.*RU.*

Team Reference Document

18/04/2019

Contents

1. Code Templates 1
1.1. Basic Configuration 1
1.2. C++ Header 1
1.3. Java Template 2
2. Data Structures 2
2.1. Union-Find 2
2.2. Segment Tree 2
2.3. Fenwick Tree 2
2.4. Matrix 3
2.5. AVL Tree 3
2.6. Cartesian Tree 4
2.7. Heap 4
2.8. Dancing Links 4
2.9. Misof Tree 4
2.10. k-d Tree 5
2.11. Sqrt Decomposition 5
2.12. Monotonic Queue 5
2.13. Convex Hull Trick 5
2.14. Sparse Table 6
3. Graphs 6
3.1. Single-Source Shortest Paths 6
3.2. All-Pairs Shortest Paths 6
3.3. Strongly Connected Components 6
3.4. Cut Points and Bridges 7
3.5. Minimum Spanning Tree 7
3.6. Topological Sort 7
3.7. Euler Path 7
3.8. Bipartite Matching 7
3.9. Maximum Flow 8
3.10. Minimum Cost Maximum Flow 8
3.11. All Pairs Maximum Flow 8
3.12. Heavy-Light Decomposition 9
3.13. Centroid Decomposition 9
3.14. Least Common Ancestors, Binary Jumping 9
3.15. Tarjan's Off-line Lowest Common Ancestors Algorithm 9
3.16. Minimum Mean Weight Cycle 10
3.17. Minimum Arborescence 10
3.18. Blossom algorithm 10
3.19. Maximum Density Subgraph 10
3.20. Maximum-Weight Closure 10
3.21. Maximum Weighted Independent Set in a Bipartite Graph 11
3.22. Synchronizing word problem 11
3.23. Max flow with lower bounds on edges 11
3.24. Tutte matrix for general matching 11
4. Strings 11
4.1. The Knuth-Morris-Pratt algorithm 11
4.2. The Z algorithm 11
4.3. Trie 11
4.4. Suffix Array 11
4.5. Aho-Corasick Algorithm 11
4.6. eerTree 12
4.7. Suffix Automaton 12
4.8. Hashing 12
5. Mathematics 12
5.1. Fraction 12
5.2. Big Integer 13
5.3. Binomial Coefficients 14
5.4. Euclidean algorithm 14
5.5. Trial Division Primality Testing 14
5.6. Miller-Rabin Primality Test 14
5.7. Pollard's ρ algorithm 14
5.8. Sieve of Eratosthenes 14
5.9. Divisor Sieve 14
5.10. Modular Exponentiation 14
5.11. Modular Multiplicative Inverse 14
5.12. Primitive Root 14
5.13. Chinese Remainder Theorem 14
5.14. Linear Congruence Solver 15
5.15. Tonelli-Shanks algorithm 15
5.16. Numeric Integration 15
5.17. Fast Fourier Transform 15
5.18. Number-Theoretic Transform 15
5.19. Fast Hadamard Transform 15
5.20. Tridiagonal Matrix Algorithm 16
5.21. Mertens Function 16
5.22. Summatory Phi 16
5.23. Prime π 16
5.24. Josephus problem 16
5.25. Number of Integer Points under Line 16
5.26. Numbers and Sequences 16
5.27. Game Theory 16
6. Geometry 16
6.1. Primitives 16
6.2. Lines 16
6.3. Circles 17
6.4. Polygon 17
6.5. Convex Hull 17
6.6. Line Segment Intersection 17
6.7. Great-Circle Distance 17
6.8. Triangle Circumcenter 17
6.9. Closest Pair of Points 18
6.10. 3D Primitives 18
6.11. Polygon Centroid 18
6.12. Rotating Calipers 18
6.13. Rectilinear Minimum Spanning Tree 19
6.14. Line upper/lower envelope 19
6.15. Formulas 19
7. Other Algorithms 19
7.1. 2SAT 19
7.2. DPLL Algorithm 20
7.3. Stable Marriage 20
7.4. Algorithm X 20
7.5. nth Permutation 20
7.6. Cycle-Finding 20
7.7. Longest Increasing Subsequence 20
7.8. Dates 20
7.9. Simulated Annealing 21
7.10. Simplex 21
7.11. Fast Square Testing 21
7.12. Fast Input Reading 22
7.13. 128-bit Integer 22
7.14. Bit Hacks 22
7.15. The Twelvefold Way 23
8. Useful Information 24
9. Misc 24
9.1. Debugging Tips 24
9.2. Solution Ideas 24
10. Formulas 25
10.1. Physics 25
10.2. Markov Chains 25
10.3. Burnside's Lemma 25
10.4. Bézout's identity 25
10.5. Misc 25
Practice Contest Checklist 26

1. Code Templates

1.1. Basic Configuration.

1.1.1. *.bashrc.*

```
xset r rate 150 100 ---------------------------------------//dd
set -o vi -------------------------------------------------//f4
 ----------------------------------------------------------//08
function check { ------------------------------------------//f6
- IFS= ----------------------------------------------------//17
- s="" ----------------------------------------------------//22
- cat $1 | while read l; do -------------------------------//81
--- s="$s$(echo $l | sed 's/\s//g')\n" --------------------//d0
--- h=$(echo -ne "$s" | md5sum) ---------------------------//27
--- echo "${h:0:2}  $l" -----------------------------------//1b
- done ----------------------------------------------------//aa
} ---------------------------------------------------------//a7
# setxkbmap -option caps:escape dvorak is -----------------//18
# setxkbmap en_US -----------------------------------------//97
alias c='cd' ----------------------------------------------//7e
alias l='ls -lh' ------------------------------------------//36
alias la='ls -lah' ----------------------------------------//7c
```

ProTip™: setxkbmap dvorak on qwerty: `o.yqtxmal ekrpat`

1.1.2. *.vimrc.*

```
set nocp et sw=4 ts=4 sts=4 si cindent hi=1000 nu ru noeb -//2f
set showcmd showmode --------------------------------------//a9
syn on | colorscheme slate --------------------------------//bf
```

1.2. C++ Header. A C++ header.

```
#include <bits/stdc++.h> ----------------------------------//84
using namespace std; --------------------------------------//16
template <class T> int size(const T &x) { return x.size(); }
#define rep(i,a,b) for (__typeof(a) i=(a); i<(b); ++i) ----//6c
#define iter(it,c) for (__typeof((c).begin()) \ ----------//fa
```

```
- it = (c).begin(); it != (c).end(); ++it) ----------------//ed
typedef pair<int, int> ii; ---------------------------------//f2
typedef vector<int> vi; ------------------------------------//0a
typedef vector<ii> vii; ------------------------------------//66
typedef long long ll; --------------------------------------//6e
const int INF = ~(1<<31); ----------------------------------//e7
 -----------------------------------------------------------//96
const double EPS = 1e-9; -----------------------------------//5e
const double pi = acos(-1); --------------------------------//5d
typedef unsigned long long ull; ----------------------------//fd
typedef vector<vi> vvi; ------------------------------------//10
typedef vector<vii> vvii; ----------------------------------//7f
template <class T> T smod(T a, T b) { ----------------------//6f
- return (a % b + b) % b; } --------------------------------//24
```

1.3. **Java Template.** A Java template.

```
import java.util.*; ----------------------------------------//37
import java.math.*; ----------------------------------------//89
import java.io.*; ------------------------------------------//28
public class Main { ----------------------------------------//cb
- public static void main(String[] args) throws Exception {//c3
--- Scanner in = new Scanner(System.in); -------------------//a3
--- PrintWriter out = new PrintWriter(System.out, false); -//00
--- // code -------------------------------------------------//60
--- out.flush(); } } ---------------------------------------//72
```

2. Data Structures

2.1. **Union-Find.** An implementation of the Union-Find disjoint sets data structure.

```
struct union_find { ----------------------------------------//42
- vi p; union_find(int n) : p(n, -1) { } -------------------//28
- int find(int x) { return p[x] < 0 ? x : p[x] = find(p[x]); }
- bool unite(int x, int y) { -------------------------------//6c
--- int xp = find(x), yp = find(y); ------------------------//64
--- if (xp == yp) return false; ----------------------------//0b
--- if (p[xp] > p[yp]) swap(xp,yp); ------------------------//78
--- p[xp] += p[yp], p[yp] = xp; ----------------------------//88
--- return true; } -----------------------------------------//1f
- int size(int x) { return -p[find(x)]; } }; ---------------//b9
```

2.2. **Segment Tree.** An implementation of a Segment Tree.

```
#ifndef STNODE ---------------------------------------------//3c
#define STNODE ---------------------------------------------//69
struct node { ----------------------------------------------//89
- int l, r; ------------------------------------------------//bf
- ll x, lazy; ----------------------------------------------//b4
- node() {} ------------------------------------------------//5b
- node(int _l, int _r) : l(_l), r(_r), x(0), lazy(0) { } --//c9
- node(int _l, int _r, ll _x) : node(_l,_r) { x = _x; } ---//16
- node(node a, node b) : node(a.l,b.r) { x = a.x + b.x; } -//77
- void update(ll v) { x = v; } -----------------------------//13
- void range_update(ll v) { lazy = v; } --------------------//b5
- void apply() { x += lazy * (r - l + 1); lazy = 0; } -----//e6
- void push(node &u) { u.lazy += lazy; } }; ----------------//eb
#endif -----------------------------------------------------//fc
#ifndef STNODE ---------------------------------------------//3c
#define STNODE ---------------------------------------------//69
struct node { ----------------------------------------------//89
- int l, r; ------------------------------------------------//bf
- int x, lazy; ---------------------------------------------//05
- node() {} ------------------------------------------------//30
- node(int _l, int _r) : l(_l), r(_r), x(INF), lazy(0) { } //ac
- node(int _l, int _r, int _x) : node(_l,_r) { x = _x; } --//d0
- node(node a, node b) : node(a.l,b.r) { x = min(a.x, b.x); }
- void update(int v) { x = v; } ----------------------------//c0
- void range_update(int v) { lazy = v; } -------------------//55
- void apply() { x += lazy; lazy = 0; } --------------------//7d
- void push(node &u) { u.lazy += lazy; } }; ----------------//5c
#endif -----------------------------------------------------//1c
#include "segment_tree_node.cpp" ---------------------------//8e
struct segment_tree { --------------------------------------//1e
- int n; ---------------------------------------------------//ad
- vector<node> arr; ----------------------------------------//37
- segment_tree() { } ---------------------------------------//ee
- segment_tree(const vector<ll> &a) : n(size(a)), arr(4*n) {
--- mk(a,0,0,n-1); } ---------------------------------------//8c
- node mk(const vector<ll> &a, int i, int l, int r) { -----//e2
--- int m = (l+r)/2; ---------------------------------------//d6
--- return arr[i] = l > r ? node(l,r) : --------------------//88
----- l == r ? node(l,r,a[l]) : ----------------------------//4c
----- node(mk(a,2*i+1,l,m),mk(a,2*i+2,m+1,r)); } -----------//49
- node update(int at, ll v, int i=0) { ---------------------//37
--- propagate(i); ------------------------------------------//15
--- int hl = arr[i].l, hr = arr[i].r; ----------------------//35
--- if (at < hl || hr < at) return arr[i]; ----------------//b1
--- if (hl == at && at == hr) { ----------------------------//bb
----- arr[i].update(v); return arr[i]; } -------------------//a4
--- return arr[i] = -----------------------------------------//20
----- node(update(at,v,2*i+1),update(at,v,2*i+2)); } ------//d0
- node query(int l, int r, int i=0) { ----------------------//10
--- propagate(i); ------------------------------------------//74
--- int hl = arr[i].l, hr = arr[i].r; ----------------------//5e
--- if (r < hl || hr < l) return node(hl,hr); --------------//1a
--- if (l <= hl && hr <= r) return arr[i]; -----------------//35
--- return node(query(l,r,2*i+1),query(l,r,2*i+2)); } -----//b6
- node range_update(int l, int r, ll v, int i=0) { --------//16
--- propagate(i); ------------------------------------------//d2
--- int hl = arr[i].l, hr = arr[i].r; ----------------------//6c
--- if (r < hl || hr < l) return arr[i]; -------------------//3c
--- if (l <= hl && hr <= r) --------------------------------//72
----- return arr[i].range_update(v), propagate(i), arr[i]; //f4
--- return arr[i] = node(range_update(l,r,v,2*i+1), -------//94
----------------------- range_update(l,r,v,2*i+2)); } ----//db
- void propagate(int i) { ----------------------------------//43
--- if (arr[i].l < arr[i].r) -------------------------------//ac
----- arr[i].push(arr[2*i+1]), arr[i].push(arr[2*i+2]); ---//a7
--- arr[i].apply(); } }; -----------------------------------//4a
```

2.2.1. *Persistent Segment Tree.*

```
int segcnt = 0; --------------------------------------------//cf
struct segment { -------------------------------------------//68
- int l, r, lid, rid, sum; ---------------------------------//fc
} segs[2000000]; -------------------------------------------//dd
int build(int l, int r) { ----------------------------------//2b
- if (l > r) return -1; ------------------------------------//4e
- int id = segcnt++; ---------------------------------------//a8
- segs[id].l = l; ------------------------------------------//90
- segs[id].r = r; ------------------------------------------//19
- if (l == r) segs[id].lid = -1, segs[id].rid = -1; -------//ee
- else { ---------------------------------------------------//fe
--- int m = (l + r) / 2; -----------------------------------//14
--- segs[id].lid = build(l , m); ---------------------------//e3
--- segs[id].rid = build(m + 1, r); } ----------------------//69
- segs[id].sum = 0; ----------------------------------------//21
- return id; } ---------------------------------------------//c5
int update(int idx, int v, int id) { -----------------------//b8
- if (id == -1) return -1; ---------------------------------//bb
- if (idx < segs[id].l || idx > segs[id].r) return id; ----//fb
- int nid = segcnt++; --------------------------------------//b3
- segs[nid].l = segs[id].l; --------------------------------//78
- segs[nid].r = segs[id].r; --------------------------------//ca
- segs[nid].lid = update(idx, v, segs[id].lid); -----------//92
- segs[nid].rid = update(idx, v, segs[id].rid); -----------//06
- segs[nid].sum = segs[id].sum + v; ------------------------//1a
- return nid; } --------------------------------------------//e6
int query(int id, int l, int r) { --------------------------//a2
- if (r < segs[id].l || segs[id].r < l) return 0; ---------//17
- if (l <= segs[id].l && segs[id].r <= r) return segs[id].sum;
- return query(segs[id].lid, l, r) ------------------------//5e
------ + query(segs[id].rid, l, r); } ---------------------//ce
```

2.3. **Fenwick Tree.** A Fenwick Tree is a data structure that represents an array of n numbers. It supports adjusting the i-th element in $O(\log n)$ time, and computing the sum of numbers in the range $i..j$ in $O(\log n)$ time. It only needs $O(n)$ space.

```
struct fenwick_tree { --------------------------------------//98
- int n; vi data; ------------------------------------------//d3
- fenwick_tree(int _n) : n(_n), data(vi(n)) { } -----------//db
- void update(int at, int by) { ----------------------------//76
--- while (at < n) data[at] += by, at |= at + 1; } --------//fb
- int query(int at) { --------------------------------------//71
--- int res = 0; -------------------------------------------//c3
--- while (at >= 0) res += data[at], at = (at & (at + 1)) - 1;
--- return res; } ------------------------------------------//e4
- int rsq(int a, int b) { return query(b) - query(a - 1); }//be
}; ---------------------------------------------------------//57
struct fenwick_tree_sq { -----------------------------------//d4
- int n; fenwick_tree x1, x0; ------------------------------//18
- fenwick_tree_sq(int _n) : n(_n), x1(fenwick_tree(n)), ---//2e
--- x0(fenwick_tree(n)) { } --------------------------------//7c
- // insert f(y) = my + c if x <= y -------------------------//17
- void update(int x, int m, int c) { -----------------------//fc
--- x1.update(x, m); x0.update(x, c); } --------------------//d6
- int query(int x) { return x*x1.query(x) + x0.query(x); } //02
}; ---------------------------------------------------------//ba
void range_update(fenwick_tree_sq &s, int a, int b, int k) {
- s.update(a, k, k * (1 - a)); s.update(b+1, -k, k * b); } //7b
```

```
int range_query(fenwick_tree_sq &s, int a, int b) { -------//83
- return s.query(b) - s.query(a-1); } ---------------------//31
```

2.4. **Matrix.** A Matrix class.

```
template <class K> bool eq(K a, K b) { return a == b; } ---//2a
template <> bool eq<double>(double a, double b) { ---------//f1
--- return abs(a - b) < EPS; } ----------------------------//14
template <class T> struct matrix { ------------------------//0c
- int rows, cols, cnt; vector<T> data; --------------------//b6
- inline T& at(int i, int j) { return data[i * cols + j]; }//53
- matrix(int r, int c) : rows(r), cols(c), cnt(r * c) { ---//f5
--- data.assign(cnt, T(0)); } -----------------------------//5b
- matrix(const matrix& other) : rows(other.rows), ---------//d8
--- cols(other.cols), cnt(other.cnt), data(other.data) { } //59
- T& operator()(int i, int j) { return at(i, j); } --------//db
- matrix<T> operator +(const matrix& other) { -------------//1f
--- matrix<T> res(*this); rep(i,0,cnt) --------------------//09
----- res.data[i] += other.data[i]; return res; } ---------//0d
- matrix<T> operator -(const matrix& other) { -------------//41
--- matrix<T> res(*this); rep(i,0,cnt) --------------------//9c
----- res.data[i] -= other.data[i]; return res; } ---------//b5
- matrix<T> operator *(T other) { -------------------------//5d
--- matrix<T> res(*this); ---------------------------------//72
--- rep(i,0,cnt) res.data[i] *= other; return res; } ------//7a
- matrix<T> operator *(const matrix& other) { -------------//98
--- matrix<T> res(rows, other.cols); ----------------------//96
--- rep(i,0,rows) rep(k,0,cols) rep(j,0,other.cols) -------//27
----- res(i, j) += at(i, k) * other.data[k * other.cols + j];
--- return res; } -----------------------------------------//11
- matrix<T> pow(ll p) { -----------------------------------//75
--- matrix<T> res(rows, cols), sq(*this); -----------------//82
--- rep(i,0,rows) res(i, i) = T(1); -----------------------//93
--- while (p) { -------------------------------------------//12
----- if (p & 1) res = res * sq; --------------------------//6e
----- p >>= 1; --------------------------------------------//8c
----- if (p) sq = sq * sq; --------------------------------//6a
--- } return res; } ---------------------------------------//81
- matrix<T> rref(T &det, int &rank) { ---------------------//0b
--- matrix<T> mat(*this); det = T(1), rank = 0; -----------//c9
--- for (int r = 0, c = 0; c < cols; c++) { ---------------//99
----- int k = r; ------------------------------------------//f0
----- rep(i,k+1,rows) if (abs(mat(i,c)) > abs(mat(k,c))) k = i;
----- if (k >= rows || eq<T>(mat(k, c), T(0))) continue; --//be
----- if (k != r) { ---------------------------------------//6a
------- det *= T(-1); -------------------------------------//1b
------- rep(i,0,cols) swap(mat.at(k, i), mat.at(r, i)); ---//f8
----- } det *= mat(r, r); rank++; -------------------------//0c
----- T d = mat(r,c); -------------------------------------//af
----- rep(i,0,cols) mat(r, i) /= d; -----------------------//b8
----- rep(i,0,rows) { -------------------------------------//dc
------- T m = mat(i, c); ----------------------------------//41
------- if (i != r && !eq<T>(m, T(0))) --------------------//64
--------- rep(j,0,cols) mat(i, j) -= m * mat(r, j); -------//6f
----- } r++; ----------------------------------------------//9a
--- } return mat; } ---------------------------------------//6e
- matrix<T> transpose() { ---------------------------------//24
--- matrix<T> res(cols, rows); ----------------------------//b7
--- rep(i,0,rows) rep(j,0,cols) res(j, i) = at(i, j); -----//48
--- return res; } }; --------------------------------------//60
```

2.5. **AVL Tree.** A fast, easily augmentable, balanced binary search tree.

```
#define AVL_MULTISET 0 ------------------------------------//b5
template <class T> ----------------------------------------//66
struct avl_tree { -----------------------------------------//b1
- struct node { -------------------------------------------//db
--- T item; node *p, *l, *r; ------------------------------//5d
--- int size, height; -------------------------------------//0d
--- node(const T &_item, node *_p = NULL) : item(_item), p(_p),
--- l(NULL), r(NULL), size(1), height(0) { } }; -----------//ad
- avl_tree() : root(NULL) { } -----------------------------//df
- node *root; ---------------------------------------------//15
- inline int sz(node *n) const { return n ? n->size : 0; } //6a
- inline int height(node *n) const { ----------------------//8c
--- return n ? n->height : -1; } --------------------------//c6
- inline bool left_heavy(node *n) const { -----------------//6c
--- return n && height(n->l) > height(n->r); } ------------//33
- inline bool right_heavy(node *n) const { ----------------//c1
--- return n && height(n->r) > height(n->l); } ------------//4d
- inline bool too_heavy(node *n) const { ------------------//33
--- return n && abs(height(n->l) - height(n->r)) > 1; } ---//39
- void delete_tree(node *n) { if (n) { --------------------//41
--- delete_tree(n->l), delete_tree(n->r); delete n; } } ---//97
- node*& parent_leg(node *n) { ----------------------------//1a
--- if (!n->p) return root; -------------------------------//6e
--- if (n->p->l == n) return n->p->l; ---------------------//d3
--- if (n->p->r == n) return n->p->r; ---------------------//dc
--- assert(false); } --------------------------------------//74
- void augment(node *n) { ---------------------------------//e6
--- if (!n) return; ---------------------------------------//44
--- n->size = 1 + sz(n->l) + sz(n->r); --------------------//2e
--- n->height = 1 + max(height(n->l), height(n->r)); } ----//0a
- #define rotate(l, r) \ -------------------------------//42
--- node *l = n->l; \ ---------------------------------//30
--- l->p = n->p; \ ------------------------------------//3d
--- parent_leg(n) = l; \ ------------------------------//c7
--- n->l = l->r; \ ------------------------------------//1e
--- if (l->r) l->r->p = n; \ --------------------------//66
--- l->r = n, n->p = l; \ -----------------------------//13
--- augment(n), augment(l) --------------------------------//be
- void left_rotate(node *n) { rotate(r, l); } -------------//96
- void right_rotate(node *n) { rotate(l, r); } ------------//cf
- void fix(node *n) { --------------------------------------//47
--- while (n) { augment(n); -------------------------------//b0
----- if (too_heavy(n)) { ---------------------------------//d9
------- if (left_heavy(n) && right_heavy(n->l)) -----------//3c
--------- left_rotate(n->l); ------------------------------//5c
------- else if (right_heavy(n) && left_heavy(n->r)) ------//d7
--------- right_rotate(n->r); -----------------------------//2e
------- if (left_heavy(n)) right_rotate(n); ---------------//71
------- else left_rotate(n); ------------------------------//fb
------- n = n->p; } ---------------------------------------//e4
----- n = n->p; } } ---------------------------------------//93
- inline int size() const { return sz(root); } ------------//13
- node* find(const T &item) const { -----------------------//c1
--- node *cur = root; -------------------------------------//84
--- while (cur) { -----------------------------------------//34
----- if (cur->item < item) cur = cur->r; -----------------//bf
----- else if (item < cur->item) cur = cur->l; ------------//ce
----- else break; } ---------------------------------------//aa
--- return cur; } -----------------------------------------//80
- node* insert(const T &item) { ---------------------------//2f
--- node *prev = NULL, **cur = &root; ---------------------//64
--- while (*cur) { ----------------------------------------//9a
----- prev = *cur; ----------------------------------------//78
----- if ((*cur)->item < item) cur = &((*cur)->r); --------//52
#if AVL_MULTISET ------------------------------------------//be
----- else cur = &((*cur)->l); ----------------------------//5a
#else -----------------------------------------------------//ce
----- else if (item < (*cur)->item) cur = &((*cur)->l); ---//63
----- else return *cur; -----------------------------------//8a
#endif ----------------------------------------------------//46
--- } -----------------------------------------------------//cc
--- node *n = new node(item, prev); -----------------------//1e
--- *cur = n, fix(n); return n; } -------------------------//5b
- void erase(const T &item) { erase(find(item)); } --------//ac
- void erase(node *n, bool free = true) { -----------------//23
--- if (!n) return; ---------------------------------------//42
--- if (!n->l && n->r) parent_leg(n) = n->r, n->r->p = n->p;
--- else if (n->l && !n->r) -------------------------------//19
----- parent_leg(n) = n->l, n->l->p = n->p; ---------------//ab
--- else if (n->l && n->r) { ------------------------------//0c
----- node *s = successor(n); -----------------------------//12
----- erase(s, false); ------------------------------------//b0
----- s->p = n->p, s->l = n->l, s->r = n->r; --------------//5e
----- if (n->l) n->l->p = s; ------------------------------//aa
----- if (n->r) n->r->p = s; ------------------------------//6c
----- parent_leg(n) = s, fix(s); --------------------------//c7
----- return; ---------------------------------------------//0e
--- } else parent_leg(n) = NULL; --------------------------//fc
--- fix(n->p), n->p = n->l = n->r = NULL; -----------------//a0
--- if (free) delete n; } ---------------------------------//f6
- node* successor(node *n) const { ------------------------//c0
--- if (!n) return NULL; ----------------------------------//07
--- if (n->r) return nth(0, n->r); ------------------------//6c
--- node *p = n->p; ---------------------------------------//ed
--- while (p && p->r == n) n = p, p = p->p; ---------------//54
--- return p; } -------------------------------------------//15
- node* predecessor(node *n) const { ----------------------//12
--- if (!n) return NULL; ----------------------------------//c7
--- if (n->l) return nth(n->l->size-1, n->l); -------------//e1
--- node *p = n->p; ---------------------------------------//11
--- while (p && p->l == n) n = p, p = p->p; ---------------//ec
--- return p; } -------------------------------------------//5e
- node* nth(int n, node *cur = NULL) const { --------------//ab
--- if (!cur) cur = root; ---------------------------------//6d
--- while (cur) { -----------------------------------------//45
----- if (n < sz(cur->l)) cur = cur->l; -------------------//2e
```

```
----- else if (n > sz(cur->l)) ----------------------------//b4
------- n -= sz(cur->l) + 1, cur = cur->r; ----------------//28
----- else break; -----------------------------------------//c5
--- } return cur; } ---------------------------------------//2d
- int count_less(node *cur) { -----------------------------//f7
--- int sum = sz(cur->l); ---------------------------------//1f
--- while (cur) { -----------------------------------------//03
----- if (cur->p && cur->p->r == cur) sum += 1 + sz(cur->p->l);
----- cur = cur->p; ---------------------------------------//b8
--- } return sum; } ---------------------------------------//32
- void clear() { delete_tree(root), root = NULL; } }; -----//b8
```

Also a very simple wrapper over the AVL tree that implements a map interface.

```
#include "avl_tree.cpp" -----------------------------------//01
template <class K, class V> struct avl_map { --------------//dc
- struct node { -------------------------------------------//58
--- K key; V value; ---------------------------------------//78
--- node(K k, V v) : key(k), value(v) { } -----------------//89
--- bool operator <(const node &other) const { ------------//bb
----- return key < other.key; } }; ------------------------//4b
- avl_tree<node> tree; ------------------------------------//f9
- V& operator [](K key) { ---------------------------------//e6
--- typename avl_tree<node>::node *n = --------------------//45
----- tree.find(node(key, V(0))); -------------------------//d6
--- if (!n) n = tree.insert(node(key, V(0))); -------------//c8
--- return n->item.value; } }; ----------------------------//1f
```

2.6. Cartesian Tree.

```
struct node { ---------------------------------------------//36
- int x, y, sz; -------------------------------------------//e5
- node *l, *r; --------------------------------------------//4d
- node(int _x, int _y) ------------------------------------//4b
--- : x(_x), y(_y), sz(1), l(NULL), r(NULL) { } }; --------//b8
int tsize(node* t) { return t ? t->sz : 0; } --------------//cb
void augment(node *t) { -----------------------------------//21
- t->sz = 1 + tsize(t->l) + tsize(t->r); } ----------------//dd
pair<node*,node*> split(node *t, int x) { -----------------//59
- if (!t) return make_pair((node*)NULL,(node*)NULL); ------//43
- if (t->x < x) { ------------------------------------------//1f
--- pair<node*,node*> res = split(t->r, x); ---------------//49
--- t->r = res.first; augment(t); -------------------------//30
--- return make_pair(t, res.second); } --------------------//16
- pair<node*,node*> res = split(t->l, x); -----------------//97
- t->l = res.second; augment(t); --------------------------//1b
- return make_pair(res.first, t); } -----------------------//ff
node* merge(node *l, node *r) { ---------------------------//e1
- if (!l) return r; if (!r) return l; ---------------------//15
- if (l->y > r->y) { ---------------------------------------//c6
--- l->r = merge(l->r, r); augment(l); return l; } --------//77
- r->l = merge(l, r->l); augment(r); return r; } ----------//56
node* find(node *t, int x) { ------------------------------//49
- while (t) { ---------------------------------------------//18
--- if (x < t->x) t = t->l; -------------------------------//55
--- else if (t->x < x) t = t->r; --------------------------//f8
--- else return t; } --------------------------------------//69
- return NULL; } ------------------------------------------//84
node* insert(node *t, int x, int y) { ---------------------//b0
- if (find(t, x) != NULL) return t; -----------------------//f4
- pair<node*,node*> res = split(t, x); --------------------//9f
- return merge(res.first, ---------------------------------//5a
----- merge(new node(x, y), res.second)); } ---------------//3f
node* erase(node *t, int x) { -----------------------------//be
- if (!t) return NULL; ------------------------------------//44
- if (t->x < x) t->r = erase(t->r, x); --------------------//17
- else if (x < t->x) t->l = erase(t->l, x); ---------------//07
- else { node *old = t; t = merge(t->l, t->r); delete old; }
- if (t) augment(t); return t; } --------------------------//a1
int kth(node *t, int k) { ---------------------------------//a2
- if (k < tsize(t->l)) return kth(t->l, k); ---------------//cd
- else if (k == tsize(t->l)) return t->x; -----------------//fe
- else return kth(t->r, k - tsize(t->l) - 1); } -----------//2c
```

