

# Triangulation

Beyond  $\Omega(n \log n)$

- Reflexivity

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## $\Omega(n \log n)$ ?

❖ For a long period, people conjectured that ...

$\Omega(n \log n)$  is a lower bound for simple polygon triangulation

❖ Most people believed that it is true, though nobody could prove it

❖ The hints for the existence of beyond- $\Omega(n \log n)$  algorithms  
came from the fact that

there are algorithms with performances

sensitive to the  $\text{polygon shapes}$

❖ Examples include the number of  $\text{reflex vertices}$ , the  $\text{sinuosity}$ , ...

- ❖ A simple polygon can be triangulated in  $O(n + r \log r)$  time,  
after a single pass of plane sweeping,  
where  $r$  is the number of reflex vertices
- ❖ They refined the plane sweep paradigm s.t.
  - The sweep line stops only at the  $r$  reflex vertices;  
//Thus only  $O(r)$ , in stead of  $O(n)$ , vertices need to be sorted
  - The sweep line may break into pieces,  
some of which may lag behind others

## Reflexivity Sensitivity

❖ It should be pointed out that,

since an  $n$ -gon may have up to  $r = n - 3$  reflex vertices

this does not beat  $\Omega(n \log n)$  in the worst cases

❖ Nevertheless,

it could be a significant gain in practice

❖ Moreover, it was the first hint that

perhaps an  $\Theta(n \log n)$  algorithms might be achievable

