.find(a->first) == st->next.end())---------// d7
--------------------st = st->fail;----------------------------------------------// 3f
----------------if (!st) st = go;-----------------------------------------------// e7
----------------s->fail = st->next[a->first];-----------------------------------// 29
----------------if (s->fail) {--------------------------------------------------// 3b
--------------------if (!s->out) s->out = s->fail->out;-------------------------// 80
--------------------else {------------------------------------------------------// ed
------------------------out_node* out = s->out;---------------------------------// bf
------------------------while (out->next) out = out->next;----------------------// ca
------------------------out->next = s->fail->out;-------------------------------// 65
--------------------}-----------------------------------------------------------// cf
----------------}---------------------------------------------------------------// 9f
------------}-------------------------------------------------------------------// 61
--------}-----------------------------------------------------------------------// e8
----}---------------------------------------------------------------------------// 91
----vector<string> search(string s) {------------------------------------------// 8d
--------vector<string> res;-----------------------------------------------------// ef
--------go_node *cur = go;------------------------------------------------------// 61
--------foreach(c, s) {---------------------------------------------------------// 6c
------------while (cur && cur->next.find(*c) == cur->next.end())----------------// 1f
----------------cur = cur->fail;------------------------------------------------// 9e
------------if (!cur) cur = go;-------------------------------------------------// 2f
------------cur = cur->next[*c];------------------------------------------------// 58
------------if (!cur) cur = go;-------------------------------------------------// 3f
------------for (out_node *out = cur->out; out; out = out->next)----------------// e0
----------------res.push_back(out->keyword);------------------------------------// 0d
--------}-----------------------------------------------------------------------// fd
--------return res;-------------------------------------------------------------// c1
----}---------------------------------------------------------------------------// e4
};------------------------------------------------------------------------------// 32
```

4.4. **The Z algorithm.** Given a string S, $Z_i(S)$ is the longest substring of S starting at i that is also a prefix of S. The Z algorithm computes these Z values in $O(n)$ time, where $n = |S|$. Z values can, for example, be used to find all occurrences of a pattern P in a string T in linear time. This is accomplished by computing Z values of $S = TP$, and looking for all i such that $Z_i \geq |T|$.

```
int* z_values(const string &s) {------------------------------------------------// 4d
----int n = size(s);------------------------------------------------------------// 97
----int* z = new int[n];--------------------------------------------------------// c4
----int l = 0, r = 0;-----------------------------------------------------------// 1c
----z[0] = n;-------------------------------------------------------------------// 98
----for (int i = 1; i < n; i++) {-----------------------------------------------// 7e
--------z[i] = 0;---------------------------------------------------------------// c9
--------if (i > r) {------------------------------------------------------------// 26
------------l = r = i;----------------------------------------------------------// a7
------------while (r < n && s[r - l] == s[r]) r++;------------------------------// ff
------------z[i] = r - l; r--;--------------------------------------------------// fc
--------} else if (z[i - l] < r - i + 1) z[i] = z[i - l];-----------------------// bf
--------else {------------------------------------------------------------------// b5
------------l = i;--------------------------------------------------------------// 02
------------while (r < n && s[r - l] == s[r]) r++;------------------------------// b3
------------z[i] = r - l; r--; } }----------------------------------------------// 8d
----return z;-------------------------------------------------------------------// 53
}-------------------------------------------------------------------------------// db
```

5. Mathematics

5.1. **Fraction.** A fraction (rational number) class. Note that numbers are stored in lowest common terms.

```
template <class T>--------------------------------------------------------------// 82
class fraction {----------------------------------------------------------------// cf
private:------------------------------------------------------------------------// 8e
----T gcd(T a, T b) { return b == T(0) ? a : gcd(b, a % b); }-------------------// 86
public:-------------------------------------------------------------------------// 0f
----T n, d;---------------------------------------------------------------------// 4b
----fraction(T n_, T d_) {------------------------------------------------------// 03
--------assert(d_ != 0);--------------------------------------------------------// 3d
--------n = n_, d = d_;---------------------------------------------------------// 06
--------if (d < T(0)) n = -n, d = -d;-------------------------------------------// be
--------T g = gcd(abs(n), abs(d));----------------------------------------------// fc
--------n /= g, d /= g; }-------------------------------------------------------// a1
----fraction(T n_) : n(n_), d(1) { }--------------------------------------------// 84
----fraction(const fraction<T>& other) : n(other.n), d(other.d) { }-------------// 01
----fraction<T> operator +(const fraction<T>& other) const {--------------------// b6
--------return fraction<T>(n * other.d + other.n * d, d * other.d);}------------// 3b
----fraction<T> operator -(const fraction<T>& other) const {--------------------// 26
--------return fraction<T>(n * other.d - other.n * d, d * other.d);}------------// 47
----fraction<T> operator *(const fraction<T>& other) const {--------------------// 38
--------return fraction<T>(n * other.n, d * other.d); }-------------------------// c5
----fraction<T> operator /(const fraction<T>& other) const {--------------------// ca
--------return fraction<T>(n * other.d, d * other.n); }-------------------------// 35
----bool operator <(const fraction<T>& other) const {---------------------------// 0c
--------return n * other.d < other.n * d; }-------------------------------------// 8c
----bool operator <=(const fraction<T>& other) const {--------------------------// 48
--------return !(other < *this); }----------------------------------------------// 86
----bool operator >(const fraction<T>& other) const {---------------------------// c9
--------return other < *this; }-------------------------------------------------// 6e
----bool operator >=(const fraction<T>& other) const {--------------------------// 4b
--------return !(*this < other); }----------------------------------------------// 57
----bool operator ==(const fraction<T>& other) const {--------------------------// 23
--------return n == other.n && d == other.d; }----------------------------------// 14
----bool operator !=(const fraction<T>& other) const {--------------------------// ec
```

```
--------return !(*this == other); }-------------------------------------------// d1
};----------------------------------------------------------------------------// 12
```

5.2. **Big Integer.** A big integer class.

