

viRUs

Team Reference Document

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1. CODE TEMPLATES

1.1. Basic Configuration.

1.1.1. .bashrc.

```
xset r rate 150 100-----// dd
set -o vi-----// f4
-----// 08
function check {------// f6
---IFS=-----// 17
---s=""-----// 22
---cat $1 | while read l; do-----// 81
-----s="$s$(echo $l | sed 's/\\s//g')\\n"-----// d0
-----h=$(echo -ne "$s" | md5sum)-----// 27
-----echo "${h:0:2} $l"-----// 1b
---done-----// aa
}-----// a7
# setxkbmap -option caps:escape dvorak is-----// 18
# setxkbmap en_US-----// 97
```

ProTip™: setxkbmap dvorak on qwerty: o.yqtxmal ekrpat

1.1.2. .vimrc.

```
set nosp et sw=4 ts=4 sts=4 si cindent hi=1000 nu ru noeb showcmd showmode----// bb
syn on | colorscheme slate-----// e5
```

1.2. C++ Header. A C++ header.

```
#include <bits/stdc++.h>-----// 84
using namespace std;-----// 16
template <class T> int size(const T &x) { return x.size(); }-----// 5f
#define rep(i,a,b) for (__typeof(a) i=(a); i<(b); ++i)-----// 6c
#define iter(it,c) for (__typeof((c).begin()) it = (c).begin(); it != (c).end(); ++it)
typedef pair<int, int> ii;-----// 44
typedef vector<int> vi;-----// 9d
typedef vector<ii> vii;-----// eb
typedef long long ll;-----// 47
const int INF = ~(1<<31); // 2147483647-----// 10
-----// b2
const double EPS = 1e-9;-----// d5
const double pi = acos(-1);-----// 67
typedef unsigned long long ull;-----// ff
typedef vector<vi> vvi;-----// 4b
typedef vector<vii> vvii;-----// 36
template <class T> T mod(T a, T b) { return (a % b + b) % b; }-----// d5
```

1.3. Java Template. A Java template.

```
import java.util.*;-----// 37
import java.math.*;-----// 89
import java.io.*;-----// 28
-----// a3
public class Main {-----// 17
---public static void main(String[] args) throws Exception {-----// 02
-----Scanner in = new Scanner(System.in);-----// ef
-----PrintWriter out = new PrintWriter(System.out, false);-----// 62
```

```
-----// code-----// e6
-----out.flush();-----// 56
---}------// 79
}------// 00
```

2. DATA STRUCTURES

2.1. Union-Find.

```
struct union_find {------// 42
---vi p; union_find(int n) : p(n, -1) { }-----// 28
---int find(int x) { return p[x] < 0 ? x : p[x] = find(p[x]); }-----// ba
---bool unite(int x, int y) {------// 6c
-----int xp = find(x), yp = find(y);-----// 64
-----if (xp == yp) return false;-----// 0b
-----if (p[xp] > p[yp]) swap(xp,yp);-----// 78
-----p[xp] += p[yp], p[yp] = xp;-----// 88
-----return true; }-----// 1f
---int size(int x) { return -p[find(x)]; } }-----// b9
```

2.2. Segment Tree.

```
#ifdef SEG_MIN-----// 03
const int ID = INF;-----// 56
int f(int a, int b) { return min(a, b); }-----// 4f
#else-----// 0e
const int ID = 0;-----// 3e
int f(int a, int b) { return a + b; }-----// dd
#endif-----// 16
struct segment_tree {------// ab
---int n; vi data, lazy;-----// dd
---segment_tree() {}-----// 93
---segment_tree(const vi &arr) : n(size(arr)), data(4*n), lazy(4*n,INF) {-----// f1
---mk(arr, 0, n-1, 0); }-----// e9
---int mk(const vi &arr, int l, int r, int i) {-----// 12
---if (l == r) return data[i] = arr[l];-----// 5b
---int m = (l + r) / 2;-----// de
---return data[i] = f(mk(arr, l, m, 2*i+1), mk(arr, m+1, r, 2*i+2)); }-----// 0a
---int query(int a, int b) { return q(a, b, 0, n-1, 0); }-----// f6
---int q(int a, int b, int l, int r, int i) {-----// 22
---propagate(l, r, i);-----// 12
---if (r < a || b < l) return ID;-----// c7
---if (a <= l && r <= b) return data[i];-----// ce
---int m = (l + r) / 2;-----// 7a
---return f(q(a, b, l, m, 2*i+1), q(a, b, m+1, r, 2*i+2)); }-----// 5c
---void update(int i, int v) { u(i, v, 0, n-1, 0); }-----// 90
---int u(int i, int v, int l, int r, int j) {-----// 02
---propagate(l, r, j);-----// ae
---if (r < i || i < l) return data[j];-----// 92
---if (l == i && r == i) return data[j] = v;-----// 4a
---int m = (l + r) / 2;-----// cb
---return data[j] = f(u(i, v, l, m, 2*j+1), u(i, v, m+1, r, 2*j+2)); }-----// 34
---void range_update(int a, int b, int v) { ru(a, b, v, 0, n-1, 0); }-----// 71
---int ru(int a, int b, int v, int l, int r, int i) {-----// e0
```

```
-----propagate(l, r, i);-----// 19
-----if (l > r) return ID;-----// cc
-----if (r < a || b < l) return data[i];-----// d9
-----if (a <= l && r <= b) return (lazy[i] = v) * (r - l + 1) + data[i];-----// 06
-----int m = (l + r) / 2;-----// cc
-----return data[i] = f(ru(a, b, v, l, m, 2*i+1),-----// cc
-----ru(a, b, v, m+1, r, 2*i+2));-----// 2b
-----}-----// 0b
---void propagate(int l, int r, int i) {-----// a7
---if (l > r || lazy[i] == INF) return;-----// 5f
---data[i] += lazy[i] * (r - l + 1);-----// 44
---if (l < r) {-----// 28
---if (lazy[2*i+1] == INF) lazy[2*i+1] = lazy[i];-----// 4e
---else lazy[2*i+1] += lazy[i];-----// 1e
---if (lazy[2*i+2] == INF) lazy[2*i+2] = lazy[i];-----// de
---else lazy[2*i+2] += lazy[i];-----// 74
---}-----// 1f
---lazy[i] = INF;-----// f8
---}-----// 41
};-----// ae
```

2.2.1. Persistent Segment Tree.

```
int segcnt = 0;-----// cf
struct segment {-----// 68
---int l, r, lid, rid, sum;-----// fc
} segs[2000000];-----// dd
int build(int l, int r) {-----// 2b
---if (l > r) return -1;-----// 4e
---int id = segcnt++;-----// a8
---segs[id].l = l;-----// 90
---segs[id].r = r;-----// 19
---if (l == r) segs[id].lid = -1, segs[id].rid = -1;-----// ee
---else {-----// fe
---int m = (l + r) / 2;-----// 14
---segs[id].lid = build(l, m);-----// e3
---segs[id].rid = build(m + 1, r); }-----// 69
---segs[id].sum = 0;-----// 21
---return id; }-----// c5
int update(int idx, int v, int id) {-----// b8
---if (id == -1) return -1;-----// bb
---if (idx < segs[id].l || idx > segs[id].r) return id;-----// fb
---int nid = segcnt++;-----// b3
---segs[nid].l = segs[id].l;-----// 78
---segs[nid].r = segs[id].r;-----// ca
---segs[nid].lid = update(idx, v, segs[id].lid);-----// 92
---segs[nid].rid = update(idx, v, segs[id].rid);-----// 06
---segs[nid].sum = segs[id].sum + v;-----// 1a
---return nid; }-----// e6
int query(int id, int l, int r) {-----// a2
---if (r < segs[id].l || segs[id].r < l) return 0;-----// 17
```

```
---if (l <= segs[id].l && segs[id].r <= r) return segs[id].sum;-----// ad
---return query(segs[id].lid, l, r) + query(segs[id].rid, l, r); }-----// ee
```

2.3. Fenwick Tree.

```
struct fenwick_tree {-----// 98
---int n; vi data;-----// d3
---fenwick_tree(int _n) : n(_n), data(vi(n)) { }-----// db
---void update(int at, int by) {-----// 76
---while (at < n) data[at] += by, at |= at + 1; }-----// fb
---int query(int at) {-----// 71
---int res = 0;-----// c3
---while (at >= 0) res += data[at], at = (at & (at + 1)) - 1;-----// 37
---return res; }-----// e4
---int rsq(int a, int b) { return query(b) - query(a - 1); }-----// be
};-----// 57
struct fenwick_tree_sq {-----// d4
---int n; fenwick_tree x1, x0;-----// 18
---fenwick_tree_sq(int _n) : n(_n), x1(fenwick_tree(n)),-----// 2e
---x0(fenwick_tree(n)) { }-----// 7c
---// insert f(y) = my + c if x <= y-----// 17
---void update(int x, int m, int c) { x1.update(x, m); x0.update(x, c); }-----// 45
---int query(int x) { return x*x1.query(x) + x0.query(x); }-----// 73
};-----// 13
void range_update(fenwick_tree_sq &s, int a, int b, int k) {-----// 89
---s.update(a, k, k * (1 - a)); s.update(b+1, -k, k * b); }-----// 7f
int range_query(fenwick_tree_sq &s, int a, int b) {-----// 15
---return s.query(b) - s.query(a-1); }-----// f3
```

2.4. Matrix.

```
template <class K> bool eq(K a, K b) { return a == b; }-----// 2a
template <> bool eq<double>(double a, double b) { return abs(a - b) < EPS; }-----// a7
template <class T> struct matrix {-----// 0a
---int rows, cols, cnt; vector<T> data;-----// a1
---inline T& at(int i, int j) { return data[i * cols + j]; }-----// 5c
---matrix(int r, int c) : rows(r), cols(c), cnt(r * c) {-----// 56
---data.assign(cnt, T(0)); }-----// e3
---matrix(const matrix& other) : rows(other.rows), cols(other.cols),-----// b5
---cnt(other.cnt), data(other.data) { }-----// c1
---T& operator()(int i, int j) { return at(i, j); }-----// 29
---matrix<T> operator +(const matrix& other) {-----// 33
---matrix<T> res(*this); rep(i,0,cnt) res.data[i] += other.data[i];-----// f8
---return res; }-----// 09
---matrix<T> operator -(const matrix& other) {-----// 91
---matrix<T> res(*this); rep(i,0,cnt) res.data[i] -= other.data[i];-----// 7b
---return res; }-----// 9a
---matrix<T> operator *(T other) {-----// 99
---matrix<T> res(*this); rep(i,0,cnt) res.data[i] *= other;-----// 05
---return res; }-----// 8c
---matrix<T> operator *(const matrix& other) {-----// 31
---matrix<T> res(rows, other.cols);-----// 4c
---rep(i,0,rows) rep(j,0,other.cols) rep(k,0,cols)-----// ae
---res(i, j) += at(i, k) * other.data[k * other.cols + j];-----// 17
```

```
-----return res; }-----// 65
---matrix<T> pow(int p) {-----// 53
-----matrix<T> res(rows, cols), sq(*this);-----// 87
---rep(i,0,rows) res(i, i) = T(1);-----// 9d
-----while (p) {-----// 79
-----if (p & 1) res = res * sq;-----// 62
-----p >>= 1;-----// 79
-----if (p) sq = sq * sq;-----// 35
-----} return res; }-----// 22
---matrix<T> rref(T &det, int &rank) {-----// 2a
---matrix<T> mat(*this); det = T(1), rank = max(rows, cols);-----// 7a
---for (int r = 0, c = 0; c < cols; c++) {-----// 8e
---int k = r;-----// 5b
---while (k < rows && eq<T>(mat(k, c), T(0))) k++;-----// 3e
---if (k >= rows) { rank--; continue; }-----// 1a
---if (k != r) {-----// c4
---det *= T(-1);-----// 55
---rep(i,0,cols)-----// e1
---swap(mat.at(k, i), mat.at(r, i));-----// 7d
---} det *= mat(r, r);-----// b6
---T d = mat(r,c);-----// 66
---rep(i,0,cols) mat(r, i) /= d;-----// d1
---rep(i,0,rows) {-----// f6
---T m = mat(i, c);-----// 05
---if (i != r && !eq<T>(m, T(0)))-----// 1a
---rep(j,0,cols) mat(i, j) -= m * mat(r, j);-----// 7b
---} r++;-----// c5
---} return mat; }-----// b3
---matrix<T> transpose() {-----// 59
---matrix<T> res(cols, rows);-----// 5b
---rep(i,0,rows) rep(j,0,cols) res(j, i) = at(i, j);-----// 92
---return res; } }-----// df
```

2.5. Cartesian Tree.

```
struct node {-----// 36
---int x, y, sz;-----// e5
---node *l, *r;-----// 4d
---node(int _x, int _y) : x(_x), y(_y), sz(1), l(NULL), r(NULL) { } }-----// 19
int tsize(node* t) { return t ? t->sz : 0; }-----// 42
void augment(node *t) { t->sz = 1 + tsize(t->l) + tsize(t->r); }-----// 1d
pair<node*,node*> split(node *t, int x) {-----// 1d
---if (!t) return make_pair((node*)NULL,(node*)NULL);-----// fd
---if (t->x < x) {-----// 0a
---pair<node*,node*> res = split(t->r, x);-----// b4
---t->r = res.first; augment(t);-----// 4d
---return make_pair(t, res.second); }-----// e0
---pair<node*,node*> res = split(t->l, x);-----// b7
---t->l = res.second; augment(t);-----// 74
---return make_pair(res.first, t); }-----// 46
node* merge(node *l, node *r) {-----// 3c
---if (!l) return r; if (!r) return l;-----// f0
```

```
---if (l->y > r->y) { l->r = merge(l->r, r); augment(l); return l; }-----// be
---r->l = merge(l, r->l); augment(r); return r; }-----// c0
node* find(node *t, int x) {-----// b4
---while (t) {-----// 51
---if (x < t->x) t = t->l;-----// 32
---else if (t->x < x) t = t->r;-----// da
---else return t; }-----// 0b
---return NULL; }-----// ae
node* insert(node *t, int x, int y) {-----// 78
---if (find(t, x) != NULL) return t;-----// 2f
---pair<node*,node*> res = split(t, x);-----// ca
---return merge(res.first, merge(new node(x, y), res.second)); }-----// 0d
node* erase(node *t, int x) {-----// 4d
---if (!t) return NULL;-----// 7b
---if (t->x < x) t->r = erase(t->r, x);-----// 7c
---else if (x < t->x) t->l = erase(t->l, x);-----// 48
---else { node *old = t; t = merge(t->l, t->r); delete old; }-----// 22
---if (t) augment(t); return t; }-----// a3
int kth(node *t, int k) {-----// b3
---if (k < tsize(t->l)) return kth(t->l, k);-----// 64
---else if (k == tsize(t->l)) return t->x;-----// 97
---else return kth(t->r, k - tsize(t->l) - 1); }-----// f0
```

2.6. Misof Tree.

```
#define BITS 15-----// 7b
struct misof_tree {-----// fe
---int cnt[BITS][1<<BITS];-----// aa
---misof_tree() { memset(cnt, 0, sizeof(cnt)); }-----// b0
---void insert(int x) { for (int i = 0; i < BITS; cnt[i++][x]++, x >>= 1); }-----// 5a
---void erase(int x) { for (int i = 0; i < BITS; cnt[i++][x]--, x >>= 1); }-----// 49
---int nth(int n) {-----// 8a
---int res = 0;-----// a4
---for (int i = BITS-1; i >= 0; i--)-----// 99
---if (cnt[i][res <= 1] <= n) n -= cnt[i][res], res |= 1;-----// f4
---return res;-----// 3a
}-----// b5
};-----// 0a
```

