09-E-03

Arrangement

Duality: Minimum Area Triangle

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MAT

- Given a set P of n points in the plane,

  find 3 points forming the triangle of minimum area

- Naive algorithm:

  - checks all possible triangles

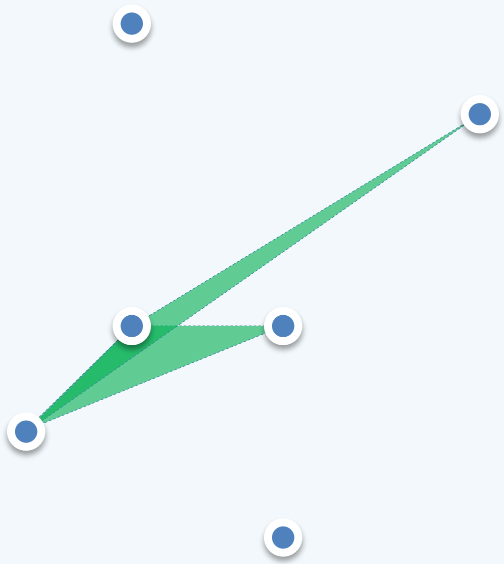
  - needs $\boxed{\mathcal{O}(n^3)}$ time

- Any faster algorithm?

- Dual arrangement!

  construct the dual arrangement $\mathcal{A}(\mathcal{D}(P))$ in the dual space

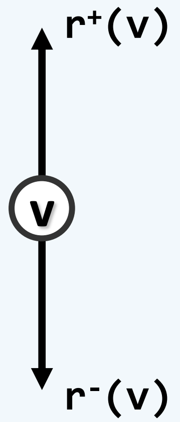
  //say, by RIC algorithm in $\boxed{\mathcal{O}(n^2)}$ time



Ray-Shooting

- Denote the two vertical rays from v

  as $r^+(v)$ and $r^-(v)$ resp.



- For each vertex v of A(H),

  find $l^+(v)$ / $l^-(v)$

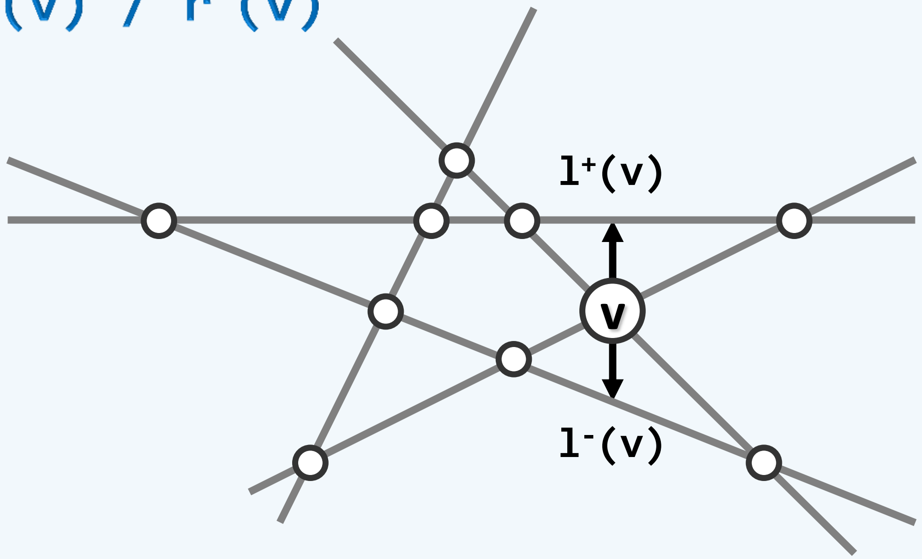
  //the 1st line of H intersecting $r^+(v)$ / $r^-(v)$

- Claim:

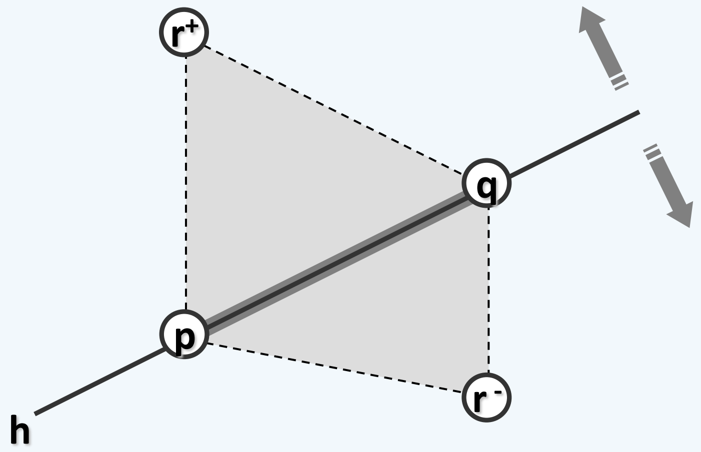
  these $\boxed{\mathcal{O}(n^2)}$ pairs of lines

  can be determined

  in $\boxed{\mathcal{O}(n^2)}$ time //how?

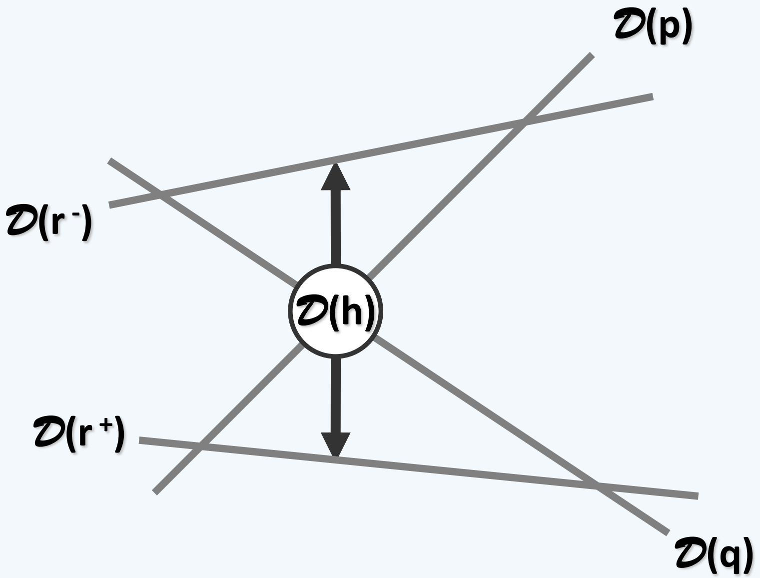


Ray-Shooting vs. MAT



- Consider two fixed points p and q in P ...
- Each of the other $\boxed{n-2}$ points defines a triangle with segment $\boxed{pq}$
- A point r defines the triangle with $\boxed{\text{minimum}}$ area iff r lies $\boxed{\text{nearest}}$ to the line $\boxed{h = pq}$

  //what does this mean in dual space?



- In the dual space, the condition translates to
  - $l^+(\ \mathcal{D}(p) \cap \mathcal{D}(q)\ ) = \mathcal{D}(\ r^-\ )$
  - $l^-(\ \mathcal{D}(p) \cap \mathcal{D}(q)\ ) = \mathcal{D}(\ r^+\ )$
- Therefore, MAT can also be solved in $\boxed{\mathcal{O}(n^2)}$ time