```
struct intx {-----------------------------------------------------------------// cf
----intx() { normalize(1); }--------------------------------------------------// 6c
----intx(string n) { init(n); }-----------------------------------------------// b9
----intx(int n) { stringstream ss; ss << n; init(ss.str()); }-----------------// 36
----intx(const intx& other) : sign(other.sign), data(other.data) { }----------// 3b
----int sign;-----------------------------------------------------------------// 26
----vector<unsigned int> data;------------------------------------------------// 19
----static const int dcnt = 9;------------------------------------------------// 12
----static const unsigned int radix = 1000000000U;----------------------------// f0
----int size() const { return data.size(); }----------------------------------// 29
----void init(string n) {-----------------------------------------------------// 13
--------intx res; res.data.clear();-------------------------------------------// 4e
--------if (n.empty()) n = "0";-----------------------------------------------// 99
--------if (n[0] == '-') res.sign = -1, n = n.substr(1);----------------------// 3b
--------for (int i = n.size() - 1; i >= 0; i -= intx::dcnt) {-----------------// e7
------------unsigned int digit = 0;--------------------------------------------// 98
------------for (int j = intx::dcnt - 1; j >= 0; j--) {-----------------------// 72
----------------int idx = i - j;----------------------------------------------// cd
----------------if (idx < 0) continue;----------------------------------------// 52
----------------digit = digit * 10 + (n[idx] - '0');--------------------------// 1f
------------}-----------------------------------------------------------------// c0
------------res.data.push_back(digit);----------------------------------------// 07
--------}---------------------------------------------------------------------// fb
--------data = res.data;------------------------------------------------------// 7d
--------normalize(res.sign);--------------------------------------------------// 76
----}-------------------------------------------------------------------------// 6e
----intx& normalize(int nsign) {----------------------------------------------// 3b
--------if (data.empty()) data.push_back(0);----------------------------------// fa
--------for (int i = data.size() - 1; i > 0 && data[i] == 0; i--)-------------// 27
------------data.erase(data.begin() + i);-------------------------------------// 67
--------sign = data.size() == 1 && data[0] == 0 ? 1 : nsign;------------------// ff
--------return *this;---------------------------------------------------------// 40
----}-------------------------------------------------------------------------// ac
----friend ostream& operator <<(ostream& outs, const intx& n) {---------------// 0d
--------if (n.sign < 0) outs << '-';------------------------------------------// c0
--------bool first = true;----------------------------------------------------// 33
--------for (int i = n.size() - 1; i >= 0; i--) {-----------------------------// 63
------------if (first) outs << n.data[i], first = false;----------------------// 33
------------else {------------------------------------------------------------// 1f
----------------unsigned int cur = n.data[i];---------------------------------// 0f
----------------stringstream ss; ss << cur;-----------------------------------// 8c
----------------string s = ss.str();------------------------------------------// 64
----------------int len = s.size();-------------------------------------------// 0d
----------------while (len < intx::dcnt) outs << '0', len++;------------------// 0a
----------------outs << s;----------------------------------------------------// 97
------------}-----------------------------------------------------------------// f7
--------}---------------------------------------------------------------------// e9
--------return outs;----------------------------------------------------------// cf
```

```
----}-------------------------------------------------------------------------// b9
----string to_string() const { stringstream ss; ss << *this; return ss.str(); }// fc
----bool operator <(const intx& b) const {------------------------------------// 21
--------if (sign != b.sign) return sign < b.sign;-----------------------------// cf
--------if (size() != b.size())-----------------------------------------------// 4d
------------return sign == 1 ? size() < b.size() : size() > b.size();---------// 4d
--------for (int i = size() - 1; i >= 0; i--) if (data[i] != b.data[i])-------// 35
----------------return sign == 1 ? data[i] < b.data[i] : data[i] > b.data[i];--// 27
--------return false;---------------------------------------------------------// ca
----}-------------------------------------------------------------------------// 32
----intx operator -() const { intx res(*this); res.sign *= -1; return res; }---// 9d
----friend intx abs(const intx &n) { return n < 0 ? -n : n; }-----------------// 02
----intx operator +(const intx& b) const {------------------------------------// f8
--------if (sign > 0 && b.sign < 0) return *this - (-b);----------------------// 36
--------if (sign < 0 && b.sign > 0) return b - (-*this);----------------------// 70
--------if (sign < 0 && b.sign < 0) return -((-*this) + (-b));----------------// 59
--------intx c; c.data.clear();-----------------------------------------------// 18
--------unsigned long long carry = 0;-----------------------------------------// 5c
--------for (int i = 0; i < size() || i < b.size() || carry; i++) {-----------// e3
------------carry += (i < size() ? data[i] : 0ULL) +--------------------------// 91
----------------(i < b.size() ? b.data[i] : 0ULL);----------------------------// 0c
------------c.data.push_back(carry % intx::radix);----------------------------// 86
------------carry /= intx::radix;---------------------------------------------// fd
--------}---------------------------------------------------------------------// 50
--------return c.normalize(sign);---------------------------------------------// 20
----}-------------------------------------------------------------------------// 70
----intx operator -(const intx& b) const {------------------------------------// 53
--------if (sign > 0 && b.sign < 0) return *this + (-b);----------------------// 8f
--------if (sign < 0 && b.sign > 0) return -(-*this + b);---------------------// 1b
--------if (sign < 0 && b.sign < 0) return (-b) - (-*this);-------------------// a1
--------if (*this < b) return -(b - *this);-----------------------------------// 36
--------intx c; c.data.clear();-----------------------------------------------// 6b
--------long long borrow = 0;-------------------------------------------------// f8
--------for (int i = 0; i < size(); i++) {------------------------------------// a7
------------borrow = data[i] - borrow - (i < b.size() ? b.data[i] : 0ULL);----// a9
------------c.data.push_back(borrow < 0 ? intx::radix + borrow : borrow);-----// ed
------------borrow = borrow < 0 ? 1 : 0;--------------------------------------// 0d
--------}---------------------------------------------------------------------// fa
--------return c.normalize(sign);---------------------------------------------// 35
----}-------------------------------------------------------------------------// 85
----intx operator *(const intx& b) const {------------------------------------// bd
--------intx c; c.data.assign(size() + b.size() + 1, 0);----------------------// d0
--------for (int i = 0; i < size(); i++) {------------------------------------// 7a
------------long long carry = 0;----------------------------------------------// 20
------------for (int j = 0; j < b.size() || carry; j++) {---------------------// c0
----------------if (j < b.size()) carry += (long long)data[i] * b.data[j];----// af
----------------carry += c.data[i + j];---------------------------------------// 18
----------------c.data[i + j] = carry % intx::radix;--------------------------// 86
----------------carry /= intx::radix;-----------------------------------------// 05
------------}-----------------------------------------------------------------// 68
--------}---------------------------------------------------------------------// 9e
```

```
--------return c.normalize(sign * b.sign);--------------------------------------// de
----}---------------------------------------------------------------------------// c6
----friend pair<intx,intx> divmod(const intx& n, const intx& d) {--------------// fb
--------assert(!(d.size() == 1 && d.data[0] == 0));-----------------------------// e9
--------intx q, r; q.data.assign(n.size(), 0);----------------------------------// ca
--------for (int i = n.size() - 1; i >= 0; i--) {-------------------------------// 1a
------------r.data.insert(r.data.begin(), 0);-----------------------------------// c7
------------r = r + n.data[i];--------------------------------------------------// e6
------------long long k = 0;----------------------------------------------------// cc
------------if (d.size() < r.size())--------------------------------------------// b9
----------------k = (long long)intx::radix * r.data[d.size()];------------------// f7
------------if (d.size() - 1 < r.size()) k += r.data[d.size() - 1];-------------// 06
------------k /= d.data.back();-------------------------------------------------// b7
------------r = r - abs(d) * k;-------------------------------------------------// 15
------------while (r < 0) r = r + abs(d), k--;----------------------------------// 11
------------q.data[i] = k;------------------------------------------------------// d4
--------}-----------------------------------------------------------------------// 2f
--------return pair<intx, intx>(q.normalize(n.sign * d.sign), r);---------------// a1
----}---------------------------------------------------------------------------// 1b
----intx operator /(const intx& d) const {--------------------------------------// a2
--------return divmod(*this,d).first; }-----------------------------------------// 1e
----intx operator %(const intx& d) const {--------------------------------------// 07
--------return divmod(*this,d).second * sign; }---------------------------------// 5a
};------------------------------------------------------------------------------// 38
```

5.2.1. *Fast Multiplication.* Fast multiplication for the big integer using Fast Fourier Transform.