2.7. Sqrt Decomposition.

```
struct segment {-----// b2
---vi arr;-----// 8c
---segment(vi _arr) : arr(_arr) { } }-----// 11
vector<segment> T;-----// a1
int K;-----// dc
void rebuild() {-----// 17
---int cnt = 0;-----// 14
---rep(i,0,size(T))-----// b1
---cnt += size(T[i].arr);-----// d1
---K = static_cast<int>(ceil(sqrt(cnt)) + 1e-9);-----// 4c
---vi arr(cnt);-----// 14
---for (int i = 0, at = 0; i < size(T); i++)-----// 79
---rep(j,0,size(T[i].arr))-----// a4
```

```
-----arr[at++] = T[i].arr[j];-----// f7
---T.clear();-----// 4c
---for (int i = 0; i < cnt; i += K)-----// 79
-----T.push_back(segment(vi(arr.begin()+i, arr.begin()+min(i+K, cnt))));-----// f0
}-----// 03
int split(int at) {-----// 71
---int i = 0;-----// 8a
---while (i < size(T) && at >= size(T[i].arr))-----// 6c
-----at -= size(T[i].arr), i++;-----// 9a
---if (i >= size(T)) return size(T);-----// 83
---if (at == 0) return i;-----// 49
---T.insert(T.begin() + i + 1, segment(vi(T[i].arr.begin() + at, T[i].arr.end())));
---T[i] = segment(vi(T[i].arr.begin(), T[i].arr.begin() + at));-----// af
---return i + 1;-----// ac
}-----// ea
void insert(int at, int v) {-----// 5f
---vi arr; arr.push_back(v);-----// 6a
---T.insert(T.begin() + split(at), segment(arr));-----// 67
}-----// cc
void erase(int at) {-----// be
---int i = split(at); split(at + 1);-----// da
---T.erase(T.begin() + i);-----// 6b
}-----// 4b
```

2.8. Monotonic Queue.

```
struct min_stack {-----// d8
---stack<int> S, M;-----// fe
---void push(int x) {-----// 20
---S.push(x);-----// e2
---M.push(M.empty() ? x : min(M.top(), x)); }-----// 92
---int top() { return S.top(); }-----// f1
---int mn() { return M.top(); }-----// 02
---void pop() { S.pop(); M.pop(); }-----// fd
---bool empty() { return S.empty(); }-----// d2
};-----// 74
struct min_queue {-----// b4
---min_stack inp, outp;-----// 3d
---void push(int x) { inp.push(x); }-----// 6b
---void fix() {-----// 5d
-----if (outp.empty()) while (!inp.empty())-----// 3b
-----outp.push(inp.top()), inp.pop();-----// 8e
---}-----// 3f
---int top() { fix(); return outp.top(); }-----// dc
---int mn() {-----// 39
-----if (inp.empty()) return outp.mn();-----// 01
-----if (outp.empty()) return inp.mn();-----// 90
-----return min(inp.mn(), outp.mn()); }-----// 97
---void pop() { fix(); outp.pop(); }-----// 4f
---bool empty() { return inp.empty() && outp.empty(); }-----// 65
};-----// 60
```

2.9. Convex Hull Trick.

```
struct convex_hull_trick {-----// 16
---vector<pair<double,double> > h;-----// b4
---double intersect(int i) {-----// 9b
-----return (h[i+1].second-h[i].second)/(h[i].first-h[i+1].first); }-----// b9
---void add(double m, double b) {-----// a4
---h.push_back(make_pair(m,b));-----// f9
---while (size(h) >= 3) {-----// f6
---int n = size(h);-----// d8
---if (intersect(n-3) < intersect(n-2)) break;-----// 07
---swap(h[n-2], h[n-1]);-----// bf
---h.pop_back(); } }-----// 4b
---double get_min(double x) {-----// b0
---int lo = 0, hi = size(h) - 2, res = -1;-----// 5b
---while (lo <= hi) {-----// 24
---int mid = lo + (hi - lo) / 2;-----// 5a
---if (intersect(mid) <= x) res = mid, lo = mid + 1;-----// 1d
---else hi = mid - 1; }-----// b6
---return h[res+1].first * x + h[res+1].second; } }-----// 84
```

3. GRAPHS

3.1. Single-Source Shortest Paths.

3.1.1. Dijkstra’s algorithm.

```
int *dist, *dad;-----// 46
struct cmp {-----// a5
---bool operator()(int a, int b) {-----// bb
---return dist[a] != dist[b] ? dist[a] < dist[b] : a < b; }-----// e6
};-----// 41
pair<int*, int*> dijkstra(int n, int s, vii *adj) {-----// 53
---dist = new int[n];-----// 84
---dad = new int[n];-----// 05
---rep(i,0,n) dist[i] = INF, dad[i] = -1;-----// 80
---set<int, cmp> pq;-----// 98
---dist[s] = 0, pq.insert(s);-----// 1f
---while (!pq.empty()) {-----// 47
---int cur = *pq.begin(); pq.erase(pq.begin());-----// 58
---rep(i,0,size(adj[cur])) {-----// a6
---int nxt = adj[cur][i].first,-----// a4
---ndist = dist[cur] + adj[cur][i].second;-----// 3a
---if (ndist < dist[nxt]) pq.erase(nxt),-----// 2d
---dist[nxt] = ndist, dad[nxt] = cur, pq.insert(nxt);-----// eb
---}-----// d2
---}-----// df
---return pair<int*, int*>(dist, dad);-----// e3
}-----// 9b
```

3.1.2. Bellman-Ford algorithm.

```
int* bellman_ford(int n, int s, vii* adj, bool& has_negative_cycle) {-----// cf
---has_negative_cycle = false;-----// 47
---int* dist = new int[n];-----// 7f
---rep(i,0,n) dist[i] = i == s ? 0 : INF;-----// df
---rep(i,0,n-1) rep(j,0,n) if (dist[j] != INF)-----// 4d
```

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-----rep(k,0,size(adj[j]))-----// 88

-----dist[adj[j]][k].first] = min(dist[adj[j]][k].first,-----// e1

-----dist[j] + adj[j][k].second);-----// 18

---rep(j,0,n) rep(k,0,size(adj[j]))-----// f8

---if (dist[j] + adj[j][k].second < dist[adj[j]][k].first)-----// 37

-----has_negative_cycle = true;-----// f1

---return dist;-----// 78

}-----// a9

3.1.3. IDA* algorithm.

int n, cur[100], pos;-----// 48

int calch() {-----// 88

---int h = 0;-----// 4a

---rep(i,0,n) if (cur[i] != 0) h += abs(i - cur[i]);-----// 9b

---return h;-----// c6

}-----// c8

int dfs(int d, int g, int prev) {-----// 12

---int h = calch();-----// 5d

---if (g + h > d) return g + h;-----// 15

---if (h == 0) return 0;-----// ff

---int mn = INF;-----// 7e

---rep(di,-2,3) {-----// 0d

-----if (di == 0) continue;-----// 0a

-----int nxt = pos + di;-----// 76

-----if (nxt == prev) continue;-----// 39

-----if (0 <= nxt && nxt < n) {-----// 68

-----swap(cur[pos], cur[nxt]);-----// 35

-----swap(pos,nxt);-----// 64

-----mn = min(mn, dfs(d, g+1, nxt));-----// 22

-----swap(pos,nxt);-----// 84

-----swap(cur[pos], cur[nxt]);-----// 3b

-----}-----// 46

-----if (mn == 0) break;-----// 8f

---}-----// d3

---return mn;-----// da

}-----// f8

int idastar() {-----// 22

---rep(i,0,n) if (cur[i] == 0) pos = i;-----// 6b

---int d = calch();-----// 38

---while (true) {-----// 18

-----int nd = dfs(d, 0, -1);-----// 42

-----if (nd == 0 || nd == INF) return d;-----// b5

-----d = nd;-----// f7

---}-----// f9

}-----// 82

3.2. Strongly Connected Components.

3.2.1. Kosaraju’s algorithm.

#include "../data-structures/union_find.cpp"-----// 5e

-----// 11

vector<bool> visited;-----// 66

vi order;-----// 9b

-----// a5

void scc_dfs(const vvi &adj, int u) {-----// a1

---int v; visited[u] = true;-----// e3

---rep(i,0,size(adj[u]))-----// 2d

-----if (!visited[v = adj[u][i]]) scc_dfs(adj, v);-----// a2

---order.push_back(u);-----// 02

}-----// 53

-----// 63

pair<union_find, vi> scc(const vvi &adj) {-----// c2

---int n = size(adj), u, v;-----// f8

---order.clear();-----// 20

---union_find uf(n);-----// a8

---vi dag;-----// 61

---vvi rev(n);-----// c5

---rep(i,0,n) rep(j,0,size(adj[i])) rev[adj[i][j]].push_back(i);-----// 7e

---visited.resize(n), fill(visited.begin(), visited.end(), false);-----// 80

---rep(i,0,n) if (!visited[i]) scc_dfs(rev, i);-----// 4e

---fill(visited.begin(), visited.end(), false);-----// 59

---stack<int> S;-----// bb

---for (int i = n-1; i >= 0; i--) {-----// 96

-----if (visited[order[i]]) continue;-----// db

-----S.push(order[i]), dag.push_back(order[i]);-----// 68

-----while (!S.empty()) {-----// 9e

-----visited[u = S.top()] = true, S.pop(), uf.unite(u, order[i]);-----// b3

-----rep(j,0,size(adj[u])) if (!visited[v = adj[u][j]]) S.push(v);-----// 1b

-----}-----// 61

---}-----// 57

---return pair<union_find, vi>(uf, dag);-----// 2b

}-----// 92

3.3. Cut Points and Bridges.

#define MAXN 5000-----// f7

int low[MAXN], num[MAXN], curnum;-----// d7

void dfs(const vvi &adj, vi &cp, vii &bri, int u, int p) {-----// 22

---low[u] = num[u] = curnum++;-----// a3

---int cnt = 0; bool found = false;-----// 97

---rep(i,0,size(adj[u])) {-----// ae

-----int v = adj[u][i];-----// 56

-----if (num[v] == -1) {-----// 3b

-----dfs(adj, cp, bri, v, u);-----// ba

-----low[u] = min(low[u], low[v]);-----// be

-----cnt++;-----// e0

-----found = found || low[v] >= num[u];-----// 30

-----if (low[v] > num[u]) bri.push_back(ii(u, v));-----// bf

-----} else if (p != v) low[u] = min(low[u], num[v]); }-----// 76

---if (found && (p != -1 || cnt > 1)) cp.push_back(u); }-----// 3e

pair<vi,vii> cut_points_and_bridges(const vvi &adj) {-----// 76

---int n = size(adj);-----// c8

---vi cp; vii bri;-----// fb

---memset(num, -1, n << 2);-----// 45


```

--- curnum = 0;-----// 07
--- rep(i,0,n) if (num[i] == -1) dfs(adj, cp, bri, i, -1);-----// 7e
--- return make_pair(cp, bri); }-----// 4c

```

3.4. Euler Path.

```

#define MAXV 1000-----// 2f
#define MAXE 5000-----// 87
vi adj[MAXV];-----// ff
int n, m, indeg[MAXV], outdeg[MAXV], res[MAXE + 1];-----// 49
ii start_end() {-----// 30
--- int start = -1, end = -1, any = 0, c = 0;-----// 74
--- rep(i,0,n) {-----// 20
----- if (outdeg[i] > 0) any = i;-----// 63
----- if (indeg[i] + 1 == outdeg[i]) start = i, c++;-----// 5a
----- else if (indeg[i] == outdeg[i] + 1) end = i, c++;-----// 13
----- else if (indeg[i] != outdeg[i]) return ii(-1, -1);-----// c1
--- }-----// ed
--- if ((start == -1) != (end == -1) || (c != 2 && c != 0)) return ii(-1, -1);-----// 54
--- if (start == -1) start = end = any;-----// 5e
--- return ii(start, end);-----// a2
}-----// eb
bool euler_path() {-----// b4
--- ii se = start_end();-----// 8a
--- int cur = se.first, at = m + 1;-----// b6
--- if (cur == -1) return false;-----// ac
--- stack<int> s;-----// 1c
--- while (true) {-----// b3
----- if (outdeg[cur] == 0) {-----// 0d
----- res[at] = cur;-----// bd
----- if (s.empty()) break;-----// c6
----- cur = s.top(); s.pop();-----// 06
----- } else s.push(cur), cur = adj[cur][outdeg[cur]];-----// 9e
--- }-----// a4
--- return at == 0;-----// ac
}-----// 22

```

3.5. Bipartite Matching.

3.5.1. Alternating Paths algorithm.

```

vi* adj;-----// cc
bool* done;-----// b1
int* owner;-----// 26
int alternating_path(int left) {-----// da
--- if (done[left]) return 0;-----// 08
--- done[left] = true;-----// f2
--- rep(i,0,size(adj[left])) {-----// 1b
----- int right = adj[left][i];-----// 46
----- if (owner[right] == -1 || alternating_path(owner[right])) {-----// f6
----- owner[right] = left; return 1;-----// f2
----- } }-----// 88
--- return 0; }-----// 41

```

3.5.2. Hopcroft-Karp algorithm. Running time is $O(|E|\sqrt{|V|})$.

```

#define MAXN 5000-----// f7
int dist[MAXN+1], q[MAXN+1];-----// b8
#define dist(v) dist[v == -1 ? MAXN : v]-----// 0f
struct bipartite_graph {-----// 2b
--- int N, M, *L, *R; vi *adj;-----// fc
--- bipartite_graph(int _N, int _M) : N(_N), M(_M),-----// 8d
--- L(new int[N]), R(new int[M]), adj(new vi[N]) {}-----// cd
--- bipartite_graph() { delete[] adj; delete[] L; delete[] R; }-----// 89
--- bool bfs() {-----// f5
--- int l = 0, r = 0;-----// 37
--- rep(v,0,N) if (L[v] == -1) dist(v) = 0, q[r++] = v;-----// f9
--- else dist(v) = INF;-----// aa
--- dist(-1) = INF;-----// f2
--- while (l < r) {-----// ba
--- int v = q[l++];-----// 50
--- if (dist(v) < dist(-1)) {-----// f1
--- iter(u, adj[v]) if (dist(R[*u]) == INF)-----// 9b
--- dist(R[*u]) = dist(v) + 1, q[r++] = R[*u];-----// 79
--- }-----// b8
--- }-----// 0d
--- return dist(-1) != INF;-----// 43
}-----// 2c
--- bool dfs(int v) {-----// 26
--- if (v != -1) {-----// d8
--- iter(u, adj[v])-----// 99
--- if (dist(R[*u]) == dist(v) + 1)-----// 74
--- if (dfs(R[*u])) {-----// 40
--- R[*u] = v, L[v] = *u;-----// 47
--- return true;-----// a2
--- }-----// 17
--- dist(v) = INF;-----// 62
--- return false;-----// 3c
--- }-----// 3d
--- return true;-----// ae
}-----// 0f
--- void add_edge(int i, int j) { adj[i].push_back(j); }-----// 92
--- int maximum_matching() {-----// a2
--- int matching = 0;-----// 71
--- memset(L, -1, sizeof(int) * N);-----// 72
--- memset(R, -1, sizeof(int) * M);-----// bf
--- while (bfs()) rep(i,0,N)-----// 3e
--- matching += L[i] == -1 && dfs(i);-----// 1d
--- return matching;-----// ec
}-----// 8b
};-----// b7

```

3.6. Maximum Flow.

3.6.1. Dinic's algorithm. An implementation of Dinic's algorithm that runs in $O(|V|^2|E|)$.

```

#define MAXV 2000-----// ba
int q[MAXV], d[MAXV];-----// e6