2.7. Heap.

An implementation of a binary heap.

```
#define RESIZE --------------------------------------------//d0
#define SWP(x,y) tmp = x, x = y, y = tmp ------------------//fb
struct default_int_cmp { ----------------------------------//8d
- default_int_cmp() { } -----------------------------------//35
- bool operator ()(const int &a, const int &b) { ----------//1a
--- return a < b; } }; ------------------------------------//d9
template <class Compare = default_int_cmp> struct heap { --//3d
- int len, count, *q, *loc, tmp; --------------------------//24
- Compare _cmp; -------------------------------------------//63
- inline bool cmp(int i, int j) { return _cmp(q[i], q[j]); }
- inline void swp(int i, int j) { -------------------------//28
--- SWP(q[i], q[j]), SWP(loc[q[i]], loc[q[j]]); } ---------//27
- void swim(int i) { --------------------------------------//36
--- while (i > 0) { ---------------------------------------//05
----- int p = (i - 1) / 2; --------------------------------//71
----- if (!cmp(i, p)) break; ------------------------------//7f
----- swp(i, p), i = p; } } -------------------------------//32
- void sink(int i) { --------------------------------------//ec
--- while (true) { ----------------------------------------//ee
----- int l = 2*i + 1, r = l + 1; -------------------------//32
----- if (l >= count) break; ------------------------------//be
----- int m = r >= count || cmp(l, r) ? l : r; ------------//81
----- if (!cmp(m, i)) break; ------------------------------//a4
----- swp(m, i), i = m; } } -------------------------------//d8
- heap(int init_len = 128) --------------------------------//98
--- : count(0), len(init_len), _cmp(Compare()) { ----------//9b
--- q = new int[len], loc = new int[len]; -----------------//47
--- memset(loc, 255, len << 2); } -------------------------//d5
- ~heap() { delete[] q; delete[] loc; } -------------------//36
- void push(int n, bool fix = true) { ---------------------//53
--- if (len == count || n >= len) { -----------------------//97
#ifdef RESIZE ---------------------------------------------//85
----- int newlen = 2 * len; -------------------------------//d6
----- while (n >= newlen) newlen *= 2; --------------------//22
----- int *newq = new int[newlen], *newloc = new int[newlen];
----- rep(i,0,len) newq[i] = q[i], newloc[i] = loc[i]; ----//50
----- memset(newloc + len, 255, (newlen - len) << 2); -----//f5
----- delete[] q, delete[] loc; ---------------------------//66
----- loc = newloc, q = newq, len = newlen; ---------------//f0
#else -----------------------------------------------------//6e
----- assert(false); --------------------------------------//91
#endif ----------------------------------------------------//35
--- } -----------------------------------------------------//d6
--- assert(loc[n] == -1); ---------------------------------//b5
--- loc[n] = count, q[count++] = n; -----------------------//4d
--- if (fix) swim(count-1); } -----------------------------//b5
- void pop(bool fix = true) { -----------------------------//70
--- assert(count > 0); ------------------------------------//e9
--- loc[q[0]] = -1, q[0] = q[--count], loc[q[0]] = 0; -----//71
--- if (fix) sink(0); -------------------------------------//d4
- } -------------------------------------------------------//0b
- int top() { assert(count > 0); return q[0]; } -----------//ae
- void heapify() { for (int i = count - 1; i > 0; i--) ----//35
--- if (cmp(i, (i - 1) / 2)) swp(i, (i - 1) / 2); } -------//e4
- void update_key(int n) { --------------------------------//be
--- assert(loc[n] != -1), swim(loc[n]), sink(loc[n]); } ---//48
- bool empty() { return count == 0; } ---------------------//1a
- int size() { return count; } ----------------------------//45
- void clear() { count = 0, memset(loc, 255, len << 2); }};//a7
```

2.8. Dancing Links.

An implementation of Donald Knuth's Dancing Links data structure. A linked list supporting deletion and restoration of elements.

```
template <class T> ----------------------------------------//82
struct dancing_links { ------------------------------------//9e
- struct node { -------------------------------------------//62
--- T item; -----------------------------------------------//dd
--- node *l, *r; ------------------------------------------//32
--- node(const T &_item, node *_l = NULL, node *_r = NULL) //6d
----- : item(_item), l(_l), r(_r) { -----------------------//6d
----- if (l) l->r = this; ---------------------------------//97
----- if (r) r->l = this; } }; ----------------------------//37
- node *front, *back; -------------------------------------//f7
- dancing_links() { front = back = NULL; } ----------------//cb
- node *push_back(const T &item) { ------------------------//4a
--- back = new node(item, back, NULL); --------------------//5c
--- if (!front) front = back; -----------------------------//7b
--- return back; } ----------------------------------------//55
- node *push_front(const T &item) { -----------------------//c0
--- front = new node(item, NULL, front); ------------------//a0
--- if (!back) back = front; ------------------------------//8b
--- return front; } ---------------------------------------//95
- void erase(node *n) { -----------------------------------//c3
--- if (!n->l) front = n->r; else n->l->r = n->r; ---------//38
--- if (!n->r) back = n->l; else n->r->l = n->l; } --------//8e
- void restore(node *n) { ---------------------------------//0e
--- if (!n->l) front = n; else n->l->r = n; ---------------//f4
--- if (!n->r) back = n; else n->r->l = n; } }; -----------//6d
```

2.9. Misof Tree.

A simple tree data structure for inserting, erasing, and querying the nth largest element.

```
#define BITS 15 -----------------------------------------//7b
struct misof_tree { ---------------------------------------//fe
- int cnt[BITS][1<<BITS]; ---------------------------------//aa
- misof_tree() { memset(cnt, 0, sizeof(cnt)); } -----------//b0
- void insert(int x) { ------------------------------------//7f
```

```
--- for (int i = 0; i < BITS; cnt[i++][x]++, x >>= 1); } --//e2
- void erase(int x) { -------------------------------------//c8
--- for (int i = 0; i < BITS; cnt[i++][x]--, x >>= 1); } --//d4
- int nth(int n) { ----------------------------------------//c4
--- int res = 0; ------------------------------------------//cb
--- for (int i = BITS-1; i >= 0; i--) ---------------------//ba
----- if (cnt[i][res <<= 1] <= n) n -= cnt[i][res], res |= 1;
--- return res; } }; --------------------------------------//89
```

2.10. k-d **Tree.** A k-dimensional tree supporting fast construction, adding points, and nearest neighbor queries.

```
#define INC(c) ((c) == K - 1 ? 0 : (c) + 1) --------------//77
template <int K> struct kd_tree { -------------------------//93
- struct pt { ---------------------------------------------//99
--- double coord[K]; --------------------------------------//31
--- pt() {} -----------------------------------------------//96
--- pt(double c[K]) { rep(i,0,K) coord[i] = c[i]; } -------//37
--- double dist(const pt &other) const { ------------------//16
----- double sum = 0.0; -----------------------------------//0c
----- rep(i,0,K) sum += pow(coord[i] - other.coord[i], 2.0);
----- return sqrt(sum); } }; ------------------------------//68
- struct cmp { --------------------------------------------//8c
--- int c; ------------------------------------------------//fa
--- cmp(int _c) : c(_c) {} --------------------------------//28
--- bool operator ()(const pt &a, const pt &b) { ----------//8e
----- for (int i = 0, cc; i <= K; i++) { ------------------//24
------- cc = i == 0 ? c : i - 1; --------------------------//ae
------- if (abs(a.coord[cc] - b.coord[cc]) > EPS) ---------//ad
--------- return a.coord[cc] < b.coord[cc]; ---------------//ed
----- } ---------------------------------------------------//5d
----- return false; } }; ----------------------------------//a4
- struct bb { ---------------------------------------------//f1
--- pt from, to; ------------------------------------------//26
--- bb(pt _from, pt _to) : from(_from), to(_to) {} --------//9c
--- double dist(const pt &p) { ----------------------------//74
----- double sum = 0.0; -----------------------------------//48
----- rep(i,0,K) { ----------------------------------------//d2
------- if (p.coord[i] < from.coord[i]) -------------------//ff
--------- sum += pow(from.coord[i] - p.coord[i], 2.0); ----//07
------- else if (p.coord[i] > to.coord[i]) ----------------//50
--------- sum += pow(p.coord[i] - to.coord[i], 2.0); ------//45
----- } ---------------------------------------------------//e8
----- return sqrt(sum); } ---------------------------------//df
--- bb bound(double l, int c, bool left) { ----------------//67
----- pt nf(from.coord), nt(to.coord); --------------------//af
----- if (left) nt.coord[c] = min(nt.coord[c], l); --------//48
----- else nf.coord[c] = max(nf.coord[c], l); -------------//14
----- return bb(nf, nt); } }; -----------------------------//97
- struct node { -------------------------------------------//7f
--- pt p; node *l, *r; ------------------------------------//2c
--- node(pt _p, node *_l, node *_r) -----------------------//a9
----- : p(_p), l(_l), r(_r) {  } }; -----------------------//92
- node *root; ---------------------------------------------//dd
- // kd_tree() : root(NULL) { } ---------------------------//f8
- kd_tree(vector<pt> pts) { -------------------------------//03
--- root = construct(pts, 0, size(pts) - 1, 0); } ---------//0e
- node* construct(vector<pt> &pts, int from, int to, int c) {
--- if (from > to) return NULL; ---------------------------//22
--- int mid = from + (to - from) / 2; ---------------------//cd
--- nth_element(pts.begin() + from, pts.begin() + mid, ----//01
--------- pts.begin() + to + 1, cmp(c)); ------------------//4e
--- return new node(pts[mid], -----------------------------//4f
----------- construct(pts, from, mid - 1, INC(c)), --------//af
----------- construct(pts, mid + 1, to, INC(c))); } -------//00
- bool contains(const pt &p) { return _con(p, root, 0); } -//51
- bool _con(const pt &p, node *n, int c) { ----------------//34
--- if (!n) return false; ---------------------------------//da
--- if (cmp(c)(p, n->p)) return _con(p, n->l, INC(c)); ----//57
--- if (cmp(c)(n->p, p)) return _con(p, n->r, INC(c)); ----//65
--- return true; } ----------------------------------------//c8
- void insert(const pt &p) { _ins(p, root, 0); } ---------//a0
- void _ins(const pt &p, node* &n, int c) { ---------------//a9
--- if (!n) n = new node(p, NULL, NULL); ------------------//f9
--- else if (cmp(c)(p, n->p)) _ins(p, n->l, INC(c)); ------//4f
--- else if (cmp(c)(n->p, p)) _ins(p, n->r, INC(c)); } ----//4e
- void clear() { _clr(root); root = NULL; } ---------------//66
- void _clr(node *n) { ------------------------------------//f6
--- if (n) _clr(n->l), _clr(n->r), delete n; } -----------//3c
- pair<pt, bool> nearest_neighbour(const pt &p, -----------//6f
----- bool allow_same=true) { -----------------------------//78
--- double mn = INFINITY, cs[K]; --------------------------//7f
--- rep(i,0,K) cs[i] = -INFINITY; -------------------------//d9
--- pt from(cs); ------------------------------------------//e8
--- rep(i,0,K) cs[i] = INFINITY; -------------------------//19
--- pt to(cs), resp; --------------------------------------//c1
--- _nn(p, root, bb(from, to), mn, resp, 0, allow_same); --//79
--- return make_pair(resp, !std::isinf(mn)); } -----------//44
- void _nn(const pt &p, node *n, bb b, --------------------//96
----- double &mn, pt &resp, int c, bool same) { -----------//e6
--- if (!n || b.dist(p) > mn) return; ---------------------//5d
--- bool l1 = true, l2 = false; ---------------------------//89
--- if ((same || p.dist(n->p) > EPS) && p.dist(n->p) < mn) //a3
------- mn = p.dist(resp = n->p); -------------------------//92
--- node *n1 = n->l, *n2 = n->r; --------------------------//52
--- rep(i,0,2) { ------------------------------------------//5c
----- if (i == 1 || cmp(c)(n->p, p)) swap(n1,n2),swap(l1,l2);
----- _nn(p, n1, b.bound(n->p.coord[c], c, l1), mn, -------//f1
--------- resp, INC(c), same); } } }; ---------------------//cd
```

2.11. **Sqrt Decomposition.** Design principle that supports many operations in amortized $\sqrt{n}$ per operation.

```
struct segment { ------------------------------------------//b2
- vi arr; -------------------------------------------------//8c
- segment(vi _arr) : arr(_arr) { } }; ---------------------//11
vector&lt;segment&gt; T; ----------------------------------------//a1
int K; ----------------------------------------------------//dc
void rebuild() { ------------------------------------------//17
- int cnt = 0; --------------------------------------------//14
- rep(i,0,size(T)) ----------------------------------------//b1
--- cnt += size(T[i].arr); --------------------------------//d1
- K = static_cast<int>(ceil(sqrt(cnt)) + 1e-9); -----------//4c
- vi arr(cnt); --------------------------------------------//14
- for (int i = 0, at = 0; i < size(T); i++) ---------------//79
--- rep(j,0,size(T[i].arr)) -------------------------------//a4
----- arr[at++] = T[i].arr[j]; ----------------------------//f7
- T.clear(); ----------------------------------------------//4c
- for (int i = 0; i < cnt; i += K) ------------------------//79
--- T.push_back(segment(vi(arr.begin()+i, -----------------//13
----------------------- arr.begin()+min(i+K, cnt)))); } //d5
int split(int at) { ---------------------------------------//13
- int i = 0; ----------------------------------------------//b5
- while (i < size(T) && at >= size(T[i].arr)) -------------//ea
--- at -= size(T[i].arr), i++; ----------------------------//e8
- if (i >= size(T)) return size(T); -----------------------//df
- if (at == 0) return i; -----------------------------------//42
- T.insert(T.begin() + i + 1, -----------------------------//bc
----- segment(vi(T[i].arr.begin() + at, T[i].arr.end()))); //34
- T[i] = segment(vi(T[i].arr.begin(), T[i].arr.begin() + at));
- return i + 1; } -----------------------------------------//87
void insert(int at, int v) { ------------------------------//9a
- vi arr; arr.push_back(v); --------------------------------//f3
- T.insert(T.begin() + split(at), segment(arr)); } --------//e7
void erase(int at) { --------------------------------------//06
- int i = split(at); split(at + 1); -----------------------//ec
- T.erase(T.begin() + i); } -------------------------------//a9
```

2.12. **Monotonic Queue.** A queue that supports querying for the minimum element. Useful for sliding window algorithms.

```
struct min_stack { ----------------------------------------//d8
- stack<int> S, M; ----------------------------------------//fe
- void push(int x) { --------------------------------------//20
--- S.push(x); --------------------------------------------//e2
--- M.push(M.empty() ? x : min(M.top(), x)); } ------------//92
- int top() { return S.top(); } ---------------------------//f1
- int mn() { return M.top(); } ----------------------------//02
- void pop() { S.pop(); M.pop(); } ------------------------//fd
- bool empty() { return S.empty(); } }; -------------------//ed
struct min_queue { ----------------------------------------//90
- min_stack inp, outp; ------------------------------------//ed
- void push(int x) { inp.push(x); } -----------------------//b3
- void fix() { --------------------------------------------//0a
--- if (outp.empty()) while (!inp.empty()) ----------------//76
----- outp.push(inp.top()), inp.pop(); } ------------------//67
- int top() { fix(); return outp.top(); } -----------------//c0
- int mn() { ----------------------------------------------//79
--- if (inp.empty()) return outp.mn(); --------------------//d2
--- if (outp.empty()) return inp.mn(); --------------------//6e
--- return min(inp.mn(), outp.mn()); } --------------------//c3
- void pop() { fix(); outp.pop(); } -----------------------//61
- bool empty() { return inp.empty() && outp.empty(); } }; -//89
```

2.13. **Convex Hull Trick.** If converting to integers, look out for division by 0 and $\pm\infty$.

```
struct convex_hull_trick { --------------------------------//16
- vector<pair<double,double> > h; -------------------------//b4
- double intersect(int i) { -------------------------------//9b
--- return (h[i+1].second-h[i].second) / ------------------//43
----- (h[i].first-h[i+1].first); } ------------------------//2e
- void add(double m, double b) { --------------------------//c4
```

```
--- h.push_back(make_pair(m,b)); ---------------------------//67
--- while (size(h) >= 3) { ---------------------------------//85
----- int n = size(h); -------------------------------------//b0
----- if (intersect(n-3) < intersect(n-2)) break; ---------//b3
----- swap(h[n-2], h[n-1]); --------------------------------//1c
----- h.pop_back(); } } ------------------------------------//1f
- double get_min(double x) { -------------------------------//ad
--- int lo = 0, hi = size(h) - 2, res = -1; ---------------//51
--- while (lo <= hi) { -------------------------------------//87
----- int mid = lo + (hi - lo) / 2; -----------------------//5e
----- if (intersect(mid) <= x) res = mid, lo = mid + 1; ---//d3
----- else hi = mid - 1; } ---------------------------------//28
--- return h[res+1].first * x + h[res+1].second; } }; -----//f6
```

And dynamic variant:

```
const ll is_query = -(1LL<<62); ----------------------------//49
struct Line { ----------------------------------------------//f1
- ll m, b; -------------------------------------------------//28
- mutable function<const Line*()> succ; --------------------//44
- bool operator<(const Line& rhs) const { ------------------//28
--- if (rhs.b != is_query) return m < rhs.m; ---------------//1e
--- const Line* s = succ(); --------------------------------//90
--- if (!s) return 0; --------------------------------------//c5
--- ll x = rhs.m; ------------------------------------------//ce
--- return b - s->b < (s->m - m) * x; } }; -----------------//67
// will maintain upper hull for maximum --------------------//d4
struct HullDynamic : public multiset<Line> { ---------------//90
- bool bad(iterator y) { -----------------------------------//a9
--- auto z = next(y); --------------------------------------//39
--- if (y == begin()) { ------------------------------------//ad
----- if (z == end()) return 0; ----------------------------//ed
----- return y->m == z->m && y->b <= z->b; } ---------------//57
--- auto x = prev(y); --------------------------------------//42
--- if (z == end()) return y->m == x->m && y->b <= x->b; --//20
--- return (x->b - y->b)*(z->m - y->m) >= ------------------//97
--------- (y->b - z->b)*(y->m - x->m); } -------------------//1f
- void insert_line(ll m, ll b) { ---------------------------//7b
--- auto y = insert({ m, b }); -----------------------------//24
--- y->succ = [=] { return next(y) == end() ? 0 : &*next(y); };
--- if (bad(y)) { erase(y); return; } ----------------------//ab
--- while (next(y) != end() && bad(next(y))) erase(next(y));
--- while (y != begin() && bad(prev(y))) erase(prev(y)); } //8e
- ll eval(ll x) { ------------------------------------------//1e
--- auto l = *lower_bound((Line) { x, is_query }); ---------//ef
--- return l.m * x + l.b; } }; -----------------------------//08
```

2.14. Sparse Table.

```
struct sparse_table { vvi m; -------------------------------//ed
- sparse_table(vi arr) { -----------------------------------//cd
--- m.push_back(arr); --------------------------------------//cb
--- for (int k = 0; (1<<(++k)) <= size(arr); ) { -----------//19
----- m.push_back(vi(size(arr)-(1<<k)+1)); -----------------//8e
----- rep(i,0,size(arr)-(1<<k)+1) --------------------------//fd
------- m[k][i] = min(m[k-1][i], m[k-1][i+(1<<(k-1))]); } }//05
- int query(int l, int r) { --------------------------------//e1
--- int k = 0; while (1<<(k+1) <= r-l+1) k++; --------------//fa
--- return min(m[k][l], m[k][r-(1<<k)+1]); } }; ------------//70
```

3. Graphs

3.1. Single-Source Shortest Paths.

3.1.1. *Dijkstra's algorithm.* An implementation of Dijkstra's algorithm. It runs in $\Theta(|E| \log |V|)$ time.

```
int *dist, *dad; -------------------------------------------//46
struct cmp { -----------------------------------------------//a5
- bool operator()(int a, int b) { --------------------------//bb
--- return dist[a] != dist[b] ? dist[a] < dist[b] : a < b; }
}; ---------------------------------------------------------//41
pair<int*, int*> dijkstra(int n, int s, vii *adj) { -------//53
- dist = new int[n]; ---------------------------------------//84
- dad = new int[n]; ----------------------------------------//05
- rep(i,0,n) dist[i] = INF, dad[i] = -1; -------------------//80
- set<int, cmp> pq; ----------------------------------------//98
- dist[s] = 0, pq.insert(s); -------------------------------//1f
- while (!pq.empty()) { ------------------------------------//47
--- int cur = *pq.begin(); pq.erase(pq.begin()); ----------//58
--- rep(i,0,size(adj[cur])) { ------------------------------//a6
----- int nxt = adj[cur][i].first, -------------------------//a4
------- ndist = dist[cur] + adj[cur][i].second; -----------//3a
----- if (ndist < dist[nxt]) pq.erase(nxt), ---------------//2d
------- dist[nxt] = ndist, dad[nxt] = cur, pq.insert(nxt); //eb
--- } } ----------------------------------------------------//e5
- return pair<int*, int*>(dist, dad); } --------------------//8b
```

3.1.2. *Bellman-Ford algorithm.* The Bellman-Ford algorithm solves the single-source shortest paths problem in $O(|V||E|)$ time. It is slower than Dijkstra's algorithm, but it works on graphs with negative edges and has the ability to detect negative cycles, neither of which Dijkstra's algorithm can do.

```
int* bellman_ford(int n, int s, vii* adj, bool& ncycle) { -//07
- ncycle = false; ------------------------------------------//00
- int* dist = new int[n]; ----------------------------------//62
- rep(i,0,n) dist[i] = i == s ? 0 : INF; -------------------//a6
- rep(i,0,n-1) rep(j,0,n) if (dist[j] != INF) --------------//f1
--- rep(k,0,size(adj[j])) ----------------------------------//20
----- dist[adj[j][k].first] = min(dist[adj[j][k].first], --//c2
--------- dist[j] + adj[j][k].second); ---------------------//2a
- rep(j,0,n) rep(k,0,size(adj[j])) -------------------------//c2
--- if (dist[j] + adj[j][k].second < dist[adj[j][k].first])//dd
----- ncycle = true; ---------------------------------------//f2
- return dist; } -------------------------------------------//73
```

3.1.3. *IDA* algorithm.*

```
int n, cur[100], pos; --------------------------------------//48
int calch() { ----------------------------------------------//88
- int h = 0; -----------------------------------------------//4a
- rep(i,0,n) if (cur[i] != 0) h += abs(i - cur[i]); -------//9b
- return h; } ----------------------------------------------//f8
int dfs(int d, int g, int prev) { --------------------------//e5
- int h = calch(); -----------------------------------------//ef
- if (g + h > d) return g + h; -----------------------------//39
- if (h == 0) return 0; ------------------------------------//f6
- int mn = INF; --------------------------------------------//44
- rep(di,-2,3) { -------------------------------------------//61
--- if (di == 0) continue; ---------------------------------//ab
--- int nxt = pos + di; ------------------------------------//45
--- if (nxt == prev) continue; -----------------------------//fc
--- if (0 <= nxt && nxt < n) { -----------------------------//82
----- swap(cur[pos], cur[nxt]); ----------------------------//9c
----- swap(pos,nxt); ---------------------------------------//af
----- mn = min(mn, dfs(d, g+1, nxt)); ----------------------//63
----- swap(pos,nxt); ---------------------------------------//8c
----- swap(cur[pos], cur[nxt]); } --------------------------//e1
--- if (mn == 0) break; } ----------------------------------//5a
- return mn; } ---------------------------------------------//89
int idastar() { --------------------------------------------//49
- rep(i,0,n) if (cur[i] == 0) pos = i; ---------------------//0a
- int d = calch(); -----------------------------------------//57
- while (true) { -------------------------------------------//de
--- int nd = dfs(d, 0, -1); --------------------------------//2a
--- if (nd == 0 || nd == INF) return d; --------------------//bd
--- d = nd; } } --------------------------------------------//7a
```

3.2. All-Pairs Shortest Paths.

3.2.1. *Floyd-Warshall algorithm.* The Floyd-Warshall algorithm solves the all-pairs shortest paths problem in $O(|V|^3)$ time.

```
void floyd_warshall(int** arr, int n) { --------------------//21
- rep(k,0,n) rep(i,0,n) rep(j,0,n) -------------------------//af
--- if (arr[i][k] != INF && arr[k][j] != INF) -------------//84
----- arr[i][j] = min(arr[i][j], arr[i][k] + arr[k][j]); } //66
```

3.3. Strongly Connected Components.

3.3.1. *Kosaraju's algorithm.* Kosarajus's algorithm finds strongly connected components of a directed graph in $O(|V| + |E|)$ time. Returns a Union-Find of the SCCs, as well as a topological ordering of the SCCs. Note that the ordering specifies a random element from each SCC, not the UF parents!

```
#include "../data-structures/union_find.cpp" ---------------//5e
vector<bool> visited; --------------------------------------//ab
vi order; --------------------------------------------------//b0
void scc_dfs(const vvi &adj, int u) { ----------------------//f8
- int v; visited[u] = true; --------------------------------//82
- rep(i,0,size(adj[u])) ------------------------------------//59
--- if (!visited[v = adj[u][i]]) scc_dfs(adj, v); ----------//c8
- order.push_back(u); } ------------------------------------//c9
pair<union_find, vi> scc(const vvi &adj) { -----------------//59
- int n = size(adj), u, v; ---------------------------------//3e
- order.clear(); -------------------------------------------//09
- union_find uf(n); vi dag; vvi rev(n); --------------------//bf
- rep(i,0,n) rep(j,0,size(adj[i])) rev[adj[i][j]].push_back(i);
- visited.resize(n); ---------------------------------------//60
- fill(visited.begin(), visited.end(), false); ------------//96
- rep(i,0,n) if (!visited[i]) scc_dfs(rev, i); ------------//35
- fill(visited.begin(), visited.end(), false); ------------//17
- stack<int> S; --------------------------------------------//e3
- for (int i = n-1; i >= 0; i--) { -------------------------//ee
--- if (visited[order[i]]) continue; -----------------------//99
--- S.push(order[i]), dag.push_back(order[i]); ------------//91
--- while (!S.empty()) { -----------------------------------//9e
----- visited[u = S.top()] = true, S.pop(); ---------------//5b
----- uf.unite(u, order[i]); -------------------------------//81
```

```
----- rep(j,0,size(adj[u])) ------------------------------//c5
------- if (!visited[v = adj[u][j]]) S.push(v); } } -------//d0
- return pair<union_find, vi>(uf, dag); } -----------------//04
```

3.4. Cut Points and Bridges.

```
#define MAXN 5000 -------------------------------------------//f7
int low[MAXN], num[MAXN], curnum; -------------------------//d7
void dfs(const vvi &adj, vi &cp, vii &bri, int u, int p) { //22
- low[u] = num[u] = curnum++; -----------------------------//a3
- int cnt = 0; bool found = false; ------------------------//97
- rep(i,0,size(adj[u])) { ---------------------------------//ae
--- int v = adj[u][i]; ------------------------------------//56
--- if (num[v] == -1) { -----------------------------------//3b
----- dfs(adj, cp, bri, v, u); ----------------------------//ba
----- low[u] = min(low[u], low[v]); -----------------------//be
----- cnt++; ----------------------------------------------//e0
----- found = found || low[v] >= num[u]; ------------------//30
----- if (low[v] > num[u]) bri.push_back(ii(u, v)); -------//bf
--- } else if (p != v) low[u] = min(low[u], num[v]); } ----//76
- if (found && (p != -1 || cnt > 1)) cp.push_back(u); } ---//3e
pair<vi,vii> cut_points_and_bridges(const vvi &adj) { -----//76
- int n = size(adj); --------------------------------------//c8
- vi cp; vii bri; -----------------------------------------//fb
- memset(num, -1, n << 2); --------------------------------//45
- curnum = 0; ---------------------------------------------//07
- rep(i,0,n) if (num[i] == -1) dfs(adj, cp, bri, i, -1); --//7e
- return make_pair(cp, bri); } ----------------------------//4c
```