```
#include "intx.cpp"-------------------------------------------------------------// 83
#include "fft.cpp"--------------------------------------------------------------// 13
--------------------------------------------------------------------------------// e0
intx fastmul(const intx &an, const intx &bn) {----------------------------------// ab
----string as = an.to_string(), bs = bn.to_string();----------------------------// 32
----int n = size(as), m = size(bs), l = 1,--------------------------------------// dc
--------len = 5, radix = 100000,------------------------------------------------// 4f
--------*a = new int[n], alen = 0,----------------------------------------------// b8
--------*b = new int[m], blen = 0;----------------------------------------------// 0a
----memset(a, 0, n << 2);-------------------------------------------------------// 1d
----memset(b, 0, m << 2);-------------------------------------------------------// 01
----for (int i = n - 1; i >= 0; i -= len, alen++)-------------------------------// 6e
--------for (int j = min(len - 1, i); j >= 0; j--)-----------------------------// 43
------------a[alen] = a[alen] * 10 + as[i - j] - '0';---------------------------// 14
----for (int i = m - 1; i >= 0; i -= len, blen++)-------------------------------// b6
--------for (int j = min(len - 1, i); j >= 0; j--)-----------------------------// ae
------------b[blen] = b[blen] * 10 + bs[i - j] - '0';---------------------------// 9b
----while (l < 2*max(alen,blen)) l <<= 1;---------------------------------------// 51
----cpx *A = new cpx[l], *B = new cpx[l];---------------------------------------// 0d
----for (int i = 0; i < l; i++) A[i] = cpx(i < alen ? a[i] : 0, 0);-------------// 35
----for (int i = 0; i < l; i++) B[i] = cpx(i < blen ? b[i] : 0, 0);-------------// 66
----fft(A, l); fft(B, l);-------------------------------------------------------// f9
----for (int i = 0; i < l; i++) A[i] *= B[i];-----------------------------------// e7
----fft(A, l, true);------------------------------------------------------------// d3
----ull *data = new ull[l];-----------------------------------------------------// e7
----for (int i = 0; i < l; i++) data[i] = (ull)(round(real(A[i])));-------------// 06
----for (int i = 0; i < l - 1; i++)---------------------------------------------// 90
--------if (data[i] >= (unsigned int)(radix)) {---------------------------------// 44
------------data[i+1] += data[i] / radix;---------------------------------------// e4
------------data[i] %= radix;---------------------------------------------------// bd
--------}-----------------------------------------------------------------------// 5d
----int stop = l-1;-------------------------------------------------------------// cb
----while (stop > 0 && data[stop] == 0) stop--;---------------------------------// 97
----stringstream ss;------------------------------------------------------------// 42
----ss << data[stop];-----------------------------------------------------------// 96
----for (int i = stop - 1; i >= 0; i--)-----------------------------------------// bd
--------ss << setfill('0') << setw(len) << data[i];-----------------------------// b6
----delete[] A; delete[] B;-----------------------------------------------------// f7
----delete[] a; delete[] b;-----------------------------------------------------// 7e
----delete[] data;--------------------------------------------------------------// 6a
----return intx(ss.str());------------------------------------------------------// 38
}-------------------------------------------------------------------------------// d9
```

5.3. **Binomial Coefficients.** The binomial coefficient $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ is the number of ways to choose k items out of a total of n items.

```
int nck(int n, int k) {---------------------------------------------------------// f6
----if (n - k < k) k = n - k;---------------------------------------------------// 18
----int res = 1;----------------------------------------------------------------// cb
----for (int i = 1; i <= k; i++) res = res * (n - (k - i)) / i;-----------------// bd
----return res;-----------------------------------------------------------------// e4
}-------------------------------------------------------------------------------// 03
```

5.4. **Euclidean algorithm.** The Euclidean algorithm computes the greatest common divisor of two integers a, b.

```
int gcd(int a, int b) { return b == 0 ? a : gcd(b, a % b); }--------------------// d9
```

The extended Euclidean algorithm computes the greatest common divisor d of two integers a, b and also finds two integers x, y such that $a \times x + b \times y = d$.

```
int egcd(int a, int b, int& x, int& y) {----------------------------------------// 85
----if (b == 0) { x = 1; y = 0; return a; }-------------------------------------// 7b
----else {----------------------------------------------------------------------// 00
--------int d = egcd(b, a % b, x, y);-------------------------------------------// 34
--------x -= a / b * y;---------------------------------------------------------// 4a
--------swap(x, y);-------------------------------------------------------------// 26
--------return d;---------------------------------------------------------------// db
----}---------------------------------------------------------------------------// 9e
}-------------------------------------------------------------------------------// 40
```

5.5. **Trial Division Primality Testing.** An optimized trial division to check whether an integer is prime.

```
bool is_prime(int n) {----------------------------------------------------------// 6c
----if (n < 2) return false;----------------------------------------------------// c9
----if (n < 4) return true;-----------------------------------------------------// d9
----if (n % 2 == 0 || n % 3 == 0) return false;---------------------------------// 0f
----if (n < 25) return true;----------------------------------------------------// ef
----int s = static_cast<int>(sqrt(static_cast<double>(n)));---------------------// 64
----for (int i = 5; i <= s; i += 6)---------------------------------------------// 6c
--------if (n % i == 0 || n % (i + 2) == 0) return false;-----------------------// e9
----return true; }--------------------------------------------------------------// 43
```

5.6. Sieve of Eratosthenes. An optimized implementation of Eratosthenes' Sieve.

```
vi prime_sieve(int n) {----------------------------------------------------// 40
----int mx = (n - 3) >> 1, sq, v, i = -1;----------------------------------// 27
----vi primes;-------------------------------------------------------------// 8f
----bool* prime = new bool[mx + 1];----------------------------------------// ef
----memset(prime, 1, mx + 1);----------------------------------------------// 28
----if (n >= 2) primes.push_back(2);---------------------------------------// f4
----while (++i <= mx) if (prime[i]) {--------------------------------------// 73
--------primes.push_back(v = (i << 1) + 3);--------------------------------// be
--------if ((sq = i * ((i << 1) + 6) + 3) > mx) break;---------------------// 2d
--------for (int j = sq; j <= mx; j += v) prime[j] = false; }--------------// 2e
----while (++i <= mx) if (prime[i]) primes.push_back((i << 1) + 3);--------// 29
----delete[] prime; // can be used for O(1) lookup-------------------------// 36
----return primes; }-------------------------------------------------------// 72
```

5.7. Modular Multiplicative Inverse. A function to find a modular multiplicative inverse.

```
#include "egcd.cpp"--------------------------------------------------------// 55
---------------------------------------------------------------------------// e8
int mod_inv(int a, int m) {------------------------------------------------// 49
----int x, y, d = egcd(a, m, x, y);----------------------------------------// 3e
----if (d != 1) return -1;-------------------------------------------------// 20
----return x < 0 ? x + m : x;----------------------------------------------// 3c
}--------------------------------------------------------------------------// 69
```

5.8. Modular Exponentiation. A function to perform fast modular exponentiation.

```
template <class T>---------------------------------------------------------// 82
T mod_pow(T b, T e, T m) {-------------------------------------------------// aa
----T res = T(1);----------------------------------------------------------// 85
----while (e) {------------------------------------------------------------// b7
--------if (e & T(1)) res = mod(res * b, m);-------------------------------// 41
--------b = mod(b * b, m), e >>= T(1); }-----------------------------------// b3
----return res;------------------------------------------------------------// eb
}--------------------------------------------------------------------------// c5
```

5.9. Chinese Remainder Theorem. An implementation of the Chinese Remainder Theorem.

