03-H-05

Triangulation

Beyond $\Omega(n\log n)$

- $\mathcal{O}(n*\log\log n)$

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Breakthrough

- It was Tarjan and Van Wyk

  who first broke the $\boxed{\Omega(n\log n)}$ bound

- [Tarjan & Van Wyk, 1986]

  A simple polygon can be triangulated in $\boxed{\mathcal{O}(n*\log\log n)}$ time

- R. E. Tarjan and C. J. Van Wyk

  An $\boxed{\mathcal{O}(n*\log\log n)}$-Time Algorithm for Triangulating Simple Polygons

  AT&T Bell Lab. Manuscript (1986)

Algorithm

1) divides the polygon, instead of the polygonal chain

2) uses $\boxed{\text{Finger Search Tree}}$ data structure (Brown & Tarjan, 1980)

3) uses linear sorting of Jordan Sequence (Hoffman et al., 1985)

❖ However, the complicated data structures and search techniques

prevent Tarjan's $\boxed{\mathcal{O}(n*\log\log n)}$ algorithm

from entering applications

Kirkpatrick, Klare & Tarjan

- In 1990,

  Kirkpatrick et al. gave another $\boxed{O(n*\log\log n)}$-time algorithm

  with $\boxed{\text{much simpler}}$ data structures

- They
  - extended the concept of HVP to WP (Warp-around Partition) and
  - introduced the concept of the "k-uniform partition"

- A $\boxed{\text{k-uniform partition}}$ of a chain

  is a partition (with all divisions occurring at vertices)

  into sub-chains of length between $\boxed{k/2}$ and $\boxed{k}$

Kirkpatrick, Klare & Tarjan

❖ For $\boxed{k = \lceil n^{2/3} \rceil}$, given

- a $\boxed{k}$-uniform partition of an $\boxed{n}$-vertex non-degenerate polygonal chain P an
- the WP's of each sub-chains,

we can compute the WP of P in $\boxed{\mathcal{O}(n)}$ time

❖ If the WP of each part of the k-uniform partition is computed recursively,

then $T(n) \leq (n/k) \times T(k) + \mathcal{O}(n) \leq n^{1/3} \times T(n^{2/3}) + \mathcal{O}(n)$

$T(n)/n \leq T(n^{2/3})/n^{2/3} + \mathcal{O}(1) = \boxed{\mathcal{O}(\log\log n)}$

❖ The WP (as well as the triangulation) of a simple polygon

can be computed in $\boxed{\mathcal{O}(n * \log\log n)}$ time