viRUs

Team Reference Document

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Contents

1. Code Templates

1.1. Basic Configuration.

1.1.1. *.bashrc.*

```
xset r rate 150 100------------------------------------------------------------// dd
set -o vi----------------------------------------------------------------------// f4
setxkbmap -option caps:escape--------------------------------------------------// bb
dvorak() { setxkbmap dvorak is; }----------------------------------------------// e6
james() { setxkbmap en_US; }---------------------------------------------------// c7
alias "h.soav"="dvorak"--------------------------------------------------------// 7d
alias "ham.o"="james"----------------------------------------------------------// 4e
-------------------------------------------------------------------------------// f4
function check {---------------------------------------------------------------// 46
----IFS=-----------------------------------------------------------------------// b6
----s=""-----------------------------------------------------------------------// 31
----cat $1 | while read l; do--------------------------------------------------// 39
--------s="$s$(echo $l | sed 's/\s//g')\n"-------------------------------------// cc
--------h=$(echo -ne "$s" | md5sum)--------------------------------------------// 83
--------echo "${h:0:2}  $l"----------------------------------------------------// eb
----done-----------------------------------------------------------------------// 74
}------------------------------------------------------------------------------// 08
```

ProTip™: setxkbmap dvorak on qwerty: `o.yqtxmal ekrpat`

1.1.2. *.vimrc.*

```
set nocp et sw=4 ts=4 sts=4 si cindent hi=1000 nu ru noeb showcmd showmode-----// bb
syn on | colorscheme slate-----------------------------------------------------// e5
```

1.2. C++ Header.

A C++ header.

```
#include <bits/stdc++.h>-------------------------------------------------------// 84
using namespace std;-----------------------------------------------------------// 16
template <class T> int size(const T &x) { return x.size(); }-------------------// 5f
#define rep(i,a,b) for (__typeof(a) i=(a); i<(b); ++i)-------------------------// 6c
#define iter(it,c) for (__typeof((c).begin()) it = (c).begin(); it != (c).end(); ++it)
typedef pair<int, int> ii;-----------------------------------------------------// 44
typedef vector<int> vi;--------------------------------------------------------// 9d
typedef vector<ii> vii;--------------------------------------------------------// eb
typedef long long ll;----------------------------------------------------------// 47
const int INF = ~(1<<31); // 2147483647----------------------------------------// 10
-------------------------------------------------------------------------------// b2
const double EPS = 1e-9;-------------------------------------------------------// d5
const double pi = acos(-1);----------------------------------------------------// 67
typedef unsigned long long ull;------------------------------------------------// ff
typedef vector<vi> vvi;--------------------------------------------------------// 4b
typedef vector<vii> vvii;------------------------------------------------------// 36
template <class T> T mod(T a, T b) { return (a % b + b) % b; }-----------------// d5
```

1.3. Java Template.

A Java template.

```
import java.util.*;------------------------------------------------------------// 37
import java.math.*;------------------------------------------------------------// 89
import java.io.*;--------------------------------------------------------------// 28
-------------------------------------------------------------------------------// a3
public class Main {------------------------------------------------------------// 17
----public static void main(String[] args) throws Exception {------------------// 02
--------Scanner in = new Scanner(System.in);-----------------------------------// ef
--------PrintWriter out = new PrintWriter(System.out, false);------------------// 62
--------// code----------------------------------------------------------------// e6
--------out.flush();-----------------------------------------------------------// 56
----}--------------------------------------------------------------------------// 79
}------------------------------------------------------------------------------// 00
```

2. Data Structures

2.1. Union-Find.

An implementation of the Union-Find disjoint sets data structure.

```
struct union_find {------------------------------------------------------------// 42
----vi p; union_find(int n) : p(n, -1) { }-------------------------------------// 28
----int find(int x) { return p[x] < 0 ? x : p[x] = find(p[x]); }---------------// ba
----bool unite(int x, int y) {-------------------------------------------------// 6c
--------int xp = find(x), yp = find(y);----------------------------------------// 64
--------if (xp == yp) return false;--------------------------------------------// 0b
--------if (p[xp] > p[yp]) swap(xp,yp);----------------------------------------// 78
--------p[xp] += p[yp], p[yp] = xp;--------------------------------------------// 88
--------return true; }---------------------------------------------------------// 1f
----int size(int x) { return -p[find(x)]; } };---------------------------------// b9
```

2.2. Segment Tree.

An implementation of a Segment Tree.

```
#ifdef SEG_MIN-----------------------------------------------------------------// 03
const int ID = INF;------------------------------------------------------------// 56
int f(int a, int b) { return min(a, b); }--------------------------------------// 4f
#else--------------------------------------------------------------------------// 0e
const int ID = 0;--------------------------------------------------------------// 3e
int f(int a, int b) { return a + b; }------------------------------------------// dd
#endif-------------------------------------------------------------------------// 16
struct segment_tree {----------------------------------------------------------// ab
----int n; vi data, lazy;------------------------------------------------------// dd
----segment_tree() {}----------------------------------------------------------// 93
----segment_tree(const vi &arr) : n(size(arr)), data(4*n), lazy(4*n,INF) {-----// f1
--------mk(arr, 0, n-1, 0); }--------------------------------------------------// e9
----int mk(const vi &arr, int l, int r, int i) {-------------------------------// 12
--------if (l == r) return data[i] = arr[l];-----------------------------------// 5b
--------int m = (l + r) / 2;---------------------------------------------------// de
--------return data[i] = f(mk(arr, l, m, 2*i+1), mk(arr, m+1, r, 2*i+2)); }----// 0a
----int query(int a, int b) { return q(a, b, 0, n-1, 0); }---------------------// f6
----int q(int a, int b, int l, int r, int i) {---------------------------------// 22
--------propagate(l, r, i);----------------------------------------------------// 12
--------if (r < a || b < l) return ID;-----------------------------------------// c7
--------if (a <= l && r <= b) return data[i];----------------------------------// ce
--------int m = (l + r) / 2;---------------------------------------------------// 7a
--------return f(q(a, b, l, m, 2*i+1), q(a, b, m+1, r, 2*i+2)); }--------------// 5c
----void update(int i, int v) { u(i, v, 0, n-1, 0); }--------------------------// 90
----int u(int i, int v, int l, int r, int j) {---------------------------------// 02
--------propagate(l, r, j);----------------------------------------------------// ae
--------if (r < i || i < l) return data[j];------------------------------------// 92
--------if (l == i && r == i) return data[j] = v;------------------------------// 4a
--------int m = (l + r) / 2;---------------------------------------------------// cb
```

```
--------return data[j] = f(u(i, v, l, m, 2*j+1), u(i, v, m+1, r, 2*j+2)); }----// 34
----void range_update(int a, int b, int v) { ru(a, b, v, 0, n-1, 0); }---------// 71
----int ru(int a, int b, int v, int l, int r, int i) {-------------------------// e0
--------propagate(l, r, i);----------------------------------------------------// 19
--------if (l > r) return ID;--------------------------------------------------// cc
--------if (r < a || b < l) return data[i];------------------------------------// d9
--------if (a <= l && r <= b) return (lazy[i] = v) * (r - l + 1) + data[i];----// 06
--------int m = (l + r) / 2;---------------------------------------------------// cc
--------return data[i] = f(ru(a, b, v, l, m, 2*i+1),---------------------------// cc
------------------------   ru(a, b, v, m+1, r, 2*i+2));------------------------// 2b
----}--------------------------------------------------------------------------// 0b
----void propagate(int l, int r, int i) {--------------------------------------// a7
--------if (l > r || lazy[i] == INF) return;-----------------------------------// 5f
--------data[i] += lazy[i] * (r - l + 1);--------------------------------------// 44
--------if (l < r) {-----------------------------------------------------------// 28
------------if (lazy[2*i+1] == INF) lazy[2*i+1] = lazy[i];---------------------// 4e
------------else lazy[2*i+1] += lazy[i];---------------------------------------// 1e
------------if (lazy[2*i+2] == INF) lazy[2*i+2] = lazy[i];---------------------// de
------------else lazy[2*i+2] += lazy[i];---------------------------------------// 74
--------}----------------------------------------------------------------------// 1f
--------lazy[i] = INF;---------------------------------------------------------// f8
----}--------------------------------------------------------------------------// 41
};-----------------------------------------------------------------------------// ae
```

2.2.1. *Persistent Segment Tree.*

```
int segcnt = 0;----------------------------------------------------------------// cf
struct segment {---------------------------------------------------------------// 68
----int l, r, lid, rid, sum;---------------------------------------------------// fc
} segs[2000000];---------------------------------------------------------------// dd
int build(int l, int r) {------------------------------------------------------// 2b
----if (l > r) return -1;------------------------------------------------------// 4e
----int id = segcnt++;---------------------------------------------------------// a8
----segs[id].l = l;------------------------------------------------------------// 90
----segs[id].r = r;------------------------------------------------------------// 19
----if (l == r) segs[id].lid = -1, segs[id].rid = -1;--------------------------// ee
----else {---------------------------------------------------------------------// fe
--------int m = (l + r) / 2;---------------------------------------------------// 14
--------segs[id].lid = build(l , m);-------------------------------------------// e3
--------segs[id].rid = build(m + 1, r); }--------------------------------------// 69
----segs[id].sum = 0;----------------------------------------------------------// 21
----return id; }---------------------------------------------------------------// c5
int update(int idx, int v, int id) {-------------------------------------------// b8
----if (id == -1) return -1;---------------------------------------------------// bb
----if (idx < segs[id].l || idx > segs[id].r) return id;-----------------------// fb
----int nid = segcnt++;--------------------------------------------------------// b3
----segs[nid].l = segs[id].l;--------------------------------------------------// 78
----segs[nid].r = segs[id].r;--------------------------------------------------// ca
----segs[nid].lid = update(idx, v, segs[id].lid);------------------------------// 92
----segs[nid].rid = update(idx, v, segs[id].rid);------------------------------// 06
----segs[nid].sum = segs[id].sum + v;------------------------------------------// 1a
----return nid; }--------------------------------------------------------------// e6
int query(int id, int l, int r) {----------------------------------------------// a2
----if (r < segs[id].l || segs[id].r < l) return 0;----------------------------// 17
----if (l <= segs[id].l && segs[id].r <= r) return segs[id].sum;---------------// ad
----return query(segs[id].lid, l, r) + query(segs[id].rid, l, r); }------------// ee
```

2.3. **Fenwick Tree.** A Fenwick Tree is a data structure that represents an array of n numbers. It supports adjusting the i-th element in $O(\log n)$ time, and computing the sum of numbers in the range $i..j$ in $O(\log n)$ time. It only needs $O(n)$ space.

```
struct fenwick_tree {----------------------------------------------------------// 98
----int n; vi data;------------------------------------------------------------// d3
----fenwick_tree(int _n) : n(_n), data(vi(n)) { }------------------------------// db
----void update(int at, int by) {----------------------------------------------// 76
--------while (at < n) data[at] += by, at |= at + 1; }-------------------------// fb
----int query(int at) {--------------------------------------------------------// 71
--------int res = 0;-----------------------------------------------------------// c3
--------while (at >= 0) res += data[at], at = (at & (at + 1)) - 1;-------------// 37
--------return res; }----------------------------------------------------------// e4
----int rsq(int a, int b) { return query(b) - query(a - 1); }------------------// be
};-----------------------------------------------------------------------------// 57
struct fenwick_tree_sq {-------------------------------------------------------// d4
----int n; fenwick_tree x1, x0;------------------------------------------------// 18
----fenwick_tree_sq(int _n) : n(_n), x1(fenwick_tree(n)),----------------------// 2e
--------x0(fenwick_tree(n)) { }------------------------------------------------// 7c
----// insert f(y) = my + c if x <= y------------------------------------------// 17
----void update(int x, int m, int c) { x1.update(x, m); x0.update(x, c); }-----// 45
----int query(int x) { return x*x1.query(x) + x0.query(x); }-------------------// 73
};-----------------------------------------------------------------------------// 13
void range_update(fenwick_tree_sq &s, int a, int b, int k) {-------------------// 89
----s.update(a, k, k * (1 - a)); s.update(b+1, -k, k * b); }-------------------// 7f
int range_query(fenwick_tree_sq &s, int a, int b) {----------------------------// 15
----return s.query(b) - s.query(a-1); }----------------------------------------// f3
```

2.4. **Matrix.** A Matrix class.

```
template <class K> bool eq(K a, K b) { return a == b; }------------------------// 2a
template <> bool eq<double>(double a, double b) { return abs(a - b) < EPS; }---// a7
template <class T> struct matrix {---------------------------------------------// 0a
----int rows, cols, cnt; vector<T> data;---------------------------------------// a1
----inline T& at(int i, int j) { return data[i * cols + j]; }------------------// 5c
----matrix(int r, int c) : rows(r), cols(c), cnt(r * c) {----------------------// 56
--------data.assign(cnt, T(0)); }----------------------------------------------// e3
----matrix(const matrix& other) : rows(other.rows), cols(other.cols),----------// b5
--------cnt(other.cnt), data(other.data) { }-----------------------------------// c1
----T& operator()(int i, int j) { return at(i, j); }---------------------------// 29
----matrix<T> operator +(const matrix& other) {--------------------------------// 33
--------matrix<T> res(*this); rep(i,0,cnt) res.data[i] += other.data[i];-------// f8
--------return res; }----------------------------------------------------------// 09
----matrix<T> operator -(const matrix& other) {--------------------------------// 91
--------matrix<T> res(*this); rep(i,0,cnt) res.data[i] -= other.data[i];-------// 7b
--------return res; }----------------------------------------------------------// 9a
----matrix<T> operator *(T other) {--------------------------------------------// 99
--------matrix<T> res(*this); rep(i,0,cnt) res.data[i] *= other;---------------// 05
--------return res; }----------------------------------------------------------// 8c
```

```
----matrix<T> operator *(const matrix& other) {----------------------------------// 31
--------matrix<T> res(rows, other.cols);-----------------------------------------// 4c
--------rep(i,0,rows) rep(j,0,other.cols) rep(k,0,cols)--------------------------// ae
----------------res(i, j) += at(i, k) * other.data[k * other.cols + j];----------// 17
--------return res; }------------------------------------------------------------// 65
----matrix<T> pow(int p) {-------------------------------------------------------// 53
--------matrix<T> res(rows, cols), sq(*this);------------------------------------// 87
--------rep(i,0,rows) res(i, i) = T(1);------------------------------------------// 9d
--------while (p) {--------------------------------------------------------------// 79
------------if (p & 1) res = res * sq;-------------------------------------------// 62
------------p >>= 1;-------------------------------------------------------------// 79
------------if (p) sq = sq * sq;-------------------------------------------------// 35
--------} return res; }----------------------------------------------------------// 22
----matrix<T> rref(T &det, int &rank) {------------------------------------------// 2a
--------matrix<T> mat(*this); det = T(1), rank = max(rows, cols);----------------// 7a
--------for (int r = 0, c = 0; c < cols; c++) {----------------------------------// 8e
------------int k = r;-----------------------------------------------------------// 5b
------------while (k < rows && eq<T>(mat(k, c), T(0))) k++;----------------------// 3e
------------if (k >= rows) { rank--; continue; }---------------------------------// 1a
------------if (k != r) {--------------------------------------------------------// c4
----------------det *= T(-1);----------------------------------------------------// 55
----------------rep(i,0,cols)----------------------------------------------------// e1
--------------------swap(mat.at(k, i), mat.at(r, i));----------------------------// 7d
------------} det *= mat(r, r);--------------------------------------------------// b6
------------T d = mat(r,c);------------------------------------------------------// 66
------------rep(i,0,cols) mat(r, i) /= d;----------------------------------------// d1
------------rep(i,0,rows) {------------------------------------------------------// f6
----------------T m = mat(i, c);-------------------------------------------------// 05
----------------if (i != r && !eq<T>(m, T(0)))-----------------------------------// 1a
--------------------rep(j,0,cols) mat(i, j) -= m * mat(r, j);--------------------// 7b
------------} r++;---------------------------------------------------------------// c5
--------} return mat; }----------------------------------------------------------// b3
----matrix<T> transpose() {------------------------------------------------------// 59
--------matrix<T> res(cols, rows);-----------------------------------------------// 5b
--------rep(i,0,rows) rep(j,0,cols) res(j, i) = at(i, j);------------------------// 92
--------return res; } };---------------------------------------------------------// df
```

2.5. **AVL Tree.** A fast, easily augmentable, balanced binary search tree.

```
#define AVL_MULTISET 0---------------------------------------------------------------// b5
-------------------------------------------------------------------------------------// 61
template <class T>-------------------------------------------------------------------// 22
struct avl_tree {--------------------------------------------------------------------// 30
----struct node {--------------------------------------------------------------------// 8f
--------T item; node *p, *l, *r;-----------------------------------------------------// a9
--------int size, height;------------------------------------------------------------// 47
--------node(const T &_item, node *_p = NULL) : item(_item), p(_p),------------------// ed
--------l(NULL), r(NULL), size(1), height(0) { } };----------------------------------// 27
----avl_tree() : root(NULL) { }------------------------------------------------------// b4
----node *root;----------------------------------------------------------------------// 4e
----inline int sz(node *n) const { return n ? n->size : 0; }-------------------------// 4f
----inline int height(node *n) const { return n ? n->height : -1; }------------------// d2
----inline bool left_heavy(node *n) const {------------------------------------------// 8e
--------return n && height(n->l) > height(n->r); }-----------------------------------// dc
----inline bool right_heavy(node *n) const {-----------------------------------------// 14
--------return n && height(n->r) > height(n->l); }-----------------------------------// 24
----inline bool too_heavy(node *n) const {-------------------------------------------// c4
--------return n && abs(height(n->l) - height(n->r)) > 1; }--------------------------// 10
----void delete_tree(node *n) {------------------------------------------------------// 47
--------if (n) { delete_tree(n->l), delete_tree(n->r); delete n; } }-----------------// e2
----node*& parent_leg(node *n) {-----------------------------------------------------// f6
--------if (!n->p) return root;------------------------------------------------------// f4
--------if (n->p->l == n) return n->p->l;--------------------------------------------// 98
--------if (n->p->r == n) return n->p->r;--------------------------------------------// 68
--------assert(false); }-------------------------------------------------------------// 0f
----void augment(node *n) {----------------------------------------------------------// d2
--------if (!n) return;--------------------------------------------------------------// b8
--------n->size = 1 + sz(n->l) + sz(n->r);-------------------------------------------// 26
--------n->height = 1 + max(height(n->l), height(n->r)); }---------------------------// f0
----#define rotate(l, r) \--------------------------------------------------------------// 08
--------node *l = n->l; \------------------------------------------------------------// af
--------l->p = n->p; \---------------------------------------------------------------// ff
--------parent_leg(n) = l; \---------------------------------------------------------// 1f
--------n->l = l->r; \---------------------------------------------------------------// 26
--------if (l->r) l->r->p = n; \-----------------------------------------------------// f1
--------l->r = n, n->p = l; \--------------------------------------------------------// b0
--------augment(n), augment(l)-------------------------------------------------------// 42
----void left_rotate(node *n) { rotate(r, l); }--------------------------------------// a8
----void right_rotate(node *n) { rotate(l, r); }-------------------------------------// b5
----void fix(node *n) {--------------------------------------------------------------// be
--------while (n) { augment(n);------------------------------------------------------// fb
------------if (too_heavy(n)) {------------------------------------------------------// b5
----------------if (left_heavy(n) && right_heavy(n->l)) left_rotate(n->l);-----------// a3
----------------else if (right_heavy(n) && left_heavy(n->r))-------------------------// 76
--------------------right_rotate(n->r);----------------------------------------------// 12
----------------if (left_heavy(n)) right_rotate(n);----------------------------------// 8a
----------------else left_rotate(n);-------------------------------------------------// 2e
----------------n = n->p; }----------------------------------------------------------// f5
------------n = n->p; } }------------------------------------------------------------// 86
----inline int size() const { return sz(root); }-------------------------------------// 15
----node* find(const T &item) const {------------------------------------------------// 8f
--------node *cur = root;------------------------------------------------------------// 37
--------while (cur) {----------------------------------------------------------------// a4
------------if (cur->item < item) cur = cur->r;--------------------------------------// 8b
------------else if (item < cur->item) cur = cur->l;---------------------------------// 38
------------else break; }------------------------------------------------------------// ae
--------return cur; }----------------------------------------------------------------// b7
----node* insert(const T &item) {----------------------------------------------------// 5f
--------node *prev = NULL, **cur = &root;--------------------------------------------// f7
--------while (*cur) {---------------------------------------------------------------// 82
------------prev = *cur;-------------------------------------------------------------// 1c
------------if ((*cur)->item < item) cur = &((*cur)->r);-----------------------------// 54
```

```
#if AVL_MULTISET---------------------------------------------------------------// b5
------------else cur = &((*cur)->l);-------------------------------------------// e4
#else--------------------------------------------------------------------------// 58
------------else if (item < (*cur)->item) cur = &((*cur)->l);------------------// 89
------------else return *cur;--------------------------------------------------// 65
#endif-------------------------------------------------------------------------// 03
--------}----------------------------------------------------------------------// be
--------node *n = new node(item, prev);----------------------------------------// 2b
--------*cur = n, fix(n); return n; }------------------------------------------// 2a
----void erase(const T &item) { erase(find(item)); }---------------------------// fa
----void erase(node *n, bool free = true) {------------------------------------// 7d
--------if (!n) return;--------------------------------------------------------// ca
--------if (!n->l && n->r) parent_leg(n) = n->r, n->r->p = n->p;---------------// c8
--------else if (n->l && !n->r) parent_leg(n) = n->l, n->l->p = n->p;----------// 52
--------else if (n->l && n->r) {-----------------------------------------------// 9a
------------node *s = successor(n);--------------------------------------------// 91
------------erase(s, false);---------------------------------------------------// 83
------------s->p = n->p, s->l = n->l, s->r = n->r;-----------------------------// 4b
------------if (n->l) n->l->p = s;---------------------------------------------// f4
------------if (n->r) n->r->p = s;---------------------------------------------// 85
------------parent_leg(n) = s, fix(s);-----------------------------------------// a6
------------return;------------------------------------------------------------// 9c
--------} else parent_leg(n) = NULL;-------------------------------------------// bb
--------fix(n->p), n->p = n->l = n->r = NULL;----------------------------------// e3
--------if (free) delete n; }--------------------------------------------------// 18
----node* successor(node *n) const {-------------------------------------------// 4c
--------if (!n) return NULL;---------------------------------------------------// f3
--------if (n->r) return nth(0, n->r);-----------------------------------------// 38
--------node *p = n->p;--------------------------------------------------------// a0
--------while (p && p->r == n) n = p, p = p->p;--------------------------------// 36
--------return p; }------------------------------------------------------------// 0e
----node* predecessor(node *n) const {-----------------------------------------// 64
--------if (!n) return NULL;---------------------------------------------------// 88
--------if (n->l) return nth(n->l->size-1, n->l);------------------------------// 92
--------node *p = n->p;--------------------------------------------------------// 05
--------while (p && p->l == n) n = p, p = p->p;--------------------------------// 90
--------return p; }------------------------------------------------------------// 42
----node* nth(int n, node *cur = NULL) const {---------------------------------// e3
--------if (!cur) cur = root;--------------------------------------------------// 9f
--------while (cur) {----------------------------------------------------------// e3
------------if (n < sz(cur->l)) cur = cur->l;----------------------------------// f6
------------else if (n > sz(cur->l)) n -= sz(cur->l) + 1, cur = cur->r;--------// 83
------------else break;--------------------------------------------------------// 29
--------} return cur; }--------------------------------------------------------// c4
----int count_less(node *cur) {------------------------------------------------// 02
--------int sum = sz(cur->l);--------------------------------------------------// 80
--------while (cur) {----------------------------------------------------------// 18
------------if (cur->p && cur->p->r == cur) sum += 1 + sz(cur->p->l);----------// b5
------------cur = cur->p;------------------------------------------------------// 08
--------} return sum; }--------------------------------------------------------// 69
----void clear() { delete_tree(root), root = NULL; } };------------------------// d2
```