```
#include "egcd.cpp"--------------------------------------------------------// 55
int crt(const vi& as, const vi& ns) {--------------------------------------// c3
----int cnt = size(as), N = 1, x = 0, r, s, l;-----------------------------// 55
----for (int i = 0; i < cnt; i++) N *= ns[i]; -----------------------------// 88
----for (int i = 0; i < cnt; i++)------------------------------------------// f9
--------egcd(ns[i], l = N/ns[i], r, s), x += as[i] * s * l;----------------// b0
----return mod(x, N); }----------------------------------------------------// 9e
```

5.10. Linear Congruence Solver. A function that returns all solutions to $ax \equiv b \pmod{n}$, modulo n.

```
#include "egcd.cpp"--------------------------------------------------------// 55
vi linear_congruence(int a, int b, int n) {--------------------------------// c8
----int x, y, d = egcd(a, n, x, y);----------------------------------------// 7a
----vi res;----------------------------------------------------------------// f5
----if (b % d != 0) return res;--------------------------------------------// 30
----int x0 = mod(b / d * x, n);--------------------------------------------// 48
----for (int k = 0; k < d; k++) res.push_back(mod(x0 + k * n / d, n));-----// 21
----return res;------------------------------------------------------------// 03
}--------------------------------------------------------------------------// 1c
```

5.11. Numeric Integration. Numeric integration using Simpson's rule.

```
double integrate(double (*f)(double), double a, double b,------------------// 76
--------double delta = 1e-6) {---------------------------------------------// c0
----if (abs(a - b) < delta)------------------------------------------------// 38
--------return (b-a)/8 *---------------------------------------------------// 56
------------(f(a) + 3*f((2*a+b)/3) + 3*f((a+2*b)/3) + f(b));---------------// e1
----return integrate(f, a,-------------------------------------------------// 64
------------(a+b)/2, delta) + integrate(f, (a+b)/2, b, delta);-------------// 0c
}--------------------------------------------------------------------------// 4b
```

5.12. Fast Fourier Transform. The Cooley-Tukey algorithm for quickly computing the discrete Fourier transform. Note that this implementation only handles powers of two, make sure to pad with zeros.

```
#include <complex>---------------------------------------------------------// 8e
typedef complex<long double> cpx;------------------------------------------// 25
void fft(cpx *x, int n, bool inv=false) {----------------------------------// 23
----for (int i = 0, j = 0; i < n; i++) {-----------------------------------// f2
--------if (i < j) swap(x[i], x[j]);---------------------------------------// 5c
--------int m = n>>1;------------------------------------------------------// e5
--------while (1 <= m && m <= j) j -= m, m >>= 1;--------------------------// fe
--------j += m;------------------------------------------------------------// ab
----}----------------------------------------------------------------------// 1e
----for (int mx = 1; mx < n; mx <<= 1) {-----------------------------------// 9d
--------cpx wp = exp(cpx(0, (inv ? -1 : 1) * pi / mx)), w = 1;-------------// 60
--------for (int m = 0; m < mx; m++, w *= wp) {----------------------------// 40
------------for (int i = m; i < n; i += mx << 1) {-------------------------// 33
----------------cpx t = x[i + mx] * w;-------------------------------------// f5
----------------x[i + mx] = x[i] - t;--------------------------------------// ac
----------------x[i] += t;-------------------------------------------------// c7
------------}--------------------------------------------------------------// 6d
--------}------------------------------------------------------------------// c2
----}----------------------------------------------------------------------// 70
----if (inv) for (int i = 0; i < n; i++) x[i] /= cpx(n);-------------------// 3e
}--------------------------------------------------------------------------// 7d
```

5.13. Formulas.

- Number of ways to choose k objects from a total of n objects where order matters and each item can only be chosen once: $P_k^n = \frac{n!}{(n-k)!}$
- Number of ways to choose k objects from a total of n objects where order matters and each item can be chosen multiple times: n^k
- Number of permutations of n objects, where there are n_1 objects of type 1, n_2 objects of type 2, ..., n_k objects of type k: $\binom{n}{n_1,n_2,\ldots,n_k} = \frac{n!}{n_1!\times n_2!\times\cdots\times n_k!}$
- Number of ways to choose k objects from a total of n objects where order does not matter and each item can only be chosen once: $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k} = \binom{n}{n-k} = \prod_{i=1}^{k} \frac{n-(k-i)}{i} = \frac{n!}{k!(n-k)!}$, $\binom{n}{0} = 1$, $\binom{0}{k} = 0$
- Number of ways to choose k objects from a total of n objects where order does not matter and each item can be chosen multiple times: $f_k^n = \binom{n+k-1}{k} = \frac{(n+k-1)!}{k!(n-1)!}$
- Number of integer solutions to $x_1 + x_2 + \cdots + x_n = k$ where $x_i \geq 0$: f_k^n
- Number of subsets of a set with n elements: 2^n

- $|A \cup B| = |A| + |B| - |A \cap B|$
- $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$
- Number of ways to walk from the lower-left corner to the upper-right corner of an $n \times m$ grid by walking only up and to the right: $\binom{n+m}{m}$
- Number of strings with n sets of brackets such that the brackets are balanced: $C_n = \sum_{k=0}^{n-1} C_k C_{n-1-k} = \frac{1}{n+1}\binom{2n}{n}$
- Number of triangulations of a convex polygon with n points, number of rooted binary trees with $n+1$ vertices, number of paths across an $n \times n$ lattice which do not rise above the main diagonal: C_n
- Number of permutations of n objects with exactly k ascending sequences or *runs*: $\left\langle {n \atop k} \right\rangle = \left\langle {n \atop n-k-1} \right\rangle = k\left\langle {n-1 \atop k} \right\rangle + (n-k+1)\left\langle {n-1 \atop k-1} \right\rangle = \sum_{i=0}^{k}(-1)^i\binom{n+1}{i}(k+1-i)^n, \left\langle {n \atop 0} \right\rangle = \left\langle {n \atop n-1} \right\rangle = 1$
- Number of permutations of n objects with exactly k cycles: $\left[{n \atop k}\right] = \left[{n-1 \atop k-1}\right] + (n-1)\left[{n-1 \atop k}\right]$
- Number of ways to partition n objects into k sets: $\left\{{n \atop k}\right\} = k\left\{{n-1 \atop k}\right\} + \left\{{n-1 \atop k-1}\right\}, \left\{{n \atop 0}\right\} = \left\{{n \atop n}\right\} = 1$
- Number of permutations of length n that have no fixed points (derangements): $D_0 = 1, D_1 = 0, D_n = (n-1)(D_{n-1} + D_{n-2})$
- Number of permutations of length n that have exactly k fixed points: $\binom{n}{k} D_{n-k}$
- **Heron's formula:** A triangle with side lengths a, b, c has area $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$.
- **Pick's theorem:** A polygon on an integer grid containing i lattice points and having b lattice points on the boundary has area $i + \frac{b}{2} - 1$.
- **Divisor sigma:** The sum of divisors of n to the xth power is $\sigma_x(n) = \prod_{i=0}^{r} \frac{p_i^{(a_i+1)x}-1}{p_i^x - 1}$ where $n = \prod_{i=0}^{r} p_i^{a_i}$ is the prime factorization.
- **Divisor count:** A special case of the above is $\sigma_0(n) = \prod_{i=0}^{r}(a_i + 1)$.
- **Euler's totient:** The number of integers less than n that are comprime to n are $n \prod_{p|n}\left(1 - \frac{1}{p}\right)$ where each p is a distinct prime factor of n.
- **König's theorem:** In any bipartite graph, the number of edges in a maximum matching is equal to the number of vertices in a minimum vertex cover.
- The number of vertices of a graph is equal to its minimum vertex cover number plus the size of a maximum independent set.

6. Geometry

6.1. **Primitives.** Geometry primitives.