3.5. Minimum Spanning Tree.

3.5.1. *Kruskal's algorithm.*

```
#include "../data-structures/union_find.cpp" --------------//5e
vector<pair<int, ii> > mst(int n, -------------------------//42
--- vector<pair<int, ii> > edges) { -----------------------//64
- union_find uf(n); ---------------------------------------//96
- sort(edges.begin(), edges.end()); -----------------------//c3
- vector<pair<int, ii> > res; -----------------------------//8c
- rep(i,0,size(edges)) ------------------------------------//b0
--- if (uf.find(edges[i].second.first) != -----------------//2d
------- uf.find(edges[i].second.second)) { ----------------//e8
----- res.push_back(edges[i]); ----------------------------//1d
----- uf.unite(edges[i].second.first, ---------------------//33
-------------- edges[i].second.second); } -----------------//65
- return res; } -------------------------------------------//d0
```

3.6. Topological Sort.

3.6.1. *Modified Depth-First Search.*

```
void tsort_dfs(int cur, char* color, const vvi& adj, ------//d5
--- stack<int>& res, bool& cyc) { -------------------------//b8
- color[cur] = 1; -----------------------------------------//b7
- rep(i,0,size(adj[cur])) { -------------------------------//70
--- int nxt = adj[cur][i]; --------------------------------//c7
--- if (color[nxt] == 0) ----------------------------------//97
----- tsort_dfs(nxt, color, adj, res, cyc); ---------------//5c
--- else if (color[nxt] == 1) -----------------------------//75
----- cyc = true; -----------------------------------------//ae
--- if (cyc) return; } ------------------------------------//5c
- color[cur] = 2; -----------------------------------------//91
- res.push(cur); } ----------------------------------------//a0
vi tsort(int n, vvi adj, bool& cyc) { ---------------------//9a
- cyc = false; --------------------------------------------//a1
- stack<int> S; -------------------------------------------//64
- vi res; -------------------------------------------------//a1
- char* color = new char[n]; ------------------------------//5d
- memset(color, 0, n); ------------------------------------//5c
- rep(i,0,n) { --------------------------------------------//a6
--- if (!color[i]) { --------------------------------------//1a
----- tsort_dfs(i, color, adj, S, cyc); -------------------//c1
----- if (cyc) return res; } } ----------------------------//6b
- while (!S.empty()) res.push_back(S.top()), S.pop(); -----//bf
- return res; } -------------------------------------------//60
```

3.7. Euler Path.

Finds an euler path (or circuit) in a directed graph, or reports that none exist.

```
#define MAXV 1000 -------------------------------------------//2f
#define MAXE 5000 -------------------------------------------//87
vi adj[MAXV]; ---------------------------------------------//ff
int n, m, indeg[MAXV], outdeg[MAXV], res[MAXE + 1]; -------//49
ii start_end() { ------------------------------------------//30
- int start = -1, end = -1, any = 0, c = 0; ---------------//74
- rep(i,0,n) { --------------------------------------------//20
--- if (outdeg[i] > 0) any = i; ---------------------------//63
--- if (indeg[i] + 1 == outdeg[i]) start = i, c++; --------//5a
--- else if (indeg[i] == outdeg[i] + 1) end = i, c++; -----//13
--- else if (indeg[i] != outdeg[i]) return ii(-1,-1); } ---//ba
- if ((start == -1) != (end == -1) || (c != 2 && c != 0)) -//89
--- return ii(-1,-1); -------------------------------------//9c
- if (start == -1) start = end = any; ---------------------//4c
- return ii(start, end); } --------------------------------//bb
bool euler_path() { ---------------------------------------//4d
- ii se = start_end(); ------------------------------------//11
- int cur = se.first, at = m + 1; -------------------------//ca
- if (cur == -1) return false; ----------------------------//eb
- stack<int> s; -------------------------------------------//6c
- while (true) { ------------------------------------------//73
--- if (outdeg[cur] == 0) { -------------------------------//3f
----- res[--at] = cur; ------------------------------------//5e
----- if (s.empty()) break; -------------------------------//c5
----- cur = s.top(); s.pop(); -----------------------------//17
--- } else s.push(cur), cur = adj[cur][--outdeg[cur]]; } --//77
- return at == 0; } ---------------------------------------//32
```

And an undirected version, which finds a cycle.

```
multiset<int> adj[1010]; ----------------------------------//8c
list<int> L; ----------------------------------------------//9f
list<int>::iterator euler(int at, int to, -----------------//80
--- list<int>::iterator it) { -----------------------------//b4
- if (at == to) return it; --------------------------------//b8
- L.insert(it, at), --it; ---------------------------------//ef
- while (!adj[at].empty()) { ------------------------------//d0
--- int nxt = *adj[at].begin(); ---------------------------//a9
--- adj[at].erase(adj[at].find(nxt)); ---------------------//56
--- adj[nxt].erase(adj[nxt].find(at)); --------------------//b7
--- if (to == -1) { ---------------------------------------//7b
----- it = euler(nxt, at, it); ----------------------------//be
----- L.insert(it, at); -----------------------------------//82
----- --it; -----------------------------------------------//36
--- } else { ----------------------------------------------//c9
----- it = euler(nxt, to, it); ----------------------------//d7
----- to = -1; } } ----------------------------------------//15
- return it; } --------------------------------------------//73
// euler(0,-1,L.begin()) ----------------------------------//fd
```

3.8. Bipartite Matching.

3.8.1. *Alternating Paths algorithm.*

The alternating paths algorithm solves bipartite matching in $O(mn^2)$ time, where m, n are the number of vertices on the left and right side of the bipartite graph, respectively.

```
vi* adj; --------------------------------------------------//cc
bool* done; -----------------------------------------------//b1
int* owner; -----------------------------------------------//26
int alternating_path(int left) { --------------------------//da
- if (done[left]) return 0; -------------------------------//08
- done[left] = true; --------------------------------------//f2
- rep(i,0,size(adj[left])) { ------------------------------//1b
--- int right = adj[left][i]; -----------------------------//46
--- if (owner[right] == -1 || -----------------------------//b6
------- alternating_path(owner[right])) { -----------------//82
----- owner[right] = left; return 1; } } ------------------//9b
- return 0; } ---------------------------------------------//7c
```

3.8.2. *Hopcroft-Karp algorithm.*

An implementation of Hopcroft-Karp algorithm for bipartite matching. Running time is $O(|E|\sqrt{|V|})$.

```
#define MAXN 5000 -------------------------------------------//f7
int dist[MAXN+1], q[MAXN+1]; ------------------------------//b8
#define dist(v) dist[v == -1 ? MAXN : v] ------------------//0f
struct bipartite_graph { ----------------------------------//2b
- int N, M, *L, *R; vi *adj; ------------------------------//fc
- bipartite_graph(int _N, int _M) : N(_N), M(_M), ---------//8d
--- L(new int[N]), R(new int[M]), adj(new vi[N]) {} -------//cd
- ~bipartite_graph() { delete[] adj; delete[] L; delete[] R; }
- bool bfs() { --------------------------------------------//f5
--- int l = 0, r = 0; ------------------------------------//37
--- rep(v,0,N) if(L[v] == -1) dist(v) = 0, q[r++] = v; ----//f9
----- else dist(v) = INF; ---------------------------------//aa
--- dist(-1) = INF; ---------------------------------------//f2
--- while(l < r) { ----------------------------------------//ba
----- int v = q[l++]; -------------------------------------//50
----- if(dist(v) < dist(-1)) { ----------------------------//f1
------- iter(u, adj[v]) if(dist(R[*u]) == INF) ------------//9b
--------- dist(R[*u]) = dist(v) + 1, q[r++] = R[*u]; } } --//78
--- return dist(-1) != INF; } -----------------------------//e3
- bool dfs(int v) { ---------------------------------------//7d
--- if(v != -1) { -----------------------------------------//3e
----- iter(u, adj[v]) -------------------------------------//af
------- if(dist(R[*u]) == dist(v) + 1) --------------------//21
--------- if(dfs(R[*u])) { --------------------------------//cd
----------- R[*u] = v, L[v] = *u; -------------------------//0f
----------- return true; } --------------------------------//b7
----- dist(v) = INF; --------------------------------------//dd
----- return false; } -------------------------------------//40
```

```
--- return true; } ------------------------------------------//4a
- void add_edge(int i, int j) { adj[i].push_back(j); } ----//69
- int maximum_matching() { ---------------------------------//9a
--- int matching = 0; --------------------------------------//f3
--- memset(L, -1, sizeof(int) * N); -----------------------//c3
--- memset(R, -1, sizeof(int) * M); -----------------------//bd
--- while(bfs()) rep(i,0,N) --------------------------------//db
----- matching += L[i] == -1 && dfs(i); -------------------//27
--- return matching; } }; ----------------------------------//e1
```

3.8.3. *Minimum Vertex Cover in Bipartite Graphs.*

```
#include "hopcroft_karp.cpp" -------------------------------//05
vector<bool> alt; ------------------------------------------//cc
void dfs(bipartite_graph &g, int at) { ---------------------//14
- alt[at] = true; ------------------------------------------//df
- iter(it,g.adj[at]) { -------------------------------------//9f
--- alt[*it + g.N] = true; ---------------------------------//68
--- if (g.R[*it] != -1 && !alt[g.R[*it]]) dfs(g, g.R[*it]); } }
vi mvc_bipartite(bipartite_graph &g) { ---------------------//b1
- vi res; g.maximum_matching(); ----------------------------//fd
- alt.assign(g.N + g.M,false); -----------------------------//14
- rep(i,0,g.N) if (g.L[i] == -1) dfs(g, i); ---------------//ff
- rep(i,0,g.N) if (!alt[i]) res.push_back(i); -------------//66
- rep(i,0,g.M) if (alt[g.N + i]) res.push_back(g.N + i); --//30
- return res; } --------------------------------------------//c4
```

3.9. Maximum Flow.

3.9.1. *Dinic's algorithm.* An implementation of Dinic's algorithm that runs in $O(|V|^2|E|)$. It computes the maximum flow of a flow network.

```
#define MAXV 2000 ----------------------------------------------//ba
int q[MAXV], d[MAXV]; --------------------------------------//e6
struct flow_network { --------------------------------------//12
- struct edge { int v, nxt, cap; --------------------------//63
--- edge(int _v, int _cap, int _nxt) ----------------------//d4
----- : v(_v), nxt(_nxt), cap(_cap) { } }; ----------------//e9
- int n, *head, *curh; vector<edge> e, e_store; -----------//e8
- flow_network(int _n) : n(_n) { --------------------------//54
--- curh = new int[n]; -------------------------------------//8c
--- memset(head = new int[n], -1, n*sizeof(int)); } -------//c6
- void reset() { e = e_store; } ---------------------------//37
- void add_edge(int u, int v, int uv, int vu=0) { ---------//e4
--- e.push_back(edge(v, uv, head[u])); head[u] = size(e)-1;//70
--- e.push_back(edge(u, vu, head[v])); head[v] = size(e)-1; }
- int augment(int v, int t, int f) { ----------------------//6b
--- if (v == t) return f; ----------------------------------//29
--- for (int &i = curh[v], ret; i != -1; i = e[i].nxt) ----//1c
----- if (e[i].cap > 0 && d[e[i].v] + 1 == d[v]) ---------//fa
------- if ((ret = augment(e[i].v, t, min(f, e[i].cap))) > 0)
--------- return (e[i].cap -= ret, e[i^1].cap += ret, ret);//94
--- return 0; } --------------------------------------------//bc
- int max_flow(int s, int t, bool res=true) { -------------//b5
--- e_store = e; -------------------------------------------//81
--- int l, r, f = 0, x; ------------------------------------//50
--- while (true) { -----------------------------------------//f7
----- memset(d, -1, n*sizeof(int)); -----------------------//63
----- l = r = 0, d[q[r++] = t] = 0; -----------------------//1b
----- while (l < r) ----------------------------------------//20
------- for (int v = q[l++], i = head[v]; i != -1; i=e[i].nxt)
--------- if (e[i^1].cap > 0 && d[e[i].v] == -1) ---------//4c
----------- d[q[r++] = e[i].v] = d[v]+1; -----------------//2d
----- if (d[s] == -1) break; -------------------------------//f8
----- memcpy(curh, head, n * sizeof(int)); ----------------//e4
----- while ((x = augment(s, t, INF)) != 0) f += x; } -----//af
--- if (res) reset(); --------------------------------------//1f
--- return f; } }; -----------------------------------------//b1
```

3.9.2. *Edmonds Karp's algorithm.* An implementation of Edmonds Karp's algorithm that runs in $O(|V||E|^2)$. It computes the maximum flow of a flow network.

```
#define MAXV 2000 ----------------------------------------------//ba
int q[MAXV], p[MAXV], d[MAXV]; -----------------------------//22
struct flow_network { --------------------------------------//cf
- struct edge { int v, nxt, cap; --------------------------//95
--- edge(int _v, int _cap, int _nxt) ----------------------//52
----- : v(_v), nxt(_nxt), cap(_cap) { } }; ----------------//60
- int n, *head; vector<edge> e, e_store; ------------------//ea
- flow_network(int _n) : n(_n) { --------------------------//ea
--- memset(head = new int[n], -1, n*sizeof(int)); } -------//07
- void reset() { e = e_store; } ---------------------------//4e
- void add_edge(int u, int v, int uv, int vu=0) { ---------//19
--- e.push_back(edge(v, uv, head[u])); head[u] = size(e)-1;//5c
--- e.push_back(edge(u, vu, head[v])); head[v] = size(e)-1; }
- int max_flow(int s, int t, bool res=true) { -------------//d6
--- e_store = e; -------------------------------------------//81
--- int l, r, v, f = 0; ------------------------------------//a0
--- while (true) { -----------------------------------------//46
----- memset(d, -1, n*sizeof(int)); -----------------------//65
----- memset(p, -1, n*sizeof(int)); -----------------------//e8
----- l = r = 0, d[q[r++] = s] = 0; -----------------------//6e
----- while (l < r) ----------------------------------------//f3
------- for (int u = q[l++], i = head[u]; i != -1; i=e[i].nxt)
--------- if (e[i].cap > 0 && -----------------------------//bb
------------- (d[v = e[i].v] == -1 || d[u] + 1 < d[v])) ---//93
----------- d[v] = d[u] + 1, p[q[r++] = v] = i; -----------//7c
----- if (p[t] == -1) break; -------------------------------//b0
----- int at = p[t], x = INF; ------------------------------//64
----- while (at != -1) -------------------------------------//3e
------- x = min(x, e[at].cap), at = p[e[at^1].v]; ---------//81
----- at = p[t], f += x; -----------------------------------//de
----- while (at != -1) -------------------------------------//4b
------- e[at].cap -= x, e[at^1].cap += x, at = p[e[at^1].v]; }
--- if (res) reset(); --------------------------------------//98
--- return f; } }; -----------------------------------------//d6
```

3.10. **Minimum Cost Maximum Flow.** An implementation of Edmonds Karp's algorithm, modified to find shortest path to augment each time (instead of just any path). It computes the maximum flow of a flow network, and when there are multiple maximum flows, finds the maximum flow with minimum cost. Running time is $O(|V|^2|E|\log|V|)$.

```
#define MAXV 2000 ----------------------------------------------//ba
int d[MAXV], p[MAXV], pot[MAXV]; ---------------------------//80
struct cmp { bool operator ()(int i, int j) { -------------//d2
--- return d[i] == d[j] ? i < j : d[i] < d[j]; } }; -------//3d
struct flow_network { --------------------------------------//09
- struct edge { int v, nxt, cap, cost; --------------------//56
--- edge(int _v, int _cap, int _cost, int _nxt) -----------//c1
----- : v(_v), nxt(_nxt), cap(_cap), cost(_cost) { } }; ---//17
- int n; vi head; vector<edge> e, e_store; ----------------//84
- flow_network(int _n) : n(_n), head(n,-1) { } ------------//00
- void reset() { e = e_store; } ---------------------------//8b
- void add_edge(int u, int v, int cost, int uv, int vu=0) {//60
--- e.push_back(edge(v, uv, cost, head[u])); --------------//e0
--- head[u] = size(e)-1; -----------------------------------//51
--- e.push_back(edge(u, vu, -cost, head[v])); -------------//b2
--- head[v] = size(e)-1; } ---------------------------------//2b
- ii min_cost_max_flow(int s, int t, bool res=true) { -----//d6
--- e_store = e; -------------------------------------------//d8
--- memset(pot, 0, n*sizeof(int)); ------------------------//cf
--- rep(it,0,n-1) rep(i,0,size(e)) if (e[i].cap > 0) ------//13
----- pot[e[i].v] = ----------------------------------------//b9
------- min(pot[e[i].v], pot[e[i^1].v] + e[i].cost); -----//45
--- int v, f = 0, c = 0; -----------------------------------//9c
--- while (true) { -----------------------------------------//91
----- memset(d, -1, n*sizeof(int)); -----------------------//a9
----- memset(p, -1, n*sizeof(int)); -----------------------//ae
----- set<int, cmp> q; -------------------------------------//ba
----- d[s] = 0; q.insert(s); -------------------------------//22
----- while (!q.empty()) { ---------------------------------//0a
------- int u = *q.begin(); --------------------------------//e7
------- q.erase(q.begin()); --------------------------------//61
------- for (int i = head[u]; i != -1; i = e[i].nxt) { ----//63
--------- if (e[i].cap == 0) continue; ---------------------//20
--------- int cd = d[u] + e[i].cost + pot[u] - pot[v = e[i].v];
--------- if (d[v] == -1 || cd < d[v]) { ------------------//c1
----------- q.erase(v); ------------------------------------//cb
----------- d[v] = cd; p[v] = i; ---------------------------//1d
----------- q.insert(v); } } } -----------------------------//d3
----- if (p[t] == -1) break; -------------------------------//2b
----- int at = p[t], x = INF; ------------------------------//26
----- while (at != -1) -------------------------------------//8d
------- x = min(x, e[at].cap), at = p[e[at^1].v]; ---------//d4
----- at = p[t], f += x; -----------------------------------//1c
----- while (at != -1) -------------------------------------//25
------- e[at].cap -= x, e[at^1].cap += x, at = p[e[at^1].v];
----- c += x * (d[t] + pot[t] - pot[s]); -------------------//e3
----- rep(i,0,n) if (p[i] != -1) pot[i] += d[i]; } --------//78
--- if (res) reset(); --------------------------------------//a6
--- return ii(f, c); } }; ----------------------------------//e4
```

3.11. All Pairs Maximum Flow.

3.11.1. *Gomory-Hu Tree.* An implementation of the Gomory-Hu Tree. The spanning tree is constructed using Gusfield's algorithm in $O(|V|^2)$ plus $|V|-1$ times the time it takes to calculate the maximum flow. If Dinic's algorithm is used to calculate the max flow, the running time is $O(|V|^3|E|)$. NOTE: Not sure if it works correctly with disconnected graphs.

```
#include "dinic.cpp" ---------------------------------------//58
bool same[MAXV]; -------------------------------------------//35
pair<vii, vvi> construct_gh_tree(flow_network &g) { -------//2f
```

```
- int n = g.n, v; -----------------------------------------//40
- vii par(n, ii(0, 0)); vvi cap(n, vi(n, -1)); ------------//03
- rep(s,1,n) { --------------------------------------------//03
--- int l = 0, r = 0; -------------------------------------//50
--- par[s].second = g.max_flow(s, par[s].first, false); ---//12
--- memset(d, 0, n * sizeof(int)); ------------------------//a1
--- memset(same, 0, n * sizeof(bool)); --------------------//61
--- d[q[r++] = s] = 1; ------------------------------------//d9
--- while (l < r) { ---------------------------------------//4b
----- same[v = q[l++]] = true; ----------------------------//3b
----- for (int i = g.head[v]; i != -1; i = g.e[i].nxt) ----//55
------- if (g.e[i].cap > 0 && d[g.e[i].v] == 0) -----------//d4
--------- d[q[r++] = g.e[i].v] = 1; } ---------------------//a7
--- rep(i,s+1,n) ------------------------------------------//3f
----- if (par[i].first == par[s].first && same[i]) --------//2f
------- par[i].first = s; ---------------------------------//fb
--- g.reset(); } ------------------------------------------//43
- rep(i,0,n) { --------------------------------------------//d3
--- int mn = INF, cur = i; --------------------------------//10
--- while (true) { ----------------------------------------//42
----- cap[cur][i] = mn; -----------------------------------//48
----- if (cur == 0) break; --------------------------------//b7
----- mn = min(mn, par[cur].second), cur = par[cur].first; } }
- return make_pair(par, cap); } ---------------------------//d9
int compute_max_flow(int s, int t, const pair<vii, vvi> &gh) {
- int cur = INF, at = s; ----------------------------------//af
- while (gh.second[at][t] == -1) --------------------------//59
--- cur = min(cur, gh.first[at].second), ------------------//b2
--- at = gh.first[at].first; ------------------------------//04
- return min(cur, gh.second[at][t]); } --------------------//aa
```

3.12. Heavy-Light Decomposition.

```
#include "../data-structures/segment_tree.cpp" ------------//16
const int ID = 0; -----------------------------------------//fa
int f(int a, int b) { return a + b; } ---------------------//e6
struct HLD { ----------------------------------------------//e3
- int n, curhead, curloc; ---------------------------------//1c
- vi sz, head, parent, loc; -------------------------------//b6
- vvi adj; segment_tree values; ---------------------------//e3
- HLD(int _n) : n(_n), sz(n, 1), head(n), -----------------//1a
-------------- parent(n, -1), loc(n), adj(n) { ------------//d0
--- vector<ll> tmp(n, ID); values = segment_tree(tmp); } --//0d
- void add_edge(int u, int v) { ---------------------------//c2
--- adj[u].push_back(v); adj[v].push_back(u); } -----------//7f
- void update_cost(int u, int v, int c) { -----------------//55
--- if (parent[v] == u) swap(u, v); assert(parent[u] == v);//53
--- values.update(loc[u], c); } ---------------------------//3b
- int csz(int u) { ----------------------------------------//4f
--- rep(i,0,size(adj[u])) if (adj[u][i] != parent[u]) -----//42
----- sz[u] += csz(adj[parent[adj[u][i]] = u][i]); --------//f2
--- return sz[u]; } ---------------------------------------//4d
- void part(int u) { --------------------------------------//33
--- head[u] = curhead; loc[u] = curloc++; -----------------//b5
--- int best = -1; ----------------------------------------//de
--- rep(i,0,size(adj[u])) ---------------------------------//5b
----- if (adj[u][i] != parent[u] && -----------------------//dd
--------- (best == -1 || sz[adj[u][i]] > sz[best])) -------//50
------- best = adj[u][i]; ---------------------------------//7d
--- if (best != -1) part(best); ---------------------------//56
--- rep(i,0,size(adj[u])) ---------------------------------//b6
----- if (adj[u][i] != parent[u] && adj[u][i] != best) ----//b4
------- part(curhead = adj[u][i]); } ----------------------//af
- void build(int r = 0) { ---------------------------------//f6
--- curloc = 0, csz(curhead = r), part(r); } --------------//86
- int lca(int u, int v) { ---------------------------------//7c
--- vi uat, vat; int res = -1; ----------------------------//2c
--- while (u != -1) uat.push_back(u), u = parent[head[u]]; //c0
--- while (v != -1) vat.push_back(v), v = parent[head[v]]; //48
--- u = size(uat) - 1, v = size(vat) - 1; -----------------//6b
--- while (u >= 0 && v >= 0 && head[uat[u]] == head[vat[v]])
----- res = (loc[uat[u]] < loc[vat[v]] ? uat[u] : vat[v]), //ba
----- u--, v--; -------------------------------------------//ce
--- return res; } -----------------------------------------//2f
- int query_upto(int u, int v) { int res = ID; ------------//71
--- while (head[u] != head[v]) ----------------------------//c5
----- res = f(res, values.query(loc[head[u]], loc[u]).x), -//b7
----- u = parent[head[u]]; --------------------------------//1b
--- return f(res, values.query(loc[v] + 1, loc[u]).x); } --//9b
- int query(int u, int v) { int l = lca(u, v); ------------//06
--- return f(query_upto(u, l), query_upto(v, l)); } }; ----//30
```

3.13. Centroid Decomposition.

```
#define MAXV 100100 ---------------------------------------//86
#define LGMAXV 20 -----------------------------------------//aa
int jmp[MAXV][LGMAXV], ------------------------------------//6d
- path[MAXV][LGMAXV], -------------------------------------//9d
- sz[MAXV], seph[MAXV], -----------------------------------//cf
- shortest[MAXV]; -----------------------------------------//6b
struct centroid_decomposition { ---------------------------//99
- int n; vvi adj; -----------------------------------------//e9
- centroid_decomposition(int _n) : n(_n), adj(n) { } ------//46
- void add_edge(int a, int b) { ---------------------------//84
--- adj[a].push_back(b); adj[b].push_back(a); } -----------//65
- int dfs(int u, int p) { ---------------------------------//dd
--- sz[u] = 1; --------------------------------------------//bf
--- rep(i,0,size(adj[u])) ---------------------------------//ef
----- if (adj[u][i] != p) sz[u] += dfs(adj[u][i], u); -----//9d
--- return sz[u]; } ---------------------------------------//bb
- void makepaths(int sep, int u, int p, int len) { --------//fe
--- jmp[u][seph[sep]] = sep, path[u][seph[sep]] = len; ----//19
--- int bad = -1; -----------------------------------------//f6
--- rep(i,0,size(adj[u])) { -------------------------------//c5
----- if (adj[u][i] == p) bad = i; ------------------------//38
----- else makepaths(sep, adj[u][i], u, len + 1); ---------//93
--- } -----------------------------------------------------//b9
--- if (p == sep) -----------------------------------------//a0
----- swap(adj[u][bad], adj[u].back()), adj[u].pop_back(); }
- void separate(int h=0, int u=0) { -----------------------//6e
--- dfs(u,-1); int sep = u; -------------------------------//29
--- down: iter(nxt,adj[sep]) ------------------------------//c2
----- if (sz[*nxt] < sz[sep] && sz[*nxt] > sz[u]/2) { -----//09
------- sep = *nxt; goto down; } --------------------------//5d
--- seph[sep] = h, makepaths(sep, sep, -1, 0); ------------//5d
--- rep(i,0,size(adj[sep])) separate(h+1, adj[sep][i]); } -//7c
- void paint(int u) { -------------------------------------//f1
--- rep(h,0,seph[u]+1) ------------------------------------//da
----- shortest[jmp[u][h]] = min(shortest[jmp[u][h]], ------//77
------------------------------- path[u][h]); } ------------//b2
- int closest(int u) { ------------------------------------//ec
--- int mn = INF/2; ---------------------------------------//1f
--- rep(h,0,seph[u]+1) ------------------------------------//80
----- mn = min(mn, path[u][h] + shortest[jmp[u][h]]); -----//5c
--- return mn; } }; ---------------------------------------//82
```

3.14. Least Common Ancestors, Binary Jumping.

```
struct node { --------------------------------------------//36
- node *p, *jmp[20]; --------------------------------------//24
- int depth; ----------------------------------------------//10
- node(node *_p = NULL) : p(_p) { -------------------------//78
--- depth = p ? 1 + p->depth : 0; -------------------------//3b
--- memset(jmp, 0, sizeof(jmp)); --------------------------//64
--- jmp[0] = p; -------------------------------------------//64
--- for (int i = 1; (1<<i) <= depth; i++) -----------------//a8
----- jmp[i] = jmp[i-1]->jmp[i-1]; } }; -------------------//3b
node* st[100000]; -----------------------------------------//65
node* lca(node *a, node *b) { -----------------------------//29
- if (!a || !b) return NULL; ------------------------------//cd
- if (a->depth < b->depth) swap(a,b); ---------------------//fe
- for (int j = 19; j >= 0; j--) ---------------------------//b3
--- while (a->depth - (1<<j) >= b->depth) a = a->jmp[j]; --//c0
- if (a == b) return a; -----------------------------------//08
- for (int j = 19; j >= 0; j--) ---------------------------//11
--- while (a->depth >= (1<<j) && a->jmp[j] != b->jmp[j]) --//f0
----- a = a->jmp[j], b = b->jmp[j]; -----------------------//d0
- return a->p; } ------------------------------------------//c5
```

3.15. Tarjan's Off-line Lowest Common Ancestors Algorithm.

```
#include "../data-structures/union_find.cpp" --------------//5e
struct tarjan_olca { --------------------------------------//87
- int *ancestor; ------------------------------------------//39
- vi *adj, answers; ---------------------------------------//dd
- vii *queries; -------------------------------------------//66
- bool *colored; ------------------------------------------//97
- union_find uf; ------------------------------------------//70
- tarjan_olca(int n, vi *_adj) : adj(_adj), uf(n) { -------//78
--- colored = new bool[n]; --------------------------------//8d
--- ancestor = new int[n]; --------------------------------//f2
--- queries = new vii[n]; ---------------------------------//3e
--- memset(colored, 0, n); } ------------------------------//78
- void query(int x, int y) { ------------------------------//29
--- queries[x].push_back(ii(y, size(answers))); -----------//5e
--- queries[y].push_back(ii(x, size(answers))); -----------//07
--- answers.push_back(-1); } ------------------------------//74
- void process(int u) { -----------------------------------//38
--- ancestor[u] = u; --------------------------------------//a8
--- rep(i,0,size(adj[u])) { -------------------------------//24
----- int v = adj[u][i]; ----------------------------------//2d
----- process(v); -----------------------------------------//0f
----- uf.unite(u,v); --------------------------------------//14
```

```
----- ancestor[uf.find(u)] = u; } -------------------------//f7
--- colored[u] = true; ------------------------------------//cf
--- rep(i,0,size(queries[u])) { ---------------------------//28
----- int v = queries[u][i].first; ------------------------//2d
----- if (colored[v]) { -----------------------------------//23
------- answers[queries[u][i].second] = ancestor[uf.find(v)];
----- } } } }; --------------------------------------------//0b
```

