Machine Learning Model

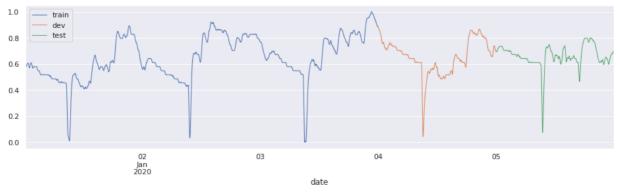
This notebook develops a few machine learning model to forecast the time-series data provided by 'time_series.csv'. Time series are a form of sequential data, which makes recurrent neural networks suitable for the task. Various network architectures are considered, involving Conv1D, LSTM and/or Dense layers.

```
In [1]: import pandas as pd
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt
import tensorflow as tf

from temp_ml_scripts import *
```

Load the time series

```
In [2]: # Load time series data
        time_series = pd.read_csv('../input/temp1-data-preparation/time_series.csv', index_col='date').asf
        req('T')['temp']
        # Chop series for developing
        time_series = time_series.loc['2020-01-01':'2020-01-05'] # 5 days
        # Normalize the data (particlularly important for NNs)
        min value = np.min(time series)
        max_value = np.max(time_series)
        time_series = (time_series - min_value)/(max_value - min_value)
        def un_normalize_series(series):
            original = series * (max_value - min_value) + min_value
            return original
        # Size of dev and test sets (in same units as time series: minutes)
        size_dev = 60 * 24 * 1 # one day
        size_test = 60 * 24 * 1 # one day
        # Split time series into train, dev and test sets
        time_series_train, time_series_dev, time_series_test = train_dev_test_split(time_series, size_dev,
        size_test)
        # for implementation, see temp-utilities.py
        # Plot the time series
        plot_time_series(time_series_train, time_series_dev, time_series_test)
        # for implementation, see temp-utilities.py
```



Train neural network

To train a neural network, it is required to construct a batch of input features x and a batch of output labels y. For plotting the time-series, it will be convenient to keep track of time in t as well.

```
In [3]: # Define the number of time steps fed into the model and the number of subsequent time steps predicted by the model
n_steps_in = 5  # this number will be varied as a model parameter
n_steps_out = 10  # this number will be kept constant to match the mathematical analysis in the notebook temp2-mathematical-models.ipynb

# Construct train, dev and test sets (X, y)
X_train, y_train, t_train = split_sequence(time_series_train, n_steps_in, n_steps_out) # for implementation of split_sequence(), see temp-ml-scripts.py
X_dev, y_dev, t_dev = split_sequence(time_series_dev, n_steps_in, n_steps_out)
X_test, y_test, t_test = split_sequence(time_series_test, n_steps_in, n_steps_out)

print('train shapes (X,y,t) : ', X_train.shape, y_train.shape, t_train.shape)
print('dev shapes (X,y,t) : ', X_dev.shape, y_dev.shape, t_dev.shape)
print('test shapes (X,y,t) : (4292, 5, 1) (4292, 10) (4292, 10)
dev shapes (X,y,t) : (4292, 5, 1) (4292, 10) (4292, 10)
test shapes (X,y,t) : (1426, 5, 1) (1426, 10) (1426, 10)
test shapes (X,y,t) : (1426, 5, 1) (1426, 10) (1426, 10)
```

Define an initial model with two layers:

- 1. LSTM (Long Short-Term Memory) layer with n_steps_in one-dimensional input features and a 20-dimensional hidden state.
- 2. Dense layer with n_steps_out real numbers in its output.

Model: "sequential"

Layer (type)	Output	Shape	Param #
lstm (LSTM)	(None,	20)	1760
dense (Dense)	(None,	10)	210
Total params: 1,970 Trainable params: 1,970 Non-trainable params: 0			

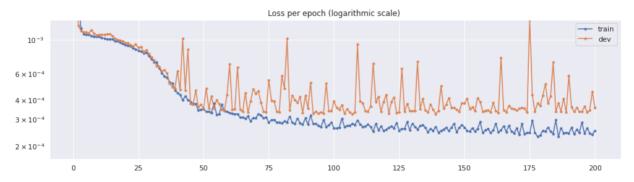
The model contains a very modest number of O(2k) trainable parameters.

Next, the model is compiled to minimize the 'mean-squared-error' loss function using the efficient adaptive 'adam' optimizer.

A model checkpoint callback is used to store the version of the model (i.e. state of weights) that achieved the lowest dev loss.

```
In [5]: # Compile the model
        model.compile(optimizer='adam', loss='mse')
        # Callbacks
        em = EpochMonitor()
        mc = tf.keras.callbacks.ModelCheckpoint('best_model.h5',
                                                 monitor='val_loss',
                                                 mode='min',
                                                 verbose=0,
                                                 save best only=True)
        # Train the model
        training_history = model.fit(X_train, y_train,
                                      validation_data=(X_dev, y_dev),
                                      epochs=200,
                                      verbose=0.
                                      callbacks=[em, mc])
        # Plot the loss during training
        # for implementation of plot training loss(), see temp-ml-scripts.py
        plot_training_loss(training_history, hide_first=2, lin=False)
```

Epoch 200



The train loss is decreasing with the number of epochs, and starting to level out, so that more training helps increasingly less. The dev loss is larger than the train loss, as would be expected. The gap between the two remains roughly constant, indicating that no overfitting is likely to be taking place yet.