Also a very simple wrapper over the AVL tree that implements a map interface.

```
#include "avl_tree.cpp"--------------------------------------------------------// 01
template <class K, class V> struct avl_map {-----------------------------------// dc
----struct node {--------------------------------------------------------------// 58
--------K key; V value;--------------------------------------------------------// 78
--------node(K k, V v) : key(k), value(v) { }----------------------------------// 89
--------bool operator <(const node &other) const { return key < other.key; } };// ba
----avl_tree<node> tree;-------------------------------------------------------// 17
----V& operator [](K key) {----------------------------------------------------// 95
--------typename avl_tree<node>::node *n = tree.find(node(key, V(0)));---------// 3e
--------if (!n) n = tree.insert(node(key, V(0)));------------------------------// 2d
--------return n->item.value;--------------------------------------------------// 0b
----}--------------------------------------------------------------------------// 41
};-----------------------------------------------------------------------------// 2e
```

2.6. **Heap.** An implementation of a binary heap.

```
#define RESIZE-----------------------------------------------------------------// d0
#define SWP(x,y) tmp = x, x = y, y = tmp---------------------------------------// fb
struct default_int_cmp {-------------------------------------------------------// 8d
----default_int_cmp() { }------------------------------------------------------// 35
----bool operator ()(const int &a, const int &b) { return a < b; } };----------// e9
template <class Compare = default_int_cmp> struct heap {-----------------------// 42
----int len, count, *q, *loc, tmp;---------------------------------------------// 07
----Compare _cmp;--------------------------------------------------------------// a5
----inline bool cmp(int i, int j) { return _cmp(q[i], q[j]); }-----------------// e2
----inline void swp(int i, int j) {--------------------------------------------// 3b
--------SWP(q[i], q[j]), SWP(loc[q[i]], loc[q[j]]); }--------------------------// bd
----void swim(int i) {---------------------------------------------------------// b5
--------while (i > 0) {--------------------------------------------------------// 70
------------int p = (i - 1) / 2;-----------------------------------------------// b8
------------if (!cmp(i, p)) break;---------------------------------------------// 2f
------------swp(i, p), i = p; } }----------------------------------------------// 20
----void sink(int i) {---------------------------------------------------------// 40
--------while (true) {---------------------------------------------------------// 07
------------int l = 2*i + 1, r = l + 1;----------------------------------------// 85
------------if (l >= count) break;---------------------------------------------// d9
------------int m = r >= count || cmp(l, r) ? l : r;---------------------------// db
------------if (!cmp(m, i)) break;---------------------------------------------// 4e
------------swp(m, i), i = m; } }----------------------------------------------// 36
----heap(int init_len = 128) : count(0), len(init_len), _cmp(Compare()) {-----// 05
--------q = new int[len], loc = new int[len];----------------------------------// bc
--------memset(loc, 255, len << 2); }------------------------------------------// 45
----~heap() { delete[] q; delete[] loc; }--------------------------------------// 23
----void push(int n, bool fix = true) {----------------------------------------// b8
--------if (len == count || n >= len) {----------------------------------------// dc
#ifdef RESIZE------------------------------------------------------------------// 0a
------------int newlen = 2 * len;----------------------------------------------// 85
------------while (n >= newlen) newlen *= 2;-----------------------------------// 54
------------int *newq = new int[newlen], *newloc = new int[newlen];------------// 9f
------------rep(i,0,len) newq[i] = q[i], newloc[i] = loc[i];-------------------// 53
------------memset(newloc + len, 255, (newlen - len) << 2);--------------------// a6
```

```
------------delete[] q, delete[] loc;-------------------------------------------// 7a
------------loc = newloc, q = newq, len = newlen;-------------------------------// 80
#else---------------------------------------------------------------------------// 82
------------assert(false);------------------------------------------------------// 46
#endif--------------------------------------------------------------------------// 5c
--------}-----------------------------------------------------------------------// 34
--------assert(loc[n] == -1);---------------------------------------------------// 71
--------loc[n] = count, q[count++] = n;-----------------------------------------// 98
--------if (fix) swim(count-1); }-----------------------------------------------// 70
----void pop(bool fix = true) {-------------------------------------------------// 2e
--------assert(count > 0);------------------------------------------------------// 7b
--------loc[q[0]] = -1, q[0] = q[--count], loc[q[0]] = 0;-----------------------// 71
--------if (fix) sink(0);-------------------------------------------------------// 80
----}---------------------------------------------------------------------------// b2
----int top() { assert(count > 0); return q[0]; }-------------------------------// d9
----void heapify() { for (int i = count - 1; i > 0; i--)------------------------// 77
--------if (cmp(i, (i - 1) / 2)) swp(i, (i - 1) / 2); }-------------------------// cc
----void update_key(int n) {----------------------------------------------------// 86
--------assert(loc[n] != -1), swim(loc[n]), sink(loc[n]); }---------------------// d9
----bool empty() { return count == 0; }-----------------------------------------// 77
----int size() { return count; }------------------------------------------------// 74
----void clear() { count = 0, memset(loc, 255, len << 2); } };------------------// 99
```

2.7. **Dancing Links.** An implementation of Donald Knuth's Dancing Links data structure. A linked list supporting deletion and restoration of elements.

```
template <class T>--------------------------------------------------------------// 82
struct dancing_links {----------------------------------------------------------// 9e
----struct node {---------------------------------------------------------------// 62
--------T item;-----------------------------------------------------------------// dd
--------node *l, *r;------------------------------------------------------------// 32
--------node(const T &_item, node *_l = NULL, node *_r = NULL)------------------// 6d
------------: item(_item), l(_l), r(_r) {---------------------------------------// 6d
------------if (l) l->r = this;-------------------------------------------------// 97
------------if (r) r->l = this;-------------------------------------------------// 81
--------}-----------------------------------------------------------------------// 2d
----};--------------------------------------------------------------------------// d3
----node *front, *back;---------------------------------------------------------// aa
----dancing_links() { front = back = NULL; }------------------------------------// 72
----node *push_back(const T &item) {--------------------------------------------// 83
--------back = new node(item, back, NULL);--------------------------------------// c4
--------if (!front) front = back;-----------------------------------------------// d2
--------return back;------------------------------------------------------------// c0
----}---------------------------------------------------------------------------// a9
----node *push_front(const T &item) {-------------------------------------------// 4a
--------front = new node(item, NULL, front);------------------------------------// 47
--------if (!back) back = front;------------------------------------------------// 10
--------return front;-----------------------------------------------------------// cf
----}---------------------------------------------------------------------------// b6
----void erase(node *n) {-------------------------------------------------------// a0
--------if (!n->l) front = n->r; else n->l->r = n->r;---------------------------// ab
--------if (!n->r) back = n->l; else n->r->l = n->l;----------------------------// 1b
----}---------------------------------------------------------------------------// 7b
----void restore(node *n) {-----------------------------------------------------// 82
--------if (!n->l) front = n; else n->l->r = n;---------------------------------// a5
--------if (!n->r) back = n; else n->r->l = n;----------------------------------// 9d
----}---------------------------------------------------------------------------// eb
};------------------------------------------------------------------------------// 5e
```

2.8. **Misof Tree.** A simple tree data structure for inserting, erasing, and querying the nth largest element.

```
#define BITS 15-----------------------------------------------------------------// 7b
struct misof_tree {-------------------------------------------------------------// fe
----int cnt[BITS][1<<BITS];-----------------------------------------------------// aa
----misof_tree() { memset(cnt, 0, sizeof(cnt)); }-------------------------------// b0
----void insert(int x) { for (int i = 0; i < BITS; cnt[i++][x]++, x >>= 1); }---// 5a
----void erase(int x) { for (int i = 0; i < BITS; cnt[i++][x]--, x >>= 1); }----// 49
----int nth(int n) {------------------------------------------------------------// 8a
--------int res = 0;------------------------------------------------------------// a4
--------for (int i = BITS-1; i >= 0; i--)---------------------------------------// 99
------------if (cnt[i][res <<= 1] <= n) n -= cnt[i][res], res |= 1;-------------// f4
--------return res;-------------------------------------------------------------// 3a
----}---------------------------------------------------------------------------// b5
};------------------------------------------------------------------------------// 0a
```

2.9. **k-d Tree.** A k-dimensional tree supporting fast construction, adding points, and nearest neighbor queries. NOTE: Not completely stable, occasionally segfaults.

```
#define INC(c) ((c) == K - 1 ? 0 : (c) + 1)-------------------------------------// 77
template <int K> struct kd_tree {-----------------------------------------------// 93
----struct pt {-----------------------------------------------------------------// 99
--------double coord[K];--------------------------------------------------------// 31
--------pt() {}-----------------------------------------------------------------// 96
--------pt(double c[K]) { rep(i,0,K) coord[i] = c[i]; }-------------------------// 37
--------double dist(const pt &other) const {------------------------------------// 16
------------double sum = 0.0;---------------------------------------------------// 0c
------------rep(i,0,K) sum += pow(coord[i] - other.coord[i], 2.0);--------------// f3
------------return sqrt(sum); } };----------------------------------------------// 68
----struct cmp {----------------------------------------------------------------// 8c
--------int c;------------------------------------------------------------------// fa
--------cmp(int _c) : c(_c) {}--------------------------------------------------// 28
--------bool operator ()(const pt &a, const pt &b) {----------------------------// 8e
------------for (int i = 0, cc; i <= K; i++) {----------------------------------// 24
----------------cc = i == 0 ? c : i - 1;----------------------------------------// ae
----------------if (abs(a.coord[cc] - b.coord[cc]) > EPS)-----------------------// ad
--------------------return a.coord[cc] < b.coord[cc];---------------------------// ed
------------}-------------------------------------------------------------------// 5d
------------return false; } };--------------------------------------------------// a4
----struct bb {-----------------------------------------------------------------// f1
--------pt from, to;------------------------------------------------------------// 26
--------bb(pt _from, pt _to) : from(_from), to(_to) {}--------------------------// 9c
--------double dist(const pt &p) {----------------------------------------------// 74
------------double sum = 0.0;---------------------------------------------------// 48
------------rep(i,0,K) {--------------------------------------------------------// d2
----------------if (p.coord[i] < from.coord[i])---------------------------------// ff
```

```
--------------------sum += pow(from.coord[i] - p.coord[i], 2.0);---------------// 07
----------------else if (p.coord[i] > to.coord[i])-----------------------------// 50
--------------------sum += pow(p.coord[i] - to.coord[i], 2.0);---------------// 45
------------}------------------------------------------------------------------// e8
------------return sqrt(sum); }------------------------------------------------// df
--------bb bound(double l, int c, bool left) {---------------------------------// 67
------------pt nf(from.coord), nt(to.coord);-----------------------------------// af
------------if (left) nt.coord[c] = min(nt.coord[c], l);-----------------------// 48
------------else nf.coord[c] = max(nf.coord[c], l);----------------------------// 14
------------return bb(nf, nt); } };-------------------------------------------// 97
----struct node {--------------------------------------------------------------// 7f
--------pt p; node *l, *r;-----------------------------------------------------// 2c
--------node(pt _p, node *_l, node *_r) : p(_p), l(_l), r(_r) {  } };----------// 84
----node *root;----------------------------------------------------------------// 62
----// kd_tree() : root(NULL) { }----------------------------------------------// 50
----kd_tree(vector<pt> pts) { root = construct(pts, 0, size(pts) - 1, 0); }----// 8a
----node* construct(vector<pt> &pts, int from, int to, int c) {----------------// 8d
--------if (from > to) return NULL;--------------------------------------------// 21
--------int mid = from + (to - from) / 2;--------------------------------------// b3
--------nth_element(pts.begin() + from, pts.begin() + mid,---------------------// 56
--------------------pts.begin() + to + 1, cmp(c));-----------------------------// a5
--------return new node(pts[mid], construct(pts, from, mid - 1, INC(c)),-------// 39
------------------------construct(pts, mid + 1, to, INC(c))); }---------------// 3a
----bool contains(const pt &p) { return _con(p, root, 0); }--------------------// 59
----bool _con(const pt &p, node *n, int c) {-----------------------------------// 70
--------if (!n) return false;--------------------------------------------------// b4
--------if (cmp(c)(p, n->p)) return _con(p, n->l, INC(c));---------------------// 2b
--------if (cmp(c)(n->p, p)) return _con(p, n->r, INC(c));---------------------// ec
--------return true; }---------------------------------------------------------// b5
----void insert(const pt &p) { _ins(p, root, 0); }-----------------------------// 09
----void _ins(const pt &p, node* &n, int c) {----------------------------------// 40
--------if (!n) n = new node(p, NULL, NULL);-----------------------------------// 98
--------else if (cmp(c)(p, n->p)) _ins(p, n->l, INC(c));-----------------------// ed
--------else if (cmp(c)(n->p, p)) _ins(p, n->r, INC(c)); }---------------------// 91
----void clear() { _clr(root); root = NULL; }----------------------------------// dd
----void _clr(node *n) { if (n) _clr(n->l), _clr(n->r), delete n; }------------// 17
----pt nearest_neighbour(const pt &p, bool allow_same=true) {------------------// 0f
--------assert(root);----------------------------------------------------------// 47
--------double mn = INFINITY, cs[K];-------------------------------------------// 0d
--------rep(i,0,K) cs[i] = -INFINITY;------------------------------------------// 56
--------pt from(cs);-----------------------------------------------------------// f0
--------rep(i,0,K) cs[i] = INFINITY;-------------------------------------------// 8c
--------pt to(cs);-------------------------------------------------------------// ad
--------return _nn(p, root, bb(from, to), mn, 0, allow_same).first;------------// f6
----}--------------------------------------------------------------------------// 79
----pair<pt, bool> _nn(--------------------------------------------------------// a1
------------const pt &p, node *n, bb b, double &mn, int c, bool same) {--------// a6
--------if (!n || b.dist(p) > mn) return make_pair(pt(), false);---------------// e4
--------bool found = same || p.dist(n->p) > EPS, l1 = true, l2 = false;--------// 59
--------pt resp = n->p;--------------------------------------------------------// 92
--------if (found) mn = min(mn, p.dist(resp));---------------------------------// 67
--------node *n1 = n->l, *n2 = n->r;-------------------------------------------// b3
--------rep(i,0,2) {-----------------------------------------------------------// af
------------if (i == 1 || cmp(c)(n->p, p)) swap(n1, n2), swap(l1, l2);---------// 1f
------------pair<pt, bool> res =-----------------------------------------------// a4
----------------_nn(p, n1, b.bound(n->p.coord[c], c, l1), mn, INC(c), same);---// a8
------------if (res.second && (!found || p.dist(res.first) < p.dist(resp)))----// cd
----------------resp = res.first, found = true;--------------------------------// 15
--------}----------------------------------------------------------------------// 24
--------return make_pair(resp, found); } };-----------------------------------// c5
```

2.10. **Sqrt Decomposition.** Design principle that supports many operations in amortized $\sqrt{n}$ per operation.

```
struct segment {---------------------------------------------------------------// b2
----vi arr;--------------------------------------------------------------------// 8c
----segment(vi _arr) : arr(_arr) { } };----------------------------------------// 11
vector&lt;segment&gt; T;-------------------------------------------------------------// a1
int K;-------------------------------------------------------------------------// dc
void rebuild() {---------------------------------------------------------------// 17
----int cnt = 0;---------------------------------------------------------------// 14
----rep(i,0,size(T))-----------------------------------------------------------// b1
--------cnt += size(T[i].arr);-------------------------------------------------// d1
----K = static_cast<int>(ceil(sqrt(cnt)) + 1e-9);------------------------------// 4c
----vi arr(cnt);---------------------------------------------------------------// 14
----for (int i = 0, at = 0; i < size(T); i++)----------------------------------// 79
--------rep(j,0,size(T[i].arr))------------------------------------------------// a4
------------arr[at++] = T[i].arr[j];-------------------------------------------// f7
----T.clear();-----------------------------------------------------------------// 4c
----for (int i = 0; i < cnt; i += K)-------------------------------------------// 79
--------T.push_back(segment(vi(arr.begin()+i, arr.begin()+min(i+K, cnt))));----// f0
}------------------------------------------------------------------------------// 03
int split(int at) {------------------------------------------------------------// 71
----int i = 0;-----------------------------------------------------------------// 8a
----while (i < size(T) && at >= size(T[i].arr))--------------------------------// 6c
--------at -= size(T[i].arr), i++;---------------------------------------------// 9a
----if (i >= size(T)) return size(T);------------------------------------------// 83
----if (at == 0) return i;-----------------------------------------------------// 49
----T.insert(T.begin() + i + 1, segment(vi(T[i].arr.begin() + at, T[i].arr.end())));
----T[i] = segment(vi(T[i].arr.begin(), T[i].arr.begin() + at));---------------// af
----return i + 1;--------------------------------------------------------------// ac
}------------------------------------------------------------------------------// ea
void insert(int at, int v) {---------------------------------------------------// 5f
----vi arr; arr.push_back(v);--------------------------------------------------// 6a
----T.insert(T.begin() + split(at), segment(arr));-----------------------------// 67
}------------------------------------------------------------------------------// cc
void erase(int at) {-----------------------------------------------------------// be
----int i = split(at); split(at + 1);------------------------------------------// da
----T.erase(T.begin() + i);----------------------------------------------------// 6b
}------------------------------------------------------------------------------// 4b
```

2.11. **Monotonic Queue.** A queue that supports querying for the minimum element. Useful for sliding window algorithms.

```
struct min_stack { // d8
----stack<int> S, M; // fe
----void push(int x) { // 20
--------S.push(x); // e2
--------M.push(M.empty() ? x : min(M.top(), x)); } // 92
----int top() { return S.top(); } // f1
----int mn() { return M.top(); } // 02
----void pop() { S.pop(); M.pop(); } // fd
----bool empty() { return S.empty(); } // d2
}; // 74
struct min_queue { // b4
----min_stack inp, outp; // 3d
----void push(int x) { inp.push(x); } // 6b
----void fix() { // 5d
--------if (outp.empty()) while (!inp.empty()) // 3b
------------outp.push(inp.top()), inp.pop(); // 8e
----} // 3f
----int top() { fix(); return outp.top(); } // dc
----int mn() { // 39
--------if (inp.empty()) return outp.mn(); // 01
--------if (outp.empty()) return inp.mn(); // 90
--------return min(inp.mn(), outp.mn()); } // 97
----void pop() { fix(); outp.pop(); } // 4f
----bool empty() { return inp.empty() && outp.empty(); } // 65
}; // 60
```

2.12. **Convex Hull Trick.**

```
struct convex_hull_trick { // 16
----vector<pair<double,double> > h; // b4
----double intersect(int i) { // 9b
--------return (h[i+1].second-h[i].second)/(h[i].first-h[i+1].first); } // b9
----void add(double m, double b) { // a4
--------h.push_back(make_pair(m,b)); // f9
--------while (size(h) >= 3) { // f6
------------int n = size(h); // d8
------------if (intersect(n-3) < intersect(n-2)) break; // 07
------------swap(h[n-2], h[n-1]); // bf
------------h.pop_back(); } } // 4b
----double get_min(double x) { // b0
--------int lo = 0, hi = size(h) - 2, res = -1; // 5b
--------while (lo <= hi) { // 24
------------int mid = lo + (hi - lo) / 2; // 5a
------------if (intersect(mid) <= x) res = mid, lo = mid + 1; // 1d
------------else hi = mid - 1; } // b6
--------return h[res+1].first * x + h[res+1].second; } }; // 84
```

3. Graphs

3.1. Single-Source Shortest Paths.

3.1.1. *Dijkstra's algorithm.* An implementation of Dijkstra's algorithm. It runs in $\Theta(|E|\log|V|)$ time.

```
int *dist, *dad; // 46
struct cmp { // a5
----bool operator()(int a, int b) { // bb
--------return dist[a] != dist[b] ? dist[a] < dist[b] : a < b; } // e6
}; // 41
pair<int*, int*> dijkstra(int n, int s, vii *adj) { // 53
----dist = new int[n]; // 84
----dad = new int[n]; // 05
----rep(i,0,n) dist[i] = INF, dad[i] = -1; // 80
----set<int, cmp> pq; // 98
----dist[s] = 0, pq.insert(s); // 1f
----while (!pq.empty()) { // 47
--------int cur = *pq.begin(); pq.erase(pq.begin()); // 58
--------rep(i,0,size(adj[cur])) { // a6
------------int nxt = adj[cur][i].first, // a4
----------------ndist = dist[cur] + adj[cur][i].second; // 3a
------------if (ndist < dist[nxt]) pq.erase(nxt), // 2d
----------------dist[nxt] = ndist, dad[nxt] = cur, pq.insert(nxt); // eb
--------} // d2
----} // df
----return pair<int*, int*>(dist, dad); // e3
} // 9b
```

3.1.2. *Bellman-Ford algorithm.* The Bellman-Ford algorithm solves the single-source shortest paths problem in $O(|V||E|)$ time. It is slower than Dijkstra's algorithm, but it works on graphs with negative edges and has the ability to detect negative cycles, neither of which Dijkstra's algorithm can do.

```
int* bellman_ford(int n, int s, vii* adj, bool& has_negative_cycle) { // cf
----has_negative_cycle = false; // 47
----int* dist = new int[n]; // 7f
----rep(i,0,n) dist[i] = i == s ? 0 : INF; // df
----rep(i,0,n-1) rep(j,0,n) if (dist[j] != INF) // 4d
--------rep(k,0,size(adj[j])) // 88
------------dist[adj[j][k].first] = min(dist[adj[j][k].first], // e1
--------------------dist[j] + adj[j][k].second); // 18
----rep(j,0,n) rep(k,0,size(adj[j])) // f8
--------if (dist[j] + adj[j][k].second < dist[adj[j][k].first]) // 37
------------has_negative_cycle = true; // f1
----return dist; // 78
} // a9
```

3.1.3. *IDA* algorithm.*

```
int n, cur[100], pos; // 48
int calch() { // 88
----int h = 0; // 4a
----rep(i,0,n) if (cur[i] != 0) h += abs(i - cur[i]); // 9b
----return h; // c6
} // c8
int dfs(int d, int g, int prev) { // 12
----int h = calch(); // 5d
----if (g + h > d) return g + h; // 15
----if (h == 0) return 0; // ff
----int mn = INF; // 7e
----rep(di,-2,3) { // 0d
```

```
--------if (di == 0) continue;--------------------------------------------------// 0a
--------int nxt = pos + di;-----------------------------------------------------// 76
--------if (nxt == prev) continue;----------------------------------------------// 39
--------if (0 <= nxt && nxt < n) {----------------------------------------------// 68
------------swap(cur[pos], cur[nxt]);------------------------------------------// 35
------------swap(pos,nxt);------------------------------------------------------// 64
------------mn = min(mn, dfs(d, g+1, nxt));-------------------------------------// 22
------------swap(pos,nxt);------------------------------------------------------// 84
------------swap(cur[pos], cur[nxt]);------------------------------------------// 3b
--------}-----------------------------------------------------------------------// 46
--------if (mn == 0) break;-----------------------------------------------------// 8f
----}---------------------------------------------------------------------------// d3
----return mn;------------------------------------------------------------------// da
}-------------------------------------------------------------------------------// f8
int idastar() {-----------------------------------------------------------------// 22
----rep(i,0,n) if (cur[i] == 0) pos = i;----------------------------------------// 6b
----int d = calch();------------------------------------------------------------// 38
----while (true) {--------------------------------------------------------------// 18
--------int nd = dfs(d, 0, -1);-------------------------------------------------// 42
--------if (nd == 0 || nd == INF) return d;-------------------------------------// b5
--------d = nd;-----------------------------------------------------------------// f7
----}---------------------------------------------------------------------------// f9
}-------------------------------------------------------------------------------// 82
```

3.2. All-Pairs Shortest Paths.

3.2.1. *Floyd-Warshall algorithm.* The Floyd-Warshall algorithm solves the all-pairs shortest paths problem in $O(|V|^3)$ time.

```
void floyd_warshall(int** arr, int n) {-----------------------------------------// 21
----rep(k,0,n) rep(i,0,n) rep(j,0,n)--------------------------------------------// af
--------if (arr[i][k] != INF && arr[k][j] != INF)-------------------------------// 84
------------arr[i][j] = min(arr[i][j], arr[i][k] + arr[k][j]);------------------// 39
}-------------------------------------------------------------------------------// bf
```

