09-B

Arrangement

Applications

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I. (Endpoint) Visibility Graphs

❖ Given S a set of n disjoint segments

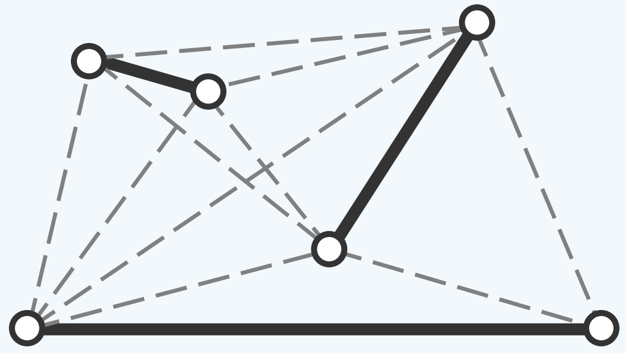
with no 3 endpoints collinear,

EVG(S) has

1) a node for each endpoint,

2) an arc for each segment, and

3) an arc between x and y if open segment xy intersects no segments in S



❖ Naive algorithm constructs EVG(S) in $\boxed{\mathcal{O}(n^3)}$ time

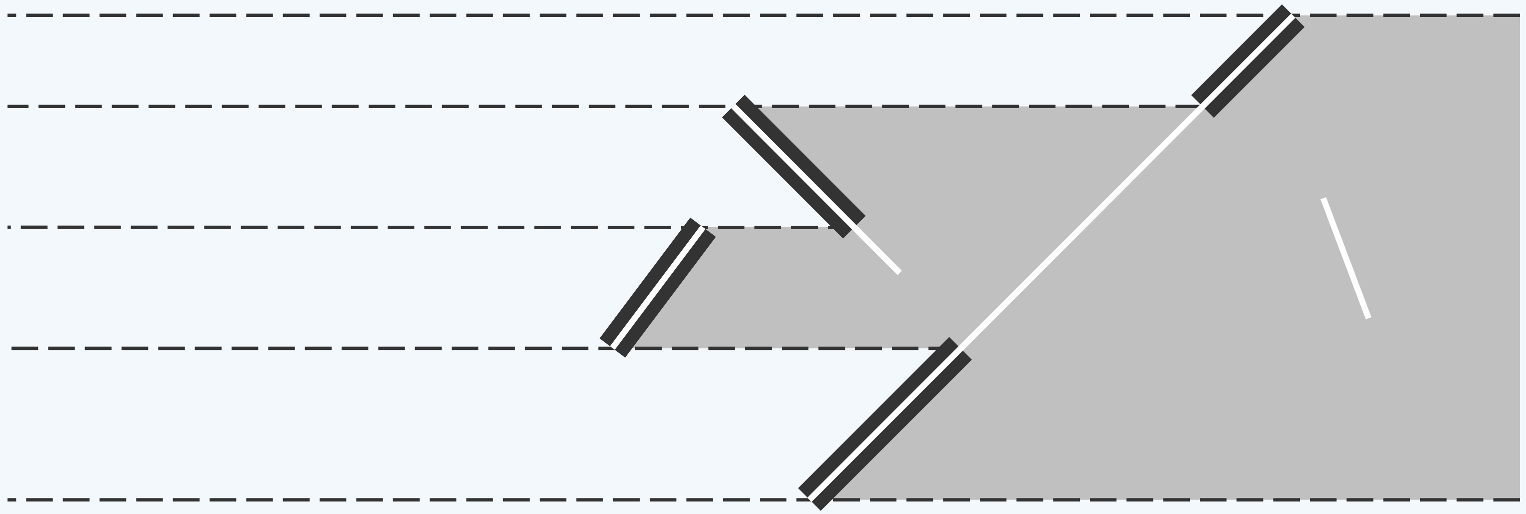
❖ Employing arrangements leads to an $\boxed{\mathcal{O}(n^2)}$ algorithm //O'Rourke 1987

II. Hidden Surface Removal

- Determine surfaces in a 3D scene hidden from the viewpoint and generate a 2D graphics image

- The first worst-case optimal $\boxed{\Theta(n^2)}$ algorithm is based on arrangements

  //McKenna 1987 (CGIC, section 6.7.2)



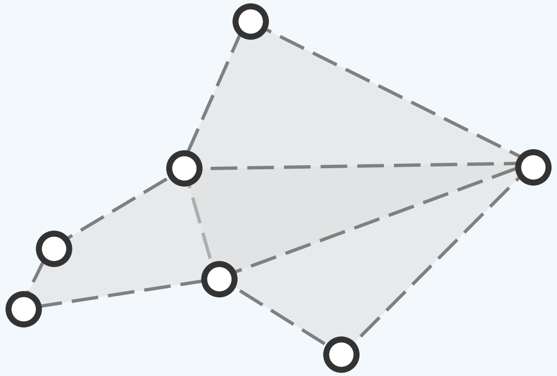
III. Largest Empty Convex Polygon

❖ Given P a set of n points in the plane, find the LECP of P

Polygon : vertices are drawn from P

Largest : with the most vertices

Empty : contains no points of P inside



❖ Using arrangements, an LECP can be found in $\boxed{\mathcal{O}(n^3)}$ time

//Edelsbrunner & Guibas 1989

//Dobkin , Edelsbrunner & Overmars 1990

IV. Ham-Sandwich Cuts

- [Ham-Sandwich Theorem]

  Let $P_1, \ldots, P_d$ be d finite sets of points in $\mathcal{E}^d$.

  There exists a hyperplane h that simultaneously bisects $P_1, \ldots, P_d$

- Using arrangements, 2D HSC can be found in $\boxed{\text{linear}}$ time

- A DAC algorithm can then solve $\boxed{\text{Red–Blue matching}}$ in $\boxed{\mathcal{O}(n\log n)}$ time.