Math 223: Homework 9 (Due Wednesday, April 25, 2018)

H.9.1. (a) Let T be a tree with 15 vertices and 6 leaves. Assume very vertex of T has degree 1, 2, or 6. How many vertices of degree 6 does T have? Why?

(b) Each of the following is the degree sequence of a graph. For each, is it the degree sequence of a tree? Why or why not?

(i) $4, 4, 3, 3, 2, 2, 2, 2, 2, 2, 2, 2, 2$

(ii) $3, 3, 2, 2, 1, 1$

(iii) $3, 3, 2, 2, 1, 1, 1, 1$

(iv) $3, 3, 2, 2, 1, 1, 1, 1, 1, 1$

(c) Give an example of a graph with 10 vertices, 9 edges, and at least two vertices of degree 1 that is not a tree.

H.9.2. In each of the following problems, give one of the following:

- A matching including all of the elements of X.
- A system of distinct representatives.
- A violation of the condition in Hall's Theorem.
- A vertex cover small enough to prove, using König's Theorem, there is not a matching including all of the elements of X. (You can also use this for SDR problems if you convert to a graph problem first.)

(a) Exercise 12.2.3b, Page 695.

(b) Exercise 12.2.3c, Page 695.

(c) Exercise 12.2.6a, Pages 695-696.

(d) Exercise 12.2.6c, Page 696.

H.9.3. Exercise 12.2.9, Page 696.

H.9.4. You must do parts (a), (b), and (c) below. You only need to do *one* of (d) or (e). You may do the other for one bonus point. If you do both, indicate which one you want to count as normal and which as bonus. If there's no indication, I'll count (d).

(a) Show that the complement of an independent set is a vertex cover.

(b) Show that the complement of a vertex cover is an independent set.

(c) Suppose G is a bipartite graph with bipartition A, B where $|A| \leq |B|$. Suppose $\alpha(G) > |B|$. Does G have a matching including all of the vertices in A? Why or why not?

(d) Exercise 12.2.19, Page 297. (*Hint: Use induction.*)

(e) Exercise 12.2.24, Page 298.

H.9.5. Exercise 12.8.5, Page 733. (Table 12.9 is on Page 734.)

H.9.6. Using induction on the number of vertices, prove that a tree on n vertices has $n-1$ edges. (*Hint: What happens to a tree when you delete a vertex of degree d?*)