```

```
struct flow_network {-----// 12
--struct edge {-----// 1e
-----int v, cap, nxt;-----// ab
-----edge() { }-----// 38
-----edge(int _v, int _cap, int _nxt) : v(_v), cap(_cap), nxt(_nxt) { }-----// bc
--};-----// 6e
--int n, ecnt, *head, *curh;-----// 46
--vector<edge> e, e_store;-----// 1f
--flow_network(int _n, int m = -1) : n(_n), ecnt(0) {-----// d3
-----e.reserve(2 * (m == -1 ? n : m));-----// 24
-----head = new int[n], curh = new int[n];-----// 6b
-----memset(head, -1, n * sizeof(int));-----// 56
--}-----// 77
--void destroy() { delete[] head; delete[] curh; }-----// f6
--void reset() { e = e_store; }-----// 87
--void add_edge(int u, int v, int uv, int vu = 0) {-----// cd
-----e.push_back(edge(v, uv, head[u])); head[u] = ecnt++;-----// c9
-----e.push_back(edge(u, vu, head[v])); head[v] = ecnt++;-----// 89
--}-----// 14
--int augment(int v, int t, int f) {-----// 3f
-----if (v == t) return f;-----// 6d
-----for (int &i = curh[v], ret; i != -1; i = e[i].nxt)-----// f9
-----if (e[i].cap > 0 && d[e[i].v] + 1 == d[v])-----// cc
-----if ((ret = augment(e[i].v, t, min(f, e[i].cap))) > 0)-----// 1f
-----return (e[i].cap -= ret, e[i^1].cap += ret, ret);-----// ac
-----return 0;-----// 19
--}-----// fd
--int max_flow(int s, int t, bool res = true) {-----// 31
-----if(s == t) return 0;-----// 9d
-----e_store = e;-----// 57
-----int f = 0, x, l, r;-----// 0e
-----while (true) {-----// b5
-----memset(d, -1, n * sizeof(int));-----// a8
-----l = r = 0, d[q[r++] = t] = 0;-----// 0e
-----while (l < r)-----// 7a
-----for (int v = q[l++], i = head[v]; i != -1; i = e[i].nxt)-----// a2
-----if (e[i^1].cap > 0 && d[e[i].v] == -1)-----// 29
-----d[q[r++] = e[i].v] = d[v]+1;-----// 28
-----if (d[s] == -1) break;-----// a0
-----memcpy(curh, head, n * sizeof(int));-----// 10
-----while ((x = augment(s, t, INF)) != 0) f += x;-----// a6
-----}-----// 96
-----if (res) reset();-----// 21
-----return f;-----// b6
--}-----// 1b
};-----// 3b

struct cmp {-----// d1
--bool operator()(int i, int j) {-----// 8a
-----return d[i] == d[j] ? i < j : d[i] < d[j];-----// 89
--}-----// df
};-----// cf
struct flow_network {-----// eb
--struct edge {-----// 9a
-----int v, cap, cost, nxt;-----// ad
-----edge(int _v, int _cap, int _cost, int _nxt)-----// ec
-----: v(_v), cap(_cap), cost(_cost), nxt(_nxt) { }-----// c4
--};-----// ad
--int n, ecnt, *head;-----// 46
--vector<edge> e, e_store;-----// 4b
--flow_network(int _n, int m = -1) : n(_n), ecnt(0) {-----// dd
-----e.reserve(2 * (m == -1 ? n : m));-----// e6
-----memset(head = new int[n], -1, n << 2);-----// 6c
--}-----// f3
--void destroy() { delete[] head; }-----// ac
--void reset() { e = e_store; }-----// 88
--void add_edge(int u, int v, int cost, int uv, int vu=0) {-----// b4
-----e.push_back(edge(v, uv, cost, head[u])); head[u] = ecnt++;-----// 43
-----e.push_back(edge(u, vu, -cost, head[v])); head[v] = ecnt++;-----// 53
--}-----// 16
--ii min_cost_max_flow(int s, int t, bool res = true) {-----// 6d
-----if (s == t) return ii(0, 0);-----// 34
-----e_store = e;-----// 70
-----memset(pot, 0, n << 2);-----// 24
-----int f = 0, c = 0, v;-----// d4
-----while (true) {-----// 29
-----memset(d, -1, n << 2);-----// fd
-----memset(p, -1, n << 2);-----// b7
-----set<int, cmp> q;-----// d8
-----q.insert(s); d[s] = 0;-----// 1d
-----while (!q.empty()) {-----// 04
-----int u = *q.begin();-----// dd
-----q.erase(q.begin());-----// 20
-----for (int i = head[u]; i != -1; i = e[i].nxt) {-----// 02
-----if (e[i].cap == 0) continue;-----// 1c
-----int cd = d[u] + e[i].cost + pot[u] - pot[v = e[i].v];-----// 1d
-----if (d[v] == -1 || cd < d[v]) {-----// d2
-----if (q.find(v) != q.end()) q.erase(q.find(v));-----// e2
-----d[v] = cd; p[v] = i;-----// f7
-----q.insert(v);-----// 74
-----}-----// 6c
-----}-----// 1b
-----}-----// da
-----if (p[t] == -1) break;-----// 09
-----int x = INF, at = p[t];-----// e8
-----while (at != -1) x = min(x, e[at].cap), at = p[e[at^1].v];-----// 32
-----at = p[t], f += x;-----// 43
-----while (at != -1)-----// 53
```

3.7. Minimum Cost Maximum Flow. Running time is $O(|V|^2|E|\log|V|)$. NOTE: Doesn't work on negative weights!

```
#define MAXV 2000-----// ba
int d[MAXV], p[MAXV], pot[MAXV];-----// 80
```



```
-----e[at].cap -= x, e[at^1].cap += x, at = p[e[at^1].v];-----// 95
-----c += x * (d[t] + pot[t] - pot[s]);-----// 44
-----rep(i,0,n) if (p[i] != -1) pot[i] += d[i];-----// 86
-----}-----// 4e
-----if (res) reset();-----// d7
-----return ii(f, c);-----// 9f
-----}-----// 4c
};-----// ec

A second implementation that is slower but works on negative weights.

struct flow_network {-----// 81
--struct mcmf_edge {-----// f6
--int u, v;-----// e1
--ll w, c;-----// b4
--mcmf_edge* rev;-----// 9d
--mcmf_edge(int _u, int _v, ll _w, ll _c, mcmf_edge* _rev = NULL) {-----// ea
--u = _u; v = _v; w = _w; c = _c; rev = _rev;-----// 83
--}-----// 02
--};-----// b9
--int n;-----// b4
--vector<pair<int, pair<ll, ll> > >* adj;-----// 72
--flow_network(int _n) {-----// 55
--n = _n;-----// fa
--adj = new vector<pair<int, pair<ll, ll> > >[n];-----// bb
--}-----// bd
--void add_edge(int u, int v, ll cost, ll cap) {-----// 79
--adj[u].push_back(make_pair(v, make_pair(cap, cost)));-----// c8
--}-----// ed
--pair<ll,ll> min_cost_max_flow(int s, int t) {-----// ea
--vector<mcmf_edge*>* g = new vector<mcmf_edge*>[n];-----// ce
--for (int i = 0; i < n; i++) {-----// 57
--for (int j = 0; j < size(adj[i]); j++) {-----// 37
--mcmf_edge *cur = new mcmf_edge(i, adj[i][j].first,-----// 21
--adj[i][j].second.first, adj[i][j].second.second),-----// 56
--*rev = new mcmf_edge(adj[i][j].first, i, 0,-----// 48
--adj[i][j].second.second, cur);-----// b1
--cur->rev = rev;-----// ef
--g[i].push_back(cur);-----// 1d
--g[adj[i][j].first].push_back(rev);-----// 05
--}-----// ba
--}-----// 83
--ll flow = 0, cost = 0;-----// 68
--mcmf_edge** back = new mcmf_edge*[n];-----// e5
--ll* dist = new ll[n];-----// 50
--while (true) {-----// 65
--for (int i = 0; i < n; i++) back[i] = NULL, dist[i] = INF;-----// d0
--dist[s] = 0;-----// 5e
--for (int i = 0; i < n - 1; i++)-----// be
--for (int j = 0; j < n; j++)-----// 6e
--if (dist[j] != INF)-----// e3
--for (int k = 0; k < size(g[j]); k++)-----// 85
--if (g[j][k]->w > 0 && dist[j] + g[j][k]->c <-----// 7f
--dist[g[j][k]->v)) {-----// 6d
--dist[g[j][k]->v] = dist[j] + g[j][k]->c;-----// cf
--back[g[j][k]->v] = g[j][k];-----// 3d
--}-----// f3
--mcmf_edge* cure = back[t];-----// b4
--if (cure == NULL) break;-----// ab
--ll cap = INF;-----// 7a
--while (true) {-----// ad
--cap = min(cap, cure->w);-----// c3
--if (cure->u == s) break;-----// 82
--cure = back[cure->u];-----// 45
--}-----// 91
--assert(cap > 0 && cap < INF);-----// ae
--cure = back[t];-----// b9
--while (true) {-----// 2a
--cost += cap * cure->c;-----// f8
--cure->w -= cap;-----// d1
--cure->rev->w += cap;-----// cf
--if (cure->u == s) break;-----// 8c
--cure = back[cure->u];-----// 60
--}-----// 09
--flow += cap;-----// f2
--}-----// be
--// instead of deleting g, we could also-----// e0
--// use it to get info about the actual flow-----// 6c
--for (int i = 0; i < n; i++)-----// eb
--for (int j = 0; j < size(g[i]); j++)-----// 82
--delete g[i][j];-----// 06
--delete[] g;-----// 23
--delete[] back;-----// 5a
--delete[] dist;-----// b9
--return make_pair(flow, cost);-----// ec
--}-----// ad
};-----// bf

3.8. All Pairs Maximum Flow.

3.8.1. Gomory-Hu Tree. An implementation of the Gomory-Hu Tree. The spanning tree is constructed
using Gusfield's algorithm in  $O(|V|^2)$  plus  $|V| - 1$  times the time it takes to calculate the maximum
flow. If Dinic's algorithm is used to calculate the max flow, the running time is  $O(|V|^3|E|)$ . NOTE:
Not sure if it works correctly with disconnected graphs.

#include "dinic.cpp"-----// 58
-----// 25
bool same[MAXV];-----// 59
pair<vii, vvi> construct_gh_tree(flow_network &g) {-----// 77
--int n = g.n, v;-----// 5d
--vii par(n, ii(0, 0)); vvi cap(n, vi(n, -1));-----// 49
--rep(s,1,n) {-----// 9e
--int l = 0, r = 0;-----// 08
--par[s].second = g.max_flow(s, par[s].first, false);-----// 54
--memset(d, 0, n * sizeof(int));-----// c8
--memset(same, 0, n * sizeof(bool));-----// c9
```

```

-----if (best != -1) part(best);-----// c4
-----rep(i,0,size(adj[u]))-----// 92
-----if (adj[u][i] != parent[u] && adj[u][i] != best)-----// e8
-----part(curhead = adj[u][i]); }-----// 88
void build(int r = 0) { curloc = 0, csz(curhead = r), part(r); }-----// 78
int lca(int u, int v) {-----// 74
    vi uat, vat; int res = -1;-----// 43
    while (u != -1) uat.push_back(u), u = parent[head[u]];-----// 51
    while (v != -1) vat.push_back(v), v = parent[head[v]];-----// 6d
    u = size(uat) - 1, v = size(vat) - 1;-----// 8a
    while (u >= 0 && v >= 0 && head[uat[u]] == head[vat[v]])-----// ae
        res = (loc[uat[u]] < loc[vat[v]] ? uat[u] : vat[v]), u--, v--;-----// a2
    return res; }-----// 91
int query_upto(int u, int v) { int res = ID;-----// 72
    while (head[u] != head[v])-----// 69
        res = f(res, values.query(loc[head[u]], loc[u])),-----// a4
        u = parent[head[u]];-----// 8c
    return f(res, values.query(loc[v] + 1, loc[u])); }-----// ea
int query(int u, int v) { int l = lca(u, v);-----// 53
    return f(query_upto(u, l), query_upto(v, l)); } }-----// 5b

```

3.10. Centroid Decomposition.

```
#define MAXV 100100 // 86
#define LGMAXV 20 // aa
int jmp[MAXV][LGMAXV], // 6d
    path[MAXV][LGMAXV], // 9d
    sz[MAXV], seph[MAXV], // cf
    shortest[MAXV]; // 6b
struct centroid_decomposition { // 99
    int n; vvi adj; // e9
    centroid_decomposition(int _n) : n(_n), adj(n) {} // 46
    void add_edge(int a, int b) { adj[a].push_back(b), adj[b].push_back(a); } // bc
    int dfs(int u, int p) { // 8f
        sz[u] = 1; // c8
        rep(i, 0, size(adj[u])) if (adj[u][i] != p) sz[u] += dfs(adj[u][i], u); // 78
        return sz[u]; } // f4
    void makepaths(int sep, int u, int p, int len) { // 84
        jmp[u][seph[sep]] = sep, path[u][seph[sep]] = len; // d9
        int bad = -1; // af
        rep(i, 0, size(adj[u])) { // f4
            if (adj[u][i] == p) bad = i; // cf
            else makepaths(sep, adj[u][i], u, len + 1); // f2
        } // 8a
        if (p == sep) swap(adj[u][bad], adj[u].back(), adj[u].pop_back()); // 07
    }
    void separate(int h=0, int u=0) { // 03
        dfs(u, -1); int sep = u; // b5
        down: iter(nxt, adj[sep]) // 04
            if (sz[*nxt] < sz[sep] && sz[*nxt] > sz[u]/2) { // db
                sep = *nxt; goto down; } // 1a
        seph[sep] = h, makepaths(sep, sep, -1, 0); // ed
        rep(i, 0, size(adj[sep])) separate(h+1, adj[sep][i]); } // 90
```

```
void paint(int u) {-----// bd
    rep(h,0,seph[u]+1)-----// c5
    shortest[jmp[u][h]] = min(shortest[jmp[u][h]], path[u][h]); }-----// 11
int closest(int u) {-----// 91
    int mn = INF/2;-----// fe
    rep(h,0,seph[u]+1) mn = min(mn, path[u][h] + shortest[jmp[u][h]]);-----// 3e
    return mn; } }-----// 13
```

3.11. Least Common Ancestors, Binary Jumping.

```
struct node {-----// 36
    node *p, *jmp[20];-----// 24
    int depth;-----// 10
    node(node *_p = NULL) : p(_p) {-----// 78
        depth = p ? 1 + p->depth : 0;-----// 3b
        memset(jmp, 0, sizeof(jmp));-----// 64
        jmp[0] = p;-----// 64
        for (int i = 1; (1<<i) <= depth; i++)-----// a8
            jmp[i] = jmp[i-1]->jmp[i-1]; } }-----// 3b
node* st[100000];-----// 65
node* lca(node *a, node *b) {-----// 29
    if (!a || !b) return NULL;-----// cd
    if (a->depth < b->depth) swap(a,b);-----// fe
    for (int j = 19; j >= 0; j--)-----// b3
        while (a->depth - (1<<j) >= b->depth) a = a->jmp[j];-----// c0
    if (a == b) return a;-----// 08
    for (int j = 19; j >= 0; j--)-----// 11
        while (a->depth >= (1<<j) && a->jmp[j] != b->jmp[j])-----// f0
            a = a->jmp[j], b = b->jmp[j];-----// d0
    return a->p; }-----// c5
```

3.12. Tarjan’s Off-line Lowest Common Ancestors Algorithm.

```
#include "../data-structures/union_find.cpp"-----// 5e
struct tarjan_olca {-----// 87
    int *ancestor;-----// 39
    vi *adj, answers;-----// dd
    vii *queries;-----// 66
    bool *colored;-----// 97
    union_find uf;-----// 70
    tarjan_olca(int n, vi *_adj) : adj(_adj), uf(n) {-----// 78
        colored = new bool[n];-----// 8d
        ancestor = new int[n];-----// f2
        queries = new vii[n];-----// 3e
        memset(colored, 0, n);-----// 6e
    }-----// 6b
    void query(int x, int y) {-----// d3
        queries[x].push_back(ii(y, size(answers)));-----// a0
        queries[y].push_back(ii(x, size(answers)));-----// 14
        answers.push_back(-1);-----// ca
    }-----// 6b
    void process(int u) {-----// 85
        ancestor[u] = u;-----// 1a
        rep(i,0,size(adj[u])) {-----// ce
```

```
int v = adj[u][i];-----// dd
        process(v);-----// e8
        uf.unite(u,v);-----// 55
        ancestor[uf.find(u)] = u;-----// 1d
    }-----// 57
    colored[u] = true;-----// b9
    rep(i,0,size(queries[u])) {-----// d7
        int v = queries[u][i].first;-----// 89
        if (colored[v]) {-----// cb
            answers[queries[u][i].second] = ancestor[uf.find(v)];-----// 63
        }-----// d0
    }-----// 40
}-----// a9
};-----// 1e
```

3.13. Maximum Density Subgraph. Given (weighted) undirected graph G . Binary search density. If g is current density, construct flow network: (S, u, m) , $(u, T, m + 2g - d_u)$, $(u, v, 1)$, where m is a large constant (larger than sum of edge weights). Run floating-point max-flow. If minimum cut has empty S -component, then maximum density is smaller than g , otherwise it’s larger. Distance between valid densities is at least $1/(n(n - 1))$. Edge case when density is 0. This also works for weighted graphs by replacing d_u be the weighted degree, and doing more iterations (if weights are not integers).