3.16. **Minimum Mean Weight Cycle.** Given a strongly connected directed graph, finds the cycle of minimum mean weight. If you have a graph that is not strongly connected, run this on each strongly connected component.

```
double min_mean_cycle(vector<vector<pair<int,double>>> adj){
- int n = size(adj); double mn = INFINITY; ----------------//dc
- vector<vector<double> > arr(n+1, vector<double>(n, mn)); //ce
- arr[0][0] = 0; ------------------------------------------//59
- rep(k,1,n+1) rep(j,0,n) iter(it,adj[j]) -----------------//b3
--- arr[k][it->first] = min(arr[k][it->first], ------------//d2
--------------------------- it->second + arr[k-1][j]); ----//9a
- rep(k,0,n) { --------------------------------------------//d3
--- double mx = -INFINITY; --------------------------------//b4
--- rep(i,0,n) mx = max(mx, (arr[n][i]-arr[k][i])/(n-k)); -//bc
--- mn = min(mn, mx); } -----------------------------------//2b
- return mn; } --------------------------------------------//cf
```

3.17. **Minimum Arborescence.** Given a weighted directed graph, finds a subset of edges of minimum total weight so that there is a unique path from the root r to each vertex. Returns a vector of size n, where the ith element is the edge for the ith vertex. The answer for the root is undefined!

```
#include "../data-structures/union_find.cpp" --------------//5e
struct arborescence { -------------------------------------//fa
- int n; union_find uf; -----------------------------------//70
- vector<vector<pair<ii,int> > > adj; ---------------------//b7
- arborescence(int _n) : n(_n), uf(n), adj(n) { } ---------//45
- void add_edge(int a, int b, int c) { --------------------//68
--- adj[b].push_back(make_pair(ii(a,b),c)); } -------------//8b
- vii find_min(int r) { -----------------------------------//88
--- vi vis(n,-1), mn(n,INF); vii par(n); ------------------//74
--- rep(i,0,n) { ------------------------------------------//10
----- if (uf.find(i) != i) continue; ---------------------//9c
----- int at = i; -----------------------------------------//67
----- while (at != r && vis[at] == -1) { ------------------//57
------- vis[at] = i; --------------------------------------//21
------- iter(it,adj[at]) if (it->second < mn[at] && -------//4a
----------- uf.find(it->first.first) != at) ---------------//b9
--------- mn[at] = it->second, par[at] = it->first; -------//aa
------- if (par[at] == ii(0,0)) return vii(); -------------//a9
------- at = uf.find(par[at].first); } --------------------//8a
----- if (at == r || vis[at] != i) continue; --------------//4e
----- union_find tmp = uf; vi seq; ------------------------//ec
----- do { seq.push_back(at); at = uf.find(par[at].first); //0b
----- } while (at != seq.front()); ------------------------//bc
----- iter(it,seq) uf.unite(*it,seq[0]); ------------------//a5
----- int c = uf.find(seq[0]); ----------------------------//21
----- vector<pair<ii,int> > nw; ---------------------------//4a
----- iter(it,seq) iter(jt,adj[*it]) ----------------------//2b
------- nw.push_back(make_pair(jt->first, -----------------//c0
------------- jt->second - mn[*it])); ---------------------//ea
----- adj[c] = nw; ----------------------------------------//c2
----- vii rest = find_min(r); -----------------------------//40
----- if (size(rest) == 0) return rest; -------------------//1d
----- ii use = rest[c]; -----------------------------------//cc
----- rest[at = tmp.find(use.second)] = use; --------------//63
----- iter(it,seq) if (*it != at) -------------------------//19
------- rest[*it] = par[*it]; -----------------------------//05
----- return rest; } --------------------------------------//d6
--- return par; } }; --------------------------------------//25
```

3.18. **Blossom algorithm.** Finds a maximum matching in an arbitrary graph in $O(|V|^4)$ time. Be vary of loop edges.

```
#define MAXV 300 ------------------------------------------//3c
bool marked[MAXV], emarked[MAXV][MAXV]; -------------------//3a
int S[MAXV]; ----------------------------------------------//f4
vi find_augmenting_path(const vector<vi> &adj,const vi &m){//38
- int n = size(adj), s = 0; -------------------------------//cd
- vi par(n,-1), height(n), root(n,-1), q, a, b; -----------//ba
- memset(marked,0,sizeof(marked)); ------------------------//35
- memset(emarked,0,sizeof(emarked)); ----------------------//31
- rep(i,0,n) if (m[i] >= 0) emarked[i][m[i]] = true; ------//c3
------------ else root[i] = i, S[s++] = i; ----------------//c6
- while (s) { ---------------------------------------------//0b
--- int v = S[--s]; ---------------------------------------//d8
--- iter(wt,adj[v]) { -------------------------------------//c2
----- int w = *wt; ----------------------------------------//70
----- if (emarked[v][w]) continue; ------------------------//18
----- if (root[w] == -1) { --------------------------------//77
------- int x = S[s++] = m[w]; ----------------------------//e5
------- par[w]=v, root[w]=root[v], height[w]=height[v]+1; -//fd
------- par[x]=w, root[x]=root[w], height[x]=height[w]+1; -//ae
----- } else if (height[w] % 2 == 0) { --------------------//55
------- if (root[v] != root[w]) { -------------------------//75
--------- while (v != -1) q.push_back(v), v = par[v]; -----//9f
--------- reverse(q.begin(), q.end()); --------------------//2f
--------- while (w != -1) q.push_back(w), w = par[w]; -----//8f
--------- return q; ---------------------------------------//51
------- } else { ------------------------------------------//e5
--------- int c = v; --------------------------------------//e1
--------- while (c != -1) a.push_back(c), c = par[c]; -----//5c
--------- c = w; ------------------------------------------//5f
--------- while (c != -1) b.push_back(c), c = par[c]; -----//bf
--------- while (!a.empty()&&!b.empty()&&a.back()==b.back())
----------- c = a.back(), a.pop_back(), b.pop_back(); -----//df
--------- memset(marked,0,sizeof(marked)); ----------------//74
--------- fill(par.begin(), par.end(), 0); ----------------//39
--------- iter(it,a) par[*it] = 1; iter(it,b) par[*it] = 1;//19
--------- par[c] = s = 1; ---------------------------------//42
--------- rep(i,0,n) root[par[i] = par[i] ? 0 : s++] = i; -//90
--------- vector<vi> adj2(s); -----------------------------//2c
--------- rep(i,0,n) iter(it,adj[i]) { --------------------//85
----------- if (par[*it] == 0) continue; ------------------//c1
----------- if (par[i] == 0) { ----------------------------//7f
------------- if (!marked[par[*it]])  { -------------------//5a
--------------- adj2[par[i]].push_back(par[*it]); ---------//e6
--------------- adj2[par[*it]].push_back(par[i]); ---------//35
--------------- marked[par[*it]] = true; } ----------------//67
----------- } else adj2[par[i]].push_back(par[*it]); } ----//bd
--------- vi m2(s, -1); -----------------------------------//c2
--------- if (m[c] != -1) m2[m2[par[m[c]]] = 0] = par[m[c]];
--------- rep(i,0,n) if(par[i]!=0&&m[i]!=-1&&par[m[i]]!=0) //61
----------- m2[par[i]] = par[m[i]]; -----------------------//3c
--------- vi p = find_augmenting_path(adj2, m2); ----------//09
--------- int t = 0; --------------------------------------//53
--------- while (t < size(p) && p[t]) t++; ----------------//b8
--------- if (t == size(p)) { -----------------------------//d8
----------- rep(i,0,size(p)) p[i] = root[p[i]]; -----------//8d
----------- return p; } -----------------------------------//21
--------- if (!p[0] || (m[c] != -1 && p[t+1] != par[m[c]]))//ee
----------- reverse(p.begin(), p.end()), t = size(p)-t-1; -//ae
--------- rep(i,0,t) q.push_back(root[p[i]]); -------------//72
--------- iter(it,adj[root[p[t-1]]]) { --------------------//e6
----------- if (par[*it] != (s = 0)) continue; ------------//f6
----------- a.push_back(c), reverse(a.begin(), a.end()); --//05
----------- iter(jt,b) a.push_back(*jt); ------------------//45
----------- while (a[s] != *it) s++; ----------------------//dd
----------- if ((height[*it] & 1) ^ (s < size(a) - size(b)))
------------- reverse(a.begin(), a.end()), s = size(a)-s-1;//d1
----------- while(a[s]!=c)q.push_back(a[s]),s=(s+1)%size(a);
----------- q.push_back(c); -------------------------------//70
----------- rep(i,t+1,size(p)) q.push_back(root[p[i]]); ---//ff
----------- return q; } } } -------------------------------//67
----- emarked[v][w] = emarked[w][v] = true; } -------------//30
--- marked[v] = true; } return q; } -----------------------//2d
vii max_matching(const vector<vi> &adj) { -----------------//6e
- vi m(size(adj), -1), ap; vii res, es; -------------------//96
- rep(i,0,size(adj)) iter(it,adj[i]) es.emplace_back(i,*it);
- random_shuffle(es.begin(), es.end()); -------------------//57
- iter(it,es) if (m[it->first] == -1 && m[it->second] == -1)
--- m[it->first] = it->second, m[it->second] = it->first; -//63
- do { ap = find_augmenting_path(adj, m); -----------------//36
------ rep(i,0,size(ap)) m[m[ap[i^1]] = ap[i]] = ap[i^1]; -//61
- } while (!ap.empty()); ----------------------------------//29
- rep(i,0,size(m)) if (i < m[i]) res.emplace_back(i, m[i]);//da
- return res; } -------------------------------------------//79
```

3.19. **Maximum Density Subgraph.** Given (weighted) undirected graph G. Binary search density. If g is current density, construct flow network: (S,u,m), $(u,T,m+2g-d_u)$, $(u,v,1)$, where m is a large constant (larger than sum of edge weights). Run floating-point max-flow. If minimum cut has empty S-component, then maximum density is smaller than g, otherwise it's larger. Distance between valid densities is at least $1/(n(n-1))$. Edge case when density is 0. This also works for weighted graphs by replacing d_u by the weighted degree, and doing more iterations (if weights are not integers).

3.20. **Maximum-Weight Closure.** Given a vertex-weighted directed graph G. Turn the graph into a flow network, adding weight ∞ to each edge. Add vertices S,T. For each vertex v of weight w, add edge (S,v,w) if $w \geq 0$, or edge $(v,T,-w)$ if $w < 0$. Sum of positive weights minus minimum $S-T$ cut is the answer. Vertices reachable from S are in the closure. The maximum-weight closure is the same as the complement of the minimum-weight closure on the graph with edges reversed.

3.21. **Maximum Weighted Independent Set in a Bipartite Graph.** This is the same as the minimum weighted vertex cover. Solve this by constructing a flow network with edges $(S, u, w(u))$ for $u \in L$, $(v, T, w(v))$ for $v \in R$ and (u, v, ∞) for $(u, v) \in E$. The minimum S,T-cut is the answer. Vertices adjacent to a cut edge are in the vertex cover.

3.22. **Synchronizing word problem.** A DFA has a synchronizing word (an input sequence that moves all states to the same state) iff. each pair of states has a synchronizing word. That can be checked using reverse DFS over pairs of states. Finding the shortest synchronizing word is NP-complete.

3.23. **Max flow with lower bounds on edges.** Change edge $(u, v, l \leq f \leq c)$ to $(u, v, f \leq c - l)$. Add edge (t, s, ∞). Create super-nodes S, T. Let $M(u) = \sum_v l(v, u) - \sum_v l(u, v)$. If $M(u) < 0$, add edge $(u, T, -M(u))$, else add edge $(S, u, M(u))$. Max flow from S to T. If all edges from S are saturated, then we have a feasible flow. Continue running max flow from s to t in original graph.

3.24. **Tutte matrix for general matching.** Create an $n \times n$ matrix A. For each edge (i, j), $i < j$, let $A_{ij} = x_{ij}$ and $A_{ji} = -x_{ij}$. All other entries are 0. The determinant of A is zero iff. the graph has a perfect matching. A randomized algorithm uses the Schwartz–Zippel lemma to check if it is zero.

4. Strings

4.1. **The Knuth-Morris-Pratt algorithm.** An implementation of the Knuth-Morris-Pratt algorithm. Runs in $O(n + m)$ time, where n and m are the lengths of the string and the pattern.

```
int* compute_pi(const string &t) { -------------------------//a2
- int m = t.size(); ----------------------------------------//8b
- int *pit = new int[m + 1]; -------------------------------//8e
- if (0 <= m) pit[0] = 0; ----------------------------------//42
- if (1 <= m) pit[1] = 0; ----------------------------------//34
- rep(i,2,m+1) { -------------------------------------------//0f
--- for (int j = pit[i - 1]; ; j = pit[j]) { ---------------//b5
----- if (t[j] == t[i - 1]) { pit[i] = j + 1; break; } ----//21
----- if (j == 0) { pit[i] = 0; break; } } } ---------------//18
- return pit; } --------------------------------------------//3f
int string_match(const string &s, const string &t) { ------//47
- int n = s.size(), m = t.size(); --------------------------//7b
- int *pit = compute_pi(t); --------------------------------//20
- for (int i = 0, j = 0; i < n; ) { ------------------------//3b
--- if (s[i] == t[j]) { ------------------------------------//80
----- i++; j++; --------------------------------------------//5e
----- if (j == m) { ----------------------------------------//3d
------- return i - m; --------------------------------------//34
------- // or j = pit[j]; ---------------------------------//5a
----- } } --------------------------------------------------//08
--- else if (j > 0) j = pit[j]; ----------------------------//13
--- else i++; } -------------------------------------------//d3
- delete[] pit; return -1; } -------------------------------//e6
```

4.2. **The Z algorithm.** Given a string S, $Z_i(S)$ is the longest substring of S starting at i that is also a prefix of S. The Z algorithm computes these Z values in $O(n)$ time, where $n = |S|$. Z values can, for example, be used to find all occurrences of a pattern P in a string T in linear time. This is accomplished by computing Z values of $S = PT$, and looking for all i such that $Z_i \geq |P|$.

```
int* z_values(const string &s) { ---------------------------//4d
- int n = size(s); -----------------------------------------//97
- int* z = new int[n]; -------------------------------------//c4
- int l = 0, r = 0; ----------------------------------------//1c
- z[0] = n; ------------------------------------------------//98
- rep(i,1,n) { ---------------------------------------------//b2
--- z[i] = 0; ----------------------------------------------//4c
--- if (i > r) { -------------------------------------------//6d
----- l = r = i; -------------------------------------------//24
----- while (r < n && s[r - l] == s[r]) r++; ---------------//68
----- z[i] = r - l; r--; -----------------------------------//07
--- } else if (z[i - l] < r - i + 1) z[i] = z[i - l]; -----//6f
--- else { -------------------------------------------------//a8
----- l = i; -----------------------------------------------//55
----- while (r < n && s[r - l] == s[r]) r++; ---------------//2c
----- z[i] = r - l; r--; } } -------------------------------//13
- return z; } ----------------------------------------------//d0
```

4.3. **Trie.** A Trie class.

```
template <class T> -----------------------------------------//82
struct trie { ----------------------------------------------//4a
- struct node { --------------------------------------------//39
--- map<T, node*> children; --------------------------------//82
--- int prefixes, words; -----------------------------------//ff
--- node() { prefixes = words = 0; } }; --------------------//16
- node* root; ----------------------------------------------//97
- trie() : root(new node()) {  } ---------------------------//d2
- template <class I> ---------------------------------------//2f
- void insert(I begin, I end) { ----------------------------//3b
--- node* cur = root; --------------------------------------//ae
--- while (true) { -----------------------------------------//03
----- cur->prefixes++; -------------------------------------//6c
----- if (begin == end) { cur->words++; break; } -----------//df
----- else { -----------------------------------------------//51
------- T head = *begin; -----------------------------------//8f
------- typename map<T, node*>::const_iterator it; ---------//ff
------- it = cur->children.find(head); ---------------------//57
------- if (it == cur->children.end()) { -------------------//f7
--------- pair<T, node*> nw(head, new node()); -------------//66
--------- it = cur->children.insert(nw).first; -------------//c5
------- } begin++, cur = it->second; } } } -----------------//68
- template<class I> ----------------------------------------//51
- int countMatches(I begin, I end) { -----------------------//84
--- node* cur = root; --------------------------------------//88
--- while (true) { -----------------------------------------//5b
----- if (begin == end) return cur->words; -----------------//61
----- else { -----------------------------------------------//c1
------- T head = *begin; -----------------------------------//75
------- typename map<T, node*>::const_iterator it; ---------//00
------- it = cur->children.find(head); ---------------------//c6
------- if (it == cur->children.end()) return 0; -----------//06
------- begin++, cur = it->second; } } } -------------------//85
- template<class I> ----------------------------------------//e7
- int countPrefixes(I begin, I end) { ----------------------//7d
--- node* cur = root; --------------------------------------//c6
--- while (true) { -----------------------------------------//ac
----- if (begin == end) return cur->prefixes; --------------//33
----- else { -----------------------------------------------//85
------- T head = *begin; -----------------------------------//0e
------- typename map<T, node*>::const_iterator it; ---------//6e
------- it = cur->children.find(head); ---------------------//40
------- if (it == cur->children.end()) return 0; -----------//18
------- begin++, cur = it->second; } } } }; ----------------//7a
```

4.4. **Suffix Array.** An $O(n \log^2 n)$ construction of a Suffix Tree.

```
struct entry { ii nr; int p; }; ----------------------------//f9
bool operator <(const entry &a, const entry &b) { ----------//58
- return a.nr < b.nr; } ------------------------------------//61
struct suffix_array { --------------------------------------//e7
- string s; int n; vvi P; vector<entry> L; vi idx; ---------//30
- suffix_array(string _s) : s(_s), n(size(s)) { ------------//ea
--- L = vector<entry>(n), P.push_back(vi(n)), idx = vi(n); //99
--- rep(i,0,n) P[0][i] = s[i]; -----------------------------//5c
--- for (int stp = 1, cnt = 1; cnt >> 1 < n; stp++, cnt <<= 1){
----- P.push_back(vi(n)); ----------------------------------//76
----- rep(i,0,n) -------------------------------------------//f6
------- L[L[i].p = i].nr = ii(P[stp - 1][i], ---------------//f0
----------- i + cnt < n ? P[stp - 1][i + cnt] : -1); ------//27
----- sort(L.begin(), L.end()); ----------------------------//3e
----- rep(i,0,n) -------------------------------------------//ad
------- P[stp][L[i].p] = i > 0 && --------------------------//bd
--------- L[i].nr == L[i - 1].nr ? P[stp][L[i - 1].p] : i; }
--- rep(i,0,n) idx[P[size(P) - 1][i]] = i; } ---------------//33
- int lcp(int x, int y) { ----------------------------------//54
--- int res = 0; -------------------------------------------//85
--- if (x == y) return n - x; ------------------------------//0a
--- for (int k = size(P) - 1; k >= 0 && x < n && y < n; k--)
----- if (P[k][x] == P[k][y]) ------------------------------//2b
------- x += 1 << k, y += 1 << k, res += 1 << k; -----------//a4
--- return res; } }; ---------------------------------------//67
```

4.5. **Aho-Corasick Algorithm.** An implementation of the Aho-Corasick algorithm. Constructs a state machine from a set of keywords which can be used to search a string for any of the keywords.

```
struct aho_corasick { --------------------------------------//78
- struct out_node { ----------------------------------------//3e
--- string keyword; out_node *next; ------------------------//f0
--- out_node(string k, out_node *n) ------------------------//20
----- : keyword(k), next(n) { } }; -------------------------//3f
- struct go_node { -----------------------------------------//7a
--- map<char, go_node*> next; ------------------------------//44
--- out_node *out; go_node *fail; --------------------------//9c
--- go_node() { out = NULL; fail = NULL; } }; --------------//39
- go_node *go; ---------------------------------------------//b8
- aho_corasick(vector<string> keywords) { ------------------//e5
--- go = new go_node(); ------------------------------------//59
--- iter(k, keywords) { ------------------------------------//18
```

```
----- go_node *cur = go; ---------------------------------//8f
----- iter(c, *k) ---------------------------------------//62
------- cur = cur->next.find(*c) != cur->next.end() ? -----//c4
--------- cur->next[*c] : (cur->next[*c] = new go_node()); //f9
----- cur->out = new out_node(*k, cur->out); } -----------//d6
--- queue<go_node*> q; -----------------------------------//9a
--- iter(a, go->next) q.push(a->second); ------------------//8f
--- while (!q.empty()) { ---------------------------------//d1
----- go_node *r = q.front(); q.pop(); -------------------//f0
----- iter(a, r->next) { ---------------------------------//a9
------- go_node *s = a->second; --------------------------//ac
------- q.push(s); ---------------------------------------//35
------- go_node *st = r->fail; ---------------------------//44
------- while (st && st->next.find(a->first) == ----------//91
----------- st->next.end()) st = st->fail; ---------------//2b
------- if (!st) st = go; --------------------------------//33
------- s->fail = st->next[a->first]; --------------------//ad
------- if (s->fail) { -----------------------------------//36
--------- if (!s->out) s->out = s->fail->out; ------------//02
--------- else { -----------------------------------------//cc
----------- out_node* out = s->out; ----------------------//70
----------- while (out->next) out = out->next; -----------//7f
----------- out->next = s->fail->out; } } } } } ----------//dc
- vector<string> search(string s) { ----------------------//34
--- vector<string> res; ----------------------------------//43
--- go_node *cur = go; -----------------------------------//4c
--- iter(c, s) { -----------------------------------------//75
----- while (cur && cur->next.find(*c) == cur->next.end()) //95
------- cur = cur->fail; ---------------------------------//c0
----- if (!cur) cur = go; --------------------------------//1f
----- cur = cur->next[*c]; -------------------------------//63
----- if (!cur) cur = go; --------------------------------//d1
----- for (out_node *out = cur->out; out; out = out->next) //aa
------- res.push_back(out->keyword); } -------------------//ec
--- return res; } }; -------------------------------------//87
```

4.6. **eerTree.** Constructs an eerTree in $O(n)$, one character at a time.

```
#define MAXN 100100 ---------------------------------------//29
#define SIGMA 26 ------------------------------------------//e2
#define BASE 'a' ------------------------------------------//a1
char *s = new char[MAXN]; --------------------------------//db
struct state { -------------------------------------------//33
- int len, link, to[SIGMA]; ------------------------------//24
} *st = new state[MAXN+2]; -------------------------------//57
struct eertree { -----------------------------------------//78
- int last, sz, n; ---------------------------------------//ba
- eertree() : last(1), sz(2), n(0) { ---------------------//83
--- st[0].len = st[0].link = -1; -------------------------//3f
--- st[1].len = st[1].link = 0; } ------------------------//34
- int extend() { -----------------------------------------//c2
--- char c = s[n++]; int p = last; -----------------------//25
--- while (n - st[p].len - 2 < 0 || c != s[n - st[p].len - 2])
----- p = st[p].link; ------------------------------------//b0
--- if (!st[p].to[c-BASE]) { -----------------------------//f4
----- int q = last = sz++; -------------------------------//ff
----- st[p].to[c-BASE] = q; ------------------------------//b9
----- st[q].len = st[p].len + 2; -------------------------//c3
----- do { p = st[p].link; -------------------------------//80
----- } while (p != -1 && (n < st[p].len + 2 || ----------//74
------------- c != s[n - st[p].len - 2])); ---------------//93
----- if (p == -1) st[q].link = 1; -----------------------//e8
----- else st[q].link = st[p].to[c-BASE]; ----------------//bf
----- return 1; } ----------------------------------------//0a
--- last = st[p].to[c-BASE]; -----------------------------//63
--- return 0; } }; ---------------------------------------//b6
```

4.7. **Suffix Automaton.** Minimum automata that accepts all suffixes of a string with $O(n)$ construction. The automata itself is a DAG therefore suitable for DP, examples are counting unique substrings, occurrences of substrings and suffix.

```
// TODO: Add longest common subsring ---------------------//0e
const int MAXL = 100000; ---------------------------------//31
struct suffix_automaton { --------------------------------//e0
- vi  len, link, occur, cnt; -----------------------------//78
- vector<map<char,int> > next; ---------------------------//90
- vector<bool> isclone; ----------------------------------//7b
- ll *occuratleast; --------------------------------------//f2
- int sz, last; ------------------------------------------//7d
- string s; ----------------------------------------------//f2
- suffix_automaton() : len(MAXL*2), link(MAXL*2), --------//36
--- occur(MAXL*2), next(MAXL*2), isclone(MAXL*2) { clear(); }
- void clear() { sz = 1; last = len[0] = 0; link[0] = -1; -//91
--------------- next[0].clear(); isclone[0] = false; } ---//21
- bool issubstr(string other){ ---------------------------//46
--- for(int i = 0, cur = 0; i < size(other); ++i){ -------//2e
----- if(cur == -1) return false; cur = next[cur][other[i]]; }
--- return true; } ---------------------------------------//3e
- void extend(char c){ int cur = sz++; len[cur] = len[last]+1;
--- next[cur].clear(); isclone[cur] = false; int p = last; //3d
--- for(; p != -1 && !next[p].count(c); p = link[p]) -----//10
----- next[p][c] = cur; ----------------------------------//41
--- if(p == -1){ link[cur] = 0; } ------------------------//40
--- else{ int q = next[p][c]; ----------------------------//67
----- if(len[p] + 1 == len[q]){ link[cur] = q; } ---------//d2
----- else { int clone = sz++; isclone[clone] = true; ----//56
------- len[clone] = len[p] + 1; -------------------------//71
------- link[clone] = link[q]; next[clone] = next[q]; ----//6d
------- for(; p != -1 && next[p].count(c) && next[p][c] == q;
------------- p = link[p]){ ------------------------------//8c
-------- next[p][c] = clone; } ---------------------------//70
------- link[q] = link[cur] = clone; ---------------------//16
----- } } last = cur; } ----------------------------------//0f
- void count(){ ------------------------------------------//ef
--- cnt=vi(sz, -1); stack<ii> S; S.push(ii(0,0)); --------//8a
--- map<char,int>::iterator i; ---------------------------//81
--- while(!S.empty()){ -----------------------------------//20
----- ii cur = S.top(); S.pop(); -------------------------//09
----- if(cur.second){ ------------------------------------//bb
------- for(i = next[cur.first].begin(); -----------------//e2
---------- i != next[cur.first].end();++i){ --------------//32
-------- cnt[cur.first] += cnt[(*i).second]; } } --------//f1
----- else if(cnt[cur.first] == -1){ ---------------------//8f
------- cnt[cur.first] = 1; S.push(ii(cur.first, 1)); ----//9e
------- for(i = next[cur.first].begin(); -----------------//7e
----------- i != next[cur.first].end();++i){ -------------//4c
--------- S.push(ii((*i).second, 0)); } } } } ------------//55
- string lexicok(ll k){ ----------------------------------//ef
--- int st = 0; string res; map<char,int>::iterator i; ---//7f
--- while(k){ --------------------------------------------//68
----- for(i = next[st].begin(); i != next[st].end(); ++i){ //7e
------- if(k <= cnt[(*i).second]){ st = (*i).second; -----//ed
--------- res.push_back((*i).first); k--; break; ---------//61
------- } else { k -= cnt[(*i).second]; } } } ------------//7d
--- return res; } ----------------------------------------//32
- void countoccur(){ --------------------------------------//a6
--- for(int i = 0; i < sz; ++i){ occur[i] = 1 - isclone[i]; }
--- vii states(sz); --------------------------------------//23
--- for(int i = 0; i < sz; ++i){ states[i] = ii(len[i],i); }
--- sort(states.begin(), states.end()); ------------------//25
--- for(int i = size(states)-1; i >= 0; --i){ ------------//34
----- int v = states[i].second; --------------------------//20
----- if(link[v] != -1) { occur[link[v]] += occur[v]; }}}};//cf
```

4.8. **Hashing.** Modulus should be a large prime. Can also use multiple instances with different moduli to minimize chance of collision.

```
struct hasher { int b = 311, m; vi h, p; -----------------//61
- hasher(string s, int _m) -------------------------------//1a
--- : m(_m), h(size(s)+1), p(size(s)+1) { ----------------//9d
--- p[0] = 1; h[0] = 0; ----------------------------------//0d
--- rep(i,0,size(s)) p[i+1] = (ll)p[i] * b % m; ----------//17
--- rep(i,0,size(s)) h[i+1] = ((ll)h[i] * b + s[i]) % m; } //7c
- int hash(int l, int r) { -------------------------------//f2
--- return (h[r+1] + m - (ll)h[l] * p[r-l+1] % m) % m; } };//6e
```