```
#include <complex>-------------------------------------------------------------// 8e
#define P(p) const point &p----------------------------------------------------// b8
#define L(p0, p1) P(p0), P(p1)-------------------------------------------------// 30
typedef complex<double> point;-------------------------------------------------// e1
double dot(P(a), P(b)) { return real(conj(a) * b); }---------------------------// a9
double cross(P(a), P(b)) { return imag(conj(a) * b); }-------------------------// ff
point rotate(P(p), P(about), double radians) {---------------------------------// e1
----return (p - about) * exp(point(0, radians)) + about; }---------------------// cb
point reflect(P(p), L(about1, about2)) {---------------------------------------// c0
----point z = p - about1, w = about2 - about1;---------------------------------// 39
----return conj(z / w) * w + about1; }-----------------------------------------// 03
point proj(P(u), P(v)) { return dot(u, v) / dot(u, u) * u; }-------------------// fc
bool parallel(L(a, b), L(p, q)) { return abs(cross(b - a, q - p)) < EPS; }-----// 6d
double ccw(P(a), P(b), P(c)) { return cross(b - a, c - b); }-------------------// ca
bool collinear(P(a), P(b), P(c)) { return abs(ccw(a, b, c)) < EPS; }-----------// 75
bool collinear(L(a, b), L(p, q)) {---------------------------------------------// 66
----return abs(ccw(a, b, p)) < EPS && abs(ccw(a, b, q)) < EPS;  }--------------// d6
double angle(P(a), P(b), P(c)) {-----------------------------------------------// d0
----return acos(dot(b - a, c - b) / abs(b - a) / abs(c - b)); }----------------// cc
double signed_angle(P(a), P(b), P(c)) {----------------------------------------// fe
----return asin(cross(b - a, c - b) / abs(b - a) / abs(c - b)); }--------------// 9e
double progress(P(p), L(a, b)) {-----------------------------------------------// d2
----if (abs(real(a) - real(b)) < EPS)------------------------------------------// 9e
--------return (imag(p) - imag(a)) / (imag(b) - imag(a));----------------------// 35
----else return (real(p) - real(a)) / (real(b) - real(a)); }-------------------// 2c
bool intersect(L(a, b), L(p, q), point &res, bool segment = false) {-----------// d6
----// NOTE: check for parallel/collinear lines before calling this function---// 02
----point r = b - a, s = q - p;------------------------------------------------// 79
----double c = cross(r, s), t = cross(p - a, s) / c, u = cross(p - a, r) / c;--// a8
----if (segment && (t < 0-EPS || t > 1+EPS || u < 0-EPS || u > 1+EPS))---------// ae
--------return false;----------------------------------------------------------// a3
----res = a + t * r;-----------------------------------------------------------// ca
----return true;---------------------------------------------------------------// 17
}------------------------------------------------------------------------------// 0f
point closest_point(L(a, b), P(c), bool segment = false) {---------------------// a1
----if (segment) {-------------------------------------------------------------// c2
--------if (dot(b - a, c - b) > 0) return b;-----------------------------------// b5
--------if (dot(a - b, c - a) > 0) return a;-----------------------------------// cf
----}--------------------------------------------------------------------------// 61
----double t = dot(c - a, b - a) / norm(b - a);--------------------------------// aa
----return a + t * (b - a);----------------------------------------------------// 7a
}------------------------------------------------------------------------------// e5
double line_segment_distance(L(a,b), L(c,d)) {---------------------------------// 99
----double x = INFINITY;-------------------------------------------------------// 83
----if (abs(a - b) < EPS && abs(c - d) < EPS) x = abs(a - c);------------------// df
----else if (abs(a - b) < EPS) x = abs(a - closest_point(c, d, a, true));------// da
----else if (abs(c - d) < EPS) x = abs(c - closest_point(a, b, c, true));------// 52
----else if ((ccw(a, b, c) < 0) != (ccw(a, b, d) < 0) &&-----------------------// ee
------------ (ccw(c, d, a) < 0) != (ccw(c, d, b) < 0)) x = 0;------------------// 79
----else {---------------------------------------------------------------------// 38
--------x = min(x, abs(a - closest_point(c,d, a, true)));----------------------// f3
--------x = min(x, abs(b - closest_point(c,d, b, true)));----------------------// ec
--------x = min(x, abs(c - closest_point(a,b, c, true)));----------------------// 36
--------x = min(x, abs(d - closest_point(a,b, d, true)));----------------------// e5
----}--------------------------------------------------------------------------// 72
----return x;------------------------------------------------------------------// 0d
}------------------------------------------------------------------------------// b3
```

6.2. **Polygon.** Polygon primitives.

```
#include "primitives.cpp"------------------------------------------------------// e0
typedef vector<point> polygon;-------------------------------------------------// b3
double polygon_area_signed(polygon p) {----------------------------------------// 31
----double area = 0; int cnt = size(p);----------------------------------------// a2
----for (int i = 1; i + 1 < cnt; i++)------------------------------------------// d2
--------area += cross(p[i] - p[0], p[i + 1] - p[0]);---------------------------// 7e
----return area / 2; }---------------------------------------------------------// e1
double polygon_area(polygon p) { return abs(polygon_area_signed(p)); }---------// 25
```

```
#define CHK(f,a,b,c) (f(a) < f(b) && f(b) <= f(c) && ccw(a,c,b) < 0)-----------// b2
int point_in_polygon(polygon p, point q) {--------------------------------------// 58
----int n = size(p); bool in = false; double d;---------------------------------// 06
----for (int i = 0, j = n - 1; i < n; j = i++)----------------------------------// 77
--------if (collinear(p[i], q, p[j]) &&-----------------------------------------// a5
------------0 <= (d = progress(q, p[i], p[j])) && d <= 1)-----------------------// b9
------------return 0;-----------------------------------------------------------// cc
----for (int i = 0, j = n - 1; i < n; j = i++)----------------------------------// 6f
--------if (CHK(real, p[i], q, p[j]) || CHK(real, p[j], q, p[i]))---------------// 1f
------------in = !in;-----------------------------------------------------------// b2
----return in ? -1 : 1; }-------------------------------------------------------// 77
// pair<polygon, polygon> cut_polygon(const polygon &poly, point a, point b) {-// 7b
//---- polygon left, right;-----------------------------------------------------// 6b
//---- point it(-100, -100);----------------------------------------------------// c9
//---- for (int i = 0, cnt = poly.size(); i < cnt; i++) {-----------------------// 28
//-------- int j = i == cnt-1 ? 0 : i + 1;--------------------------------------// 8e
//-------- point p = poly[i], q = poly[j];--------------------------------------// 19
//-------- if (ccw(a, b, p) <= 0) left.push_back(p);----------------------------// 12
//-------- if (ccw(a, b, p) >= 0) right.push_back(p);---------------------------// e3
//-------- // myintersect = intersect where-------------------------------------// 24
//-------- // (a,b) is a line, (p,q) is a line segment--------------------------// f2
//-------- if (myintersect(a, b, p, q, it))-------------------------------------// f0
//------------ left.push_back(it), right.push_back(it);-------------------------// 21
//---- }------------------------------------------------------------------------// 5e
//---- return pair<polygon, polygon>(left, right);------------------------------// 1d
// }----------------------------------------------------------------------------// 37
```

6.3. **Convex Hull.** An algorithm that finds the Convex Hull of a set of points.