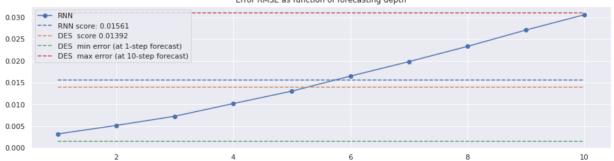
```
In [6]: # Load best state of the model
    best_model = tf.keras.models.load_model('best_model.h5')

# Forecasting
    # for implementation of direct_forecast(), see temp-ml-scripts.py
    forecast = direct_forecast(X_dev, y_dev, t_dev, best_model)

# Scoring
    # for implementation of evaluate_model_performance(), see temp-ml-scripts.py
    errors, score = evaluate_model_performance(forecast)

# Compare score to best model from preceding notebook
    # for implementation of compare_score(), see temp-ml-scripts.py
    compare_score(errors, score)

Error RMSE as function of forecasting depth
```



This simple LSTM model already performs comparably to the best model from the previous notebook temp2-mathematical-models.ipynb. The challenge is to improve upon the present model through tuning of the model hyperparameters.

Hyperparameter tuning

The hyperparameter tuning is set up as follows:

- Parameters and their ranges of values are specified in a dictionary param_values. These include fixed ('global') parameters n_steps_out, min_n_epochs, max_n_epochs, patience as well as variable parameters, such as model_name, n_steps_in, hidden_dim,...
- A custom function <code>select_model()</code> defines models, which are selected with the parameter <code>model_name</code> .
- The training and evaluation of each model in the parameter search is carried out by the custom helper function <code>grid_search()</code> . Remarks:
 - A ModelCheckpoint() callback is used to keep the best performance of each model on the dev set over the whole training.
 - A custom EarlyStoppingAfter() callback is used to train each model at least min_n_epochs epochs, after which the training is stopped once the dev loss has not lowered its all-time lowest value during the last patience epochs, or once max_n_epochs epochs have elapsed. This method allows for an efficient and reasonably fair comparison between models of different complexity, which require different amounts of training.

```
In [7]: def grid search(param values):
            Performs a grid search over the specified parameter ranges, training and evaluating an RNN in
         each instance
            Input:
            param values - dictionary with ('parameter name', [list of values]) key-values pairs, dict
            Output:
            table - record of input and output of each run, pandas.DataFrame
            # Create a parameter grid
            param_grid = ParameterGrid(param_values)
            # Separate variable from constant parameters (for printing purpose)
            variable_param = dict()
            constant param = dict()
            for (key,val) in param_values.items():
                if len(val)>1:
                    variable param[key] = val
                else:
                    constant_param[key] = val
            # Initialization
            results = []
            run = 1
            # Grid search
            for param in param_grid:
                time1 = datetime.now()
                # Select model
                model = select model(param)
                # Compile model
                model.compile(optimizer='adam', loss='mse')
                # Callbacks
                em = EpochMonitor()
                es = EarlyStoppingAfter(monitor='val loss',
                                         mode='min',
                                         verbose=0,
                                         patience=param['patience'],
                                         start_epoch=param['min_n_epochs'])
                mc = tf.keras.callbacks.ModelCheckpoint('best model.h5',
                                                         monitor='val_loss',
                                                         mode='min',
                                                         verbose=0.
                                                         save_best_only=True)
                # Construct train, dev and test sets (X, y)
                X_train, y_train, t_train = split_sequence(time_series_train, param['n_steps_in'], param[
         'n steps out'])
                X_dev, y_dev, t_dev = split_sequence(time_series_dev, param['n_steps_in'], param['n_steps_
        out'])
                # Train model
                train_hist = model.fit(X_train, y_train,
                                        validation data=(X dev, y dev),
                                        epochs=param['max_n_epochs'],
                                        verbose=0,
                                        callbacks=[em, es, mc])
                train_hist = train_hist.history
                n_epochs = len(train_hist['loss'])
                 # Load best state of the model
                best model = tf.keras.models.load model('best model.h5')
                # Forecast
                # for implementation of direct_forecast(), see temp-ml-scripts.py
                forecast = direct_forecast(X_dev, y_dev, t_dev, best_model)
                # Score
                # for implementation of evaluate model performance(), see temp-ml-scripts.py
                errors, score = evaluate_model_performance(forecast)
                # Store result
                col = list(param.keys())
col = col + ['neurons', 'n_epochs', 'train_hist', 'model', 'forecast', 'errors', 'score']
                res = list(param.values())
                res = res + [model.count_params(), n_epochs, train_hist, best_model, forecast, errors, sco
        re1
                results.append(res)
                time2 = datetime.now()
                deltat = time2 - time1
                print('{run:2}/{tot:2}
                                        {time} {epo:3} epochs {neu:7} neurons score = {sco:.5} {pa
        r}'.format(
                    run=run,
                    tot=len(param_grid),
                    time=str(deltat).split('.', 2)[0],
```

```
epo=n_epochs,
    neu=model.count_params(),
    sco=score,
    par={key:param[key] for key in variable_param.keys()}
))
run=run+1

# Collect result in a table and sort by score
table = pd.DataFrame(np.array(results), columns=col)
table = table.sort_values('score')
return table
```