5. Mathematics

5.1. **Fraction.** A fraction (rational number) class. Note that numbers are stored in lowest common terms.

```
template <class T> struct fraction { ---------------------//27
- T gcd(T a, T b) { return b == T(0) ? a : gcd(b, a % b); }//fe
- T n, d; ------------------------------------------------//6a
- fraction(T n_=T(0), T d_=T(1)) { -----------------------//be
--- assert(d_ != 0); -------------------------------------//41
--- n = n_, d = d_; --------------------------------------//d7
--- if (d < T(0)) n = -n, d = -d; ------------------------//ac
--- T g = gcd(abs(n), abs(d)); ---------------------------//bb
--- n /= g, d /= g; } ------------------------------------//55
- fraction(const fraction<T>& other) ---------------------//e3
--- : n(other.n), d(other.d) { } -------------------------//fa
- fraction<T> operator +(const fraction<T>& other) const { //d9
--- return fraction<T>(n * other.d + other.n * d, --------//bd
-------------------- d * other.d);} ----------------------//99
- fraction<T> operator -(const fraction<T>& other) const { //ae
--- return fraction<T>(n * other.d - other.n * d, --------//4a
-------------------- d * other.d); } ---------------------//8c
- fraction<T> operator *(const fraction<T>& other) const { //ea
--- return fraction<T>(n * other.n, d * other.d); } ------//65
- fraction<T> operator /(const fraction<T>& other) const { //52
--- return fraction<T>(n * other.d, d * other.n); } ------//af
```

```
- bool operator <(const fraction<T>& other) const { -------//f6
--- return n * other.d < other.n * d; } -------------------//d9
- bool operator <=(const fraction<T>& other) const { ------//77
--- return !(other < *this); } ----------------------------//bc
- bool operator >(const fraction<T>& other) const { -------//2c
--- return other < *this; } -------------------------------//04
- bool operator >=(const fraction<T>& other) const { ------//db
--- return !(*this < other); } ----------------------------//89
- bool operator ==(const fraction<T>& other) const { ------//c9
--- return n == other.n && d == other.d; } ----------------//02
- bool operator !=(const fraction<T>& other) const { ------//a4
--- return !(*this == other); } }; ------------------------//12
```

5.2. **Big Integer.** A big integer class.

```
struct intx { --------------------------------------------//cf
- intx() { normalize(1); } --------------------------------//6c
- intx(string n) { init(n); } -----------------------------//b9
- intx(int n) { stringstream ss; ss << n; init(ss.str()); }//36
- intx(const intx& other) ---------------------------------//a6
--- : sign(other.sign), data(other.data) { } --------------//3d
- int sign; -----------------------------------------------//de
- vector<unsigned int> data; ------------------------------//e7
- static const int dcnt = 9; ------------------------------//1a
- static const unsigned int radix = 1000000000U; ----------//5d
- int size() const { return data.size(); } ----------------//54
- void init(string n) { -----------------------------------//b4
--- intx res; res.data.clear(); ---------------------------//29
--- if (n.empty()) n = "0"; -------------------------------//fc
--- if (n[0] == '-') res.sign = -1, n = n.substr(1); ------//8a
--- for (int i = n.size() - 1; i >= 0; i -= intx::dcnt) { -//b8
----- unsigned int digit = 0; -----------------------------//91
----- for (int j = intx::dcnt - 1; j >= 0; j--) { ---------//b1
------- int idx = i - j; ----------------------------------//08
------- if (idx < 0) continue; ----------------------------//03
------- digit = digit * 10 + (n[idx] - '0'); } -----------//c8
----- res.data.push_back(digit); } ------------------------//6a
--- data = res.data; --------------------------------------//70
--- normalize(res.sign); } --------------------------------//4e
- intx& normalize(int nsign) { ----------------------------//65
--- if (data.empty()) data.push_back(0); ------------------//97
--- for (int i = data.size() - 1; i > 0 && data[i] == 0; i--)
----- data.erase(data.begin() + i); -----------------------//26
--- sign = data.size() == 1 && data[0] == 0 ? 1 : nsign; --//dc
--- return *this; } ---------------------------------------//b5
- friend ostream& operator <<(ostream& outs, const intx& n) {
--- if (n.sign < 0) outs << '-'; --------------------------//3e
--- bool first = true; ------------------------------------//cb
--- for (int i = n.size() - 1; i >= 0; i--) { ------------//7a
----- if (first) outs << n.data[i], first = false; --------//29
----- else { ----------------------------------------------//b3
------- unsigned int cur = n.data[i]; ---------------------//f8
------- stringstream ss; ss << cur; -----------------------//85
------- string s = ss.str(); ------------------------------//47
------- int len = s.size(); -------------------------------//34
------- while (len < intx::dcnt) outs << '0', len++; ------//c6
------- outs << s; } } ------------------------------------//93
--- return outs; } ----------------------------------------//0f
- string to_string() const { ------------------------------//38
--- stringstream ss; ss << *this; return ss.str(); } ------//51
- bool operator <(const intx& b) const { ------------------//24
--- if (sign != b.sign) return sign < b.sign; -------------//20
--- if (size() != b.size()) -------------------------------//ca
----- return sign == 1 ? size() < b.size() : size() > b.size();
--- for (int i = size() - 1; i >= 0; i--) -----------------//73
----- if (data[i] != b.data[i]) ---------------------------//14
------- return sign == 1 ? data[i] < b.data[i] ------------//2a
------------------------ : data[i] > b.data[i]; -----------//0c
--- return false; } ---------------------------------------//ba
- intx operator -() const { -------------------------------//bc
--- intx res(*this); res.sign *= -1; return res; } --------//19
- friend intx abs(const intx &n) { return n < 0 ? -n : n; }//61
- intx operator +(const intx& b) const { ------------------//cc
--- if (sign > 0 && b.sign < 0) return *this - (-b); ------//46
--- if (sign < 0 && b.sign > 0) return b - (-*this); ------//d7
--- if (sign < 0 && b.sign < 0) return -((-*this) + (-b)); //ae
--- intx c; c.data.clear(); -------------------------------//51
--- unsigned long long carry = 0; -------------------------//35
--- for (int i = 0; i < size() || i < b.size() || carry; i++) {
----- carry += (i < size() ? data[i] : 0ULL) + ------------//f0
------- (i < b.size() ? b.data[i] : 0ULL); ----------------//b6
----- c.data.push_back(carry % intx::radix); --------------//39
----- carry /= intx::radix; } -----------------------------//51
--- return c.normalize(sign); } ---------------------------//95
- intx operator -(const intx& b) const { ------------------//35
--- if (sign > 0 && b.sign < 0) return *this + (-b); ------//b4
--- if (sign < 0 && b.sign > 0) return -(-*this + b); -----//59
--- if (sign < 0 && b.sign < 0) return (-b) - (-*this); ---//84
--- if (*this < b) return -(b - *this); -------------------//7f
--- intx c; c.data.clear(); -------------------------------//46
--- long long borrow = 0; ---------------------------------//05
--- rep(i,0,size()) { -------------------------------------//9f
----- borrow = data[i] - borrow ---------------------------//a4
--------------------- - (i < b.size() ? b.data[i] : 0ULL);//aa
----- c.data.push_back(borrow < 0 ? intx::radix + borrow --//13
-------------------------------- : borrow); ---------------//d1
----- borrow = borrow < 0 ? 1 : 0; } ----------------------//1b
--- return c.normalize(sign); } ---------------------------//8a
- intx operator *(const intx& b) const { ------------------//c3
--- intx c; c.data.assign(size() + b.size() + 1, 0); ------//7d
--- rep(i,0,size()) { -------------------------------------//c0
----- long long carry = 0; --------------------------------//f6
----- for (int j = 0; j < b.size() || carry; j++) { -------//c8
------- if (j < b.size()) ---------------------------------//bc
--------- carry += (long long)data[i] * b.data[j]; --------//37
------- carry += c.data[i + j]; ---------------------------//5c
------- c.data[i + j] = carry % intx::radix; --------------//cd
------- carry /= intx::radix; } } -------------------------//ef
--- return c.normalize(sign * b.sign); } ------------------//ca
- friend pair<intx,intx> divmod(const intx& n, const intx& d) {
--- assert(!(d.size() == 1 && d.data[0] == 0)); -----------//67
--- intx q, r; q.data.assign(n.size(), 0); ----------------//e2
--- for (int i = n.size() - 1; i >= 0; i--) { -------------//76
----- r.data.insert(r.data.begin(), 0); -------------------//2a
----- r = r + n.data[i]; ----------------------------------//58
----- long long k = 0; ------------------------------------//6a
----- if (d.size() < r.size()) ----------------------------//01
------- k = (long long)intx::radix * r.data[d.size()]; ----//0d
----- if (d.size() - 1 < r.size()) k += r.data[d.size() - 1];
----- k /= d.data.back(); ---------------------------------//61
----- r = r - abs(d) * k; ---------------------------------//e4
----- // if (r < 0) for (ll t = 1LL << 62; t >= 1; t >>= 1) {
----- //     intx dd = abs(d) * t; ------------------------//3b
----- //     while (r + dd < 0) r = r + dd, k -= t; } -----//bb
----- while (r < 0) r = r + abs(d), k--; ------------------//b2
----- q.data[i] = k; } ------------------------------------//eb
--- return pair<intx, intx>(q.normalize(n.sign * d.sign), r); }
- intx operator /(const intx& d) const { ------------------//20
--- return divmod(*this,d).first; } -----------------------//c2
- intx operator %(const intx& d) const { ------------------//d9
--- return divmod(*this,d).second * sign; } }; ------------//28
```

5.2.1. *Fast Multiplication.* Fast multiplication for the big integer using Fast Fourier Transform.

```
#include "intx.cpp" ---------------------------------------//83
#include "fft.cpp" ----------------------------------------//13
intx fastmul(const intx &an, const intx &bn) { ------------//03
- string as = an.to_string(), bs = bn.to_string(); --------//fe
- int n = size(as), m = size(bs), l = 1, ------------------//a6
--- len = 5, radix = 100000, ------------------------------//b5
--- *a = new int[n], alen = 0, ----------------------------//4b
--- *b = new int[m], blen = 0; ----------------------------//c3
- memset(a, 0, n << 2); -----------------------------------//1d
- memset(b, 0, m << 2); -----------------------------------//d1
- for (int i = n - 1; i >= 0; i -= len, alen++) -----------//22
--- for (int j = min(len - 1, i); j >= 0; j--) ------------//3e
----- a[alen] = a[alen] * 10 + as[i - j] - '0'; -----------//31
- for (int i = m - 1; i >= 0; i -= len, blen++) -----------//f3
--- for (int j = min(len - 1, i); j >= 0; j--) ------------//a4
----- b[blen] = b[blen] * 10 + bs[i - j] - '0'; -----------//36
- while (l < 2*max(alen,blen)) l <<= 1; -------------------//8e
- cpx *A = new cpx[l], *B = new cpx[l]; -------------------//7d
- rep(i,0,l) A[i] = cpx(i < alen ? a[i] : 0, 0); ----------//01
- rep(i,0,l) B[i] = cpx(i < blen ? b[i] : 0, 0); ----------//d1
- fft(A, l); fft(B, l); -----------------------------------//77
- rep(i,0,l) A[i] *= B[i]; --------------------------------//78
- fft(A, l, true); ----------------------------------------//4b
- ull *data = new ull[l]; ---------------------------------//ab
- rep(i,0,l) data[i] = (ull)(round(real(A[i]))); ----------//f4
- rep(i,0,l-1) --------------------------------------------//a0
--- if (data[i] >= (unsigned int)(radix)) { ---------------//8f
----- data[i+1] += data[i] / radix; -----------------------//b1
----- data[i] %= radix; } ---------------------------------//7d
- int stop = l-1; -----------------------------------------//f5
- while (stop > 0 && data[stop] == 0) stop--; -------------//36
- stringstream ss; ----------------------------------------//75
- ss << data[stop]; ---------------------------------------//e9
- for (int i = stop - 1; i >= 0; i--) ---------------------//99
--- ss << setfill('0') << setw(len) << data[i]; -----------//8d
```

```
- delete[] A; delete[] B; ----------------------------------//ad
- delete[] a; delete[] b; ----------------------------------//5b
- delete[] data; -------------------------------------------//1e
- return intx(ss.str()); } ---------------------------------//cf
```

5.3. **Binomial Coefficients.** The binomial coefficient $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ is the number of ways to choose k items out of a total of n items. Also contains an implementation of Lucas' theorem for computing the answer modulo a prime p. Use modular multiplicative inverse if needed, and be very careful of overflows.

```
int nck(int n, int k) { ------------------------------------//f6
- if (n < k) return 0; -------------------------------------//55
- k = min(k, n - k); ---------------------------------------//bd
- int res = 1; ---------------------------------------------//e6
- rep(i,1,k+1) res = res * (n - (k - i)) / i; -------------//4d
- return res; } --------------------------------------------//0e
int nck(int n, int k, int p) { -----------------------------//94
- int res = 1; ---------------------------------------------//30
- while (n || k) { -----------------------------------------//84
--- res = nck(n % p, k % p) % p * res % p; -----------------//33
--- n /= p, k /= p; } --------------------------------------//bf
- return res; } --------------------------------------------//f4
```

5.4. **Euclidean algorithm.** The Euclidean algorithm computes the greatest common divisor of two integers a, b.

```
ll gcd(ll a, ll b) { return b == 0 ? a : gcd(b, a % b); } -//39
```

The extended Euclidean algorithm computes the greatest common divisor d of two integers a, b and also finds two integers x, y such that $a \times x + b \times y = d$.

```
ll egcd(ll a, ll b, ll& x, ll& y) { ------------------------//e0
- if (b == 0) { x = 1; y = 0; return a; } ------------------//8b
- ll d = egcd(b, a % b, x, y); -----------------------------//6a
- x -= a / b * y; swap(x, y); return d; } ------------------//95
```

5.5. **Trial Division Primality Testing.** An optimized trial division to check whether an integer is prime.

```
bool is_prime(int n) { -------------------------------------//6c
- if (n < 2) return false; ---------------------------------//c9
- if (n < 4) return true; ----------------------------------//d9
- if (n % 2 == 0 || n % 3 == 0) return false; -------------//0f
- if (n < 25) return true; ---------------------------------//ef
- for (int i = 5; i*i <= n; i += 6) -----------------------//38
--- if (n % i == 0 || n % (i + 2) == 0) return false; -----//69
- return true; } -------------------------------------------//b1
```

5.6. **Miller-Rabin Primality Test.** The Miller-Rabin probabilistic primality test.

```
#include "mod_pow.cpp" -------------------------------------//c7
bool is_probable_prime(ll n, int k) { ----------------------//be
- if (~n & 1) return n == 2; -------------------------------//d1
- if (n <= 3) return n == 3; -------------------------------//39
- int s = 0; ll d = n - 1; ---------------------------------//37
- while (~d & 1) d >>= 1, s++; -----------------------------//35
- while (k--) { --------------------------------------------//c8
--- ll a = (n - 3) * rand() / RAND_MAX + 2; ----------------//06
--- ll x = mod_pow(a, d, n); -------------------------------//64
--- if (x == 1 || x == n - 1) continue; --------------------//9b
--- bool ok = false; ---------------------------------------//03
--- rep(i,0,s-1) { -----------------------------------------//13
----- x = (x * x) % n; -------------------------------------//90
----- if (x == 1) return false; ----------------------------//5c
----- if (x == n - 1) { ok = true; break; } ----------------//a1
--- } ------------------------------------------------------//3a
--- if (!ok) return false; ---------------------------------//37
- } return true; } -----------------------------------------//fe
```

5.7. **Pollard's ρ algorithm.**

```
// public static int[] seeds = new int[] {2,3,5,7,11,13,1031};
// public static BigInteger rho(BigInteger n, --------------//8a
//                              BigInteger seed) { ---------//3e
//     int i = 0, ------------------------------------------//a5
//         k = 2; ------------------------------------------//ad
//     BigInteger x = seed, --------------------------------//4f
//                y = seed; --------------------------------//8b
//     while (i < 1000000) { -------------------------------//9f
//         i++; --------------------------------------------//e3
//         x = (x.multiply(x).add(n) -----------------------//83
//               .subtract(BigInteger.ONE)).mod(n); --------//3f
//         BigInteger d = y.subtract(x).abs().gcd(n); ------//d0
//         if (!d.equals(BigInteger.ONE) && !d.equals(n)) {//47
//             return d; } ---------------------------------//32
//         if (i == k) { -----------------------------------//5e
//             y = x; --------------------------------------//f0
//             k = k*2; } } --------------------------------//23
//     return BigInteger.ONE; } ----------------------------//25
```

5.8. **Sieve of Eratosthenes.** An optimized implementation of Eratosthenes' Sieve.

```
vi prime_sieve(int n) { ------------------------------------//40
- int mx = (n - 3) >> 1, sq, v, i = -1; --------------------//27
- vi primes; -----------------------------------------------//8f
- bool* prime = new bool[mx + 1]; --------------------------//ef
- memset(prime, 1, mx + 1); --------------------------------//28
- if (n >= 2) primes.push_back(2); -------------------------//f4
- while (++i <= mx) if (prime[i]) { ------------------------//73
--- primes.push_back(v = (i << 1) + 3); --------------------//be
--- if ((sq = i * ((i << 1) + 6) + 3) > mx) break; ---------//2d
--- for (int j = sq; j <= mx; j += v) prime[j] = false; } -//2e
- while (++i <= mx) ----------------------------------------//52
--- if (prime[i]) primes.push_back((i << 1) + 3); ----------//ff
- delete[] prime; // can be used for O(1) lookup ----------//ae
- return primes; } -----------------------------------------//a8
```

5.9. **Divisor Sieve.** A O(n) prime sieve. Computes the smallest divisor of any number up to n.

```
vi divisor_sieve(int n) { ----------------------------------//7f
- vi mnd(n+1, 2), ps; --------------------------------------//ca
- if (n >= 2) ps.push_back(2); -----------------------------//79
- mnd[0] = 0; ----------------------------------------------//3d
- for (int k = 1; k <= n; k += 2) mnd[k] = k; --------------//b1
- for (int k = 3; k <= n; k += 2) { ------------------------//d9
--- if (mnd[k] == k) ps.push_back(k); ----------------------//7c
--- rep(i,1,size(ps)) --------------------------------------//3d
----- if (ps[i] > mnd[k] || ps[i]*k > n) break; ------------//6f
----- else mnd[ps[i]*k] = ps[i]; } -------------------------//06
- return ps; } ---------------------------------------------//06
```

5.10. **Modular Exponentiation.** A function to perform fast modular exponentiation.

```
template <class T> -----------------------------------------//82
T mod_pow(T b, T e, T m) { ---------------------------------//aa
- T res = T(1); --------------------------------------------//85
- while (e) { ----------------------------------------------//b7
--- if (e & T(1)) res = smod(res * b, m); ------------------//6d
--- b = smod(b * b, m), e >>= T(1); } ----------------------//12
- return res; } --------------------------------------------//86
```

5.11. **Modular Multiplicative Inverse.** A function to find a modular multiplicative inverse. Alternatively use `mod_pow(a,m-2,m)` when m is prime.

```
#include "egcd.cpp" ----------------------------------------//55
ll mod_inv(ll a, ll m) { -----------------------------------//0a
- ll x, y, d = egcd(a, m, x, y); ---------------------------//db
- return d == 1 ? smod(x,m) : -1; } ------------------------//7a
```

A sieve version:

```
vi inv_sieve(int n, int p) { -------------------------------//40
- vi inv(n,1); ---------------------------------------------//d7
- rep(i,2,n) inv[i] = (p - (ll)(p/i) * inv[p%i] % p) % p; -//fe
- return inv; } --------------------------------------------//14
```

5.12. **Primitive Root.**

```
#include "mod_pow.cpp" -------------------------------------//c7
ll primitive_root(ll m) { ----------------------------------//8a
- vector<ll> div; ------------------------------------------//f2
- for (ll i = 1; i*i <= m-1; i++) { ------------------------//ca
--- if ((m-1) % i == 0) { ----------------------------------//85
----- if (i < m) div.push_back(i); -------------------------//fd
----- if (m/i < m) div.push_back(m/i); } } -----------------//f2
- rep(x,2,m) { ---------------------------------------------//57
--- bool ok = true; ----------------------------------------//17
--- iter(it,div) if (mod_pow<ll>(x, *it, m) == 1) { -------//48
----- ok = false; break; } ---------------------------------//e5
--- if (ok) return x; } ------------------------------------//00
- return -1; } ---------------------------------------------//a8
```

5.13. **Chinese Remainder Theorem.** An implementation of the Chinese Remainder Theorem.

```
#include "egcd.cpp" ----------------------------------------//55
ll crt(vector<ll> &as, vector<ll> &ns) { -------------------//72
- ll cnt = size(as), N = 1, x = 0, r, s, l; ----------------//ce
- rep(i,0,cnt) N *= ns[i]; ---------------------------------//6a
- rep(i,0,cnt) egcd(ns[i], l = N/ns[i], r, s), x += as[i]*s*l;
- return smod(x, N); } -------------------------------------//80
pair<ll,ll> gcrt(vector<ll> &as, vector<ll> &ns) { ---------//30
- map<ll,pair<ll,ll> > ms; ---------------------------------//79
- rep(at,0,size(as)) { -------------------------------------//45
--- ll n = ns[at]; -----------------------------------------//48
--- for (ll i = 2; i*i <= n; i = i == 2 ? 3 : i + 2) { ----//d5
----- ll cur = 1; ------------------------------------------//88
----- while (n % i == 0) n /= i, cur *= i; -----------------//38
----- if (cur > 1 && cur > ms[i].first) --------------------//97
```

```
------- ms[i] = make_pair(cur, as[at] % cur); } -----------//af
--- if (n > 1 && n > ms[n].first) -------------------------//0d
----- ms[n] = make_pair(n, as[at] % n); } -----------------//6f
- vector<ll> as2, ns2; ll n = 1; --------------------------//cc
- iter(it,ms) { -------------------------------------------//6e
--- as2.push_back(it->second.second); ---------------------//f8
--- ns2.push_back(it->second.first); ----------------------//2b
--- n *= it->second.first; } ------------------------------//ba
- ll x = crt(as2,ns2); ------------------------------------//57
- rep(i,0,size(as)) if (smod(x,ns[i]) != smod(as[i],ns[i]))//d6
----- return ii(0,0); -------------------------------------//e6
- return make_pair(x,n); } --------------------------------//e1
```

5.14. **Linear Congruence Solver.** Given $ax \equiv b \pmod n$, returns (t, m) such that all solutions are given by $x \equiv t \pmod m$. No solutions iff $(0, 0)$ is returned.

```
#include "egcd.cpp" ---------------------------------------//55
pair<ll,ll> linear_congruence(ll a, ll b, ll n) { ---------//62
- ll x, y, d = egcd(smod(a,n), n, x, y); ------------------//17
- if ((b = smod(b,n)) % d != 0) return ii(0,0); -----------//5a
- return make_pair(smod(b / d * x, n),n/d); } -------------//3d
```

5.15. **Tonelli-Shanks algorithm.** Given prime p and integer $1 \leq n < p$, returns the square root r of n modulo p. There is also another solution given by $-r$ modulo p.

```
#include "mod_pow.cpp" ------------------------------------//c7
ll legendre(ll a, ll p) { ---------------------------------//27
- if (a % p == 0) return 0; -------------------------------//29
- if (p == 2) return 1; -----------------------------------//9a
- return mod_pow(a, (p-1)/2, p) == 1 ? 1 : -1; } ----------//65
ll tonelli_shanks(ll n, ll p) { ---------------------------//e0
- assert(legendre(n,p) == 1); -----------------------------//46
- if (p == 2) return 1; -----------------------------------//2d
- ll s = 0, q = p-1, z = 2; -------------------------------//66
- while (~q & 1) s++, q >>= 1; ----------------------------//a7
- if (s == 1) return mod_pow(n, (p+1)/4, p); --------------//a7
- while (legendre(z,p) != -1) z++; ------------------------//25
- ll c = mod_pow(z, q, p), --------------------------------//65
--- r = mod_pow(n, (q+1)/2, p), ---------------------------//b5
--- t = mod_pow(n, q, p), ---------------------------------//5c
--- m = s; ------------------------------------------------//01
- while (t != 1) { ----------------------------------------//44
--- ll i = 1, ts = (ll)t*t % p; ---------------------------//55
--- while (ts != 1) i++, ts = ((ll)ts * ts) % p; ----------//16
--- ll b = mod_pow(c, 1LL<<(m-i-1), p); -------------------//6c
--- r = (ll)r * b % p; ------------------------------------//4f
--- t = (ll)t * b % p * b % p; ----------------------------//78
--- c = (ll)b * b % p; ------------------------------------//31
--- m = i; } ----------------------------------------------//b2
- return r; } ---------------------------------------------//48
```

5.16. **Numeric Integration.** Numeric integration using Simpson's rule.

```
double integrate(double (*f)(double), double a, double b, -//76
--- double delta = 1e-6) { --------------------------------//c0
- if (abs(a - b) < delta) ---------------------------------//38
--- return (b-a)/8 * --------------------------------------//56
----- (f(a) + 3*f((2*a+b)/3) + 3*f((a+2*b)/3) + f(b)); ----//e1
- return integrate(f, a, ----------------------------------//64
----- (a+b)/2, delta) + integrate(f, (a+b)/2, b, delta); } //a3
```

5.17. **Fast Fourier Transform.** The Cooley-Tukey algorithm for quickly computing the discrete Fourier transform. The `fft` function only supports powers of twos. The `czt` function implements the Chirp Z-transform and supports any size, but is slightly slower.

```
#include <complex> ----------------------------------------//8e
typedef complex<long double> cpx; -------------------------//25
// NOTE: n must be a power of two -------------------------//14
void fft(cpx *x, int n, bool inv=false) { -----------------//36
- for (int i = 0, j = 0; i < n; i++) { --------------------//f9
--- if (i < j) swap(x[i], x[j]); --------------------------//44
--- int m = n>>1; -----------------------------------------//9c
--- while (1 <= m && m <= j) j -= m, m >>= 1; -------------//fe
--- j += m; } ---------------------------------------------//83
- for (int mx = 1; mx < n; mx <<= 1) { --------------------//16
--- cpx wp = exp(cpx(0, (inv ? -1 : 1) * pi / mx)), w = 1; //5c
--- for (int m = 0; m < mx; m++, w *= wp) { ---------------//82
----- for (int i = m; i < n; i += mx << 1) { --------------//23
------- cpx t = x[i + mx] * w; ----------------------------//44
------- x[i + mx] = x[i] - t; -----------------------------//da
------- x[i] += t; } } } ----------------------------------//57
- if (inv) rep(i,0,n) x[i] /= cpx(n); } -------------------//50
void czt(cpx *x, int n, bool inv=false) { -----------------//0d
- int len = 2*n+1; ----------------------------------------//c5
- while (len & (len - 1)) len &= len - 1; -----------------//1b
- len <<= 1; ----------------------------------------------//d4
- cpx w = exp(-2.0L * pi / n * cpx(0,1)), -----------------//d5
--- *c = new cpx[n], *a = new cpx[len], -------------------//09
--- *b = new cpx[len]; ------------------------------------//78
- rep(i,0,n) c[i] = pow(w, (inv ? -1.0 : 1.0)*i*i/2); -----//da
- rep(i,0,n) a[i] = x[i] * c[i], b[i] = 1.0L/c[i]; --------//67
- rep(i,0,n-1) b[len - n + i + 1] = 1.0L/c[n-i-1]; --------//4c
- fft(a, len); fft(b, len); -------------------------------//1d
- rep(i,0,len) a[i] *= b[i]; ------------------------------//a6
- fft(a, len, true); --------------------------------------//96
- rep(i,0,n) { --------------------------------------------//29
--- x[i] = c[i] * a[i]; -----------------------------------//43
--- if (inv) x[i] /= cpx(n); } ----------------------------//ed
- delete[] a; ---------------------------------------------//f7
- delete[] b; ---------------------------------------------//94
- delete[] c; } -------------------------------------------//2c
```

