

# **Delaunay Triangulation**

**Proximity Graph**

**- Gabriel Graph**

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## Definition

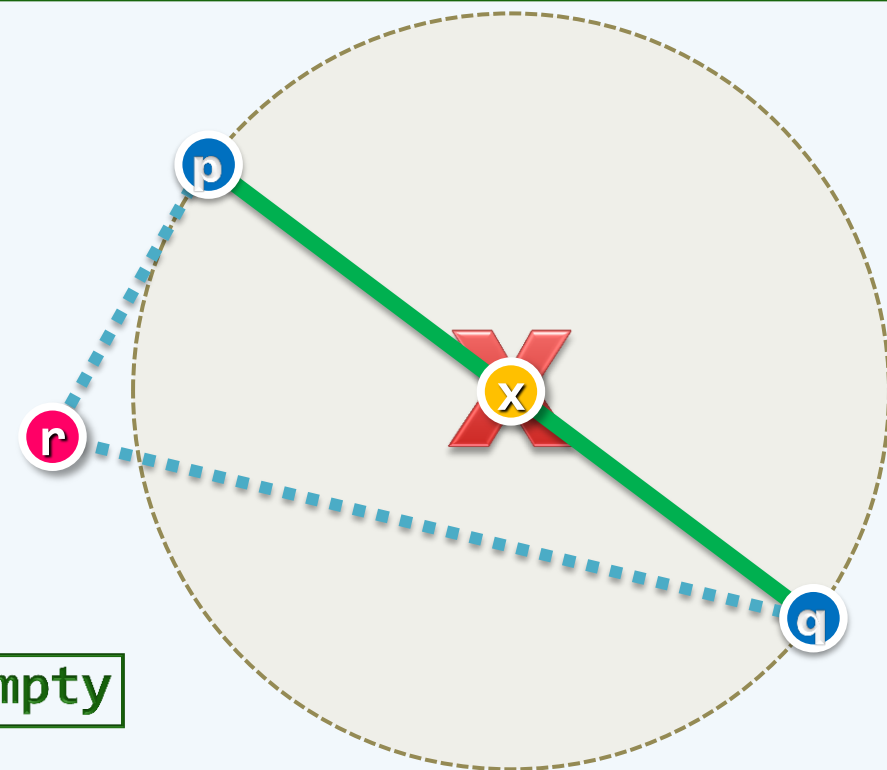
❖ For any  $p, q \in S$ ,  $pq \in GG(S)$  iff

$$|pq|^2 = \min\{ |pr|^2 + |rq|^2 \mid r \in S \}$$

❖  $pq$  belongs to  $GG(S)$  iff

- the **disk** taking  $pq$  as its diameter is **empty**
- $pq$  spans an **acute** angle w.r.t. every  $r \in S \setminus \{p, q\}$
- $pq$  intersects with the common boundary between  $\text{Cell}(p)$  and  $\text{Cell}(q)$

❖ The second equivalent definition implies that  $GG(S) \subseteq DT(S)$



## Construction

❖ [Matula, 1980]

GG(S) can be constructed from  $\text{VD}(S)$

in  $\mathcal{O}(n)$  time

❖ By the third equivalent definition of GG(S),  
we can

- construct  $\text{VD}(S)$  and  $\mathcal{O}(n \log n)$
- intersect each candidate segment  $pq$   
with the common boundary between  $\text{Cell}(p)$  and  $\text{Cell}(q)$   $\mathcal{O}(n)$