Search 1: vary number of input steps and hidden dimension

The initial model features two key parameters that can be tuned:

- the number of input time steps n_steps_in
- the dimension of the hidden state hidden_dim

The initial model uses the values $n_steps_{in} = 5$ and $hidden_dim = 20$ without motivation. Here, both parameters are varied in search of a better performing model.

```
In [8]: # Define models
         def select_model(param):
             Selects a model from a list of different neural networks
             # The same model as defined above, but with adjustable parameters:
             if param['model_name'] == 'mod1':
                 model = tf.keras.models.Sequential([
                     tf.keras.layers.LSTM(param['hidden_dim'], activation='relu', input_shape=(param['n_ste
         ps_in'], 1)),
                      tf.keras.layers.Dense(param['n_steps_out'], activation='linear')
                 ])
             return model
         # Define parameter ranges
         param_values = {
             # always keep fixed
             'n_steps_out' : [10],
'min_n_epochs' : [200],
'max_n_epochs' : [200],
             'patience' : [0],
             # keep fixed
             'model_name' : ['mod1'],
             # vary
             'n_steps_in': [3,5,10,20],
'hidden_dim': [20,40,60,80],
         }
         # Run search
         search1 = grid_search(param_values) # for implementation of grid_search(), see temp-ml-scripts.py
         search1.to_csv('search1.csv')
         search1[['neurons','n_epochs','n_steps_in','hidden_dim','score']]
```

1/16 20}	0:01:28	200 epochs	1970 neurons	score = 0.019467	<pre>{'n_steps_in': 3, 'hidden_dim':</pre>
2/16 20}	0:01:54	200 epochs	1970 neurons	score = 0.015297	<pre>{'n_steps_in': 5, 'hidden_dim':</pre>
3/16 m': 20}	0:02:47	200 epochs	1970 neurons	score = 0.016801	{'n_steps_in': 10, 'hidden_di
4/16 m': 20}	0:04:36	200 epochs	1970 neurons	score = 0.014655	{'n_steps_in': 20, 'hidden_di
5/16 40}	0:01:34	200 epochs	7130 neurons	score = 0.013973	<pre>{'n_steps_in': 3, 'hidden_dim':</pre>
6/16 40}	0:01:57	200 epochs	7130 neurons	score = 0.016042	<pre>{'n_steps_in': 5, 'hidden_dim':</pre>
7/16 m': 40}	0:02:51	200 epochs	7130 neurons	score = 0.015274	{'n_steps_in': 10, 'hidden_di
8/16 m': 40}	0:04:52	200 epochs	7130 neurons	score = 0.013775	{'n_steps_in': 20, 'hidden_di
9/16 60}	0:01:41	200 epochs	15490 neurons	score = 0.014903	<pre>{'n_steps_in': 3, 'hidden_dim':</pre>
10/16 60}	0:02:04	200 epochs	15490 neurons	score = 0.015454	<pre>{'n_steps_in': 5, 'hidden_dim':</pre>
11/16 m': 60}	0:03:09	200 epochs	15490 neurons	score = 0.014439	{'n_steps_in': 10, 'hidden_di
12/16 m': 60}		200 epochs	15490 neurons	score = 0.013208	{'n_steps_in': 20, 'hidden_di
13/16 80}	0:01:50	200 epochs	27050 neurons	score = 0.013995	<pre>{'n_steps_in': 3, 'hidden_dim':</pre>
14/16 80}	0:02:24	200 epochs	27050 neurons	score = 0.015762	<pre>{'n_steps_in': 5, 'hidden_dim':</pre>
15/16 m': 80}	0:03:51	200 epochs	27050 neurons	score = 0.013046	{'n_steps_in': 10, 'hidden_di
16/16 m': 80}	0:06:20	200 epochs	27050 neurons	score = 0.013258	{'n_steps_in': 20, 'hidden_di

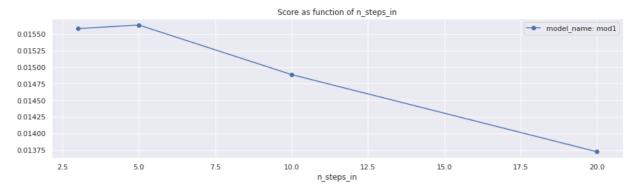
Out[8]:

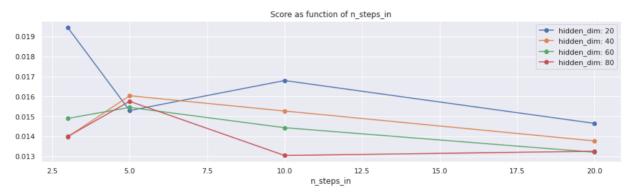
	neurons	n_epochs	n_steps_in	hidden_dim	score
14	27050	200	10	80	0.0130463
11	15490	200	20	60	0.0132082
15	27050	200	20	80	0.0132585
7	7130	200	20	40	0.0137749
4	7130	200	3	40	0.0139732
12	27050	200	3	80	0.0139952
10	15490	200	10	60	0.0144391
3	1970	200	20	20	0.014655
8	15490	200	3	60	0.0149026
6	7130	200	10	40	0.0152741
1	1970	200	5	20	0.015297
9	15490	200	5	60	0.0154536
13	27050	200	5	80	0.0157617
5	7130	200	5	40	0.0160418
2	1970	200	10	20	0.016801
0	1970	200	3	20	0.0194674

