09-A

Arrangement

Introduction

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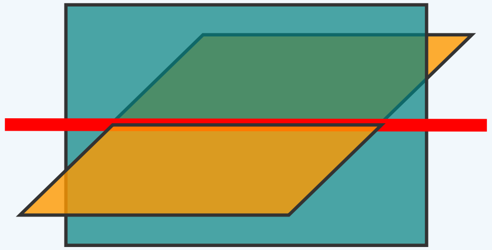
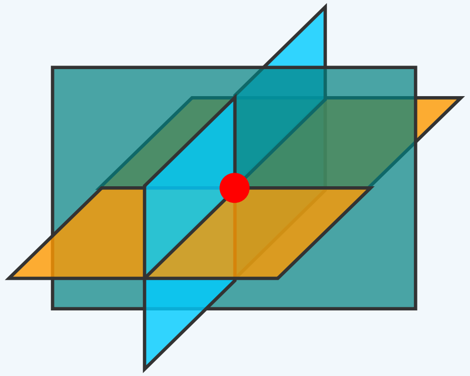
k-Flat

❖ The affine hull of $\boxed{k+1}$ affinely independent points is called a $\boxed{\text{k-flat}}$

0-flat:	point
1-flat:	line
2-flat:	plane
...	
(d-1)-flat:	hyperplane
d-flat:	$\mathcal{E}^d$
(-1)-flat:	defined as the empty set $\varnothing$





❖ Each k-flat in $\mathcal{E}^d$ is the intersection of $\boxed{\text{at least } \boxed{d-k}}$ hyperplanes