3.14. Maximum Weighted Independent Set in a Bipartite Graph. This is the same as the minimum weighted vertex cover. Solve this by constructing a flow network with edges $(S, u, w(u))$ for $u \in L$, $(v, T, w(v))$ for $v \in R$ and (u, v, ∞) for $(u, v) \in E$. The minimum S, T -cut is the answer. Vertices adjacent to a cut edge are in the vertex cover.

4. STRINGS

4.1. The Knuth-Morris-Pratt algorithm.

```
int* compute_pi(const string &t) {-----// a2
    int m = t.size();-----// 8b
    int *pit = new int[m + 1];-----// 8e
    if (0 <= m) pit[0] = 0;-----// 42
    if (1 <= m) pit[1] = 0;-----// 34
    rep(i,2,m+1) {-----// 0f
        for (int j = pit[i - 1]; ; j = pit[j]) {-----// b5
            if (t[j] == t[i - 1]) { pit[i] = j + 1; break; }-----// 21
            if (j == 0) { pit[i] = 0; break; }-----// 95
        }-----// c9
    }-----// eb
    return pit; }-----// e8
int string_match(const string &s, const string &t) {-----// 9e
    int n = s.size(), m = t.size();-----// 92
    int *pit = compute_pi(t);-----// 72
    for (int i = 0, j = 0; i < n; ) {-----// 27
        if (s[i] == t[j]) {-----// 73
            i++; j++;-----// 7e
            if (j == m) {-----// de
                return i - m;-----// e9
            }-----// ce
            // or j = pit[j];-----// 85
        }-----// 35
```

```
-----else if (j > 0) j = pit[j];-----// 43
-----else i++; }-----// b8
----delete[] pit; return -1; }-----// e3
```

4.2. The Z algorithm.

```
int* z_values(const string &s) {-----// 4d
----int n = size(s);-----// 97
----int* z = new int[n];-----// c4
----int l = 0, r = 0;-----// 1c
----z[0] = n;-----// 98
----rep(i,1,n) {-----// b2
-----z[i] = 0;-----// 4c
-----if (i > r) {-----// 6d
-----l = r = i;-----// 24
-----while (r < n && s[r - l] == s[r]) r++;-----// 68
-----z[i] = r - l; r--;-----// 07
----} else if (z[i - l] < r - i + 1) z[i] = z[i - l];-----// 6f
----else {-----// a8
-----l = i;-----// 55
-----while (r < n && s[r - l] == s[r]) r++;-----// 2c
-----z[i] = r - l; r--; } }-----// 13
----return z;-----// 78
}-----// 16
```

4.3. Suffix Array. An $O(n \log^2 n)$ construction of a Suffix Tree.

```
struct entry { ii nr; int p; };-----// f9
bool operator <(const entry &a, const entry &b) { return a.nr < b.nr; }-----// 77
struct suffix_array {-----// 87
----string s; int n; vvi P; vector<entry> L; vi idx;-----// b6
----suffix_array(string _s) : s(_s), n(size(s)) {-----// a3
-----L = vector<entry>(n), P.push_back(vi(n)), idx = vi(n);-----// 12
-----rep(i,0,n) P[0][i] = s[i];-----// 5c
-----for (int stp = 1, cnt = 1; cnt >> 1 < n; stp++, cnt <= 1) {-----// 86
-----P.push_back(vi(n));-----// 53
-----rep(i,0,n)-----// 6f
-----L[L[i].p = i].nr = ii(P[stp - 1][i],-----// e2
-----i + cnt < n ? P[stp - 1][i + cnt] : -1);-----// 43
-----sort(L.begin(), L.end());-----// 5f
-----rep(i,0,n)-----// a8
-----P[stp][L[i].p] = i > 0 &&-----// 3a
-----L[i].nr == L[i - 1].nr ? P[stp][L[i - 1].p] : i;-----// 55
-----}-----// 8b
-----rep(i,0,n) idx[P[size(P) - 1][i]] = i;-----// 17
----}-----// d9
----int lcp(int x, int y) {-----// 71
----int res = 0;-----// d6
----if (x == y) return n - x;-----// bc
----for (int k = size(P) - 1; k >= 0 && x < n && y < n; k--)-----// fe
----if (P[k][x] == P[k][y]) x += 1 << k, y += 1 << k, res += 1 << k;-----// b7
----return res;-----// bc
----}-----// f1
};-----// f6
```

4.4. Aho-Corasick Algorithm.

```
struct aho_corasick {-----// 78
----struct out_node {-----// 3e
-----string keyword; out_node *next;-----// f0
-----out_node(string k, out_node *n) : keyword(k), next(n) { }-----// 26
----};-----// b9
----struct go_node {-----// 40
----map<char, go_node*> next;-----// 6b
----out_node *out; go_node *fail;-----// 3e
----go_node() { out = NULL; fail = NULL; }-----// 0f
----};-----// c0
----go_node *go;-----// b8
----aho_corasick(vector<string> keywords) {-----// 4b
----go = new go_node();-----// 77
----iter(k, keywords) {-----// f2
----go_node *cur = go;-----// a2
----iter(c, *k)-----// 6e
----cur = cur->next.find(*c) != cur->next.end() ? cur->next[*c] :-----// 97
----(cur->next[*c] = new go_node());-----// af
----cur->out = new out_node(*k, cur->out);-----// 3f
----}-----// eb
----queue<go_node*> q;-----// 2c
----iter(a, go->next) q.push(a->second);-----// db
----while (!q.empty()) {-----// 07
----go_node *r = q.front(); q.pop();-----// e0
----iter(a, r->next) {-----// 18
----go_node *s = a->second;-----// 55
----q.push(s);-----// b5
----go_node *st = r->fail;-----// 53
----while (st && st->next.find(a->first) == st->next.end())-----// 0e
----st = st->fail;-----// b3
----if (!st) st = go;-----// 0b
----s->fail = st->next[a->first];-----// c1
----if (s->fail) {-----// 98
----if (!s->out) s->out = s->fail->out;-----// ad
----else {-----// 5b
----out_node* out = s->out;-----// b8
----while (out->next) out = out->next;-----// b4
----out->next = s->fail->out;-----// 62
----}-----// a6
----}-----// 81
----}-----// 55
----}-----// bf
----}-----// de
----vector<string> search(string s) {-----// c4
----vector<string> res;-----// 79
----go_node *cur = go;-----// 85
----iter(c, s) {-----// 57
----while (cur && cur->next.find(*c) == cur->next.end())-----// df
----cur = cur->fail;-----// b1
----if (!cur) cur = go;-----// 92
```

```
-----cur = cur->next[*c];-----// 97
-----if (!cur) cur = go;-----// 01
-----for (out_node *out = cur->out; out; out = out->next)-----// d7
-----res.push_back(out->keyword);-----// 7c
-----}-----// 56
-----return res;-----// 6b
}-----// 3e
};-----// de
```

4.5. eerTree.

```
#define MAXN 100100-----// 29
#define SIGMA 26-----// e2
#define BASE 'a'-----// a1
char *s = new char[MAXN];-----// db
struct state {-----// 33
    int len, link, to[SIGMA];-----// 24
} *st = new state[MAXN+2];-----// 57
struct eertree {-----// 78
    int last, sz, n;-----// ba
    eertree() : last(1), sz(2), n(0) {-----// 83
        st[0].len = st[0].link = -1;-----// 3f
        st[1].len = st[1].link = 0; }-----// 34
    int extend() {-----// c2
        char c = s[n++]; int p = last;-----// 25
        while (n - st[p].len - 2 < 0 || c != s[n - st[p].len - 2]) p = st[p].link;
        if (!st[p].to[c-BASE]) {-----// 82
            int q = last = sz++;-----// 42
            st[p].to[c-BASE] = q;-----// fc
            st[q].len = st[p].len + 2;-----// c5
            do { p = st[p].link;-----// 04
            } while (p != -1 && (n < st[p].len + 2 || c != s[n - st[p].len - 2]));
            if (p == -1) st[q].link = 1;-----// 77
            else st[q].link = st[p].to[c-BASE];-----// 6a
            return 1; }-----// 29
        last = st[p].to[c-BASE];-----// 42
        return 0; } }-----// ec
```

4.6. Suffix Automaton. Minimum automata that accepts all suffixes of a string with $O(n)$ construction. The automata itself is a DAG therefore suitable for DP, examples are counting unique substrings, occurrences of substrings and suffix.

```
// TODO: Add longest common subsring-----// 0e
const int MAXL = 100000;-----// 31
struct suffix_automaton {-----// e0
    vi len, link, occur, cnt;-----// 78
    vector<map<char,int>> next;-----// 90
    vector<bool> isclone;-----// 7b
    ll *occuratleast;-----// f2
    int sz, last;-----// 7d
    string s;-----// f2
    suffix_automaton() : len(MAXL*2), link(MAXL*2), occur(MAXL*2), next(MAXL*2),
    isclone(MAXL*2) { clear(); }-----// a3
    void clear(){ sz = 1; last = len[0] = 0; link[0] = -1; next[0].clear();-----// aa
```

```
isclone[0] = false; }-----// 26
bool issubstr(string other){-----// 3b
    for(int i = 0, cur = 0; i < size(other); ++i){-----// 7f
        if(cur == -1) return false; cur = next[cur][other[i]]; }-----// 54
    return true; }-----// 1a
void extend(char c){ int cur = sz++; len[cur] = len[last] + 1;-----// 1d
    next[cur].clear(); isclone[cur] = false; int p = last;-----// a9
    for(; p != -1 && !next[p].count(c); p = link[p]){ next[p][c] = cur; }-----// 6f
    if(p == -1){ link[cur] = 0; }-----// 18
    else{ int q = next[p][c];-----// 34
        if(len[p] + 1 == len[q]){ link[cur] = q; }-----// 4d
        else { int clone = sz++; isclone[clone] = true;-----// 57
            len[clone] = len[p] + 1;-----// 8c
            link[clone] = link[q]; next[clone] = next[q];-----// 76
            for(; p != -1 && next[p].count(c) && next[p][c] == q; p = link[p]){
                next[p][c] = clone; }-----// 32
            link[q] = link[cur] = clone;-----// 73
        } } last = cur; }-----// b9
    void count(){-----// e7
        cnt=vi(sz, -1); stack<ii> S; S.push(ii(0,0));map<char,int>::iterator i;-----// 56
        while(!S.empty()){-----// 4c
            ii cur = S.top(); S.pop();-----// 67
            if(cur.second){-----// 78
                for(i = next[cur.first].begin();i != next[cur.first].end();++i){
                    cnt[cur.first] += cnt[(*i).second]; } }-----// da
                else if(cnt[cur.first] == -1){-----// 99
                    cnt[cur.first] = 1; S.push(ii(cur.first, 1));-----// bd
                    for(i = next[cur.first].begin();i != next[cur.first].end();++i){
                        S.push(ii((*i).second, 0)); } } } }-----// 61
string lexicok(ll k){-----// 8b
    int st = 0; string res; map<char,int>::iterator i;-----// cf
    while(k){ for(i = next[st].begin(); i != next[st].end(); ++i){-----// 69
        if(k <= cnt[(*i).second]){ st = (*i).second;-----// ec
            res.push_back((*i).first); k--; break;-----// 63
        } else { k -= cnt[(*i).second]; } } }-----// ee
    return res; }-----// 0b
    void countoccur(){-----// ad
        for(int i = 0; i < sz; ++i){ occur[i] = 1 - isclone[i]; }-----// 1b
        vii states(sz);-----// dc
        for(int i = 0; i < sz; ++i){ states[i] = ii(len[i],i); }-----// 97
        sort(states.begin(), states.end());-----// 8d
        for(int i = size(states)-1; i >= 0; --i){ int v = states[i].second;-----// a4
            if(link[v] != -1) { occur[link[v]] += occur[v]; } } }-----// cc
};-----// 32
-----// 56
```

4.7. Hashing. Modulus should be a large prime. Can also use multiple instances with different moduli to minimize chance of collision.

```
struct hasher { int b = 311, m; vi h, p;-----// 61
hasher(string s, int _m) : m(_m), h(size(s)+1), p(size(s)+1) {-----// f6
    p[0] = 1; h[0] = 0;-----// d3
```

```
-----rep(i,0,size(s)) p[i+1] = ((ll)p[i] * b % m;-----// 8a
-----rep(i,0,size(s)) h[i+1] = ((ll)h[i] * b + s[i]) % m; }-----// 10
----int hash(int l, int r) {-----// b2
-----return (h[r+1] + m - (ll)h[l] * p[r-l+1] % m) % m; } }-----// 26
```

5. MATHEMATICS

5.1. **Binomial Coefficients.** The binomial coefficient $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ is the number of ways to choose k items out of a total of n items. Also contains an implementation of Lucas' theorem for computing the answer modulo a prime p .

```
int nck(int n, int k) {-----// f6
---if (n < k) return 0;-----// 55
---k = min(k, n - k);-----// bd
---int res = 1;-----// e6
---rep(i,1,k+1) res = res * (n - (k - i)) / i;-----// 4d
---return res;-----// 1f
}-----// 6c
int nck(int n, int k, int p) {-----// cf
---int res = 1;-----// 5c
---while (n || k) {-----// e2
---res *= nck(n % p, k % p);-----// cc
---res %= p, n /= p, k /= p;-----// 0a
---}-----// d9
---return res;-----// 30
}-----// 0a
```

5.2. Euclidean algorithm.

```
int gcd(int a, int b) { return b == 0 ? a : gcd(b, a % b); }-----// d9
```

The extended Euclidean algorithm computes the greatest common divisor d of two integers a, b and also finds two integers x, y such that $a \times x + b \times y = d$.

```
int egcd(int a, int b, int& x, int& y) {-----// 85
---if (b == 0) { x = 1; y = 0; return a; }-----// 7b
---else {-----// 00
---int d = egcd(b, a % b, x, y);-----// 34
---x -= a / b * y;-----// 4a
---swap(x, y);-----// 26
---return d;-----// db
---}-----// 9e
}-----// 40
```

5.3. Trial Division Primality Testing.

```
bool is_prime(int n) {-----// 6c
---if (n < 2) return false;-----// c9
---if (n < 4) return true;-----// d9
---if (n % 2 == 0 || n % 3 == 0) return false;-----// 0f
---if (n < 25) return true;-----// ef
---int s = static_cast<int>(sqrt(static_cast<double>(n)));-----// 64
---for (int i = 5; i <= s; i += 6)-----// 6c
-----if (n % i == 0 || n % (i + 2) == 0) return false;-----// e9
---return true; }-----// 43
```

5.4. Miller-Rabin Primality Test.

```
#include "mod_pow.cpp"-----// c7
bool is_probable_prime(ll n, int k) {-----// be
---if (~n & 1) return n == 2;-----// d1
---if (n <= 3) return n == 3;-----// 39
---int s = 0; ll d = n - 1;-----// 37
---while (~d & 1) d >= 1, s++;-----// 35
---while (k--) {-----// c8
-----ll a = (n - 3) * rand() / RAND_MAX + 2;-----// 06
-----ll x = mod_pow(a, d, n);-----// 64
-----if (x == 1 || x == n - 1) continue;-----// 9b
-----bool ok = false;-----// 03
-----rep(i,0,s-1) {-----// 13
-----x = (x * x) % n;-----// 90
-----if (x == 1) return false;-----// 5c
-----if (x == n - 1) { ok = true; break; }-----// a1
-----}-----// 3a
-----if (!ok) return false;-----// 37
---} return true; }-----// fe
```