5.18. **Number-Theoretic Transform.** Other possible moduli: 2 113 929 217 (2^{25}), 2 013 265 920 268 435 457 (2^{28}, with $g = 5$).

```
#include "../mathematics/primitive_root.cpp" --------------//8c
int mod = 998244353, g = primitive_root(mod), -------------//9c
- ginv = mod_pow<ll>(g, mod-2, mod), ----------------------//7e
- inv2 = mod_pow<ll>(2, mod-2, mod); ----------------------//5b
#define MAXN (1<<22) --------------------------------------//29
struct Num { ----------------------------------------------//bf
- int x; --------------------------------------------------//5b
- Num(ll _x=0) { x = (_x%mod+mod)%mod; } ------------------//6f
- Num operator +(const Num &b) { return x + b.x; } --------//55
- Num operator -(const Num &b) const { return x - b.x; } --//c5
- Num operator *(const Num &b) const { return (ll)x * b.x; }
- Num operator /(const Num &b) const { --------------------//5e
--- return (ll)x * b.inv().x; } ---------------------------//f1
- Num inv() const { return mod_pow<ll>((ll)x, mod-2, mod); }
- Num pow(int p) const { return mod_pow<ll>((ll)x, p, mod); }
} T1[MAXN], T2[MAXN]; -------------------------------------//47
void ntt(Num x[], int n, bool inv = false) { --------------//d6
- Num z = inv ? ginv : g; ---------------------------------//22
- z = z.pow((mod - 1) / n); -------------------------------//6b
- for (ll i = 0, j = 0; i < n; i++) { ---------------------//8e
--- if (i < j) swap(x[i], x[j]); --------------------------//0c
--- ll k = n>>1; ------------------------------------------//e1
--- while (1 <= k && k <= j) j -= k, k >>= 1; -------------//dd
--- j += k; } ---------------------------------------------//ee
- for (int mx = 1, p = n/2; mx < n; mx <<= 1, p >>= 1) { --//23
--- Num wp = z.pow(p), w = 1; -----------------------------//af
--- for (int k = 0; k < mx; k++, w = w*wp) { --------------//2b
----- for (int i = k; i < n; i += mx << 1) { --------------//32
------- Num t = x[i + mx] * w; ----------------------------//82
------- x[i + mx] = x[i] - t; -----------------------------//67
------- x[i] = x[i] + t; } } } ----------------------------//b9
- if (inv) { ----------------------------------------------//64
--- Num ni = Num(n).inv(); --------------------------------//91
--- rep(i,0,n) { x[i] = x[i] * ni; } } } ------------------//7f
void inv(Num x[], Num y[], int l) { -----------------------//1e
- if (l == 1) { y[0] = x[0].inv(); return; } --------------//5b
- inv(x, y, l>>1); ----------------------------------------//7e
- // NOTE: maybe l<<2 instead of l<<1 ----------------------//e6
- rep(i,l>>1,l<<1) T1[i] = y[i] = 0; ----------------------//2b
- rep(i,0,l) T1[i] = x[i]; --------------------------------//60
- ntt(T1, l<<1); ntt(y, l<<1); ----------------------------//4c
- rep(i,0,l<<1) y[i] = y[i]*2 - T1[i] * y[i] * y[i]; ------//14
- ntt(y, l<<1, true); } -----------------------------------//18
void sqrt(Num x[], Num y[], int l) { ----------------------//9f
- if (l == 1) { assert(x[0].x == 1); y[0] = 1; return; } --//5d
- sqrt(x, y, l>>1); ---------------------------------------//7b
- inv(y, T2, l>>1); ---------------------------------------//50
- rep(i,l>>1,l<<1) T1[i] = T2[i] = 0; ---------------------//56
- rep(i,0,l) T1[i] = x[i]; --------------------------------//e6
- ntt(T2, l<<1); ntt(T1, l<<1); ---------------------------//25
- rep(i,0,l<<1) T2[i] = T1[i] * T2[i]; --------------------//6b
- ntt(T2, l<<1, true); ------------------------------------//9d
- rep(i,0,l) y[i] = (y[i] + T2[i]) * inv2; } --------------//9d
```

5.19. **Fast Hadamard Transform.** Computes the Hadamard transform of the given array. Can be used to compute the `XOR`-convolution of arrays, exactly like with FFT. For `AND`-convolution, use $(x + y, y)$ and $(x - y, y)$. For `OR`-convolution, use $(x, x + y)$ and $(x, -x + y)$. **Note**: Size of array must be a power of 2.

```
void fht(vi &arr, bool inv=false, int l=0, int r=-1) { ----//f7
- if (r == -1) { fht(arr,inv,0,size(arr)); return; } ------//e5
- if (l+1 == r) return; -----------------------------------//3c
- int k = (r-l)/2; ----------------------------------------//8f
- if (!inv) fht(arr, inv, l, l+k), fht(arr, inv, l+k, r); -//ef
- rep(i,l,l+k) { int x = arr[i], y = arr[i+k]; ------------//93
--- if (!inv) arr[i] = x-y, arr[i+k] = x+y; ---------------//81
```

```
--- else arr[i] = (x+y)/2, arr[i+k] = (-x+y)/2; } ---------//38
- if (inv) fht(arr, inv, l, l+k), fht(arr, inv, l+k, r); } //db
```

5.20. **Tridiagonal Matrix Algorithm.** Solves a tridiagonal system of linear equations $a_i x_{i-1} + b_i x_i + c_i x_{i+1} = d_i$ where $a_1 = c_n = 0$. Beware of numerical instability.

```
#define MAXN 5000 ----------------------------------------//f7
long double A[MAXN], B[MAXN], C[MAXN], D[MAXN], X[MAXN]; --//d8
void solve(int n) { ---------------------------------------//01
- C[0] /= B[0]; D[0] /= B[0]; -----------------------------//94
- rep(i,1,n-1) C[i] /= B[i] - A[i]*C[i-1]; ----------------//6b
- rep(i,1,n) ----------------------------------------------//52
--- D[i] = (D[i] - A[i] * D[i-1]) / (B[i] - A[i] * C[i-1]);//d4
- X[n-1] = D[n-1]; ----------------------------------------//d7
- for (int i = n-2; i>=0; i--) ----------------------------//65
--- X[i] = D[i] - C[i] * X[i+1]; } ------------------------//6c
```

5.21. **Mertens Function.** Mertens function is $M(n) = \sum_{i=1}^{n} \mu(i)$. Let $L \approx (n \log \log n)^{2/3}$ and the algorithm runs in $O(n^{2/3})$.

```
#define L 9000000 ----------------------------------------//27
int mob[L], mer[L]; --------------------------------------//f1
unordered_map<ll,ll> mem; --------------------------------//30
ll M(ll n) { ---------------------------------------------//de
- if (n < L) return mer[n]; ------------------------------//1c
- if (mem.find(n) != mem.end()) return mem[n]; -----------//79
- ll ans = 0, done = 1; ----------------------------------//48
- for (ll i = 2; i*i <= n; i++) ans += M(n/i), done = i; --//41
- for (ll i = 1; i*i <= n; i++) --------------------------//35
--- ans += mer[i] * (n/i - max(done, n/(i+1))); ----------//94
- return mem[n] = 1 - ans; } -----------------------------//5c
void sieve() { -------------------------------------------//94
- for (int i = 1; i < L; i++) mer[i] = mob[i] = 1; -------//a8
- for (int i = 2; i < L; i++) { --------------------------//94
--- if (mer[i]) { ----------------------------------------//33
----- mob[i] = -1; ---------------------------------------//3c
----- for (int j = i+i; j < L; j += i) -------------------//58
------- mer[j] = 0, mob[j] = (j/i)%i == 0 ? 0 : -mob[j/i]; }
--- mer[i] = mob[i] + mer[i-1]; } } ----------------------//70
```

5.22. **Summatory Phi.** The summatory phi function $\Phi(n) = \sum_{i=1}^{n} \phi(i)$. Let $L \approx (n \log \log n)^{2/3}$ and the algorithm runs in $O(n^{2/3})$.

```
#define N 10000000 ---------------------------------------//e8
ll sp[N]; ------------------------------------------------//90
unordered_map<ll,ll> mem; --------------------------------//54
ll sumphi(ll n) { ----------------------------------------//3a
- if (n < N) return sp[n]; -------------------------------//de
- if (mem.find(n) != mem.end()) return mem[n]; -----------//4c
- ll ans = 0, done = 1; ----------------------------------//b2
- for (ll i = 2; i*i <= n; i++) ans += sumphi(n/i), done = i;
- for (ll i = 1; i*i <= n; i++) --------------------------//5a
--- ans += sp[i] * (n/i - max(done, n/(i+1))); -----------//b0
- return mem[n] = n*(n+1)/2 - ans; } ---------------------//fa
void sieve() { -------------------------------------------//55
- for (int i = 1; i < N; i++) sp[i] = i; -----------------//61
- for (int i = 2; i < N; i++) { --------------------------//f4
--- if (sp[i] == i) { ------------------------------------//e3
----- sp[i] = i-1; ---------------------------------------//d9
----- for (int j = i+i; j < N; j += i) sp[j] -= sp[j] / i; }
--- sp[i] += sp[i-1]; } } --------------------------------//f3
```

5.23. **Prime π.** Returns $\pi(\lfloor n/k \rfloor)$ for all $1 \le k \le n$, where $\pi(n)$ is the number of primes $\le n$. Can also be modified to accumulate any multiplicative function over the primes.

```
#include "prime_sieve.cpp" -------------------------------//3d
unordered_map<ll,ll> primepi(ll n) { ---------------------//73
#define f(n) (1) ------------------------------------------//34
#define F(n) (n) ------------------------------------------//99
- ll st = 1, *dp[3], k = 0; ------------------------------//a7
- while (st*st < n) st++; --------------------------------//bd
- vi ps = prime_sieve(st); -------------------------------//ae
- ps.push_back(st+1); ------------------------------------//21
- rep(i,0,3) dp[i] = new ll[2*st]; -----------------------//5a
- ll *pre = new ll[size(ps)-1]; --------------------------//dc
- rep(i,0,size(ps)-1) ------------------------------------//a5
--- pre[i] = f(ps[i]) + (i == 0 ? f(1) : pre[i-1]); -------//eb
#define L(i) ((i)<st?(i)+1:n/(2*st-(i))) -----------------//67
#define I(l) ((l)<st?(l)-1:2*st-n/(l)) -------------------//da
- rep(i,0,2*st) { ----------------------------------------//8a
--- ll cur = L(i); ---------------------------------------//e6
--- while ((ll)ps[k]*ps[k] <= cur) k++; ------------------//96
--- dp[2][i] = k, dp[1][i] = F(L(i)), dp[0][i] = 0; } -----//cf
- for (int j = 0, start = 0; start < 2*st; j++) { --------//f9
--- rep(i,start,2*st) { ----------------------------------//4b
----- if (j >= dp[2][i]) { start++; continue; } ----------//18
----- ll s = j == 0 ? f(1) : pre[j-1]; -------------------//c2
----- int l = I(L(i)/ps[j]); -----------------------------//35
----- dp[j&1][i] = dp[~j&1][i] ---------------------------//14
------- - f(ps[j]) * (dp[~min(j,(int)dp[2][l])&1][l] - s); //61
--- } } --------------------------------------------------//c0
- unordered_map<ll,ll> res; ------------------------------//23
- rep(i,0,2*st) res[L(i)] = dp[~dp[2][i]&1][i]-f(1); ------//20
- delete[] pre; rep(i,0,3) delete[] dp[i]; ---------------//9d
- return res; } ------------------------------------------//6d
```

5.24. **Josephus problem.** Last man standing out of n if every kth is killed. Zero-based, and does not kill 0 on first pass.

```
int J(int n, int k) { ------------------------------------//27
- if (n == 1) return 0; ----------------------------------//e8
- if (k == 1) return n-1; --------------------------------//21
- if (n < k) return (J(n-1,k)+k)%n; ----------------------//31
- int np = n - n/k; --------------------------------------//b4
- return k*((J(np,k)+np-n%k%np)%np) / (k-1); } -----------//dd
```

5.25. **Number of Integer Points under Line.** Count the number of integer solutions to $Ax + By \le C$, $0 \le x \le n$, $0 \le y$. In other words, evaluate the sum $\sum_{x=0}^{n} \left\lfloor \frac{C-Ax}{B} + 1 \right\rfloor$. To count all solutions, let $n = \left\lfloor \frac{c}{a} \right\rfloor$. In any case, it must hold that $C - nA \ge 0$. Be very careful about overflows.

```
ll floor_sum(ll n, ll a, ll b, ll c) { -------------------//db
- if (c == 0) return 1; ----------------------------------//42
- if (c < 0) return 0; -----------------------------------//1c
- if (a % b == 0) return (n+1)*(c/b+1)-n*(n+1)/2*a/b; -----//88
- if (a >= b) return floor_sum(n,a%b,b,c)-a/b*n*(n+1)/2; --//bb
- ll t = (c-a*n+b)/b; ------------------------------------//c6
- return floor_sum((c-b*t)/b,b,a,c-b*t)+t*(n+1); } -------//9b
```

5.26. **Numbers and Sequences.** Some random prime numbers: 1031, 32771, 1048583, 33554467, 1073741827, 34359738421, 1099511627791, 35184372088891, 1125899906842679, 36028797018963971.

More random prime numbers: $10^3 + \{-9, -3, 9, 13\}$, $10^6 + \{-17, 3, 33\}$, $10^9 + \{7, 9, 21, 33, 87\}$.

Some maximal divisor counts:

840	32
720 720	240
735 134 400	1 344
963 761 198 400	6 720
866 421 317 361 600	26 880
897 612 484 786 617 600	103 680

5.27. **Game Theory.** Useful identity:

$$\oplus_{x=0}^{a-1} x = [0, a-1, 1, a][a\%4]$$

6. Geometry

6.1. **Primitives.** Geometry primitives.

```
#define P(p) const point &p ---------------------------------//2e
#define L(p0, p1) P(p0), P(p1) ------------------------------//cf
#define C(p0, r) P(p0), double r ----------------------------//f1
#define PP(pp) pair<point,point> &pp ------------------------//e5
typedef complex<double> point; ------------------------------//6a
double dot(P(a), P(b)) { return real(conj(a) * b); } -------//d2
double cross(P(a), P(b)) { return imag(conj(a) * b); } -----//8a
point rotate(P(p), double radians = pi / 2, ----------------//98
------------ P(about) = point(0,0)) { ----------------------//19
- return (p - about) * exp(point(0, radians)) + about; } --//9b
point reflect(P(p), L(about1, about2)) { -------------------//f7
- point z = p - about1, w = about2 - about1; ---------------//3f
- return conj(z / w) * w + about1; } -----------------------//b3
point proj(P(u), P(v)) { return dot(u, v) / dot(u, u) * u; }
point normalize(P(p), double k = 1.0) { --------------------//05
- return abs(p) == 0 ? point(0,0) : p / abs(p) * k; } ------//f7
double ccw(P(a), P(b), P(c)) { return cross(b - a, c - b); }
bool collinear(P(a), P(b), P(c)) { -------------------------//9e
- return abs(ccw(a, b, c)) < EPS; } ------------------------//51
double angle(P(a), P(b), P(c)) { ---------------------------//45
- return acos(dot(b - a, c - b) / abs(b - a) / abs(c - b)); }
double signed_angle(P(a), P(b), P(c)) { --------------------//3a
- return asin(cross(b - a, c - b) / abs(b - a) / abs(c - b)); }
double angle(P(p)) { return atan2(imag(p), real(p)); } -----//00
point perp(P(p)) { return point(-imag(p), real(p)); } ------//22
double progress(P(p), L(a, b)) { ---------------------------//af
- if (abs(real(a) - real(b)) < EPS) ------------------------//78
--- return (imag(p) - imag(a)) / (imag(b) - imag(a)); -----//76
- else return (real(p) - real(a)) / (real(b) - real(a)); } //c2
```

6.2. **Lines.** Line related functions.

```
#include "primitives.cpp" ----------------------------------//e0
bool collinear(L(a, b), L(p, q)) { -------------------------//7c
- return abs(ccw(a, b, p)) < EPS && abs(ccw(a, b, q)) < EPS; }
bool parallel(L(a, b), L(p, q)) { --------------------------//58
- return abs(cross(b - a, q - p)) < EPS; } -----------------//9c
point closest_point(L(a, b), P(c), bool segment = false) { //c7
- if (segment) { -------------------------------------------//2d
--- if (dot(b - a, c - b) > 0) return b; -------------------//dd
```

```
--- if (dot(a - b, c - a) > 0) return a; ------------------//69
- } --------------------------------------------------------//a3
- double t = dot(c - a, b - a) / norm(b - a); -------------//c3
- return a + t * (b - a); } --------------------------------//f3
double line_segment_distance(L(a,b), L(c,d)) { ------------//17
- double x = INFINITY; -------------------------------------//cf
- if (abs(a - b) < EPS && abs(c - d) < EPS) x = abs(a - c);//eb
- else if (abs(a - b) < EPS) -------------------------------//cd
--- x = abs(a - closest_point(c, d, a, true)); ------------//81
- else if (abs(c - d) < EPS) -------------------------------//b9
--- x = abs(c - closest_point(a, b, c, true)); ------------//b0
- else if ((ccw(a, b, c) < 0) != (ccw(a, b, d) < 0) && ----//48
------ (ccw(c, d, a) < 0) != (ccw(c, d, b) < 0)) x = 0; ---//0f
- else { ---------------------------------------------------//2c
--- x = min(x, abs(a - closest_point(c,d, a, true))); -----//0e
--- x = min(x, abs(b - closest_point(c,d, b, true))); -----//f1
--- x = min(x, abs(c - closest_point(a,b, c, true))); -----//72
--- x = min(x, abs(d - closest_point(a,b, d, true))); -----//ff
- } --------------------------------------------------------//8b
- return x; } ----------------------------------------------//b6
bool intersect(L(a,b), L(p,q), point &res, bool seg=false) {
- // NOTE: check parallel/collinear before ----------------//7e
- point r = b - a, s = q - p; ------------------------------//51
- double c = cross(r, s), ----------------------------------//f0
-------- t = cross(p - a, s) / c, u = cross(p - a, r) / c; //7d
- if (seg && ------------------------------------------------//a6
----- (t < 0-EPS || t > 1+EPS || u < 0-EPS || u > 1+EPS)) -//c9
--- return false; ------------------------------------------//1e
- res = a + t * r; -----------------------------------------//ab
- return true; } -------------------------------------------//6f
```

6.3. **Circles.** Circle related functions.

```
#include "lines.cpp" ---------------------------------------//d3
int intersect(C(A, rA), C(B, rB), point &r1, point &r2) { -//41
- double d = abs(B - A); -----------------------------------//5c
- if ((rA + rB) <  (d - EPS) || d < abs(rA - rB) - EPS) ---//4e
--- return 0; ----------------------------------------------//27
- double a = (rA*rA - rB*rB + d*d) / 2 / d, ---------------//1d
-------- h = sqrt(rA*rA - a*a); ----------------------------//e0
- point v = normalize(B - A, a), ---------------------------//81
------- u = normalize(rotate(B-A), h); ---------------------//83
- r1 = A + v + u, r2 = A + v - u; --------------------------//12
- return 1 + (abs(u) >= EPS); } ----------------------------//28
int intersect(L(A, B), C(O, r), point &r1, point &r2) { ---//cc
- point H = proj(B-A, O-A) + A; double h = abs(H-O); ------//b1
- if (r < h - EPS) return 0; -------------------------------//fe
- point v = normalize(B-A, sqrt(r*r - h*h)); --------------//77
- r1 = H + v, r2 = H - v; ----------------------------------//ce
- return 1 + (abs(v) > EPS); } -----------------------------//a4
int tangent(P(A), C(O, r), point &r1, point &r2) { --------//51
- point v = O - A; double d = abs(v); ----------------------//30
- if (d < r - EPS) return 0; -------------------------------//fc
- double alpha = asin(r / d), L = sqrt(d*d - r*r); --------//93
- v = normalize(v, L); --------------------------------------//01
- r1 = A + rotate(v, alpha), r2 = A + rotate(v, -alpha); --//10
- return 1 + (abs(v) > EPS); } -----------------------------//0c
void tangent_outer(point A, double rA, --------------------//b7
------------------ point B, double rB, PP(P), PP(Q)) { ----//ae
- if (rA - rB > EPS) { swap(rA, rB); swap(A, B); } --------//4f
- double theta = asin((rB - rA)/abs(A - B)); --------------//1e
- point v = rotate(B - A, theta + pi/2), -------------------//0c
------- u = rotate(B - A, -(theta + pi/2)); ---------------//4d
- u = normalize(u, rA); ------------------------------------//4e
- P.first = A + normalize(v, rA); --------------------------//d4
- P.second = B + normalize(v, rB); -------------------------//ad
- Q.first = A + normalize(u, rA); --------------------------//1c
- Q.second = B + normalize(u, rB); } -----------------------//dc
```

6.4. **Polygon.** Polygon primitives.

```
#include "primitives.cpp" ----------------------------------//e0
typedef vector<point> polygon; -----------------------------//b3
double polygon_area_signed(polygon p) { -------------------//31
- double area = 0; int cnt = size(p); ---------------------//a2
- rep(i,1,cnt-1) area += cross(p[i] - p[0], p[i + 1] - p[0]);
- return area / 2; } ---------------------------------------//66
double polygon_area(polygon p) { --------------------------//a3
- return abs(polygon_area_signed(p)); } -------------------//71
#define CHK(f,a,b,c) \ -------------------------------------//08
--- (f(a) < f(b) && f(b) <= f(c) && ccw(a,c,b) < 0) -------//c3
int point_in_polygon(polygon p, point q) { ----------------//87
- int n = size(p); bool in = false; double d; -------------//84
- for (int i = 0, j = n - 1; i < n; j = i++) --------------//32
--- if (collinear(p[i], q, p[j]) && ------------------------//f3
----- 0 <= (d = progress(q, p[i], p[j])) && d <= 1) -------//c8
----- return 0; --------------------------------------------//a2
- for (int i = 0, j = n - 1; i < n; j = i++) --------------//b3
--- if (CHK(real, p[i], q, p[j]) || CHK(real, p[j], q, p[i]))
----- in = !in; --------------------------------------------//44
- return in ? -1 : 1; } ------------------------------------//aa
// pair<polygon, polygon> cut_polygon(const polygon &poly, //08
//                                  point a, point b) { -//61
//     polygon left, right; --------------------------------//f4
//     point it(-100, -100); -------------------------------//22
//     for (int i = 0, cnt = poly.size(); i < cnt; i++) { -//81
//         int j = i == cnt-1 ? 0 : i + 1; -----------------//78
//         point p = poly[i], q = poly[j]; -----------------//4c
//         if (ccw(a, b, p) <= 0) left.push_back(p); ------//75
//         if (ccw(a, b, p) >= 0) right.push_back(p); -----//1b
//         // myintersect = intersect where ----------------//ab
//         // (a,b) is a line, (p,q) is a line segment ----//96
//         if (myintersect(a, b, p, q, it)) ----------------//58
//             left.push_back(it), right.push_back(it); } -//5e
//     return pair<polygon, polygon>(left, right); } ------//04
```

6.5. **Convex Hull.** An algorithm that finds the Convex Hull of a set of points. NOTE: Doesn't work on some weird edge cases. (A small case that included three collinear lines would return the same point on both the upper and lower hull.)

```
#include "polygon.cpp" -------------------------------------//58
#define MAXN 1000 ----------------------------------------//09
point hull[MAXN]; ------------------------------------------//43
bool cmp(const point &a, const point &b) { ----------------//32
- return abs(real(a) - real(b)) > EPS ? -------------------//44
--- real(a) < real(b) : imag(a) < imag(b); } --------------//40
int convex_hull(polygon p) { ------------------------------//cd
- int n = size(p), l = 0; ----------------------------------//67
- sort(p.begin(), p.end(), cmp); ---------------------------//3d
- rep(i,0,n) { ---------------------------------------------//e4
--- if (i > 0 && p[i] == p[i - 1]) continue; --------------//c7
--- while (l >= 2 && ---------------------------------------//7f
------- ccw(hull[l - 2], hull[l - 1], p[i]) >= 0) l--; ----//92
--- hull[l++] = p[i]; } ------------------------------------//46
- int r = l; -----------------------------------------------//65
- for (int i = n - 2; i >= 0; i--) { -----------------------//c6
--- if (p[i] == p[i + 1]) continue; ------------------------//51
--- while (r - l >= 1 && ------------------------------------//e1
------- ccw(hull[r - 2], hull[r - 1], p[i]) >= 0) r--; ----//b3
--- hull[r++] = p[i]; } ------------------------------------//d4
- return l == 1 ? 1 : r - 1; } -----------------------------//f9
```

6.6. **Line Segment Intersection.** Computes the intersection between two line segments.

```
#include "lines.cpp" ---------------------------------------//d3
bool line_segment_intersect(L(a, b), L(c, d), point &A, ---//bf
------------------------------------------- point &B) { -//5f
- if (abs(a - b) < EPS && abs(c - d) < EPS) { -------------//4f
--- A = B = a; return abs(a - d) < EPS; } -----------------//cf
- else if (abs(a - b) < EPS) { -----------------------------//8d
--- A = B = a; double p = progress(a, c,d); ---------------//e0
--- return 0.0 <= p && p <= 1.0 ----------------------------//94
----- && (abs(a - c) + abs(d - a) - abs(d - c)) < EPS; } --//53
- else if (abs(c - d) < EPS) { -----------------------------//83
--- A = B = c; double p = progress(c, a,b); ---------------//8a
--- return 0.0 <= p && p <= 1.0 ----------------------------//35
----- && (abs(c - a) + abs(b - c) - abs(b - a)) < EPS; } --//28
- else if (collinear(a,b, c,d)) { --------------------------//e6
--- double ap = progress(a, c,d), bp = progress(b, c,d); --//b8
--- if (ap > bp) swap(ap, bp); -----------------------------//a5
--- if (bp < 0.0 || ap > 1.0) return false; ---------------//11
--- A = c + max(ap, 0.0) * (d - c); ------------------------//09
--- B = c + min(bp, 1.0) * (d - c); ------------------------//78
--- return true; } -----------------------------------------//65
- else if (parallel(a,b, c,d)) return false; ---------------//c1
- else if (intersect(a,b, c,d, A, true)) { -----------------//8b
--- B = A; return true; } ----------------------------------//e4
- return false; } -------------------------------------------//14
```

6.7. **Great-Circle Distance.** Computes the distance between two points (given as latitude/longitude coordinates) on a sphere of radius r.

```
double gc_distance(double pLat, double pLong, -------------//7b
-------- double qLat, double qLong, double r) { -----------//a4
- pLat *= pi / 180; pLong *= pi / 180; ---------------------//ee
- qLat *= pi / 180; qLong *= pi / 180; ---------------------//75
- return r * acos(cos(pLat) * cos(qLat) * cos(pLong - qLong) +
--------- sin(pLat) * sin(qLat)); } -----------------------//e5
```

6.8. **Triangle Circumcenter.** Returns the unique point that is the same distance from all three points. It is also the center of the unique circle that goes through all three points.

```
#include "primitives.cpp" ---------------------------------//e0
point circumcenter(point a, point b, point c) { -----------//76
- b -= a, c -= a; -----------------------------------------//41
- return a + ----------------------------------------------//c0
--- perp(b * norm(c) - c * norm(b)) / 2.0 / cross(b, c); } //97
```

6.9. Closest Pair of Points. A sweep line algorithm for computing the distance between the closest pair of points.

```
#include "primitives.cpp" ---------------------------------//e0
-----------------------------------------------------------//85
struct cmpx { bool operator ()(const point &a, ------------//5e
------------------------------ const point &b) { ----------//d7
--- return abs(real(a) - real(b)) > EPS ? -----------------//41
----- real(a) < real(b) : imag(a) < imag(b); } }; ---------//45
struct cmpy { bool operator ()(const point &a, ------------//a1
------------------------------ const point &b) { ----------//2c
- return abs(imag(a) - imag(b)) > EPS ? -------------------//f1
----- imag(a) < imag(b) : real(a) < real(b); } }; ---------//8e
double closest_pair(vector<point> pts) { ------------------//2c
- sort(pts.begin(), pts.end(), cmpx()); -------------------//18
- set<point, cmpy> cur; -----------------------------------//ea
- set<point, cmpy>::const_iterator it, jt; ----------------//20
- double mn = INFINITY; -----------------------------------//91
- for (int i = 0, l = 0; i < size(pts); i++) { ------------//5d
--- while (real(pts[i]) - real(pts[l]) > mn) --------------//4a
----- cur.erase(pts[l++]); --------------------------------//da
--- it = cur.lower_bound(point(-INFINITY, imag(pts[i]) - mn));
--- jt = cur.upper_bound(point(INFINITY, imag(pts[i]) + mn));
--- while (it != jt) mn = min(mn, abs(*it - pts[i])), it++;//94
--- cur.insert(pts[i]); } ---------------------------------//f6
- return mn; } --------------------------------------------//95
```

