

viRUs

Team Reference Document

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1. CODE TEMPLATES

1.1. Basic Configuration.

1.1.1. .bashrc.

```
xset r rate 150 100-----// dd
set -o vi-----// f4
-----// 08
function check {------// f6
---IFS=-----// 17
---s=""-----// 22
---cat $1 | while read l; do-----// 81
-----s="$s$(echo $l | sed 's/\\s//g')\\n"-----// d0
-----h=$(echo -ne "$s" | md5sum)-----// 27
-----echo "${h:0:2} $l"-----// 1b
---done-----// aa
}-----// a7
# setxkbmap -option caps:escape dvorak is-----// 18
# setxkbmap en_US-----// 97
```

ProTip™: setxkbmap dvorak on qwerty: o.yqtxmal ekrpat

1.1.2. .vimrc.

```
set nosp et sw=4 ts=4 sts=4 si cindent hi=1000 nu ru noeb showcmd showmode----// bb
syn on | colorscheme slate-----// e5
```

1.2. C++ Header. A C++ header.

```
#include <bits/stdc++.h>-----// 84
using namespace std;-----// 16
template <class T> int size(const T &x) { return x.size(); }-----// 5f
#define rep(i,a,b) for (__typeof(a) i=(a); i<(b); ++i)-----// 6c
#define iter(it,c) for (__typeof((c).begin()) it = (c).begin(); it != (c).end(); ++it)
typedef pair<int, int> ii;-----// 44
typedef vector<int> vi;-----// 9d
typedef vector<ii> vii;-----// eb
typedef long long ll;-----// 47
const int INF = ~(1<<31); // 2147483647-----// 10
-----// b2
const double EPS = 1e-9;-----// d5
const double pi = acos(-1);-----// 67
typedef unsigned long long ull;-----// ff
typedef vector<vi> vvi;-----// 4b
typedef vector<vii> vvii;-----// 36
template <class T> T mod(T a, T b) { return (a % b + b) % b; }-----// d5
```

1.3. Java Template. A Java template.

```
import java.util.*;-----// 37
import java.math.*;-----// 89
import java.io.*;-----// 28
-----// a3
public class Main {-----// 17
---public static void main(String[] args) throws Exception {-----// 02
-----Scanner in = new Scanner(System.in);-----// ef
-----PrintWriter out = new PrintWriter(System.out, false);-----// 62
```

```
-----// code-----// e6
-----out.flush();-----// 56
---}------// 79
}------// 00
```

2. DATA STRUCTURES

2.1. Union-Find.

```
struct union_find {------// 42
---vi p; union_find(int n) : p(n, -1) { }-----// 28
---int find(int x) { return p[x] < 0 ? x : p[x] = find(p[x]); }-----// ba
---bool unite(int x, int y) {------// 6c
-----int xp = find(x), yp = find(y);-----// 64
-----if (xp == yp) return false;-----// 0b
-----if (p[xp] > p[yp]) swap(xp,yp);-----// 78
-----p[xp] += p[yp], p[yp] = xp;-----// 88
-----return true; }-----// 1f
---int size(int x) { return -p[find(x)]; } }-----// b9
```

2.2. Segment Tree.

```
#ifdef SEG_MIN-----// 03
const int ID = INF;-----// 56
int f(int a, int b) { return min(a, b); }-----// 4f
#else-----// 0e
const int ID = 0;-----// 3e
int f(int a, int b) { return a + b; }-----// dd
#endif-----// 16
struct segment_tree {------// ab
---int n; vi data, lazy;-----// dd
---segment_tree() {}-----// 93
---segment_tree(const vi &arr) : n(size(arr)), data(4*n), lazy(4*n,INF) {-----// f1
---mk(arr, 0, n-1, 0); }-----// e9
---int mk(const vi &arr, int l, int r, int i) {-----// 12
---if (l == r) return data[i] = arr[l];-----// 5b
---int m = (l + r) / 2;-----// de
---return data[i] = f(mk(arr, l, m, 2*i+1), mk(arr, m+1, r, 2*i+2)); }-----// 0a
---int query(int a, int b) { return q(a, b, 0, n-1, 0); }-----// f6
---int q(int a, int b, int l, int r, int i) {-----// 22
---propagate(l, r, i);-----// 12
---if (r < a || b < l) return ID;-----// c7
---if (a <= l && r <= b) return data[i];-----// ce
---int m = (l + r) / 2;-----// 7a
---return f(q(a, b, l, m, 2*i+1), q(a, b, m+1, r, 2*i+2)); }-----// 5c
---void update(int i, int v) { u(i, v, 0, n-1, 0); }-----// 90
---int u(int i, int v, int l, int r, int j) {-----// 02
---propagate(l, r, j);-----// ae
---if (r < i || i < l) return data[j];-----// 92
---if (l == i && r == i) return data[j] = v;-----// 4a
---int m = (l + r) / 2;-----// cb
---return data[j] = f(u(i, v, l, m, 2*j+1), u(i, v, m+1, r, 2*j+2)); }-----// 34
---void range_update(int a, int b, int v) { ru(a, b, v, 0, n-1, 0); }-----// 71
---int ru(int a, int b, int v, int l, int r, int i) {-----// e0
```

```
-----propagate(l, r, i);-----// 19
-----if (l > r) return ID;-----// cc
-----if (r < a || b < l) return data[i];-----// d9
-----if (a <= l && r <= b) return (lazy[i] = v) * (r - l + 1) + data[i];-----// 06
-----int m = (l + r) / 2;-----// cc
-----return data[i] = f(ru(a, b, v, l, m, 2*i+1),-----// cc
-----ru(a, b, v, m+1, r, 2*i+2));-----// 2b
-----}-----// 0b
---void propagate(int l, int r, int i) {-----// a7
---if (l > r || lazy[i] == INF) return;-----// 5f
---data[i] += lazy[i] * (r - l + 1);-----// 44
---if (l < r) {-----// 28
---if (lazy[2*i+1] == INF) lazy[2*i+1] = lazy[i];-----// 4e
---else lazy[2*i+1] += lazy[i];-----// 1e
---if (lazy[2*i+2] == INF) lazy[2*i+2] = lazy[i];-----// de
---else lazy[2*i+2] += lazy[i];-----// 74
---}-----// 1f
---lazy[i] = INF;-----// f8
---}-----// 41
};-----// ae
```

2.2.1. Persistent Segment Tree.

```
int segcnt = 0;-----// cf
struct segment {-----// 68
---int l, r, lid, rid, sum;-----// fc
} segs[2000000];-----// dd
int build(int l, int r) {-----// 2b
---if (l > r) return -1;-----// 4e
---int id = segcnt++;-----// a8
---segs[id].l = l;-----// 90
---segs[id].r = r;-----// 19
---if (l == r) segs[id].lid = -1, segs[id].rid = -1;-----// ee
---else {-----// fe
---int m = (l + r) / 2;-----// 14
---segs[id].lid = build(l, m);-----// e3
---segs[id].rid = build(m + 1, r); }-----// 69
---segs[id].sum = 0;-----// 21
---return id; }-----// c5
int update(int idx, int v, int id) {-----// b8
---if (id == -1) return -1;-----// bb
---if (idx < segs[id].l || idx > segs[id].r) return id;-----// fb
---int nid = segcnt++;-----// b3
---segs[nid].l = segs[id].l;-----// 78
---segs[nid].r = segs[id].r;-----// ca
---segs[nid].lid = update(idx, v, segs[id].lid);-----// 92
---segs[nid].rid = update(idx, v, segs[id].rid);-----// 06
---segs[nid].sum = segs[id].sum + v;-----// 1a
---return nid; }-----// e6
int query(int id, int l, int r) {-----// a2
---if (r < segs[id].l || segs[id].r < l) return 0;-----// 17
```

```
---if (l <= segs[id].l && segs[id].r <= r) return segs[id].sum;-----// ad
---return query(segs[id].lid, l, r) + query(segs[id].rid, l, r); }-----// ee
```

2.3. Fenwick Tree.

```
struct fenwick_tree {-----// 98
---int n; vi data;-----// d3
---fenwick_tree(int _n) : n(_n), data(vi(n)) { }-----// db
---void update(int at, int by) {-----// 76
---while (at < n) data[at] += by, at |= at + 1; }-----// fb
---int query(int at) {-----// 71
---int res = 0;-----// c3
---while (at >= 0) res += data[at], at = (at & (at + 1)) - 1;-----// 37
---return res; }-----// e4
---int rsq(int a, int b) { return query(b) - query(a - 1); }-----// be
};-----// 57
struct fenwick_tree_sq {-----// d4
---int n; fenwick_tree x1, x0;-----// 18
---fenwick_tree_sq(int _n) : n(_n), x1(fenwick_tree(n)),-----// 2e
---x0(fenwick_tree(n)) { }-----// 7c
---// insert f(y) = my + c if x <= y-----// 17
---void update(int x, int m, int c) { x1.update(x, m); x0.update(x, c); }-----// 45
---int query(int x) { return x*x1.query(x) + x0.query(x); }-----// 73
};-----// 13
void range_update(fenwick_tree_sq &s, int a, int b, int k) {-----// 89
---s.update(a, k, k * (1 - a)); s.update(b+1, -k, k * b); }-----// 7f
int range_query(fenwick_tree_sq &s, int a, int b) {-----// 15
---return s.query(b) - s.query(a-1); }-----// f3
```

2.4. Matrix.

```
template <class K> bool eq(K a, K b) { return a == b; }-----// 2a
template <> bool eq<double>(double a, double b) { return abs(a - b) < EPS; }-----// a7
template <class T> struct matrix {-----// 0a
---int rows, cols, cnt; vector<T> data;-----// a1
---inline T& at(int i, int j) { return data[i * cols + j]; }-----// 5c
---matrix(int r, int c) : rows(r), cols(c), cnt(r * c) {-----// 56
---data.assign(cnt, T(0)); }-----// e3
---matrix(const matrix& other) : rows(other.rows), cols(other.cols),-----// b5
---cnt(other.cnt), data(other.data) { }-----// c1
---T& operator()(int i, int j) { return at(i, j); }-----// 29
---matrix<T> operator +(const matrix& other) {-----// 33
---matrix<T> res(*this); rep(i,0,cnt) res.data[i] += other.data[i];-----// f8
---return res; }-----// 09
---matrix<T> operator -(const matrix& other) {-----// 91
---matrix<T> res(*this); rep(i,0,cnt) res.data[i] -= other.data[i];-----// 7b
---return res; }-----// 9a
---matrix<T> operator *(T other) {-----// 99
---matrix<T> res(*this); rep(i,0,cnt) res.data[i] *= other;-----// 05
---return res; }-----// 8c
---matrix<T> operator *(const matrix& other) {-----// 31
---matrix<T> res(rows, other.cols);-----// 4c
---rep(i,0,rows) rep(j,0,other.cols) rep(k,0,cols)-----// ae
---res(i, j) += at(i, k) * other.data[k * other.cols + j];-----// 17
```

```
-----return res; }-----// 65
---matrix<T> pow(int p) {-----// 53
-----matrix<T> res(rows, cols), sq(*this);-----// 87
---rep(i,0,rows) res(i, i) = T(1);-----// 9d
-----while (p) {-----// 79
-----if (p & 1) res = res * sq;-----// 62
-----p >>= 1;-----// 79
-----if (p) sq = sq * sq;-----// 35
-----} return res; }-----// 22
---matrix<T> rref(T &det, int &rank) {-----// 2a
---matrix<T> mat(*this); det = T(1), rank = max(rows, cols);-----// 7a
---for (int r = 0, c = 0; c < cols; c++) {-----// 8e
---int k = r;-----// 5b
---while (k < rows && eq<T>(mat(k, c), T(0))) k++;-----// 3e
---if (k >= rows) { rank--; continue; }-----// 1a
---if (k != r) {-----// c4
---det *= T(-1);-----// 55
---rep(i,0,cols)-----// e1
---swap(mat.at(k, i), mat.at(r, i));-----// 7d
---} det *= mat(r, r);-----// b6
---T d = mat(r,c);-----// 66
---rep(i,0,cols) mat(r, i) /= d;-----// d1
---rep(i,0,rows) {-----// f6
---T m = mat(i, c);-----// 05
---if (i != r && !eq<T>(m, T(0)))-----// 1a
---rep(j,0,cols) mat(i, j) -= m * mat(r, j);-----// 7b
---} r++;-----// c5
---} return mat; }-----// b3
---matrix<T> transpose() {-----// 59
---matrix<T> res(cols, rows);-----// 5b
---rep(i,0,rows) rep(j,0,cols) res(j, i) = at(i, j);-----// 92
---return res; } }-----// df
```

2.5. Cartesian Tree.

```
struct node {-----// 36
---int x, y, sz;-----// e5
---node *l, *r;-----// 4d
---node(int _x, int _y) : x(_x), y(_y), sz(1), l(NULL), r(NULL) { } }-----// 19
int tsize(node* t) { return t ? t->sz : 0; }-----// 42
void augment(node *t) { t->sz = 1 + tsize(t->l) + tsize(t->r); }-----// 1d
pair<node*,node*> split(node *t, int x) {-----// 1d
---if (!t) return make_pair((node*)NULL,(node*)NULL);-----// fd
---if (t->x < x) {-----// 0a
---pair<node*,node*> res = split(t->r, x);-----// b4
---t->r = res.first; augment(t);-----// 4d
---return make_pair(t, res.second); }-----// e0
---pair<node*,node*> res = split(t->l, x);-----// b7
---t->l = res.second; augment(t);-----// 74
---return make_pair(res.first, t); }-----// 46
node* merge(node *l, node *r) {-----// 3c
---if (!l) return r; if (!r) return l;-----// f0
```

```
---if (l->y > r->y) { l->r = merge(l->r, r); augment(l); return l; }-----// be
---r->l = merge(l, r->l); augment(r); return r; }-----// c0
node* find(node *t, int x) {-----// b4
---while (t) {-----// 51
---if (x < t->x) t = t->l;-----// 32
---else if (t->x < x) t = t->r;-----// da
---else return t; }-----// 0b
---return NULL; }-----// ae
node* insert(node *t, int x, int y) {-----// 78
---if (find(t, x) != NULL) return t;-----// 2f
---pair<node*,node*> res = split(t, x);-----// ca
---return merge(res.first, merge(new node(x, y), res.second)); }-----// 0d
node* erase(node *t, int x) {-----// 4d
---if (!t) return NULL;-----// 7b
---if (t->x < x) t->r = erase(t->r, x);-----// 7c
---else if (x < t->x) t->l = erase(t->l, x);-----// 48
---else { node *old = t; t = merge(t->l, t->r); delete old; }-----// 22
---if (t) augment(t); return t; }-----// a3
int kth(node *t, int k) {-----// b3
---if (k < tsize(t->l)) return kth(t->l, k);-----// 64
---else if (k == tsize(t->l)) return t->x;-----// 97
---else return kth(t->r, k - tsize(t->l) - 1); }-----// f0
```

2.6. Misof Tree.

```
#define BITS 15-----// 7b
struct misof_tree {-----// fe
---int cnt[BITS][1<<BITS];-----// aa
---misof_tree() { memset(cnt, 0, sizeof(cnt)); }-----// b0
---void insert(int x) { for (int i = 0; i < BITS; cnt[i++][x]++, x >>= 1); }-----// 5a
---void erase(int x) { for (int i = 0; i < BITS; cnt[i++][x]--, x >>= 1); }-----// 49
---int nth(int n) {-----// 8a
---int res = 0;-----// a4
---for (int i = BITS-1; i >= 0; i--)-----// 99
---if (cnt[i][res <= 1] <= n) n -= cnt[i][res], res |= 1;-----// f4
---return res;-----// 3a
}-----// b5
};-----// 0a
```