Vertical Flat

- A $\boxed{\text{line}}$ is called $\boxed{\text{vertical}}$

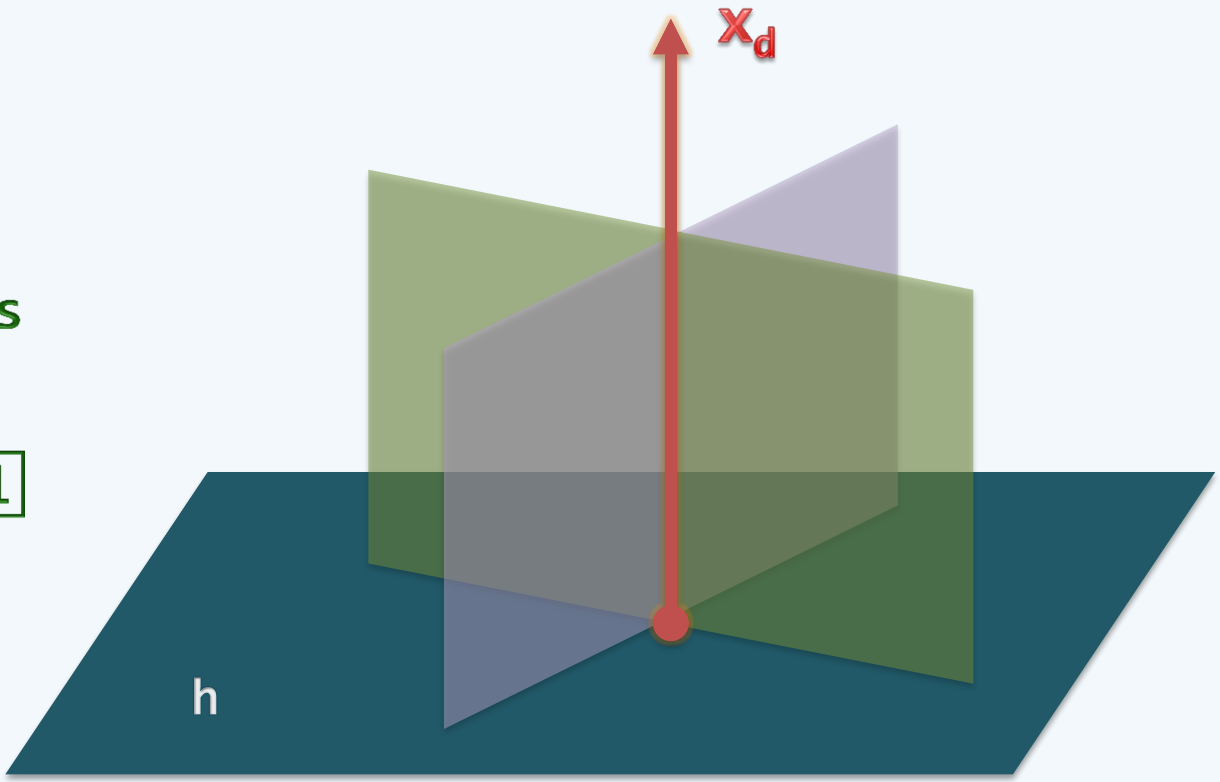
  if

  it is parallel to the x_d-axis

- A $\boxed{\text{k-flat}}$ is called $\boxed{\text{vertical}}$

  if

  it contains a vertical line

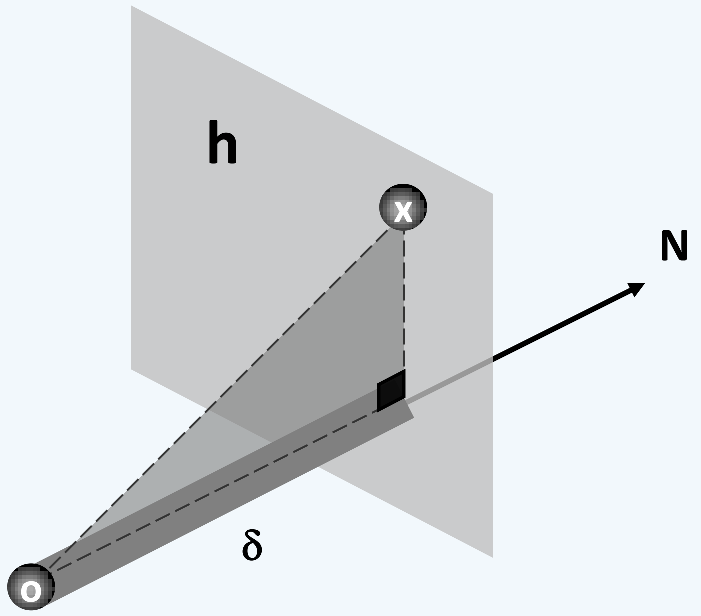


Hyperplane

- $h(N, \delta) = \{ x \in \mathcal{E}^d \mid N^T x = \delta \}$ where

  N is a normalized vector in $\mathcal{E}^d$, and

  $\delta \in R$



- Each hyperplane in $\mathcal{E}^d$ is itself

  a ($\boxed{d - 1}$)-dimensional Euclidean space

  e.g., lines in the plane, or

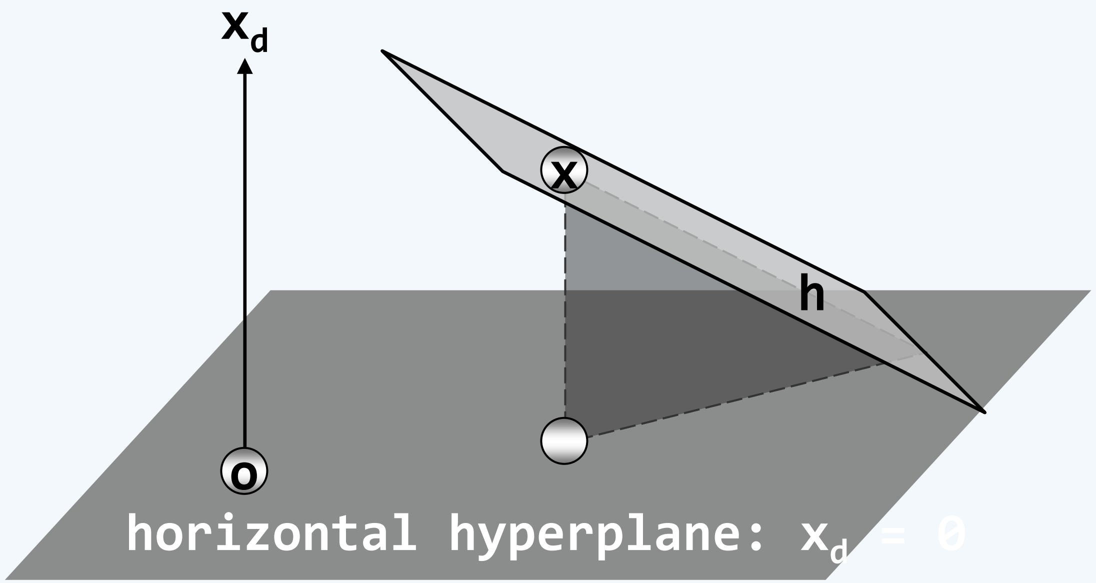
  planes in the space

Non-vertical Hyperplane

- $h \subset \mathcal{E}^d$ is not vertical iff

  there exist d reals $\boxed{\eta_1, \ldots, \eta_d} \in R$ s.t.

