04-J

Voronoi Diagram

Construction

- Naive

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Idea

- Compute all the Voronoi cells

  one after another ...

- For each cell,

  we introduce the bisectors

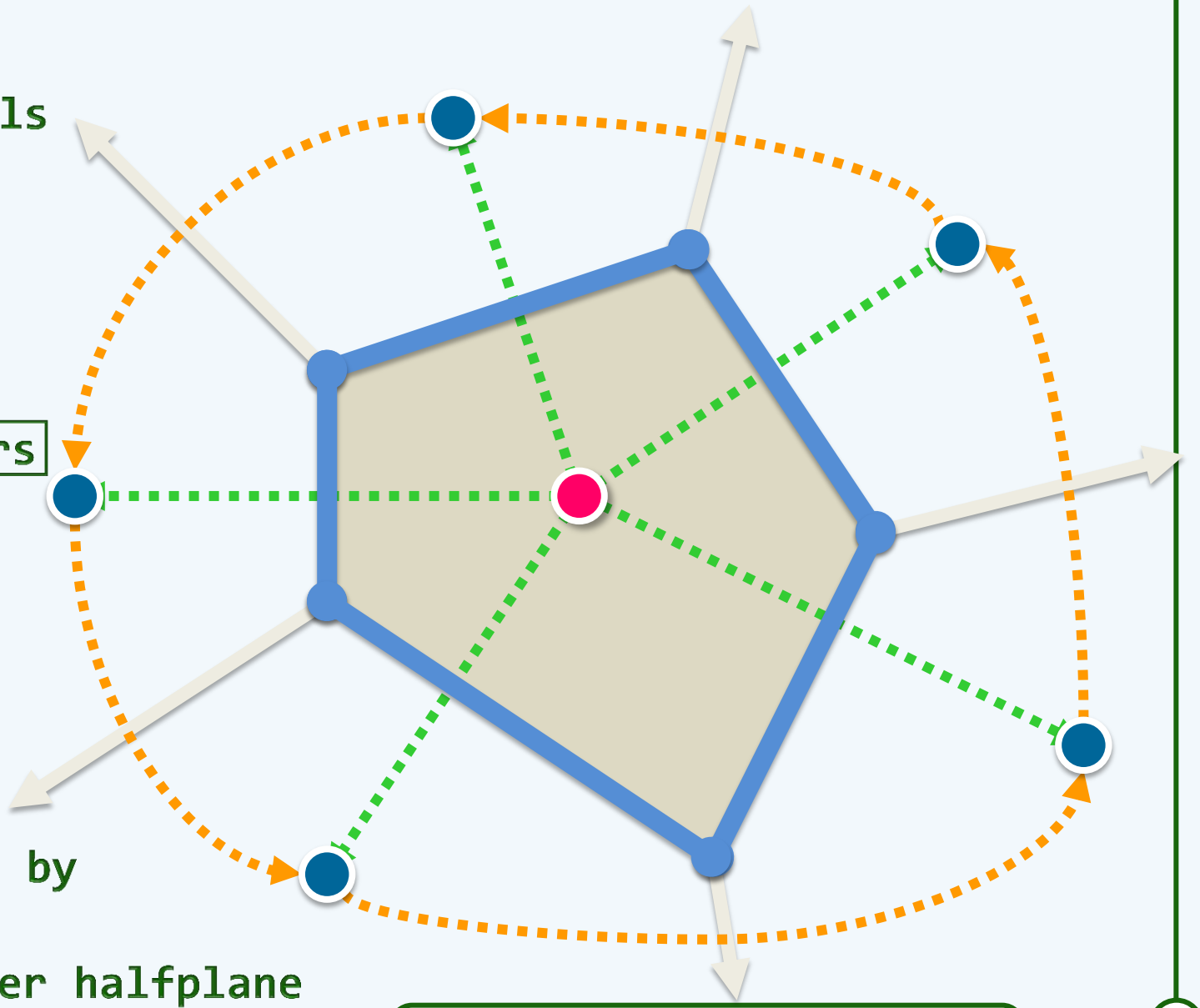
  (potentially) defining it

  in an incremental manner

- For each bisector,

  the cell would be refined by

  intersecting with a proper halfplane



Algorithm

❖ Initialize VD as an empty VD, represented as a list of Voronoi cells

```
For each p ∈ S
    initialize VC, the Voronoi Cell for p, as the entire plane
    for each q ∈ P\{p}
        find b(p, q) the bisector between p and q
        //let h(p, q) be the halfspace bounded by b(p, q) and containing p
        refine VC by intersecting it with h(p, q)
    append VC to VD
return(VD)
```

Complexity

- 1) each refinement iteration costs $O(n)$ time

  2) each cell needs $O(n)$ refinement iterations; and

  3) there are altogether $O(n)$ cells //It follows that ...

- The brute-force algorithm constructs a VD of n sites in $O(n^3)$ time

  //this is far from the lower bound

- Using **linear programming**,

  the brute-force algorithm constructs a VD of n sites

  in $O(n^2 \log n)$ time //this is, however, just a small improvement