09-D

Arrangement

Planar Arrangement

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Arrangement of Lines

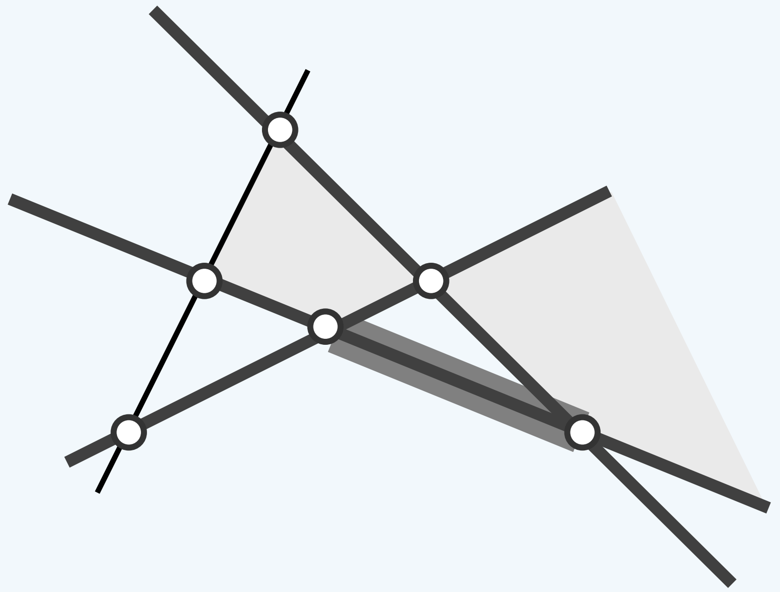
❖ Let L be a set of n lines in the plane

❖ The arrangement $\mathcal{A}(L)$ is

the plane subdivision induced by L

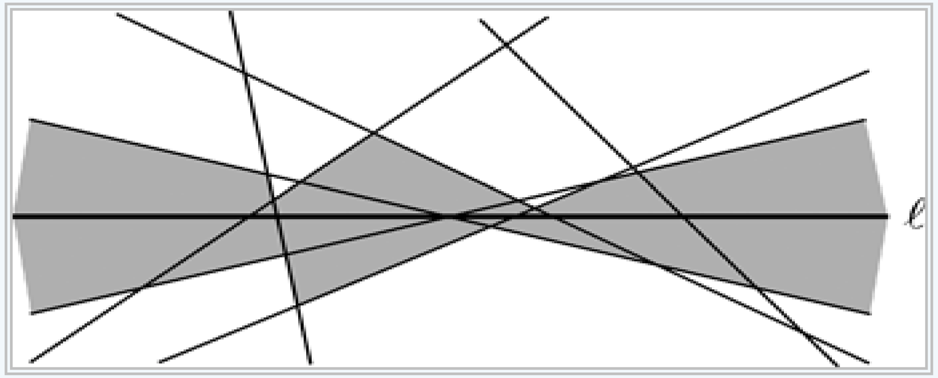
❖ Complexity: for the arrangement of n lines

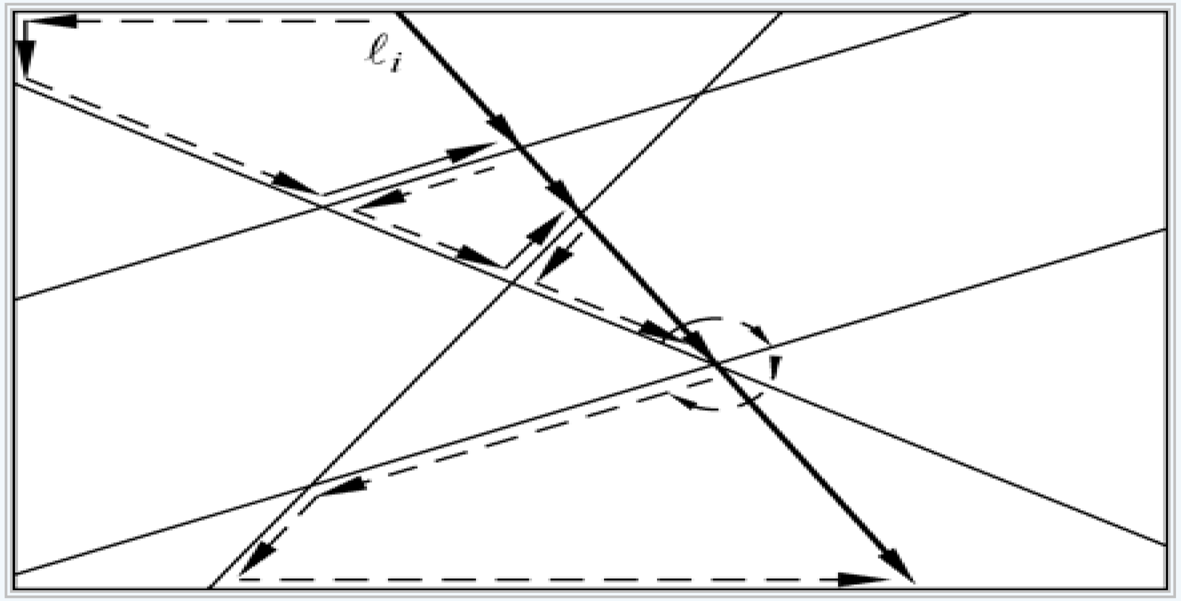
#vertices + #edges + #faces = $\Theta(n^2)$



Constructing Planar Arrangements

- Goal: to construct an representation of $\mathcal{A}(L)$ as a $\boxed{\text{DCEL}}$
- Plane sweep needs $\boxed{\Theta(n^2\log n)}$ time //$\Theta(n^2)$ events $\times$ $\Theta(\log n)$ time each
- RIC algorithm runs in $\boxed{\mathcal{O}(n^2)}$ time //This implies that ...
- The insertion of each line can be done in $\boxed{\mathcal{O}(n)}$ time //how to?
  1) How many cells are involved?
  2) What's their total complexity?