- Plot the results of each run
- Plot the training history of each run
- Plot the best score in comparison to the mathematical model

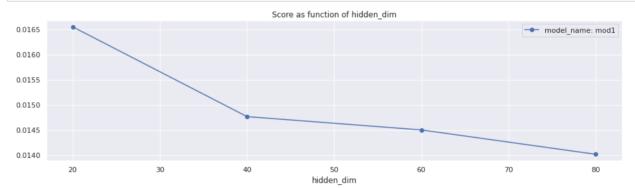
```
In [9]: # Set data to plot
    data = search1
    best = data.iloc[0]
```

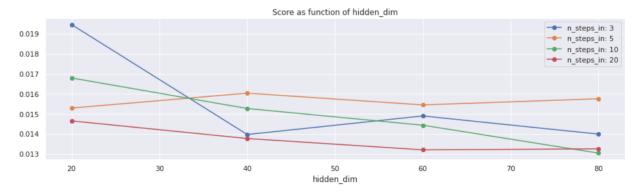
In [10]: # Plot score vs n_steps_in
 plot_score(data, variable='n_steps_in', category='model_name') # averaged over hidden_dim
 plot_score(data, variable='n_steps_in', category='hidden_dim')





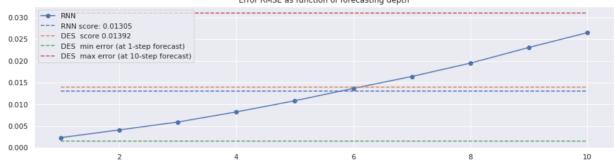
In [11]: # Plot score vs hidden_dim
 plot_score(data, variable='hidden_dim', category='model_name') # averaged over n_steps_in
 plot_score(data, variable='hidden_dim', category='n_steps_in')





```
In [ ]: # # Plot training histories
    # labels = ['n_steps_in', 'hidden_dim']
    # for i in range(len(data)):
    # title = ' run '+str(i+1)
    # for lab in labels: title=title+' '+lab+': '+str(data[lab].loc[i])
    # plot_training_loss(data['train_hist'].loc[i], hide_first=2, title=title, lin=False)
```

```
In [13]: # Construct train and dev sets
         X_train, y_train, t_train = split_sequence(time_series_train, best['n_steps_in'], best['n_steps_ou
         t'1)
         X_dev, y_dev, t_dev = split_sequence(time_series_dev, best['n_steps_in'], best['n_steps_out'])
         # Forecasting and scoring
         forecast = direct_forecast(X_dev, y_dev, t_dev, best['model']) # for implementation of direct_for
         ecast(), see temp-ml-scripts.py
         errors, score = evaluate model performance(forecast)
                                                                            # for implementation of evaluate m
         odel_performance(), see temp-ml-scripts.py
                                                                            # for implementation of compare sc
         compare score(errors, score)
         ore(), see temp-ml-scripts.py
                                                Error RMSE as function of forecasting depth
          0.030
                   RNN score: 0.01305
```



```
In [14]: # Best score (lowest) in current search
    print('best score: ',best['score'])
    print('n_steps_in: ',best['n_steps_in'])
    print('hidden_dim: ',best['hidden_dim'])

    best score: 0.013046260658448596
    n_steps_in: 10
    hidden_dim: 80
```

For the estimated best combination of n_steps_out and hidden_dim, this RNN model outperforms the mathematical DES model.

Search 2: Vary architectures and number of input steps

Looking for better performance, a next step would be to change the neural network architecture itself. In this search, three different architectures are considered, schematically:

- LSTM + Dense
- LSTM + LSTM + Dense
- Conv1D + LSTM + Dense

and each are considered with a few values of n_steps_in .

```
In [15]: # Define models
         def select_model(param):
             Selects a model from a list of different neural networks
             # Three different recurrent neural network architectures with a comparable number of neurons ~
         O(30k-40k)
             # Same network architecture as above
             if param['model_name'] == 'mod1':
                 model = tf.keras.models.Sequential([
                     tf.keras.layers.LSTM(100, activation='relu', input_shape=(param['n_steps_in'], 1)),
                     tf.keras.layers.Dense(param['n_steps_out'], activation='linear')
                 ])
             # Deeper network: stack of two LSTM layers
             if param['model_name'] == 'mod2':
                 model = tf.keras.models.Sequential([
                     tf.keras.layers.LSTM(60, activation='relu', return sequences=True, input shape=(param[
         'n_steps_in'], 1)),
                     tf.keras.layers.LSTM(50, activation='relu'),
                     tf.keras.layers.Dense(param['n_steps_out'], activation='linear')
                 ])
             # Deeper network: preprocess input with ConvlD, before feeding into LSTM
             if param['model_name'] == 'mod3':
                 model = tf.keras.models.Sequential([
                     tf.keras.layers.ConvlD(filters=128, kernel size=5, strides=1, padding='causal', activa
         tion='relu', input_shape=(param['n_steps_in'], 1)),
                     tf.keras.layers.LSTM(50, activation='relu'),
                     tf.keras.layers.Dense(param['n_steps_out'], activation='linear')
                 ])
             return model
         for name in ['mod1','mod2','mod3']:
             print(name,select_model({'model_name':name,'n_steps_in':20,'n_steps_out':10}).count_params())
```