```
#include "polygon.cpp"----------------------------------------------------------// 58
#define MAXN 1000---------------------------------------------------------------// 09
point hull[MAXN];---------------------------------------------------------------// 43
bool cmp(const point &a, const point &b) {--------------------------------------// 32
----return abs(real(a) - real(b)) > EPS ?---------------------------------------// 44
--------real(a) < real(b) : imag(a) < imag(b); }--------------------------------// 40
int convex_hull(polygon p) {----------------------------------------------------// cd
----int n = size(p), l = 0;-----------------------------------------------------// 67
----sort(p.begin(), p.end(), cmp);----------------------------------------------// 3d
----for (int i = 0; i < n; i++) {-----------------------------------------------// 6f
--------if (i > 0 && p[i] == p[i - 1]) continue;--------------------------------// b2
--------while (l >= 2 && ccw(hull[l - 2], hull[l - 1], p[i]) >= 0) l--;---------// 20
--------hull[l++] = p[i];-------------------------------------------------------// f7
----}---------------------------------------------------------------------------// d8
----int r = l;------------------------------------------------------------------// 59
----for (int i = n - 2; i >= 0; i--) {------------------------------------------// 16
--------if (p[i] == p[i + 1]) continue;-----------------------------------------// c7
--------while (r - l >= 1 && ccw(hull[r - 2], hull[r - 1], p[i]) >= 0) r--;----// 9f
--------hull[r++] = p[i];-------------------------------------------------------// 6d
----}---------------------------------------------------------------------------// 74
----return l == 1 ? 1 : r - 1;--------------------------------------------------// 6d
}-------------------------------------------------------------------------------// 79
```

6.4. **Line Segment Intersection.** Computes the intersection between two line segments.

```
#include "primitives.cpp"-------------------------------------------------------// e0
bool line_segment_intersect(L(a, b), L(c, d), point &A, point &B) {-------------// 6c
----if (abs(a - b) < EPS && abs(c - d) < EPS) {---------------------------------// db
--------A = B = a; return abs(a - d) < EPS; }-----------------------------------// ee
----else if (abs(a - b) < EPS) {------------------------------------------------// 03
--------A = B = a; double p = progress(a, c,d);---------------------------------// c9
--------return 0.0 <= p && p <= 1.0---------------------------------------------// 8a
------------&& (abs(a - c) + abs(d - a) - abs(d - c)) < EPS; }------------------// 27
----else if (abs(c - d) < EPS) {------------------------------------------------// 26
--------A = B = c; double p = progress(c, a,b);---------------------------------// d9
--------return 0.0 <= p && p <= 1.0---------------------------------------------// 8e
------------&& (abs(c - a) + abs(b - c) - abs(b - a)) < EPS; }------------------// 4f
----else if (collinear(a,b, c,d)) {---------------------------------------------// bc
--------double ap = progress(a, c,d), bp = progress(b, c,d);--------------------// a7
--------if (ap > bp) swap(ap, bp);----------------------------------------------// b1
--------if (bp < 0.0 || ap > 1.0) return false;--------------------------------// 0c
--------A = c + max(ap, 0.0) * (d - c);-----------------------------------------// f6
--------B = c + min(bp, 1.0) * (d - c);-----------------------------------------// 5c
--------return true; }----------------------------------------------------------// ab
----else if (parallel(a,b, c,d)) return false;----------------------------------// ca
----else if (intersect(a,b, c,d, A, true)) {------------------------------------// 10
--------B = A; return true; }---------------------------------------------------// bf
----return false;---------------------------------------------------------------// b7
}-------------------------------------------------------------------------------// 8b
--------------------------------------------------------------------------------// e6
```

6.5. **Great-Circle Distance.** Computes the distance between two points (given as latitude/longitude coordinates) on a sphere of radius r.

```
double gc_distance(double pLat, double pLong,-----------------------------------// 7b
----------------  double qLat, double qLong, double r) {------------------------// a4
----pLat *= pi / 180; pLong *= pi / 180;----------------------------------------// ee
----qLat *= pi / 180; qLong *= pi / 180;----------------------------------------// 75
----return r * acos(cos(pLat) * cos(qLat) * cos(pLong - qLong) +----------------// e3
--------------------sin(pLat) * sin(qLat));-------------------------------------// 1e
--------------------------------------------------------------------------------// 60
}-------------------------------------------------------------------------------// 3f
```

6.6. **Formulas.** Let $a = (a_x, a_y)$ and $b = (b_x, b_y)$ be two-dimensional vectors.

- $a \cdot b = |a||b| \cos\theta$, where θ is the angle between a and b.
- $a \times b = |a||b| \sin\theta$, where θ is the signed angle between a and b.
- $a \times b$ is equal to the area of the parallelogram with two of its sides formed by a and b. Half of that is the area of the triangle formed by a and b.

7. Other Algorithms

7.1. **Binary Search.** An implementation of binary search that finds a real valued root of the continous function f on the interval $[a, b]$, with a maximum error of ε.

```
double binary_search_continuous(double low, double high,------------------------// 8e
--------double eps, double (*f)(double)) {--------------------------------------// c0
----while (true) {--------------------------------------------------------------// 3a
--------double mid = (low + high) / 2, cur = f(mid);---------------------------// 75
--------if (abs(cur) < eps) return mid;-----------------------------------------// 76
--------else if (0 < cur) high = mid;-------------------------------------------// e5
```

```
--------else low = mid;--------------------------------------------------------// a7
----}--------------------------------------------------------------------------// b5
}------------------------------------------------------------------------------// cb
```

Another implementation that takes a binary predicate f, and finds an integer value x on the integer interval $[a, b]$ such that $f(x) \wedge \neg f(x-1)$.

```
int binary_search_discrete(int low, int high, bool (*f)(int)) {----------------// 51
----assert(low <= high);-------------------------------------------------------// 19
----while (low < high) {-------------------------------------------------------// a3
--------int mid = low + (high - low) / 2;--------------------------------------// 04
--------if (f(mid)) high = mid;------------------------------------------------// ca
--------else low = mid + 1;----------------------------------------------------// 03
----}--------------------------------------------------------------------------// 9b
----assert(f(low));------------------------------------------------------------// 42
----return low;----------------------------------------------------------------// a6
}------------------------------------------------------------------------------// d3
```

7.2. **Ternary Search.** Given a function f that is first monotonically increasing and then monotonically decreasing, ternary search finds the x such that $f(x)$ is maximized.

```
template <class F>-------------------------------------------------------------// d1
double ternary_search_continuous(double lo, double hi, double eps, F f) {------// e7
----while (hi - lo > eps) {----------------------------------------------------// 3e
--------double m1 = lo + (hi - lo) / 3, m2 = hi - (hi - lo) / 3;---------------// e8
--------if (f(m1) < f(m2)) lo = m1;--------------------------------------------// 1d
--------else hi = m2;----------------------------------------------------------// b3
----}--------------------------------------------------------------------------// bb
----return hi;-----------------------------------------------------------------// fa
}------------------------------------------------------------------------------// 66
```

7.3. **2SAT.** A fast 2SAT solver.