3.3. Strongly Connected Components.

3.3.1. *Kosaraju's algorithm.* Kosarajus's algorithm finds strongly connected components of a directed graph in $O(|V| + |E|)$ time.

```
#include "../data-structures/union_find.cpp"------------------------------------// 5e
--------------------------------------------------------------------------------// 11
vector<bool> visited;-----------------------------------------------------------// 66
vi order;-----------------------------------------------------------------------// 9b
--------------------------------------------------------------------------------// a5
void scc_dfs(const vvi &adj, int u) {-------------------------------------------// a1
----int v; visited[u] = true;---------------------------------------------------// e3
----rep(i,0,size(adj[u]))-------------------------------------------------------// 2d
--------if (!visited[v = adj[u][i]]) scc_dfs(adj, v);----------------------------// a2
----order.push_back(u);---------------------------------------------------------// 02
}-------------------------------------------------------------------------------// 53
--------------------------------------------------------------------------------// 63
pair<union_find, vi> scc(const vvi &adj) {--------------------------------------// c2
----int n = size(adj), u, v;----------------------------------------------------// f8
----order.clear();--------------------------------------------------------------// 20
----union_find uf(n);-----------------------------------------------------------// a8
----vi dag;---------------------------------------------------------------------// 61
----vvi rev(n);-----------------------------------------------------------------// c5
----rep(i,0,n) rep(j,0,size(adj[i])) rev[adj[i][j]].push_back(i);---------------// 7e
----visited.resize(n), fill(visited.begin(), visited.end(), false);-------------// 80
----rep(i,0,n) if (!visited[i]) scc_dfs(rev, i);--------------------------------// 4e
----fill(visited.begin(), visited.end(), false);--------------------------------// 59
----stack<int> S;---------------------------------------------------------------// bb
----for (int i = n-1; i >= 0; i--) {--------------------------------------------// 96
--------if (visited[order[i]]) continue;----------------------------------------// db
--------S.push(order[i]), dag.push_back(order[i]);------------------------------// 68
--------while (!S.empty()) {----------------------------------------------------// 9e
------------visited[u = S.top()] = true, S.pop(), uf.unite(u, order[i]);--------// b3
------------rep(j,0,size(adj[u])) if (!visited[v = adj[u][j]]) S.push(v);-------// 1b
--------}-----------------------------------------------------------------------// 61
----}---------------------------------------------------------------------------// 57
----return pair<union_find, vi>(uf, dag);---------------------------------------// 2b
}-------------------------------------------------------------------------------// 92
```

3.4. Cut Points and Bridges.

```
#define MAXN 5000---------------------------------------------------------------// f7
int low[MAXN], num[MAXN], curnum;-----------------------------------------------// d7
void dfs(const vvi &adj, vi &cp, vii &bri, int u, int p) {----------------------// 22
----low[u] = num[u] = curnum++;-------------------------------------------------// a3
----int cnt = 0; bool found = false;--------------------------------------------// 97
----rep(i,0,size(adj[u])) {-----------------------------------------------------// ae
--------int v = adj[u][i];------------------------------------------------------// 56
--------if (num[v] == -1) {-----------------------------------------------------// 3b
------------dfs(adj, cp, bri, v, u);--------------------------------------------// ba
------------low[u] = min(low[u], low[v]);---------------------------------------// be
------------cnt++;--------------------------------------------------------------// e0
------------found = found || low[v] >= num[u];----------------------------------// 30
------------if (low[v] > num[u]) bri.push_back(ii(u, v));-----------------------// bf
--------} else if (p != v) low[u] = min(low[u], num[v]); }----------------------// 76
----if (found && (p != -1 || cnt > 1)) cp.push_back(u); }-----------------------// 3e
pair<vi,vii> cut_points_and_bridges(const vvi &adj) {---------------------------// 76
----int n = size(adj);----------------------------------------------------------// c8
----vi cp; vii bri;-------------------------------------------------------------// fb
----memset(num, -1, n << 2);----------------------------------------------------// 45
----curnum = 0;-----------------------------------------------------------------// 07
----rep(i,0,n) if (num[i] == -1) dfs(adj, cp, bri, i, -1);----------------------// 7e
----return make_pair(cp, bri); }------------------------------------------------// 4c
```

3.5. Minimum Spanning Tree.

3.5.1. *Kruskal's algorithm.*

```
#include "../data-structures/union_find.cpp"------------------------------------// 5e
--------------------------------------------------------------------------------// 11
// n is the number of vertices--------------------------------------------------// 18
// edges is a list of edges of the form (weight, (a, b))------------------------// c6
// the edges in the minimum spanning tree are returned on the same form---------// 4d
vector<pair<int, ii> > mst(int n, vector<pair<int, ii> > edges) {---------------// a7
```

```
----union_find uf(n); // 04
----sort(edges.begin(), edges.end()); // 51
----vector<pair<int, ii> > res; // 71
----rep(i,0,size(edges)) // 97
--------if (uf.find(edges[i].second.first) != // bd
----------------uf.find(edges[i].second.second)) { // 85
------------res.push_back(edges[i]); // d3
------------uf.unite(edges[i].second.first, edges[i].second.second); // 6c
--------} // 37
----return res; // cb
} // 50
```

3.6. Topological Sort.

3.6.1. *Modified Depth-First Search.*

```
void tsort_dfs(int cur, char* color, const vvi& adj, stack<int>& res, // ca
--------bool& has_cycle) { // a8
----color[cur] = 1; // 5b
----rep(i,0,size(adj[cur])) { // c4
--------int nxt = adj[cur][i]; // c1
--------if (color[nxt] == 0) // dd
------------tsort_dfs(nxt, color, adj, res, has_cycle); // 12
--------else if (color[nxt] == 1) // 78
------------has_cycle = true; // c8
--------if (has_cycle) return; // 87
----} // 57
----color[cur] = 2; // 61
----res.push(cur); // 7e
} // c8
// 5e
vi tsort(int n, vvi adj, bool& has_cycle) { // 7f
----has_cycle = false; // 38
----stack<int> S; // 4f
----vi res; // a4
----char* color = new char[n]; // ba
----memset(color, 0, n); // 95
----rep(i,0,n) { // 6e
--------if (!color[i]) { // f5
------------tsort_dfs(i, color, adj, S, has_cycle); // 71
------------if (has_cycle) return res; // 14
--------} // fe
----} // 5e
----while (!S.empty()) res.push_back(S.top()), S.pop(); // 28
----return res; // 2b
} // c0
```

3.7. Euler Path.

Finds an euler path (or circuit) in a directed graph, or reports that none exist.

```
#define MAXV 1000 // 2f
#define MAXE 5000 // 87
vi adj[MAXV]; // ff
int n, m, indeg[MAXV], outdeg[MAXV], res[MAXE + 1]; // 49
ii start_end() { // 30
----int start = -1, end = -1, any = 0, c = 0; // 74
----rep(i,0,n) { // 20
--------if (outdeg[i] > 0) any = i; // 63
--------if (indeg[i] + 1 == outdeg[i]) start = i, c++; // 5a
--------else if (indeg[i] == outdeg[i] + 1) end = i, c++; // 13
--------else if (indeg[i] != outdeg[i]) return ii(-1,-1); // c1
----} // ed
----if ((start == -1) != (end == -1) || (c != 2 && c != 0)) return ii(-1,-1); // 54
----if (start == -1) start = end = any; // 5e
----return ii(start, end); // a2
} // eb
bool euler_path() { // b4
----ii se = start_end(); // 8a
----int cur = se.first, at = m + 1; // b6
----if (cur == -1) return false; // ac
----stack<int> s; // 1c
----while (true) { // b3
--------if (outdeg[cur] == 0) { // 0d
------------res[--at] = cur; // bd
------------if (s.empty()) break; // c6
------------cur = s.top(); s.pop(); // 06
--------} else s.push(cur), cur = adj[cur][--outdeg[cur]]; // 9e
----} // a4
----return at == 0; // ac
} // 22
```

3.8. Bipartite Matching.

3.8.1. *Alternating Paths algorithm.*

The alternating paths algorithm solves bipartite matching in $O(mn^2)$ time, where m, n are the number of vertices on the left and right side of the bipartite graph, respectively.

```
vi* adj; // cc
bool* done; // b1
int* owner; // 26
int alternating_path(int left) { // da
----if (done[left]) return 0; // 08
----done[left] = true; // f2
----rep(i,0,size(adj[left])) { // 1b
--------int right = adj[left][i]; // 46
--------if (owner[right] == -1 || alternating_path(owner[right])) { // f6
------------owner[right] = left; return 1; // f2
--------} } // 88
----return 0; } // 41
```

3.8.2. *Hopcroft-Karp algorithm.*

An implementation of Hopcroft-Karp algorithm for bipartite matching. Running time is $O(|E|\sqrt{|V|})$.

```
#define MAXN 5000 // f7
int dist[MAXN+1], q[MAXN+1]; // b8
#define dist(v) dist[v == -1 ? MAXN : v] // 0f
struct bipartite_graph { // 2b
----int N, M, *L, *R; vi *adj; // fc
----bipartite_graph(int _N, int _M) : N(_N), M(_M), // 8d
```

```
--------L(new int[N]), R(new int[M]), adj(new vi[N]) {}------------------------// cd
----~bipartite_graph() { delete[] adj; delete[] L; delete[] R; }---------------// 89
----bool bfs() {---------------------------------------------------------------// f5
--------int l = 0, r = 0;------------------------------------------------------// 37
--------rep(v,0,N) if(L[v] == -1) dist(v) = 0, q[r++] = v;---------------------// f9
------------else dist(v) = INF;------------------------------------------------// aa
--------dist(-1) = INF;--------------------------------------------------------// f2
--------while(l < r) {---------------------------------------------------------// ba
------------int v = q[l++];----------------------------------------------------// 50
------------if(dist(v) < dist(-1)) {-------------------------------------------// f1
----------------iter(u, adj[v]) if(dist(R[*u]) == INF)-------------------------// 9b
--------------------dist(R[*u]) = dist(v) + 1, q[r++] = R[*u];-----------------// 79
------------}------------------------------------------------------------------// b8
--------}----------------------------------------------------------------------// 0d
--------return dist(-1) != INF;------------------------------------------------// 43
----}--------------------------------------------------------------------------// 2c
----bool dfs(int v) {----------------------------------------------------------// 26
--------if(v != -1) {----------------------------------------------------------// d8
------------iter(u, adj[v])----------------------------------------------------// 99
----------------if(dist(R[*u]) == dist(v) + 1)---------------------------------// 74
--------------------if(dfs(R[*u])) {-------------------------------------------// 40
------------------------R[*u] = v, L[v] = *u;----------------------------------// 47
------------------------return true;-------------------------------------------// a2
--------------------}----------------------------------------------------------// 17
------------dist(v) = INF;-----------------------------------------------------// 62
------------return false;------------------------------------------------------// 3c
--------}----------------------------------------------------------------------// 3d
--------return true;-----------------------------------------------------------// ae
----}--------------------------------------------------------------------------// 0f
----void add_edge(int i, int j) { adj[i].push_back(j); }-----------------------// 92
----int maximum_matching() {---------------------------------------------------// a2
--------int matching = 0;------------------------------------------------------// 71
--------memset(L, -1, sizeof(int) * N);----------------------------------------// 72
--------memset(R, -1, sizeof(int) * M);----------------------------------------// bf
--------while(bfs()) rep(i,0,N)------------------------------------------------// 3e
------------matching += L[i] == -1 && dfs(i);----------------------------------// 1d
--------return matching;-------------------------------------------------------// ec
----}--------------------------------------------------------------------------// 8b
};-----------------------------------------------------------------------------// b7
```

3.9. Maximum Flow.

3.9.1. *Dinic's algorithm.* An implementation of Dinic's algorithm that runs in $O(|V|^2|E|)$. It computes the maximum flow of a flow network.

```
#define MAXV 2000------------------------------------------------------------// ba
int q[MAXV], d[MAXV];----------------------------------------------------------// e6
struct flow_network {----------------------------------------------------------// 12
----struct edge {--------------------------------------------------------------// 1e
--------int v, cap, nxt;-------------------------------------------------------// ab
--------edge() { }-------------------------------------------------------------// 38
--------edge(int _v, int _cap, int _nxt) : v(_v), cap(_cap), nxt(_nxt) { }-----// bc
----};-------------------------------------------------------------------------// 6e
----int n, ecnt, *head, *curh;-------------------------------------------------// 46
----vector<edge> e, e_store;---------------------------------------------------// 1f
----flow_network(int _n, int m = -1) : n(_n), ecnt(0) {------------------------// d3
--------e.reserve(2 * (m == -1 ? n : m));--------------------------------------// 24
--------head = new int[n], curh = new int[n];----------------------------------// 6b
--------memset(head, -1, n * sizeof(int));-------------------------------------// 56
----}--------------------------------------------------------------------------// 77
----void destroy() { delete[] head; delete[] curh; }---------------------------// f6
----void reset() { e = e_store; }----------------------------------------------// 87
----void add_edge(int u, int v, int uv, int vu = 0) {--------------------------// cd
--------e.push_back(edge(v, uv, head[u])); head[u] = ecnt++;-------------------// c9
--------e.push_back(edge(u, vu, head[v])); head[v] = ecnt++;-------------------// 89
----}--------------------------------------------------------------------------// 14
----int augment(int v, int t, int f) {-----------------------------------------// 3f
--------if (v == t) return f;--------------------------------------------------// 6d
--------for (int &i = curh[v], ret; i != -1; i = e[i].nxt)---------------------// f9
------------if (e[i].cap > 0 && d[e[i].v] + 1 == d[v])-------------------------// cc
----------------if ((ret = augment(e[i].v, t, min(f, e[i].cap))) > 0)----------// 1f
--------------------return (e[i].cap -= ret, e[i^1].cap += ret, ret);----------// ac
--------return 0;--------------------------------------------------------------// 19
----}--------------------------------------------------------------------------// fd
----int max_flow(int s, int t, bool res = true) {------------------------------// 31
--------if(s == t) return 0;---------------------------------------------------// 9d
--------e_store = e;-----------------------------------------------------------// 57
--------int f = 0, x, l, r;----------------------------------------------------// 0e
--------while (true) {---------------------------------------------------------// b5
------------memset(d, -1, n * sizeof(int));------------------------------------// a8
------------l = r = 0, d[q[r++] = t] = 0;--------------------------------------// 0e
------------while (l < r)------------------------------------------------------// 7a
----------------for (int v = q[l++], i = head[v]; i != -1; i = e[i].nxt)-------// a2
--------------------if (e[i^1].cap > 0 && d[e[i].v] == -1)---------------------// 29
------------------------d[q[r++] = e[i].v] = d[v]+1;---------------------------// 28
------------if (d[s] == -1) break;---------------------------------------------// a0
------------memcpy(curh, head, n * sizeof(int));-------------------------------// 10
------------while ((x = augment(s, t, INF)) != 0) f += x;----------------------// a6
--------}----------------------------------------------------------------------// 96
--------if (res) reset();------------------------------------------------------// 21
--------return f;--------------------------------------------------------------// b6
----}--------------------------------------------------------------------------// 1b
};-----------------------------------------------------------------------------// 3b
```

3.9.2. *Edmonds Karp's algorithm.* An implementation of Edmonds Karp's algorithm that runs in $O(|V||E|^2)$. It computes the maximum flow of a flow network.

```
#define MAXV 2000------------------------------------------------------------// ba
int q[MAXV], d[MAXV], p[MAXV];-------------------------------------------------// 7b
struct flow_network {----------------------------------------------------------// 5e
----struct edge {--------------------------------------------------------------// fc
--------int v, cap, nxt;-------------------------------------------------------// cb
--------edge(int _v, int _cap, int _nxt) : v(_v), cap(_cap), nxt(_nxt) {  }----// 7a
----};-------------------------------------------------------------------------// 31
----int n, ecnt, *head;--------------------------------------------------------// 39
```

```
----vector<edge> e, e_store;----------------------------------------------------// ea
----flow_network(int _n, int m = -1) : n(_n), ecnt(0) {-------------------------// 34
--------e.reserve(2 * (m == -1 ? n : m));---------------------------------------// 92
--------memset(head = new int[n], -1, n << 2);----------------------------------// 58
----}---------------------------------------------------------------------------// 3a
----void destroy() { delete[] head; }-------------------------------------------// d5
----void reset() { e = e_store; }-----------------------------------------------// 1b
----void add_edge(int u, int v, int uv, int vu=0) {-----------------------------// 7c
--------e.push_back(edge(v, uv, head[u])); head[u] = ecnt++;--------------------// 4c
--------e.push_back(edge(u, vu, head[v])); head[v] = ecnt++;--------------------// bc
----}---------------------------------------------------------------------------// ef
----int max_flow(int s, int t, bool res = true) {-------------------------------// 12
--------if (s == t) return 0;---------------------------------------------------// d6
--------e_store = e;------------------------------------------------------------// 9e
--------int f = 0, l, r, v;-----------------------------------------------------// 6f
--------while (true) {----------------------------------------------------------// 42
------------memset(d, -1, n << 2);----------------------------------------------// 3b
------------memset(p, -1, n << 2);----------------------------------------------// 92
------------l = r = 0, d[q[r++] = s] = 0;---------------------------------------// 5f
------------while (l < r)-------------------------------------------------------// 2c
----------------for (int u = q[l++], i = head[u]; i != -1; i = e[i].nxt)--------// c6
--------------------if (e[i].cap > 0 &&-----------------------------------------// 8a
------------------------(d[v = e[i].v] == -1 || d[u] + 1 < d[v]))---------------// 2f
------------------------d[v] = d[u] + 1, p[q[r++] = v] = i;---------------------// d5
------------if (p[t] == -1) break;----------------------------------------------// 4f
------------int x = INF, at = p[t];---------------------------------------------// b1
------------while (at != -1) x = min(x, e[at].cap), at = p[e[at^1].v];----------// 8a
------------at = p[t], f += x;--------------------------------------------------// 2d
------------while (at != -1)----------------------------------------------------// cd
----------------e[at].cap -= x, e[at^1].cap += x, at = p[e[at^1].v];------------// 2e
--------}-----------------------------------------------------------------------// 47
--------if (res) reset();-------------------------------------------------------// 3b
--------return f;---------------------------------------------------------------// bc
----}---------------------------------------------------------------------------// 05
};------------------------------------------------------------------------------// 75
```

3.10. **Minimum Cost Maximum Flow.** An implementation of Edmonds Karp's algorithm, modified to find shortest path to augment each time (instead of just any path). It computes the maximum flow of a flow network, and when there are multiple maximum flows, finds the maximum flow with minimum cost. Running time is $O(|V|^2|E|\log|V|)$. NOTE: Doesn't work on negative weights!

```
#define MAXV 2000-------------------------------------------------------------// ba
int d[MAXV], p[MAXV], pot[MAXV];------------------------------------------------// 80
struct cmp {--------------------------------------------------------------------// d1
----bool operator ()(int i, int j) {--------------------------------------------// 8a
--------return d[i] == d[j] ? i < j : d[i] < d[j];------------------------------// 89
----}---------------------------------------------------------------------------// df
};------------------------------------------------------------------------------// cf
struct flow_network {-----------------------------------------------------------// eb
----struct edge {---------------------------------------------------------------// 9a
--------int v, cap, cost, nxt;--------------------------------------------------// ad
--------edge(int _v, int _cap, int _cost, int _nxt)-----------------------------// ec
------------: v(_v), cap(_cap), cost(_cost), nxt(_nxt) { }----------------------// c4
----};--------------------------------------------------------------------------// ad
----int n, ecnt, *head;---------------------------------------------------------// 46
----vector<edge> e, e_store;----------------------------------------------------// 4b
----flow_network(int _n, int m = -1) : n(_n), ecnt(0) {-------------------------// dd
--------e.reserve(2 * (m == -1 ? n : m));---------------------------------------// e6
--------memset(head = new int[n], -1, n << 2);----------------------------------// 6c
----}---------------------------------------------------------------------------// f3
----void destroy() { delete[] head; }-------------------------------------------// ac
----void reset() { e = e_store; }-----------------------------------------------// 88
----void add_edge(int u, int v, int cost, int uv, int vu=0) {-------------------// b4
--------e.push_back(edge(v, uv, cost, head[u])); head[u] = ecnt++;--------------// 43
--------e.push_back(edge(u, vu, -cost, head[v])); head[v] = ecnt++;-------------// 53
----}---------------------------------------------------------------------------// 16
----ii min_cost_max_flow(int s, int t, bool res = true) {-----------------------// 6d
--------if (s == t) return ii(0, 0);--------------------------------------------// 34
--------e_store = e;------------------------------------------------------------// 70
--------memset(pot, 0, n << 2);-------------------------------------------------// 24
--------int f = 0, c = 0, v;----------------------------------------------------// d4
--------while (true) {----------------------------------------------------------// 29
------------memset(d, -1, n << 2);----------------------------------------------// fd
------------memset(p, -1, n << 2);----------------------------------------------// b7
------------set<int, cmp> q;----------------------------------------------------// d8
------------q.insert(s); d[s] = 0;----------------------------------------------// 1d
------------while (!q.empty()) {------------------------------------------------// 04
----------------int u = *q.begin();---------------------------------------------// dd
----------------q.erase(q.begin());---------------------------------------------// 20
----------------for (int i = head[u]; i != -1; i = e[i].nxt) {------------------// 02
--------------------if (e[i].cap == 0) continue;--------------------------------// 1c
--------------------int cd = d[u] + e[i].cost + pot[u] - pot[v = e[i].v];------// 1d
--------------------if (d[v] == -1 || cd < d[v]) {------------------------------// d2
------------------------if (q.find(v) != q.end()) q.erase(q.find(v));-----------// e2
------------------------d[v] = cd; p[v] = i;------------------------------------// f7
------------------------q.insert(v);--------------------------------------------// 74
--------------------}-----------------------------------------------------------// 6c
----------------}---------------------------------------------------------------// 1b
------------}-------------------------------------------------------------------// da
------------if (p[t] == -1) break;----------------------------------------------// 09
------------int x = INF, at = p[t];---------------------------------------------// e8
------------while (at != -1) x = min(x, e[at].cap), at = p[e[at^1].v];----------// 32
------------at = p[t], f += x;--------------------------------------------------// 43
------------while (at != -1)----------------------------------------------------// 53
----------------e[at].cap -= x, e[at^1].cap += x, at = p[e[at^1].v];------------// 95
------------c += x * (d[t] + pot[t] - pot[s]);----------------------------------// 44
------------rep(i,0,n) if (p[i] != -1) pot[i] += d[i];--------------------------// 86
--------}-----------------------------------------------------------------------// 4e
--------if (res) reset();-------------------------------------------------------// d7
--------return ii(f, c);--------------------------------------------------------// 9f
----}---------------------------------------------------------------------------// 4c
};------------------------------------------------------------------------------// ec
```