5.5. Pollard's ρ algorithm.

```
// public static int[] seeds = new int[] {2,3,5,7,11,13,1031};-----// 1d
// public static BigInteger rho(BigInteger n, BigInteger seed) {-----// 03
//---- int i = 0,-----// 00
//----- k = 2;-----// 79
//---- BigInteger x = seed,-----// cc
//-----y = seed;-----// 31
//---- while (i < 1000000) {-----// 10
//----- i++;-----// 8c
//----- x = (x.multiply(x).add(n).subtract(BigInteger.ONE)).mod(n);-----// 74
//----- BigInteger d = y.subtract(x).abs().gcd(n);-----// ce
//----- if (!d.equals(BigInteger.ONE) && !d.equals(n)) {-----// b9
//----- return d;-----// 3b
//----- }-----// 7c
//----- if (i == k) {-----// 2c
//----- y = x;-----// 89
//----- k = k*2;-----// 1d
//----- }-----// 10
//---- }-----// 96
//---- return BigInteger.ONE;-----// 62
// }-----// d7
```

5.6. Sieve of Eratosthenes.

```
vi prime_sieve(int n) {-----// 40
---int mx = (n - 3) >> 1, sq, v, i = -1;-----// 27
---vi primes;-----// 8f
---bool* prime = new bool[mx + 1];-----// ef
---memset(prime, 1, mx + 1);-----// 28
---if (n >= 2) primes.push_back(2);-----// f4
---while (++i <= mx) if (prime[i]) {-----// 73
---primes.push_back(v = (i << 1) + 3);-----// be
---if ((sq = i * ((i << 1) + 6) + 3) > mx) break;-----// 2d
```



```
-----for (int j = sq; j <= mx; j += v) prime[j] = false; }-----// 2e
---while (++i <= mx) if (prime[i]) primes.push_back((i < 1) + 3);-----// 29
---delete[] prime; // can be used for O(1) lookup-----// 36
---return primes; }-----// 72
```

5.7. Divisor Sieve.

```
vi divisor_sieve(int n) {-----// 7f
---vi minimalDiv(n+1, 2), primes;-----// 37
---if(n>=2) primes.push_back(2);-----// 27
---minimalDiv[0] = 0;-----// 02
---for(int k=1;k<=n;k+=2) minimalDiv[k] = k;-----// e6
---for(int k=3;k<=n;k+=2) {-----// 5d
-----if(minimalDiv[k] == k) primes.push_back(k);-----// 75
-----rep(i, 1, size(primes))-----// 49
-----if(primes[i] > minimalDiv[k] || primes[i]*k > n) break;-----// 53
-----else minimalDiv[primes[i]*k] = primes[i];-----// 90
---}-----// 9d
---return primes; }-----// 93
-----// a8
```

5.8. Modular Multiplicative Inverse.

```
#include "egcd.cpp"-----// 55
-----// e8
int mod_inv(int a, int m) {-----// 49
---int x, y, d = egcd(a, m, x, y);-----// 3e
---if (d != 1) return -1;-----// 20
---return x < 0 ? x + m : x;-----// 3c
}-----// 69
```

5.9. Primitive Root.

```
#include "mod_pow.cpp"-----// c7
ll primitive_root(ll m) {-----// 8a
---vector<ll> div;-----// f2
---for (ll i = 1; i*i <= m-1; i++) {-----// ca
-----if ((m-1) % i == 0) {-----// 85
-----if (i < m) div.push_back(i);-----// fd
-----if (m/i < m) div.push_back(m/i); } }-----// f2
---rep(x,2,m) {-----// 57
---bool ok = true;-----// 17
---iter(it,div) if (mod_pow(x, *it, m) == 1) { ok = false; break; }-----// a4
---if (ok) return x; }-----// 55
---return -1; }-----// 15
```

5.10. Chinese Remainder Theorem.

```
#include "egcd.cpp"-----// 55
int crt(const vi& as, const vi& ns) {-----// c3
---int cnt = size(as), N = 1, x = 0, r, s, l;-----// 55
---rep(i,0,cnt) N *= ns[i];-----// b1
---rep(i,0,cnt) egcd(ns[i], l = N/ns[i], r, s), x += as[i] * s * l;-----// 21
---return mod(x, N); }-----// b2
```

5.11. Linear Congruence Solver. A function that returns all solutions to $ax \equiv b \pmod n$, modulo n .

```
#include "egcd.cpp"-----// 55
vi linear_congruence(int a, int b, int n) {-----// c8
---int x, y, d = egcd(a, n, x, y);-----// 7a
---vi res;-----// f5
---if (b % d != 0) return res;-----// 30
---int x0 = mod(b / d * x, n);-----// 48
---rep(k,0,d) res.push_back(mod(x0 + k * n / d, n));-----// 7e
---return res;-----// fe
}-----// c0
```

5.12. Numeric Integration.

```
double integrate(double (*f)(double), double a, double b,-----// 76
---double delta = 1e-6) {-----// c0
---if (abs(a - b) < delta)-----// 38
---return (b-a)/8 *-----// 56
---((f(a) + 3*f((2*a+b)/3) + 3*f((a+2*b)/3) + f(b));-----// e1
---return integrate(f, a,-----// 64
---(a+b)/2, delta) + integrate(f, (a+b)/2, b, delta);-----// 0c
}-----// 4b
```

5.13. Fast Fourier Transform. The Cooley-Tukey algorithm for quickly computing the discrete Fourier transform. The fft function only supports powers of twos. The czt function implements the Chirp Z-transform and supports any size, but is slightly slower.

```
#include <complex>-----// 8e
typedef complex<long double> cpx;-----// 25
// NOTE: n must be a power of two-----// 14
void fft(cpx *x, int n, bool inv=false) {-----// 36
---for (int i = 0, j = 0; i < n; i++) {-----// f9
---if (i < j) swap(x[i], x[j]);-----// 44
---int m = n>>1;-----// 9c
---while (1 <= m && m <= j) j -= m, m >>= 1;-----// fe
---j += m;-----// 11
---}-----// d0
---for (int mx = 1; mx < n; mx <= 1) {-----// 15
---cpx wp = exp(cpx(0, (inv ? -1 : 1) * pi / mx)), w = 1;-----// 79
---for (int m = 0; m < mx; m++, w *= wp) {-----// dc
---for (int i = m; i < n; i += mx << 1) {-----// 6a
---cpx t = x[i + mx] * w;-----// 12
---x[i + mx] = x[i] - t;-----// 73
---x[i] += t;-----// 0e
---}-----// 14
---}-----// a4
---}-----// bf
---if (inv) rep(i,0,n) x[i] /= cpx(n);-----// 16
}-----// 1c
void czt(cpx *x, int n, bool inv=false) {-----// c5
---int len = 2*n+1;-----// bc
---while (len & (len - 1)) len &= len - 1;-----// 65
---len <= 1;-----// 21
---cpx w = exp(-2.0L * pi / n * cpx(0,1)),-----// 45
```

```
-----*c = new cpx[n], *a = new cpx[len],-----// 4e
-----*b = new cpx[len];-----// 30
---rep(i,0,n) c[i] = pow(w, (inv ? -1.0 : 1.0)*i/2);-----// 9e
---rep(i,0,n) a[i] = x[i] * c[i], b[i] = 1.0L/c[i];-----// e9
---rep(i,0,n-1) b[len - n + i + 1] = 1.0L/c[n-i-1];-----// 9f
---fft(a, len); fft(b, len);-----// 63
---rep(i,0,len) a[i] *= b[i];-----// 58
---fft(a, len, true);-----// 2d
---rep(i,0,n) {-----// ff
-----x[i] = c[i] * a[i];-----// 77
-----if (inv) x[i] /= cpx(n);-----// b1
-----}-----// 27
---delete[] a;-----// 0a
---delete[] b;-----// 5c
---delete[] c;-----// f8
}-----// c6
```

5.14. Number-Theoretic Transform.

```
#include "../mathematics/primitive_root.cpp"-----// 8c
int mod = 998244353, g = primitive_root(mod),-----// 9c
---ginv = mod_pow(g, mod-2, mod), inv2 = mod_pow(2, mod-2, mod);-----// 75
#define MAXN (1<<22)-----// 94
struct Num {-----// c5
---int x;-----// 02
---Num(ll _x=0) { x = (_x%mod+mod)%mod; }-----// 1b
---Num operator +(const Num &b) { return x + b.x; }-----// 08
---Num operator -(const Num &b) const { return x - b.x; }-----// 89
---Num operator *(const Num &b) const { return (ll)x * b.x; }-----// e3
---Num operator /(const Num &b) const { return (ll)x * b.inv().x; }-----// 2a
---Num inv() const { return mod_pow((ll)x, mod-2, mod); }-----// d3
---Num pow(int p) const { return mod_pow((ll)x, p, mod); }-----// d5
} T1[MAXN], T2[MAXN];-----// d5
void ntt(Num x[], int n, bool inv = false) {-----// 24
---Num z = inv ? ginv : g;-----// 00
---z = z.pow((mod - 1) / n);-----// 45
---for (ll i = 0, j = 0; i < n; i++) {-----// fe
---if (i < j) swap(x[i], x[j]);-----// eb
---ll k = n>>1;-----// 02
---while (1 <= k && k <= j) j -= k, k >= 1;-----// d2
---j += k; }-----// c4
---for (int mx = 1, p = n/2; mx < n; mx <= 1, p >= 1) {-----// a2
---Num wp = z.pow(p), w = 1;-----// 35
---for (int k = 0; k < mx; k++, w = w*wp) {-----// 42
---for (int i = k; i < n; i += mx << 1) {-----// 9a
---Num t = x[i + mx] * w;-----// 22
---x[i + mx] = x[i] - t;-----// d0
---x[i] = x[i] + t; } } }-----// 1e
---if (inv) {-----// 6c
---Num ni = Num(n).inv();-----// 18
---rep(i,0,n) { x[i] = x[i] * ni; } } }-----// 72
void inv(Num x[], Num y[], int l) {-----// ee
```

```
---if (l == 1) { y[0] = x[0].inv(); return; }-----// 71
---inv(x, y, l>>1);-----// bd
---// NOTE: maybe l<<2 instead of l<<1-----// 24
---rep(i,l>>1,l<<1) T1[i] = y[i] = 0;-----// dd
---rep(i,0,l) T1[i] = x[i];-----// f7
---ntt(T1, l<<1); ntt(y, l<<1);-----// a5
---rep(i,0,l<<1) y[i] = y[i]*2 - T1[i] * y[i] * y[i];-----// bc
---ntt(y, l<<1, true); }-----// 40
void sqrt(Num x[], Num y[], int l) {-----// 44
---if (l == 1) { assert(x[0].x == 1); y[0] = 1; return; }-----// 64
---sqrt(x, y, l>>1);-----// a7
---inv(y, T2, l>>1);-----// 3b
---rep(i,l>>1,l<<1) T1[i] = T2[i] = 0;-----// 1c
---rep(i,0,l) T1[i] = x[i];-----// 08
---ntt(T2, l<<1); ntt(T1, l<<1);-----// 7e
---rep(i,0,l<<1) T2[i] = T1[i] * T2[i];-----// 9f
---ntt(T2, l<<1, true);-----// eb
---rep(i,0,l) y[i] = (y[i] + T2[i]) * inv2; }-----// b8
```

5.15. **Tridiagonal Matrix Algorithm.** Solves a tridiagonal system of linear equations $a_i x_{i-1} + b_i x_i + c_i x_{i+1} = d_i$ where $a_1 = c_n = 0$. Beware of numerical instability.

```
#define MAXN 5000-----// f7
long double A[MAXN], B[MAXN], C[MAXN], D[MAXN], X[MAXN];-----// d8
void solve(int n) {-----// 01
---C[0] /= B[0]; D[0] /= B[0];-----// 94
---rep(i,1,n-1) C[i] /= B[i] - A[i]*C[i-1];-----// 6b
---rep(i,1,n) D[i] = (D[i] - A[i] * D[i-1]) / (B[i] - A[i] * C[i-1]);-----// 33
---X[n-1] = D[n-1];-----// c7
---for (int i = n-2; i>=0; i--) X[i] = D[i] - C[i] * X[i+1]; }-----// ad
```

5.16. **Mertens Function.** Mertens function is $M(n) = \sum_{i=1}^n \mu(i)$. Let $L \approx (n \log \log n)^{2/3}$ and the algorithm runs in $O(n^{2/3})$. Can also be easily changed to compute the summatory Φ .

```
#define L 9000000-----// 27
int mob[L], mer[L];-----// f1
unordered_map<ll,ll> mem;-----// 30
ll M(ll n) {-----// de
---if (n < L) return mer[n];-----// 1c
---if (mem.find(n) != mem.end()) return mem[n];-----// 79
---ll ans = 0, done = 1;-----// 48
---for (ll i = 2; i*i <= n; i++) ans += M(n/i), done = i;-----// 41
---for (ll i = 1; i*i <= n; i++) ans += mer[i] * (n/i - max(done, n/(i+1)));-----// 43
---return mer[n] = 1 - ans; }-----// c2
void sieve() {-----// b9
---for (int i = 1; i < L; i++) mer[i] = mob[i] = 1;-----// f7
---for (int i = 2; i < L; i++) {-----// 8e
---if (mer[i]) {-----// 8b
---mob[i] = -1;-----// e5
---for (int j = i+i; j < L; j += i)-----// f0
---mer[j] = 0, mob[j] = (j/i)%i == 0 ? 0 : -mob[j/i];-----// 26
---}-----// aa
---mer[i] = mob[i] + mer[i-1]; } }-----// 3b
```

5.17. **Numbers and Sequences.** Some random prime numbers: 1031, 32771, 1048583, 33554467, 1073741827, 34359738421, 1099511627791, 35184372088891, 1125899906842679, 36028797018963971.

6. GEOMETRY

6.1. Primitives.

```
#define P(p) const point &p-----// 2e
#define L(p0, p1) P(p0), P(p1)-----// cf
#define C(p0, r) P(p0), double r-----// f1
#define PP(pp) pair<point,point> &pp-----// e5
typedef complex<double> point;-----// 6a
double dot(P(a), P(b)) { return real(conj(a) * b); }-----// d2
double cross(P(a), P(b)) { return imag(conj(a) * b); }-----// 8a
point rotate(P(p), double radians = pi / 2, P(about) = point(0,0)) {-----// 23
----return (p - about) * exp(point(0, radians)) + about; }-----// 25
point reflect(P(p), L(about1, about2)) {-----// 50
----point z = p - about1, w = about2 - about1;-----// 8b
----return conj(z / w) * w + about1; }-----// 83
point proj(P(u), P(v)) { return dot(u, v) / dot(u, u) * u; }-----// e7
point normalize(P(p), double k = 1.0) {-----// 5f
----return abs(p) == 0 ? point(0,0) : p / abs(p) * k; }-----// 4a
double ccw(P(a), P(b), P(c)) { return cross(b - a, c - b); }-----// 27
bool collinear(P(a), P(b), P(c)) { return abs(ccw(a, b, c)) < EPS; }-----// b3
double angle(P(a), P(b), P(c)) {-----// 61
----return acos(dot(b - a, c - b) / abs(b - a) / abs(c - b)); }-----// c7
double signed_angle(P(a), P(b), P(c)) {-----// 4a
----return asin(cross(b - a, c - b) / abs(b - a) / abs(c - b)); }-----// 40
double angle(P(p)) { return atan2(imag(p), real(p)); }-----// e6
point perp(P(p)) { return point(-imag(p), real(p)); }-----// d9
double progress(P(p), L(a, b)) {-----// b3
----if (abs(real(a) - real(b)) < EPS)-----// 5e
----return (imag(p) - imag(a)) / (imag(b) - imag(a));-----// 5e
----else return (real(p) - real(a)) / (real(b) - real(a)); }-----// 31
-----// 53
-----// 46
```