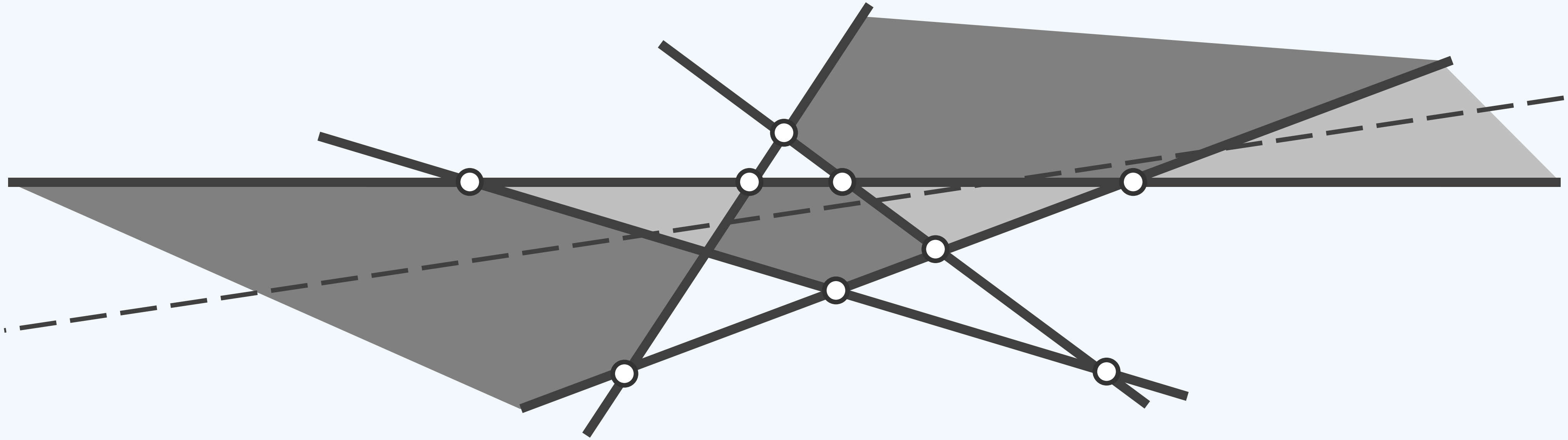
Zone Theorem

- The zone of a line l in arrangement $\mathcal{A}(L)$ is the set of faces of $\mathcal{A}(L)$ intersecting l

- The complexity of a zone is the total complexity of all its faces

  //i.e. the total $\boxed{\text{sum}}$ of #edges and/or #vertices of these faces

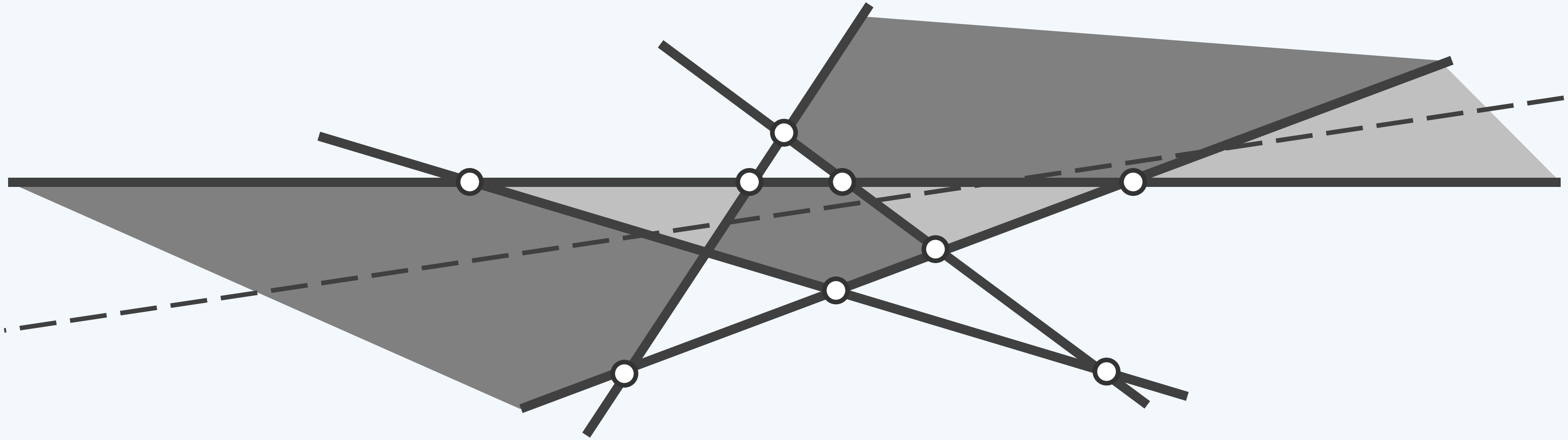
  //note that an edge may be counted $\boxed{\text{twice}}$, and a vertex more than $\boxed{\text{once}}$



Zone Theorem

❖ [CGAA-8.3]

The complexity of a zone of a line in an arrangement of n lines is $\boxed{\mathcal{O}(n)}$



Complexity of RIC

❖ The time required to insert l_i is linear in the complexity of l_i's zone

namely, $\boxed{\mathcal{O}(i - 1)}$ by the Zone Theorem

❖ The RIC algorithm

1) constructs the arrangement of n lines in the plane in $\boxed{\mathcal{O}(n^2)}$ time and

2) is optimal in terms of the worst case complexity