2.7. Sqrt Decomposition.

```
struct segment {-----// b2
---vi arr;-----// 8c
---segment(vi _arr) : arr(_arr) { } }-----// 11
vector<segment> T;-----// a1
int K;-----// dc
void rebuild() {-----// 17
---int cnt = 0;-----// 14
---rep(i,0,size(T))-----// b1
---cnt += size(T[i].arr);-----// d1
---K = static_cast<int>(ceil(sqrt(cnt)) + 1e-9);-----// 4c
---vi arr(cnt);-----// 14
---for (int i = 0, at = 0; i < size(T); i++)-----// 79
---rep(j,0,size(T[i].arr))-----// a4
```

```
-----arr[at++] = T[i].arr[j];-----// f7
---T.clear();-----// 4c
---for (int i = 0; i < cnt; i += K)-----// 79
-----T.push_back(segment(vi(arr.begin()+i, arr.begin()+min(i+K, cnt))));-----// f0
}-----// 03
int split(int at) {-----// 71
---int i = 0;-----// 8a
---while (i < size(T) && at >= size(T[i].arr))-----// 6c
-----at -= size(T[i].arr), i++;-----// 9a
---if (i >= size(T)) return size(T);-----// 83
---if (at == 0) return i;-----// 49
---T.insert(T.begin() + i + 1, segment(vi(T[i].arr.begin() + at, T[i].arr.end())));
---T[i] = segment(vi(T[i].arr.begin(), T[i].arr.begin() + at));-----// af
---return i + 1;-----// ac
}-----// ea
void insert(int at, int v) {-----// 5f
---vi arr; arr.push_back(v);-----// 6a
---T.insert(T.begin() + split(at), segment(arr));-----// 67
}-----// cc
void erase(int at) {-----// be
---int i = split(at); split(at + 1);-----// da
---T.erase(T.begin() + i);-----// 6b
}-----// 4b
```

2.8. Monotonic Queue.

```
struct min_stack {-----// d8
---stack<int> S, M;-----// fe
---void push(int x) {-----// 20
---S.push(x);-----// e2
---M.push(M.empty() ? x : min(M.top(), x)); }-----// 92
---int top() { return S.top(); }-----// f1
---int mn() { return M.top(); }-----// 02
---void pop() { S.pop(); M.pop(); }-----// fd
---bool empty() { return S.empty(); }-----// d2
};-----// 74
struct min_queue {-----// b4
---min_stack inp, outp;-----// 3d
---void push(int x) { inp.push(x); }-----// 6b
---void fix() {-----// 5d
-----if (outp.empty()) while (!inp.empty())-----// 3b
-----outp.push(inp.top()), inp.pop();-----// 8e
---}-----// 3f
---int top() { fix(); return outp.top(); }-----// dc
---int mn() {-----// 39
-----if (inp.empty()) return outp.mn();-----// 01
-----if (outp.empty()) return inp.mn();-----// 90
-----return min(inp.mn(), outp.mn()); }-----// 97
---void pop() { fix(); outp.pop(); }-----// 4f
---bool empty() { return inp.empty() && outp.empty(); }-----// 65
};-----// 60
```

2.9. Convex Hull Trick.

```
struct convex_hull_trick {-----// 16
---vector<pair<double,double> > h;-----// b4
---double intersect(int i) {-----// 9b
-----return (h[i+1].second-h[i].second)/(h[i].first-h[i+1].first); }-----// b9
---void add(double m, double b) {-----// a4
---h.push_back(make_pair(m,b));-----// f9
---while (size(h) >= 3) {-----// f6
---int n = size(h);-----// d8
---if (intersect(n-3) < intersect(n-2)) break;-----// 07
---swap(h[n-2], h[n-1]);-----// bf
---h.pop_back(); } }-----// 4b
---double get_min(double x) {-----// b0
---int lo = 0, hi = size(h) - 2, res = -1;-----// 5b
---while (lo <= hi) {-----// 24
---int mid = lo + (hi - lo) / 2;-----// 5a
---if (intersect(mid) <= x) res = mid, lo = mid + 1;-----// 1d
---else hi = mid - 1; }-----// b6
---return h[res+1].first * x + h[res+1].second; } }-----// 84
```

3. GRAPHS

3.1. Single-Source Shortest Paths.

3.1.1. Dijkstra’s algorithm.

```
int *dist, *dad;-----// 46
struct cmp {-----// a5
---bool operator()(int a, int b) {-----// bb
---return dist[a] != dist[b] ? dist[a] < dist[b] : a < b; }-----// e6
};-----// 41
pair<int*, int*> dijkstra(int n, int s, vii *adj) {-----// 53
---dist = new int[n];-----// 84
---dad = new int[n];-----// 05
---rep(i,0,n) dist[i] = INF, dad[i] = -1;-----// 80
---set<int, cmp> pq;-----// 98
---dist[s] = 0, pq.insert(s);-----// 1f
---while (!pq.empty()) {-----// 47
---int cur = *pq.begin(); pq.erase(pq.begin());-----// 58
---rep(i,0,size(adj[cur])) {-----// a6
---int nxt = adj[cur][i].first,-----// a4
---ndist = dist[cur] + adj[cur][i].second;-----// 3a
---if (ndist < dist[nxt]) pq.erase(nxt),-----// 2d
---dist[nxt] = ndist, dad[nxt] = cur, pq.insert(nxt);-----// eb
---}-----// d2
---}-----// df
---return pair<int*, int*>(dist, dad);-----// e3
}-----// 9b
```

3.1.2. Bellman-Ford algorithm.

```
int* bellman_ford(int n, int s, vii* adj, bool& has_negative_cycle) {-----// cf
---has_negative_cycle = false;-----// 47
---int* dist = new int[n];-----// 7f
---rep(i,0,n) dist[i] = i == s ? 0 : INF;-----// df
---rep(i,0,n-1) rep(j,0,n) if (dist[j] != INF)-----// 4d
```

```

vi order;-----// 9b
-----// a5
void scc_dfs(const vvi &adj, int u) {-----// a1
----int v; visited[u] = true;-----// e3
----rep(i,0,size(adj[u]))-----// 2d
-----if (!visited[v = adj[u][i]]) scc_dfs(adj, v);-----// a2
----order.push_back(u);-----// 02
}-----// 53

```

```
pair<union find, vi> scc(const vi &adj) {-----// c2
```

```

--- int n = size(adj), u, v;-----// f8
--- order.clear();-----// 20
--- union_find uf(n);-----// a8
--- vi dag;-----// 61
--- vvi rev(n);-----// c5
--- rep(i,0,n) rep(j,0,size(adj[i])) rev[adj[i][j]].push_back(i);-----// 7e
--- visited.resize(n), fill(visited.begin(), visited.end(), false);-----// 80
--- rep(i,0,n) if (!visited[i]) scc_dfs(rev, i);-----// 4e
--- fill(visited.begin(), visited.end(), false);-----// 59
--- stack<int> S;-----// bb
--- for (int i = n-1; i >= 0; i--) {-----// 96
---     if (visited[order[i]]) continue;-----// db
---     S.push(order[i]), dag.push_back(order[i]);-----// 68
---     while (!S.empty()) {-----// 9e
---         visited[u = S.top()] = true, S.pop(), uf.unite(u, order[i]);-----// b3
---         rep(j,0,size(adj[u])) if (!visited[v = adj[u][j]]) S.push(v);-----// 1b
---     }-----// 61
--- }-----// 57
--- return pair<union_find, vi>(uf, dag);-----// 2b
}-----// 92

```

3.3. Cut Points and Bridges.

```
#define MAXN 5000-----// f7
int low[MAXN], num[MAXN], curnum;-----// d7
void dfs(const vvi &adj, vi &cp, vii &bri, int u, int p) {-// 22
    low[u] = num[u] = curnum++;-----// a3
    int cnt = 0; bool found = false;-----// 97
    rep(i, 0, size(adj[u])) {-// ae
        int v = adj[u][i];-----// 56
        if (num[v] == -1) {-// 3b
            dfs(adj, cp, bri, v, u);-----// ba
            low[u] = min(low[u], low[v]);-----// be
            cnt++;-----// e0
            found = found || low[v] >= num[u];-----// 30
            if (low[v] > num[u]) bri.push_back(ii(u, v));-----// bf
        } else if (p != v) low[u] = min(low[u], num[v]); }-----// 76
    if (found && (p != -1 || cnt > 1)) cp.push_back(u); }-----// 3e
pair<vi, vii> cut_points_and_bridges(const vvi &adj) {-// 76
    int n = size(adj);-----// c8
    vi cp; vii bri;-----// fb
    memset(num, -1, n << 2);-----// 45
```

```
#include "../data-structures/union_find.cpp"-----// 5e
-----// 11
vector<bool> visited;-----// 66
```



```

--- curnum = 0;-----// 07
--- rep(i,0,n) if (num[i] == -1) dfs(adj, cp, bri, i, -1);-----// 7e
--- return make_pair(cp, bri); }-----// 4c

```

3.4. Euler Path.

```

#define MAXV 1000-----// 2f
#define MAXE 5000-----// 87
vi adj[MAXV];-----// ff
int n, m, indeg[MAXV], outdeg[MAXV], res[MAXE + 1];-----// 49
ii start_end() {-----// 30
--- int start = -1, end = -1, any = 0, c = 0;-----// 74
--- rep(i,0,n) {-----// 20
----- if (outdeg[i] > 0) any = i;-----// 63
----- if (indeg[i] + 1 == outdeg[i]) start = i, c++;-----// 5a
----- else if (indeg[i] == outdeg[i] + 1) end = i, c++;-----// 13
----- else if (indeg[i] != outdeg[i]) return ii(-1, -1);-----// c1
--- }-----// ed
--- if ((start == -1) != (end == -1) || (c != 2 && c != 0)) return ii(-1, -1);-----// 54
--- if (start == -1) start = end = any;-----// 5e
--- return ii(start, end);-----// a2
}-----// eb
bool euler_path() {-----// b4
--- ii se = start_end();-----// 8a
--- int cur = se.first, at = m + 1;-----// b6
--- if (cur == -1) return false;-----// ac
--- stack<int> s;-----// 1c
--- while (true) {-----// b3
----- if (outdeg[cur] == 0) {-----// 0d
----- res[at] = cur;-----// bd
----- if (s.empty()) break;-----// c6
----- cur = s.top(); s.pop();-----// 06
----- } else s.push(cur), cur = adj[cur][--outdeg[cur]];-----// 9e
--- }-----// a4
--- return at == 0;-----// ac
}-----// 22

```

3.5. Bipartite Matching.

3.5.1. Alternating Paths algorithm.

```

vi* adj;-----// cc
bool* done;-----// b1
int* owner;-----// 26
int alternating_path(int left) {-----// da
--- if (done[left]) return 0;-----// 08
--- done[left] = true;-----// f2
--- rep(i,0,size(adj[left])) {-----// 1b
----- int right = adj[left][i];-----// 46
----- if (owner[right] == -1 || alternating_path(owner[right])) {-----// f6
----- owner[right] = left; return 1;-----// f2
----- } }-----// 88
--- return 0; }-----// 41

```

3.5.2. Hopcroft-Karp algorithm. Running time is $O(|E|\sqrt{|V|})$.

```

#define MAXN 5000-----// f7
int dist[MAXN+1], q[MAXN+1];-----// b8
#define dist(v) dist[v == -1 ? MAXN : v]-----// 0f
struct bipartite_graph {-----// 2b
--- int N, M, *L, *R; vi *adj;-----// fc
--- bipartite_graph(int _N, int _M) : N(_N), M(_M),-----// 8d
--- L(new int[N]), R(new int[M]), adj(new vi[N]) {}-----// cd
--- bipartite_graph() { delete[] adj; delete[] L; delete[] R; }-----// 89
--- bool bfs() {-----// f5
----- int l = 0, r = 0;-----// 37
----- rep(v,0,N) if (L[v] == -1) dist(v) = 0, q[r++] = v;-----// f9
----- else dist(v) = INF;-----// aa
----- dist(-1) = INF;-----// f2
----- while (l < r) {-----// ba
----- int v = q[l++];-----// 50
----- if (dist(v) < dist(-1)) {-----// f1
----- iter(u, adj[v]) if (dist(R[*u]) == INF)-----// 9b
----- dist(R[*u]) = dist(v) + 1, q[r++] = R[*u];-----// 79
----- }-----// b8
----- }-----// 0d
--- return dist(-1) != INF;-----// 43
}-----// 2c
--- bool dfs(int v) {-----// 26
--- if (v != -1) {-----// d8
--- iter(u, adj[v])-----// 99
--- if (dist(R[*u]) == dist(v) + 1)-----// 74
--- if (dfs(R[*u])) {-----// 40
--- R[*u] = v, L[v] = *u;-----// 47
--- return true;-----// a2
--- }-----// 17
--- dist(v) = INF;-----// 62
--- return false;-----// 3c
--- }-----// 3d
--- return true;-----// ae
}-----// 0f
--- void add_edge(int i, int j) { adj[i].push_back(j); }-----// 92
--- int maximum_matching() {-----// a2
--- int matching = 0;-----// 71
--- memset(L, -1, sizeof(int) * N);-----// 72
--- memset(R, -1, sizeof(int) * M);-----// bf
--- while (bfs()) rep(i,0,N)-----// 3e
--- matching += L[i] == -1 && dfs(i);-----// 1d
--- return matching;-----// ec
}-----// 8b
};-----// b7

```

3.6. Maximum Flow.

3.6.1. Dinic's algorithm. An implementation of Dinic's algorithm that runs in $O(|V|^2|E|)$.

```

#define MAXV 2000-----// ba
int q[MAXV], d[MAXV];-----// e6