6.2. Lines.

```
#include "primitives.cpp"-----// e0
-----// 85
bool collinear(L(a, b), L(p, q)) {-----// 2f
----return abs(ccw(a, b, p)) < EPS && abs(ccw(a, b, q)) < EPS; }-----// 3e
bool parallel(L(a, b), L(p, q)) { return abs(cross(b - a, q - p)) < EPS; }-----// 8d
point closest_point(L(a, b), P(c), bool segment = false) {-----// f2
----if (segment) {-----// f4
-----if (dot(b - a, c - b) > 0) return b;-----// 88
-----if (dot(a - b, c - a) > 0) return a;-----// 75
-----}-----// ce
----double t = dot(c - a, b - a) / norm(b - a);-----// 62
----return a + t * (b - a);-----// 6e
}-----// 8c
double line_segment_distance(L(a,b), L(c,d)) {-----// f3
----double x = INFINITY;-----// 64
```

```
----if (abs(a - b) < EPS && abs(c - d) < EPS) x = abs(a - c);-----// a5
----else if (abs(a - b) < EPS) x = abs(a - closest_point(c, d, a, true));-----// 23
----else if (abs(c - d) < EPS) x = abs(c - closest_point(a, b, c, true));-----// 53
----else if ((ccw(a, b, c) < 0) != (ccw(a, b, d) < 0) &&-----// 6d
----- (ccw(c, d, a) < 0) != (ccw(c, d, b) < 0)) x = 0;-----// bf
----else {-----// e1
-----x = min(x, abs(a - closest_point(c,d, a, true)));-----// 29
-----x = min(x, abs(b - closest_point(c,d, b, true)));-----// fe
-----x = min(x, abs(c - closest_point(a,b, c, true)));-----// 81
-----x = min(x, abs(d - closest_point(a,b, d, true)));-----// e4
-----}-----// c5
----return x;-----// b7
}-----// 27
bool intersect(L(a, b), L(p, q), point &res, bool segment = false) {-----// d2
----// NOTE: check for parallel/collinear lines before calling this function-----// 1b
----point r = b - a, s = q - p;-----// 34
----double c = cross(r, s), t = cross(p - a, s) / c, u = cross(p - a, r) / c;-----// 0b
----if (segment && (t < 0-EPS || t > 1+EPS || u < 0-EPS || u > 1+EPS))-----// e4
----return false;-----// e3
----res = a + t * r;-----// 47
----return true;-----// 05
}-----// 44
-----// cc
```

6.3. Circles.

```
#include "primitives.cpp"-----// e0
int intersect(C(A, rA), C(B, rB), point & res1, point & res2) {-----// 3b
----double d = abs(B - A);-----// 5c
----if ((rA + rB) < (d - EPS) || d < abs(rA - rB) - EPS) return 0;-----// 39
----double a = (rA*rA - rB*rB + d*d) / 2 / d, h = sqrt(rA*rA - a*a);-----// 9b
----point v = normalize(B - A, a), u = normalize(rotate(B-A), h);-----// 79
----res1 = A + v + u, res2 = A + v - u;-----// 24
----if (abs(u) < EPS) return 1; return 2;-----// 82
}-----// bb
int intersect(L(A, B), C(0, r), point & res1, point & res2) {-----// 0e
----double h = abs(0 - closest_point(A, B, 0));-----// 24
----if(r < h - EPS) return 0;-----// df
----point H = proj(0 - A, B - A) + A, v = normalize((B - A), sqrt(r*r - h*h));// 19
----res1 = H + v; res2 = H - v;-----// 40
----if(abs(v) < EPS) return 1; return 2;-----// 37
}-----// 46
int tangent(P(A), C(0, r), point & res1, point & res2) {-----// aa
----point v = 0 - A; double d = abs(v);-----// 49
----if (d < r - EPS) return 0;-----// ca
----double alpha = asin(r / d), L = sqrt(d*d - r*r);-----// 3f
----v = normalize(v, L);-----// 3f
----res1 = A + rotate(v, alpha); res2 = A + rotate(v, -alpha);-----// be
----if (abs(r - d) < EPS || abs(v) < EPS) return 1;-----// bb
----return 2;-----// b9
}-----// f4
void tangent_outer(point A, double rA, point B, double rB, PP(P), PP(Q)) {-----// 83
```

```
---if (rA - rB > EPS) { swap(rA, rB); swap(A, B); }-----// 82
---double theta = asin((rB - rA)/abs(A - B));-----// 37
---point v = rotate(B - A, theta + pi/2), u = rotate(B - A, -(theta + pi/2));--// 00
---u = normalize(u, rA);-----// 44
---P.first = A + normalize(v, rA); P.second = B + normalize(v, rB);-----// d2
---Q.first = A + normalize(u, rA); Q.second = B + normalize(u, rB);-----// 23
}-----// b5
```

6.4. Polygon.

```
#include "primitives.cpp"-----// e0
typedef vector<point> polygon;-----// b3
double polygon_area_signed(polygon p) {-----// 31
---double area = 0; int cnt = size(p);-----// a2
---rep(i,1,cnt-1) area += cross(p[i] - p[0], p[i + 1] - p[0]);-----// 51
---return area / 2; }-----// 66
double polygon_area(polygon p) { return abs(polygon_area_signed(p)); }-----// a4
#define CHK(f,a,b,c) (f(a) < f(b) && f(b) <= f(c) && ccw(a,c,b) < 0)-----// 8f
int point_in_polygon(polygon p, point q) {-----// 5d
---int n = size(p); bool in = false; double d;-----// 69
---for (int i = 0, j = n - 1; i < n; j = i++)-----// f3
---if (collinear(p[i], q, p[j]) &&-----// 9d
---0 <= (d = progress(q, p[i], p[j])) && d <= 1)-----// 4b
---return 0;-----// b3
---for (int i = 0, j = n - 1; i < n; j = i++)-----// 67
---if (CHK(real, p[i], q, p[j]) || CHK(real, p[j], q, p[i]))-----// b4
---in = !in;-----// ff
---return in ? -1 : 1; }-----// ba
// pair<polygon, polygon> cut_polygon(const polygon &poly, point a, point b) {--// 0d
//---- polygon left, right;-----// 0a
//---- point it(-100, -100);-----// 5b
//---- for (int i = 0, cnt = poly.size(); i < cnt; i++) {-----// 70
//----- int j = i == cnt-1 ? 0 : i + 1;-----// 02
//----- point p = poly[i], q = poly[j];-----// 44
//----- if (ccw(a, b, p) <= 0) left.push_back(p);-----// 8d
//----- if (ccw(a, b, p) >= 0) right.push_back(p);-----// 43
//----- // myintersect = intersect where-----// ba
//----- // (a,b) is a line, (p,q) is a line segment-----// 7e
//----- if (myintersect(a, b, p, q, it))-----// 6f
//----- left.push_back(it), right.push_back(it);-----// 8a
//----- }-----// e0
//---- return pair<polygon, polygon>(left, right);-----// 3d
// }-----// 07
```

6.5. Convex Hull. NOTE: Doesn't work on some weird edge cases. (A small case that included three collinear lines would return the same point on both the upper and lower hull.)

```
#include "polygon.cpp"-----// 58
#define MAXN 1000-----// 09
point hull[MAXN];-----// 43
bool cmp(const point &a, const point &b) {-----// 32
---return abs(real(a) - real(b)) > EPS ?-----// 44
---real(a) < real(b) : imag(a) < imag(b); }-----// 40
int convex_hull(polygon p) {-----// cd
```

```
---int n = size(p), l = 0;-----// 67
---sort(p.begin(), p.end(), cmp);-----// 3d
---rep(i,0,n) {-----// e4
---if (i > 0 && p[i] == p[i - 1]) continue;-----// c7
---while (l >= 2 && ccw(hull[l - 2], hull[l - 1], p[i]) >= 0) l--;-----// 62
---hull[l++] = p[i];-----// bd
---}-----// d2
---int r = l;-----// 30
---for (int i = n - 2; i >= 0; i--) {-----// 59
---if (p[i] == p[i + 1]) continue;-----// af
---while (r - l >= 1 && ccw(hull[r - 2], hull[r - 1], p[i]) >= 0) r--;-----// 4d
---hull[r++] = p[i];-----// f5
---}-----// f6
---return l == 1 ? 1 : r - 1;-----// a6
}-----// 6d
```

6.6. Line Segment Intersection.

```
#include "primitives.cpp"-----// e0
#include "lines.cpp"-----// 54
bool line_segment_intersect(L(a, b), L(c, d), point &A, point &B) {-----// 34
---if (abs(a - b) < EPS && abs(c - d) < EPS) {-----// 74
---A = B = a; return abs(a - d) < EPS; }-----// 37
---else if (abs(a - b) < EPS) {-----// 07
---A = B = a; double p = progress(a, c,d);-----// 6c
---return 0.0 <= p && p <= 1.0-----// 4c
---&& (abs(a - c) + abs(d - a) - abs(d - c)) < EPS; }-----// d2
---else if (abs(c - d) < EPS) {-----// a0
---A = B = c; double p = progress(c, a,b);-----// c0
---return 0.0 <= p && p <= 1.0-----// fb
---&& (abs(c - a) + abs(b - c) - abs(b - a)) < EPS; }-----// 15
---else if (collinear(a,b, c,d)) {-----// 49
---double ap = progress(a, c,d), bp = progress(b, c,d);-----// 34
---if (ap > bp) swap(ap, bp);-----// e0
---if (bp < 0.0 || ap > 1.0) return false;-----// 14
---A = c + max(ap, 0.0) * (d - c);-----// 14
---B = c + min(bp, 1.0) * (d - c);-----// 26
---return true; }-----// bd
---else if (parallel(a,b, c,d)) return false;-----// fc
---else if (intersect(a,b, c,d, A, true)) {-----// 78
---B = A; return true; }-----// 7d
---return false;-----// 4b
}-----// 05
}-----// 83
```

6.7. Great-Circle Distance. Computes the distance between two points (given as latitude/longitude coordinates) on a sphere of radius *r*.

```
double gc_distance(double pLat, double pLong,-----// 7b
---double qLat, double qLong, double r) {-----// a4
---pLat *= pi / 180; pLong *= pi / 180;-----// ee
---qLat *= pi / 180; qLong *= pi / 180;-----// 75
---return r * acos(cos(pLat) * cos(qLat) * cos(pLong - qLong) +-----// e3
---sin(pLat) * sin(qLat));-----// 1e
```

```
-----// 60
}-----// 3f
```

6.8. **Triangle Circumcenter.** Returns the unique point that is the same distance from all three points. It is also the center of the unique circle that goes through all three points.

```
#include "primitives.cpp"-----// e0
point circumcenter(point a, point b, point c) {-----// 76
---b -= a, c -= a;-----// 41
---return a + perp(b * norm(c) - c * norm(b)) / 2.0 / cross(b, c);-----// 7a
}-----// c3
```

6.9. **Closest Pair of Points.**

```
#include "primitives.cpp"-----// e0
-----// 85
struct cmpx { bool operator()(const point &a, const point &b) {-----// 01
-----return abs(real(a) - real(b)) > EPS ?-----// e9
-----real(a) < real(b) : imag(a) < imag(b); } };-----// 53
struct cmpy { bool operator()(const point &a, const point &b) {-----// 6f
---return abs(imag(a) - imag(b)) > EPS ?-----// 0b
-----imag(a) < imag(b) : real(a) < real(b); } };-----// a4
double closest_pair(vector<point> pts) {-----// f1
---sort(pts.begin(), pts.end(), cmpx());-----// 0c
---set<point, cmpy> cur;-----// bd
---set<point, cmpy>::const_iterator it, jt;-----// a6
---double mn = INFINITY;-----// f9
---for (int i = 0, l = 0; i < size(pts); i++) {-----// ac
---while (real(pts[i]) - real(pts[l]) > mn) cur.erase(pts[l++]);-----// 8b
-----it = cur.lower_bound(point(-INFINITY, imag(pts[i]) - mn));-----// fc
-----jt = cur.upper_bound(point(INFINITY, imag(pts[i]) + mn));-----// 39
-----while (it != jt) mn = min(mn, abs(*it - pts[i])), it++;-----// 09
-----cur.insert(pts[i]); }-----// 82
---return mn; }-----// 4c
```

6.10. **3D Primitives.**

```
#define P(p) const point3d &p-----// a7
#define L(p0, p1) P(p0), P(p1)-----// 0f
#define PL(p0, p1, p2) P(p0), P(p1), P(p2)-----// 67
struct point3d {-----// 63
---double x, y, z;-----// e6
---point3d() : x(0), y(0), z(0) {}-----// af
---point3d(double _x, double _y, double _z) : x(_x), y(_y), z(_z) {}-----// fc
---point3d operator+(P(p)) const {-----// 17
-----return point3d(x + p.x, y + p.y, z + p.z); }-----// 8e
---point3d operator-(P(p)) const {-----// fb
-----return point3d(x - p.x, y - p.y, z - p.z); }-----// 83
---point3d operator-() const {-----// 89
-----return point3d(-x, -y, -z); }-----// d4
---point3d operator*(double k) const {-----// 4d
-----return point3d(x * k, y * k, z * k); }-----// fd
---point3d operator/(double k) const {-----// 95
-----return point3d(x / k, y / k, z / k); }-----// 58
---double operator%(P(p)) const {-----// d1
```

```
-----return x * p.x + y * p.y + z * p.z; }-----// 09
point3d operator*(P(p)) const {-----// 4f
---return point3d(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x); }-----// ed
double length() const {-----// 3e
---return sqrt(*this % *this); }-----// 05
double distTo(P(p)) const {-----// dd
---return (*this - p).length(); }-----// 57
double distTo(P(A), P(B)) const {-----// bd
---// A and B must be two different points-----// 4e
---return ((*this - A) * (*this - B)).length() / A.distTo(B); }-----// 6e
point3d normalize(double k = 1) const {-----// db
---// length() must not return 0-----// 3c
---return (*this) * (k / length()); }-----// d4
point3d getProjection(P(A), P(B)) const {-----// 86
---point3d v = B - A;-----// 64
---return A + v.normalize((v % (*this - A)) / v.length()); }-----// 53
point3d rotate(P(normal)) const {-----// 55
---// normal must have length 1 and be orthogonal to the vector-----// eb
---return (*this) * normal; }-----// 5c
point3d rotate(double alpha, P(normal)) const {-----// 21
---return (*this) * cos(alpha) + rotate(normal) * sin(alpha); }-----// 82
point3d rotatePoint(P(0), P(axe), double alpha) const {-----// 7a
---point3d Z = axe.normalize(axe % (*this - 0));-----// ba
---return 0 + Z + (*this - 0 - Z).rotate(alpha, 0); }-----// 38
bool isZero() const {-----// 64
---return abs(x) < EPS && abs(y) < EPS && abs(z) < EPS; }-----// 15
bool isOnLine(L(A, B)) const {-----// 30
---return ((A - *this) * (B - *this)).isZero(); }-----// 58
bool isInSegment(L(A, B)) const {-----// f1
---return isOnLine(A, B) && ((A - *this) % (B - *this)) < EPS; }-----// d9
bool isInSegmentStrictly(L(A, B)) const {-----// 0e
---return isOnLine(A, B) && ((A - *this) % (B - *this)) < -EPS; }-----// ba
double getAngle() const {-----// 0f
---return atan2(y, x); }-----// 40
double getAngle(P(u)) const {-----// d5
---return atan2((*this * u).length(), *this % u); }-----// 79
bool isOnPlane(PL(A, B, C)) const {-----// 8e
---return abs((A - *this) * (B - *this) % (C - *this)) < EPS; } };-----// 74
int line_line_intersect(L(A, B), L(C, D), point3d &0){-----// dc
---if (abs((B - A) * (C - A) % (D - A)) > EPS) return 0;-----// 6a
---if (((A - B) * (C - D)).length() < EPS)-----// 79
---return A.isOnLine(C, D) ? 2 : 0;-----// 09
point3d normal = ((A - B) * (C - B)).normalize();-----// bc
double s1 = (C - A) * (D - A) % normal;-----// 68
0 = A + ((B - A) / (s1 + ((D - B) * (C - B) % normal))) * s1;-----// 56
return 1; }-----// a7
int line_plane_intersect(L(A, B), PL(C, D, E), point3d & 0) {-----// 09
---double V1 = (C - A) * (D - A) % (E - A);-----// c1
---double V2 = (D - B) * (C - B) % (E - B);-----// 29
---if (abs(V1 + V2) < EPS)-----// 81
---return A.isOnPlane(C, D, E) ? 2 : 0;-----// d5
```



```
---0 = A + ((B - A) / (V1 + V2)) * V1;-----// 38 // if (h > 1) {-----// 26
---return 1; }-----// ce //---- int a = 0,-----// e6
bool plane_plane_intersect(P(A), P(nA), P(B), P(nB), point3d &P, point3d &Q) {--// 5a //----- b = 0;-----// df
---point3d n = nA * nB;-----// 49 //---- rep(i,0,h) {-----// 1d
---if (n.isZero()) return false;-----// 03 //----- if (hull[i].first < hull[a].first)-----// ac
---point3d v = n * nA;-----// d7 //----- a = i;-----// b1
---P = A + (n * nA) * ((B - A) % nB / (v % nB));--// 1a //----- if (hull[i].first > hull[b].first)-----// 02
---Q = P + n;-----// 9c //----- b = i;-----// 84
---return true; }-----// 1a //---- }-----// 1e
//---- caliper A(hull[a], pi/2), B(hull[b], 3*pi/2);-----// 60
//---- double done = 0;-----// 3c
//---- while (true) {-----// 31
//----- mx = max(mx, abs(point(hull[a].first,hull[a].second)-----// e3
//----- - point(hull[b].first, hull[b].second)));-----// 24
//----- double tha = A.angle_to(hull[(a+1)%h]),-----// 57
//----- thb = B.angle_to(hull[(b+1)%h]);-----// f1
//----- if (tha <= thb) {-----// 91
//----- A.rotate(tha);-----// c9
//----- B.rotate(tha);-----// f4
//----- a = (a+1) % h;-----// d4
//----- A.move_to(hull[a]);-----// b3
//----- } else {-----// 56
//----- A.rotate(thb);-----// 56
//----- B.rotate(thb);-----// 38
//----- b = (b+1) % h;-----// 96
//----- B.move_to(hull[b]);-----// 38
//----- }-----// bc
//----- done += min(tha, thb);-----// d2
//----- if (done > pi) {-----// c2
//----- break;-----// e8
//----- }-----// 37
//---- }-----// ac
// }-----// 9c
```