mod1 41810 mod2 37590 mod3 37078

```
In [16]: # Define parameter ranges
         param_values = {
             # always keep fixed
             'n_steps_out' : [10],
             'min_n_epochs' : [200],
'max_n_epochs' : [200],
             'patience' : [0],
             # varv
             'model_name' : ['mod1','mod2','mod3'],
             'n_steps_in': [5,10,20]
         }
         # Run grid search
         search2 = grid_search(param_values)
         search2.to_csv('search2.csv')
         search2[['neurons','n_epochs','model_name','n_steps_in','score']]
                                                                              {'model_name': 'mod1', 'n_steps
          1/9
                0:02:41
                            200 epochs
                                         41810 neurons
                                                         score = 0.015034
         _in': 5}
          2/ 9
                 0:04:20
                            200 epochs
                                         41810 neurons
                                                          score = 0.013529
                                                                              {'model_name': 'mod1', 'n_steps
         _in': 10}
          3/ 9 0:07:33
                            200 epochs
                                         41810 neurons
                                                         score = 0.012665
                                                                              {'model_name': 'mod1', 'n_steps
         _in': 20}
          4/9
                 0:03:34
                            200 epochs
                                         37590 neurons
                                                          score = 0.014552
                                                                              {'model name': 'mod2', 'n steps
         _in': 5}
          _
5/ 9
                 0:05:39
                            200 epochs
                                         37590 neurons
                                                          score = 0.012213
                                                                              {'model_name': 'mod2', 'n_steps
         _in': 10}
6/ 9 0:10:03
                                                          score = 0.011758
                                                                              {'model_name': 'mod2', 'n_steps
                            200 epochs
                                         37590 neurons
         _in': 20}
          _
7/ 9
                 0:02:23
                                                                              {'model_name': 'mod3', 'n_steps
                            200 epochs
                                         37078 neurons
                                                          score = 0.013453
          _in': 5}
          8/ 9 0:03:32
                                         37078 neurons
                                                         score = 0.011682
                                                                              {'model name': 'mod3', 'n steps
                            200 epochs
         _in': 10}
          9/ 9 0:05:51
                                         37078 neurons
                                                                              {'model name': 'mod3', 'n steps
                            200 epochs
                                                         score = 0.011858
         _in': 20}
```

Out[16]:

	neurons	n_epochs	model_name	n_steps_in	score
7	37078	200	mod3	10	0.0116822
5	37590	200	mod2	20	0.0117579
8	37078	200	mod3	20	0.0118578
4	37590	200	mod2	10	0.0122131
2	41810	200	mod1	20	0.0126651
6	37078	200	mod3	5	0.0134535
1	41810	200	mod1	10	0.013529
3	37590	200	mod2	5	0.0145517
0	41810	200	mod1	5	0.0150335

- Plot the results of each run
- · Plot the training history of each run
- · Plot the best score in comparison to the mathematical model

```
In [17]: # Set data to plot
    data = search2
    best = data.iloc[0]
```

```
In [18]: # Plot score vs model name
          plot score(data, variable='n steps in', category='model name')
                                                         Score as function of n steps in
           0.0150
                                                                                                      model name: mod1
                                                                                                         model_name: mod2
           0.0145
                                                                                                         model_name: mod3
           0.0140
           0.0135
           0.0130
           0.0125
           0.0120
                           6
                                                                            14
                                                                                         16
                                                                                                     18
                                                                                                                  20
                                                                n steps in
          # # Plot training histories
 In [ ]:
           # labels = ['model_name', 'n_steps_in']
          # for i in range(len(data)):
                 title = 'run '+str(i+1)
          #
                 for lab in labels: title=title+'
                                                       '+lab+': '+str(data[lab].loc[i])
                 plot_training_loss(data['train_hist'].loc[i], hide_first=2, title=title, lin=False)
In [20]: # Construct train and dev sets
          X_train, y_train, t_train = split_sequence(time_series_train, best['n_steps_in'], best['n_steps_ou
          t'])
          X_dev, y_dev, t_dev = split_sequence(time_series_dev, best['n_steps_in'], best['n_steps_out'])
          # Forecasting and scoring
          forecast = direct_forecast(X_dev, y_dev, t_dev, best['model']) # for implementation of direct_for
          ecast(), see temp-ml-scripts.py
          errors, score = evaluate_model_performance(forecast)
                                                                                  # for implementation of evaluate m
          odel performance(), see temp-ml-scripts.py
          compare_score(errors, score)
                                                                                  # for implementation of compare_sc
          ore(), see temp-ml-scripts.py
                                                    Error RMSE as function of forecasting depth
                 - RNN
           0.030
                    RNN score: 0.01168
                    DES score 0.01392
           0.025
                 --- DES min error (at 1-step forecast)
--- DES max error (at 10-step forecast)
           0.020
           0.015
           0.010
           0.005
           0.000
In [21]: # Best score (lowest) in current search
          print('best score: ',best['score'])
          print('model_name: ',best['model_name'])
          print('n_steps_in: ',best['n_steps_in'])
          best score: 0.011682168575362629 model_name: mod3
```

The third model, Conv1D + LSTM + Dense, has shown significant performance improvement.