```

```
struct flow_network {-----// 12
--struct edge {-----// 1e
-----int v, cap, nxt;-----// ab
-----edge() { }-----// 38
-----edge(int _v, int _cap, int _nxt) : v(_v), cap(_cap), nxt(_nxt) { }-----// bc
--};-----// 6e
--int n, ecnt, *head, *curh;-----// 46
--vector<edge> e, e_store;-----// 1f
--flow_network(int _n, int m = -1) : n(_n), ecnt(0) {-----// d3
-----e.reserve(2 * (m == -1 ? n : m));-----// 24
-----head = new int[n], curh = new int[n];-----// 6b
-----memset(head, -1, n * sizeof(int));-----// 56
--}-----// 77
--void destroy() { delete[] head; delete[] curh; }-----// f6
--void reset() { e = e_store; }-----// 87
--void add_edge(int u, int v, int uv, int vu = 0) {-----// cd
-----e.push_back(edge(v, uv, head[u])); head[u] = ecnt++;-----// c9
-----e.push_back(edge(u, vu, head[v])); head[v] = ecnt++;-----// 89
--}-----// 14
--int augment(int v, int t, int f) {-----// 3f
-----if (v == t) return f;-----// 6d
-----for (int &i = curh[v], ret; i != -1; i = e[i].nxt)-----// f9
-----if (e[i].cap > 0 && d[e[i].v] + 1 == d[v])-----// cc
-----if ((ret = augment(e[i].v, t, min(f, e[i].cap))) > 0)-----// 1f
-----return (e[i].cap -= ret, e[i^1].cap += ret, ret);-----// ac
-----return 0;-----// 19
--}-----// fd
--int max_flow(int s, int t, bool res = true) {-----// 31
-----if(s == t) return 0;-----// 9d
-----e_store = e;-----// 57
-----int f = 0, x, l, r;-----// 0e
-----while (true) {-----// b5
-----memset(d, -1, n * sizeof(int));-----// a8
-----l = r = 0, d[q[r++] = t] = 0;-----// 0e
-----while (l < r)-----// 7a
-----for (int v = q[l++], i = head[v]; i != -1; i = e[i].nxt)-----// a2
-----if (e[i^1].cap > 0 && d[e[i].v] == -1)-----// 29
-----d[q[r++] = e[i].v] = d[v]+1;-----// 28
-----if (d[s] == -1) break;-----// a0
-----memcpy(curh, head, n * sizeof(int));-----// 10
-----while ((x = augment(s, t, INF)) != 0) f += x;-----// a6
-----}-----// 96
-----if (res) reset();-----// 21
-----return f;-----// b6
--}-----// 1b
};-----// 3b

struct cmp {-----// d1
--bool operator()(int i, int j) {-----// 8a
-----return d[i] == d[j] ? i < j : d[i] < d[j];-----// 89
--}-----// df
};-----// cf
struct flow_network {-----// eb
--struct edge {-----// 9a
-----int v, cap, cost, nxt;-----// ad
-----edge(int _v, int _cap, int _cost, int _nxt)-----// ec
-----: v(_v), cap(_cap), cost(_cost), nxt(_nxt) { }-----// c4
--};-----// ad
--int n, ecnt, *head;-----// 46
--vector<edge> e, e_store;-----// 4b
--flow_network(int _n, int m = -1) : n(_n), ecnt(0) {-----// dd
-----e.reserve(2 * (m == -1 ? n : m));-----// e6
-----memset(head = new int[n], -1, n << 2);-----// 6c
--}-----// f3
--void destroy() { delete[] head; }-----// ac
--void reset() { e = e_store; }-----// 88
--void add_edge(int u, int v, int cost, int uv, int vu=0) {-----// b4
-----e.push_back(edge(v, uv, cost, head[u])); head[u] = ecnt++;-----// 43
-----e.push_back(edge(u, vu, -cost, head[v])); head[v] = ecnt++;-----// 53
--}-----// 16
--ii min_cost_max_flow(int s, int t, bool res = true) {-----// 6d
-----if (s == t) return ii(0, 0);-----// 34
-----e_store = e;-----// 70
-----memset(pot, 0, n << 2);-----// 24
-----int f = 0, c = 0, v;-----// d4
-----while (true) {-----// 29
-----memset(d, -1, n << 2);-----// fd
-----memset(p, -1, n << 2);-----// b7
-----set<int, cmp> q;-----// d8
-----q.insert(s); d[s] = 0;-----// 1d
-----while (!q.empty()) {-----// 04
-----int u = *q.begin();-----// dd
-----q.erase(q.begin());-----// 20
-----for (int i = head[u]; i != -1; i = e[i].nxt) {-----// 02
-----if (e[i].cap == 0) continue;-----// 1c
-----int cd = d[u] + e[i].cost + pot[u] - pot[v = e[i].v];-----// 1d
-----if (d[v] == -1 || cd < d[v]) {-----// d2
-----if (q.find(v) != q.end()) q.erase(q.find(v));-----// e2
-----d[v] = cd; p[v] = i;-----// f7
-----q.insert(v);-----// 74
-----}-----// 6c
-----}-----// 1b
-----}-----// da
-----if (p[t] == -1) break;-----// 09
-----int x = INF, at = p[t];-----// e8
-----while (at != -1) x = min(x, e[at].cap), at = p[e[at^1].v];-----// 32
-----at = p[t], f += x;-----// 43
-----while (at != -1)-----// 53
```

3.7. Minimum Cost Maximum Flow. Running time is $O(|V|^2|E|\log|V|)$. NOTE: Doesn't work on negative weights!

```
#define MAXV 2000-----// ba
int d[MAXV], p[MAXV], pot[MAXV];-----// 80
```



```
-----e[at].cap -= x, e[at^1].cap += x, at = p[e[at^1].v];-----// 95
-----c += x * (d[t] + pot[t] - pot[s]);-----// 44
-----rep(i,0,n) if (p[i] != -1) pot[i] += d[i];-----// 86
-----}-----// 4e
-----if (res) reset();-----// d7
-----return ii(f, c);-----// 9f
-----}-----// 4c
};-----// ec

A second implementation that is slower but works on negative weights.

struct flow_network {-----// 81
--struct mcmf_edge {-----// f6
--int u, v;-----// e1
--ll w, c;-----// b4
--mcmf_edge* rev;-----// 9d
--mcmf_edge(int _u, int _v, ll _w, ll _c, mcmf_edge* _rev = NULL) {-----// ea
--u = _u; v = _v; w = _w; c = _c; rev = _rev;-----// 83
--}-----// 02
--};-----// b9
--int n;-----// b4
--vector<pair<int, pair<ll, ll> > >* adj;-----// 72
--flow_network(int _n) {-----// 55
--n = _n;-----// fa
--adj = new vector<pair<int, pair<ll, ll> > >[n];-----// bb
--}-----// bd
--void add_edge(int u, int v, ll cost, ll cap) {-----// 79
--adj[u].push_back(make_pair(v, make_pair(cap, cost)));-----// c8
--}-----// ed
--pair<ll,ll> min_cost_max_flow(int s, int t) {-----// ea
--vector<mcmf_edge*>* g = new vector<mcmf_edge*>[n];-----// ce
--for (int i = 0; i < n; i++) {-----// 57
--for (int j = 0; j < size(adj[i]); j++) {-----// 37
--mcmf_edge *cur = new mcmf_edge(i, adj[i][j].first,-----// 21
--adj[i][j].second.first, adj[i][j].second.second),-----// 56
--*rev = new mcmf_edge(adj[i][j].first, i, 0,-----// 48
--adj[i][j].second.second, cur);-----// b1
--cur->rev = rev;-----// ef
--g[i].push_back(cur);-----// 1d
--g[adj[i][j].first].push_back(rev);-----// 05
--}-----// ba
--}-----// 83
--ll flow = 0, cost = 0;-----// 68
--mcmf_edge** back = new mcmf_edge*[n];-----// e5
--ll* dist = new ll[n];-----// 50
--while (true) {-----// 65
--for (int i = 0; i < n; i++) back[i] = NULL, dist[i] = INF;-----// d0
--dist[s] = 0;-----// 5e
--for (int i = 0; i < n - 1; i++)-----// be
--for (int j = 0; j < n; j++)-----// 6e
--if (dist[j] != INF)-----// e3
--for (int k = 0; k < size(g[j]); k++)-----// 85
--if (g[j][k]->w > 0 && dist[j] + g[j][k]->c <-----// 7f
--dist[g[j][k]->v)) {-----// 6d
--dist[g[j][k]->v] = dist[j] + g[j][k]->c;-----// cf
--back[g[j][k]->v] = g[j][k];-----// 3d
--}-----// f3
--mcmf_edge* cure = back[t];-----// b4
--if (cure == NULL) break;-----// ab
--ll cap = INF;-----// 7a
--while (true) {-----// ad
--cap = min(cap, cure->w);-----// c3
--if (cure->u == s) break;-----// 82
--cure = back[cure->u];-----// 45
--}-----// 91
--assert(cap > 0 && cap < INF);-----// ae
--cure = back[t];-----// b9
--while (true) {-----// 2a
--cost += cap * cure->c;-----// f8
--cure->w -= cap;-----// d1
--cure->rev->w += cap;-----// cf
--if (cure->u == s) break;-----// 8c
--cure = back[cure->u];-----// 60
--}-----// 09
--flow += cap;-----// f2
--}-----// be
--// instead of deleting g, we could also-----// e0
--// use it to get info about the actual flow-----// 6c
--for (int i = 0; i < n; i++)-----// eb
--for (int j = 0; j < size(g[i]); j++)-----// 82
--delete g[i][j];-----// 06
--delete[] g;-----// 23
--delete[] back;-----// 5a
--delete[] dist;-----// b9
--return make_pair(flow, cost);-----// ec
--}-----// ad
};-----// bf

3.8. All Pairs Maximum Flow.

3.8.1. Gomory-Hu Tree. An implementation of the Gomory-Hu Tree. The spanning tree is constructed
using Gusfield's algorithm in  $O(|V|^2)$  plus  $|V| - 1$  times the time it takes to calculate the maximum
flow. If Dinic's algorithm is used to calculate the max flow, the running time is  $O(|V|^3|E|)$ . NOTE:
Not sure if it works correctly with disconnected graphs.

#include "dinic.cpp"-----// 58
-----// 25
bool same[MAXV];-----// 59
pair<vii, vvi> construct_gh_tree(flow_network &g) {-----// 77
--int n = g.n, v;-----// 5d
--vii par(n, ii(0, 0)); vvi cap(n, vi(n, -1));-----// 49
--rep(s,1,n) {-----// 9e
--int l = 0, r = 0;-----// 08
--par[s].second = g.max_flow(s, par[s].first, false);-----// 54
--memset(d, 0, n * sizeof(int));-----// c8
--memset(same, 0, n * sizeof(bool));-----// c9
```

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```
-----d[q[r++] = s] = 1;-----// dd
-----while (l < r) {-----// 45
-----same[v = q[l++]] = true;-----// c5
-----for (int i = g.head[v]; i != -1; i = g.e[i].nxt)-----// 66
-----if (g.e[i].cap > 0 && d[g.e[i].v] == 0)-----// 21
-----d[q[r++] = g.e[i].v] = 1;-----// dd
-----}-----// 44
-----rep(i,s+1,n)-----// 71
-----if (par[i].first == par[s].first && same[i]) par[i].first = s;-----// 97
-----g.reset();-----// d8
-----}-----// 93
rep(i,0,n) {-----// 14
    int mn = INF, cur = i;-----// 59
    while (true) {-----// b8
        cap[cur][i] = mn;-----// 8d
        if (cur == 0) break;-----// fb
        mn = min(mn, par[cur].second), cur = par[cur].first;-----// 4d
    }-----// aa
}-----// 90
return make_pair(par, cap);-----// 62
}-----// b3
int compute_max_flow(int s, int t, const pair<vii, vvi> &gh) {-----// 93
    if (s == t) return 0;-----// 33
    int cur = INF, at = s;-----// e7
    while (gh.second[at][t] == -1)-----// 42
        cur = min(cur, gh.first[at].second), at = gh.first[at].first;-----// 8d
    return min(cur, gh.second[at][t]);-----// 54
}-----// 46
```

3.9. Heavy-Light Decomposition.

```
#include "../data-structures/segment_tree.cpp"-----// 16
struct HLD {-----// 25
    int n, curhead, curloc;-----// d9
    vi sz, head, parent, loc;-----// 81
    vvi adj; segment_tree values;-----// 13
    HLD(int _n) : n(_n), sz(n, 1), head(n), parent(n, -1), loc(n), adj(n) {-----// 1c
        vi tmp(n, ID); values = segment_tree(tmp); }-----// f0
    void add_edge(int u, int v) { adj[u].push_back(v), adj[v].push_back(u); }-----// 77
    void update_cost(int u, int v, int c) {-----// 7b
        if (parent[v] == u) swap(u, v); assert(parent[u] == v);-----// db
        values.update(loc[u], c); }-----// 50
    int csz(int u) {-----// 7c
        rep(i,0,size(adj[u])) if (adj[u][i] != parent[u])-----// a5
            sz[u] += csz(adj[parent[adj[u][i]] = u][i]);-----// c2
        return sz[u]; }-----// 75
    void part(int u) {-----// c3
        head[u] = curhead; loc[u] = curloc++;-----// 63
        int best = -1;-----// 27
        rep(i,0,size(adj[u]))-----// 49
            if (adj[u][i] != parent[u] && (best == -1 || sz[adj[u][i]] > sz[best]))-----
                best = adj[u][i];-----// 26
```

```
-----if (best != -1) part(best);-----// c4
        rep(i,0,size(adj[u]))-----// 92
            if (adj[u][i] != parent[u] && adj[u][i] != best)-----// e8
                part(curhead = adj[u][i]); }-----// 88
    void build(int r = 0) { curloc = 0, csz(curhead = r), part(r); }-----// 78
    int lca(int u, int v) {-----// 74
        vi uat, vat; int res = -1;-----// 43
        while (u != -1) uat.push_back(u), u = parent[head[u]];-----// 51
        while (v != -1) vat.push_back(v), v = parent[head[v]];-----// 6d
        u = size(uat) - 1, v = size(vat) - 1;-----// 8a
        while (u >= 0 && v >= 0 && head[uat[u]] == head[vat[v]])-----// ae
            res = (loc[uat[u]] < loc[vat[v]] ? uat[u] : vat[v]), u--, v--;-----// a2
        return res; }-----// 91
    int query_upto(int u, int v) { int res = ID;-----// 72
        while (head[u] != head[v])-----// 69
            res = f(res, values.query(loc[head[u]], loc[u])),-----// a4
            u = parent[head[u]];-----// 8c
        return f(res, values.query(loc[v] + 1, loc[u])); }-----// ea
    int query(int u, int v) { int l = lca(u, v);-----// 53
        return f(query_upto(u, l), query_upto(v, l)); } }-----// 5b
```

3.10. Centroid Decomposition.

```
#define MAXV 100100-----// 86
#define LGMAXV 20-----// aa
int jmp[MAXV][LGMAXV],-----// 6d
    path[MAXV][LGMAXV],-----// 9d
    sz[MAXV], seph[MAXV],-----// cf
    shortest[MAXV];-----// 6b
struct centroid_decomposition {-----// 99
    int n; vvi adj;-----// e9
    centroid_decomposition(int _n) : n(_n), adj(n) { }-----// 46
    void add_edge(int a, int b) { adj[a].push_back(b), adj[b].push_back(a); }-----// bc
    int dfs(int u, int p) {-----// 8f
        sz[u] = 1;-----// c8
        rep(i,0,size(adj[u])) if (adj[u][i] != p) sz[u] += dfs(adj[u][i], u);-----// 78
        return sz[u]; }-----// f4
    void makepaths(int sep, int u, int p, int len) {-----// 84
        jmp[u][seph[sep]] = sep, path[u][seph[sep]] = len;-----// d9
        int bad = -1;-----// af
        rep(i,0,size(adj[u])) {-----// f4
            if (adj[u][i] == p) bad = i;-----// cf
            else makepaths(sep, adj[u][i], u, len + 1);-----// f2
        }-----// 8a
        if (p == sep) swap(adj[u][bad], adj[u].back()), adj[u].pop_back(); }-----// 07
    void separate(int h=0, int u=0) {-----// 03
        dfs(u, -1); int sep = u;-----// b5
        down: iter(nxt,adj[sep])-----// 04
            if (sz[*nxt] < sz[sep] && sz[*nxt] > sz[u]/2) {-----// db
                sep = *nxt; goto down; }-----// 1a
        seph[sep] = h, makepaths(sep, sep, -1, 0);-----// ed
        rep(i,0,size(adj[sep])) separate(h+1, adj[sep][i]); }-----// 90
```

```
void paint(int u) {-----// bd
    rep(h,0,seph[u]+1)-----// c5
    shortest[jmp[u][h]] = min(shortest[jmp[u][h]], path[u][h]); }-----// 11
int closest(int u) {-----// 91
    int mn = INF/2;-----// fe
    rep(h,0,seph[u]+1) mn = min(mn, path[u][h] + shortest[jmp[u][h]]);-----// 3e
    return mn; } }-----// 13
```

3.11. Least Common Ancestors, Binary Jumping.

```
struct node {-----// 36
    node *p, *jmp[20];-----// 24
    int depth;-----// 10
    node(node *_p = NULL) : p(_p) {-----// 78
        depth = p ? 1 + p->depth : 0;-----// 3b
        memset(jmp, 0, sizeof(jmp));-----// 64
        jmp[0] = p;-----// 64
        for (int i = 1; (1<<i) <= depth; i++)-----// a8
            jmp[i] = jmp[i-1]->jmp[i-1]; } }-----// 3b
node* st[100000];-----// 65
node* lca(node *a, node *b) {-----// 29
    if (!a || !b) return NULL;-----// cd
    if (a->depth < b->depth) swap(a,b);-----// fe
    for (int j = 19; j >= 0; j--)-----// b3
        while (a->depth - (1<<j) >= b->depth) a = a->jmp[j];-----// c0
    if (a == b) return a;-----// 08
    for (int j = 19; j >= 0; j--)-----// 11
        while (a->depth >= (1<<j) && a->jmp[j] != b->jmp[j])-----// f0
            a = a->jmp[j], b = b->jmp[j];-----// d0
    return a->p; }-----// c5
```

3.12. Tarjan’s Off-line Lowest Common Ancestors Algorithm.