```
#include "../graph/scc.cpp"----------------------------------------------------// c3
-------------------------------------------------------------------------------// 63
bool two_sat(int n, const vii& clauses, vi& all_truthy) {----------------------// f4
----all_truthy.clear();--------------------------------------------------------// 31
----vvi adj(2*n+1);------------------------------------------------------------// 7b
----for (int i = 0; i < size(clauses); i++) {----------------------------------// 9b
--------adj[-clauses[i].first + n].push_back(clauses[i].second + n);-----------// 17
--------if (clauses[i].first != clauses[i].second)-----------------------------// 87
------------adj[-clauses[i].second + n].push_back(clauses[i].first + n);-------// 93
----}--------------------------------------------------------------------------// d8
----pair<union_find, vi> res = scc(adj);---------------------------------------// 9f
----union_find scc = res.first;------------------------------------------------// 42
----vi dag = res.second;-------------------------------------------------------// 58
----vi truth(2*n+1, -1);-------------------------------------------------------// 00
----for (int i = 2*n; i >= 0; i--) {-------------------------------------------// f4
--------int cur = order[i] - n, p = scc.find(cur + n), o = scc.find(-cur + n);-// 5a
--------if (cur == 0) continue;------------------------------------------------// 26
--------if (p == o) return false;----------------------------------------------// 33
--------if (truth[p] == -1) truth[p] = 1;--------------------------------------// c3
--------truth[cur + n] = truth[p];---------------------------------------------// b3
--------truth[o] = 1 - truth[p];-----------------------------------------------// 80
--------if (truth[p] == 1) all_truthy.push_back(cur);--------------------------// 5c
----}--------------------------------------------------------------------------// d9
----return true;---------------------------------------------------------------// eb
}------------------------------------------------------------------------------// 61
```

7.4. **Stable Marriage.** The Gale-Shapley algorithm for solving the stable marriage problem.

```
vi stable_marriage(int n, int** m, int** w) {----------------------------------// e4
----queue<int> q;--------------------------------------------------------------// f6
----vi at(n, 0), eng(n, -1), res(n, -1); vvi inv(n, vi(n));--------------------// c3
----for (int i = 0; i < n; i++) for (int j = 0; j < n; j++)--------------------// 05
--------inv[i][w[i][j]] = j;---------------------------------------------------// b9
----for (int i = 0; i < n; i++) q.push(i);-------------------------------------// fe
----while (!q.empty()) {-------------------------------------------------------// 55
--------int curm = q.front(); q.pop();-----------------------------------------// ab
--------for (int &i = at[curm]; i < n; i++) {----------------------------------// 9a
------------int curw = m[curm][i];---------------------------------------------// cf
------------if (eng[curw] == -1) { }-------------------------------------------// 35
------------else if (inv[curw][curm] < inv[curw][eng[curw]])-------------------// 10
----------------q.push(eng[curw]);---------------------------------------------// 8c
------------else continue;-----------------------------------------------------// b4
------------res[eng[curw] = curm] = curw, ++i; break;--------------------------// 5e
--------}----------------------------------------------------------------------// 24
----}--------------------------------------------------------------------------// b8
----return res;----------------------------------------------------------------// 95
}------------------------------------------------------------------------------// 03
```

7.5. **Algorithm X.** An implementation of Knuth's Algorithm X, using dancing links. Solves the Exact Cover problem.

```
bool handle_solution(vi rows) { return false; }--------------------------------// 63
struct exact_cover {-----------------------------------------------------------// 95
----struct node {--------------------------------------------------------------// 7e
--------node *l, *r, *u, *d, *p;-----------------------------------------------// 19
--------int row, col, size;----------------------------------------------------// ae
--------node(int row, int col) : row(row), col(col) {--------------------------// 68
------------size = 0; l = r = u = d = p = NULL; }------------------------------// 8f
----};-------------------------------------------------------------------------// 9e
----int rows, cols, *sol;------------------------------------------------------// 54
----bool **arr;----------------------------------------------------------------// 4a
----node *head;----------------------------------------------------------------// c2
----exact_cover(int rows, int cols) : rows(rows), cols(cols), head(NULL) {-----// ce
--------arr = new bool*[rows];-------------------------------------------------// 15
--------sol = new int[rows];---------------------------------------------------// 69
--------for (int i = 0; i < rows; i++)-----------------------------------------// c7
------------arr[i] = new bool[cols], memset(arr[i], 0, cols);------------------// 68
----}--------------------------------------------------------------------------// 8b
----void set_value(int row, int col, bool val = true) { arr[row][col] = val; }-// af
----void setup() {-------------------------------------------------------------// a8
--------node ***ptr = new node**[rows + 1];------------------------------------// da
--------for (int i = 0; i <= rows; i++) {--------------------------------------// ce
------------ptr[i] = new node*[cols];------------------------------------------// cc
------------for (int j = 0; j < cols; j++)-------------------------------------// 56
----------------if (i == rows || arr[i][j]) ptr[i][j] = new node(i, j);--------// 95
----------------else ptr[i][j] = NULL;-----------------------------------------// 40
--------}----------------------------------------------------------------------// b0
```

```
        for (int i = 0; i <= rows; i++) {                    // 80
            for (int j = 0; j < cols; j++) {                 // 86
                if (!ptr[i][j]) continue;                    // 76
                int ni = i + 1, nj = j + 1;                  // 34
                while (true) {                               // 7f
                    if (ni == rows + 1) ni = 0;              // 54
                    if (ni == rows || arr[ni][j]) break;     // 77
                    ++ni;                                    // c8
                }                                            // 47
                ptr[i][j]->d = ptr[ni][j];                   // a9
                ptr[ni][j]->u = ptr[i][j];                   // c0
                while (true) {                               // 0d
                    if (nj == cols) nj = 0;                  // a7
                    if (i == rows || arr[i][nj]) break;      // e9
                    ++nj;                                    // a6
                }                                            // 4e
                ptr[i][j]->r = ptr[i][nj];                   // b3
                ptr[i][nj]->l = ptr[i][j];                   // 46
            }                                                // 83
        }                                                    // b4
        head = new node(rows, -1);                           // 80
        head->r = ptr[rows][0];                              // b9
        ptr[rows][0]->l = head;                              // c1
        head->l = ptr[rows][cols - 1];                       // 28
        ptr[rows][cols - 1]->r = head;                       // 83
        for (int j = 0; j < cols; j++) {                     // 02
            int cnt = -1;                                    // 36
            for (int i = 0; i <= rows; i++)                  // 56
                if (ptr[i][j]) cnt++, ptr[i][j]->p = ptr[rows][j]; // 05
            ptr[rows][j]->size = cnt;                        // d4
        }                                                    // 8f
        for (int i = 0; i <= rows; i++) delete[] ptr[i];     // cd
        delete[] ptr;                                        // 42
    }                                                        // a9
    #define COVER(c, i, j) \                                 // 23
        c->r->l = c->l, c->l->r = c->r; \                    // 83
        for (node *i = c->d; i != c; i = i->d) \             // 5c
            for (node *j = i->r; j != i; j = j->r) \         // 0e
                j->d->u = j->u, j->u->d = j->d, j->p->size--; // 5a
    #define UNCOVER(c, i, j) \                               // 17
        for (node *i = c->u; i != c; i = i->u) \             // 98
            for (node *j = i->l; j != i; j = j->l) \         // 7d
                j->p->size++, j->d->u = j->u->d = j; \       // be
        c->r->l = c->l->r = c;                               // bb
    bool search(int k = 0) {                                 // 4f
        if (head == head->r) {                               // a7
            vi res(k);                                       // 4f
            for (int i = 0; i < k; i++) res[i] = sol[i];     // c0
            sort(res.begin(), res.end());                    // 3e
            return handle_solution(res);                     // dc
        }                                                    // 1d
        node *c = head->r, *tmp = head->r;                   // a6
        for ( ; tmp != head; tmp = tmp->r) if (tmp->size < c->size) c = tmp; // 1e
        if (c == c->d) return false;                         // 17
        COVER(c, i, j);                                      // 61
        bool found = false;                                  // 6e
        for (node *r = c->d; !found && r != c; r = r->d) {   // 1e
            sol[k] = r->row;                                 // 0b
            for (node *j = r->r; j != r; j = j->r) { COVER(j->p, a, b); } // 3a
            found = search(k + 1);                           // f4
            for (node *j = r->l; j != r; j = j->l) { UNCOVER(j->p, a, b); } // 8a
        }                                                    // a1
        UNCOVER(c, i, j);                                    // 64
        return found;                                        // ff
    }                                                        // 06
};                                                           // 10
```

7.6. **nth Permutation.** A very fast algorithm for computing the nth permutation of the list $\{0, 1, \ldots, k-1\}$.