6.11. Polygon Centroid.

```
#include "polygon.cpp"-----// 58
point polygon_centroid(polygon p) {-----// 79
---double cx = 0.0, cy = 0.0;-----// d5
---double mnx = 0.0, mny = 0.0;-----// 22
---int n = size(p);-----// 2d
---rep(i,0,n)-----// 08
//----- mnx = min(mnx, real(p[i])),-----// c6
//----- mny = min(mny, imag(p[i]));-----// 84
---rep(i,0,n)-----// 3f
---p[i] = point(real(p[i]) - mnx, imag(p[i]) - mny);-----// 49
---rep(i,0,n) {-----// 3c
//----- int j = (i + 1) % n;-----// 5b
//----- cx += (real(p[i]) + real(p[j])) * cross(p[i], p[j]);-----// 4f
//----- cy += (imag(p[i]) + imag(p[j])) * cross(p[i], p[j]); }-----// 4a
---return point(cx, cy) / 6.0 / polygon_area_signed(p) + point(mnx, mny); }--// a1
```

6.12. Rotating Calipers.

```
#include "primitives.cpp"-----// e0
struct caliper {-----// 8e
---ii pt;-----// 05
---double angle;-----// d4
---caliper(ii _pt, double _angle) : pt(_pt), angle(_angle) { }-----// 35
---double angle_to(ii pt2) {-----// 8b
//----- double x = angle - atan2(pt2.second - pt.second, pt2.first - pt.first);--// 1e
//----- while (x >= pi) x -= 2*pi;-----// 4a
//----- while (x <= -pi) x += 2*pi;-----// a3
//----- return x; }-----// 7d
---void rotate(double by) {-----// 57
---angle -= by;-----// 5d
---while (angle < 0) angle += 2*pi;-----// 03
---}-----// 20
---void move_to(ii pt2) { pt = pt2; }-----// 37
---double dist(const caliper &other) {-----// 68
---point a(pt.first,pt.second),-----// d7
//----- b = a + exp(point(0,angle)) * 10.0,-----// 2e
//----- c(other.pt.first, other.pt.second);-----// 71
---return abs(c - closest_point(a, b, c));-----// 58
---} };;-----// 4b
//-----// c5
```

```
// int h = convex_hull(pts);-----// 9c
// double mx = 0;-----// f1
```

6.13. Formulas. Let $a = (a_x, a_y)$ and $b = (b_x, b_y)$ be two-dimensional vectors.

- $a \cdot b = |a||b| \cos \theta$, where θ is the angle between a and b .
- $a \times b = |a||b| \sin \theta$, where θ is the signed angle between a and b .
- $a \times b$ is equal to the area of the parallelogram with two of its sides formed by a and b . Half of that is the area of the triangle formed by a and b .
- **Euler’s formula:** $V - E + F = 2$
- Side lengths a, b, c can form a triangle iff. $a + b > c$, $b + c > a$ and $a + c > b$.
- Sum of internal angles of a regular convex n -gon is $(n - 2)\pi$.

7. OTHER ALGORITHMS

7.1. 2SAT.

```
#include "../graph/scc.cpp"-----// c3
-----// 63
bool two_sat(int n, const vii& clauses, vi& all_truthy) {-----// f4
---all_truthy.clear();-----// 31
---vvi adj(2*n+1);-----// 7b
---rep(i,0,size(closures)) {-----// 76
```


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-----adj[-clauses[i].first + n].push_back(clauses[i].second + n);-----// eb	----exact_cover(int _rows, int _cols) : rows(_rows), cols(_cols), head(NULL) {--// b6
-----if (clauses[i].first != clauses[i].second)-----// bc	-----arr = new bool*[rows];-----// cf
-----adj[-clauses[i].second + n].push_back(clauses[i].first + n);-----// f0	-----sol = new int[rows];-----// 5f
---}-----// da	-----rep(i,0,rows)-----// 9b
---pair<union_find, vi> res = scc(adj);-----// 00	-----arr[i] = new bool[cols], memset(arr[i], 0, cols);-----// dd
---union_find scc = res.first;-----// 20	---}-----// 21
---vi dag = res.second;-----// ed	----void set_value(int row, int col, bool val = true) { arr[row][col] = val; }--// 9e
---vi truth(2*n+1, -1);-----// c7	----void setup() {--// a3
---for (int i = 2*n; i >= 0; i--) {-----// 50	-----node ***ptr = new node**[rows + 1];-----// bd
-----int cur = order[i] - n, p = scc.find(cur + n), o = scc.find(-cur + n);--// 4f	-----rep(i,0,rows+1) {-----// 76
-----if (cur == 0) continue;-----// cd	-----ptr[i] = new node*[cols];-----// eb
-----if (p == o) return false;-----// d0	-----rep(j,0,cols)-----// cd
-----if (truth[p] == -1) truth[p] = 1;-----// d3	-----if (i == rows arr[i][j]) ptr[i][j] = new node(i, j);-----// 16
-----truth[cur + n] = truth[p];-----// 50	-----else ptr[i][j] = NULL;-----// d2
-----truth[o] = 1 - truth[p];-----// 8c	---}-----// ac
-----if (truth[p] == 1) all_truthy.push_back(cur);-----// 55	-----rep(i,0,rows+1) {-----// fc
---}-----// c3	-----rep(j,0,cols) {-----// 51
---return true;-----// eb	-----if (!ptr[i][j]) continue;-----// f7
}-----// 6b	-----int ni = i + 1, nj = j + 1;-----// 7a
	-----while (true) {-----// fc
	-----if (ni == rows + 1) ni = 0;-----// 4c
	-----if (ni == rows arr[ni][j]) break;-----// 8d
	-----++ni;-----// 68
	---}-----// ad
	-----ptr[i][j]->d = ptr[ni][j];-----// 84
	-----ptr[ni][j]->u = ptr[i][j];-----// 66
	-----while (true) {-----// 7f
	-----if (nj == cols) nj = 0;-----// de
	-----if (i == rows arr[i][nj]) break;-----// 4c
	-----++nj;-----// c5
	---}-----// 72
	-----ptr[i][j]->r = ptr[i][nj];-----// 60
	-----ptr[i][nj]->l = ptr[i][j];-----// 82
	---}-----// 0b
	---}-----// 16
	-----head = new node(rows, -1);-----// 66
	-----head->r = ptr[rows][0];-----// 3e
	-----ptr[rows][0]->l = head;-----// 8c
	-----head->l = ptr[rows][cols - 1];-----// 6a
	-----ptr[rows][cols - 1]->r = head;-----// c1
	-----rep(j,0,cols) {-----// 92
	-----int cnt = -1;-----// d4
	-----rep(i,0,rows+1)-----// bd
	-----if (ptr[i][j]) cnt++, ptr[i][j]->p = ptr[rows][j];-----// f3
	-----ptr[rows][j]->size = cnt;-----// c2
	---}-----// b9
	-----rep(i,0,rows+1) delete[] ptr[i];-----// a5
	-----delete[] ptr;-----// 72
	---}-----// 19
	----#define COVER(c, i, j) □-----// 91
	-----c->r->l = c->l, c->l->r = c->r; □-----// 82
7.2. Stable Marriage.	
vi stable_marriage(int n, int** m, int** w) {-----// e4	
---queue<int> q;-----// f6	
---vi at(n, 0), eng(n, -1), res(n, -1); vvi inv(n, vi(n));-----// c3	
---rep(i,0,n) rep(j,0,n) inv[i][w[i][j]] = j;-----// f1	
---rep(i,0,n) q.push(i);-----// d8	
---while (!q.empty()) {-----// 68	
-----int curm = q.front(); q.pop();-----// e2	
-----for (int &i = at[curm]; i < n; i++) {-----// 7e	
-----int curw = m[curm][i];-----// 95	
-----if (eng[curw] == -1) { }-----// f7	
-----else if (inv[curw][curm] < inv[curw][eng[curw]])-----// d6	
-----q.push(eng[curw]);-----// 2e	
-----else continue;-----// 1d	
-----res[eng[curw] = curm] = curw, ++i; break;-----// a1	
---}-----// c4	
---}-----// 3d	
---return res;-----// 42	
}-----// bf	
7.3. Algorithm X.	
bool handle_solution(vi rows) { return false; }-----// 63	
struct exact_cover {-----// 95	
---struct node {-----// 7e	
-----node *l, *r, *u, *d, *p;-----// 19	
-----int row, col, size;-----// ae	
-----node(int _row, int _col) : row(_row), col(_col) {-----// c9	
-----size = 0; l = r = u = d = p = NULL; }-----// c3	
---};-----// c1	
---int rows, cols, *sol;-----// 7b	
---bool **arr;-----// e6	
---node *head;-----// fe	

```

-----for (node *i = c->d; i != c; i = i->d)  62
-----for (node *j = i->r; j != i; j = j->r)  26
-----j->d->u = j->u, j->u->d = j->d, j->p->size--;  c1
--#define UNCOVER(c, i, j)  89
-----for (node *i = c->u; i != c; i = i->u)  f0
-----for (node *j = i->l; j != i; j = j->l)  7b
-----j->p->size++, j->d->u = j->u->d = j;  65
-----c->r->l = c->l->r = c;  0e
--bool search(int k = 0) {  f9
--if (head == head->r) {  75
--vi res(k);  90
--rep(i,0,k) res[i] = sol[i];  2a
--sort(res.begin(), res.end());  63
--return handle_solution(res);  11
--}  3d
--node *c = head->r, *tmp = head->r;  a3
--for ( ; tmp != head; tmp = tmp->r) if (tmp->size < c->size) c = tmp;  41
--if (c == c->d) return false;  02
--COVER(c, i, j);  f6
--bool found = false;  8d
--for (node *r = c->d; !found && r != c; r = r->d) {  78
--sol[k] = r->row;  c0
--for (node *j = r->r; j != r; j = j->r) { COVER(j->p, a, b); }  f9
--found = search(k + 1);  fb
--for (node *j = r->l; j != r; j = j->l) { UNCOVER(j->p, a, b); }  87
--}  7c
--UNCOVER(c, i, j);  a7
--return found;  c0
--}  d2
};  d7

```

7.4. nth Permutation.

```

vector<int> nth_permutation(int cnt, int n) {  78
--vector<int> idx(cnt), per(cnt), fac(cnt);  9e
--rep(i,0,cnt) idx[i] = i;  bc
--rep(i,1,cnt+1) fac[i - 1] = n % i, n /= i;  2b
--for (int i = cnt - 1; i >= 0; i--)  f9
--per[cnt - i - 1] = idx[fac[i]], idx.erase(idx.begin() + fac[i]);  ee
--return per;  ab
}  37

```

7.5. Cycle-Finding.

```

ii find_cycle(int x0, int (*f)(int)) {  a5
--int t = f(x0), h = f(t), mu = 0, lam = 1;  8d
--while (t != h) t = f(t), h = f(f(h));  79
--h = x0;  04
--while (t != h) t = f(t), h = f(h), mu++;  9d
--h = f(t);  00
--while (t != h) h = f(h), lam++;  5e
--return ii(mu, lam);  b4
}  42

```

7.6. Dates.

```

int intToDay(int jd) { return jd % 7; }  89
int dateToInt(int y, int m, int d) {  96
--return 1461 * (y + 4800 + (m - 14) / 12) / 4 +  a8
--367 * (m - 2 - (m - 14) / 12 * 12) / 12  d1
--3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +  be
--d - 32075;  e0
}  fa
void intToDate(int jd, int &y, int &m, int &d) {  a1
--int x, n, i, j;  00
--x = jd + 68569;  11
--n = 4 * x / 146097;  2f
--x -= (146097 * n + 3) / 4;  58
--i = (4000 * (x + 1)) / 1461001;  0d
--x -= 1461 * i / 4 - 31;  09
--j = 80 * x / 2447;  3d
--d = x - 2447 * j / 80;  eb
--x = j / 11;  b7
--m = j + 2 - 12 * x;  82
--y = 100 * (n - 49) + i + x;  70
}  af

```

7.7. **Simulated Annealing.** An example use of Simulated Annealing to find a permutation of length n that maximizes $\sum_{i=1}^{n-1} |p_i - p_{i+1}|$.

```

double curtime() { return static_cast<double>(clock()) / CLOCKS_PER_SEC; }  9d
int simulated_annealing(int n, double seconds) {  54
--default_random_engine rng;  67
--uniform_real_distribution<double> randfloat(0.0, 1.0);  ed
--uniform_int_distribution<int> randint(0, n - 2);  bb
--  88
--// random initial solution  22
--vi sol(n);  33
--rep(i,0,n) sol[i] = i + 1;  ee
--random_shuffle(sol.begin(), sol.end());  1e
--  5b
--// initialize score  11
--int score = 0;  4d
--rep(i,1,n) score += abs(sol[i] - sol[i-1]);  74
--  25
--int iters = 0;  4d
--double T0 = 100.0, T1 = 0.001,  f4
--progress = 0, temp = T0,  8b
--starttime = curtime();  a2
--while (true) {  db
--if (!(iters & ((1 << 4) - 1))) {  e8
--progress = (curtime() - starttime) / seconds;  a0
--temp = T0 * pow(T1 / T0, progress);  12
--if (progress > 1.0) break;  48
--}  1a
--  88
--// random mutation  84

```

```
-----int a = randint(rng);-----// f7
-----// 02
-----// compute delta for mutation-----// 4e
-----int delta = 0;-----// 10
-----if (a > 0) delta += abs(sol[a+1] - sol[a-1]) - abs(sol[a] - sol[a-1]);--// 21
-----if (a+2 < n) delta += abs(sol[a] - sol[a+2]) - abs(sol[a+1] - sol[a+2]);
-----// 22
-----// maybe apply mutation-----// 4d
-----if (delta >= 0 || randfloat(rng) < exp(delta / temp)) {------// a6
-----swap(sol[a], sol[a+1]);-----// ce
-----score += delta;-----// 64
-----// if (score >= target) return;-----// a6
-----}-----// 85
-----iters++;-----// 3c
-----}-----// ec
-----return score;-----// d0
}-----// ec
```

8. USEFUL INFORMATION

8.1. Fast Input Reading.

```
void readn(register int *n) {------// dc
---int sign = 1;-----// 32
---register char c;-----// a5
---*n = 0;-----// 35
---while((c = getc_unlocked(stdin)) != '\n') {------// f3
---switch(c) {------// 0c
---case '-': sign = -1; break;-----// 28
---case ' ': goto hell;-----// fd
---case '\n': goto hell;-----// 79
---default: *n *= 10; *n += c - '0'; break;-----// c0
---}-----// 2d
---}-----// c3
hell:-----// ba
---*n *= sign;-----// a0
}-----// 67
```

8.2. Bit Hacks.

```
int snoob(int x) {------// 73
---int y = x & -x, z = x + y;-----// 12
---return z | ((x ^ z) >> 2) / y;-----// 97
}-----// 14
```

9. Misc

9.1. Debugging Tips.

- Stack overflow? Recursive DFS on tree that is actually a long path?
- Floating-point numbers
 - Getting NaN? Make sure `acos` etc. are not getting values out of their range (perhaps `1+eps`).
 - Rounding negative numbers?
 - Outputting in scientific notation?
- Wrong Answer?

