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Triangulation

Triangulating Monotone Polygons

- Monotonicity Testing

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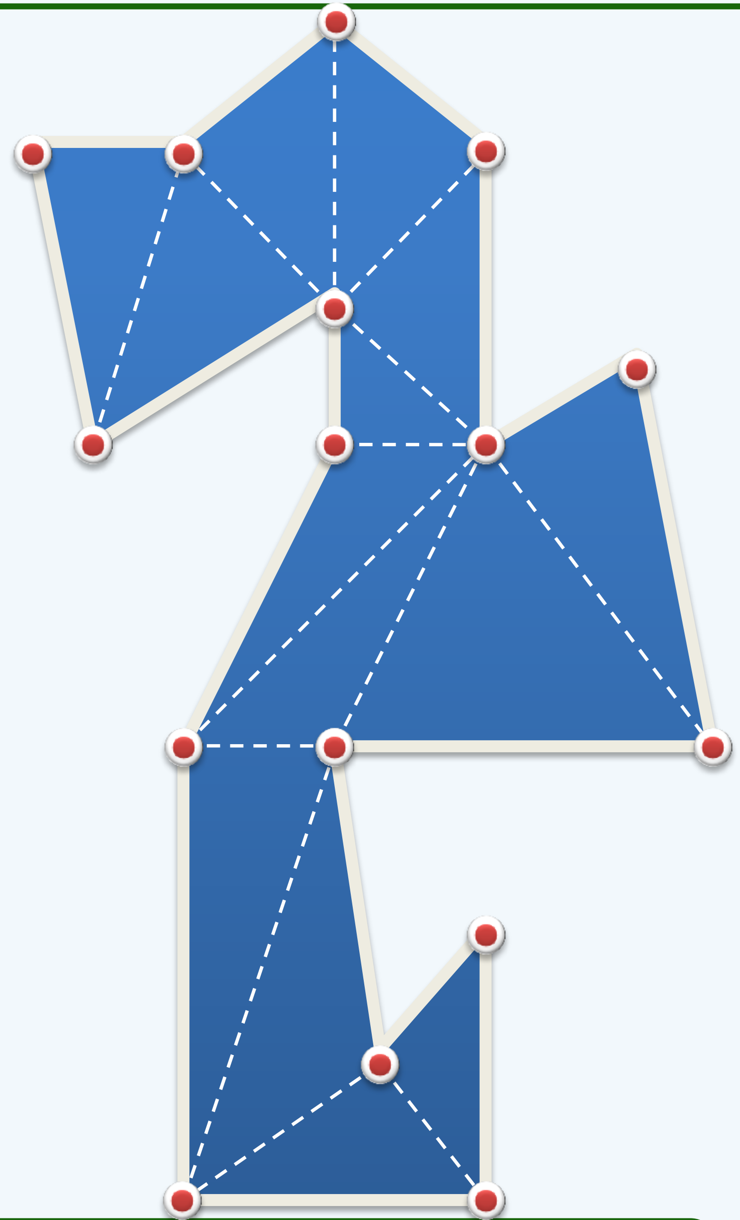
Simple Polygon

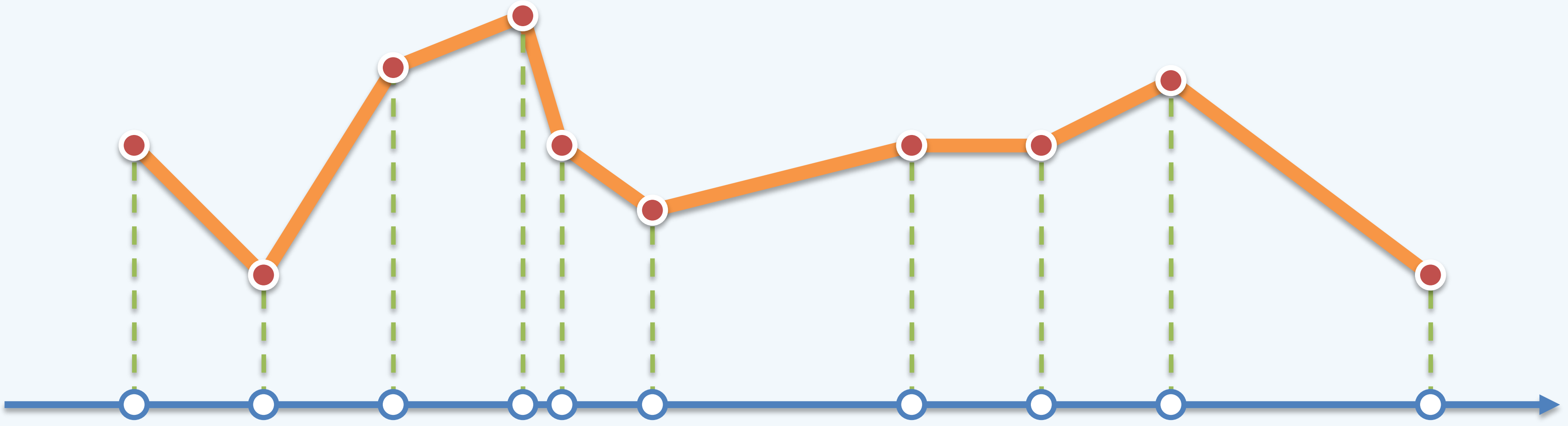
- Triangulating simple polygons may be the most fundamental technique in many applications
- Here we consider only simple polygons $\boxed{\text{without holes}}$
- Before introducing algorithms for triangulating a general simple polygon, let's first consider how to triangulate a special class of polygons, namely, the $\boxed{\text{monotone polygons}}$



Monotone Chain

- Let $M = \{ p_1, \ldots, p_k \}$ be a polygonal chain, and L a line
- If the projections of $\{ p_1, \ldots, p_k \}$ onto L are ordered the $\boxed{\text{same}}$ as in M,

  then M is called to be $\boxed{\text{monotone}}$ w.r.t. L



- M is called $\boxed{\text{monotone}}$ if it is monotone w.r.t. at least $\boxed{\text{one}}$ line

Monotone Polygon

- A polygon is called $\boxed{\text{monotone}}$ if
  it consists of 2 chains w.r.t. a $\boxed{\text{same}}$ line
- Here we use the convention that
  the direction for monotonicity is the $\boxed{y}$-axis
- Hence the 2 monotone chains are referred to as
  the $\boxed{\text{left}}$ & $\boxed{\text{right}}$ chains

Monotonicity Testing

- It's trivial that

  whether a polygon is monotone

  w.r.t. a $\boxed{\text{given}}$ direction

  can be determined in $\boxed{\mathcal{O}(n)}$ time

- But a "$\boxed{\text{harder}}$" problem would be ...

- Given a simple polygon P,

  determine whether or not P is monotone

  w.r.t. $\boxed{\text{some}}$ direction

Monotonicity Testing

- [Preparata & Supowit, 1981]

  Whether a polygon is monotone (w.r.t. some direction)

  can be determined in $\boxed{\text{linear}}$ time

- In fact, Preparata and Supowit's algorithm gives

  a description of $\boxed{\text{all}}$ directions of monotonicity

  in $\boxed{\mathcal{O}(n)}$ time

- Optimal partitioning of a chain into monotone pieces

  ...

- Determining whether a $\boxed{\text{polyhedral surface}}$ is monotone

  ...