```
#include "../data-structures/union_find.cpp"-----// 5e
struct tarjan_olca {-----// 87
    int *ancestor;-----// 39
    vi *adj, answers;-----// dd
    vii *queries;-----// 66
    bool *colored;-----// 97
    union_find uf;-----// 70
    tarjan_olca(int n, vi *_adj) : adj(_adj), uf(n) {-----// 78
        colored = new bool[n];-----// 8d
        ancestor = new int[n];-----// f2
        queries = new vii[n];-----// 3e
        memset(colored, 0, n);-----// 6e
    }-----// 6b
    void query(int x, int y) {-----// d3
        queries[x].push_back(ii(y, size(answers)));-----// a0
        queries[y].push_back(ii(x, size(answers)));-----// 14
        answers.push_back(-1);-----// ca
    }-----// 6b
    void process(int u) {-----// 85
        ancestor[u] = u;-----// 1a
        rep(i,0,size(adj[u])) {-----// ce
```

```
int v = adj[u][i];-----// dd
        process(v);-----// e8
        uf.unite(u,v);-----// 55
        ancestor[uf.find(u)] = u;-----// 1d
    }-----// 57
    colored[u] = true;-----// b9
    rep(i,0,size(queries[u])) {-----// d7
        int v = queries[u][i].first;-----// 89
        if (colored[v]) {-----// cb
            answers[queries[u][i].second] = ancestor[uf.find(v)];-----// 63
        }-----// d0
    }-----// 40
}-----// a9
};-----// 1e
```

3.13. Maximum Density Subgraph. Given (weighted) undirected graph G . Binary search density. If g is current density, construct flow network: (S, u, m) , $(u, T, m + 2g - d_u)$, $(u, v, 1)$, where m is a large constant (larger than sum of edge weights). Run floating-point max-flow. If minimum cut has empty S -component, then maximum density is smaller than g , otherwise it’s larger. Distance between valid densities is at least $1/(n(n - 1))$. Edge case when density is 0. This also works for weighted graphs by replacing d_u be the weighted degree, and doing more iterations (if weights are not integers).

3.14. Maximum Weighted Independent Set in a Bipartite Graph. This is the same as the minimum weighted vertex cover. Solve this by constructing a flow network with edges $(S, u, w(u))$ for $u \in L$, $(v, T, w(v))$ for $v \in R$ and (u, v, ∞) for $(u, v) \in E$. The minimum S, T -cut is the answer. Vertices adjacent to a cut edge are in the vertex cover.

4. STRINGS

4.1. The Knuth-Morris-Pratt algorithm.

```
int* compute_pi(const string &t) {-----// a2
    int m = t.size();-----// 8b
    int *pit = new int[m + 1];-----// 8e
    if (0 <= m) pit[0] = 0;-----// 42
    if (1 <= m) pit[1] = 0;-----// 34
    rep(i,2,m+1) {-----// 0f
        for (int j = pit[i - 1]; ; j = pit[j]) {-----// b5
            if (t[j] == t[i - 1]) { pit[i] = j + 1; break; }-----// 21
            if (j == 0) { pit[i] = 0; break; }-----// 95
        }-----// c9
    }-----// eb
    return pit; }-----// e8
int string_match(const string &s, const string &t) {-----// 9e
    int n = s.size(), m = t.size();-----// 92
    int *pit = compute_pi(t);-----// 72
    for (int i = 0, j = 0; i < n; ) {-----// 27
        if (s[i] == t[j]) {-----// 73
            i++; j++;-----// 7e
            if (j == m) {-----// de
                return i - m;-----// e9
            }-----// ce
            // or j = pit[j];-----// 85
        }-----// 35
```

```
-----else if (j > 0) j = pit[j];-----// 43
-----else i++; }-----// b8
----delete[] pit; return -1; }-----// e3
```

4.2. The Z algorithm.

```
int* z_values(const string &s) {-----// 4d
----int n = size(s);-----// 97
----int* z = new int[n];-----// c4
----int l = 0, r = 0;-----// 1c
----z[0] = n;-----// 98
----rep(i,1,n) {-----// b2
-----z[i] = 0;-----// 4c
-----if (i > r) {-----// 6d
-----l = r = i;-----// 24
-----while (r < n && s[r - l] == s[r]) r++;-----// 68
-----z[i] = r - l; r--;-----// 07
-----} else if (z[i - l] < r - i + 1) z[i] = z[i - l];-----// 6f
-----else {-----// a8
-----l = i;-----// 55
-----while (r < n && s[r - l] == s[r]) r++;-----// 2c
-----z[i] = r - l; r--; } }-----// 13
----return z;-----// 78
}-----// 16
```

4.3. Suffix Array. An $O(n \log^2 n)$ construction of a Suffix Tree.

```
struct entry { ii nr; int p; };-----// f9
bool operator <(const entry &a, const entry &b) { return a.nr < b.nr; }-----// 77
struct suffix_array {-----// 87
----string s; int n; vvi P; vector<entry> L; vi idx;-----// b6
----suffix_array(string _s) : s(_s), n(size(s)) {-----// a3
-----L = vector<entry>(n), P.push_back(vi(n)), idx = vi(n);-----// 12
-----rep(i,0,n) P[0][i] = s[i];-----// 5c
-----for (int stp = 1, cnt = 1; cnt >> 1 < n; stp++, cnt <= 1) {-----// 86
-----P.push_back(vi(n));-----// 53
-----rep(i,0,n)-----// 6f
-----L[L[i].p = i].nr = ii(P[stp - 1][i],-----// e2
-----i + cnt < n ? P[stp - 1][i + cnt] : -1);-----// 43
-----sort(L.begin(), L.end());-----// 5f
-----rep(i,0,n)-----// a8
-----P[stp][L[i].p] = i > 0 &&-----// 3a
-----L[i].nr == L[i - 1].nr ? P[stp][L[i - 1].p] : i;-----// 55
-----}-----// 8b
-----rep(i,0,n) idx[P[size(P) - 1][i]] = i;-----// 17
-----}-----// d9
----int lcp(int x, int y) {-----// 71
----int res = 0;-----// d6
----if (x == y) return n - x;-----// bc
----for (int k = size(P) - 1; k >= 0 && x < n && y < n; k--)-----// fe
----if (P[k][x] == P[k][y]) x += 1 << k, y += 1 << k, res += 1 << k;-----// b7
----return res;-----// bc
----}-----// f1
};-----// f6
```

4.4. Aho-Corasick Algorithm.

```
struct aho_corasick {-----// 78
----struct out_node {-----// 3e
-----string keyword; out_node *next;-----// f0
-----out_node(string k, out_node *n) : keyword(k), next(n) { }-----// 26
----};-----// b9
----struct go_node {-----// 40
----map<char, go_node*> next;-----// 6b
----out_node *out; go_node *fail;-----// 3e
----go_node() { out = NULL; fail = NULL; }-----// 0f
----};-----// c0
----go_node *go;-----// b8
----aho_corasick(vector<string> keywords) {-----// 4b
----go = new go_node();-----// 77
----iter(k, keywords) {-----// f2
----go_node *cur = go;-----// a2
----iter(c, *k)-----// 6e
----cur = cur->next.find(*c) != cur->next.end() ? cur->next[*c] :-----// 97
----(cur->next[*c] = new go_node());-----// af
----cur->out = new out_node(*k, cur->out);-----// 3f
----}-----// eb
----queue<go_node*> q;-----// 2c
----iter(a, go->next) q.push(a->second);-----// db
----while (!q.empty()) {-----// 07
----go_node *r = q.front(); q.pop();-----// e0
----iter(a, r->next) {-----// 18
----go_node *s = a->second;-----// 55
----q.push(s);-----// b5
----go_node *st = r->fail;-----// 53
----while (st && st->next.find(a->first) == st->next.end())-----// 0e
----st = st->fail;-----// b3
----if (!st) st = go;-----// 0b
----s->fail = st->next[a->first];-----// c1
----if (s->fail) {-----// 98
----if (!s->out) s->out = s->fail->out;-----// ad
----else {-----// 5b
----out_node* out = s->out;-----// b8
----while (out->next) out = out->next;-----// b4
----out->next = s->fail->out;-----// 62
----}-----// a6
----}-----// 81
----}-----// 55
----}-----// bf
----}-----// de
----vector<string> search(string s) {-----// c4
----vector<string> res;-----// 79
----go_node *cur = go;-----// 85
----iter(c, s) {-----// 57
----while (cur && cur->next.find(*c) == cur->next.end())-----// df
----cur = cur->fail;-----// b1
----if (!cur) cur = go;-----// 92
```

```
-----cur = cur->next[*c];-----// 97
-----if (!cur) cur = go;-----// 01
-----for (out_node *out = cur->out; out; out = out->next)-----// d7
-----res.push_back(out->keyword);-----// 7c
-----}-----// 56
-----return res;-----// 6b
}-----// 3e
};-----// de
```

4.5. eerTree.

```
#define MAXN 100100-----// 29
#define SIGMA 26-----// e2
#define BASE 'a'-----// a1
char *s = new char[MAXN];-----// db
struct state {-----// 33
    int len, link, to[SIGMA];-----// 24
} *st = new state[MAXN+2];-----// 57
struct eertree {-----// 78
    int last, sz, n;-----// ba
    eertree() : last(1), sz(2), n(0) {-----// 83
        st[0].len = st[0].link = -1;-----// 3f
        st[1].len = st[1].link = 0; }-----// 34
    int extend() {-----// c2
        char c = s[n++]; int p = last;-----// 25
        while (n - st[p].len - 2 < 0 || c != s[n - st[p].len - 2]) p = st[p].link;
        if (!st[p].to[c-BASE]) {-----// 82
            int q = last = sz++;-----// 42
            st[p].to[c-BASE] = q;-----// fc
            st[q].len = st[p].len + 2;-----// c5
            do { p = st[p].link;-----// 04
            } while (p != -1 && (n < st[p].len + 2 || c != s[n - st[p].len - 2]));
            if (p == -1) st[q].link = 1;-----// 77
            else st[q].link = st[p].to[c-BASE];-----// 6a
            return 1; }-----// 29
        last = st[p].to[c-BASE];-----// 42
        return 0; } } ;-----// ec
```

4.6. Suffix Automaton. Minimum automata that accepts all suffixes of a string with $O(n)$ construction. The automata itself is a DAG therefore suitable for DP, examples are counting unique substrings, occurrences of substrings and suffix.

```
// TODO: Add longest common subsring-----// 0e
const int MAXL = 100000;-----// 31
struct suffix_automaton {-----// e0
    vi len, link, occur, cnt;-----// 78
    vector<map<char,int>> next;-----// 90
    vector<bool> isclone;-----// 7b
    ll *occuratleast;-----// f2
    int sz, last;-----// 7d
    string s;-----// f2
    suffix_automaton() : len(MAXL*2), link(MAXL*2), occur(MAXL*2), next(MAXL*2),
    isclone(MAXL*2) { clear(); }-----// a3
    void clear(){ sz = 1; last = len[0] = 0; link[0] = -1; next[0].clear(); }-----// aa
```

```
isclone[0] = false; }-----// 26
bool issubstr(string other){-----// 3b
    for(int i = 0, cur = 0; i < size(other); ++i){-----// 7f
        if(cur == -1) return false; cur = next[cur][other[i]]; }-----// 54
    return true; }-----// 1a
void extend(char c){ int cur = sz++; len[cur] = len[last] + 1;-----// 1d
    next[cur].clear(); isclone[cur] = false; int p = last;-----// a9
    for(; p != -1 && !next[p].count(c); p = link[p]){ next[p][c] = cur; }-----// 6f
    if(p == -1){ link[cur] = 0; }-----// 18
    else{ int q = next[p][c];-----// 34
        if(len[p] + 1 == len[q]){ link[cur] = q; }-----// 4d
        else { int clone = sz++; isclone[clone] = true;-----// 57
            len[clone] = len[p] + 1;-----// 8c
            link[clone] = link[q]; next[clone] = next[q];-----// 76
            for(; p != -1 && next[p].count(c) && next[p][c] == q; p = link[p]){
                next[p][c] = clone; }-----// 32
            link[q] = link[cur] = clone;-----// 73
        } } last = cur; }-----// b9
    void count(){-----// e7
        cnt=vi(sz, -1); stack<ii> S; S.push(ii(0,0));map<char,int>::iterator i;-----// 56
        while(!S.empty()){-----// 4c
            ii cur = S.top(); S.pop();-----// 67
            if(cur.second){-----// 78
                for(i = next[cur.first].begin();i != next[cur.first].end();++i){
                    cnt[cur.first] += cnt[(*i).second]; } }-----// da
                else if(cnt[cur.first] == -1){-----// 99
                    cnt[cur.first] = 1; S.push(ii(cur.first, 1));-----// bd
                    for(i = next[cur.first].begin();i != next[cur.first].end();++i){
                        S.push(ii((*i).second, 0)); } } } }-----// 61
string lexicok(ll k){-----// 8b
    int st = 0; string res; map<char,int>::iterator i;-----// cf
    while(k){ for(i = next[st].begin(); i != next[st].end(); ++i){-----// 69
        if(k <= cnt[(*i).second]){ st = (*i).second;-----// ec
            res.push_back((*i).first); k--; break;-----// 63
        } else { k -= cnt[(*i).second]; } } }-----// ee
    return res; }-----// 0b
    void countoccur(){-----// ad
        for(int i = 0; i < sz; ++i){ occur[i] = 1 - isclone[i]; }-----// 1b
        vii states(sz);-----// dc
        for(int i = 0; i < sz; ++i){ states[i] = ii(len[i],i); }-----// 97
        sort(states.begin(), states.end());-----// 8d
        for(int i = size(states)-1; i >= 0; --i){ int v = states[i].second;-----// a4
            if(link[v] != -1) { occur[link[v]] += occur[v]; } } }-----// cc
};-----// 32
-----// 56
```

4.7. Hashing. Modulus should be a large prime. Can also use multiple instances with different moduli to minimize chance of collision.

```
struct hasher { int b = 311, m; vi h, p;-----// 61
hasher(string s, int _m) : m(_m), h(size(s)+1), p(size(s)+1) {-----// f6
    p[0] = 1; h[0] = 0;-----// d3
```

```
-----rep(i,0,size(s)) p[i+1] = ((ll)p[i] * b % m;-----// 8a
-----rep(i,0,size(s)) h[i+1] = ((ll)h[i] * b + s[i]) % m; }-----// 10
---int hash(int l, int r) {-----// b2
-----return (h[r+1] + m - (ll)h[l] * p[r-l+1] % m) % m; } }-----// 26
```

5. MATHEMATICS

5.1. **Binomial Coefficients.** The binomial coefficient $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ is the number of ways to choose k items out of a total of n items. Also contains an implementation of Lucas' theorem for computing the answer modulo a prime p .

```
int nck(int n, int k) {-----// f6
---if (n < k) return 0;-----// 55
---k = min(k, n - k);-----// bd
---int res = 1;-----// e6
---rep(i,1,k+1) res = res * (n - (k - i)) / i;-----// 4d
---return res;-----// 1f
}-----// 6c
int nck(int n, int k, int p) {-----// cf
---int res = 1;-----// 5c
---while (n || k) {-----// e2
---res *= nck(n % p, k % p);-----// cc
---res %= p, n /= p, k /= p;-----// 0a
---}-----// d9
---return res;-----// 30
}-----// 0a
```

5.2. Euclidean algorithm.

```
int gcd(int a, int b) { return b == 0 ? a : gcd(b, a % b); }-----// d9
```

The extended Euclidean algorithm computes the greatest common divisor d of two integers a, b and also finds two integers x, y such that $a \times x + b \times y = d$.

