01-A-05

Convex Hull

Convexity

- Convex Hull

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Convex Combination

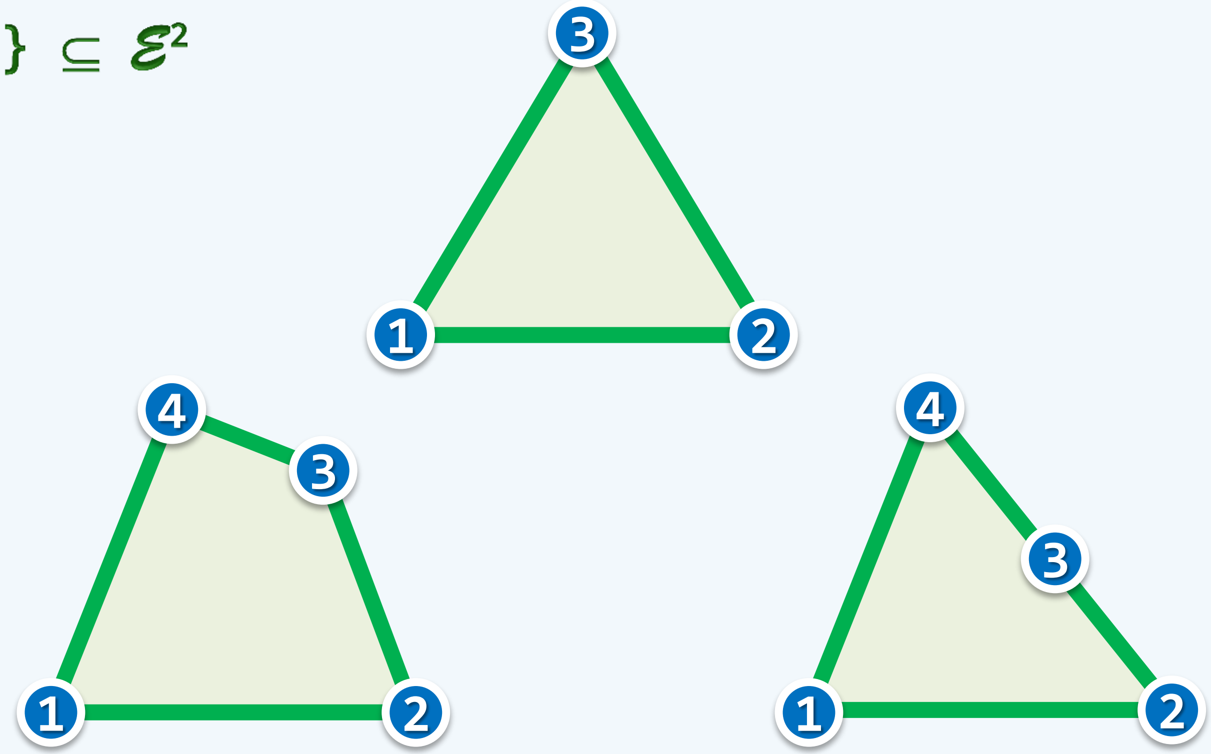
- Given a point set $S = \{p_1, \ldots, p_n\} \subseteq \mathcal{E}^2$
- Let $\lambda = \langle \lambda_1, \ldots, \lambda_n \rangle^T \in R^n$,