  $h = \{ x = (x_1, \ldots, x_d)^T \mid x_d = \eta_1 x_1 + \ldots + \eta_{d-1} x_{d-1} + \eta_d \}$



Upper/Lower Halfspaces

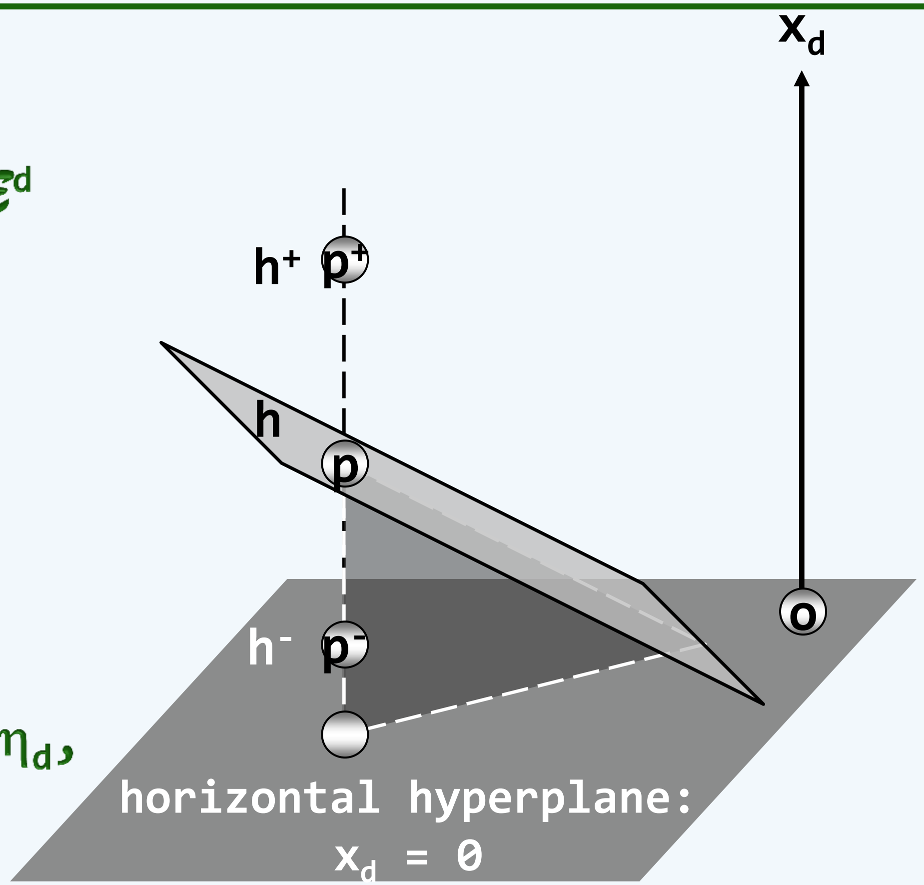
- Let h be a non-vertical hyperplane in $\mathcal{E}^d$
- $p = (x_1, \ldots, x_d)^T$ is called to lie

  $\boxed{\text{above}}/\boxed{\text{on}}/\boxed{\text{below}}$ h if

  $x_d \quad \boxed{>}/\boxed{=}/\boxed{<} \quad \eta_1 x_1 + \ldots + \eta_{d-1} x_{d-1} + \eta_d$,

- $\boxed{h^+}/\boxed{h^-} = \{\ p \in \mathcal{E}^d \mid p \text{ is ABOVE/BELOW } h\ \}$

  is called the $\boxed{\text{upper}}/\boxed{\text{lower}}$ halfspace defined by h



Arrangement

- Let S be a finite collection of geometric objects in $\mathcal{R}^d$

  e.g. hyperplanes or spheres

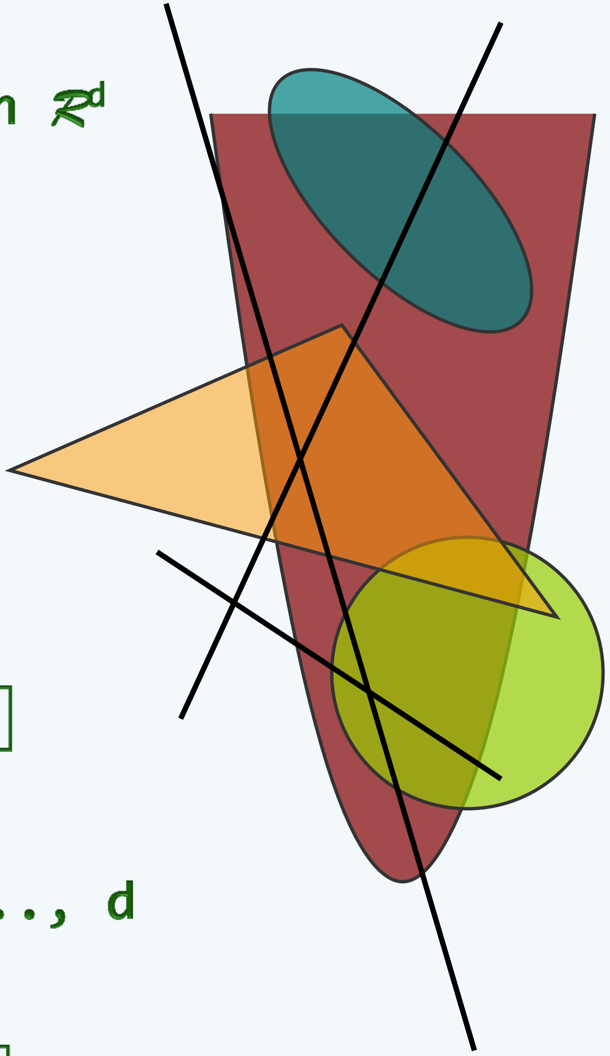
- The $\boxed{\text{arrangement}}$ $\mathcal{A}$(S) is