```
int egcd(int a, int b, int& x, int& y) {-----// 85
---if (b == 0) { x = 1; y = 0; return a; }-----// 7b
---else {-----// 00
---int d = egcd(b, a % b, x, y);-----// 34
---x -= a / b * y;-----// 4a
---swap(x, y);-----// 26
---return d;-----// db
---}-----// 9e
}-----// 40
```

5.3. Trial Division Primality Testing.

```
bool is_prime(int n) {-----// 6c
---if (n < 2) return false;-----// c9
---if (n < 4) return true;-----// d9
---if (n % 2 == 0 || n % 3 == 0) return false;-----// 0f
---if (n < 25) return true;-----// ef
---int s = static_cast<int>(sqrt(static_cast<double>(n)));-----// 64
---for (int i = 5; i <= s; i += 6)-----// 6c
-----if (n % i == 0 || n % (i + 2) == 0) return false;-----// e9
---return true; }-----// 43
```

5.4. Miller-Rabin Primality Test.

```
#include "mod_pow.cpp"-----// c7
bool is_probable_prime(ll n, int k) {-----// be
---if (~n & 1) return n == 2;-----// d1
---if (n <= 3) return n == 3;-----// 39
---int s = 0; ll d = n - 1;-----// 37
---while (~d & 1) d >= 1, s++;-----// 35
---while (k--) {-----// c8
-----ll a = (n - 3) * rand() / RAND_MAX + 2;-----// 06
-----ll x = mod_pow(a, d, n);-----// 64
-----if (x == 1 || x == n - 1) continue;-----// 9b
-----bool ok = false;-----// 03
-----rep(i,0,s-1) {-----// 13
-----x = (x * x) % n;-----// 90
-----if (x == 1) return false;-----// 5c
-----if (x == n - 1) { ok = true; break; }-----// a1
-----}-----// 3a
-----if (!ok) return false;-----// 37
---} return true; }-----// fe
```

5.5. Pollard's ρ algorithm.

```
// public static int[] seeds = new int[] {2,3,5,7,11,13,1031};-----// 1d
// public static BigInteger rho(BigInteger n, BigInteger seed) {-----// 03
//---- int i = 0,-----// 00
//---- k = 2;-----// 79
//---- BigInteger x = seed,-----// cc
//-----y = seed;-----// 31
//---- while (i < 1000000) {-----// 10
//-----i++;-----// 8c
//-----x = (x.multiply(x).add(n).subtract(BigInteger.ONE)).mod(n);-----// 74
//-----BigInteger d = y.subtract(x).abs().gcd(n);-----// ce
//-----if (!d.equals(BigInteger.ONE) && !d.equals(n)) {-----// b9
//-----return d;-----// 3b
//-----}-----// 7c
//-----if (i == k) {-----// 2c
//-----y = x;-----// 89
//-----k = k*2;-----// 1d
//-----}-----// 10
//---- }-----// 96
//---- return BigInteger.ONE;-----// 62
// }-----// d7
```

5.6. Sieve of Eratosthenes.

```
vi prime_sieve(int n) {-----// 40
---int mx = (n - 3) >> 1, sq, v, i = -1;-----// 27
---vi primes;-----// 8f
---bool* prime = new bool[mx + 1];-----// ef
---memset(prime, 1, mx + 1);-----// 28
---if (n >= 2) primes.push_back(2);-----// f4
---while (++i <= mx) if (prime[i]) {-----// 73
---primes.push_back(v = (i << 1) + 3);-----// be
---if ((sq = i * ((i << 1) + 6) + 3) > mx) break;-----// 2d
```



```
-----for (int j = sq; j <= mx; j += v) prime[j] = false; }-----// 2e
---while (++i <= mx) if (prime[i]) primes.push_back((i < 1) + 3);-----// 29
---delete[] prime; // can be used for O(1) lookup-----// 36
---return primes; }-----// 72
```

5.7. Divisor Sieve.

```
vi divisor_sieve(int n) {-----// 7f
---vi minimalDiv(n+1, 2), primes;-----// 37
---if(n>=2) primes.push_back(2);-----// 27
---minimalDiv[0] = 0;-----// 02
---for(int k=1;k<=n;k+=2) minimalDiv[k] = k;-----// e6
---for(int k=3;k<=n;k+=2) {-----// 5d
-----if(minimalDiv[k] == k) primes.push_back(k);-----// 75
-----rep(i, 1, size(primes))-----// 49
-----if(primes[i] > minimalDiv[k] || primes[i]*k > n) break;-----// 53
-----else minimalDiv[primes[i]*k] = primes[i];-----// 90
---}-----// 9d
---return primes; }-----// 93
-----// a8
```

5.8. Modular Multiplicative Inverse.

```
#include "egcd.cpp"-----// 55
-----// e8
int mod_inv(int a, int m) {-----// 49
---int x, y, d = egcd(a, m, x, y);-----// 3e
---if (d != 1) return -1;-----// 20
---return x < 0 ? x + m : x;-----// 3c
}-----// 69
```

5.9. Primitive Root.

```
#include "mod_pow.cpp"-----// c7
ll primitive_root(ll m) {-----// 8a
---vector<ll> div;-----// f2
---for (ll i = 1; i*i <= m-1; i++) {-----// ca
-----if ((m-1) % i == 0) {-----// 85
-----if (i < m) div.push_back(i);-----// fd
-----if (m/i < m) div.push_back(m/i); } }-----// f2
---rep(x,2,m) {-----// 57
---bool ok = true;-----// 17
---iter(it,div) if (mod_pow(x, *it, m) == 1) { ok = false; break; }-----// a4
---if (ok) return x; }-----// 55
---return -1; }-----// 15
```

5.10. Chinese Remainder Theorem.

```
#include "egcd.cpp"-----// 55
int crt(const vi& as, const vi& ns) {-----// c3
---int cnt = size(as), N = 1, x = 0, r, s, l;-----// 55
---rep(i,0,cnt) N *= ns[i];-----// b1
---rep(i,0,cnt) egcd(ns[i], l = N/ns[i], r, s), x += as[i] * s * l;-----// 21
---return mod(x, N); }-----// b2
```

5.11. Linear Congruence Solver. A function that returns all solutions to $ax \equiv b \pmod n$, modulo n .

```
#include "egcd.cpp"-----// 55
vi linear_congruence(int a, int b, int n) {-----// c8
---int x, y, d = egcd(a, n, x, y);-----// 7a
---vi res;-----// f5
---if (b % d != 0) return res;-----// 30
---int x0 = mod(b / d * x, n);-----// 48
---rep(k,0,d) res.push_back(mod(x0 + k * n / d, n));-----// 7e
---return res;-----// fe
}-----// c0
```

5.12. Numeric Integration.

```
double integrate(double (*f)(double), double a, double b,-----// 76
---double delta = 1e-6) {-----// c0
---if (abs(a - b) < delta)-----// 38
---return (b-a)/8 *-----// 56
---((f(a) + 3*f((2*a+b)/3) + 3*f((a+2*b)/3) + f(b));-----// e1
---return integrate(f, a,-----// 64
---(a+b)/2, delta) + integrate(f, (a+b)/2, b, delta);-----// 0c
}-----// 4b
```

5.13. Fast Fourier Transform. The Cooley-Tukey algorithm for quickly computing the discrete Fourier transform. The fft function only supports powers of twos. The czt function implements the Chirp Z-transform and supports any size, but is slightly slower.

```
#include <complex>-----// 8e
typedef complex<long double> cpx;-----// 25
// NOTE: n must be a power of two-----// 14
void fft(cpx *x, int n, bool inv=false) {-----// 36
---for (int i = 0, j = 0; i < n; i++) {-----// f9
---if (i < j) swap(x[i], x[j]);-----// 44
---int m = n>>1;-----// 9c
---while (1 <= m && m <= j) j -= m, m >>= 1;-----// fe
---j += m;-----// 11
---}-----// d0
---for (int mx = 1; mx < n; mx <= 1) {-----// 15
---cpx wp = exp(cpx(0, (inv ? -1 : 1) * pi / mx)), w = 1;-----// 79
---for (int m = 0; m < mx; m++, w *= wp) {-----// dc
---for (int i = m; i < n; i += mx << 1) {-----// 6a
---cpx t = x[i + mx] * w;-----// 12
---x[i + mx] = x[i] - t;-----// 73
---x[i] += t;-----// 0e
---}-----// 14
---}-----// a4
---}-----// bf
---if (inv) rep(i,0,n) x[i] /= cpx(n);-----// 16
}-----// 1c
void czt(cpx *x, int n, bool inv=false) {-----// c5
---int len = 2*n+1;-----// bc
---while (len & (len - 1)) len &= len - 1;-----// 65
---len <= 1;-----// 21
---cpx w = exp(-2.0L * pi / n * cpx(0,1)),-----// 45
```

```
-----*c = new cpx[n], *a = new cpx[len],-----// 4e
-----*b = new cpx[len];-----// 30
---rep(i,0,n) c[i] = pow(w, (inv ? -1.0 : 1.0)*i/2);-----// 9e
---rep(i,0,n) a[i] = x[i] * c[i], b[i] = 1.0L/c[i];-----// e9
---rep(i,0,n-1) b[len - n + i + 1] = 1.0L/c[n-i-1];-----// 9f
---fft(a, len); fft(b, len);-----// 63
---rep(i,0,len) a[i] *= b[i];-----// 58
---fft(a, len, true);-----// 2d
---rep(i,0,n) {-----// ff
-----x[i] = c[i] * a[i];-----// 77
-----if (inv) x[i] /= cpx(n);-----// b1
-----}-----// 27
---delete[] a;-----// 0a
---delete[] b;-----// 5c
---delete[] c;-----// f8
}-----// c6
```

5.14. Number-Theoretic Transform.

```
#include "../mathematics/primitive_root.cpp"-----// 8c
int mod = 998244353, g = primitive_root(mod),-----// 9c
---ginv = mod_pow(g, mod-2, mod), inv2 = mod_pow(2, mod-2, mod);-----// 75
#define MAXN (1<<22)-----// 94
struct Num {-----// c5
---int x;-----// 02
---Num(ll _x=0) { x = (_x%mod+mod)%mod; }-----// 1b
---Num operator +(const Num &b) { return x + b.x; }-----// 08
---Num operator -(const Num &b) const { return x - b.x; }-----// 89
---Num operator *(const Num &b) const { return (ll)x * b.x; }-----// e3
---Num operator /(const Num &b) const { return (ll)x * b.inv().x; }-----// 2a
---Num inv() const { return mod_pow((ll)x, mod-2, mod); }-----// d3
---Num pow(int p) const { return mod_pow((ll)x, p, mod); }-----// d5
} T1[MAXN], T2[MAXN];-----// d5
void ntt(Num x[], int n, bool inv = false) {-----// 24
---Num z = inv ? ginv : g;-----// 00
---z = z.pow((mod - 1) / n);-----// 45
---for (ll i = 0, j = 0; i < n; i++) {-----// fe
---if (i < j) swap(x[i], x[j]);-----// eb
---ll k = n>>1;-----// 02
---while (1 <= k && k <= j) j -= k, k >= 1;-----// d2
---j += k; }-----// c4
---for (int mx = 1, p = n/2; mx < n; mx <= 1, p >= 1) {-----// a2
---Num wp = z.pow(p), w = 1;-----// 35
---for (int k = 0; k < mx; k++, w = w*wp) {-----// 42
---for (int i = k; i < n; i += mx << 1) {-----// 9a
---Num t = x[i + mx] * w;-----// 22
---x[i + mx] = x[i] - t;-----// d0
---x[i] = x[i] + t; } } }-----// 1e
---if (inv) {-----// 6c
---Num ni = Num(n).inv();-----// 18
---rep(i,0,n) { x[i] = x[i] * ni; } } }-----// 72
void inv(Num x[], Num y[], int l) {-----// ee
```

```
---if (l == 1) { y[0] = x[0].inv(); return; }-----// 71
---inv(x, y, l>>1);-----// bd
---// NOTE: maybe l<<2 instead of l<<1-----// 24
---rep(i,l>>1,l<<1) T1[i] = y[i] = 0;-----// dd
---rep(i,0,l) T1[i] = x[i];-----// f7
---ntt(T1, l<<1); ntt(y, l<<1);-----// a5
---rep(i,0,l<<1) y[i] = y[i]*2 - T1[i] * y[i] * y[i];-----// bc
---ntt(y, l<<1, true); }-----// 40
void sqrt(Num x[], Num y[], int l) {-----// 44
---if (l == 1) { assert(x[0].x == 1); y[0] = 1; return; }-----// 64
---sqrt(x, y, l>>1);-----// a7
---inv(y, T2, l>>1);-----// 3b
---rep(i,l>>1,l<<1) T1[i] = T2[i] = 0;-----// 1c
---rep(i,0,l) T1[i] = x[i];-----// 08
---ntt(T2, l<<1); ntt(T1, l<<1);-----// 7e
---rep(i,0,l<<1) T2[i] = T1[i] * T2[i];-----// 9f
---ntt(T2, l<<1, true);-----// eb
---rep(i,0,l) y[i] = (y[i] + T2[i]) * inv2; }-----// b8
```

5.15. **Tridiagonal Matrix Algorithm.** Solves a tridiagonal system of linear equations $a_i x_{i-1} + b_i x_i + c_i x_{i+1} = d_i$ where $a_1 = c_n = 0$. Beware of numerical instability.

```
#define MAXN 5000-----// f7
long double A[MAXN], B[MAXN], C[MAXN], D[MAXN], X[MAXN];-----// d8
void solve(int n) {-----// 01
---C[0] /= B[0]; D[0] /= B[0];-----// 94
---rep(i,1,n-1) C[i] /= B[i] - A[i]*C[i-1];-----// 6b
---rep(i,1,n) D[i] = (D[i] - A[i] * D[i-1]) / (B[i] - A[i] * C[i-1]);-----// 33
---X[n-1] = D[n-1];-----// c7
---for (int i = n-2; i>=0; i--) X[i] = D[i] - C[i] * X[i+1]; }-----// ad
```

5.16. **Mertens Function.** Mertens function is $M(n) = \sum_{i=1}^n \mu(i)$. Let $L \approx (n \log \log n)^{2/3}$ and the algorithm runs in $O(n^{2/3})$. Can also be easily changed to compute the summatory Φ .

```
#define L 9000000-----// 27
int mob[L], mer[L];-----// f1
unordered_map<ll,ll> mem;-----// 30
ll M(ll n) {-----// de
---if (n < L) return mer[n];-----// 1c
---if (mem.find(n) != mem.end()) return mem[n];-----// 79
---ll ans = 0, done = 1;-----// 48
---for (ll i = 2; i*i <= n; i++) ans += M(n/i), done = i;-----// 41
---for (ll i = 1; i*i <= n; i++) ans += mer[i] * (n/i - max(done, n/(i+1)));-----// 43
---return mer[n] = 1 - ans; }-----// c2
void sieve() {-----// b9
---for (int i = 1; i < L; i++) mer[i] = mob[i] = 1;-----// f7
---for (int i = 2; i < L; i++) {-----// 8e
---if (mer[i]) {-----// 8b
---mob[i] = -1;-----// e5
---for (int j = i+i; j < L; j += i)-----// f0
---mer[j] = 0, mob[j] = (j/i)%i == 0 ? 0 : -mob[j/i];-----// 26
---}-----// aa
---mer[i] = mob[i] + mer[i-1]; } }-----// 3b
```

5.17. **Numbers and Sequences.** Some random prime numbers: 1031, 32771, 1048583, 33554467, 1073741827, 34359738421, 1099511627791, 35184372088891, 1125899906842679, 36028797018963971.

6. GEOMETRY

6.1. Primitives.