A second implementation that is slower but works on negative weights.

```
struct flow_network {-----------------------------------------------------------// 81
----struct mcmf_edge {-----------------------------------------------------------// f6
--------int u, v;---------------------------------------------------------------// e1
--------ll w, c;----------------------------------------------------------------// b4
--------mcmf_edge* rev;---------------------------------------------------------// 9d
--------mcmf_edge(int _u, int _v, ll _w, ll _c, mcmf_edge* _rev = NULL) {-------// ea
------------u = _u; v = _v; w = _w; c = _c; rev = _rev;-------------------------// 83
--------}-----------------------------------------------------------------------// 02
----};--------------------------------------------------------------------------// b9
----int n;----------------------------------------------------------------------// b4
----vector<pair<int, pair<ll, ll> > >* adj;-------------------------------------// 72
----flow_network(int _n) {------------------------------------------------------// 55
--------n = _n;-----------------------------------------------------------------// fa
--------adj = new vector<pair<int, pair<ll, ll> > >[n];------------------------// bb
----}---------------------------------------------------------------------------// bd
----void add_edge(int u, int v, ll cost, ll cap) {------------------------------// 79
--------adj[u].push_back(make_pair(v, make_pair(cap, cost)));-------------------// c8
----}---------------------------------------------------------------------------// ed
----pair<ll,ll> min_cost_max_flow(int s, int t) {-------------------------------// ea
--------vector<mcmf_edge*>* g = new vector<mcmf_edge*>[n];----------------------// ce
--------for (int i = 0; i < n; i++) {-------------------------------------------// 57
------------for (int j = 0; j < size(adj[i]); j++) {----------------------------// 37
----------------mcmf_edge *cur = new mcmf_edge(i, adj[i][j].first,--------------// 21
----------------------------adj[i][j].second.first, adj[i][j].second.second),--// 56
------------------------*rev = new mcmf_edge(adj[i][j].first, i, 0,-------------// 48
---------------------------------adj[i][j].second.second, cur);-----------------// b1
----------------cur->rev = rev;-------------------------------------------------// ef
----------------g[i].push_back(cur);--------------------------------------------// 1d
----------------g[adj[i][j].first].push_back(rev);------------------------------// 05
------------}-------------------------------------------------------------------// ba
--------}-----------------------------------------------------------------------// 83
--------ll flow = 0, cost = 0;--------------------------------------------------// 68
--------mcmf_edge** back = new mcmf_edge*[n];-----------------------------------// e5
--------ll* dist = new ll[n];---------------------------------------------------// 50
--------while (true) {----------------------------------------------------------// 65
------------for (int i = 0; i < n; i++) back[i] = NULL, dist[i] = INF;----------// d0
------------dist[s] = 0;--------------------------------------------------------// 5e
------------for (int i = 0; i < n - 1; i++)-------------------------------------// be
----------------for (int j = 0; j < n; j++)-------------------------------------// 6e
--------------------if (dist[j] != INF)-----------------------------------------// e3
------------------------for (int k = 0; k < size(g[j]); k++)--------------------// 85
----------------------------if (g[j][k]->w > 0 && dist[j] + g[j][k]->c <--------// 7f
------------------------------------dist[g[j][k]->v]) {-------------------------// 6d
--------------------------------dist[g[j][k]->v] = dist[j] + g[j][k]->c;--------// cf
--------------------------------back[g[j][k]->v] = g[j][k];---------------------// 3d
----------------------------}---------------------------------------------------// f3
------------mcmf_edge* cure = back[t];------------------------------------------// b4
------------if (cure == NULL) break;--------------------------------------------// ab
------------ll cap = INF;-------------------------------------------------------// 7a
------------while (true) {------------------------------------------------------// ad
----------------cap = min(cap, cure->w);----------------------------------------// c3
----------------if (cure->u == s) break;----------------------------------------// 82
----------------cure = back[cure->u];-------------------------------------------// 45
------------}-------------------------------------------------------------------// 91
------------assert(cap > 0 && cap < INF);---------------------------------------// ae
------------cure = back[t];-----------------------------------------------------// b9
------------while (true) {------------------------------------------------------// 2a
----------------cost += cap * cure->c;------------------------------------------// f8
----------------cure->w -= cap;-------------------------------------------------// d1
----------------cure->rev->w += cap;--------------------------------------------// cf
----------------if (cure->u == s) break;----------------------------------------// 8c
----------------cure = back[cure->u];-------------------------------------------// 60
------------}-------------------------------------------------------------------// 09
------------flow += cap;--------------------------------------------------------// f2
--------}-----------------------------------------------------------------------// be
--------// instead of deleting g, we could also---------------------------------// e0
--------// use it to get info about the actual flow-----------------------------// 6c
--------for (int i = 0; i < n; i++)---------------------------------------------// eb
------------for (int j = 0; j < size(g[i]); j++)--------------------------------// 82
----------------delete g[i][j];-------------------------------------------------// 06
--------delete[] g;-------------------------------------------------------------// 23
--------delete[] back;----------------------------------------------------------// 5a
--------delete[] dist;----------------------------------------------------------// b9
--------return make_pair(flow, cost);-------------------------------------------// ec
----}---------------------------------------------------------------------------// ad
};------------------------------------------------------------------------------// bf
```

3.11. **All Pairs Maximum Flow.**

3.11.1. *Gomory-Hu Tree.* An implementation of the Gomory-Hu Tree. The spanning tree is constructed using Gusfield's algorithm in $O(|V|^2)$ plus $|V|-1$ times the time it takes to calculate the maximum flow. If Dinic's algorithm is used to calculate the max flow, the running time is $O(|V|^3|E|)$. NOTE: Not sure if it works correctly with disconnected graphs.

```
#include "dinic.cpp"------------------------------------------------------------// 58
--------------------------------------------------------------------------------// 25
bool same[MAXV];----------------------------------------------------------------// 59
pair<vii, vvi> construct_gh_tree(flow_network &g) {-----------------------------// 77
----int n = g.n, v;-------------------------------------------------------------// 5d
----vii par(n, ii(0, 0)); vvi cap(n, vi(n, -1));--------------------------------// 49
----rep(s,1,n) {----------------------------------------------------------------// 9e
--------int l = 0, r = 0;-------------------------------------------------------// 08
--------par[s].second = g.max_flow(s, par[s].first, false);---------------------// 54
--------memset(d, 0, n * sizeof(int));------------------------------------------// c8
--------memset(same, 0, n * sizeof(bool));--------------------------------------// c9
--------d[q[r++] = s] = 1;------------------------------------------------------// dd
--------while (l < r) {---------------------------------------------------------// 45
------------same[v = q[l++]] = true;--------------------------------------------// c5
------------for (int i = g.head[v]; i != -1; i = g.e[i].nxt)--------------------// 66
----------------if (g.e[i].cap > 0 && d[g.e[i].v] == 0)-------------------------// 21
--------------------d[q[r++] = g.e[i].v] = 1;-----------------------------------// dd
--------}-----------------------------------------------------------------------// 44
--------rep(i,s+1,n)------------------------------------------------------------// 71
------------if (par[i].first == par[s].first && same[i]) par[i].first = s;------// 97
```

```
--------g.reset();-------------------------------------------------------------// d8
----}--------------------------------------------------------------------------// 93
----rep(i,0,n) {---------------------------------------------------------------// 14
--------int mn = INF, cur = i;-------------------------------------------------// 59
--------while (true) {---------------------------------------------------------// b8
------------cap[cur][i] = mn;--------------------------------------------------// 8d
------------if (cur == 0) break;-----------------------------------------------// fb
------------mn = min(mn, par[cur].second), cur = par[cur].first;---------------// 4d
--------}----------------------------------------------------------------------// aa
----}--------------------------------------------------------------------------// 90
----return make_pair(par, cap);------------------------------------------------// 62
}------------------------------------------------------------------------------// b3
int compute_max_flow(int s, int t, const pair<vii, vvi> &gh) {-----------------// 93
----if (s == t) return 0;------------------------------------------------------// 33
----int cur = INF, at = s;-----------------------------------------------------// e7
----while (gh.second[at][t] == -1)---------------------------------------------// 42
--------cur = min(cur, gh.first[at].second), at = gh.first[at].first;----------// 8d
----return min(cur, gh.second[at][t]);-----------------------------------------// 54
}------------------------------------------------------------------------------// 46
```

3.12. **Heavy-Light Decomposition.**

```
#include "../data-structures/segment_tree.cpp"---------------------------------// 16
struct HLD {-------------------------------------------------------------------// 25
----int n, curhead, curloc;----------------------------------------------------// d9
----vi sz, head, parent, loc;--------------------------------------------------// 81
----vvi adj; segment_tree values;----------------------------------------------// 13
----HLD(int _n) : n(_n), sz(n, 1), head(n), parent(n, -1), loc(n), adj(n) {----// 1c
--------vi tmp(n, ID); values = segment_tree(tmp); }---------------------------// f0
----void add_edge(int u, int v) { adj[u].push_back(v), adj[v].push_back(u); }--// 77
----void update_cost(int u, int v, int c) {------------------------------------// 7b
--------if (parent[v] == u) swap(u, v); assert(parent[u] == v);----------------// db
--------values.update(loc[u], c); }--------------------------------------------// 50
----int csz(int u) {-----------------------------------------------------------// 7c
--------rep(i,0,size(adj[u])) if (adj[u][i] != parent[u])----------------------// a5
------------sz[u] += csz(adj[parent[adj[u][i]] = u][i]);-----------------------// c2
--------return sz[u]; }--------------------------------------------------------// 75
----void part(int u) {---------------------------------------------------------// c3
--------head[u] = curhead; loc[u] = curloc++;----------------------------------// 63
--------int best = -1;---------------------------------------------------------// 27
--------rep(i,0,size(adj[u]))--------------------------------------------------// 49
------------if (adj[u][i] != parent[u] && (best == -1 || sz[adj[u][i]] > sz[best]))
----------------best = adj[u][i];----------------------------------------------// 26
--------if (best != -1) part(best);--------------------------------------------// c4
--------rep(i,0,size(adj[u]))--------------------------------------------------// 92
------------if (adj[u][i] != parent[u] && adj[u][i] != best)-------------------// e8
----------------part(curhead = adj[u][i]); }-----------------------------------// 88
----void build(int r = 0) { curloc = 0, csz(curhead = r), part(r); }-----------// 78
----int lca(int u, int v) {----------------------------------------------------// 74
--------vi uat, vat; int res = -1;---------------------------------------------// 43
--------while (u != -1) uat.push_back(u), u = parent[head[u]];-----------------// 51
--------while (v != -1) vat.push_back(v), v = parent[head[v]];-----------------// 6d
--------u = size(uat) - 1, v = size(vat) - 1;----------------------------------// 8a
--------while (u >= 0 && v >= 0 && head[uat[u]] == head[vat[v]])---------------// ae
------------res = (loc[uat[u]] < loc[vat[v]] ? uat[u] : vat[v]), u--, v--;-----// a2
--------return res; }----------------------------------------------------------// 91
----int query_upto(int u, int v) { int res = ID;-------------------------------// 72
--------while (head[u] != head[v])---------------------------------------------// 69
------------res = f(res, values.query(loc[head[u]], loc[u])),------------------// a4
------------u = parent[head[u]];-----------------------------------------------// 8c
--------return f(res, values.query(loc[v] + 1, loc[u])); }---------------------// ea
----int query(int u, int v) { int l = lca(u, v);-------------------------------// 53
--------return f(query_upto(u, l), query_upto(v, l)); } };---------------------// 5b
```

3.13. **Centroid Decomposition.**

```
#define MAXV 100100------------------------------------------------------------// 86
#define LGMAXV 20--------------------------------------------------------------// aa
int jmp[MAXV][LGMAXV],---------------------------------------------------------// 6d
----path[MAXV][LGMAXV],--------------------------------------------------------// 9d
----sz[MAXV], seph[MAXV],------------------------------------------------------// cf
----shortest[MAXV];------------------------------------------------------------// 6b
struct centroid_decomposition {------------------------------------------------// 99
----int n; vvi adj;------------------------------------------------------------// e9
----centroid_decomposition(int _n) : n(_n), adj(n) { }-------------------------// 46
----void add_edge(int a, int b) { adj[a].push_back(b), adj[b].push_back(a); }--// bc
----int dfs(int u, int p) {----------------------------------------------------// 8f
--------sz[u] = 1;-------------------------------------------------------------// c8
--------rep(i,0,size(adj[u])) if (adj[u][i] != p) sz[u] += dfs(adj[u][i], u);--// 78
--------return sz[u]; }--------------------------------------------------------// f4
----void makepaths(int sep, int u, int p, int len) {---------------------------// 84
--------jmp[u][seph[sep]] = sep, path[u][seph[sep]] = len;---------------------// d9
--------int bad = -1;----------------------------------------------------------// af
--------rep(i,0,size(adj[u])) {------------------------------------------------// f4
------------if (adj[u][i] == p) bad = i;---------------------------------------// cf
------------else makepaths(sep, adj[u][i], u, len + 1);------------------------// f2
--------}----------------------------------------------------------------------// 8a
--------if (p == sep) swap(adj[u][bad], adj[u].back()), adj[u].pop_back(); }---// 07
----void separate(int h=0, int u=0) {------------------------------------------// 03
--------dfs(u,-1); int sep = u;------------------------------------------------// b5
--------down: iter(nxt,adj[sep])-----------------------------------------------// 04
------------if (sz[*nxt] < sz[sep] && sz[*nxt] > sz[u]/2) {--------------------// db
----------------sep = *nxt; goto down; }---------------------------------------// 1a
--------seph[sep] = h, makepaths(sep, sep, -1, 0);-----------------------------// ed
--------rep(i,0,size(adj[sep])) separate(h+1, adj[sep][i]); }------------------// 90
----void paint(int u) {--------------------------------------------------------// bd
--------rep(h,0,seph[u]+1)-----------------------------------------------------// c5
------------shortest[jmp[u][h]] = min(shortest[jmp[u][h]], path[u][h]); }------// 11
----int closest(int u) {-------------------------------------------------------// 91
--------int mn = INF/2;--------------------------------------------------------// fe
--------rep(h,0,seph[u]+1) mn = min(mn, path[u][h] + shortest[jmp[u][h]]);----// 3e
--------return mn; } };--------------------------------------------------------// 13
```

3.14. **Tarjan's Off-line Lowest Common Ancestors Algorithm.**

```
#include "../data-structures/union_find.cpp"------------------------------------// 5e
struct tarjan_olca {------------------------------------------------------------// 87
----int *ancestor;--------------------------------------------------------------// 39
----vi *adj, answers;-----------------------------------------------------------// dd
----vii *queries;---------------------------------------------------------------// 66
----bool *colored;--------------------------------------------------------------// 97
----union_find uf;--------------------------------------------------------------// 70
----tarjan_olca(int n, vi *_adj) : adj(_adj), uf(n) {---------------------------// 78
--------colored = new bool[n];--------------------------------------------------// 8d
--------ancestor = new int[n];--------------------------------------------------// f2
--------queries = new vii[n];---------------------------------------------------// 3e
--------memset(colored, 0, n);--------------------------------------------------// 6e
----}---------------------------------------------------------------------------// 6b
----void query(int x, int y) {--------------------------------------------------// d3
--------queries[x].push_back(ii(y, size(answers)));-----------------------------// a0
--------queries[y].push_back(ii(x, size(answers)));-----------------------------// 14
--------answers.push_back(-1);--------------------------------------------------// ca
----}---------------------------------------------------------------------------// 6b
----void process(int u) {-------------------------------------------------------// 85
--------ancestor[u] = u;--------------------------------------------------------// 1a
--------rep(i,0,size(adj[u])) {-------------------------------------------------// ce
------------int v = adj[u][i];--------------------------------------------------// dd
------------process(v);---------------------------------------------------------// e8
------------uf.unite(u,v);------------------------------------------------------// 55
------------ancestor[uf.find(u)] = u;-------------------------------------------// 1d
--------}-----------------------------------------------------------------------// 57
--------colored[u] = true;------------------------------------------------------// b9
--------rep(i,0,size(queries[u])) {---------------------------------------------// d7
------------int v = queries[u][i].first;----------------------------------------// 89
------------if (colored[v]) {---------------------------------------------------// cb
----------------answers[queries[u][i].second] = ancestor[uf.find(v)];-----------// 63
------------}-------------------------------------------------------------------// d0
--------}-----------------------------------------------------------------------// 40
----}---------------------------------------------------------------------------// a9
};------------------------------------------------------------------------------// 1e
```

3.15. **Maximum Density Subgraph.** Given (weighted) undirected graph G. Binary search density. If g is current density, construct flow network: (S, u, m), $(u, T, m + 2g - d_u)$, $(u, v, 1)$, where m is a large constant (larger than sum of edge weights). Run floating-point max-flow. If minimum cut has empty S-component, then maximum density is smaller than g, otherwise it's larger. Distance between valid densities is at least $1/(n(n-1))$. Edge case when density is 0. This also works for weighted graphs by replacing d_u be the weighted degree, and doing more iterations (if weights are not integers).

3.16. **Maximum Weighted Independent Set in a Bipartite Graph.** This is the same as the minimum weighted vertex cover. Solve this by constructing a flow network with edges $(S, u, w(u))$ for $u \in L$, $(v, T, w(v))$ for $v \in R$ and (u, v, ∞) for $(u, v) \in E$. The minimum S, T-cut is the answer. Vertices adjacent to a cut edge are in the vertex cover.

4. Strings

4.1. **The Knuth-Morris-Pratt algorithm.** An implementation of the Knuth-Morris-Pratt algorithm. Runs in $O(n + m)$ time, where n and m are the lengths of the string and the pattern.

```
int* compute_pi(const string &t) {----------------------------------------------// a2
----int m = t.size();-----------------------------------------------------------// 8b
----int *pit = new int[m + 1];--------------------------------------------------// 8e
----if (0 <= m) pit[0] = 0;-----------------------------------------------------// 42
----if (1 <= m) pit[1] = 0;-----------------------------------------------------// 34
----rep(i,2,m+1) {--------------------------------------------------------------// 0f
--------for (int j = pit[i - 1]; ; j = pit[j]) {--------------------------------// b5
------------if (t[j] == t[i - 1]) { pit[i] = j + 1; break; }--------------------// 21
------------if (j == 0) { pit[i] = 0; break; }----------------------------------// 95
--------}-----------------------------------------------------------------------// c9
----}---------------------------------------------------------------------------// eb
----return pit; }---------------------------------------------------------------// e8
int string_match(const string &s, const string &t) {----------------------------// 9e
----int n = s.size(), m = t.size();---------------------------------------------// 92
----int *pit = compute_pi(t);---------------------------------------------------// 72
----for (int i = 0, j = 0; i < n; ) {-------------------------------------------// 27
--------if (s[i] == t[j]) {-----------------------------------------------------// 73
------------i++; j++;-----------------------------------------------------------// 7e
------------if (j == m) {-------------------------------------------------------// de
----------------return i - m;---------------------------------------------------// e9
----------------// or j = pit[j];-----------------------------------------------// ce
------------}-------------------------------------------------------------------// 85
--------}-----------------------------------------------------------------------// 35
--------else if (j > 0) j = pit[j];---------------------------------------------// 43
--------else i++; }-------------------------------------------------------------// b8
----delete[] pit; return -1; }--------------------------------------------------// e3
```

4.2. **The Z algorithm.** Given a string S, $Z_i(S)$ is the longest substring of S starting at i that is also a prefix of S. The Z algorithm computes these Z values in $O(n)$ time, where $n = |S|$. Z values can, for example, be used to find all occurrences of a pattern P in a string T in linear time. This is accomplished by computing Z values of $S = TP$, and looking for all i such that $Z_i \geq |T|$.

```
int* z_values(const string &s) {------------------------------------------------// 4d
----int n = size(s);------------------------------------------------------------// 97
----int* z = new int[n];--------------------------------------------------------// c4
----int l = 0, r = 0;-----------------------------------------------------------// 1c
----z[0] = n;-------------------------------------------------------------------// 98
----rep(i,1,n) {----------------------------------------------------------------// b2
--------z[i] = 0;---------------------------------------------------------------// 4c
--------if (i > r) {------------------------------------------------------------// 6d
------------l = r = i;----------------------------------------------------------// 24
------------while (r < n && s[r - l] == s[r]) r++;------------------------------// 68
------------z[i] = r - l; r--;--------------------------------------------------// 07
--------} else if (z[i - l] < r - i + 1) z[i] = z[i - l];-----------------------// 6f
--------else {------------------------------------------------------------------// a8
------------l = i;--------------------------------------------------------------// 55
------------while (r < n && s[r - l] == s[r]) r++;------------------------------// 2c
------------z[i] = r - l; r--; } }----------------------------------------------// 13
----return z;-------------------------------------------------------------------// 78
}-------------------------------------------------------------------------------// 16
```

4.3. **Trie.** A Trie class.

```
template <class T>-------------------------------------------------------------// 82
struct trie {------------------------------------------------------------------// 4a
----struct node {--------------------------------------------------------------// 39
--------map<T, node*> children;------------------------------------------------// 82
--------int prefixes, words;---------------------------------------------------// ff
--------node() { prefixes = words = 0; } };------------------------------------// 16
----node* root;----------------------------------------------------------------// 97
----trie() : root(new node()) {  }---------------------------------------------// d2
----template <class I>---------------------------------------------------------// 2f
----void insert(I begin, I end) {----------------------------------------------// 3b
--------node* cur = root;------------------------------------------------------// ae
--------while (true) {---------------------------------------------------------// 03
------------cur->prefixes++;---------------------------------------------------// 6c
------------if (begin == end) { cur->words++; break; }-------------------------// df
------------else {-------------------------------------------------------------// 51
----------------T head = *begin;-----------------------------------------------// 8f
----------------typename map<T, node*>::const_iterator it;---------------------// ff
----------------it = cur->children.find(head);---------------------------------// 57
----------------if (it == cur->children.end()) {-------------------------------// f7
--------------------pair<T, node*> nw(head, new node());-----------------------// 66
--------------------it = cur->children.insert(nw).first;-----------------------// c5
----------------} begin++, cur = it->second; } } }-----------------------------// 68
----template<class I>----------------------------------------------------------// 51
----int countMatches(I begin, I end) {-----------------------------------------// 84
--------node* cur = root;------------------------------------------------------// 88
--------while (true) {---------------------------------------------------------// 5b
------------if (begin == end) return cur->words;-------------------------------// 61
------------else {-------------------------------------------------------------// c1
----------------T head = *begin;-----------------------------------------------// 75
----------------typename map<T, node*>::const_iterator it;---------------------// 00
----------------it = cur->children.find(head);---------------------------------// c6
----------------if (it == cur->children.end()) return 0;-----------------------// 06
----------------begin++, cur = it->second; } } }-------------------------------// 85
----template<class I>----------------------------------------------------------// e7
----int countPrefixes(I begin, I end) {----------------------------------------// 7d
--------node* cur = root;------------------------------------------------------// c6
--------while (true) {---------------------------------------------------------// ac
------------if (begin == end) return cur->prefixes;----------------------------// 33
------------else {-------------------------------------------------------------// 85
----------------T head = *begin;-----------------------------------------------// 0e
----------------typename map<T, node*>::const_iterator it;---------------------// 6e
----------------it = cur->children.find(head);---------------------------------// 40
----------------if (it == cur->children.end()) return 0;-----------------------// 18
----------------begin++, cur = it->second; } } } };----------------------------// 7a
```

4.4. **Suffix Array.** An $O(n \log^2 n)$ construction of a Suffix Tree.

```
struct entry { ii nr; int p; };------------------------------------------------// f9
bool operator <(const entry &a, const entry &b) { return a.nr < b.nr; }--------// 77
struct suffix_array {----------------------------------------------------------// 87
----string s; int n; vvi P; vector<entry> L; vi idx;---------------------------// b6
----suffix_array(string _s) : s(_s), n(size(s)) {------------------------------// a3
--------L = vector<entry>(n), P.push_back(vi(n)), idx = vi(n);-----------------// 12
--------rep(i,0,n) P[0][i] = s[i];---------------------------------------------// 5c
--------for (int stp = 1, cnt = 1; cnt >> 1 < n; stp++, cnt <<= 1) {-----------// 86
------------P.push_back(vi(n));------------------------------------------------// 53
------------rep(i,0,n)---------------------------------------------------------// 6f
----------------L[L[i].p = i].nr = ii(P[stp - 1][i],---------------------------// e2
------------------------i + cnt < n ? P[stp - 1][i + cnt] : -1);---------------// 43
------------sort(L.begin(), L.end());------------------------------------------// 5f
------------rep(i,0,n)---------------------------------------------------------// a8
----------------P[stp][L[i].p] = i > 0 &&--------------------------------------// 3a
--------------------L[i].nr == L[i - 1].nr ? P[stp][L[i - 1].p] : i;-----------// 55
--------}----------------------------------------------------------------------// 8b
--------rep(i,0,n) idx[P[size(P) - 1][i]] = i;---------------------------------// 17
----}--------------------------------------------------------------------------// d9
----int lcp(int x, int y) {----------------------------------------------------// 71
--------int res = 0;-----------------------------------------------------------// d6
--------if (x == y) return n - x;----------------------------------------------// bc
--------for (int k = size(P) - 1; k >= 0 && x < n && y < n; k--)---------------// fe
------------if (P[k][x] == P[k][y]) x += 1 << k, y += 1 << k, res += 1 << k;---// b7
--------return res;------------------------------------------------------------// bc
----}--------------------------------------------------------------------------// f1
};-----------------------------------------------------------------------------// f6
```