Search 3: vary Conv1D model parameters: filters, input steps, kernel

This search focusses on the third model and attempts to optimize the parameters of the Conv1D layer

- filters
- · input steps

n_steps_in: 10

kernel size

```
1/27
      0:03:59
                 400 epochs
                               4578 neurons
                                              score = 0.012606
                                                                 {'filters': 32, 'n_steps_in':
5, 'kernel': 3}
2/27 0:04:58
                                                                 {'filters': 32, 'n steps in': 1
                 332 epochs
                                4578 neurons
                                              score = 0.011662
0, 'kernel': 3}
3/27
      0:09:46
                 400 epochs
                               4578 neurons
                                              score = 0.011982
                                                                 {'filters': 32, 'n_steps_in': 2
0, 'kernel': 3}
                                                                 {'filters': 32, 'n steps in':
4/27 0:03:53
                 400 epochs
                               4642 neurons
                                              score = 0.013564
5, 'kernel': 5}
                                                                 {'filters': 32, 'n_steps_in': 1
5/27 0:05:55
                 400 epochs
                                              score = 0.011797
                               4642 neurons
0, 'kernel': 5}
6/27 0:07:06
                                                                 {'filters': 32, 'n steps in': 2
                 284 epochs
                               4642 neurons
                                              score = 0.012197
0, 'kernel': 5}
7/27 0:03:53
                 400 epochs
                               4802 neurons
                                              score = 0.013823
                                                                 {'filters': 32, 'n_steps_in':
5, 'kernel': 10}
8/27 0:05:22
                 353 epochs
                               4802 neurons
                                              score = 0.012422
                                                                 {'filters': 32, 'n steps in': 1
0, 'kernel': 10}
                                                                 {'filters': 32, 'n steps in': 2
9/27
      0:08:10
                 326 epochs
                                4802 neurons
                                              score = 0.012344
0, 'kernel': 10}
10/27 0:03:57
                 400 epochs
                               7266 neurons
                                              score = 0.012675
                                                                 {'filters': 64, 'n_steps_in':
5, 'kernel': 3}
                                                                 {'filters': 64, 'n_steps_in': 1
11/27 0:05:22
                 342 epochs
                               7266 neurons
                                              score = 0.011739
0, 'kernel': 3}
12/27 0:06:54
                 257 epochs
                               7266 neurons
                                              score = 0.011618
                                                                 {'filters': 64, 'n steps in': 2
0, 'kernel': 3}
13/27 0:03:35
                                                                 {'filters': 64, 'n steps in':
                 329 epochs
                               7394 neurons
                                              score = 0.013882
5, 'kernel': 5}
14/27 0:05:18
                 340 epochs
                                7394 neurons
                                              score = 0.012022
                                                                 {'filters': 64, 'n_steps_in': 1
0, 'kernel': 5}
15/27
      0:09:31
                 373 epochs
                               7394 neurons
                                              score = 0.011919
                                                                 {'filters': 64, 'n_steps_in': 2
0, 'kernel': 5}
                                                                 {'filters': 64, 'n_steps_in':
16/27 0:04:10
                 400 epochs
                               7714 neurons
                                              score = 0.013433
5, 'kernel': 10}
17/27 0:05:33
                 345 epochs
                                              score = 0.012373
                                                                 {'filters': 64, 'n_steps_in': 1
                               7714 neurons
0, 'kernel': 10}
18/27 0:07:49
                 299 epochs
                               7714 neurons
                                              score = 0.012636
                                                                 {'filters': 64, 'n_steps_in': 2
0, 'kernel': 10}
19/27
      0:04:17
                 400 epochs
                               12642 neurons
                                              score = 0.011877
                                                                 {'filters': 128, 'n_steps_in':
5, 'kernel': 3}
20/27 0:05:26
                 342 epochs
                               12642 neurons
                                              score = 0.01175
                                                                 {'filters': 128, 'n_steps_in': 1
0, 'kernel': 3}
                                                                 {'filters': 128, 'n_steps_in':
21/27 0:09:48
                 373 epochs
                              12642 neurons
                                              score = 0.012328
20, 'kernel': 3}
22/27 0:02:51
                 263 epochs
                                                                 {'filters': 128, 'n_steps_in':
                              12898 neurons
                                              score = 0.012719
5, 'kernel': 5}
23/27 0:04:43
                                              score = 0.011943
                                                                 {'filters': 128, 'n_steps_in':
                 290 epochs
                              12898 neurons
10, 'kernel': 5}
24/27 0:07:45
                 281 epochs
                              12898 neurons
                                              score = 0.011558
                                                                 {'filters': 128, 'n steps in':
20, 'kernel': 5}
                 400 epochs
25/27 0:04:28
                              13538 neurons
                                              score = 0.012563
                                                                 {'filters': 128, 'n_steps_in':
5, 'kernel': 10}
26/27 0:06:50
                 400 epochs
                              13538 neurons
                                              score = 0.011382
                                                                 {'filters': 128, 'n steps in':
10, 'kernel': 10}
27/27 0:08:43
                              13538 neurons
                                                                 {'filters': 128, 'n_steps_in':
                 320 epochs
                                              score = 0.011974
20, 'kernel': 10}
```