```
vector<int> nth_permutation(int cnt, int n) {                // 78
    vector<int> idx(cnt), per(cnt), fac(cnt);                // 9e
    for (int i = 0; i < cnt; i++) idx[i] = i;                // 80
    for (int i = 1; i <= cnt; i++) fac[i - 1] = n % i, n /= i; // 04
    for (int i = cnt - 1; i >= 0; i--)                       // 52
        per[cnt - i - 1] = idx[fac[i]], idx.erase(idx.begin() + fac[i]); // 41
    return per;                                              // 84
}                                                            // 97
```

7.7. **Cycle-Finding.** An implementation of Floyd's Cycle-Finding algorithm.

```
ii find_cycle(int x0, int (*f)(int)) {                       // a5
    int t = f(x0), h = f(t), mu = 0, lam = 1;                // 8d
    while (t != h) t = f(t), h = f(f(h));                    // 79
    h = x0;                                                  // 04
    while (t != h) t = f(t), h = f(h), mu++;                 // 9d
    h = f(t);                                                // 00
    while (t != h) h = f(h), lam++;                          // 5e
    return ii(mu, lam);                                      // b4
}                                                            // 42
```

7.8. **Dates.** Functions to simplify date calculations.

```
int intToDay(int jd) { return jd % 7; }                      // 89
int dateToInt(int y, int m, int d) {                         // 96
    return 1461 * (y + 4800 + (m - 14) / 12) / 4 +           // a8
        367 * (m - 2 - (m - 14) / 12 * 12) / 12 -            // d1
        3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +         // be
        d - 32075;                                           // e0
}                                                            // fa
void intToDate(int jd, int &y, int &m, int &d) {             // a1
    int x, n, i, j;                                          // 00
    x = jd + 68569;                                          // 11
    n = 4 * x / 146097;                                      // 2f
    x -= (146097 * n + 3) / 4;                               // 58
```

```
----i = (4000 * (x + 1)) / 1461001;--------------------------------------------// 0d
----x -= 1461 * i / 4 - 31;----------------------------------------------------// 09
----j = 80 * x / 2447;---------------------------------------------------------// 3d
----d = x - 2447 * j / 80;-----------------------------------------------------// eb
----x = j / 11;----------------------------------------------------------------// b7
----m = j + 2 - 12 * x;--------------------------------------------------------// 82
----y = 100 * (n - 49) + i + x;------------------------------------------------// 70
}------------------------------------------------------------------------------// af
```

8. Useful Information

8.1. Tips & Tricks.

- How fast does our algorithm have to be? Can we use brute-force?
- Does order matter?
- Is it better to look at the problem in another way? Maybe backwards?
- Are there subproblems that are recomputed? Can we cache them?
- Do we need to remember everything we compute, or just the last few iterations of computation?
- Does it help to sort the data?
- Can we speed up lookup by using a map (tree or hash) or an array?
- Can we binary search the answer?
- Can we add vertices/edges to the graph to make the problem easier? Can we turn the graph into some other kind of a graph (perhaps a DAG, or a flow network)?
- Make sure integers are not overflowing.
- Is it better to compute the answer modulo n? Perhaps we can compute the answer modulo $m_1, m_2, \ldots, m_k$, where $m_1, m_2, \ldots, m_k$ are pairwise coprime integers, and find the real answer using CRT?
- Are there any edge cases? When $n = 0, n = -1, n = 1, n = 2^{31} - 1$ or $n = -2^{31}$? When the list is empty, or contains a single element? When the graph is empty, or contains a single vertex? When the graph contains self-loops? When the polygon is concave or non-simple?
- Can we use exponentiation by squaring?

8.2. Fast Input Reading.

If input or output is huge, sometimes it is beneficial to optimize the input reading/output writing. This can be achieved by reading all input in at once (using fread), and then parsing it manually. Output can also be stored in an output buffer and then dumped once in the end (using fwrite). A simpler, but still effective, way to achieve speed is to use the following input reading method.

```
void readn(register int *n) {--------------------------------------------------// dc
----int sign = 1;--------------------------------------------------------------// 32
----register char c;-----------------------------------------------------------// a5
----*n = 0;--------------------------------------------------------------------// 35
----while((c = getc_unlocked(stdin)) != '\n') {--------------------------------// f3
--------switch(c) {------------------------------------------------------------// 0c
------------case '-': sign = -1; break;----------------------------------------// 28
------------case ' ': goto hell;-----------------------------------------------// fd
------------case '\n': goto hell;----------------------------------------------// 79
------------default: *n *= 10; *n += c - '0'; break;---------------------------// c0
--------}----------------------------------------------------------------------// 2d
----}--------------------------------------------------------------------------// c3
hell:--------------------------------------------------------------------------// ba
----*n *= sign;----------------------------------------------------------------// a0
}------------------------------------------------------------------------------// 67
```

8.3. 128-bit Integer.

GCC has a 128-bit integer data type named `__int128`. Useful if doing multiplication of 64-bit integers, or something needing a little more than 64-bits to represent.

8.4. Worst Time Complexity.

n	Worst AC Algorithm	Comment
≤ 10	$O(n!), O(n^6)$	e.g. Enumerating a permutation
≤ 15	$O(2^n \times n^2)$	e.g. DP TSP
≤ 20	$O(2^n), O(n^5)$	e.g. DP + bitmask technique
≤ 50	$O(n^4)$	e.g. DP with 3 dimensions + $O(n)$ loop, choosing ${}_nC_k = 4$
$\leq 10^2$	$O(n^3)$	e.g. Floyd Warshall's
$\leq 10^3$	$O(n^2)$	e.g. Bubble/Selection/Insertion sort
$\leq 10^5$	$O(n \log_2 n)$	e.g. Merge sort, building a Segment tree
$\leq 10^6$	$O(n), O(\log_2 n), O(1)$	Usually, contest problems have $n \leq 10^6$ (e.g. to read input)

8.5. Bit Hacks.

- `n & -n` returns the first set bit in n.
- `n & (n - 1)` is 0 only if n is a power of two.
- `snoob(x)` returns the next integer that has the same amount of bits set as `x`. Useful for iterating through subsets of some specified size.

```
int snoob(int x) {-----------------------------------------------------------
----int y = x & -x, z = x + y;-----------------------------------------------
----return z | ((x ^ z) >> 2) / y;-------------------------------------------
}----------------------------------------------------------------------------
```