4.5. **Aho-Corasick Algorithm.** An implementation of the Aho-Corasick algorithm. Constructs a state machine from a set of keywords which can be used to search a string for any of the keywords.

```
struct aho_corasick {----------------------------------------------------------// 78
----struct out_node {----------------------------------------------------------// 3e
--------string keyword; out_node *next;----------------------------------------// f0
--------out_node(string k, out_node *n) : keyword(k), next(n) { }--------------// 26
----};-------------------------------------------------------------------------// b9
----struct go_node {-----------------------------------------------------------// 40
--------map<char, go_node*> next;----------------------------------------------// 6b
--------out_node *out; go_node *fail;------------------------------------------// 3e
--------go_node() { out = NULL; fail = NULL; }---------------------------------// 0f
----};-------------------------------------------------------------------------// c0
----go_node *go;---------------------------------------------------------------// b8
----aho_corasick(vector<string> keywords) {------------------------------------// 4b
--------go = new go_node();----------------------------------------------------// 77
--------iter(k, keywords) {----------------------------------------------------// f2
------------go_node *cur = go;-------------------------------------------------// a2
------------iter(c, *k)--------------------------------------------------------// 6e
----------------cur = cur->next.find(*c) != cur->next.end() ? cur->next[*c] :--// 97
--------------------(cur->next[*c] = new go_node());---------------------------// af
------------cur->out = new out_node(*k, cur->out);-----------------------------// 3f
--------}----------------------------------------------------------------------// eb
--------queue<go_node*> q;-----------------------------------------------------// 2c
--------iter(a, go->next) q.push(a->second);-----------------------------------// db
--------while (!q.empty()) {---------------------------------------------------// 07
------------go_node *r = q.front(); q.pop();-----------------------------------// e0
------------iter(a, r->next) {-------------------------------------------------// 18
----------------go_node *s = a->second;----------------------------------------// 55
```

```
----------------q.push(s);-----------------------------------------------------// b5
----------------go_node *st = r->fail;-----------------------------------------// 53
----------------while (st && st->next.find(a->first) == st->next.end())--------// 0e
--------------------st = st->fail;---------------------------------------------// b3
----------------if (!st) st = go;----------------------------------------------// 0b
----------------s->fail = st->next[a->first];----------------------------------// c1
----------------if (s->fail) {-------------------------------------------------// 98
--------------------if (!s->out) s->out = s->fail->out;------------------------// ad
--------------------else {-----------------------------------------------------// 5b
------------------------out_node* out = s->out;--------------------------------// b8
------------------------while (out->next) out = out->next;---------------------// b4
------------------------out->next = s->fail->out;------------------------------// 62
--------------------}----------------------------------------------------------// a6
----------------}--------------------------------------------------------------// 81
------------}------------------------------------------------------------------// 55
--------}----------------------------------------------------------------------// bf
----}--------------------------------------------------------------------------// de
----vector<string> search(string s) {------------------------------------------// c4
--------vector<string> res;----------------------------------------------------// 79
--------go_node *cur = go;-----------------------------------------------------// 85
--------iter(c, s) {-----------------------------------------------------------// 57
------------while (cur && cur->next.find(*c) == cur->next.end())---------------// df
----------------cur = cur->fail;-----------------------------------------------// b1
------------if (!cur) cur = go;------------------------------------------------// 92
------------cur = cur->next[*c];-----------------------------------------------// 97
------------if (!cur) cur = go;------------------------------------------------// 01
------------for (out_node *out = cur->out; out; out = out->next)---------------// d7
----------------res.push_back(out->keyword);-----------------------------------// 7c
--------}----------------------------------------------------------------------// 56
--------return res;------------------------------------------------------------// 6b
----}--------------------------------------------------------------------------// 3e
};-----------------------------------------------------------------------------// de
```

4.6. **Eertree.** Constructs an Eertree in $O(n)$, one character at a time.

```
#define MAXN 100100------------------------------------------------------------// 29
#define SIGMA 26---------------------------------------------------------------// e2
#define BASE 'a'---------------------------------------------------------------// a1
char *s = new char[MAXN];------------------------------------------------------// db
struct state {-----------------------------------------------------------------// 33
----int len, link, to[SIGMA];--------------------------------------------------// 24
} *st = new state[MAXN+2];-----------------------------------------------------// 57
struct eertree {---------------------------------------------------------------// 78
----int last, sz, n;-----------------------------------------------------------// ba
----eertree() : last(1), sz(2), n(0) {-----------------------------------------// 83
--------st[0].len = st[0].link = -1;-------------------------------------------// 3f
--------st[1].len = st[1].link = 0; }------------------------------------------// 34
----int extend() {-------------------------------------------------------------// c2
--------char c = s[n++]; int p = last;-----------------------------------------// 25
--------while (n - st[p].len - 2 < 0 || c != s[n - st[p].len - 2]) p = st[p].link;
--------if (!st[p].to[c-BASE]) {-----------------------------------------------// 82
------------int q = last = sz++;-----------------------------------------------// 42
------------st[p].to[c-BASE] = q;----------------------------------------------// fc
------------st[q].len = st[p].len + 2;-----------------------------------------// c5
------------do { p = st[p].link;-----------------------------------------------// 04
------------} while (p != -1 && (n < st[p].len + 2 || c != s[n - st[p].len - 2]));
------------if (p == -1) st[q].link = 1;---------------------------------------// 77
------------else st[q].link = st[p].to[c-BASE];--------------------------------// 6a
------------return 1; }--------------------------------------------------------// 29
--------last = st[p].to[c-BASE];-----------------------------------------------// 42
--------return 0; } };---------------------------------------------------------// ec
```

4.7. **Suffix Automaton.** Minimum automata that accepts all suffixes of a string with $O(n)$ construction. The automata itself is a DAG therefore suitable for DP, examples are counting unique substrings, occurrences of substrings and suffix.

```
// TODO: Add longest common subsring-------------------------------------------// 0e
const int MAXL = 100000;-------------------------------------------------------// 31
struct suffix_automaton {------------------------------------------------------// e0
----vi  len, link, occur, cnt;-------------------------------------------------// 78
----vector<map<char,int> > next;-----------------------------------------------// 90
----vector<bool> isclone;------------------------------------------------------// 7b
----ll *occuratleast;----------------------------------------------------------// f2
----int sz, last;--------------------------------------------------------------// 7d
----string s;------------------------------------------------------------------// f2
----suffix_automaton() : len(MAXL*2), link(MAXL*2), occur(MAXL*2), next(MAXL*2),
----isclone(MAXL*2) { clear(); }-----------------------------------------------// a3
----void clear(){ sz = 1; last = len[0] = 0; link[0] = -1; next[0].clear();----// aa
--------isclone[0] = false; }--------------------------------------------------// 26
----bool issubstr(string other){-----------------------------------------------// 3b
--------for(int i = 0, cur = 0; i < size(other); ++i){-------------------------// 7f
------------if(cur == -1) return false; cur = next[cur][other[i]]; }-----------// 54
--------return true; }---------------------------------------------------------// 1a
----void extend(char c){ int cur = sz++; len[cur] = len[last] + 1;-------------// 1d
--------next[cur].clear(); isclone[cur] = false; int p = last;-----------------// a9
--------for(; p != -1 && !next[p].count(c); p = link[p]){ next[p][c] = cur; }--// 6f
--------if(p == -1){ link[cur] = 0; }------------------------------------------// 18
--------else{ int q = next[p][c];----------------------------------------------// 34
------------if(len[p] + 1 == len[q]){ link[cur] = q; }-------------------------// 4d
------------else { int clone = sz++; isclone[clone] = true;--------------------// 57
----------------len[clone] = len[p] + 1;---------------------------------------// 8c
----------------link[clone] = link[q]; next[clone] = next[q];------------------// 76
----------------for(; p != -1 && next[p].count(c) && next[p][c] == q; p = link[p]){
--------------------next[p][c] = clone; }--------------------------------------// 32
----------------link[q] = link[cur] = clone;-----------------------------------// 73
------------} } last = cur; }--------------------------------------------------// b9
----void count(){--------------------------------------------------------------// e7
--------cnt=vi(sz, -1); stack<ii> S; S.push(ii(0,0));map<char,int>::iterator i;// 56
--------while(!S.empty()){-----------------------------------------------------// 4c
------------ii cur = S.top(); S.pop();-----------------------------------------// 67
------------if(cur.second){----------------------------------------------------// 78
----------------for(i = next[cur.first].begin();i != next[cur.first].end();++i){
--------------------cnt[cur.first] += cnt[(*i).second]; } }--------------------// da
------------else if(cnt[cur.first] == -1){-------------------------------------// 99
```

```
----------------cnt[cur.first] = 1; S.push(ii(cur.first, 1));------------------// bd
----------------for(i = next[cur.first].begin();i != next[cur.first].end();++i){
--------------------S.push(ii((*i).second, 0)); } } } }------------------------// 61
----string lexicok(ll k){------------------------------------------------------// 8b
--------int st = 0; string res; map<char,int>::iterator i;---------------------// cf
--------while(k){ for(i = next[st].begin(); i != next[st].end(); ++i){---------// 69
----------------if(k <= cnt[(*i).second]){ st = (*i).second; ------------------// ec
--------------------res.push_back((*i).first); k--; break;---------------------// 63
----------------} else { k -= cnt[(*i).second]; } } }--------------------------// ee
--------return res; }----------------------------------------------------------// 0b
----void countoccur(){---------------------------------------------------------// ad
--------for(int i = 0; i < sz; ++i){ occur[i] = 1 - isclone[i]; }--------------// 1b
--------vii states(sz);--------------------------------------------------------// dc
--------for(int i = 0; i < sz; ++i){ states[i] = ii(len[i],i); }---------------// 97
--------sort(states.begin(), states.end());------------------------------------// 8d
--------for(int i = size(states)-1; i >= 0; --i){ int v = states[i].second;----// a4
------------if(link[v] != -1) { occur[link[v]] += occur[v]; } } }--------------// cc
};-----------------------------------------------------------------------------// 32
-------------------------------------------------------------------------------// 56
```

5. Mathematics

5.1. Fraction. A fraction (rational number) class. Note that numbers are stored in lowest common terms.

```
template <class T> struct fraction {-------------------------------------------// 27
----T gcd(T a, T b) { return b == T(0) ? a : gcd(b, a % b); }------------------// fe
----T n, d;--------------------------------------------------------------------// 6a
----fraction(T n_, T d_) {-----------------------------------------------------// b0
--------assert(d_ != 0);-------------------------------------------------------// 8c
--------n = n_, d = d_;--------------------------------------------------------// ad
--------if (d < T(0)) n = -n, d = -d;------------------------------------------// bc
--------T g = gcd(abs(n), abs(d));---------------------------------------------// 95
--------n /= g, d /= g; }------------------------------------------------------// 53
----fraction(T n_) : n(n_), d(1) { }-------------------------------------------// ec
----fraction(const fraction<T>& other) : n(other.n), d(other.d) { }------------// a6
----fraction<T> operator +(const fraction<T>& other) const {-------------------// 24
--------return fraction<T>(n * other.d + other.n * d, d * other.d);}-----------// d1
----fraction<T> operator -(const fraction<T>& other) const {-------------------// 89
--------return fraction<T>(n * other.d - other.n * d, d * other.d);}-----------// bf
----fraction<T> operator *(const fraction<T>& other) const {-------------------// 10
--------return fraction<T>(n * other.n, d * other.d); }------------------------// b4
----fraction<T> operator /(const fraction<T>& other) const {-------------------// 33
--------return fraction<T>(n * other.d, d * other.n); }------------------------// bc
----bool operator <(const fraction<T>& other) const {--------------------------// e8
--------return n * other.d < other.n * d; }------------------------------------// cc
----bool operator <=(const fraction<T>& other) const {-------------------------// 89
--------return !(other < *this); }---------------------------------------------// 90
----bool operator >(const fraction<T>& other) const {--------------------------// e7
--------return other < *this; }------------------------------------------------// 24
----bool operator >=(const fraction<T>& other) const {-------------------------// 94
--------return !(*this < other); }---------------------------------------------// 18
----bool operator ==(const fraction<T>& other) const {-------------------------// b8
--------return n == other.n && d == other.d; }---------------------------------// 6f
----bool operator !=(const fraction<T>& other) const {-------------------------// 5d
--------return !(*this == other); } };-----------------------------------------// 8f
```

5.2. Big Integer. A big integer class.

```
struct intx {------------------------------------------------------------------// cf
----intx() { normalize(1); }---------------------------------------------------// 6c
----intx(string n) { init(n); }------------------------------------------------// b9
----intx(int n) { stringstream ss; ss << n; init(ss.str()); }------------------// 36
----intx(const intx& other) : sign(other.sign), data(other.data) { }-----------// 3b
----int sign;------------------------------------------------------------------// 26
----vector<unsigned int> data;-------------------------------------------------// 19
----static const int dcnt = 9;-------------------------------------------------// 12
----static const unsigned int radix = 1000000000U;-----------------------------// f0
----int size() const { return data.size(); }-----------------------------------// 29
----void init(string n) {------------------------------------------------------// 13
--------intx res; res.data.clear();--------------------------------------------// 4e
--------if (n.empty()) n = "0";------------------------------------------------// 99
--------if (n[0] == '-') res.sign = -1, n = n.substr(1);-----------------------// 3b
--------for (int i = n.size() - 1; i >= 0; i -= intx::dcnt) {------------------// e7
------------unsigned int digit = 0;--------------------------------------------// 98
------------for (int j = intx::dcnt - 1; j >= 0; j--) {------------------------// 72
----------------int idx = i - j;-----------------------------------------------// cd
----------------if (idx < 0) continue;-----------------------------------------// 52
----------------digit = digit * 10 + (n[idx] - '0');---------------------------// 1f
------------}------------------------------------------------------------------// c0
------------res.data.push_back(digit);-----------------------------------------// 07
--------}----------------------------------------------------------------------// fb
--------data = res.data;-------------------------------------------------------// 7d
--------normalize(res.sign);---------------------------------------------------// 76
----}--------------------------------------------------------------------------// 6e
----intx& normalize(int nsign) {-----------------------------------------------// 3b
--------if (data.empty()) data.push_back(0);-----------------------------------// fa
--------for (int i = data.size() - 1; i > 0 && data[i] == 0; i--)--------------// 27
------------data.erase(data.begin() + i);--------------------------------------// 67
--------sign = data.size() == 1 && data[0] == 0 ? 1 : nsign;-------------------// ff
--------return *this;----------------------------------------------------------// 40
----}--------------------------------------------------------------------------// ac
----friend ostream& operator <<(ostream& outs, const intx& n) {----------------// 0d
--------if (n.sign < 0) outs << '-';-------------------------------------------// c0
--------bool first = true;-----------------------------------------------------// 33
--------for (int i = n.size() - 1; i >= 0; i--) {------------------------------// 63
------------if (first) outs << n.data[i], first = false;-----------------------// 33
------------else {-------------------------------------------------------------// 1f
----------------unsigned int cur = n.data[i];----------------------------------// 0f
----------------stringstream ss; ss << cur;------------------------------------// 8c
----------------string s = ss.str();-------------------------------------------// 64
----------------int len = s.size();--------------------------------------------// 0d
----------------while (len < intx::dcnt) outs << '0', len++;-------------------// 0a
----------------outs << s;-----------------------------------------------------// 97
------------}------------------------------------------------------------------// f7
```

```
--------}------------------------------------------------------------// e9
--------return outs;-------------------------------------------------// cf
----}----------------------------------------------------------------// b9
----string to_string() const { stringstream ss; ss << *this; return ss.str(); }// fc
----bool operator <(const intx& b) const {---------------------------// 21
--------if (sign != b.sign) return sign < b.sign;--------------------// cf
--------if (size() != b.size())--------------------------------------// 4d
------------return sign == 1 ? size() < b.size() : size() > b.size();// 4d
--------for (int i = size() - 1; i >= 0; i--) if (data[i] != b.data[i])// 35
----------------return sign == 1 ? data[i] < b.data[i] : data[i] > b.data[i];--// 27
--------return false;------------------------------------------------// ca
----}----------------------------------------------------------------// 32
----intx operator -() const { intx res(*this); res.sign *= -1; return res; }---// 9d
----friend intx abs(const intx &n) { return n < 0 ? -n : n; }---------// 02
----intx operator +(const intx& b) const {---------------------------// f8
--------if (sign > 0 && b.sign < 0) return *this - (-b);-------------// 36
--------if (sign < 0 && b.sign > 0) return b - (-*this);-------------// 70
--------if (sign < 0 && b.sign < 0) return -((-*this) + (-b));-------// 59
--------intx c; c.data.clear();--------------------------------------// 18
--------unsigned long long carry = 0;--------------------------------// 5c
--------for (int i = 0; i < size() || i < b.size() || carry; i++) {--// e3
------------carry += (i < size() ? data[i] : 0ULL) +-----------------// 91
----------------(i < b.size() ? b.data[i] : 0ULL);-------------------// 0c
------------c.data.push_back(carry % intx::radix);-------------------// 86
------------carry /= intx::radix;------------------------------------// fd
--------}------------------------------------------------------------// 50
--------return c.normalize(sign);------------------------------------// 20
----}----------------------------------------------------------------// 70
----intx operator -(const intx& b) const {---------------------------// 53
--------if (sign > 0 && b.sign < 0) return *this + (-b);-------------// 8f
--------if (sign < 0 && b.sign > 0) return -(-*this + b);------------// 1b
--------if (sign < 0 && b.sign < 0) return (-b) - (-*this);----------// a1
--------if (*this < b) return -(b - *this);--------------------------// 36
--------intx c; c.data.clear();--------------------------------------// 6b
--------long long borrow = 0;----------------------------------------// f8
--------rep(i,0,size()) {--------------------------------------------// a7
------------borrow = data[i] - borrow - (i < b.size() ? b.data[i] : 0ULL);-----// a5
------------c.data.push_back(borrow < 0 ? intx::radix + borrow : borrow);------// 9b
------------borrow = borrow < 0 ? 1 : 0;-----------------------------// fb
--------}------------------------------------------------------------// dd
--------return c.normalize(sign);------------------------------------// 5c
----}----------------------------------------------------------------// 5e
----intx operator *(const intx& b) const {---------------------------// b3
--------intx c; c.data.assign(size() + b.size() + 1, 0);-------------// 3a
--------rep(i,0,size()) {--------------------------------------------// 0f
------------long long carry = 0;-------------------------------------// 15
------------for (int j = 0; j < b.size() || carry; j++) {------------// 95
----------------if (j < b.size()) carry += (long long)data[i] * b.data[j];-----// 6d
----------------carry += c.data[i + j];------------------------------// c6
----------------c.data[i + j] = carry % intx::radix;-----------------// a8
----------------carry /= intx::radix;--------------------------------// dc
------------}--------------------------------------------------------// e3
--------}------------------------------------------------------------// f0
--------return c.normalize(sign * b.sign);---------------------------// 09
----}----------------------------------------------------------------// a7
----friend pair<intx,intx> divmod(const intx& n, const intx& d) {-----// 40
--------assert(!(d.size() == 1 && d.data[0] == 0));------------------// 42
--------intx q, r; q.data.assign(n.size(), 0);-----------------------// 5e
--------for (int i = n.size() - 1; i >= 0; i--) {--------------------// 52
------------r.data.insert(r.data.begin(), 0);------------------------// cb
------------r = r + n.data[i];---------------------------------------// ea
------------long long k = 0;-----------------------------------------// dd
------------if (d.size() < r.size())---------------------------------// 4d
----------------k = (long long)intx::radix * r.data[d.size()];-------// d2
------------if (d.size() - 1 < r.size()) k += r.data[d.size() - 1];--// af
------------k /= d.data.back();--------------------------------------// 85
------------r = r - abs(d) * k;--------------------------------------// 3b
------------while (r < 0) r = r + abs(d), k--;-----------------------// a1
------------q.data[i] = k;-------------------------------------------// 9c
--------}------------------------------------------------------------// 03
--------return pair<intx, intx>(q.normalize(n.sign * d.sign), r);----// 58
----}----------------------------------------------------------------// 5f
----intx operator /(const intx& d) const {---------------------------// 23
--------return divmod(*this,d).first; }------------------------------// 33
----intx operator %(const intx& d) const {---------------------------// 16
--------return divmod(*this,d).second * sign; }----------------------// 21
};-------------------------------------------------------------------// f4
```

5.2.1. *Fast Multiplication.* Fast multiplication for the big integer using Fast Fourier Transform.

```
#include "intx.cpp"--------------------------------------------------// 83
#include "fft.cpp"---------------------------------------------------// 13
---------------------------------------------------------------------// e0
intx fastmul(const intx &an, const intx &bn) {-----------------------// ab
----string as = an.to_string(), bs = bn.to_string();-----------------// 32
----int n = size(as), m = size(bs), l = 1,---------------------------// dc
--------len = 5, radix = 100000,-------------------------------------// 4f
--------*a = new int[n], alen = 0,-----------------------------------// b8
--------*b = new int[m], blen = 0;-----------------------------------// 0a
----memset(a, 0, n << 2);--------------------------------------------// 1d
----memset(b, 0, m << 2);--------------------------------------------// 01
----for (int i = n - 1; i >= 0; i -= len, alen++)--------------------// 6e
--------for (int j = min(len - 1, i); j >= 0; j--)------------------// 43
------------a[alen] = a[alen] * 10 + as[i - j] - '0';----------------// 14
----for (int i = m - 1; i >= 0; i -= len, blen++)--------------------// b6
--------for (int j = min(len - 1, i); j >= 0; j--)------------------// ae
------------b[blen] = b[blen] * 10 + bs[i - j] - '0';----------------// 9b
----while (l < 2*max(alen,blen)) l <<= 1;----------------------------// 51
----cpx *A = new cpx[l], *B = new cpx[l];----------------------------// 0d
----rep(i,0,l) A[i] = cpx(i < alen ? a[i] : 0, 0);-------------------// ff
----rep(i,0,l) B[i] = cpx(i < blen ? b[i] : 0, 0);-------------------// 7f
----fft(A, l); fft(B, l);--------------------------------------------// 77
----rep(i,0,l) A[i] *= B[i];-----------------------------------------// 1c
```

```
----fft(A, l, true);--------------------------------// ec
----ull *data = new ull[l];-------------------------// f1
----rep(i,0,l) data[i] = (ull)(round(real(A[i])));--// e2
----rep(i,0,l-1)------------------------------------// c8
--------if (data[i] >= (unsigned int)(radix)) {-----// 03
------------data[i+1] += data[i] / radix;-----------// 48
------------data[i] %= radix;-----------------------// 94
--------}-------------------------------------------// 47
----int stop = l-1;---------------------------------// 92
----while (stop > 0 && data[stop] == 0) stop--;-----// 5b
----stringstream ss;--------------------------------// a6
----ss << data[stop];-------------------------------// f3
----for (int i = stop - 1; i >= 0; i--)-------------// 7b
--------ss << setfill('0') << setw(len) << data[i];-// 41
----delete[] A; delete[] B;-------------------------// dd
----delete[] a; delete[] b;-------------------------// 77
----delete[] data;----------------------------------// 5e
----return intx(ss.str());--------------------------// 88
}---------------------------------------------------// d8
```

5.3. **Binomial Coefficients.** The binomial coefficient $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ is the number of ways to choose k items out of a total of n items. Also contains an implementation of Lucas' theorem for computing the answer modulo a prime p.

```
int nck(int n, int k) {-----------------------------// f6
----if (n < k) return 0;----------------------------// 55
----k = min(k, n - k);------------------------------// bd
----int res = 1;------------------------------------// e6
----rep(i,1,k+1) res = res * (n - (k - i)) / i;-----// 4d
----return res;-------------------------------------// 1f
}---------------------------------------------------// 6c
int nck(int n, int k, int p) {----------------------// cf
----int res = 1;------------------------------------// 5c
----while (n || k) {--------------------------------// e2
--------res *= nck(n % p, k % p);-------------------// cc
--------res %= p, n /= p, k /= p;-------------------// 0a
----}-----------------------------------------------// d9
----return res;-------------------------------------// 30
}---------------------------------------------------// 0a
```

5.4. **Euclidean algorithm.** The Euclidean algorithm computes the greatest common divisor of two integers a, b.

```
int gcd(int a, int b) { return b == 0 ? a : gcd(b, a % b); }----// d9
```

The extended Euclidean algorithm computes the greatest common divisor d of two integers a, b and also finds two integers x, y such that $a \times x + b \times y = d$.

```
int egcd(int a, int b, int& x, int& y) {------------// 85
----if (b == 0) { x = 1; y = 0; return a; }---------// 7b
----else {------------------------------------------// 00
--------int d = egcd(b, a % b, x, y);---------------// 34
--------x -= a / b * y;-----------------------------// 4a
--------swap(x, y);---------------------------------// 26
--------return d;-----------------------------------// db
----}-----------------------------------------------// 9e
}---------------------------------------------------// 40
```