```
#define P(p) const point &p-----// 2e
#define L(p0, p1) P(p0), P(p1)-----// cf
#define C(p0, r) P(p0), double r-----// f1
#define PP(pp) pair<point,point> &pp-----// e5
typedef complex<double> point;-----// 6a
double dot(P(a), P(b)) { return real(conj(a) * b); }-----// d2
double cross(P(a), P(b)) { return imag(conj(a) * b); }-----// 8a
point rotate(P(p), double radians = pi / 2, P(about) = point(0,0)) {-----// 23
    return (p - about) * exp(point(0, radians)) + about; }-----// 25
point reflect(P(p), L(about1, about2)) {-----// 50
    point z = p - about1, w = about2 - about1;-----// 8b
    return conj(z / w) * w + about1; }-----// 83
point proj(P(u), P(v)) { return dot(u, v) / dot(u, u) * u; }-----// e7
point normalize(P(p), double k = 1.0) {-----// 5f
    return abs(p) == 0 ? point(0,0) : p / abs(p) * k; }-----// 4a
double ccw(P(a), P(b), P(c)) { return cross(b - a, c - b); }-----// 27
bool collinear(P(a), P(b), P(c)) { return abs(ccw(a, b, c)) < EPS; }-----// b3
double angle(P(a), P(b), P(c)) {-----// 61
    return acos(dot(b - a, c - b) / abs(b - a) / abs(c - b)); }-----// c7
double signed_angle(P(a), P(b), P(c)) {-----// 4a
    return asin(cross(b - a, c - b) / abs(b - a) / abs(c - b)); }-----// 40
double angle(P(p)) { return atan2(imag(p), real(p)); }-----// e6
point perp(P(p)) { return point(-imag(p), real(p)); }-----// d9
double progress(P(p), L(a, b)) {-----// b3
    if (abs(real(a) - real(b)) < EPS)-----// 5e
        return (imag(p) - imag(a)) / (imag(b) - imag(a));-----// 5e
    else return (real(p) - real(a)) / (real(b) - real(a)); }-----// 31
-----// 53
-----// 46
```

6.2. Lines.

```
#include "primitives.cpp"-----// e0
-----// 85
bool collinear(L(a, b), L(p, q)) {-----// 2f
    return abs(ccw(a, b, p)) < EPS && abs(ccw(a, b, q)) < EPS; }-----// 3e
bool parallel(L(a, b), L(p, q)) { return abs(cross(b - a, q - p)) < EPS; }-----// 8d
point closest_point(L(a, b), P(c), bool segment = false) {-----// f2
    if (segment) {-----// f4
        if (dot(b - a, c - b) > 0) return b;-----// 88
        if (dot(a - b, c - a) > 0) return a;-----// 75
    }-----// ce
    double t = dot(c - a, b - a) / norm(b - a);-----// 62
    return a + t * (b - a);-----// 6e
}-----// 8c
double line_segment_distance(L(a,b), L(c,d)) {-----// f3
    double x = INFINITY;-----// 64
```

```
if (abs(a - b) < EPS && abs(c - d) < EPS) x = abs(a - c);-----// a5
else if (abs(a - b) < EPS) x = abs(a - closest_point(c, d, a, true));-----// 23
else if (abs(c - d) < EPS) x = abs(c - closest_point(a, b, c, true));-----// 53
else if ((ccw(a, b, c) < 0) != (ccw(a, b, d) < 0) &&-----// 6d
    (ccw(c, d, a) < 0) != (ccw(c, d, b) < 0)) x = 0;-----// bf
else {-----// e1
    x = min(x, abs(a - closest_point(c,d, a, true)));-----// 29
    x = min(x, abs(b - closest_point(c,d, b, true)));-----// fe
    x = min(x, abs(c - closest_point(a,b, c, true)));-----// 81
    x = min(x, abs(d - closest_point(a,b, d, true)));-----// e4
}-----// c5
return x;-----// b7
}-----// 27
bool intersect(L(a, b), L(p, q), point &res, bool segment = false) {-----// d2
    // NOTE: check for parallel/collinear lines before calling this function-----// 1b
    point r = b - a, s = q - p;-----// 34
    double c = cross(r, s), t = cross(p - a, s) / c, u = cross(p - a, r) / c;-----// 0b
    if (segment && (t < 0-EPS || t > 1+EPS || u < 0-EPS || u > 1+EPS))-----// e4
        return false;-----// e3
    res = a + t * r;-----// 47
    return true;-----// 05
}-----// 44
-----// cc
```

6.3. Circles.

```
#include "primitives.cpp"-----// e0
int intersect(C(A, rA), C(B, rB), point & res1, point & res2) {-----// 3b
    double d = abs(B - A);-----// 5c
    if ((rA + rB) < (d - EPS) || d < abs(rA - rB) - EPS) return 0;-----// 39
    double a = (rA*rA - rB*rB + d*d) / 2 / d, h = sqrt(rA*rA - a*a);-----// 9b
    point v = normalize(B - A, a), u = normalize(rotate(B-A), h);-----// 79
    res1 = A + v + u, res2 = A + v - u;-----// 24
    if (abs(u) < EPS) return 1; return 2;-----// 82
}-----// bb
int intersect(L(A, B), C(0, r), point & res1, point & res2) {-----// 0e
    double h = abs(0 - closest_point(A, B, 0));-----// 24
    if(r < h - EPS) return 0;-----// df
    point H = proj(0 - A, B - A) + A, v = normalize((B - A), sqrt(r*r - h*h));// 19
    res1 = H + v; res2 = H - v;-----// 40
    if(abs(v) < EPS) return 1; return 2;-----// 37
}-----// 46
int tangent(P(A), C(0, r), point & res1, point & res2) {-----// aa
    point v = 0 - A; double d = abs(v);-----// 49
    if (d < r - EPS) return 0;-----// ca
    double alpha = asin(r / d), L = sqrt(d*d - r*r);-----// 3f
    v = normalize(v, L);-----// 3f
    res1 = A + rotate(v, alpha); res2 = A + rotate(v, -alpha);-----// be
    if (abs(r - d) < EPS || abs(v) < EPS) return 1;-----// bb
    return 2;-----// b9
}-----// f4
void tangent_outer(point A, double rA, point B, double rB, PP(P), PP(Q)) {-----// 83
```

```
---if (rA - rB > EPS) { swap(rA, rB); swap(A, B); }-----// 82
---double theta = asin((rB - rA)/abs(A - B));-----// 37
---point v = rotate(B - A, theta + pi/2), u = rotate(B - A, -(theta + pi/2));--// 00
---u = normalize(u, rA);-----// 44
---P.first = A + normalize(v, rA); P.second = B + normalize(v, rB);-----// d2
---Q.first = A + normalize(u, rA); Q.second = B + normalize(u, rB);-----// 23
}-----// b5
```

6.4. Polygon.

```
#include "primitives.cpp"-----// e0
typedef vector<point> polygon;-----// b3
double polygon_area_signed(polygon p) {-----// 31
---double area = 0; int cnt = size(p);-----// a2
---rep(i,1,cnt-1) area += cross(p[i] - p[0], p[i + 1] - p[0]);-----// 51
---return area / 2; }-----// 66
double polygon_area(polygon p) { return abs(polygon_area_signed(p)); }-----// a4
#define CHK(f,a,b,c) (f(a) < f(b) && f(b) <= f(c) && ccw(a,c,b) < 0)-----// 8f
int point_in_polygon(polygon p, point q) {-----// 5d
---int n = size(p); bool in = false; double d;-----// 69
---for (int i = 0, j = n - 1; i < n; j = i++)-----// f3
---if (collinear(p[i], q, p[j]) &&-----// 9d
---0 <= (d = progress(q, p[i], p[j])) && d <= 1)-----// 4b
---return 0;-----// b3
---for (int i = 0, j = n - 1; i < n; j = i++)-----// 67
---if (CHK(real, p[i], q, p[j]) || CHK(real, p[j], q, p[i]))-----// b4
---in = !in;-----// ff
---return in ? -1 : 1; }-----// ba
// pair<polygon, polygon> cut_polygon(const polygon &poly, point a, point b) {--// 0d
//---- polygon left, right;-----// 0a
//---- point it(-100, -100);-----// 5b
//---- for (int i = 0, cnt = poly.size(); i < cnt; i++) {-----// 70
//----- int j = i == cnt-1 ? 0 : i + 1;-----// 02
//----- point p = poly[i], q = poly[j];-----// 44
//----- if (ccw(a, b, p) <= 0) left.push_back(p);-----// 8d
//----- if (ccw(a, b, p) >= 0) right.push_back(p);-----// 43
//----- // myintersect = intersect where-----// ba
//----- // (a,b) is a line, (p,q) is a line segment-----// 7e
//----- if (myintersect(a, b, p, q, it))-----// 6f
//----- left.push_back(it), right.push_back(it);-----// 8a
//----- }-----// e0
//---- return pair<polygon, polygon>(left, right);-----// 3d
// }-----// 07
```

6.5. Convex Hull. NOTE: Doesn't work on some weird edge cases. (A small case that included three collinear lines would return the same point on both the upper and lower hull.)

```
#include "polygon.cpp"-----// 58
#define MAXN 1000-----// 09
point hull[MAXN];-----// 43
bool cmp(const point &a, const point &b) {-----// 32
---return abs(real(a) - real(b)) > EPS ?-----// 44
---real(a) < real(b) : imag(a) < imag(b); }-----// 40
int convex_hull(polygon p) {-----// cd
```

```
---int n = size(p), l = 0;-----// 67
---sort(p.begin(), p.end(), cmp);-----// 3d
---rep(i,0,n) {-----// e4
---if (i > 0 && p[i] == p[i - 1]) continue;-----// c7
---while (l >= 2 && ccw(hull[l - 2], hull[l - 1], p[i]) >= 0) l--;-----// 62
---hull[l++] = p[i];-----// bd
---}-----// d2
---int r = l;-----// 30
---for (int i = n - 2; i >= 0; i--) {-----// 59
---if (p[i] == p[i + 1]) continue;-----// af
---while (r - l >= 1 && ccw(hull[r - 2], hull[r - 1], p[i]) >= 0) r--;-----// 4d
---hull[r++] = p[i];-----// f5
---}-----// f6
---return l == 1 ? 1 : r - 1;-----// a6
}-----// 6d
```

6.6. Line Segment Intersection.

```
#include "primitives.cpp"-----// e0
#include "lines.cpp"-----// 54
bool line_segment_intersect(L(a, b), L(c, d), point &A, point &B) {-----// 34
---if (abs(a - b) < EPS && abs(c - d) < EPS) {-----// 74
---A = B = a; return abs(a - d) < EPS; }-----// 37
---else if (abs(a - b) < EPS) {-----// 07
---A = B = a; double p = progress(a, c,d);-----// 6c
---return 0.0 <= p && p <= 1.0-----// 4c
---&& (abs(a - c) + abs(d - a) - abs(d - c)) < EPS; }-----// d2
---else if (abs(c - d) < EPS) {-----// a0
---A = B = c; double p = progress(c, a,b);-----// c0
---return 0.0 <= p && p <= 1.0-----// fb
---&& (abs(c - a) + abs(b - c) - abs(b - a)) < EPS; }-----// 15
---else if (collinear(a,b, c,d)) {-----// 49
---double ap = progress(a, c,d), bp = progress(b, c,d);-----// 34
---if (ap > bp) swap(ap, bp);-----// e0
---if (bp < 0.0 || ap > 1.0) return false;-----// 14
---A = c + max(ap, 0.0) * (d - c);-----// 14
---B = c + min(bp, 1.0) * (d - c);-----// 26
---return true; }-----// bd
---else if (parallel(a,b, c,d)) return false;-----// fc
---else if (intersect(a,b, c,d, A, true)) {-----// 78
---B = A; return true; }-----// 7d
---return false;-----// 4b
}-----// 05
}-----// 83
```

6.7. Great-Circle Distance. Computes the distance between two points (given as latitude/longitude coordinates) on a sphere of radius *r*.

```
double gc_distance(double pLat, double pLong,-----// 7b
---double qLat, double qLong, double r) {-----// a4
---pLat *= pi / 180; pLong *= pi / 180;-----// ee
---qLat *= pi / 180; qLong *= pi / 180;-----// 75
---return r * acos(cos(pLat) * cos(qLat) * cos(pLong - qLong) +-----// e3
---sin(pLat) * sin(qLat));-----// 1e
```

```
-----// 60
}-----// 3f
```

6.8. **Triangle Circumcenter.** Returns the unique point that is the same distance from all three points. It is also the center of the unique circle that goes through all three points.

```
#include "primitives.cpp"-----// e0
point circumcenter(point a, point b, point c) {-----// 76
---b -= a, c -= a;-----// 41
---return a + perp(b * norm(c) - c * norm(b)) / 2.0 / cross(b, c);-----// 7a
}-----// c3
```

6.9. **Closest Pair of Points.**

```
#include "primitives.cpp"-----// e0
-----// 85
struct cmpx { bool operator()(const point &a, const point &b) {-----// 01
-----return abs(real(a) - real(b)) > EPS ?-----// e9
-----real(a) < real(b) : imag(a) < imag(b); } };-----// 53
struct cmpy { bool operator()(const point &a, const point &b) {-----// 6f
---return abs(imag(a) - imag(b)) > EPS ?-----// 0b
-----imag(a) < imag(b) : real(a) < real(b); } };-----// a4
double closest_pair(vector<point> pts) {-----// f1
---sort(pts.begin(), pts.end(), cmpx());-----// 0c
---set<point, cmpy> cur;-----// bd
---set<point, cmpy>::const_iterator it, jt;-----// a6
---double mn = INFINITY;-----// f9
---for (int i = 0, l = 0; i < size(pts); i++) {-----// ac
---while (real(pts[i]) - real(pts[l]) > mn) cur.erase(pts[l++]);-----// 8b
-----it = cur.lower_bound(point(-INFINITY, imag(pts[i]) - mn));-----// fc
-----jt = cur.upper_bound(point(INFINITY, imag(pts[i]) + mn));-----// 39
-----while (it != jt) mn = min(mn, abs(*it - pts[i])), it++;-----// 09
-----cur.insert(pts[i]); }-----// 82
---return mn; }-----// 4c
```

6.10. **3D Primitives.**

```
#define P(p) const point3d &p-----// a7
#define L(p0, p1) P(p0), P(p1)-----// 0f
#define PL(p0, p1, p2) P(p0), P(p1), P(p2)-----// 67
struct point3d {-----// 63
---double x, y, z;-----// e6
---point3d() : x(0), y(0), z(0) {}-----// af
---point3d(double _x, double _y, double _z) : x(_x), y(_y), z(_z) {}-----// fc
---point3d operator+(P(p)) const {-----// 17
-----return point3d(x + p.x, y + p.y, z + p.z); }-----// 8e
---point3d operator-(P(p)) const {-----// fb
-----return point3d(x - p.x, y - p.y, z - p.z); }-----// 83
---point3d operator-() const {-----// 89
-----return point3d(-x, -y, -z); }-----// d4
---point3d operator*(double k) const {-----// 4d
-----return point3d(x * k, y * k, z * k); }-----// fd
---point3d operator/(double k) const {-----// 95
-----return point3d(x / k, y / k, z / k); }-----// 58
---double operator%(P(p)) const {-----// d1
```