6.10. 3D Primitives. Three-dimensional geometry primitives.

```
#define P(p) const point3d &p -----------------------------//a7
#define L(p0, p1) P(p0), P(p1) ----------------------------//0f
#define PL(p0, p1, p2) P(p0), P(p1), P(p2) ----------------//67
struct point3d { ------------------------------------------//63
- double x, y, z; -----------------------------------------//e6
- point3d() : x(0), y(0), z(0) {} -------------------------//af
- point3d(double _x, double _y, double _z) ----------------//ab
--- : x(_x), y(_y), z(_z) {} ------------------------------//8a
- point3d operator+(P(p)) const { -------------------------//30
--- return point3d(x + p.x, y + p.y, z + p.z); } ----------//25
- point3d operator-(P(p)) const { -------------------------//2c
--- return point3d(x - p.x, y - p.y, z - p.z); } ----------//04
- point3d operator-() const { -----------------------------//30
--- return point3d(-x, -y, -z); } -------------------------//48
- point3d operator*(double k) const { ---------------------//56
--- return point3d(x * k, y * k, z * k); } ----------------//99
- point3d operator/(double k) const { ---------------------//d2
--- return point3d(x / k, y / k, z / k); } ----------------//75
- double operator%(P(p)) const { --------------------------//69
--- return x * p.x + y * p.y + z * p.z; } -----------------//b2
- point3d operator*(P(p)) const { -------------------------//50
--- return point3d(y*p.z - z*p.y, -------------------------//2b
------------------ z*p.x - x*p.z, x*p.y - y*p.x); } -------//26
- double length() const { ---------------------------------//25
--- return sqrt(*this % *this); } -------------------------//7c
- double distTo(P(p)) const { -----------------------------//c1
--- return (*this - p).length(); } ------------------------//5e
- double distTo(P(A), P(B)) const { -----------------------//dc
--- // A and B must be two different points ---------------//63
--- return ((*this - A) * (*this - B)).length() / A.distTo(B);}
- point3d normalize(double k = 1) const { -----------------//90
--- // length() must not return 0 -------------------------//3d
--- return (*this) * (k / length()); } --------------------//61
- point3d getProjection(P(A), P(B)) const { ---------------//08
--- point3d v = B - A; ------------------------------------//bf
--- return A + v.normalize((v % (*this - A)) / v.length()); }
- point3d rotate(P(normal)) const { -----------------------//69
--- //normal must have length 1 and be orthogonal to the vector
--- return (*this) * normal; } ----------------------------//f5
- point3d rotate(double alpha, P(normal)) const { ---------//89
--- return (*this) * cos(alpha) + rotate(normal) * sin(alpha);}
- point3d rotatePoint(P(O), P(axe), double alpha) const{ --//b7
--- point3d Z = axe.normalize(axe % (*this - O)); ---------//4e
--- return O + Z + (*this - O - Z).rotate(alpha, O); } ----//0f
- bool isZero() const { -----------------------------------//71
--- return abs(x) < EPS && abs(y) < EPS && abs(z) < EPS; } //91
- bool isOnLine(L(A, B)) const { --------------------------//92
--- return ((A - *this) * (B - *this)).isZero(); } --------//5b
- bool isInSegment(L(A, B)) const { -----------------------//3c
--- return isOnLine(A, B) && ((A - *this) % (B - *this))<EPS;}
- bool isInSegmentStrictly(L(A, B)) const { ---------------//47
--- return isOnLine(A, B) && ((A - *this) % (B - *this))<-EPS;}
- double getAngle() const { -------------------------------//a0
--- return atan2(y, x); } ---------------------------------//37
- double getAngle(P(u)) const { ---------------------------//5e
--- return atan2((*this * u).length(), *this % u); } ------//ed
- bool isOnPlane(PL(A, B, C)) const { ---------------------//cc
--- return ------------------------------------------------//d5
----- abs((A - *this) * (B - *this) % (C - *this)) < EPS; } };
int line_line_intersect(L(A, B), L(C, D), point3d &O){ ----//89
- if (abs((B - A) * (C - A) % (D - A)) > EPS) return 0; ---//87
- if (((A - B) * (C - D)).length() < EPS) -----------------//fb
--- return A.isOnLine(C, D) ? 2 : 0; ----------------------//65
- point3d normal = ((A - B) * (C - B)).normalize(); -------//88
- double s1 = (C - A) * (D - A) % normal; -----------------//ae
- O = A + ((B - A) / (s1 + ((D - B) * (C - B) % normal))) * s1;
- return 1; } ---------------------------------------------//e5
int line_plane_intersect(L(A, B), PL(C, D, E), point3d & O) {
- double V1 = (C - A) * (D - A) % (E - A); ----------------//a7
- double V2 = (D - B) * (C - B) % (E - B); ----------------//2c
- if (abs(V1 + V2) < EPS) ---------------------------------//4e
--- return A.isOnPlane(C, D, E) ? 2 : 0; ------------------//c3
- O = A + ((B - A) / (V1 + V2)) * V1; ---------------------//56
- return 1; } ---------------------------------------------//de
bool plane_plane_intersect(P(A), P(nA), P(B), P(nB), ------//44
--- point3d &P, point3d &Q) { -----------------------------//87
- point3d n = nA * nB; ------------------------------------//56
- if (n.isZero()) return false; ---------------------------//db
- point3d v = n * nA; -------------------------------------//ed
- P = A + (n * nA) * ((B - A) % nB / (v % nB)); -----------//49
- Q = P + n; -----------------------------------------------//85
- return true; } -------------------------------------------//c3
```

6.11. Polygon Centroid.

```
#include "polygon.cpp" ------------------------------------//58
point polygon_centroid(polygon p) { -----------------------//79
- double cx = 0.0, cy = 0.0; ------------------------------//d5
- double mnx = 0.0, mny = 0.0; ----------------------------//22
- int n = size(p); ----------------------------------------//2d
- rep(i,0,n) ----------------------------------------------//08
--- mnx = min(mnx, real(p[i])), ---------------------------//c6
--- mny = min(mny, imag(p[i])); ---------------------------//84
- rep(i,0,n) ----------------------------------------------//3f
--- p[i] = point(real(p[i]) - mnx, imag(p[i]) - mny); -----//49
- rep(i,0,n) { --------------------------------------------//3c
--- int j = (i + 1) % n; ----------------------------------//5b
--- cx += (real(p[i]) + real(p[j])) * cross(p[i], p[j]); --//4f
--- cy += (imag(p[i]) + imag(p[j])) * cross(p[i], p[j]); } //4a
- return point(cx, cy) / 6.0 / polygon_area_signed(p) -----//dd
-------- + point(mnx, mny); } -----------------------------//b5
```

6.12. Rotating Calipers.

```
#include "lines.cpp" --------------------------------------//d3
struct caliper { ------------------------------------------//6b
- ii pt; --------------------------------------------------//ff
- double angle; -------------------------------------------//44
- caliper(ii _pt, double _angle) : pt(_pt), angle(_angle) { }
- double angle_to(ii pt2) { -------------------------------//e8
--- double x = angle - atan2(pt2.second - pt.second, ------//18
------------------------------ pt2.first - pt.first); -----//92
--- while (x >= pi) x -= 2*pi; ----------------------------//37
--- while (x <= -pi) x += 2*pi; ---------------------------//86
--- return x; } -------------------------------------------//fa
- void rotate(double by) { --------------------------------//ce
--- angle -= by; ------------------------------------------//85
--- while (angle < 0) angle += 2*pi; } --------------------//48
- void move_to(ii pt2) { pt = pt2; } ----------------------//fb
- double dist(const caliper &other) { ---------------------//9c
--- point a(pt.first,pt.second), --------------------------//9c
------ b = a + exp(point(0,angle)) * 10.0, ----------------//38
------ c(other.pt.first, other.pt.second); ----------------//94
--- return abs(c - closest_point(a, b, c)); } }; ----------//bc
// int h = convex_hull(pts); ------------------------------//ff
// double mx = 0; -----------------------------------------//91
// if (h > 1) { -------------------------------------------//18
//     int a = 0, -----------------------------------------//e4
//         b = 0; -----------------------------------------//3b
//     rep(i,0,h) { ---------------------------------------//e7
//         if (hull[i].first < hull[a].first) -------------//70
//             a = i; -------------------------------------//7f
//         if (hull[i].first > hull[b].first) -------------//d3
//             b = i; } -----------------------------------//ba
//     caliper A(hull[a], pi/2), B(hull[b], 3*pi/2); ------//99
//     double done = 0; -----------------------------------//0d
//     while (true) { -------------------------------------//b0
//         mx = max(mx, abs(point(hull[a].first,hull[a].second)
//                  - point(hull[b].first,hull[b].second)));
```

```
//         double tha = A.angle_to(hull[(a+1)%h]), --------//ed
//                thb = B.angle_to(hull[(b+1)%h]); -------//dd
//         if (tha <= thb) { ----------------------------//0a
//             A.rotate(tha); ---------------------------//70
//             B.rotate(tha); ---------------------------//b6
//             a = (a+1) % h; ---------------------------//5c
//             A.move_to(hull[a]); ----------------------//70
//         } else { -------------------------------------//34
//             A.rotate(thb); ---------------------------//93
//             B.rotate(thb); ---------------------------//fb
//             b = (b+1) % h; ---------------------------//56
//             B.move_to(hull[b]); } --------------------//9f
//         done += min(tha, thb); -----------------------//2c
//         if (done > pi) { -----------------------------//ab
//             break; -----------------------------------//57
//         } } } ----------------------------------------//25
```

6.13. **Rectilinear Minimum Spanning Tree.** Given a set of n points in the plane, and the aim is to find a minimum spanning tree connecting these n points, assuming the Manhattan distance is used. The function `candidates` returns at most $4n$ edges that are a superset of the edges in a minimum spanning tree, and then one can use Kruskal's algorithm.

```
#define MAXN 100100 ----------------------------------------//29
struct RMST { ----------------------------------------------//71
- struct point { -------------------------------------------//be
--- int i; ll x, y; ----------------------------------------//a0
--- point() : i(-1) { } -----------------------------------//6e
--- ll d1() { return x + y; } -----------------------------//51
--- ll d2() { return x - y; } -----------------------------//0e
--- ll dist(point other) { ---------------------------------//b6
----- return abs(x - other.x) + abs(y - other.y); } -------//c7
--- bool operator <(const point &other) const { -----------//e5
----- return y == other.y ? x > other.x : y < other.y; } --//88
- } best[MAXN], arr[MAXN], tmp[MAXN]; ---------------------//07
- int n; --------------------------------------------------//11
- RMST() : n(0) {} ----------------------------------------//1d
- void add_point(int x, int y) { --------------------------//13
--- arr[arr[n].i = n].x = x, arr[n++].y = y; } ------------//9d
- void rec(int l, int r) { --------------------------------//42
--- if (l >= r) return; -----------------------------------//ab
--- int m = (l+r)/2; --------------------------------------//55
--- rec(l,m), rec(m+1,r); ---------------------------------//61
--- point bst; --------------------------------------------//fa
--- for (int i = l, j = m+1, k = l; i <= m || j <= r; k++) {
----- if (j > r || (i <= m && arr[i].d1() < arr[j].d1())) {//c9
------- tmp[k] = arr[i++]; --------------------------------//4f
------- if (bst.i != -1 && (best[tmp[k].i].i == -1 --------//d0
--------------------- || best[tmp[k].i].d2() < bst.d2()))//72
--------- best[tmp[k].i] = bst; ---------------------------//a2
----- } else { --------------------------------------------//2b
------- tmp[k] = arr[j++]; --------------------------------//17
------- if (bst.i == -1 || bst.d2() < tmp[k].d2()) --------//bc
--------- bst = tmp[k]; } } -------------------------------//a5
--- rep(i,l,r+1) arr[i] = tmp[i]; } -----------------------//10
- vector<pair<ll,ii> > candidates() { ---------------------//65
--- vector<pair<ll, ii> > es; -----------------------------//a6
--- rep(p,0,2) { ------------------------------------------//6f
----- rep(q,0,2) { ----------------------------------------//32
------- sort(arr, arr+n); ---------------------------------//e6
------- rep(i,0,n) best[i].i = -1; ------------------------//a8
------- rec(0,n-1); ---------------------------------------//6a
------- rep(i,0,n) { --------------------------------------//34
--------- if(best[arr[i].i].i != -1) ----------------------//af
----------- es.push_back({arr[i].dist(best[arr[i].i]), ----//90
----------------------- {arr[i].i, best[arr[i].i].i}}); --//94
--------- swap(arr[i].x, arr[i].y); -----------------------//09
--------- arr[i].x *= -1, arr[i].y *= -1; } } -------------//74
----- rep(i,0,n) arr[i].x *= -1; } ------------------------//14
--- return es; } }; ---------------------------------------//84
```

6.14. **Line upper/lower envelope.** To find the upper/lower envelope of a collection of lines $a_i + b_i x$, plot the points (b_i, a_i), add the point $(0, \pm\infty)$ (depending on if upper/lower envelope is desired), and then find the convex hull.

6.15. **Formulas.** Let $a = (a_x, a_y)$ and $b = (b_x, b_y)$ be two-dimensional vectors.

- $a \cdot b = |a||b| \cos\theta$, where θ is the angle between a and b.
- $a \times b = |a||b| \sin\theta$, where θ is the signed angle between a and b.
- $a \times b$ is equal to the area of the parallelogram with two of its sides formed by a and b. Half of that is the area of the triangle formed by a and b.
- **Euler's formula:** $V - E + F = 2$
- Side lengths a, b, c can form a triangle iff. $a + b > c$, $b + c > a$ and $a + c > b$.
- Sum of internal angles of a regular convex n-gon is $(n-2)\pi$.
- **Law of sines:** $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
- **Law of cosines:** $b^2 = a^2 + c^2 - 2ac\cos B$
- Internal tangents of circles $(c_1, r_1), (c_2, r_2)$ intersect at $(c_1 r_2 + c_2 r_1)/(r_1 + r_2)$, external intersect at $(c_1 r_2 - c_2 r_1)/(r_1 + r_2)$.

7. Other Algorithms

7.1. **2SAT.** A fast 2SAT solver.

```
struct { vi adj; int val, num, lo; bool done; } V[2*1000+100];
struct TwoSat { -------------------------------------------//01
- int n, at = 0; vi S; ------------------------------------//3a
- TwoSat(int _n) : n(_n) { --------------------------------//d8
--- rep(i,0,2*n+1) ----------------------------------------//58
----- V[i].adj.clear(), -----------------------------------//77
----- V[i].val = V[i].num = -1, V[i].done = false; } ------//9a
- bool put(int x, int v) { --------------------------------//de
--- return (V[n+x].val &= v) != (V[n-x].val &= 1-v); } ----//26
- void add_or(int x, int y) { -----------------------------//85
--- V[n-x].adj.push_back(n+y), V[n-y].adj.push_back(n+x); }//66
- int dfs(int u) { ----------------------------------------//6d
--- int br = 2, res; --------------------------------------//74
--- S.push_back(u), V[u].num = V[u].lo = at++; ------------//d0
--- iter(v,V[u].adj) { ------------------------------------//31
----- if (V[*v].num == -1) { ------------------------------//99
------- if (!(res = dfs(*v))) return 0; -------------------//08
------- br |= res, V[u].lo = min(V[u].lo, V[*v].lo); ------//82
----- } else if (!V[*v].done) -----------------------------//46
------- V[u].lo = min(V[u].lo, V[*v].num); ----------------//d9
----- br |= !V[*v].val; } ---------------------------------//0c
--- res = br - 3; -----------------------------------------//c7
--- if (V[u].num == V[u].lo) rep(i,res+1,2) { -------------//12
----- for (int j = size(S)-1; ; j--) { --------------------//bd
------- int v = S[j]; -------------------------------------//73
------- if (i) { ------------------------------------------//e0
--------- if (!put(v-n, res)) return 0; -------------------//ea
--------- V[v].done = true, S.pop_back(); -----------------//3e
------- } else res &= V[v].val; ---------------------------//48
------- if (v == u) break; } ------------------------------//77
----- res &= 1; } -----------------------------------------//5c
--- return br | !res; } -----------------------------------//4b
- bool sat() { --------------------------------------------//23
--- rep(i,0,2*n+1) ----------------------------------------//16
----- if (i != n && V[i].num == -1 && !dfs(i)) return false;
--- return true; } }; -------------------------------------//dc
```

7.2. **DPLL Algorithm.** A SAT solver that can solve a random 1000-variable SAT instance within a second.

```
#define IDX(x) ((abs(x)-1)*2+((x)>0)) ---------------------//ca
struct SAT { ----------------------------------------------//e3
- int n; --------------------------------------------------//6d
- vi cl, head, tail, val; ---------------------------------//85
- vii log; vvi w, loc; ------------------------------------//ff
- SAT() : n(0) { } ----------------------------------------//f3
- int var() { return ++n; } -------------------------------//9a
- void clause(vi vars) { ----------------------------------//5e
--- set<int> seen; iter(it,vars) { ------------------------//66
----- if (seen.find(IDX(*it)^1) != seen.end()) return; ----//f9
----- seen.insert(IDX(*it)); } ----------------------------//4f
--- head.push_back(cl.size()); ----------------------------//1d
--- iter(it,seen) cl.push_back(*it); ----------------------//ad
--- tail.push_back((int)cl.size() - 2); } -----------------//21
- bool assume(int x) { ------------------------------------//58
--- if (val[x^1]) return false; ---------------------------//07
--- if (val[x]) return true; ------------------------------//d6
--- val[x] = true; log.push_back(ii(-1, x)); --------------//9e
--- rep(i,0,w[x^1].size()) { ------------------------------//fd
----- int at = w[x^1][i], h = head[at], t = tail[at]; -----//9b
----- log.push_back(ii(at, h)); ---------------------------//5c
----- if (cl[t+1] != (x^1)) swap(cl[t], cl[t+1]); ---------//40
----- while (h < t && val[cl[h]^1]) h++; ------------------//0c
----- if ((head[at] = h) < t) { ---------------------------//68
------- w[cl[h]].push_back(w[x^1][i]); --------------------//cd
------- swap(w[x^1][i--], w[x^1].back()); -----------------//2d
------- w[x^1].pop_back(); --------------------------------//61
------- swap(cl[head[at]++], cl[t+1]); --------------------//a9
----- } else if (!assume(cl[t])) return false; } ---------//3a
--- return true; } ----------------------------------------//f7
- bool bt() { ---------------------------------------------//6e
--- int v = log.size(), x; ll b = -1; ---------------------//09
--- rep(i,0,n) if (val[2*i] == val[2*i+1]) { --------------//66
----- ll s = 0, t = 0; ------------------------------------//02
----- rep(j,0,2) { iter(it,loc[2*i]) ----------------------//9c
------- s+=1LL<<max(0,40-tail[*it]+head[*it]); swap(s,t); }//37
```

```
----- if (max(s,t) >= b) b = max(s,t), x = 2*i + (t>=s); } //8c
--- if (b == -1 || (assume(x) && bt())) return true; ------//fc
--- while (log.size() != v) { -----------------------------//88
----- int a = log.back().first, b = log.back().second; ----//f5
----- if (a == -1) val[b] = false; else head[a] = b; ------//61
----- log.pop_back(); } -----------------------------------//60
--- return assume(x^1) && bt(); } -------------------------//4f
- bool solve() { ------------------------------------------//ff
--- val.assign(2*n+1, false); -----------------------------//c5
--- w.assign(2*n+1, vi()); loc.assign(2*n+1, vi()); -------//46
--- rep(i,0,head.size()) { --------------------------------//cf
----- if (head[i] == tail[i]+2) return false; -------------//76
----- rep(at,head[i],tail[i]+2) loc[cl[at]].push_back(i); }//2a
--- rep(i,0,head.size()) if (head[i] < tail[i]+1) rep(t,0,2)
----- w[cl[tail[i]+t]].push_back(i); ----------------------//39
--- rep(i,0,head.size()) if (head[i] == tail[i]+1) --------//7f
----- if (!assume(cl[head[i]])) return false; -------------//8d
--- return bt(); } ----------------------------------------//37
- bool get_value(int x) { return val[IDX(x)]; } }; --------//97
```

7.3. **Stable Marriage.** The Gale-Shapley algorithm for solving the stable marriage problem.

```
vi stable_marriage(int n, int** m, int** w) { -------------//e4
- queue<int> q; -------------------------------------------//f6
- vi at(n, 0), eng(n, -1), res(n, -1); vvi inv(n, vi(n)); -//c3
- rep(i,0,n) rep(j,0,n) inv[i][w[i][j]] = j; --------------//f1
- rep(i,0,n) q.push(i); -----------------------------------//d8
- while (!q.empty()) { ------------------------------------//68
--- int curm = q.front(); q.pop(); ------------------------//e2
--- for (int &i = at[curm]; i < n; i++) { -----------------//7e
----- int curw = m[curm][i]; ------------------------------//95
----- if (eng[curw] == -1) { } ----------------------------//f7
----- else if (inv[curw][curm] < inv[curw][eng[curw]]) ----//d6
------- q.push(eng[curw]); --------------------------------//2e
----- else continue; --------------------------------------//1d
----- res[eng[curw] = curm] = curw, ++i; break; } } -------//34
- return res; } -------------------------------------------//1f
```

7.4. **Algorithm X.** An implementation of Knuth's Algorithm X, using dancing links. Solves the Exact Cover problem.

```
bool handle_solution(vi rows) { return false; } -----------//63
struct exact_cover { --------------------------------------//95
- struct node { -------------------------------------------//7e
--- node *l, *r, *u, *d, *p; ------------------------------//19
--- int row, col, size; -----------------------------------//ae
--- node(int _row, int _col) : row(_row), col(_col) { -----//c9
----- size = 0; l = r = u = d = p = NULL; } }; ------------//fe
- int rows, cols, *sol; -----------------------------------//b8
- bool **arr; ---------------------------------------------//ea
- node *head; ---------------------------------------------//ee
- exact_cover(int _rows, int _cols) -----------------------//fb
--- : rows(_rows), cols(_cols), head(NULL) { --------------//4e
--- arr = new bool*[rows]; --------------------------------//4a
--- sol = new int[rows]; ----------------------------------//14
--- rep(i,0,rows) -----------------------------------------//44
----- arr[i] = new bool[cols], memset(arr[i], 0, cols); } -//28
- void set_value(int row, int col, bool val = true) { -----//d7
--- arr[row][col] = val; } --------------------------------//a7
- void setup() { ------------------------------------------//ef
--- node ***ptr = new node**[rows + 1]; -------------------//9f
--- rep(i,0,rows+1) { -------------------------------------//ca
----- ptr[i] = new node*[cols]; ---------------------------//09
----- rep(j,0,cols) ---------------------------------------//42
------- if (i == rows || arr[i][j]) ptr[i][j] = new node(i,j);
------- else ptr[i][j] = NULL; } --------------------------//85
--- rep(i,0,rows+1) { -------------------------------------//58
----- rep(j,0,cols) { -------------------------------------//1d
------- if (!ptr[i][j]) continue; -------------------------//92
------- int ni = i + 1, nj = j + 1; -----------------------//50
------- while (true) { ------------------------------------//00
--------- if (ni == rows + 1) ni = 0; ---------------------//f4
--------- if (ni == rows || arr[ni][j]) break; ------------//98
--------- ++ni; } -----------------------------------------//af
------- ptr[i][j]->d = ptr[ni][j]; ------------------------//41
------- ptr[ni][j]->u = ptr[i][j]; ------------------------//5c
------- while (true) { ------------------------------------//1c
--------- if (nj == cols) nj = 0; -------------------------//24
--------- if (i == rows || arr[i][nj]) break; -------------//fa
--------- ++nj; } -----------------------------------------//8b
------- ptr[i][j]->r = ptr[i][nj]; ------------------------//85
------- ptr[i][nj]->l = ptr[i][j]; } } --------------------//10
--- head = new node(rows, -1); ----------------------------//68
--- head->r = ptr[rows][0]; -------------------------------//54
--- ptr[rows][0]->l = head; -------------------------------//f3
--- head->l = ptr[rows][cols - 1]; ------------------------//fd
--- ptr[rows][cols - 1]->r = head; ------------------------//5a
--- rep(j,0,cols) { ---------------------------------------//56
----- int cnt = -1; ---------------------------------------//34
----- rep(i,0,rows+1) -------------------------------------//44
------- if (ptr[i][j]) cnt++, ptr[i][j]->p = ptr[rows][j]; //95
----- ptr[rows][j]->size = cnt; } -------------------------//a2
--- rep(i,0,rows+1) delete[] ptr[i]; ----------------------//f3
--- delete[] ptr; } ---------------------------------------//c6
- #define COVER(c, i, j) \ --------------------------------//bf
--- c->r->l = c->l, c->l->r = c->r; \ ---------------------//b2
--- for (node *i = c->d; i != c; i = i->d) \ --------------//d5
----- for (node *j = i->r; j != i; j = j->r) \ ------------//23
------- j->d->u = j->u, j->u->d = j->d, j->p->size--; -----//c3
- #define UNCOVER(c, i, j) \ ------------------------------//67
--- for (node *i = c->u; i != c; i = i->u) \ --------------//eb
----- for (node *j = i->l; j != i; j = j->l) \ ------------//d9
------- j->p->size++, j->d->u = j->u->d = j; \ ------------//0e
--- c->r->l = c->l->r = c; --------------------------------//21
- bool search(int k = 0) { --------------------------------//6f
--- if (head == head->r) { --------------------------------//6d
----- vi res(k); ------------------------------------------//ec
----- rep(i,0,k) res[i] = sol[i]; -------------------------//46
----- sort(res.begin(), res.end()); -----------------------//3d
----- return handle_solution(res); } ----------------------//68
--- node *c = head->r, *tmp = head->r; --------------------//2a
--- for ( ; tmp != head; tmp = tmp->r) --------------------//2f
----- if (tmp->size < c->size) c = tmp; -------------------//28
--- if (c == c->d) return false; --------------------------//3b
--- COVER(c, i, j); ---------------------------------------//70
--- bool found = false; -----------------------------------//7f
--- for (node *r = c->d; !found && r != c; r = r->d) { ----//63
----- sol[k] = r->row; ------------------------------------//13
----- for (node *j = r->r; j != r; j = j->r) { ------------//71
------- COVER(j->p, a, b); } ------------------------------//96
----- found = search(k + 1); ------------------------------//1c
----- for (node *j = r->l; j != r; j = j->l) { ------------//1e
------- UNCOVER(j->p, a, b); } } --------------------------//2b
--- UNCOVER(c, i, j); -------------------------------------//48
--- return found; } }; ------------------------------------//5f
```

7.5. ***n*th Permutation.** A very fast algorithm for computing the nth permutation of the list $\{0, 1, \ldots, k-1\}$.

```
vector<int> nth_permutation(int cnt, int n) { -------------//78
- vector<int> idx(cnt), per(cnt), fac(cnt); ---------------//9e
- rep(i,0,cnt) idx[i] = i; --------------------------------//bc
- rep(i,1,cnt+1) fac[i - 1] = n % i, n /= i; --------------//2b
- for (int i = cnt - 1; i >= 0; i--) ----------------------//f9
--- per[cnt - i - 1] = idx[fac[i]], -----------------------//a8
--- idx.erase(idx.begin() + fac[i]); ----------------------//39
- return per; } -------------------------------------------//a8
```

7.6. **Cycle-Finding.** An implementation of Floyd's Cycle-Finding algorithm.

```
ii find_cycle(int x0, int (*f)(int)) { --------------------//a5
- int t = f(x0), h = f(t), mu = 0, lam = 1; ---------------//8d
- while (t != h) t = f(t), h = f(f(h)); -------------------//79
- h = x0; -------------------------------------------------//04
- while (t != h) t = f(t), h = f(h), mu++; ----------------//9d
- h = f(t); -----------------------------------------------//00
- while (t != h) h = f(h), lam++; -------------------------//5e
- return ii(mu, lam); } -----------------------------------//14
```

7.7. **Longest Increasing Subsequence.**

```
vi lis(vi arr) { ------------------------------------------//99
- if (arr.empty()) return vi(); ---------------------------//3c
- vi seq, back(size(arr)), ans; ---------------------------//0d
- rep(i,0,size(arr)) { ------------------------------------//10
--- int res = 0, lo = 1, hi = size(seq); ------------------//7d
--- while (lo <= hi) { ------------------------------------//54
----- int mid = (lo+hi)/2; --------------------------------//27
----- if (arr[seq[mid-1]] < arr[i]) res = mid, lo = mid + 1;
----- else hi = mid - 1; } --------------------------------//78
--- if (res < size(seq)) seq[res] = i; --------------------//cf
--- else seq.push_back(i); --------------------------------//10
--- back[i] = res == 0 ? -1 : seq[res-1]; } ---------------//5b
- int at = seq.back(); ------------------------------------//25
- while (at != -1) ans.push_back(at), at = back[at]; ------//d3
- reverse(ans.begin(), ans.end()); ------------------------//4a
- return ans; } -------------------------------------------//70
```

7.8. **Dates.** Functions to simplify date calculations.

```
int intToDay(int jd) { return jd % 7; } -------------------//89
int dateToInt(int y, int m, int d) { ----------------------//96
- return 1461 * (y + 4800 + (m - 14) / 12) / 4 + ----------//a8
```

```
--- 367 * (m - 2 - (m - 14) / 12 * 12) / 12 - -------------//d1
--- 3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 + ----------//be
--- d - 32075; } -------------------------------------------//b6
void intToDate(int jd, int &y, int &m, int &d) { ----------//64
- int x, n, i, j; ------------------------------------------//e5
- x = jd + 68569; ------------------------------------------//97
- n = 4 * x / 146097; --------------------------------------//54
- x -= (146097 * n + 3) / 4; -------------------------------//dc
- i = (4000 * (x + 1)) / 1461001; --------------------------//ac
- x -= 1461 * i / 4 - 31; ----------------------------------//33
- j = 80 * x / 2447; ---------------------------------------//f8
- d = x - 2447 * j / 80; -----------------------------------//44
- x = j / 11; ----------------------------------------------//24
- m = j + 2 - 12 * x; --------------------------------------//67
- y = 100 * (n - 49) + i + x; } ----------------------------//d1
```