5.5. **Trial Division Primality Testing.** An optimized trial division to check whether an integer is prime.

```
bool is_prime(int n) {------------------------------// 6c
----if (n < 2) return false;------------------------// c9
----if (n < 4) return true;-------------------------// d9
----if (n % 2 == 0 || n % 3 == 0) return false;-----// 0f
----if (n < 25) return true;------------------------// ef
----int s = static_cast<int>(sqrt(static_cast<double>(n)));----// 64
----for (int i = 5; i <= s; i += 6)-----------------// 6c
--------if (n % i == 0 || n % (i + 2) == 0) return false;------// e9
----return true; }----------------------------------// 43
```

5.6. **Miller-Rabin Primality Test.** The Miller-Rabin probabilistic primality test.

```
#include "mod_pow.cpp"------------------------------// c7
bool is_probable_prime(ll n, int k) {---------------// be
----if (~n & 1) return n == 2;----------------------// d1
----if (n <= 3) return n == 3;----------------------// 39
----int s = 0; ll d = n - 1;------------------------// 37
----while (~d & 1) d >>= 1, s++;--------------------// 35
----while (k--) {-----------------------------------// c8
--------ll a = (n - 3) * rand() / RAND_MAX + 2;-----// 06
--------ll x = mod_pow(a, d, n);--------------------// 64
--------if (x == 1 || x == n - 1) continue;---------// 9b
--------bool ok = false;----------------------------// 03
--------rep(i,0,s-1) {------------------------------// 13
------------x = (x * x) % n;------------------------// 90
------------if (x == 1) return false;---------------// 5c
------------if (x == n - 1) { ok = true; break; }---// a1
--------}-------------------------------------------// 3a
--------if (!ok) return false;----------------------// 37
----} return true; }--------------------------------// fe
```

5.7. **Pollard's ρ algorithm.**

```
// public static int[] seeds = new int[] {2,3,5,7,11,13,1031};-----// 1d
// public static BigInteger rho(BigInteger n, BigInteger seed) {---// 03
//---- int i = 0,--------------------------------------------------// 00
//-------- k = 2;--------------------------------------------------// 79
//---- BigInteger x = seed,----------------------------------------// cc
//----------------y = seed;----------------------------------------// 31
//---- while (i < 1000000) {---------------------------------------// 10
//-------- i++;----------------------------------------------------// 8c
//-------- x = (x.multiply(x).add(n).subtract(BigInteger.ONE)).mod(n);---// 74
//-------- BigInteger d = y.subtract(x).abs().gcd(n);--------------// ce
//-------- if (!d.equals(BigInteger.ONE) && !d.equals(n)) {--------// b9
//------------ return d;-------------------------------------------// 3b
//-------- } ------------------------------------------------------// 7c
//-------- if (i == k) {-------------------------------------------// 2c
//------------ y = x;----------------------------------------------// 89
//------------ k = k*2;--------------------------------------------// 1d
```

```
//-------- }--------------------------------------------------------------// 10
//---- }------------------------------------------------------------------// 96
//---- return BigInteger.ONE;---------------------------------------------// 62
// }----------------------------------------------------------------------// d7
```

5.8. **Sieve of Eratosthenes.** An optimized implementation of Eratosthenes' Sieve.

```
vi prime_sieve(int n) {--------------------------------------------------// 40
----int mx = (n - 3) >> 1, sq, v, i = -1;--------------------------------// 27
----vi primes;-----------------------------------------------------------// 8f
----bool* prime = new bool[mx + 1];--------------------------------------// ef
----memset(prime, 1, mx + 1);--------------------------------------------// 28
----if (n >= 2) primes.push_back(2);-------------------------------------// f4
----while (++i <= mx) if (prime[i]) {------------------------------------// 73
--------primes.push_back(v = (i << 1) + 3);------------------------------// be
--------if ((sq = i * ((i << 1) + 6) + 3) > mx) break;-------------------// 2d
--------for (int j = sq; j <= mx; j += v) prime[j] = false; }------------// 2e
----while (++i <= mx) if (prime[i]) primes.push_back((i << 1) + 3);------// 29
----delete[] prime; // can be used for O(1) lookup-----------------------// 36
----return primes; }-----------------------------------------------------// 72
```

5.9. **Modular Multiplicative Inverse.** A function to find a modular multiplicative inverse.

```
#include "egcd.cpp"------------------------------------------------------// 55
-------------------------------------------------------------------------// e8
int mod_inv(int a, int m) {----------------------------------------------// 49
----int x, y, d = egcd(a, m, x, y);--------------------------------------// 3e
----if (d != 1) return -1;-----------------------------------------------// 20
----return x < 0 ? x + m : x;--------------------------------------------// 3c
}------------------------------------------------------------------------// 69
```

5.10. **Modular Exponentiation.** A function to perform fast modular exponentiation.

```
template <class T>-------------------------------------------------------// 82
T mod_pow(T b, T e, T m) {-----------------------------------------------// aa
----T res = T(1);--------------------------------------------------------// 85
----while (e) {----------------------------------------------------------// b7
--------if (e & T(1)) res = mod(res * b, m);-----------------------------// 41
--------b = mod(b * b, m), e >>= T(1); }---------------------------------// b3
----return res;----------------------------------------------------------// eb
}------------------------------------------------------------------------// c5
```

5.11. **Chinese Remainder Theorem.** An implementation of the Chinese Remainder Theorem.

```
#include "egcd.cpp"------------------------------------------------------// 55
int crt(const vi& as, const vi& ns) {------------------------------------// c3
----int cnt = size(as), N = 1, x = 0, r, s, l;---------------------------// 55
----rep(i,0,cnt) N *= ns[i];---------------------------------------------// b1
----rep(i,0,cnt) egcd(ns[i], l = N/ns[i], r, s), x += as[i] * s * l;-----// 21
----return mod(x, N); }--------------------------------------------------// b2
```

5.12. **Linear Congruence Solver.** A function that returns all solutions to $ax \equiv b \pmod{n}$, modulo n.

```
#include "egcd.cpp"------------------------------------------------------// 55
vi linear_congruence(int a, int b, int n) {------------------------------// c8
----int x, y, d = egcd(a, n, x, y);--------------------------------------// 7a
----vi res;--------------------------------------------------------------// f5
----if (b % d != 0) return res;------------------------------------------// 30
----int x0 = mod(b / d * x, n);------------------------------------------// 48
----rep(k,0,d) res.push_back(mod(x0 + k * n / d, n));--------------------// 7e
----return res;----------------------------------------------------------// fe
}------------------------------------------------------------------------// c0
```

5.13. **Numeric Integration.** Numeric integration using Simpson's rule.

```
double integrate(double (*f)(double), double a, double b,----------------// 76
--------double delta = 1e-6) {-------------------------------------------// c0
----if (abs(a - b) < delta)----------------------------------------------// 38
--------return (b-a)/8 *-------------------------------------------------// 56
------------(f(a) + 3*f((2*a+b)/3) + 3*f((a+2*b)/3) + f(b));-------------// e1
----return integrate(f, a,-----------------------------------------------// 64
------------(a+b)/2, delta) + integrate(f, (a+b)/2, b, delta);-----------// 0c
}------------------------------------------------------------------------// 4b
```

5.14. **Fast Fourier Transform.** The Cooley-Tukey algorithm for quickly computing the discrete Fourier transform. The `fft` function only supports powers of twos. The `czt` function implements the Chirp Z-transform and supports any size, but is slightly slower.

```
#include <complex>-------------------------------------------------------// 8e
typedef complex<long double> cpx;----------------------------------------// 25
// NOTE: n must be a power of two----------------------------------------// 14
void fft(cpx *x, int n, bool inv=false) {--------------------------------// 36
----for (int i = 0, j = 0; i < n; i++) {---------------------------------// f9
--------if (i < j) swap(x[i], x[j]);-------------------------------------// 44
--------int m = n>>1;----------------------------------------------------// 9c
--------while (1 <= m && m <= j) j -= m, m >>= 1;------------------------// fe
--------j += m;----------------------------------------------------------// 11
----}--------------------------------------------------------------------// d0
----for (int mx = 1; mx < n; mx <<= 1) {---------------------------------// 15
--------cpx wp = exp(cpx(0, (inv ? -1 : 1) * pi / mx)), w = 1;-----------// 79
--------for (int m = 0; m < mx; m++, w *= wp) {--------------------------// dc
------------for (int i = m; i < n; i += mx << 1) {-----------------------// 6a
----------------cpx t = x[i + mx] * w;-----------------------------------// 12
----------------x[i + mx] = x[i] - t;------------------------------------// 73
----------------x[i] += t;-----------------------------------------------// 0e
------------}------------------------------------------------------------// 14
--------}----------------------------------------------------------------// a4
----}--------------------------------------------------------------------// bf
----if (inv) rep(i,0,n) x[i] /= cpx(n);----------------------------------// 16
}------------------------------------------------------------------------// 1c
void czt(cpx *x, int n, bool inv=false) {--------------------------------// c5
----int len = 2*n+1;-----------------------------------------------------// bc
----while (len & (len - 1)) len &= len - 1;------------------------------// 65
----len <<= 1;-----------------------------------------------------------// 21
----cpx w = exp(-2.0L * pi / n * cpx(0,1)),------------------------------// 45
--------*c = new cpx[n], *a = new cpx[len],------------------------------// 4e
--------*b = new cpx[len];-----------------------------------------------// 30
----rep(i,0,n) c[i] = pow(w, (inv ? -1.0 : 1.0)*i*i/2);------------------// 9e
----rep(i,0,n) a[i] = x[i] * c[i], b[i] = 1.0L/c[i];---------------------// e9
----rep(i,0,n-1) b[len - n + i + 1] = 1.0L/c[n-i-1];---------------------// 9f
----fft(a, len); fft(b, len);--------------------------------------------// 63
```

```
----rep(i,0,len) a[i] *= b[i];--------------------------------------------------// 58
----fft(a, len, true);----------------------------------------------------------// 2d
----rep(i,0,n) {----------------------------------------------------------------// ff
--------x[i] = c[i] * a[i];-----------------------------------------------------// 77
--------if (inv) x[i] /= cpx(n);-----------------------------------------------// b1
----}---------------------------------------------------------------------------// 27
----delete[] a;-----------------------------------------------------------------// 0a
----delete[] b;-----------------------------------------------------------------// 5c
----delete[] c;-----------------------------------------------------------------// f8
}-------------------------------------------------------------------------------// c6
```

5.15. **Tridiagonal Matrix Algorithm.** Solves a tridiagonal system of linear equations $a_i x_{i-1} + b_i x_i + c_i x_{i+1} = d_i$ where $a_1 = c_n = 0$. Beware of numerical instability.

```
#define MAXN 5000-------------------------------------------------------------// f7
long double A[MAXN], B[MAXN], C[MAXN], D[MAXN], X[MAXN];------------------------// d8
void solve(int n) {-------------------------------------------------------------// 01
----C[0] /= B[0]; D[0] /= B[0];-------------------------------------------------// 94
----rep(i,1,n-1) C[i] /= B[i] - A[i]*C[i-1];------------------------------------// 6b
----rep(i,1,n) D[i] = (D[i] - A[i] * D[i-1]) / (B[i] - A[i] * C[i-1]);---------// 33
----X[n-1] = D[n-1];------------------------------------------------------------// c7
----for (int i = n-2; i>=0; i--) X[i] = D[i] - C[i] * X[i+1]; }----------------// ad
```

5.16. **Markov Chains.** A Markov Chain can be represented as a weighted directed graph of states, where the weight of an edge represents the probability of transitioning over that edge in one timestep. Let $P^{(m)} = (p^{(m)}_{ij})$ be the probability matrix of transitioning from state i to state j in m timesteps, and note that $P^{(1)}$ is the adjacency matrix of the graph. **Chapman-Kolmogorov:** $p^{(m+n)}_{ij} = \sum_k p^{(m)}_{ik} p^{(n)}_{kj}$. It follows that $P^{(m+n)} = P^{(m)}P^{(n)}$ and $P^{(m)} = P^m$. If $p^{(0)}$ is the initial probability distribution (a vector), then $p^{(0)}P^{(m)}$ is the probability distribution after m timesteps.

The return times of a state i is $R_i = \{m \mid p^{(m)}_{ii} > 0\}$, and i is *aperiodic* if $\gcd(R_i) = 1$. A MC is aperiodic if any of its vertices is aperiodic. A MC is *irreducible* if the corresponding graph is strongly connected.

A distribution π is stationary if $\pi P = \pi$. If MC is irreducible then $\pi_i = 1/\mathbb{E}[T_i]$, where T_i is the expected time between two visits at i. π_j/π_i is the expected number of visits at j in between two consecutive visits at i. A MC is *ergodic* if $\lim_{m\to\infty} p^{(0)}P^m = \pi$. A MC is ergodic iff. it is irreducible and aperiodic.

A MC for a random walk in an undirected weighted graph (unweighted graph can be made weighted by adding 1-weights) has $p_{uv} = w_{uv}/\sum_x w_{ux}$. If the graph is connected, then $\pi_u = \sum_x w_{ux}/\sum_v \sum_x w_{vx}$. Such a random walk is aperiodic iff. the graph is not bipartite.

An *absorbing* MC is of the form $P = \begin{pmatrix} Q & R \\ 0 & I_r \end{pmatrix}$. Let $N = \sum_{m=0}^{\infty} Q^m = (I_t - Q)^{-1}$. Then, if starting in state i, the expected number of steps till absorption is the i-th entry in $N1$. If starting in state i, the probability of being absorbed in state j is the (i,j)-th entry of NR.

Many problems on MC can be formulated in terms of a system of recurrence relations, and then solved using Gaussian elimination.

5.17. **Burnside's Lemma.** Let G be a finite group that acts on a set X. For each g in G let X^g denote the set of elements in X that are fixed by g. Then the number of orbits

$$|X/G| = \frac{1}{|G|}\sum_{g\in G} |X^g|$$

5.18. **Bézout's identity.** If (x,y) is any solution to $ax+by=d$ (e.g. found by the Extended Euclidean Algorithm), then all solutions are given by

$$\left(x + k\frac{b}{\gcd(a,b)}, y - k\frac{a}{\gcd(a,b)}\right)$$

.

5.19. **Formulas.**

- Number of ways to choose k objects from a total of n objects where order matters and each item can only be chosen once: $P^n_k = \frac{n!}{(n-k)!}$
- Number of ways to choose k objects from a total of n objects where order matters and each item can be chosen multiple times: n^k
- Number of permutations of n objects, where there are n_1 objects of type 1, n_2 objects of type 2, ..., n_k objects of type k: $\binom{n}{n_1,n_2,\ldots,n_k} = \frac{n!}{n_1!\times n_2!\times\cdots\times n_k!}$
- Number of ways to choose k objects from a total of n objects where order does not matter and each item can only be chosen once: $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k} = \binom{n}{n-k} = \prod_{i=1}^{k}\frac{n-(k-i)}{i} = \frac{n!}{k!(n-k)!}$, $\binom{n}{0} = 1$, $\binom{0}{k} = 0$
- Number of ways to choose k objects from a total of n objects where order does not matter and each item can be chosen multiple times: $f^n_k = \binom{n+k-1}{k} = \frac{(n+k-1)!}{k!(n-1)!}$
- Number of integer solutions to $x_1 + x_2 + \cdots + x_n = k$ where $x_i \geq 0$: f^n_k
- Number of subsets of a set with n elements: 2^n
- $|A\cup B| = |A| + |B| - |A\cap B|$
- $|A\cup B\cup C| = |A| + |B| + |C| - |A\cap B| - |A\cap C| - |B\cap C| + |A\cap B\cap C|$
- Number of ways to walk from the lower-left corner to the upper-right corner of an $n\times m$ grid by walking only up and to the right: $\binom{n+m}{m}$
- Number of strings with n sets of brackets such that the brackets are balanced: $C_n = \sum_{k=0}^{n-1} C_k C_{n-1-k} = \frac{1}{n+1}\binom{2n}{n}$
- Number of triangulations of a convex polygon with n points, number of rooted binary trees with $n+1$ vertices, number of paths across an $n\times n$ lattice which do not rise above the main diagonal: C_n
- Number of permutations of n objects with exactly k ascending sequences or *runs*: $\left\langle {n \atop k} \right\rangle = \left\langle {n \atop n-k-1} \right\rangle = k\left\langle {n-1 \atop k} \right\rangle + (n-k+1)\left\langle {n-1 \atop k-1} \right\rangle = \sum_{i=0}^{k}(-1)^i\binom{n+1}{i}(k+1-i)^n$, $\left\langle {n \atop 0} \right\rangle = \left\langle {n \atop n-1} \right\rangle = 1$
- Number of permutations of n objects with exactly k cycles: $\left[{n \atop k}\right] = \left[{n-1 \atop k-1}\right] + (n-1)\left[{n-1 \atop k}\right]$
- Number of ways to partition n objects into k sets: $\left\{{n \atop k}\right\} = k\left\{{n-1 \atop k}\right\} + \left\{{n-1 \atop k-1}\right\}$, $\left\{{n \atop 0}\right\} = \left\{{n \atop n}\right\} = 1$
- Number of permutations of length n that have no fixed points (derangements): $D_0 = 1, D_1 = 0, D_n = (n-1)(D_{n-1} + D_{n-2})$
- Number of permutations of length n that have exactly k fixed points: $\binom{n}{k}D_{n-k}$
- Number of trees on n labeled vertices: n^{n-2}
- **Jacobi symbol:** $\left(\frac{a}{b}\right) = a^{(b-1)/2} \pmod b$
- **Heron's formula:** A triangle with side lengths a, b, c has area $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$.
- **Pick's theorem:** A polygon on an integer grid containing i lattice points and having b lattice points on the boundary has area $i + \frac{b}{2} - 1$.
- **Divisor sigma:** The sum of divisors of n to the xth power is $\sigma_x(n) = \prod_{i=0}^{r}\frac{p_i^{(a_i+1)x}-1}{p_i^x - 1}$ where $n = \prod_{i=0}^{r} p_i^{a_i}$ is the prime factorization.
- **Divisor count:** A special case of the above is $\sigma_0(n) = \prod_{i=0}^{r}(a_i+1)$.
- **Euler's totient:** The number of integers less than n that are comprime to n are $n\prod_{p|n}\left(1 - \frac{1}{p}\right)$ where each p is a distinct prime factor of n.

- **König's theorem:** In any bipartite graph, the number of edges in a maximum matching is equal to the number of vertices in a minimum vertex cover.
- A minumum Steiner tree for n vertices requires at most $n-2$ additional Steiner vertices.
- The number of vertices of a graph is equal to its minimum vertex cover number plus the size of a maximum independent set.
- $\gcd(2^a-1, 2^b-1) = 2^{\gcd(a,b)}-1$
- **Wilson's theorem:** $(n-1)! \equiv -1 \pmod{n}$ iff. n is prime
- **Lagrange polynomial** through points $(x_0, y_0), \ldots, (x_k, y_k)$ is $L(x) = \sum_{j=0}^{k} y_j \prod_{\substack{0\le m\le k \\ m\ne j}} \frac{x-x_m}{x_j-x_m}$
- $\sum_{k=0}^{m} (-1)^k \binom{n}{k} = (-1)^m \binom{n-1}{m}$
- $2^{\omega(n)} = O(\sqrt{n})$, where $\omega(n)$ is the number of distinct prime factors
- $\sum_{i=1}^{n} 2^{\omega(i)} = O(n \log n)$
- **Hook length formula:** If λ is a Young diagram and $h_\lambda(i,j)$ is the hook-length of cell (i,j), then then the number of Young tableux $d_\lambda = n!/\prod h_\lambda(i,j)$.
- **Möbius inversion formula:** If $f(n) = \sum_{d|n} g(d)$, then $g(n) = \sum_{d|n} \mu(d) f(n/d)$

5.20. **Numbers and Sequences.** Some random prime numbers: 1031, 32771, 1048583, 33554467, 1073741827, 34359738421, 1099511627791, 35184372088891, 1125899906842679, 36028797018963971.

6. Geometry

6.1. **Primitives.** Geometry primitives.

```
#define P(p) const point &p--------------------------------------------------// 2e
#define L(p0, p1) P(p0), P(p1)-----------------------------------------------// cf
#define C(p0, r) P(p0), double r---------------------------------------------// f1
#define PP(pp) pair<point,point> &pp-----------------------------------------// e5
typedef complex<double> point;-----------------------------------------------// 6a
double dot(P(a), P(b)) { return real(conj(a) * b); }-------------------------// d2
double cross(P(a), P(b)) { return imag(conj(a) * b); }-----------------------// 8a
point rotate(P(p), double radians = pi / 2, P(about) = point(0,0)) {---------// 23
----return (p - about) * exp(point(0, radians)) + about; }-------------------// 25
point reflect(P(p), L(about1, about2)) {-------------------------------------// 50
----point z = p - about1, w = about2 - about1;-------------------------------// 8b
----return conj(z / w) * w + about1; }---------------------------------------// 83
point proj(P(u), P(v)) { return dot(u, v) / dot(u, u) * u; }-----------------// e7
point normalize(P(p), double k = 1.0) {--------------------------------------// 5f
----return abs(p) == 0 ? point(0,0) : p / abs(p) * k; }----------------------// 4a
bool parallel(L(a, b), L(p, q)) { return abs(cross(b - a, q - p)) < EPS; }---// 07
double ccw(P(a), P(b), P(c)) { return cross(b - a, c - b); }-----------------// 29
bool collinear(P(a), P(b), P(c)) { return abs(ccw(a, b, c)) < EPS; }---------// 2b
bool collinear(L(a, b), L(p, q)) {-------------------------------------------// 70
----return abs(ccw(a, b, p)) < EPS && abs(ccw(a, b, q)) < EPS;  }------------// 59
double angle(P(a), P(b), P(c)) {---------------------------------------------// 0a
----return acos(dot(b - a, c - b) / abs(b - a) / abs(c - b)); }--------------// d4
double signed_angle(P(a), P(b), P(c)) {--------------------------------------// da
----return asin(cross(b - a, c - b) / abs(b - a) / abs(c - b)); }------------// 83
double angle(P(p)) { return atan2(imag(p), real(p)); }-----------------------// 20
point perp(P(p)) { return point(-imag(p), real(p)); }------------------------// ca
double progress(P(p), L(a, b)) {---------------------------------------------// 44
----if (abs(real(a) - real(b)) < EPS)----------------------------------------// 81
--------return (imag(p) - imag(a)) / (imag(b) - imag(a));--------------------// 55
----else return (real(p) - real(a)) / (real(b) - real(a)); }-----------------// fb
-----------------------------------------------------------------------------// 3c
```

6.2. **Lines.** Line related functions.

```
#include "primitives.cpp"----------------------------------------------------// e0
-----------------------------------------------------------------------------// 85
bool collinear(L(a, b), L(p, q)) {-------------------------------------------// 2f
----return abs(ccw(a, b, p)) < EPS && abs(ccw(a, b, q)) < EPS;  }------------// 3e
bool parallel(L(a, b), L(p, q)) { return abs(cross(b - a, q - p)) < EPS; }---// 8d
point closest_point(L(a, b), P(c), bool segment = false) {-------------------// f2
----if (segment) {-----------------------------------------------------------// f4
--------if (dot(b - a, c - b) > 0) return b;---------------------------------// 88
--------if (dot(a - b, c - a) > 0) return a;---------------------------------// 75
----}------------------------------------------------------------------------// ce
----double t = dot(c - a, b - a) / norm(b - a);------------------------------// 62
----return a + t * (b - a);--------------------------------------------------// 6e
}----------------------------------------------------------------------------// 8c
double line_segment_distance(L(a,b), L(c,d)) {-------------------------------// f3
----double x = INFINITY;-----------------------------------------------------// 64
----if (abs(a - b) < EPS && abs(c - d) < EPS) x = abs(a - c);----------------// a5
----else if (abs(a - b) < EPS) x = abs(a - closest_point(c, d, a, true));----// 23
----else if (abs(c - d) < EPS) x = abs(c - closest_point(a, b, c, true));----// 53
----else if ((ccw(a, b, c) < 0) != (ccw(a, b, d) < 0) &&---------------------// 6d
------------ (ccw(c, d, a) < 0) != (ccw(c, d, b) < 0)) x = 0;----------------// bf
----else {-------------------------------------------------------------------// e1
--------x = min(x, abs(a - closest_point(c,d, a, true)));--------------------// 29
--------x = min(x, abs(b - closest_point(c,d, b, true)));--------------------// fe
--------x = min(x, abs(c - closest_point(a,b, c, true)));--------------------// 81
--------x = min(x, abs(d - closest_point(a,b, d, true)));--------------------// e4
----}------------------------------------------------------------------------// c5
----return x;----------------------------------------------------------------// b7
}----------------------------------------------------------------------------// 27
bool intersect(L(a, b), L(p, q), point &res, bool segment = false) {---------// d2
----// NOTE: check for parallel/collinear lines before calling this function---// 1b
----point r = b - a, s = q - p;----------------------------------------------// 34
----double c = cross(r, s), t = cross(p - a, s) / c, u = cross(p - a, r) / c;--// 0b
----if (segment && (t < 0-EPS || t > 1+EPS || u < 0-EPS || u > 1+EPS))-------// e4
--------return false;--------------------------------------------------------// e3
----res = a + t * r;---------------------------------------------------------// 47
----return true;-------------------------------------------------------------// 05
}----------------------------------------------------------------------------// 44
-----------------------------------------------------------------------------// cc
```