```
-----return x * p.x + y * p.y + z * p.z; }-----// 09
point3d operator*(P(p)) const {-----// 4f
-----return point3d(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x); }-----// ed
double length() const {-----// 3e
-----return sqrt(*this % *this); }-----// 05
double distTo(P(p)) const {-----// dd
-----return (*this - p).length(); }-----// 57
double distTo(P(A), P(B)) const {-----// bd
-----// A and B must be two different points-----// 4e
-----return ((*this - A) * (*this - B)).length() / A.distTo(B); }-----// 6e
point3d normalize(double k = 1) const {-----// db
-----// length() must not return 0-----// 3c
-----return (*this) * (k / length()); }-----// d4
point3d getProjection(P(A), P(B)) const {-----// 86
point3d v = B - A;-----// 64
-----return A + v.normalize((v % (*this - A)) / v.length()); }-----// 53
point3d rotate(P(normal)) const {-----// 55
-----// normal must have length 1 and be orthogonal to the vector-----// eb
-----return (*this) * normal; }-----// 5c
point3d rotate(double alpha, P(normal)) const {-----// 21
-----return (*this) * cos(alpha) + rotate(normal) * sin(alpha); }-----// 82
point3d rotatePoint(P(0), P(axe), double alpha) const {-----// 7a
point3d Z = axe.normalize(axe % (*this - 0));-----// ba
-----return 0 + Z + (*this - 0 - Z).rotate(alpha, 0); }-----// 38
bool isZero() const {-----// 64
-----return abs(x) < EPS && abs(y) < EPS && abs(z) < EPS; }-----// 15
bool isOnLine(L(A, B)) const {-----// 30
-----return ((A - *this) * (B - *this)).isZero(); }-----// 58
bool isInSegment(L(A, B)) const {-----// f1
-----return isOnLine(A, B) && ((A - *this) % (B - *this)) < EPS; }-----// d9
bool isInSegmentStrictly(L(A, B)) const {-----// 0e
-----return isOnLine(A, B) && ((A - *this) % (B - *this)) < -EPS; }-----// ba
double getAngle() const {-----// 0f
-----return atan2(y, x); }-----// 40
double getAngle(P(u)) const {-----// d5
-----return atan2((*this * u).length(), *this % u); }-----// 79
bool isOnPlane(PL(A, B, C)) const {-----// 8e
-----return abs((A - *this) * (B - *this) % (C - *this)) < EPS; } };-----// 74
int line_line_intersect(L(A, B), L(C, D), point3d &0){-----// dc
---if (abs((B - A) * (C - A) % (D - A)) > EPS) return 0;-----// 6a
---if (((A - B) * (C - D)).length() < EPS)-----// 79
-----return A.isOnLine(C, D) ? 2 : 0;-----// 09
point3d normal = ((A - B) * (C - B)).normalize();-----// bc
double s1 = (C - A) * (D - A) % normal;-----// 68
0 = A + ((B - A) / (s1 + ((D - B) * (C - B) % normal))) * s1;-----// 56
return 1; }-----// a7
int line_plane_intersect(L(A, B), PL(C, D, E), point3d & 0) {-----// 09
double V1 = (C - A) * (D - A) % (E - A);-----// c1
double V2 = (D - B) * (C - B) % (E - B);-----// 29
if (abs(V1 + V2) < EPS)-----// 81
return A.isOnPlane(C, D, E) ? 2 : 0;-----// d5
```


6.11. Polygon Centroid.

6.12. Rotating Calipers.

6.13. Formulas. Let $a = (a_x, a_y)$ and $b = (b_x, b_y)$ be two-dimensional vectors.

- $a \cdot b = |a||b| \cos \theta$, where θ is the angle between a and b .
- $a \times b = |a||b| \sin \theta$, where θ is the signed angle between a and b .
- $a \times b$ is equal to the area of the parallelogram with two of its sides formed by a and b . Half of that is the area of the triangle formed by a and b .
- **Euler's formula:** $V - E + F = 2$
- Side lengths a, b, c can form a triangle iff. $a + b > c$, $b + c > a$ and $a + c > b$.
- Sum of internal angles of a regular convex n -gon is $(n - 2)\pi$.

7. OTHER ALGORITHMS

7.1. 2SAT.

```

----- c(other.pt.first, other.pt.second);-----// 71 #include "../graph/scc.cpp"-----// c3
-----return abs(c - closest_point(a, b, c));-----// 58-----// 63
-- } };-----// 4b bool two_sat(int n, const vii& clauses, vi& all_truthy) {-----// f4
-----// c5-----all_truthy.clear();-----// 31
// int h = convex_hull(pts);-----// 9c-----vvi adj(2*n+1);-----// 7b
// double mx = 0;-----// f1-----rep(i,0,size(courses)) {-----// 76

```



```
-----adj[-clauses[i].first + n].push_back(clauses[i].second + n);-----// eb
-----if (clauses[i].first != clauses[i].second)-----// bc
-----adj[-clauses[i].second + n].push_back(clauses[i].first + n);-----// f0
}-----// da
pair<union_find, vi> res = scc(adj);-----// 00
union_find scc = res.first;-----// 20
vi dag = res.second;-----// ed
vi truth(2*n+1, -1);-----// c7
for (int i = 2*n; i >= 0; i--) {-----// 50
    int cur = order[i] - n, p = scc.find(cur + n), o = scc.find(-cur + n);// 4f
    if (cur == 0) continue;-----// cd
    if (p == o) return false;-----// d0
    if (truth[p] == -1) truth[p] = 1;-----// d3
    truth[cur + n] = truth[p];-----// 50
    truth[o] = 1 - truth[p];-----// 8c
    if (truth[p] == 1) all_truthy.push_back(cur);-----// 55
}-----// c3
return true;-----// eb
}-----// 6b
```

7.2. Stable Marriage.

```
vi stable_marriage(int n, int** m, int** w) {-----// e4
    queue<int> q;-----// f6
    vi at(n, 0), eng(n, -1), res(n, -1); vvi inv(n, vi(n));-----// c3
    rep(i,0,n) rep(j,0,n) inv[i][w[i][j]] = j;-----// f1
    rep(i,0,n) q.push(i);-----// d8
    while (!q.empty()) {-----// 68
        int curm = q.front(); q.pop();-----// e2
        for (int &i = at[curm]; i < n; i++) {-----// 7e
            int curw = m[curm][i];-----// 95
            if (eng[curw] == -1) { }-----// f7
            else if (inv[curw][curm] < inv[curw][eng[curw]])-----// d6
                q.push(eng[curw]);-----// 2e
            else continue;-----// 1d
            res[eng[curw] = curm] = curw, ++i; break;-----// a1
        }-----// c4
    }-----// 3d
    return res;-----// 42
}-----// bf
```

7.3. Algorithm X.

```
bool handle_solution(vi rows) { return false; }-----// 63
struct exact_cover {-----// 95
    struct node {-----// 7e
        node *l, *r, *u, *d, *p;-----// 19
        int row, col, size;-----// ae
        node(int _row, int _col) : row(_row), col(_col) {-----// c9
            size = 0; l = r = u = d = p = NULL; }-----// c3
    };-----// c1
    int rows, cols, *sol;-----// 7b
    bool **arr;-----// e6
    node *head;-----// fe
```

```
exact_cover(int _rows, int _cols) : rows(_rows), cols(_cols), head(NULL) {--// b6
    arr = new bool*[rows];-----// cf
    sol = new int[rows];-----// 5f
    rep(i,0,rows)-----// 9b
        arr[i] = new bool[cols], memset(arr[i], 0, cols);-----// dd
    }-----// 21
    void set_value(int row, int col, bool val = true) { arr[row][col] = val; }--// 9e
    void setup() {-----// a3
        node ***ptr = new node**[rows + 1];-----// bd
        rep(i,0,rows+1) {-----// 76
            ptr[i] = new node*[cols];-----// eb
            rep(j,0,cols)-----// cd
                if (i == rows || arr[i][j]) ptr[i][j] = new node(i, j);-----// 16
                else ptr[i][j] = NULL;-----// d2
        }-----// ac
        rep(i,0,rows+1) {-----// fc
            rep(j,0,cols) {-----// 51
                if (!ptr[i][j]) continue;-----// f7
                int ni = i + 1, nj = j + 1;-----// 7a
                while (true) {-----// fc
                    if (ni == rows + 1) ni = 0;-----// 4c
                    if (ni == rows || arr[ni][j]) break;-----// 8d
                    ++ni;-----// 68
                }-----// ad
                ptr[i][j]->d = ptr[ni][j];-----// 84
                ptr[ni][j]->u = ptr[i][j];-----// 66
                while (true) {-----// 7f
                    if (nj == cols) nj = 0;-----// de
                    if (i == rows || arr[i][nj]) break;-----// 4c
                    ++nj;-----// c5
                }-----// 72
                ptr[i][j]->r = ptr[i][nj];-----// 60
                ptr[i][nj]->l = ptr[i][j];-----// 82
            }-----// 0b
        }-----// 16
        head = new node(rows, -1);-----// 66
        head->r = ptr[rows][0];-----// 3e
        ptr[rows][0]->l = head;-----// 8c
        head->l = ptr[rows][cols - 1];-----// 6a
        ptr[rows][cols - 1]->r = head;-----// c1
        rep(j,0,cols) {-----// 92
            int cnt = -1;-----// d4
            rep(i,0,rows+1)-----// bd
                if (ptr[i][j]) cnt++, ptr[i][j]->p = ptr[rows][j];-----// f3
            ptr[rows][j]->size = cnt;-----// c2
        }-----// b9
        rep(i,0,rows+1) delete[] ptr[i];-----// a5
        delete[] ptr;-----// 72
    }-----// 19
    #define COVER(c, i, j) □-----// 91
    c->r->l = c->l, c->l->r = c->r; □-----// 82
```

```
-----for (node *i = c->d; i != c; i = i->d) \-----// 62
-----for (node *j = i->r; j != i; j = j->r) \-----// 26
-----j->d->u = j->u, j->u->d = j->d, j->p->size--;-----// c1
--define UNCOVER(c, i, j) \-----// 89
-----for (node *i = c->u; i != c; i = i->u) \-----// f0
-----for (node *j = i->l; j != i; j = j->l) \-----// 7b
-----j->p->size++, j->d->u = j->u->d = j; \-----// 65
-----c->r->l = c->l->r = c;-----// 0e
--bool search(int k = 0) {-----// f9
--if (head == head->r) {-----// 75
--vi res(k);-----// 90
--rep(i,0,k) res[i] = sol[i];-----// 2a
--sort(res.begin(), res.end());-----// 63
--return handle_solution(res);-----// 11
--}-----// 3d
--node *c = head->r, *tmp = head->r;-----// a3
--for ( ; tmp != head; tmp = tmp->r) if (tmp->size < c->size) c = tmp;-----// 41
--if (c == c->d) return false;-----// 02
--COVER(c, i, j);-----// f6
--bool found = false;-----// 8d
--for (node *r = c->d; !found && r != c; r = r->d) {-----// 78
--sol[k] = r->row;-----// c0
--for (node *j = r->r; j != r; j = j->r) { COVER(j->p, a, b); }-----// f9
--found = search(k + 1);-----// fb
--for (node *j = r->l; j != r; j = j->l) { UNCOVER(j->p, a, b); }-----// 87
--}-----// 7c
--UNCOVER(c, i, j);-----// a7
--return found;-----// c0
--}-----// d2
};-----// d7
```

7.4. nth Permutation.

```
vector<int> nth_permutation(int cnt, int n) {-----// 78
--vector<int> idx(cnt), per(cnt), fac(cnt);-----// 9e
--rep(i,0,cnt) idx[i] = i;-----// bc
--rep(i,1,cnt+1) fac[i - 1] = n % i, n /= i;-----// 2b
--for (int i = cnt - 1; i >= 0; i--)-----// f9
--per[cnt - i - 1] = idx[fac[i]], idx.erase(idx.begin() + fac[i]);-----// ee
--return per;-----// ab
}-----// 37
```

7.5. Cycle-Finding.

```
ii find_cycle(int x0, int (*f)(int)) {-----// a5
--int t = f(x0), h = f(t), mu = 0, lam = 1;-----// 8d
--while (t != h) t = f(t), h = f(f(h));-----// 79
--h = x0;-----// 04
--while (t != h) t = f(t), h = f(h), mu++;-----// 9d
--h = f(t);-----// 00
--while (t != h) h = f(h), lam++;-----// 5e
--return ii(mu, lam);-----// b4
}-----// 42
```

7.6. Dates.

```
int intToDay(int jd) { return jd % 7; }-----// 89
int dateToInt(int y, int m, int d) {-----// 96
--return 1461 * (y + 4800 + (m - 14) / 12) / 4 +-----// a8
--367 * (m - 2 - (m - 14) / 12 * 12) / 12-----// d1
--3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +-----// be
--d - 32075;-----// e0
}-----// fa
void intToDate(int jd, int &y, int &m, int &d) {-----// a1
--int x, n, i, j;-----// 00
--x = jd + 68569;-----// 11
--n = 4 * x / 146097;-----// 2f
--x -= (146097 * n + 3) / 4;-----// 58
--i = (4000 * (x + 1)) / 1461001;-----// 0d
--x -= 1461 * i / 4 - 31;-----// 09
--j = 80 * x / 2447;-----// 3d
--d = x - 2447 * j / 80;-----// eb
--x = j / 11;-----// b7
--m = j + 2 - 12 * x;-----// 82
--y = 100 * (n - 49) + i + x;-----// 70
}-----// af
```

7.7. Simulated Annealing. An example use of Simulated Annealing to find a permutation of length n that maximizes $\sum_{i=1}^{n-1} |p_i - p_{i+1}|$.

```
double curtime() { return static_cast<double>(clock()) / CLOCKS_PER_SEC; }-----// 9d
int simulated_annealing(int n, double seconds) {-----// 54
--default_random_engine rng;-----// 67
--uniform_real_distribution<double> randfloat(0.0, 1.0);-----// ed
--uniform_int_distribution<int> randint(0, n - 2);-----// bb
--// random initial solution-----// 01
--vi sol(n);-----// 1c
--rep(i,0,n) sol[i] = i + 1;-----// 33
--random_shuffle(sol.begin(), sol.end());-----// ea
--// initialize score-----// 28
--int score = 0;-----// 7d
--rep(i,1,n) score += abs(sol[i] - sol[i-1]);-----// 61
--int iters = 0;-----// 0b
--double T0 = 100.0, T1 = 0.001,-----// 5c
--progress = 0, temp = T0,-----// 3a
--starttime = curtime();-----// d6
--while (true) {-----// 46
--if (!(iters & ((1 << 4) - 1))) {-----// 5d
--progress = (curtime() - starttime) / seconds;-----// 44
--temp = T0 * pow(T1 / T0, progress);-----// a7
--if (progress > 1.0) break; }-----// 8b
--// random mutation-----// eb
--int a = randint(rng);-----// c3
--// compute delta for mutation-----// 84
--int delta = 0;-----// 60
--if (a > 0) delta += abs(sol[a+1] - sol[a-1]) - abs(sol[a] - sol[a-1]);-----// 94
--if (a+2 < n) delta += abs(sol[a] - sol[a+2]) - abs(sol[a+1] - sol[a+2]);-----// 94
```

```
-----// maybe apply mutation-----// fb
-----if (delta >= 0 || randfloat(rng) < exp(delta / temp)) {------// 81
-----swap(sol[a], sol[a+1]);-----// b3
-----score += delta;-----// db
-----// if (score >= target) return;-----// 4d
-----}------// 5c
-----iters++; }-----// 28
----return score; }-----// ba
```

7.8. Fast Input Reading.

```
void readn(register int *n) {------// dc
----int sign = 1;-----// 32
----register char c;-----// a5
----*n = 0;-----// 35
----while((c = getc_unlocked(stdin)) != '\n') {------// f3
-----switch(c) {------// 0c
-----case '-': sign = -1; break;-----// 28
-----case ' ': goto hell;-----// fd
-----case '\n': goto hell;-----// 79
-----default: *n *= 10; *n += c - '0'; break;-----// c0
-----}------// 2d
----}------// c3
hell:-----// ba
----*n *= sign;-----// a0
}------// 67
```

7.9. Bit Hacks.