- Read the problem statement again!
- Are multiple test cases being handled correctly? Try repeating the same test case many times.
- Integer overflow?
- Think very carefully about boundaries of all input parameters
- Try out possible edge cases:
 - * $n = 0, n = -1, n = 1, n = 2^{31} - 1$ or $n = -2^{31}$
 - * List is empty, or contains a single element
 - * n is even, n is odd
 - * Graph is empty, or contains a single vertex
 - * Graph is a multigraph (loops or multiple edges)
 - * Polygon is concave or non-simple
- Is initial condition wrong for small cases?
- Are you sure the algorithm is correct?
- Explain your solution to someone.
- Are you using any functions that you don't completely understand? Maybe STL functions?
- Maybe you (or someone else) should rewrite the solution?

9.2. Solution Ideas.

- Dynamic Programming
 - Drop a parameter, recover from others
 - Swap answer and a parameter
 - Parsing CFGs: CYK Algorithm
 - Optimizations
 - * Convex hull optimization
 - $dp[i] = \min_{j < i} \{dp[j] + b[j] \times a[i]\}$
 - $b[j] \geq b[j + 1]$
 - optionally $a[i] \leq a[i + 1]$
 - $O(n^2)$ to $O(n)$
 - * Divide and conquer optimization
 - $dp[i][j] = \min_{k < j} \{dp[i - 1][k] + C[k][j]\}$
 - $A[i][j] \leq A[i][j + 1]$
 - $O(kn^2)$ to $O(kn \log n)$
 - sufficient: $C[a][c] + C[b][d] \leq C[a][d] + C[b][c]$, $a \leq b \leq c \leq d$ (QI)
 - * Knuth optimization
 - $dp[i][j] = \min_{i < k < j} \{dp[i][k] + dp[k][j] + C[i][j]\}$
 - $A[i][j - 1] \leq A[i][j] \leq A[i + 1][j]$
 - $O(n^3)$ to $O(n^2)$
 - sufficient: QI and $C[b][c] \leq C[a][d]$, $a \leq b \leq c \leq d$
 - Greedy
 - Randomized
 - Optimizations
 - Use bitset (/64)
 - Switch order of loops (cache locality)
 - Process queries offline
 - Mo's algorithm
 - Square-root decomposition
 - Precomputation
 - Efficient simulation
 - Mo's algorithm

- Sqrt decomposition
 - Store 2^k jump pointers
- Data structure techniques
 - Sqrt buckets
 - Store 2^k jump pointers
 - 2^k merging trick
- Counting
 - Inclusion-exclusion principle
 - Generating functions
- Graphs
 - Can we model the problem as a graph?
 - Can we use any properties of the graph?
 - Strongly connected components
 - Cycles (or odd cycles)
 - Bipartite (no odd cycles)
 - * Bipartite matching
 - * Hall’s marriage theorem
 - * Stable Marriage
 - Cut vertex/bridge
 - Biconnected components
 - Degrees of vertices (odd/even)
 - Trees
 - * Heavy-light decomposition
 - * Centroid decomposition
 - * Least common ancestor
 - Eulerian path/circuit
 - Chinese postman problem
 - Topological sort
 - (Min-Cost) Max Flow
 - Min Cut
 - * Maximum Density Subgraph
 - Huffman Coding
 - Min-Cost Arborescence
 - Steiner Tree
 - Kirchoff’s matrix tree theorem
 - Prüfer sequences
 - Lovász Toggle
 - Look at the DFS tree (which has no cross-edges)
- Mathematics
 - Is the function multiplicative?
 - Look for a pattern
 - Permutations
 - * Consider the cycles of the permutation
 - Functions
 - * Sum of piecewise-linear functions is a piecewise-linear function
 - * Sum of convex (concave) functions is convex (concave)
 - Modular arithmetic
 - * Chinese Remainder Theorem
 - * Linear Congruence
 - Sieve
 - System of linear equations
 - Values too big to represent?
 - * Compute using the logarithm
 - * Divide everything by some large value
- Logic
 - 2-SAT
 - XOR-SAT (Gauss elimination or Bipartite matching)
- Meet in the middle
- Only work with the smaller half ($\log(n)$)
- Strings
 - Trie (maybe over something weird, like bits)
 - Suffix array
 - Suffix automaton (+DP?)
 - Aho-Corasick
 - eertree
 - Work with $S + S$
- Hashing
- Euler tour, tree to array
- Segment trees
 - Lazy propagation
 - Persistent
 - Implicit
 - Segment tree of X
- Geometry
 - Minkowski sum (of convex sets)
 - Rotating calipers
 - Sweep line (horizontally or vertically?)
 - Sweep angle
 - Convex hull
- Fix a parameter (possibly the answer).
- Are there few distinct values?
- Binary search
- Sliding Window (+ Monotonic Queue)
- Computing a Convolution? Fast Fourier Transform
- Exact Cover (+ Algorithm X)
- Cycle-Finding
- What is the smallest set of values that identify the solution? The cycle structure of the permutation? The powers of primes in the factorization?
- Look at the complement problem
 - Minimize something instead of maximizing
- Immediately enforce necessary conditions. (All values greater than 0? Initialize them all to 1)
- Add large constant to negative numbers to make them positive
- Counting/Bucket sort

FORMULAS

#labeled rooted trees

#labeled unrooted trees

$\sum_{i=1}^n i^2 = n(n+1)(2n+1)/6$

$!n = n \times!(n-1) + (-1)^n$

$\sum_{i=1}^n \binom{n}{i} F_i = F_{2n}$

$\sum_{k=0}^n \binom{n}{m} = \binom{n+1}{m+1}$

$a \equiv b \pmod{x, y} \Rightarrow a \equiv b \pmod{\text{lcm}(x, y)}$

$ac \equiv bc \pmod{m} \Rightarrow a \equiv b \pmod{\frac{m}{\text{gcd}(c, m)}}$

$p \text{ prime} \Leftrightarrow (p-1)! \equiv -1 \pmod{p}$

$\sigma_x(n) = \prod_{i=0}^r \frac{p_i^{(a_i+1)x} - 1}{p_i^x - 1}$

$\sum_{k=0}^m (-1)^k \binom{n}{k} = (-1)^m \binom{n-1}{m}$

$2^{\omega(n)} = O(\sqrt{n})$

n^{n-1}

n^{n-2}

$\sum_{i=1}^n i^3 = n^2(n+1)^2/4$

$!n = (n-1)(!(n-1) +!(n-2))$

$\sum_i \binom{n-i}{i} = F_{n+1}$

$\sum_{d|n} \phi(d) = n$

$(\sum_{d|n} \sigma_0(d))^2 = \sum_{d|n} \sigma_0(d)^3$

$\text{gcd}(n^a - 1, n^b - 1) = n^{\text{gcd}(a, b)} - 1$

$\sigma_0(n) = \prod_{i=0}^r (a_i + 1)$

$\sum_{i=1}^n 2^{\omega(i)} = O(n \log n)$

• **Jacobi symbol:** $\left(\frac{a}{b}\right) = a^{(b-1)/2} \pmod{b}$

• **Heron’s formula:** A triangle with side lengths a, b, c has area $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$.

• **Pick’s theorem:** A polygon on an integer grid containing i lattice points and having b lattice points on the boundary has area $i + \frac{b}{2} - 1$.

• **Euler’s totient:** The number of integers less than n that are coprime to n are $n \prod_{p|n} \left(1 - \frac{1}{p}\right)$ where each p is a distinct prime factor of n .

• **König’s theorem:** In any bipartite graph $G = (L \cup R, E)$, the number of edges in a maximum matching is equal to the number of vertices in a minimum vertex cover. Let U be the set of unmatched vertices in L , and Z be the set of vertices that are either in U or are connected to U by an alternating path. Then $K = (L \setminus Z) \cup (R \cap Z)$ is the minimum vertex cover.

• A mininum Steiner tree for n vertices requires at most $n - 2$ additional Steiner vertices.

• The number of vertices of a graph is equal to its minimum vertex cover number plus the size of a maximum independent set.

• **Wilson’s theorem:** $(n-1)! \equiv -1 \pmod{n}$ iff. n is prime

• **Lagrange polynomial** through points $(x_0, y_0), \dots, (x_k, y_k)$ is $L(x) = \sum_{j=0}^k y_j \prod_{\substack{0 \leq m \leq k \\ m \neq j}} \frac{x - x_m}{x_j - x_m}$

• **Hook length formula:** If λ is a Young diagram and $h_\lambda(i, j)$ is the hook-length of cell (i, j) , then then the number of Young tableaux $d_\lambda = n! / \prod h_\lambda(i, j)$.

• **Möbius inversion formula:** If $f(n) = \sum_{d|n} g(d)$, then $g(n) = \sum_{d|n} \mu(d) f(n/d)$

9.3. **Markov Chains.** A Markov Chain can be represented as a weighted directed graph of states, where the weight of an edge represents the probability of transitioning over that edge in one timestep. Let $P^{(m)} = (p_{ij}^{(m)})$ be the probability matrix of transitioning from state i to state j in m timesteps, and note that $P^{(1)}$ is the adjacency matrix of the graph. **Chapman-Kolmogorov:** $p_{ij}^{(m+n)} = \sum_k p_{ik}^{(m)} p_{kj}^{(n)}$. It follows that $P^{(m+n)} = P^{(m)} P^{(n)}$ and $P^{(m)} = P^m$. If $p^{(0)}$ is the initial probability distribution (a vector), then $p^{(0)} P^{(m)}$ is the probability distribution after m timesteps.

The return times of a state i is $R_i = \{m \mid p_{ii}^{(m)} > 0\}$, and i is *aperiodic* if $\text{gcd}(R_i) = 1$. A MC is aperiodic if any of its vertices is aperiodic. A MC is *irreducible* if the corresponding graph is strongly connected.

A distribution π is stationary if $\pi P = \pi$. If MC is irreducible then $\pi_i = 1/\mathbb{E}[T_i]$, where T_i is the expected time between two visits at i . π_j/π_i is the expected number of visits at j in between two consecutive visits at i . A MC is *ergodic* if $\lim_{m \rightarrow \infty} p^{(0)} P^m = \pi$. A MC is ergodic iff. it is irreducible and aperiodic.

A MC for a random walk in an undirected weighted graph (unweighted graph can be made weighted by adding 1-weights) has $p_{uv} = w_{uv}/\sum_x w_{ux}$. If the graph is connected, then $\pi_u = \sum_x w_{ux}/\sum_v \sum_x w_{vx}$. Such a random walk is aperiodic iff. the graph is not bipartite.

An *absorbing* MC is of the form $P = \begin{pmatrix} Q & R \\ 0 & I_r \end{pmatrix}$. Let $N = \sum_{m=0}^\infty Q^m = (I_t - Q)^{-1}$. Then, if starting in state i , the expected number of steps till absorption is the i -th entry in $N1$. If starting in state i , the probability of being absorbed in state j is the (i, j) -th entry of NR .

Many problems on MC can be formulated in terms of a system of recurrence relations, and then solved using Gaussian elimination.

9.4. **Burnside’s Lemma.** Let G be a finite group that acts on a set X . For each g in G let X^g denote the set of elements in X that are fixed by g . Then the number of orbits

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

9.5. **Bézout’s identity.** If (x, y) is any solution to $ax + by = d$ (e.g. found by the Extended Euclidean Algorithm), then all solutions are given by

$$\left(x + k \frac{b}{\text{gcd}(a, b)}, y - k \frac{a}{\text{gcd}(a, b)}\right)$$

9.6. **The Twelffold Way.** Putting n balls into k boxes.

Balls Boxes	same same	distinct same	same distinct	distinct distinct	Remarks
-	$\text{p}_k(n)$	$\sum_{i=0}^k \left\{ \begin{matrix} n \\ i \end{matrix} \right\}$	$\binom{n+k-1}{k-1}$	k^n	$\text{p}_k(n)$: #partitions of n into $\leq k$ positive parts
size ≥ 1	$\text{p}(n, k)$	$\left\{ \begin{matrix} n \\ k \end{matrix} \right\}$	$\binom{n-1}{k-1}$	$k! \left\{ \begin{matrix} n \\ k \end{matrix} \right\}$	$\text{p}(n, k)$: #partitions of n into k positive parts
size ≤ 1	$[n \leq k]$	$[n \leq k]$	$\binom{n}{k}$	$n! \binom{k}{n}$	$[cond]$: 1 if $cond = true$, else 0

Catalan	$C_0 = 1, C_n = \frac{1}{n+1} \binom{2n}{n} = \sum_{i=0}^{n-1} C_i C_{n-i-1} = \frac{4n-2}{n+1} C_{n-1}$	
Stirling 1st kind	$[0] = 1, [n] = \begin{bmatrix} n \\ 0 \end{bmatrix} = 0, \begin{bmatrix} n \\ k \end{bmatrix} = (n-1) \begin{bmatrix} n-1 \\ k \end{bmatrix} + \begin{bmatrix} n-1 \\ k-1 \end{bmatrix}$	#perms of n objs with e
Stirling 2nd kind	$\left\{ \begin{matrix} n \\ 1 \end{matrix} \right\} = \left\{ \begin{matrix} n \\ n \end{matrix} \right\} = 1, \left\{ \begin{matrix} n \\ k \end{matrix} \right\} = k \left\{ \begin{matrix} n-1 \\ k \end{matrix} \right\} + \left\{ \begin{matrix} n-1 \\ k-1 \end{matrix} \right\}$	#ways to partition n objs
Euler	$\left\langle \begin{matrix} n \\ 0 \end{matrix} \right\rangle = \left\langle \begin{matrix} n \\ n-1 \end{matrix} \right\rangle = 1, \left\langle \begin{matrix} n \\ k \end{matrix} \right\rangle = (k+1) \left\langle \begin{matrix} n-1 \\ k \end{matrix} \right\rangle + (n-k) \left\langle \begin{matrix} n-1 \\ k-1 \end{matrix} \right\rangle$	#perms of n objs with e
Euler 2nd Order	$\left\langle\!\left\langle \begin{matrix} n \\ k \end{matrix} \right\rangle\!\right\rangle = (k+1) \left\langle\!\left\langle \begin{matrix} n-1 \\ k \end{matrix} \right\rangle\!\right\rangle + (2n-k-1) \left\langle\!\left\langle \begin{matrix} n-1 \\ k-1 \end{matrix} \right\rangle\!\right\rangle$	#perms of 1, 1, 2, 2, ..., n
Bell	$B_1 = 1, B_n = \sum_{k=0}^{n-1} B_k \binom{n-1}{k} = \sum_{k=0}^n \left\{ \begin{matrix} n \\ k \end{matrix} \right\}$	#partitions of 1.. n (Stir)

PRACTICE CONTEST CHECKLIST

- How many operations per second? Compare to local machine.
- What is the stack size?
- How to use printf/scanf with long long/long double?
- Is `__int128` available?
- Does MLE give RTE or MLE as a verdict? What about stack overflow?
- What is `RAND_MAX`?
- How does the judge handle extra spaces (or missing newlines) in the output?
- Look at documentation for programming languages.
- Try different programming languages: C++ and Java.
- Try the submit script.
- Try local programs: `i?python[23]`, `factor`.
- Try submitting with `assert(false)` and `assert(false)`.
- Return-value from `main`.
- Remove this page from the notebook.