6.3. **Circles.** Circle related functions.

```
#include "primitives.cpp"----------------------------------------------------// e0
int intersect(C(A, rA), C(B, rB), point & res1, point & res2) {--------------// 3b
----double d = abs(B - A);---------------------------------------------------// 5c
----if ((rA + rB) <  (d - EPS) || d < abs(rA - rB) - EPS) return 0;----------// 39
----double a = (rA*rA - rB*rB + d*d) / 2 / d, h = sqrt(rA*rA - a*a);---------// 9b
----point v = normalize(B - A, a), u = normalize(rotate(B-A), h);------------// 79
----res1 = A + v + u, res2 = A + v - u;--------------------------------------// 24
----if (abs(u) < EPS) return 1; return 2;------------------------------------// 82
```

```
}------------------------------------------------------------------------------// bb
-------------------------------------------------------------------------------// ce
int intersect(L(A, B), C(O, r), point & res1, point & res2) {------------------// 4d
---- double h = abs(O - closest_point(A, B, O));-------------------------------// d0
---- if(r < h - EPS) return 0;-------------------------------------------------// af
---- point H = proj(O - A, B - A) + A, v = normalize((B - A), sqrt(r*r - h*h));// 1a
---- res1 = H + v; res2 = H - v;-----------------------------------------------// fc
---- if(abs(v) < EPS) return 1; return 2;--------------------------------------// d1
}------------------------------------------------------------------------------// f8
-------------------------------------------------------------------------------// 74
int tangent(P(A), C(O, r), point & res1, point & res2) {-----------------------// ec
----point v = O - A; double d = abs(v);----------------------------------------// 62
----if (d < r - EPS) return 0;-------------------------------------------------// fc
----double alpha = asin(r / d), L = sqrt(d*d - r*r);---------------------------// f0
----v = normalize(v, L);-------------------------------------------------------// 0f
----res1 = A + rotate(v, alpha); res2 = A + rotate(v, -alpha);-----------------// 9e
----if (abs(r - d) < EPS || abs(v) < EPS) return 1;----------------------------// ee
----return 2;------------------------------------------------------------------// 4f
}------------------------------------------------------------------------------// 3c
-------------------------------------------------------------------------------// 29
void tangent_outer(point A, double rA, point B, double rB, PP(P), PP(Q)) {-----// 9e
----if (rA - rB > EPS) { swap(rA, rB); swap(A, B); }---------------------------// 9c
----double theta = asin((rB - rA)/abs(A - B));---------------------------------// 68
----point v = rotate(B - A, theta + pi/2), u = rotate(B - A, -(theta + pi/2));-// d7
----u = normalize(u, rA);------------------------------------------------------// 87
----P.first = A + normalize(v, rA); P.second = B + normalize(v, rB);-----------// 20
----Q.first = A + normalize(u, rA); Q.second = B + normalize(u, rB);-----------// 52
}------------------------------------------------------------------------------// 63
```

6.4. **Polygon.** Polygon primitives.

```
#include "primitives.cpp"------------------------------------------------------// e0
typedef vector<point> polygon;-------------------------------------------------// b3
double polygon_area_signed(polygon p) {----------------------------------------// 31
----double area = 0; int cnt = size(p);----------------------------------------// a2
----rep(i,1,cnt-1) area += cross(p[i] - p[0], p[i + 1] - p[0]);----------------// 51
----return area / 2; }---------------------------------------------------------// 66
double polygon_area(polygon p) { return abs(polygon_area_signed(p)); }---------// a4
#define CHK(f,a,b,c) (f(a) < f(b) && f(b) <= f(c) && ccw(a,c,b) < 0)-----------// 8f
int point_in_polygon(polygon p, point q) {-------------------------------------// 5d
----int n = size(p); bool in = false; double d;--------------------------------// 69
----for (int i = 0, j = n - 1; i < n; j = i++)---------------------------------// f3
--------if (collinear(p[i], q, p[j]) &&----------------------------------------// 9d
------------0 <= (d = progress(q, p[i], p[j])) && d <= 1)----------------------// 4b
------------return 0;----------------------------------------------------------// b3
----for (int i = 0, j = n - 1; i < n; j = i++)---------------------------------// 67
--------if (CHK(real, p[i], q, p[j]) || CHK(real, p[j], q, p[i]))-------------// b4
------------in = !in;----------------------------------------------------------// ff
----return in ? -1 : 1; }------------------------------------------------------// ba
// pair<polygon, polygon> cut_polygon(const polygon &poly, point a, point b) {-// 0d
//---- polygon left, right;----------------------------------------------------// 0a
//---- point it(-100, -100);---------------------------------------------------// 5b
//---- for (int i = 0, cnt = poly.size(); i < cnt; i++) {----------------------// 70
//-------- int j = i == cnt-1 ? 0 : i + 1;-------------------------------------// 02
//-------- point p = poly[i], q = poly[j];-------------------------------------// 44
//-------- if (ccw(a, b, p) <= 0) left.push_back(p);--------------------------// 8d
//-------- if (ccw(a, b, p) >= 0) right.push_back(p);-------------------------// 43
//-------- // myintersect = intersect where-----------------------------------// ba
//-------- // (a,b) is a line, (p,q) is a line segment------------------------// 7e
//-------- if (myintersect(a, b, p, q, it))-----------------------------------// 6f
//------------ left.push_back(it), right.push_back(it);-----------------------// 8a
//---- }-----------------------------------------------------------------------// e0
//---- return pair<polygon, polygon>(left, right);-----------------------------// 3d
// }---------------------------------------------------------------------------// 07
```

6.5. **Convex Hull.** An algorithm that finds the Convex Hull of a set of points. NOTE: Doesn't work on some weird edge cases. (A small case that included three collinear lines would return the same point on both the upper and lower hull.)

```
#include "polygon.cpp"---------------------------------------------------------// 58
#define MAXN 1000----------------------------------------------------------------// 09
point hull[MAXN];--------------------------------------------------------------// 43
bool cmp(const point &a, const point &b) {-------------------------------------// 32
----return abs(real(a) - real(b)) > EPS ?--------------------------------------// 44
--------real(a) < real(b) : imag(a) < imag(b); }-------------------------------// 40
int convex_hull(polygon p) {---------------------------------------------------// cd
----int n = size(p), l = 0;----------------------------------------------------// 67
----sort(p.begin(), p.end(), cmp);---------------------------------------------// 3d
----rep(i,0,n) {---------------------------------------------------------------// e4
--------if (i > 0 && p[i] == p[i - 1]) continue;-------------------------------// c7
--------while (l >= 2 && ccw(hull[l - 2], hull[l - 1], p[i]) >= 0) l--;--------// 62
--------hull[l++] = p[i];------------------------------------------------------// bd
----}--------------------------------------------------------------------------// d2
----int r = l;-----------------------------------------------------------------// 30
----for (int i = n - 2; i >= 0; i--) {-----------------------------------------// 59
--------if (p[i] == p[i + 1]) continue;----------------------------------------// af
--------while (r - l >= 1 && ccw(hull[r - 2], hull[r - 1], p[i]) >= 0) r--;----// 4d
--------hull[r++] = p[i];------------------------------------------------------// f5
----}--------------------------------------------------------------------------// f6
----return l == 1 ? 1 : r - 1;-------------------------------------------------// a6
}------------------------------------------------------------------------------// 6d
```

6.6. **Line Segment Intersection.** Computes the intersection between two line segments.

```
#include "primitives.cpp"------------------------------------------------------// e0
bool line_segment_intersect(L(a, b), L(c, d), point &A, point &B) {------------// 6c
----if (abs(a - b) < EPS && abs(c - d) < EPS) {--------------------------------// db
--------A = B = a; return abs(a - d) < EPS; }----------------------------------// ee
----else if (abs(a - b) < EPS) {-----------------------------------------------// 03
--------A = B = a; double p = progress(a, c,d);--------------------------------// c9
--------return 0.0 <= p && p <= 1.0--------------------------------------------// 8a
------------&& (abs(a - c) + abs(d - a) - abs(d - c)) < EPS; }-----------------// 27
----else if (abs(c - d) < EPS) {-----------------------------------------------// 26
--------A = B = c; double p = progress(c, a,b);--------------------------------// d9
--------return 0.0 <= p && p <= 1.0--------------------------------------------// 8e
------------&& (abs(c - a) + abs(b - c) - abs(b - a)) < EPS; }-----------------// 4f
```

```
----else if (collinear(a,b, c,d)) {---------------------------------------------// bc
--------double ap = progress(a, c,d), bp = progress(b, c,d);--------------------// a7
--------if (ap > bp) swap(ap, bp);----------------------------------------------// b1
--------if (bp < 0.0 || ap > 1.0) return false;---------------------------------// 0c
--------A = c + max(ap, 0.0) * (d - c);-----------------------------------------// f6
--------B = c + min(bp, 1.0) * (d - c);-----------------------------------------// 5c
--------return true; }----------------------------------------------------------// ab
----else if (parallel(a,b, c,d)) return false;----------------------------------// ca
----else if (intersect(a,b, c,d, A, true)) {------------------------------------// 10
--------B = A; return true; }---------------------------------------------------// bf
----return false;---------------------------------------------------------------// b7
}-------------------------------------------------------------------------------// 8b
--------------------------------------------------------------------------------// e6
```

6.7. **Great-Circle Distance.** Computes the distance between two points (given as latitude/longitude coordinates) on a sphere of radius r.

```
double gc_distance(double pLat, double pLong,-----------------------------------// 7b
----------------  double qLat, double qLong, double r) {------------------------// a4
----pLat *= pi / 180; pLong *= pi / 180;----------------------------------------// ee
----qLat *= pi / 180; qLong *= pi / 180;----------------------------------------// 75
----return r * acos(cos(pLat) * cos(qLat) * cos(pLong - qLong) +----------------// e3
--------------------sin(pLat) * sin(qLat));------------------------------------// 1e
--------------------------------------------------------------------------------// 60
}-------------------------------------------------------------------------------// 3f
```

6.8. **Triangle Circumcenter.** Returns the unique point that is the same distance from all three points. It is also the center of the unique circle that goes through all three points.

```
#include "primitives.cpp"-------------------------------------------------------// e0
point circumcenter(point a, point b, point c) {---------------------------------// 76
----b -= a, c -= a;-------------------------------------------------------------// 41
----return a + perp(b * norm(c) - c * norm(b)) / 2.0 / cross(b, c);-------------// 7a
}-------------------------------------------------------------------------------// c3
```

6.9. **Closest Pair of Points.** A sweep line algorithm for computing the distance between the closest pair of points.

```
#include "primitives.cpp"-------------------------------------------------------// e0
--------------------------------------------------------------------------------// 85
struct cmpx { bool operator ()(const point &a, const point &b) {----------------// 01
--------return abs(real(a) - real(b)) > EPS ?-----------------------------------// e9
------------real(a) < real(b) : imag(a) < imag(b); } };-------------------------// 53
struct cmpy { bool operator ()(const point &a, const point &b) {----------------// 6f
----return abs(imag(a) - imag(b)) > EPS ?---------------------------------------// 0b
------------imag(a) < imag(b) : real(a) < real(b); } };-------------------------// a4
double closest_pair(vector<point> pts) {----------------------------------------// f1
----sort(pts.begin(), pts.end(), cmpx());---------------------------------------// 0c
----set<point, cmpy> cur;-------------------------------------------------------// bd
----set<point, cmpy>::const_iterator it, jt;------------------------------------// a6
----double mn = INFINITY;-------------------------------------------------------// f9
----for (int i = 0, l = 0; i < size(pts); i++) {--------------------------------// ac
--------while (real(pts[i]) - real(pts[l]) > mn) cur.erase(pts[l++]);----------// 8b
--------it = cur.lower_bound(point(-INFINITY, imag(pts[i]) - mn));-------------// fc
--------jt = cur.upper_bound(point(INFINITY, imag(pts[i]) + mn));--------------// 39
--------while (it != jt) mn = min(mn, abs(*it - pts[i])), it++;----------------// 09
--------cur.insert(pts[i]); }---------------------------------------------------// 82
----return mn; }----------------------------------------------------------------// 4c
```

6.10. **3D Primitives.** Three-dimensional geometry primitives.

```
#define P(p) const point3d &p-------------------------------------------------// a7
#define L(p0, p1) P(p0), P(p1)------------------------------------------------// 0f
#define PL(p0, p1, p2) P(p0), P(p1), P(p2)------------------------------------// 67
struct point3d {----------------------------------------------------------------// 63
----double x, y, z;-------------------------------------------------------------// e6
----point3d() : x(0), y(0), z(0) {}---------------------------------------------// af
----point3d(double _x, double _y, double _z) : x(_x), y(_y), z(_z) {}----------// fc
----point3d operator+(P(p)) const {---------------------------------------------// 17
--------return point3d(x + p.x, y + p.y, z + p.z); }---------------------------// 8e
----point3d operator-(P(p)) const {---------------------------------------------// fb
--------return point3d(x - p.x, y - p.y, z - p.z); }---------------------------// 83
----point3d operator-() const {-------------------------------------------------// 89
--------return point3d(-x, -y, -z); }-------------------------------------------// d4
----point3d operator*(double k) const {-----------------------------------------// 4d
--------return point3d(x * k, y * k, z * k); }---------------------------------// fd
----point3d operator/(double k) const {-----------------------------------------// 95
--------return point3d(x / k, y / k, z / k); }---------------------------------// 58
----double operator%(P(p)) const {----------------------------------------------// d1
--------return x * p.x + y * p.y + z * p.z; }----------------------------------// 09
----point3d operator*(P(p)) const {---------------------------------------------// 4f
--------return point3d(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x); }---------// ed
----double length() const {-----------------------------------------------------// 3e
--------return sqrt(*this % *this); }-------------------------------------------// 05
----double distTo(P(p)) const {-------------------------------------------------// dd
--------return (*this - p).length(); }------------------------------------------// 57
----double distTo(P(A), P(B)) const {-------------------------------------------// bd
--------// A and B must be two different points--------------------------------// 4e
--------return ((*this - A) * (*this - B)).length() / A.distTo(B); }-----------// 6e
----point3d normalize(double k = 1) const {-------------------------------------// db
--------// length() must not return 0-------------------------------------------// 3c
--------return (*this) * (k / length()); }--------------------------------------// d4
----point3d getProjection(P(A), P(B)) const {-----------------------------------// 86
--------point3d v = B - A;------------------------------------------------------// 64
--------return A + v.normalize((v % (*this - A)) / v.length()); }---------------// 53
----point3d rotate(P(normal)) const {-------------------------------------------// 55
--------// normal must have length 1 and be orthogonal to the vector-----------// eb
----    return (*this) * normal; }----------------------------------------------// 5c
----point3d rotate(double alpha, P(normal)) const {-----------------------------// 21
--------return (*this) * cos(alpha) + rotate(normal) * sin(alpha); }------------// 82
----point3d rotatePoint(P(O), P(axe), double alpha) const{----------------------// 7a
--------point3d Z = axe.normalize(axe % (*this - O));--------------------------// ba
--------return O + Z + (*this - O - Z).rotate(alpha, O); }---------------------// 38
----bool isZero() const {-------------------------------------------------------// 64
--------return abs(x) < EPS && abs(y) < EPS && abs(z) < EPS; }-----------------// 15
----bool isOnLine(L(A, B)) const {----------------------------------------------// 30
--------return ((A - *this) * (B - *this)).isZero(); }--------------------------// 58
```

```
----bool isInSegment(L(A, B)) const {------------------------------------------// f1
--------return isOnLine(A, B) && ((A - *this) % (B - *this)) < EPS; }----------// d9
----bool isInSegmentStrictly(L(A, B)) const {----------------------------------// 0e
--------return isOnLine(A, B) && ((A - *this) % (B - *this)) < -EPS; }---------// ba
----double getAngle() const {--------------------------------------------------// 0f
--------return atan2(y, x); }--------------------------------------------------// 40
----double getAngle(P(u)) const {----------------------------------------------// d5
--------return atan2((*this * u).length(), *this % u); }-----------------------// 79
----bool isOnPlane(PL(A, B, C)) const {----------------------------------------// 8e
--------return abs((A - *this) * (B - *this) % (C - *this)) < EPS; } };--------// 74
int line_line_intersect(L(A, B), L(C, D), point3d &O){-------------------------// dc
----if (abs((B - A) * (C - A) % (D - A)) > EPS) return 0;----------------------// 6a
----if (((A - B) * (C - D)).length() < EPS)------------------------------------// 79
--------return A.isOnLine(C, D) ? 2 : 0;---------------------------------------// 09
----point3d normal = ((A - B) * (C - B)).normalize();--------------------------// bc
----double s1 = (C - A) * (D - A) % normal;------------------------------------// 68
----O = A + ((B - A) / (s1 + ((D - B) * (C - B) % normal))) * s1;--------------// 56
----return 1; }----------------------------------------------------------------// a7
int line_plane_intersect(L(A, B), PL(C, D, E), point3d & O) {------------------// 09
----double V1 = (C - A) * (D - A) % (E - A);-----------------------------------// c1
----double V2 = (D - B) * (C - B) % (E - B);-----------------------------------// 29
----if (abs(V1 + V2) < EPS)----------------------------------------------------// 81
--------return A.isOnPlane(C, D, E) ? 2 : 0;-----------------------------------// d5
----O = A + ((B - A) / (V1 + V2)) * V1;----------------------------------------// 38
----return 1; }----------------------------------------------------------------// ce
bool plane_plane_intersect(P(A), P(nA), P(B), P(nB), point3d &P, point3d &Q) {-// 5a
----point3d n = nA * nB;-------------------------------------------------------// 49
----if (n.isZero()) return false;----------------------------------------------// 03
----point3d v = n * nA;--------------------------------------------------------// d7
----P = A + (n * nA) * ((B - A) % nB / (v % nB));------------------------------// 1a
----Q = P + n;-----------------------------------------------------------------// 9c
----return true; }-------------------------------------------------------------// 1a
```

6.11. **Polygon Centroid.**

```
#include "polygon.cpp"---------------------------------------------------------// 58
point polygon_centroid(polygon p) {--------------------------------------------// 79
----double cx = 0.0, cy = 0.0;-------------------------------------------------// d5
----double mnx = 0.0, mny = 0.0;-----------------------------------------------// 22
----int n = size(p);-----------------------------------------------------------// 2d
----rep(i,0,n)-----------------------------------------------------------------// 08
--------mnx = min(mnx, real(p[i])),--------------------------------------------// c6
--------mny = min(mny, imag(p[i]));--------------------------------------------// 84
----rep(i,0,n)-----------------------------------------------------------------// 3f
--------p[i] = point(real(p[i]) - mnx, imag(p[i]) - mny);----------------------// 49
----rep(i,0,n) {---------------------------------------------------------------// 3c
--------int j = (i + 1) % n;---------------------------------------------------// 5b
--------cx += (real(p[i]) + real(p[j])) * cross(p[i], p[j]);-------------------// 4f
--------cy += (imag(p[i]) + imag(p[j])) * cross(p[i], p[j]); }-----------------// 4a
----return point(cx, cy) / 6.0 / polygon_area_signed(p) + point(mnx, mny); }---// a1
```

6.12. **Rotating Calipers.**

```
#include "primitives.cpp"------------------------------------------------------// e0
struct caliper {---------------------------------------------------------------// 8e
----ii pt;---------------------------------------------------------------------// 05
----double angle;--------------------------------------------------------------// d4
----caliper(ii _pt, double _angle) : pt(_pt), angle(_angle) { }----------------// 35
----double angle_to(ii pt2) {--------------------------------------------------// 8b
--------double x = angle - atan2(pt2.second - pt.second, pt2.first - pt.first);// 1e
--------while (x >= pi) x -= 2*pi;---------------------------------------------// 4a
--------while (x <= -pi) x += 2*pi;--------------------------------------------// a3
--------return x; }------------------------------------------------------------// 7d
----void rotate(double by) {---------------------------------------------------// 57
--------angle -= by;-----------------------------------------------------------// 5d
--------while (angle < 0) angle += 2*pi;---------------------------------------// 03
----}--------------------------------------------------------------------------// 20
----void move_to(ii pt2) { pt = pt2; }-----------------------------------------// 37
----double dist(const caliper &other) {----------------------------------------// 68
--------point a(pt.first,pt.second),-------------------------------------------// d7
------------  b = a + exp(point(0,angle)) * 10.0,------------------------------// 2e
------------  c(other.pt.first, other.pt.second);------------------------------// 71
--------return abs(c - closest_point(a, b, c));--------------------------------// 58
----} };-----------------------------------------------------------------------// 4b
-------------------------------------------------------------------------------// c5
// int h = convex_hull(pts);---------------------------------------------------// 9c
// double mx = 0;--------------------------------------------------------------// f1
// if (h > 1) {----------------------------------------------------------------// 26
//---- int a = 0,--------------------------------------------------------------// e6
//-------- b = 0;--------------------------------------------------------------// df
//---- rep(i,0,h) {------------------------------------------------------------// 1d
//-------- if (hull[i].first < hull[a].first)---------------------------------// ac
//------------ a = i;----------------------------------------------------------// b1
//-------- if (hull[i].first > hull[b].first)---------------------------------// 02
//------------ b = i;----------------------------------------------------------// 84
//---- }-----------------------------------------------------------------------// 1e
//---- caliper A(hull[a], pi/2), B(hull[b], 3*pi/2);---------------------------// 60
//---- double done = 0;--------------------------------------------------------// 3c
//---- while (true) {----------------------------------------------------------// 31
//-------- mx = max(mx, abs(point(hull[a].first,hull[a].second)----------------// e3
//-------------------  - point(hull[b].first, hull[b].second)));--------------// 24
//-------- double tha = A.angle_to(hull[(a+1)%h]),----------------------------// 57
//---------------- thb = B.angle_to(hull[(b+1)%h]);---------------------------// f1
//-------- if (tha <= thb) {--------------------------------------------------// 91
//------------ A.rotate(tha);-------------------------------------------------// c9
//------------ B.rotate(tha);-------------------------------------------------// f4
//------------ a = (a+1) % h;-------------------------------------------------// d4
//------------ A.move_to(hull[a]);--------------------------------------------// b3
//-------- } else {-----------------------------------------------------------// 56
//------------ A.rotate(thb);-------------------------------------------------// 56
//------------ B.rotate(thb);-------------------------------------------------// 38
//------------ b = (b+1) % h;-------------------------------------------------// 96
//------------ B.move_to(hull[b]);--------------------------------------------// 38
//-------- }------------------------------------------------------------------// bc
```

```
//-------- done += min(tha, thb);------------------------------------------------// d2
//-------- if (done > pi) {-------------------------------------------------------// c2
//------------ break;--------------------------------------------------------------// e8
//-------- }-----------------------------------------------------------------------// 37
//---- }---------------------------------------------------------------------------// ac
// }------------------------------------------------------------------------------// 9c
```

6.13. **Formulas.** Let $a = (a_x, a_y)$ and $b = (b_x, b_y)$ be two-dimensional vectors.