```
int snoob(int x) {------// 73
----int y = x & -x, z = x + y;-----// 12
----return z | ((x ^ z) >> 2) / y;-----// 97
}------// 14
```

Catalan	$C_0 = 1, C_n = \frac{1}{n+1} \binom{2n}{n} = \sum_{i=0}^{n-1} C_i C_{n-i-1} = \frac{4n-2}{n+1} C_{n-1}$	
Stirling 1st kind	$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = 1, \begin{bmatrix} n \\ 0 \end{bmatrix} = \begin{bmatrix} n \\ 0 \end{bmatrix} = 0, \begin{bmatrix} n \\ k \end{bmatrix} = (n-1) \begin{bmatrix} n-1 \\ k \end{bmatrix} + \begin{bmatrix} n-1 \\ k-1 \end{bmatrix}$	#perms of n objs with exactly k cycles
Stirling 2nd kind	$\left\{ \begin{smallmatrix} n \\ 1 \end{smallmatrix} \right\} = \left\{ \begin{smallmatrix} n \\ n \end{smallmatrix} \right\} = 1, \left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\} = k \left\{ \begin{smallmatrix} n-1 \\ k \end{smallmatrix} \right\} + \left\{ \begin{smallmatrix} n-1 \\ k-1 \end{smallmatrix} \right\}$	#ways to partition n objs into k nonempty sets
Euler	$\left\langle \begin{smallmatrix} n \\ 0 \end{smallmatrix} \right\rangle = \left\langle \begin{smallmatrix} n \\ n-1 \end{smallmatrix} \right\rangle = 1, \left\langle \begin{smallmatrix} n \\ k \end{smallmatrix} \right\rangle = (k+1) \left\langle \begin{smallmatrix} n-1 \\ k \end{smallmatrix} \right\rangle + (n-k) \left\langle \begin{smallmatrix} n-1 \\ k-1 \end{smallmatrix} \right\rangle$	#perms of n objs with exactly k ascents
Euler 2nd Order	$\left\langle\!\left\langle \begin{smallmatrix} n \\ k \end{smallmatrix} \right\rangle\!\right\rangle = (k+1) \left\langle\!\left\langle \begin{smallmatrix} n-1 \\ k \end{smallmatrix} \right\rangle\!\right\rangle + (2n-k-1) \left\langle\!\left\langle \begin{smallmatrix} n-1 \\ k-1 \end{smallmatrix} \right\rangle\!\right\rangle$	#perms of $1, 1, 2, 2, \dots, n, n$ with exactly k ascents
Bell	$B_1 = 1, B_n = \sum_{k=0}^{n-1} B_k \binom{n-1}{k} = \sum_{k=0}^n \left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\}$	#partitions of $1..n$ (Stirling 2nd, no limit on k)

#labeled rooted trees
 n^{n-1}

#labeled unrooted trees
 n^{n-2}

$\sum_{i=1}^n i^2 = n(n+1)(2n+1)/6$
 $\sum_{i=1}^n i^3 = n^2(n+1)^2/4$

$!n = n \times!(n-1) + (-1)^n$
 $!n = (n-1)!(n-1) +!(n-2)$

$\sum_{i=1}^n \binom{n}{i} F_i = F_{2n}$
 $\sum_{k=0}^n \binom{k}{m} = \binom{n+1}{m+1}$
 $\sum_i \binom{n-i}{i} = F_{n+1}$

$a \equiv b \pmod{x, y} \Rightarrow a \equiv b \pmod{\text{lcm}(x, y)}$
 $ac \equiv bc \pmod{m} \Rightarrow a \equiv b \pmod{\frac{m}{\text{gcd}(c, m)}}$
 $p \text{ prime} \Leftrightarrow (p-1)! \equiv -1 \pmod{p}$
 $\sum_{d|n} \phi(d) = n$
 $(\sum_{d|n} \sigma_0(d))^2 = \sum_{d|n} \sigma_0(d)^3$
 $\text{gcd}(n^a - 1, n^b - 1) = n^{\text{gcd}(a, b)} - 1$

$\sigma_x(n) = \prod_{i=0}^r \frac{p_i^{(a_i+1)x} - 1}{p_i^x - 1}$
 $\sum_{k=0}^m (-1)^k \binom{n}{k} = (-1)^m \binom{n-1}{m}$
 $2^{\omega(n)} = O(\sqrt{n})$
 $\sigma_0(n) = \prod_{i=0}^r (a_i + 1)$

$d = v_i t + \frac{1}{2} a t^2$
 $v_f = v_i + a t$
 $\sum_{i=1}^n 2^{\omega(i)} = O(n \log n)$
 $v_f^2 = v_i^2 + 2ad$
 $d = \frac{v_i + v_f}{2} t$

7.10. The Twelfold Way. Putting n balls into k boxes.

Balls	same	distinct	same	distinct	
Boxes	same	same	distinct	distinct	Remarks
-	$p_k(n)$	$\sum_{i=0}^k \left\{ \begin{smallmatrix} n \\ i \end{smallmatrix} \right\}$	$\binom{n+k-1}{k-1}$	k^n	$p_k(n)$: #partitions of n into $\leq k$ positive parts
size ≥ 1	$p(n, k)$	$\left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\}$	$\binom{n-1}{k-1}$	$k! \left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\}$	$p(n, k)$: #partitions of n into k positive parts
size ≤ 1	$[n \leq k]$	$[n \leq k]$	$\binom{k}{n}$	$n! \binom{k}{n}$	$[cond]$: 1 if $cond = true$, else 0

8. USEFUL INFORMATION			<ul style="list-style-type: none">· $A[i][j - 1] \leq A[i][j] \leq A[i + 1][j]$· $O(n^3)$ to $O(n^2)$· sufficient: QI and $C[b][c] \leq C[a][d], a \leq b \leq c \leq d$
9. MISC			<ul style="list-style-type: none">– Lovász Toggle– Look at the DFS tree (which has no cross-edges)
9.1. Debugging Tips.	<ul style="list-style-type: none">• Stack overflow? Recursive DFS on tree that is actually a long path?• Floating-point numbers<ul style="list-style-type: none">– Getting NaN? Make sure <code>acos</code> etc. are not getting values out of their range (perhaps <code>1+eps</code>).– Rounding negative numbers?– Outputting in scientific notation?• Wrong Answer?<ul style="list-style-type: none">– Read the problem statement again!– Are multiple test cases being handled correctly? Try repeating the same test case many times.– Integer overflow?– Think very carefully about boundaries of all input parameters– Try out possible edge cases:<ul style="list-style-type: none">* $n = 0, n = -1, n = 1, n = 2^{31} - 1$ or $n = -2^{31}$* List is empty, or contains a single element* n is even, n is odd* Graph is empty, or contains a single vertex* Graph is a multigraph (loops or multiple edges)* Polygon is concave or non-simple– Is initial condition wrong for small cases?– Are you sure the algorithm is correct?– Explain your solution to someone.– Are you using any functions that you don't completely understand? Maybe STL functions?– Maybe you (or someone else) should rewrite the solution?	<ul style="list-style-type: none">• Greedy• Randomized• Optimizations<ul style="list-style-type: none">– Use bitset (/64)– Switch order of loops (cache locality)• Process queries offline<ul style="list-style-type: none">– Mo's algorithm• Square-root decomposition• Precomputation• Efficient simulation<ul style="list-style-type: none">– Mo's algorithm– Sqrt decomposition– Store 2^k jump pointers• Data structure techniques<ul style="list-style-type: none">– Sqrt buckets– Store 2^k jump pointers– 2^k merging trick• Counting<ul style="list-style-type: none">– Inclusion-exclusion principle– Generating functions• Graphs<ul style="list-style-type: none">– Can we model the problem as a graph?– Can we use any properties of the graph?– Strongly connected components– Cycles (or odd cycles)– Bipartite (no odd cycles)<ul style="list-style-type: none">* Bipartite matching* Hall's marriage theorem* Stable Marriage– Cut vertex/bridge– Biconnected components– Degrees of vertices (odd/even)– Trees<ul style="list-style-type: none">* Heavy-light decomposition* Centroid decomposition* Least common ancestor– Eulerian path/circuit– Chinese postman problem– Topological sort– (Min-Cost) Max Flow– Min Cut<ul style="list-style-type: none">* Maximum Density Subgraph– Huffman Coding– Min-Cost Arborescence– Steiner Tree– Kirchoff's matrix tree theorem– Prüfer sequences	<ul style="list-style-type: none">• Mathematics<ul style="list-style-type: none">– Is the function multiplicative?– Look for a pattern– Permutations<ul style="list-style-type: none">* Consider the cycles of the permutation– Functions<ul style="list-style-type: none">* Sum of piecewise-linear functions is a piecewise-linear function* Sum of convex (concave) functions is convex (concave)– Modular arithmetic<ul style="list-style-type: none">* Chinese Remainder Theorem* Linear Congruence– Sieve– System of linear equations– Values too big to represent?<ul style="list-style-type: none">* Compute using the logarithm* Divide everything by some large value• Logic<ul style="list-style-type: none">– 2-SAT– XOR-SAT (Gauss elimination or Bipartite matching)• Meet in the middle• Only work with the smaller half ($\log(n)$)• Strings<ul style="list-style-type: none">– Trie (maybe over something weird, like bits)– Suffix array– Suffix automaton (+DP?)– Aho-Corasick– <code>eerTree</code>– Work with $S + S$• Hashing• Euler tour, tree to array• Segment trees<ul style="list-style-type: none">– Lazy propagation– Persistent– Implicit– Segment tree of X• Geometry<ul style="list-style-type: none">– Minkowski sum (of convex sets)– Rotating calipers– Sweep line (horizontally or vertically?)– Sweep angle– Convex hull• Fix a parameter (possibly the answer).• Are there few distinct values?• Binary search• Sliding Window (+ Monotonic Queue)• Computing a Convolution? Fast Fourier Transform• Exact Cover (+ Algorithm X)
9.2. Solution Ideas.	<ul style="list-style-type: none">• Dynamic Programming<ul style="list-style-type: none">– Drop a parameter, recover from others– Swap answer and a parameter– Parsing CFGs: CYK Algorithm– Optimizations<ul style="list-style-type: none">* Convex hull optimization<ul style="list-style-type: none">· $dp[i] = \min_{j < i} \{dp[j] + b[j] \times a[i]\}$· $b[j] \geq b[j + 1]$· optionally $a[i] \leq a[i + 1]$· $O(n^2)$ to $O(n)$* Divide and conquer optimization<ul style="list-style-type: none">· $dp[i][j] = \min_{k < j} \{dp[i - 1][k] + C[k][j]\}$· $A[i][j] \leq A[i][j + 1]$· $O(kn^2)$ to $O(kn \log n)$· sufficient: $C[a][c] + C[b][d] \leq C[a][d] + C[b][c], a \leq b \leq c \leq d$ (QI)* Knuth optimization<ul style="list-style-type: none">· $dp[i][j] = \min_{i < k < j} \{dp[i][k] + dp[k][j] + C[i][j]\}$		

- Cycle-Finding
- What is the smallest set of values that identify the solution? The cycle structure of the permutation? The powers of primes in the factorization?
- Look at the complement problem
 - Minimize something instead of maximizing
- Immediately enforce necessary conditions. (All values greater than 0? Initialize them all to 1)
- Add large constant to negative numbers to make them positive
- Counting/Bucket sort

FORMULAS

- **Jacobi symbol:** $\left(\frac{a}{b}\right) = a^{(b-1)/2} \pmod{b}$
- **Heron’s formula:** A triangle with side lengths a, b, c has area $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$.
- **Pick’s theorem:** A polygon on an integer grid containing i lattice points and having b lattice points on the boundary has area $i + \frac{b}{2} - 1$.
- **Euler’s totient:** The number of integers less than n that are coprime to n are $n \prod_{p|n} \left(1 - \frac{1}{p}\right)$ where each p is a distinct prime factor of n .
- **König’s theorem:** In any bipartite graph $G = (L \cup R, E)$, the number of edges in a maximum matching is equal to the number of vertices in a minimum vertex cover. Let U be the set of unmatched vertices in L , and Z be the set of vertices that are either in U or are connected to U by an alternating path. Then $K = (L \setminus Z) \cup (R \cap Z)$ is the minimum vertex cover.
- A minimum Steiner tree for n vertices requires at most $n - 2$ additional Steiner vertices.
- The number of vertices of a graph is equal to its minimum vertex cover number plus the size of a maximum independent set.
- **Wilson’s theorem:** $(n - 1)! \equiv -1 \pmod{n}$ iff. n is prime
- **Lagrange polynomial** through points $(x_0, y_0), \dots, (x_k, y_k)$ is
$$L(x) = \sum_{j=0}^k y_j \prod_{\substack{0 \leq m \leq k \\ m \neq j}} \frac{x - x_m}{x_j - x_m}$$
- **Hook length formula:** If λ is a Young diagram and $h_\lambda(i, j)$ is the hook-length of cell (i, j) , then the number of Young tableaux $d_\lambda = n! / \prod h_\lambda(i, j)$.
- **Möbius inversion formula:** If $f(n) = \sum_{d|n} g(d)$, then $g(n) = \sum_{d|n} \mu(d) f(n/d)$

9.3. Markov Chains. A Markov Chain can be represented as a weighted directed graph of states, where the weight of an edge represents the probability of transitioning over that edge in one timestep. Let $P^{(m)} = (p_{ij}^{(m)})$ be the probability matrix of transitioning from state i to state j in m timesteps, and note that $P^{(1)}$ is the adjacency matrix of the graph. **Chapman-Kolmogorov:** $p_{ij}^{(m+n)} = \sum_k p_{ik}^{(m)} p_{kj}^{(n)}$. It follows that $P^{(m+n)} = P^{(m)} P^{(n)}$ and $P^{(m)} = P^m$. If $p^{(0)}$ is the initial probability distribution (a vector), then $p^{(0)} P^{(m)}$ is the probability distribution after m timesteps.

The return times of a state i is $R_i = \{m \mid p_{ii}^{(m)} > 0\}$, and i is *aperiodic* if $\gcd(R_i) = 1$. A MC is aperiodic if any of its vertices is aperiodic. A MC is *irreducible* if the corresponding graph is strongly connected.

A distribution π is stationary if $\pi P = \pi$. If MC is irreducible then $\pi_i = 1/\mathbb{E}[T_i]$, where T_i is the expected time between two visits at i . π_j/π_i is the expected number of visits at j in between two consecutive visits at i . A MC is *ergodic* if $\lim_{m \rightarrow \infty} p^{(0)} P^m = \pi$. A MC is ergodic iff. it is irreducible and aperiodic.

A MC for a random walk in an undirected weighted graph (unweighted graph can be made weighted by adding 1-weights) has $p_{uv} = w_{uv} / \sum_x w_{ux}$. If the graph is connected, then $\pi_u = \sum_x w_{ux} / \sum_v \sum_x w_{vx}$. Such a random walk is aperiodic iff. the graph is not bipartite.

An *absorbing* MC is of the form $P = \begin{pmatrix} Q & R \\ 0 & I_r \end{pmatrix}$. Let $N = \sum_{m=0}^\infty Q^m = (I_t - Q)^{-1}$. Then, if starting in state i , the expected number of steps till absorption is the i -th entry in $N1$. If starting in state i , the probability of being absorbed in state j is the (i, j) -th entry of NR .

Many problems on MC can be formulated in terms of a system of recurrence relations, and then solved using Gaussian elimination.

9.4. Burnside’s Lemma. Let G be a finite group that acts on a set X . For each g in G let X^g denote the set of elements in X that are fixed by g . Then the number of orbits

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

9.5. Bézout’s identity. If (x, y) is any solution to $ax+by = d$ (e.g. found by the Extended Euclidean Algorithm), then all solutions are given by

$$\left(x + k \frac{b}{\gcd(a, b)}, y - k \frac{a}{\gcd(a, b)}\right)$$

PRACTICE CONTEST CHECKLIST

- How many operations per second? Compare to local machine.
- What is the stack size?
- How to use printf/scanf with long long/long double?
- Is `__int128` available?
- Does MLE give RTE or MLE as a verdict? What about stack overflow?
- What is `RAND_MAX`?
- How does the judge handle extra spaces (or missing newlines) in the output?
- Look at documentation for programming languages.
- Try different programming languages: C++ and Java.
- Try the submit script.
- Try local programs: `i?python[23]`, `factor`.
- Try submitting with `assert(false)` and `assert(false)`.
- Return-value from `main`.
- Remove this page from the notebook.