  $\lambda_1 + \ldots + \lambda_n = 1$, and

  $\min\{\lambda_1, \ldots, \lambda_n\} \geq 0$
- The point

$$p = [p_1, \ldots, p_n]\,\lambda$$
$$= \lambda_1 p_1 + \ldots + \lambda_n p_n$$

is called a $\boxed{\text{convex combination}}$ of S



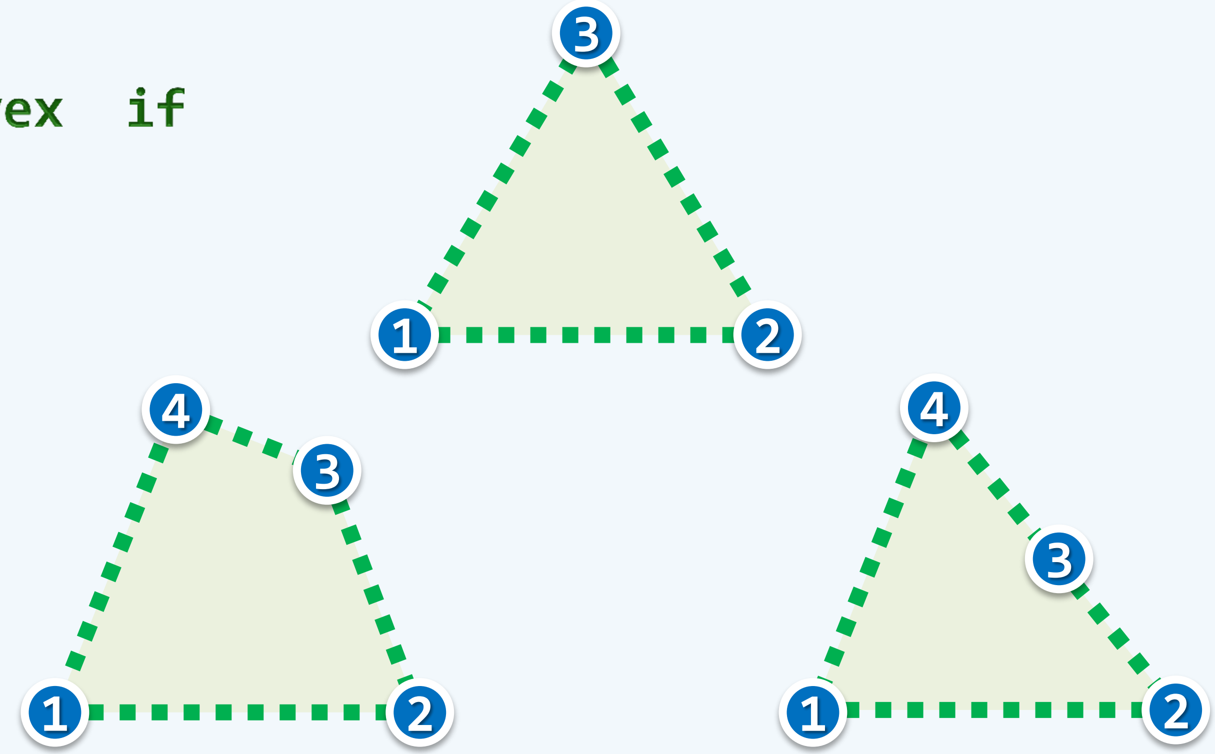
Convex Set

❖ A point set $P \subseteq \mathcal{E}^2$ is called convex if

the convex combination

of each subset of P

is still a subset of P



Convex Hull

- The convex hull of a point set $P \subseteq \mathcal{E}^2$, denoted as conv(P), consists of $\boxed{\text{all}}$ convex combinations of $\boxed{\text{all}}$ its subsets

- e.g.

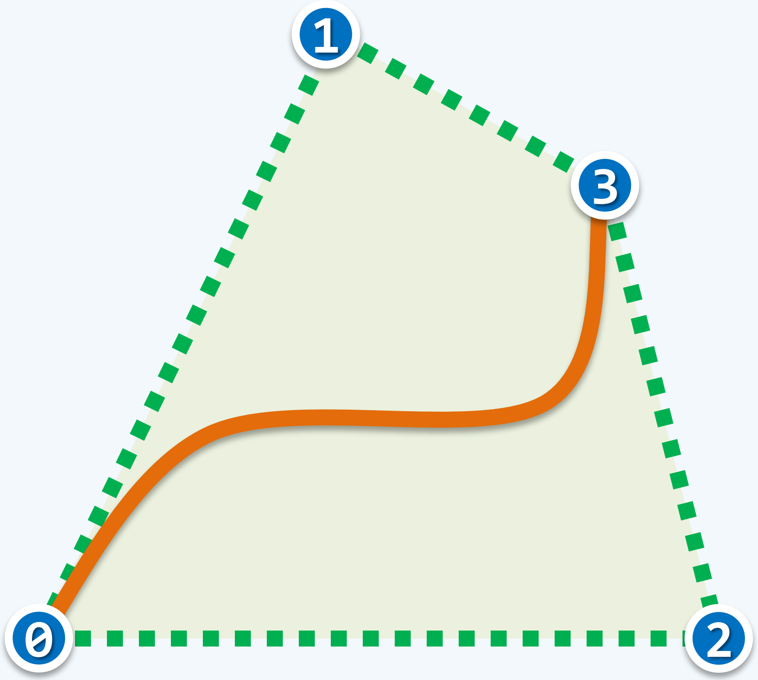
  $\boxed{\text{Bezier curve}}$ is contained within the convex hull of the control points

  - non-negativity:

    $\boxed{B_{i,n}(u) \geq 0}$ for all i

  - partition of unity:

    $\boxed{B_{0,n}(u) + B_{1,n}(u) + \ldots + B_{n,n}(u) = 1}$



Convex Hull

- conv(P) is
  - the intersection of all convex sets containing P
  - the minimum convex set containing P
  - ......
  - a $\boxed{\text{polygon}}$ for a $\boxed{\text{finite}}$ planar set P
- From the computational point of view,

  we consider only
  - a finite point set P and
  - the $\boxed{\text{boundary}}$ of $\boxed{\text{conv(P)}}$,

    denoted as $\boxed{\text{CH(P)}}$