- $a \cdot b = |a||b| \cos\theta$, where θ is the angle between a and b.
- $a \times b = |a||b| \sin\theta$, where θ is the signed angle between a and b.
- $a \times b$ is equal to the area of the parallelogram with two of its sides formed by a and b. Half of that is the area of the triangle formed by a and b.
- **Euler's formula:** $V - E + F = 2$
- Side lengths a, b, c can form a triangle iff. $a + b > c$, $b + c > a$ and $a + c > b$.
- Sum of internal angles of a regular convex n-gon is $(n - 2)\pi$.

7. Other Algorithms

7.1. **2SAT.** A fast 2SAT solver.

```
#include "../graph/scc.cpp"-------------------------------------------------------// c3
----------------------------------------------------------------------------------// 63
bool two_sat(int n, const vii& clauses, vi& all_truthy) {--------------------// f4
----all_truthy.clear();--------------------------------------------------------// 31
----vvi adj(2*n+1);--------------------------------------------------------------// 7b
----rep(i,0,size(clauses)) {-----------------------------------------------------// 76
--------adj[-clauses[i].first + n].push_back(clauses[i].second + n);-----------// eb
--------if (clauses[i].first != clauses[i].second)-------------------------------// bc
------------adj[-clauses[i].second + n].push_back(clauses[i].first + n);-------// f0
----}-----------------------------------------------------------------------------// da
----pair<union_find, vi> res = scc(adj);------------------------------------------// 00
----union_find scc = res.first;---------------------------------------------------// 20
----vi dag = res.second;----------------------------------------------------------// ed
----vi truth(2*n+1, -1);----------------------------------------------------------// c7
----for (int i = 2*n; i >= 0; i--) {----------------------------------------------// 50
--------int cur = order[i] - n, p = scc.find(cur + n), o = scc.find(-cur + n);-// 4f
--------if (cur == 0) continue;---------------------------------------------------// cd
--------if (p == o) return false;-------------------------------------------------// d0
--------if (truth[p] == -1) truth[p] = 1;-----------------------------------------// d3
--------truth[cur + n] = truth[p];------------------------------------------------// 50
--------truth[o] = 1 - truth[p];--------------------------------------------------// 8c
--------if (truth[p] == 1) all_truthy.push_back(cur);-----------------------------// 55
----}-----------------------------------------------------------------------------// c3
----return true;------------------------------------------------------------------// eb
}---------------------------------------------------------------------------------// 6b
```

7.2. **Stable Marriage.** The Gale-Shapley algorithm for solving the stable marriage problem.

```
vi stable_marriage(int n, int** m, int** w) {------------------------------------// e4
----queue<int> q;-----------------------------------------------------------------// f6
----vi at(n, 0), eng(n, -1), res(n, -1); vvi inv(n, vi(n));-----------------------// c3
----rep(i,0,n) rep(j,0,n) inv[i][w[i][j]] = j;------------------------------------// f1
----rep(i,0,n) q.push(i);---------------------------------------------------------// d8
----while (!q.empty()) {----------------------------------------------------------// 68
--------int curm = q.front(); q.pop();--------------------------------------------// e2
--------for (int &i = at[curm]; i < n; i++) {-------------------------------------// 7e
------------int curw = m[curm][i];------------------------------------------------// 95
------------if (eng[curw] == -1) { }----------------------------------------------// f7
------------else if (inv[curw][curm] < inv[curw][eng[curw]])----------------------// d6
----------------q.push(eng[curw]);------------------------------------------------// 2e
------------else continue;--------------------------------------------------------// 1d
------------res[eng[curw] = curm] = curw, ++i; break;-----------------------------// a1
--------}-------------------------------------------------------------------------// c4
----}-----------------------------------------------------------------------------// 3d
----return res;-------------------------------------------------------------------// 42
}---------------------------------------------------------------------------------// bf
```

7.3. **Algorithm X.** An implementation of Knuth's Algorithm X, using dancing links. Solves the Exact Cover problem.

```
bool handle_solution(vi rows) { return false; }-----------------------------------// 63
struct exact_cover {--------------------------------------------------------------// 95
----struct node {-----------------------------------------------------------------// 7e
--------node *l, *r, *u, *d, *p;--------------------------------------------------// 19
--------int row, col, size;-------------------------------------------------------// ae
--------node(int _row, int _col) : row(_row), col(_col) {-------------------------// c9
------------size = 0; l = r = u = d = p = NULL; }---------------------------------// c3
----};----------------------------------------------------------------------------// c1
----int rows, cols, *sol;---------------------------------------------------------// 7b
----bool **arr;-------------------------------------------------------------------// e6
----node *head;-------------------------------------------------------------------// fe
----exact_cover(int _rows, int _cols) : rows(_rows), cols(_cols), head(NULL) {-// b6
--------arr = new bool*[rows];----------------------------------------------------// cf
--------sol = new int[rows];------------------------------------------------------// 5f
--------rep(i,0,rows)-------------------------------------------------------------// 9b
------------arr[i] = new bool[cols], memset(arr[i], 0, cols);---------------------// dd
----}-----------------------------------------------------------------------------// 21
----void set_value(int row, int col, bool val = true) { arr[row][col] = val; }-// 9e
----void setup() {----------------------------------------------------------------// a3
--------node ***ptr = new node**[rows + 1];---------------------------------------// bd
--------rep(i,0,rows+1) {---------------------------------------------------------// 76
------------ptr[i] = new node*[cols];---------------------------------------------// eb
------------rep(j,0,cols)---------------------------------------------------------// cd
----------------if (i == rows || arr[i][j]) ptr[i][j] = new node(i, j);--------// 16
----------------else ptr[i][j] = NULL;--------------------------------------------// d2
--------}-------------------------------------------------------------------------// ac
--------rep(i,0,rows+1) {---------------------------------------------------------// fc
------------rep(j,0,cols) {-------------------------------------------------------// 51
----------------if (!ptr[i][j]) continue;-----------------------------------------// f7
----------------int ni = i + 1, nj = j + 1;---------------------------------------// 7a
----------------while (true) {----------------------------------------------------// fc
--------------------if (ni == rows + 1) ni = 0;-----------------------------------// 4c
--------------------if (ni == rows || arr[ni][j]) break;--------------------------// 8d
--------------------++ni;---------------------------------------------------------// 68
----------------}-----------------------------------------------------------------// ad
----------------ptr[i][j]->d = ptr[ni][j];----------------------------------------// 84
```

```
----------------ptr[ni][j]->u = ptr[i][j];-------------------------------------// 66
----------------while (true) {-------------------------------------------------// 7f
--------------------if (nj == cols) nj = 0;------------------------------------// de
--------------------if (i == rows || arr[i][nj]) break;------------------------// 4c
--------------------++nj;------------------------------------------------------// c5
----------------}--------------------------------------------------------------// 72
----------------ptr[i][j]->r = ptr[i][nj];-------------------------------------// 60
----------------ptr[i][nj]->l = ptr[i][j];-------------------------------------// 82
------------}------------------------------------------------------------------// 0b
--------}----------------------------------------------------------------------// 16
--------head = new node(rows, -1);---------------------------------------------// 66
--------head->r = ptr[rows][0];------------------------------------------------// 3e
--------ptr[rows][0]->l = head;------------------------------------------------// 8c
--------head->l = ptr[rows][cols - 1];-----------------------------------------// 6a
--------ptr[rows][cols - 1]->r = head;-----------------------------------------// c1
--------rep(j,0,cols) {--------------------------------------------------------// 92
------------int cnt = -1;------------------------------------------------------// d4
------------rep(i,0,rows+1)----------------------------------------------------// bd
----------------if (ptr[i][j]) cnt++, ptr[i][j]->p = ptr[rows][j];-------------// f3
------------ptr[rows][j]->size = cnt;------------------------------------------// c2
--------}----------------------------------------------------------------------// b9
--------rep(i,0,rows+1) delete[] ptr[i];---------------------------------------// a5
--------delete[] ptr;----------------------------------------------------------// 72
----}--------------------------------------------------------------------------// 19
----#define COVER(c, i, j) \----------------------------------------------------// 91
--------c->r->l = c->l, c->l->r = c->r; \--------------------------------------// 82
--------for (node *i = c->d; i != c; i = i->d) \-------------------------------// 62
------------for (node *j = i->r; j != i; j = j->r) \---------------------------// 26
----------------j->d->u = j->u, j->u->d = j->d, j->p->size--;------------------// c1
----#define UNCOVER(c, i, j) \--------------------------------------------------// 89
--------for (node *i = c->u; i != c; i = i->u) \-------------------------------// f0
------------for (node *j = i->l; j != i; j = j->l) \---------------------------// 7b
----------------j->p->size++, j->d->u = j->u->d = j; \-------------------------// 65
--------c->r->l = c->l->r = c;-------------------------------------------------// 0e
----bool search(int k = 0) {---------------------------------------------------// f9
--------if (head == head->r) {-------------------------------------------------// 75
------------vi res(k);---------------------------------------------------------// 90
------------rep(i,0,k) res[i] = sol[i];----------------------------------------// 2a
------------sort(res.begin(), res.end());--------------------------------------// 63
------------return handle_solution(res);---------------------------------------// 11
--------}----------------------------------------------------------------------// 3d
--------node *c = head->r, *tmp = head->r;-------------------------------------// a3
--------for ( ; tmp != head; tmp = tmp->r) if (tmp->size < c->size) c = tmp;---// 41
--------if (c == c->d) return false;-------------------------------------------// 02
--------COVER(c, i, j);--------------------------------------------------------// f6
--------bool found = false;----------------------------------------------------// 8d
--------for (node *r = c->d; !found && r != c; r = r->d) {---------------------// 78
------------sol[k] = r->row;---------------------------------------------------// c0
------------for (node *j = r->r; j != r; j = j->r) { COVER(j->p, a, b); }------// f9
------------found = search(k + 1);---------------------------------------------// fb
------------for (node *j = r->l; j != r; j = j->l) { UNCOVER(j->p, a, b); }----// 87
--------}----------------------------------------------------------------------// 7c
--------UNCOVER(c, i, j);------------------------------------------------------// a7
--------return found;----------------------------------------------------------// c0
----}--------------------------------------------------------------------------// d2
};-----------------------------------------------------------------------------// d7
```

7.4. **nth Permutation.** A very fast algorithm for computing the nth permutation of the list $\{0, 1, \ldots, k-1\}$.

```
vector<int> nth_permutation(int cnt, int n) {----------------------------------// 78
----vector<int> idx(cnt), per(cnt), fac(cnt);----------------------------------// 9e
----rep(i,0,cnt) idx[i] = i;---------------------------------------------------// bc
----rep(i,1,cnt+1) fac[i - 1] = n % i, n /= i;---------------------------------// 2b
----for (int i = cnt - 1; i >= 0; i--)-----------------------------------------// f9
--------per[cnt - i - 1] = idx[fac[i]], idx.erase(idx.begin() + fac[i]);-------// ee
----return per;----------------------------------------------------------------// ab
}------------------------------------------------------------------------------// 37
```

7.5. **Cycle-Finding.** An implementation of Floyd's Cycle-Finding algorithm.

```
ii find_cycle(int x0, int (*f)(int)) {-----------------------------------------// a5
----int t = f(x0), h = f(t), mu = 0, lam = 1;----------------------------------// 8d
----while (t != h) t = f(t), h = f(f(h));--------------------------------------// 79
----h = x0;--------------------------------------------------------------------// 04
----while (t != h) t = f(t), h = f(h), mu++;-----------------------------------// 9d
----h = f(t);------------------------------------------------------------------// 00
----while (t != h) h = f(h), lam++;--------------------------------------------// 5e
----return ii(mu, lam);--------------------------------------------------------// b4
}------------------------------------------------------------------------------// 42
```

7.6. **Dates.** Functions to simplify date calculations.

```
int intToDay(int jd) { return jd % 7; }----------------------------------------// 89
int dateToInt(int y, int m, int d) {-------------------------------------------// 96
----return 1461 * (y + 4800 + (m - 14) / 12) / 4 +-----------------------------// a8
--------367 * (m - 2 - (m - 14) / 12 * 12) / 12 -------------------------------// d1
--------3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +---------------------------// be
--------d - 32075;-------------------------------------------------------------// e0
}------------------------------------------------------------------------------// fa
void intToDate(int jd, int &y, int &m, int &d) {-------------------------------// a1
----int x, n, i, j;------------------------------------------------------------// 00
----x = jd + 68569;------------------------------------------------------------// 11
----n = 4 * x / 146097;--------------------------------------------------------// 2f
----x -= (146097 * n + 3) / 4;-------------------------------------------------// 58
----i = (4000 * (x + 1)) / 1461001;--------------------------------------------// 0d
----x -= 1461 * i / 4 - 31;----------------------------------------------------// 09
----j = 80 * x / 2447;---------------------------------------------------------// 3d
----d = x - 2447 * j / 80;-----------------------------------------------------// eb
----x = j / 11;----------------------------------------------------------------// b7
----m = j + 2 - 12 * x;--------------------------------------------------------// 82
----y = 100 * (n - 49) + i + x;------------------------------------------------// 70
}------------------------------------------------------------------------------// af
```

7.7. **Simulated Annealing.** An example use of Simulated Annealing to find a permutation of length n that maximizes $\sum_{i=1}^{n-1} |p_i - p_{i+1}|$.

```
double curtime() { return static_cast<double>(clock()) / CLOCKS_PER_SEC; }-----// 9d
int simulated_annealing(int n, double seconds) {--------------------------------// 54
----default_random_engine rng;--------------------------------------------------// 67
----uniform_real_distribution<double> randfloat(0.0, 1.0);----------------------// ed
----uniform_int_distribution<int> randint(0, n - 2);----------------------------// bb
--------------------------------------------------------------------------------// 88
----// random initial solution-------------------------------------------------// 22
----vi sol(n);------------------------------------------------------------------// 33
----rep(i,0,n) sol[i] = i + 1;--------------------------------------------------// ee
----random_shuffle(sol.begin(), sol.end());-------------------------------------// 1e
--------------------------------------------------------------------------------// 5b
----// initialize score--------------------------------------------------------// 11
----int score = 0;--------------------------------------------------------------// 4d
----rep(i,1,n) score += abs(sol[i] - sol[i-1]);---------------------------------// 74
--------------------------------------------------------------------------------// 25
----int iters = 0;--------------------------------------------------------------// 4d
----double T0 = 100.0, T1 = 0.001,----------------------------------------------// f4
--------   progress = 0, temp = T0,--------------------------------------------// 8b
--------   starttime = curtime();----------------------------------------------// a2
----while (true) {--------------------------------------------------------------// db
--------if (!(iters & ((1 << 4) - 1))) {----------------------------------------// e8
------------progress = (curtime() - starttime) / seconds;-----------------------// a0
------------temp = T0 * pow(T1 / T0, progress);---------------------------------// 12
------------if (progress > 1.0) break;------------------------------------------// 48
--------}-----------------------------------------------------------------------// 1a
--------------------------------------------------------------------------------// 88
--------// random mutation-----------------------------------------------------// 84
--------int a = randint(rng);---------------------------------------------------// f7
--------------------------------------------------------------------------------// 02
--------// compute delta for mutation------------------------------------------// 4e
--------int delta = 0;----------------------------------------------------------// 10
--------if (a > 0) delta += abs(sol[a+1] - sol[a-1]) - abs(sol[a] - sol[a-1]);-// 21
--------if (a+2 < n) delta += abs(sol[a] - sol[a+2]) - abs(sol[a+1] - sol[a+2]);
--------------------------------------------------------------------------------// 22
--------// maybe apply mutation------------------------------------------------// 4d
--------if (delta >= 0 || randfloat(rng) < exp(delta / temp)) {----------------// a6
------------swap(sol[a], sol[a+1]);---------------------------------------------// ce
------------score += delta;-----------------------------------------------------// 64
------------// if (score >= target) return;------------------------------------// a6
--------}-----------------------------------------------------------------------// 85
--------iters++;----------------------------------------------------------------// 3c
----}---------------------------------------------------------------------------// ec
----return score;---------------------------------------------------------------// d0
}-------------------------------------------------------------------------------// ec
```

8. Useful Information

8.1. **Tips & Tricks.**

- How fast does our algorithm have to be? Can we use brute-force?
- Does order matter?
- Is it better to look at the problem in another way? Maybe backwards?
- Are there subproblems that are recomputed? Can we cache them?
- Do we need to remember everything we compute, or just the last few iterations of computation?
- Does it help to sort the data?
- Can we speed up lookup by using a map (tree or hash) or an array?
- Can we binary search the answer?
- Can we add vertices/edges to the graph to make the problem easier? Can we turn the graph into some other kind of a graph (perhaps a DAG, or a flow network)?
- Make sure integers are not overflowing.
- Is it better to compute the answer modulo n? Perhaps we can compute the answer modulo $m_1, m_2, \ldots, m_k$, where $m_1, m_2, \ldots, m_k$ are pairwise coprime integers, and find the real answer using CRT?
- Are there any edge cases? When $n = 0, n = -1, n = 1, n = 2^{31} - 1$ or $n = -2^{31}$? When the list is empty, or contains a single element? When the graph is empty, or contains a single vertex? When the graph contains self-loops? When the polygon is concave or non-simple?
- Can we use exponentiation by squaring?

8.2. **Fast Input Reading.** If input or output is huge, sometimes it is beneficial to optimize the input reading/output writing. This can be achieved by reading all input in at once (using fread), and then parsing it manually. Output can also be stored in an output buffer and then dumped once in the end (using fwrite). A simpler, but still effective, way to achieve speed is to use the following input reading method.

```
void readn(register int *n) {---------------------------------------------------// dc
----int sign = 1;---------------------------------------------------------------// 32
----register char c;------------------------------------------------------------// a5
----*n = 0;---------------------------------------------------------------------// 35
----while((c = getc_unlocked(stdin)) != '\n') {---------------------------------// f3
--------switch(c) {-------------------------------------------------------------// 0c
------------case '-': sign = -1; break;-----------------------------------------// 28
------------case ' ': goto hell;------------------------------------------------// fd
------------case '\n': goto hell;-----------------------------------------------// 79
------------default: *n *= 10; *n += c - '0'; break;----------------------------// c0
--------}-----------------------------------------------------------------------// 2d
----}---------------------------------------------------------------------------// c3
hell:---------------------------------------------------------------------------// ba
----*n *= sign;-----------------------------------------------------------------// a0
}-------------------------------------------------------------------------------// 67
```

8.3. **128-bit Integer.** GCC has a 128-bit integer data type named `__int128`. Useful if doing multiplication of 64-bit integers, or something needing a little more than 64-bits to represent.

8.4. **Worst Time Complexity.**

n	Worst AC Algorithm	Comment
≤ 10	$O(n!), O(n^6)$	e.g. Enumerating a permutation
≤ 15	$O(2^n \times n^2)$	e.g. DP TSP
≤ 20	$O(2^n), O(n^5)$	e.g. DP + bitmask technique
≤ 50	$O(n^4)$	e.g. DP with 3 dimensions + $O(n)$ loop, choosing ${}_nC_k = 4$
$\leq 10^2$	$O(n^3)$	e.g. Floyd Warshall's
$\leq 10^3$	$O(n^2)$	e.g. Bubble/Selection/Insertion sort
$\leq 10^5$	$O(n \log_2 n)$	e.g. Merge sort, building a Segment tree
$\leq 10^6$	$O(n), O(\log_2 n), O(1)$	Usually, contest problems have $n \leq 10^6$ (e.g. to read input)

8.5. Bit Hacks.

- `n & -n` returns the first set bit in n.
- `n & (n - 1)` is 0 only if n is a power of two.
- `snoob(x)` returns the next integer that has the same amount of bits set as `x`. Useful for iterating through subsets of some specified size.

```
int snoob(int x) {-------------------------------------------------------------// 73
----int y = x & -x, z = x + y;-------------------------------------------------// 12
----return z | ((x ^ z) >> 2) / y;---------------------------------------------// 97
}------------------------------------------------------------------------------// 14
```

9. Misc

9.1. Debugging Tips.

- Stack overflow? Recursive DFS on tree that is actually a long path?
- Floating-point numbers
  - Getting `NaN`? Make sure `acos` etc. are not getting values out of their range (perhaps `1+eps`).
  - Rounding negative numbers?
  - Outputting in scientific notation?

9.2. Solution Ideas.

- Dynamic Programming
  - Drop a parameter, recover from others
  - Swap answer and a parameter
  - Parsing CFGs: CYK Algorithm
  - Optimizations
    - Convex hull optimization
      - $\mathrm{dp}[i] = \min_{j<i}\{\mathrm{dp}[j] + b[j] \times a[i]\}$
      - $b[j] \geq b[j+1]$
      - optionally $a[i] \leq a[i+1]$
      - $O(n^2)$ to $O(n)$
    - Divide and conquer optimization
      - $\mathrm{dp}[i][j] = \min_{k<j}\{\mathrm{dp}[i-1][k] + C[k][j]\}$
      - $A[i][j] \leq A[i][j+1]$
      - $O(kn^2)$ to $O(kn \log n)$
      - sufficient: $C[a][c] + C[b][d] \leq C[a][d] + C[b][c]$, $a \leq b \leq c \leq d$ (QI)
    - Knuth optimization
      - $\mathrm{dp}[i][j] = \min_{i<k<j}\{\mathrm{dp}[i][k] + \mathrm{dp}[k][j] + C[i][j]\}$
      - $A[i][j-1] \leq A[i][j] \leq A[i+1][j]$
      - $O(n^3)$ to $O(n^2)$
      - sufficient: QI and $C[b][c] \leq C[a][d]$, $a \leq b \leq c \leq d$
- Greedy
- Randomized
- Optimizations
  - Use bitset (/64)
  - Switch order of loops (cache locality)
- Process queries offline
  - Mo's algorithm
- Square-root decomposition
- Precomputation
- Efficient simulation
  - Mo's algorithm
  - Sqrt decomposition
  - Store 2^k jump pointers
- Data structure techniques
  - Sqrt buckets
  - Store 2^k jump pointers
  - 2^k merging trick
- Counting
  - Inclusion-exclusion principle
  - Generating functions
- Graphs
  - Can we model the problem as a graph?
  - Can we use any properties of the graph?
  - Strongly connected components
  - Cycles (or odd cycles)
  - Bipartite (no odd cycles)
    - Bipartite matching
    - Hall's marriage theorem
    - Stable Marriage
  - Cut vertex/bridge
  - Biconnected components
  - Degrees of vertices (odd/even)
  - Trees
    - Heavy-light decomposition
    - Centroid decomposition
    - Least common ancestor
  - Eulerian path/circuit
  - Chinese postman problem
  - Topological sort
  - (Min-Cost) Max Flow
  - Min Cut
    - Maximum Density Subgraph
  - Huffman Coding
  - Min-Cost Arborescence
  - Steiner Tree
  - Kirchoff's matrix tree theorem
  - Prüfer sequences
- Mathematics
  - Is the function multiplicative?
  - Look for a pattern
  - Permutations
    - Consider the cycles of the permutation
  - Functions
    - Sum of piecewise-linear functions is a piecewise-linear function
    - Sum of convex (concave) functions is convex (concave)
  - Modular arithmetic
    - Chinese Remainder Theorem
    - Linear Congruence
  - Sieve
  - System of linear equations
- Logic
  - 2-SAT

  - XOR-SAT (Gauss elimination or Bipartite matching)
- Meet in the middle
- Only work with the smaller half ($\log(n)$)
- Strings
  - Trie (maybe over something weird, like bits)
  - Suffix array
  - Suffix automaton (+DP?)
  - Aho-Corasick
  - eerTree
  - Work with $S + S$
- Hashing
- Euler tour, tree to array
- Segment trees
  - Lazy propagation
  - Persistent
  - Implicit
  - Segment tree of X
- Geometry
  - Minkowski sum (of convex sets)
  - Rotating calipers
  - Sweep line (horizontally or vertically?)
  - Sweep angle
  - Convex hull
- Fix a parameter (possibly the answer).
- Are there few distinct values?
- Binary search
- Sliding Window (+ Monotonic Queue)
- Computing a Convolution? Fast Fourier Transform
- Exact Cover (+ Algorithm X)
- Cycle-Finding
- What is the smallest set of values that identify the solution? The cycle structure of the permutation? The powers of primes in the factorization?