  the decomposition of $\mathcal{R}^d$ induced by S where

  each maximal connected $\boxed{\text{open}}$ set is called a $\boxed{\text{cell}}$

- A $\boxed{\text{k-cell}}$ is a cell of dimension k, for k = 0, 1, ..., d

- The $\boxed{\text{closure}}$ of a k-cell of A(H) is called a $\boxed{\text{k-face}}$



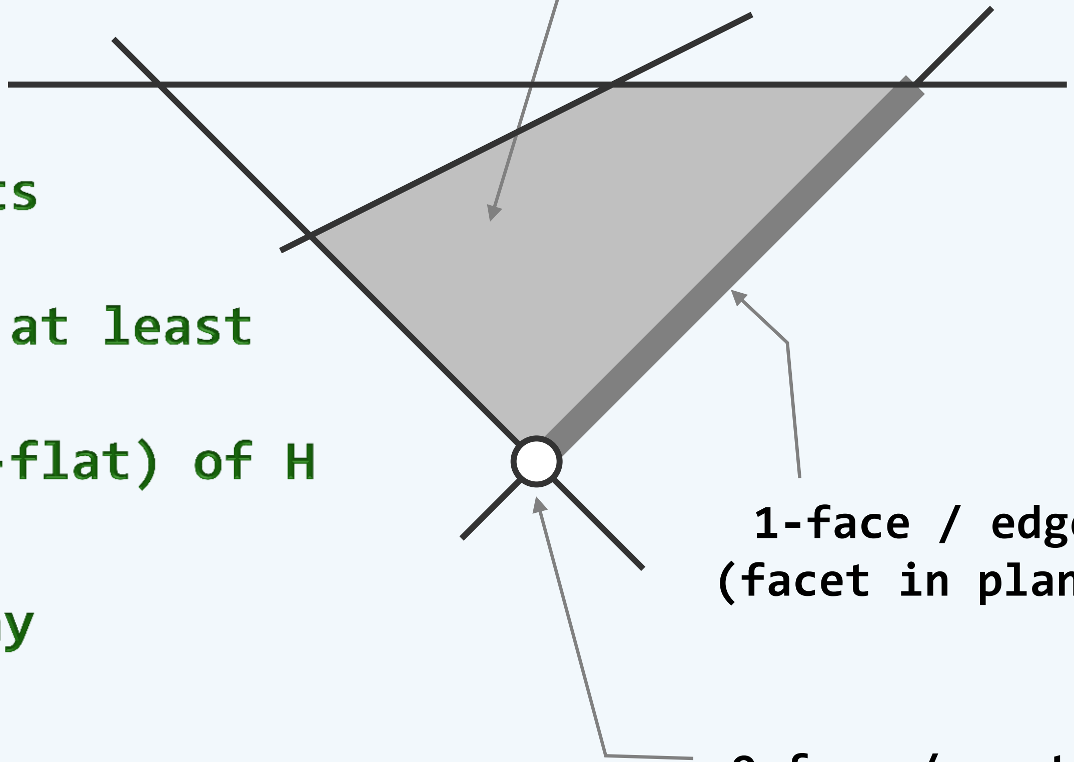
Arrangement of Hyperplanes

- Let A(H) be an arrangement of n hyperplanes in $\mathcal{E}^d$
- Each k-face of A(H) is

  a maximal connected set of points

  lying in the intersection of at least

  d-k hyperplanes (i.e. a k-flat) of H



- If (k-1)-face f $\subset$ k-face g, we say

  f is a $\boxed{\text{subface}}$ of g and

  there is an $\boxed{\text{incidence}}$ between f and g