7.9. **Simulated Annealing.** An example use of Simulated Annealing to find a permutation of length n that maximizes $\sum_{i=1}^{n-1} |p_i - p_{i+1}|$.

```
double curtime() { -----------------------------------------//1c
- return static_cast<double>(clock()) / CLOCKS_PER_SEC; } -//49
int simulated_annealing(int n, double seconds) { ----------//60
- default_random_engine rng; -------------------------------//6b
- uniform_real_distribution<double> randfloat(0.0, 1.0); --//06
- uniform_int_distribution<int> randint(0, n - 2); --------//15
- // random initial solution -------------------------------//14
- vi sol(n); -----------------------------------------------//12
- rep(i,0,n) sol[i] = i + 1; -------------------------------//74
- random_shuffle(sol.begin(), sol.end()); ------------------//68
- // initialize score --------------------------------------//24
- int score = 0; -------------------------------------------//e7
- rep(i,1,n) score += abs(sol[i] - sol[i-1]); -------------//58
- int iters = 0; -------------------------------------------//2e
- double T0 = 100.0, T1 = 0.001, --------------------------//e7
----- progress = 0, temp = T0, -----------------------------//fb
----- starttime = curtime(); -------------------------------//84
- while (true) { -------------------------------------------//ff
--- if (!(iters & ((1 << 4) - 1))) { -----------------------//46
----- progress = (curtime() - starttime) / seconds; -------//e9
----- temp = T0 * pow(T1 / T0, progress); -----------------//cc
----- if (progress > 1.0) break; } -------------------------//36
--- // random mutation -------------------------------------//6a
--- int a = randint(rng); ----------------------------------//87
--- // compute delta for mutation -------------------------//e8
--- int delta = 0; -----------------------------------------//06
--- if (a > 0) delta += abs(sol[a+1] - sol[a-1]) ----------//c3
--------------------- - abs(sol[a] - sol[a-1]); -----------//a1
--- if (a+2 < n) delta += abs(sol[a] - sol[a+2]) ----------//b4
----------------------- - abs(sol[a+1] - sol[a+2]); -------//69
--- // maybe apply mutation --------------------------------//36
--- if (delta >= 0 || randfloat(rng) < exp(delta / temp)) {//06
----- swap(sol[a], sol[a+1]); ------------------------------//78
----- score += delta; --------------------------------------//92
----- // if (score >= target) return; ---------------------//35
--- } ------------------------------------------------------//3a
--- iters++; } ---------------------------------------------//7a
- return score; } ------------------------------------------//c8
```

7.10. **Simplex.**

```
typedef long double DOUBLE; --------------------------------//c6
typedef vector<DOUBLE> VD; ---------------------------------//c3
typedef vector<VD> VVD; ------------------------------------//ae
typedef vector<int> VI; ------------------------------------//51
const DOUBLE EPS = 1e-9; -----------------------------------//66
struct LPSolver { ------------------------------------------//65
 int m, n; -------------------------------------------------//1c
 VI B, N; --------------------------------------------------//a0
 VVD D; ----------------------------------------------------//db
 LPSolver(const VVD &A, const VD &b, const VD &c) : -------//4f
- m(b.size()), n(c.size()), --------------------------------//53
- N(n + 1), B(m), D(m + 2, VD(n + 2)) { --------------------//d4
- for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) -//5e
--- D[i][j] = A[i][j]; -------------------------------------//4f
- for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1;//58
--- D[i][n + 1] = b[i]; } ----------------------------------//44
- for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
- N[n] = -1; D[m + 1][n] = 1; } ----------------------------//8d
 void Pivot(int r, int s) { --------------------------------//77
- double inv = 1.0 / D[r][s]; ------------------------------//22
- for (int i = 0; i < m + 2; i++) if (i != r) -------------//4c
-- for (int j = 0; j < n + 2; j++) if (j != s) ------------//9f
--- D[i][j] -= D[r][j] * D[i][s] * inv; --------------------//5b
- for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] *= inv;
- for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] *= -inv;
- D[r][s] = inv; -------------------------------------------//28
- swap(B[r], N[s]); } --------------------------------------//a4
 bool Simplex(int phase) { ---------------------------------//17
- int x = phase == 1 ? m + 1 : m; --------------------------//e9
- while (true) { -------------------------------------------//15
-- int s = -1; ---------------------------------------------//59
-- for (int j = 0; j <= n; j++) { --------------------------//d1
--- if (phase == 2 && N[j] == -1) continue; ----------------//f2
--- if (s == -1 || D[x][j] < D[x][s] || -------------------//f8
------- D[x][j] == D[x][s] && N[j] < N[s]) s = j; } -------//ed
-- if (D[x][s] > -EPS) return true; ------------------------//35
-- int r = -1; ---------------------------------------------//2a
-- for (int i = 0; i < m; i++) { ---------------------------//d6
--- if (D[i][s] < EPS) continue; ---------------------------//57
--- if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / --//d4
------- D[r][s] || (D[i][n + 1] / D[i][s]) == (D[r][n + 1] /
------- D[r][s]) && B[i] < B[r]) r = i; } ------------------//62
-- if (r == -1) return false; ------------------------------//e3
-- Pivot(r, s); } } ----------------------------------------//fe
 DOUBLE Solve(VD &x) { -------------------------------------//b2
- int r = 0; -----------------------------------------------//f8
- for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1])
--- r = i; -------------------------------------------------//b4
- if (D[r][n + 1] < -EPS) { --------------------------------//39
-- Pivot(r, n); --------------------------------------------//e1
-- if (!Simplex(1) || D[m + 1][n + 1] < -EPS) -------------//0e
---- return -numeric_limits<DOUBLE>::infinity(); ----------//49
-- for (int i = 0; i < m; i++) if (B[i] == -1) { ----------//85
--- int s = -1; --------------------------------------------//8d
--- for (int j = 0; j <= n; j++) --------------------------//9f
---- if (s == -1 || D[i][j] < D[i][s] || -----------------//90
-------- D[i][j] == D[i][s] && N[j] < N[s]) ---------------//c8
------ s = j; ----------------------------------------------//d4
--- Pivot(i, s); } } ---------------------------------------//2f
- if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
- x = VD(n); -----------------------------------------------//87
- for (int i = 0; i < m; i++) if (B[i] < n) ---------------//e9
--- x[B[i]] = D[i][n + 1]; ---------------------------------//bb
- return D[m][n + 1]; } }; ---------------------------------//30
// Two-phase simplex algorithm for solving linear programs //c3
// of the form --------------------------------------------//21
//     maximize     c^T x ---------------------------------//1d
//     subject to   Ax <= b -------------------------------//6e
//                  x >= 0 --------------------------------//44
// INPUT: A -- an m x n matrix ----------------------------//23
//        b -- an m-dimensional vector --------------------//81
//        c -- an n-dimensional vector --------------------//e5
//        x -- a vector where the optimal solution will be //17
//             stored -------------------------------------//83
// OUTPUT: value of the optimal solution (infinity if -----//d5
//                  unbounded above, nan if infeasible) --//7d
// To use this code, create an LPSolver object with A, b, -//ea
// and c as arguments.  Then, call Solve(x). --------------//2a
// #include <iostream> ------------------------------------//56
// #include <iomanip> -------------------------------------//e6
// #include <vector> --------------------------------------//55
// #include <cmath> ---------------------------------------//a2
// #include <limits> --------------------------------------//ca
// using namespace std; -----------------------------------//21
// int main() { -------------------------------------------//27
//   const int m = 4; -------------------------------------//86
//   const int n = 3; -------------------------------------//b7
//   DOUBLE _A[m][n] = { ----------------------------------//8a
//     { 6, -1, 0 }, --------------------------------------//66
//     { -1, -5, 0 }, -------------------------------------//57
//     { 1, 5, 1 }, ---------------------------------------//6f
//     { -1, -5, -1 } -------------------------------------//0c
//   }; ---------------------------------------------------//06
//   DOUBLE _b[m] = { 10, -4, 5, -5 }; --------------------//80
//   DOUBLE _c[n] = { 1, -1, 0 }; -------------------------//c9
//   VVD A(m); --------------------------------------------//5f
//   VD b(_b, _b + m); ------------------------------------//14
//   VD c(_c, _c + n); ------------------------------------//78
//   for (int i = 0; i < m; i++) A[i] = VD(_A[i], _A[i] + n);
//   LPSolver solver(A, b, c); ----------------------------//e5
//   VD x; ------------------------------------------------//c9
//   DOUBLE value = solver.Solve(x); ----------------------//c3
//   cerr << "VALUE: " << value << endl; // VALUE: 1.29032 //fc
//   cerr << "SOLUTION:"; // SOLUTION: 1.74194 0.451613 1 -//3a
//   for (size_t i = 0; i < x.size(); i++) cerr << " " << x[i];
//   cerr << endl; ----------------------------------------//5f
//   return 0; --------------------------------------------//61
// } ------------------------------------------------------//ab
```

7.11. **Fast Square Testing.** An optimized test for square integers.

```
long long M; -----------------------------------------------//a7
void init_is_square() { -----------------------------------//cd
- rep(i,0,64) M |= 1ULL << (63-(i*i)%64); } ---------------//a6
inline bool is_square(ll x) { -----------------------------//14
- if ((M << x) >= 0) return false; ------------------------//14
- int c = __builtin_ctz(x); -------------------------------//49
- if (c & 1) return false; --------------------------------//b0
- x >>= c; ------------------------------------------------//13
- if ((x&7) - 1) return false; ----------------------------//1f
- ll r = sqrt(x); -----------------------------------------//21
- return r*r == x; } --------------------------------------//2a
```

7.12. **Fast Input Reading.** If input or output is huge, sometimes it is beneficial to optimize the input reading/output writing. This can be achieved by reading all input in at once (using fread), and then parsing it manually. Output can also be stored in an output buffer and then dumped once in the end (using fwrite). A simpler, but still effective, way to achieve speed is to use the following input reading method.

```
void readn(register int *n) { -----------------------------//dc
- int sign = 1; -------------------------------------------//32
- register char c; ----------------------------------------//a5
- *n = 0; -------------------------------------------------//35
- while((c = getc_unlocked(stdin)) != '\n') { -------------//f3
--- switch(c) { -------------------------------------------//0c
----- case '-': sign = -1; break; -------------------------//28
----- case ' ': goto hell; --------------------------------//fd
----- case '\n': goto hell; -------------------------------//79
----- default: *n *= 10; *n += c - '0'; break; } } --------//bc
hell: -----------------------------------------------------//a8
- *n *= sign; } -------------------------------------------//67
```

7.13. **128-bit Integer.** GCC has a 128-bit integer data type named `__int128`. Useful if doing multiplication of 64-bit integers, or something needing a little more than 64-bits to represent. There's also `__float128`.

7.14. **Bit Hacks.**

```
int snoob(int x) { ----------------------------------------//73
- int y = x & -x, z = x + y; ------------------------------//12
- return z | ((x ^ z) >> 2) / y; } ------------------------//3d
```

Catalan	$C_0 = 1,\ C_n = \frac{1}{n+1}\binom{2n}{n} = \sum_{i=0}^{n-1} C_i C_{n-i-1} = \frac{4n-2}{n+1} C_{n-1}$	
Stirling 1st kind	$\left[{0 \atop 0}\right] = 1,\ \left[{n \atop 0}\right] = \left[{0 \atop n}\right] = 0,\ \left[{n \atop k}\right] = (n-1)\left[{n-1 \atop k}\right] + \left[{n-1 \atop k-1}\right]$	#perms of n objs with exactly k cycles
Stirling 2nd kind	$\left\{{n \atop 1}\right\} = \left\{{n \atop n}\right\} = 1,\ \left\{{n \atop k}\right\} = k\left\{{n-1 \atop k}\right\} + \left\{{n-1 \atop k-1}\right\}$	#ways to partition n objs into k nonempty sets
Euler	$\left\langle{n \atop 0}\right\rangle = \left\langle{n \atop n-1}\right\rangle = 1,\ \left\langle{n \atop k}\right\rangle = (k+1)\left\langle{n-1 \atop k}\right\rangle + (n-k)\left\langle{n-1 \atop k-1}\right\rangle$	#perms of n objs with exactly k ascents
Euler 2nd Order	$\left\langle\!\left\langle{n \atop k}\right\rangle\!\right\rangle = (k+1)\left\langle\!\left\langle{n-1 \atop k}\right\rangle\!\right\rangle + (2n-k-1)\left\langle\!\left\langle{n-1 \atop k-1}\right\rangle\!\right\rangle$	#perms of $1, 1, 2, 2, ..., n, n$ with exactly k ascents
Bell	$B_1 = 1,\ B_n = \sum_{k=0}^{n-1} B_k \binom{n-1}{k} = \sum_{k=0}^{n} \left\{{n \atop k}\right\}$	#partitions of $1..n$ (Stirling 2nd, no limit on k)

#labeled rooted trees	n^{n-1}
#labeled unrooted trees	n^{n-2}
#forests of k rooted trees	$\frac{k}{n}\binom{n}{k}n^{n-k}$
$\sum_{i=1}^{n} i^2 = n(n+1)(2n+1)/6$	$\sum_{i=1}^{n} i^3 = n^2(n+1)^2/4$
$!n = n\times!(n-1) + (-1)^n$	$!n = (n-1)(!(n-1)+!(n-2))$
$\sum_{i=1}^{n}\binom{n}{i}F_i = F_{2n}$	$\sum_i \binom{n-i}{i} = F_{n+1}$
$\sum_{k=0}^{n}\binom{k}{m} = \binom{n+1}{m+1}$	$x^k = \sum_{i=0}^{k} i!\left\{{k \atop i}\right\}\binom{x}{i} = \sum_{i=0}^{k}\left\langle{k \atop i}\right\rangle\binom{x+i}{k}$
$a \equiv b \pmod{x, y} \Rightarrow a \equiv b \pmod{\mathrm{lcm}(x, y)}$	$\sum_{d\mid n}\phi(d) = n$
$ac \equiv bc \pmod{m} \Rightarrow a \equiv b \pmod{\frac{m}{\gcd(c,m)}}$	$(\sum_{d\mid n}\sigma_0(d))^2 = \sum_{d\mid n}\sigma_0(d)^3$
p prime $\Leftrightarrow (p-1)! \equiv -1 \pmod{p}$	$\gcd(n^a - 1, n^b - 1) = n^{\gcd(a,b)} - 1$
$\sigma_x(n) = \prod_{i=0}^{r}\frac{p_i^{(a_i+1)x}-1}{p_i^x - 1}$	$\sigma_0(n) = \prod_{i=0}^{r}(a_i + 1)$
$\sum_{k=0}^{m}(-1)^k\binom{n}{k} = (-1)^m\binom{n-1}{m}$	
$2^{\omega(n)} = O(\sqrt{n})$	$\sum_{i=1}^{n} 2^{\omega(i)} = O(n \log n)$
$d = v_i t + \frac{1}{2}at^2$	$v_f^2 = v_i^2 + 2ad$
$v_f = v_i + at$	$d = \frac{v_i + v_f}{2}t$

7.15. **The Twelvefold Way.** Putting n balls into k boxes.

Balls	same	distinct	same	distinct	
Boxes	same	same	distinct	distinct	Remarks
-	$\mathrm{p}_k(n)$	$\sum_{i=0}^{k}\left\{{n \atop i}\right\}$	$\binom{n+k-1}{k-1}$	k^n	$\mathrm{p}_k(n)$: #partitions of n into $\leq k$ positive parts
size ≥ 1	$\mathrm{p}(n, k)$	$\left\{{n \atop k}\right\}$	$\binom{n-1}{k-1}$	$k!\left\{{n \atop k}\right\}$	$\mathrm{p}(n, k)$: #partitions of n into k positive parts
size ≤ 1	$[n \leq k]$	$[n \leq k]$	$\binom{k}{n}$	$n!\binom{k}{n}$	$[cond]$: 1 if $cond = true$, else 0

8. Useful Information

9. Misc

9.1. Debugging Tips.

- Stack overflow? Recursive DFS on tree that is actually a long path?
- Floating-point numbers
  - Getting `NaN`? Make sure `acos` etc. are not getting values out of their range (perhaps `1+eps`).
  - Rounding negative numbers?
  - Outputting in scientific notation?
- Wrong Answer?
  - Read the problem statement again!
  - Are multiple test cases being handled correctly? Try repeating the same test case many times.
  - Integer overflow?
  - Think very carefully about boundaries of all input parameters
  - Try out possible edge cases:
    - $n = 0, n = -1, n = 1, n = 2^{31} - 1$ or $n = -2^{31}$
    - List is empty, or contains a single element
    - n is even, n is odd
    - Graph is empty, or contains a single vertex
    - Graph is a multigraph (loops or multiple edges)
    - Polygon is concave or non-simple
  - Is initial condition wrong for small cases?
  - Are you sure the algorithm is correct?
  - Explain your solution to someone.
  - Are you using any functions that you don't completely understand? Maybe STL functions?
  - Maybe you (or someone else) should rewrite the solution?
  - Can the input line be empty?
- Run-Time Error?
  - Is it actually Memory Limit Exceeded?

9.2. Solution Ideas.

- Dynamic Programming
  - Parsing CFGs: CYK Algorithm
  - Drop a parameter, recover from others
  - Swap answer and a parameter
  - When grouping: try splitting in two
  - 2^k trick
  - When optimizing
    - Convex hull optimization
      - $\text{dp}[i] = \min_{j<i}\{\text{dp}[j] + b[j] \times a[i]\}$
      - $b[j] \geq b[j+1]$
      - optionally $a[i] \leq a[i+1]$
      - $O(n^2)$ to $O(n)$
    - Divide and conquer optimization
      - $\text{dp}[i][j] = \min_{k<j}\{\text{dp}[i-1][k] + C[k][j]\}$
      - $A[i][j] \leq A[i][j+1]$
      - $O(kn^2)$ to $O(kn \log n)$
      - sufficient: $C[a][c] + C[b][d] \leq C[a][d] + C[b][c]$, $a \leq b \leq c \leq d$ (QI)
    - Knuth optimization
      - $\text{dp}[i][j] = \min_{i<k<j}\{\text{dp}[i][k] + \text{dp}[k][j] + C[i][j]\}$
      - $A[i][j-1] \leq A[i][j] \leq A[i+1][j]$
      - $O(n^3)$ to $O(n^2)$
      - sufficient: QI and $C[b][c] \leq C[a][d]$, $a \leq b \leq c \leq d$
- Greedy
- Randomized
- Optimizations
  - Use bitset (/64)
  - Switch order of loops (cache locality)
- Process queries offline
  - Mo's algorithm
- Square-root decomposition
- Precomputation
- Efficient simulation
  - Mo's algorithm
  - Sqrt decomposition
  - Store 2^k jump pointers
- Data structure techniques
  - Sqrt buckets
  - Store 2^k jump pointers
  - 2^k merging trick
- Counting
  - Inclusion-exclusion principle
  - Generating functions
- Graphs
  - Can we model the problem as a graph?
  - Can we use any properties of the graph?
  - Strongly connected components
  - Cycles (or odd cycles)
  - Bipartite (no odd cycles)
    - Bipartite matching
    - Hall's marriage theorem
    - Stable Marriage
  - Cut vertex/bridge
  - Biconnected components
  - Degrees of vertices (odd/even)
  - Trees
    - Heavy-light decomposition
    - Centroid decomposition
    - Least common ancestor
    - Centers of the tree
  - Eulerian path/circuit
  - Chinese postman problem
  - Topological sort
  - (Min-Cost) Max Flow
  - Min Cut
    - Maximum Density Subgraph
  - Huffman Coding
  - Min-Cost Arborescence
  - Steiner Tree
  - Kirchoff's matrix tree theorem
  - Prüfer sequences
  - Lovász Toggle
  - Look at the DFS tree (which has no cross-edges)
  - Is the graph a DFA or NFA?
    - Is it the Synchronizing word problem?
- Mathematics
  - Is the function multiplicative?
  - Look for a pattern
  - Permutations
    - Consider the cycles of the permutation
  - Functions
    - Sum of piecewise-linear functions is a piecewise-linear function
    - Sum of convex (concave) functions is convex (concave)
  - Modular arithmetic
    - Chinese Remainder Theorem
    - Linear Congruence
  - Sieve
  - System of linear equations
  - Values too big to represent?
    - Compute using the logarithm
    - Divide everything by some large value
  - Linear programming
    - Is the dual problem easier to solve?
  - Can the problem be modeled as a different combinatorial problem? Does that simplify calculations?
- Logic
  - 2-SAT
  - XOR-SAT (Gauss elimination or Bipartite matching)
- Meet in the middle
- Only work with the smaller half ($\log(n)$)
- Strings
  - Trie (maybe over something weird, like bits)
  - Suffix array
  - Suffix automaton (+DP?)
  - Aho-Corasick
  - eerTree
  - Work with $S + S$
- Hashing
- Euler tour, tree to array
- Segment trees
  - Lazy propagation
  - Persistent
  - Implicit
  - Segment tree of X
- Geometry
  - Minkowski sum (of convex sets)
  - Rotating calipers
  - Sweep line (horizontally or vertically?)
  - Sweep angle
  - Convex hull
- Fix a parameter (possibly the answer).
- Are there few distinct values?
- Binary search
- Sliding Window (+ Monotonic Queue)
- Computing a Convolution? Fast Fourier Transform
- Computing a 2D Convolution? FFT on each row, and then on each column
- Exact Cover (+ Algorithm X)
- Cycle-Finding
- What is the smallest set of values that identify the solution? The cycle structure of the permutation? The powers of primes in the factorization?
- Look at the complement problem

- Minimize something instead of maximizing
- Immediately enforce necessary conditions. (All values greater than 0? Initialize them all to 1)
- Add large constant to negative numbers to make them positive
- Counting/Bucket sort

10. Formulas

- **Legendre symbol:** $\left(\frac{a}{b}\right) = a^{(b-1)/2} \pmod b$, b odd prime.
- **Heron's formula:** A triangle with side lengths a, b, c has area $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$.
- **Pick's theorem:** A polygon on an integer grid strictly containing i lattice points and having b lattice points on the boundary has area $i + \frac{b}{2} - 1$. (Nothing similar in higher dimensions)
- **Euler's totient:** The number of integers less than n that are coprime to n are $n\prod_{p|n}\left(1-\frac{1}{p}\right)$ where each p is a distinct prime factor of n.
- **König's theorem:** In any bipartite graph $G = (L \cup R, E)$, the number of edges in a maximum matching is equal to the number of vertices in a minimum vertex cover. Let U be the set of unmatched vertices in L, and Z be the set of vertices that are either in U or are connected to U by an alternating path. Then $K = (L \setminus Z) \cup (R \cap Z)$ is the minimum vertex cover.
- A minumum Steiner tree for n vertices requires at most $n-2$ additional Steiner vertices.
- The number of vertices of a graph is equal to its minimum vertex cover number plus the size of a maximum independent set.
- **Lagrange polynomial** through points $(x_0, y_0), \ldots, (x_k, y_k)$ is $L(x) = \sum_{j=0}^{k} y_j \prod_{\substack{0 \le m \le k \\ m \ne j}} \frac{x - x_m}{x_j - x_m}$
- **Hook length formula:** If λ is a Young diagram and $h_\lambda(i,j)$ is the hook-length of cell (i,j), then then the number of Young tableux $d_\lambda = n!/\prod h_\lambda(i,j)$.
- **Möbius inversion formula:** If $f(n) = \sum_{d|n} g(d)$, then $g(n) = \sum_{d|n} \mu(d) f(n/d)$. If $f(n) = \sum_{m=1}^{n} g(\lfloor n/m \rfloor)$, then $g(n) = \sum_{m=1}^{n} \mu(m) f(\lfloor \frac{n}{m} \rfloor)$.
- #primitive pythagorean triples with hypotenuse $< n$ approx $n/(2\pi)$.
- **Frobenius Number:** largest number which can't be expressed as a linear combination of numbers $a_1, \ldots, a_n$ with non-negative coefficients. $g(a_1, a_2) = a_1 a_2 - a_1 - a_2$, $N(a_1, a_2) = (a_1 - 1)(a_2 - 1)/2$. $g(d \cdot a_1, d \cdot a_2, a_3) = d \cdot g(a_1, a_2, a_3) + a_3(d-1)$. An integer $x > (\max_i a_i)^2$ can be expressed in such a way iff. $x \mid \gcd(a_1, \ldots, a_n)$.

10.1. Physics.

- **Snell's law:** $\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$

10.2. Markov Chains.

A Markov Chain can be represented as a weighted directed graph of states, where the weight of an edge represents the probability of transitioning over that edge in one timestep. Let $P^{(m)} = (p_{ij}^{(m)})$ be the probability matrix of transitioning from state i to state j in m timesteps, and note that $P^{(1)}$ is the adjacency matrix of the graph. **Chapman-Kolmogorov:** $p_{ij}^{(m+n)} = \sum_k p_{ik}^{(m)} p_{kj}^{(n)}$. It follows that $P^{(m+n)} = P^{(m)} P^{(n)}$ and $P^{(m)} = P^m$. If $p^{(0)}$ is the initial probability distribution (a vector), then $p^{(0)} P^{(m)}$ is the probability distribution after m timesteps.

The return times of a state i is $R_i = \{m \mid p_{ii}^{(m)} > 0\}$, and i is *aperiodic* if $\gcd(R_i) = 1$. A MC is aperiodic if any of its vertices is aperiodic. A MC is *irreducible* if the corresponding graph is strongly connected.

A distribution π is stationary if $\pi P = \pi$. If MC is irreducible then $\pi_i = 1/\mathbb{E}[T_i]$, where T_i is the expected time between two visits at i. π_j/π_i is the expected number of visits at j in between two consecutive visits at i. A MC is *ergodic* if $\lim_{m\to\infty} p^{(0)} P^m = \pi$. A MC is ergodic iff. it is irreducible and aperiodic.

A MC for a random walk in an undirected weighted graph (unweighted graph can be made weighted by adding 1-weights) has $p_{uv} = w_{uv}/\sum_x w_{ux}$. If the graph is connected, then $\pi_u = \sum_x w_{ux}/\sum_v \sum_x w_{vx}$. Such a random walk is aperiodic iff. the graph is not bipartite.

An *absorbing* MC is of the form $P = \begin{pmatrix} Q & R \\ 0 & I_r \end{pmatrix}$. Let $N = \sum_{m=0}^{\infty} Q^m = (I_t - Q)^{-1}$. Then, if starting in state i, the expected number of steps till absorption is the i-th entry in $N1$. If starting in state i, the probability of being absorbed in state j is the (i,j)-th entry of NR.

Many problems on MC can be formulated in terms of a system of recurrence relations, and then solved using Gaussian elimination.

10.3. Burnside's Lemma.

Let G be a finite group that acts on a set X. For each g in G let X^g denote the set of elements in X that are fixed by g. Then the number of orbits

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

$$Z(S_n) = \frac{1}{n} \sum_{l=1}^{n} a_l Z(S_{n-l})$$

10.4. Bézout's identity.

If (x, y) is any solution to $ax + by = d$ (e.g. found by the Extended Euclidean Algorithm), then all solutions are given by

$$\left(x + k\frac{b}{\gcd(a,b)}, y - k\frac{a}{\gcd(a,b)}\right)$$

10.5. Misc.

10.5.1. *Determinants and PM.*

$$det(A) = \sum_{\sigma \in S_n} \operatorname{sgn}(\sigma) \prod_{i=1}^{n} a_{i,\sigma(i)}$$

$$perm(A) = \sum_{\sigma \in S_n} \prod_{i=1}^{n} a_{i,\sigma(i)}$$

$$pf(A) = \frac{1}{2^n n!} \sum_{\sigma \in S_{2n}} \operatorname{sgn}(\sigma) \prod_{i=1}^{n} a_{\sigma(2i-1),\sigma(2i)}$$

$$= \sum_{M \in \mathrm{PM}(n)} \operatorname{sgn}(M) \prod_{(i,j) \in M} a_{i,j}$$

10.5.2. *BEST Theorem.*

Count directed Eulerian cycles. Number of OST given by Kirchoff's Theorem (remove r/c with root) $\#\mathrm{OST}(G, r) \cdot \prod_v (d_v - 1)!$

10.5.3. *Primitive Roots.*

Only exists when n is $2, 4, p^k, 2p^k$, where p odd prime. Assume n prime. Number of primitive roots $\phi(\phi(n))$ Let g be primitive root. All primitive roots are of the form g^k where $k, \phi(p)$ are coprime.
k-roots: $g^{i \cdot \phi(n)/k}$ for $0 \le i < k$

10.5.4. *Sum of primes.*

For any multiplicative f:

$$S(n, p) = S(n, p-1) - f(p) \cdot (S(n/p, p-1) - S(p-1, p-1))$$

10.5.5. *Floor.*

$$\lfloor \lfloor x/y \rfloor / z \rfloor = \lfloor x/(yz) \rfloor$$

$$x\%y = x - y\lfloor x/y \rfloor$$

Practice Contest Checklist

- How many operations per second? Compare to local machine.
- What is the stack size?
- How to use printf/scanf with long long/long double?
- Are `__int128` and `__float128` available?
- Does MLE give RTE or MLE as a verdict? What about stack overflow?
- What is `RAND_MAX`?
- How does the judge handle extra spaces (or missing newlines) in the output?
- Look at documentation for programming languages.
- Try different programming languages: C++, Java and Python.
- Try the submit script.
- Try local programs: i?python[23], factor.
- Try submitting with `assert(false)` and `assert(true)`.
- Return-value from `main`.
- Look for directory with sample test cases.
- Make sure printing works.
- Remove this page from the notebook.