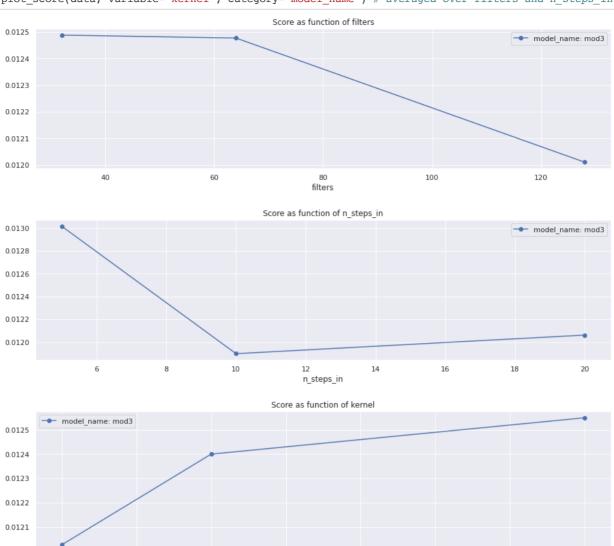
Out[23]:

	neurons	n_epochs	filters	n_steps_in	kernel	score
25	13538	400	128	10	10	0.0113822
23	12898	281	128	20	5	0.011558
11	7266	257	64	20	3	0.0116181
1	4578	332	32	10	3	0.0116621
10	7266	342	64	10	3	0.0117389
19	12642	342	128	10	3	0.0117503
4	4642	400	32	10	5	0.011797
18	12642	400	128	5	3	0.011877
14	7394	373	64	20	5	0.0119194
22	12898	290	128	10	5	0.011943
26	13538	320	128	20	10	0.0119741
2	4578	400	32	20	3	0.0119817
13	7394	340	64	10	5	0.0120216
5	4642	284	32	20	5	0.012197
20	12642	373	128	20	3	0.0123278
8	4802	326	32	20	10	0.0123439
16	7714	345	64	10	10	0.0123727
7	4802	353	32	10	10	0.0124217
24	13538	400	128	5	10	0.0125631
0	4578	400	32	5	3	0.0126055
17	7714	299	64	20	10	0.0126355
9	7266	400	64	5	3	0.012675
21	12898	263	128	5	5	0.0127193
15	7714	400	64	5	10	0.0134331
3	4642	400	32	5	5	0.0135644
6	4802	400	32	5	10	0.0138235
12	7394	329	64	5	5	0.0138816

- Plot the results of each run
- Plot the training history of each run
- Plot the best score in comparison to the mathematical model

```
In [24]: # Set data to plot
data = search3
best = data.iloc[0]
```

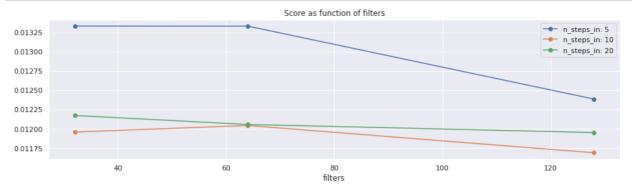
In [25]: # Plot score
plot_score(data, variable='filters', category='model_name') # averaged over n_steps_in and kernel
plot_score(data, variable='n_steps_in', category='model_name') # averaged over filters and kernel
plot_score(data, variable='kernel', category='model_name') # averaged over filters and n_steps_in

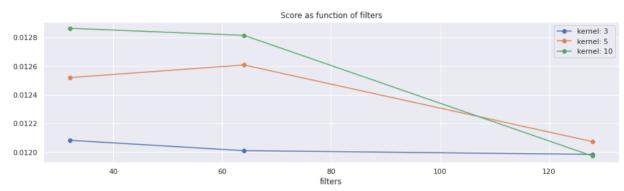


kernel

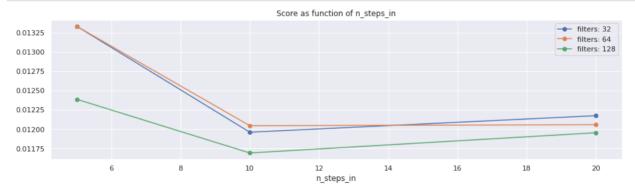
10

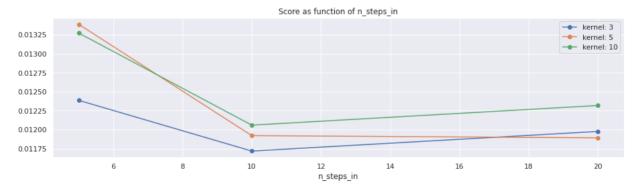
In [26]: # More precise plots
 plot_score(data, variable='filters', category='n_steps_in') # averaged over kernel
 plot_score(data, variable='filters', category='kernel') # averaged over n_steps_in



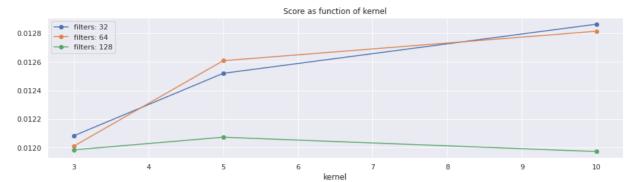


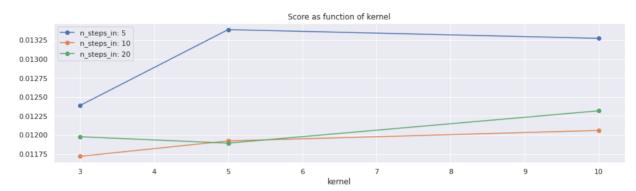
In [27]: # More precise plots
 plot_score(data, variable='n_steps_in', category='filters') # averaged over kernel
 plot_score(data, variable='n_steps_in', category='kernel') # averaged over filters





```
In [28]: # More precise plots
    plot_score(data, variable='kernel', category='filters') # averaged over n_steps_in
    plot_score(data, variable='kernel', category='n_steps_in') # averaged over filters
```





```
In [29]: # # Plot training histories
# labels = ['filters', 'n_steps_in', 'kernel']
# for i in range(len(data)):
# title = ' run '+str(i+1)
# for lab in labels: title=title+' '+lab+': '+str(data[lab].loc[i])
# plot_training_loss(data['train_hist'].loc[i], hide_first=2, title=title, lin=False)
```

```
In [30]: # Construct train and dev sets
X_train, y_train, t_train = split_sequence(time_series_train, best['n_steps_in'], best['n_steps_ou t'])
X_dev, y_dev, t_dev = split_sequence(time_series_dev, best['n_steps_in'], best['n_steps_out'])
# Forecasting and scoring
forecast = direct_forecast(X_dev, y_dev, t_dev, best['model']) # for implementation of direct_for ecast(), see temp-ml-scripts.py
errors, score = evaluate_model_performance(forecast) # for implementation of evaluate_m odel_performance(), see temp-ml-scripts.py
compare_score(errors, score) # for implementation of compare_score(), see temp-ml-scripts.py
```

```
Error RMSE as function of forecasting depth
        --- RNN
0.030
             RNN score: 0.01138
             DES score 0.01392
0.025
             DES min error (at 1-step forecast)
         --- DES max error (at 10-step forecast)
0.020
0.015
0.010
0.005
0.000
                            2
                                                          4
                                                                                        6
                                                                                                                      8
                                                                                                                                                   10
```

```
In [31]: # Best score (lowest) in current search
    print('best score: ',best['score'])
    print('model_name: ',best['model_name'])
    print('filters: ',best['filters'])
    print('n_steps_in: ',best['n_steps_in'])
    print('kernel: ',best['kernel'])
```

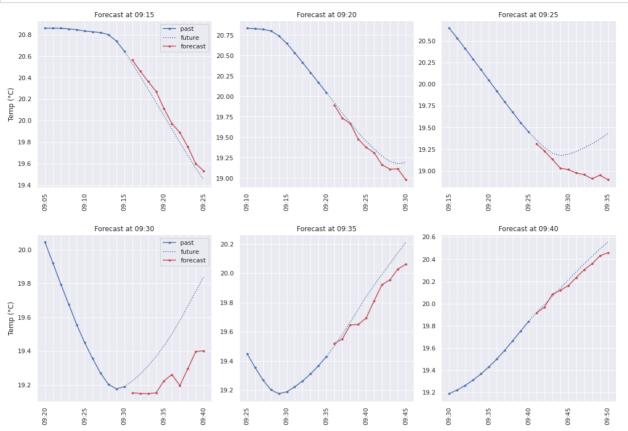
best score: 0.011382175017157109
model_name: mod3
filters: 128
n_steps_in: 10
kernel: 10

Demonstration of best model

Load the best model from all runs in all searches

```
In [32]: searches = [search1, search2, search3]
         best_score = 999
         for search in searches:
            cand = search.iloc[0]
             print('>',cand['score'])
             if cand['score'] < best score:</pre>
                 best_score = cand['score']
                 best = cand
         print('\nBest score: ',best_score)
         print('\nModel details:\n'+str(best.iloc[:-5]))
         > 0.013046260658448596
         > 0.011682168575362629
         > 0.011382175017157109
         Best score: 0.011382175017157109
         Model details:
                        128
         filters
         hidden_dim
                           20
         kernel
                           10
         max_n_epochs 400
min n epochs 200
         max_n_epochs
         min_n_epo.
model_name
                         mod3
                         10
10
50
         n_steps_in
         n_steps_out
         patience
                    13538
         neurons
         n epochs
                          400
         Name: 25, dtype: object
In [33]: best_model = best['model']
         n_steps_in = best['n_steps_in']
         n_steps_out = best['n_steps_out']
         best model.save('selected best model.h5')
         # Construct dev and test sets
         X_dev, y_dev, t_dev = split_sequence(time_series_dev, n_steps_in, n_steps_out)
         X_test, y_test, t_test = split_sequence(time_series_test, n_steps_in, n_steps_out)
         # Demonstrate model on test set
         past = time_series_dev.iloc[-n_steps_out:]
                                                                         # past = last steps of dev...
         past = past.append(time_series_test.iloc[:n_steps_out])
                                                                         # ... and first steps of test
                                                                         # un-normalize to degree Centigrad
         past = un_normalize_series(past)
         e
         # Forecasting
         forecast = direct_forecast(X_test, y_test, t_test, best_model) # for implementation of direct_for
         ecast(), see temp-ml-scripts.py
         forecast = un_normalize_series(forecast)
                                                                         # un-normalize to degree Centigrad
```

In [34]: # Forecasting in a valley
Demonstration(past, forecast, cols=3, rows=2, start=557-10, step=5)



In [35]: # Forecasting on a bumpy downhill slope
Demonstration(past, forecast, cols=3, rows=2, start=662-10, step=5)

