Triangulation

Beyond Ω(nlogn)

- Trapezoidalization

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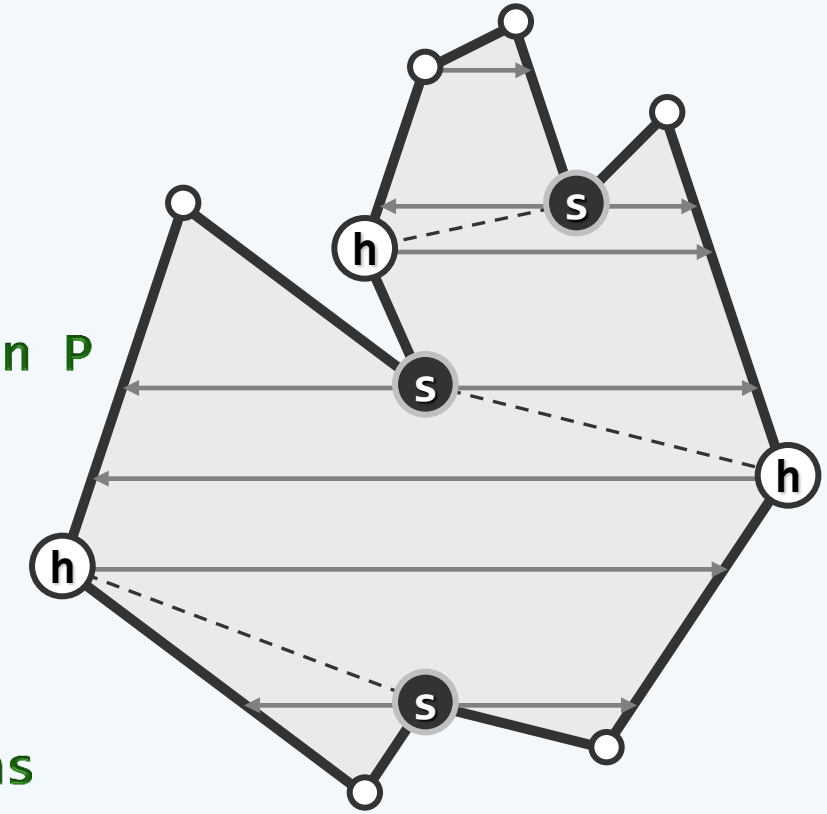
Definition

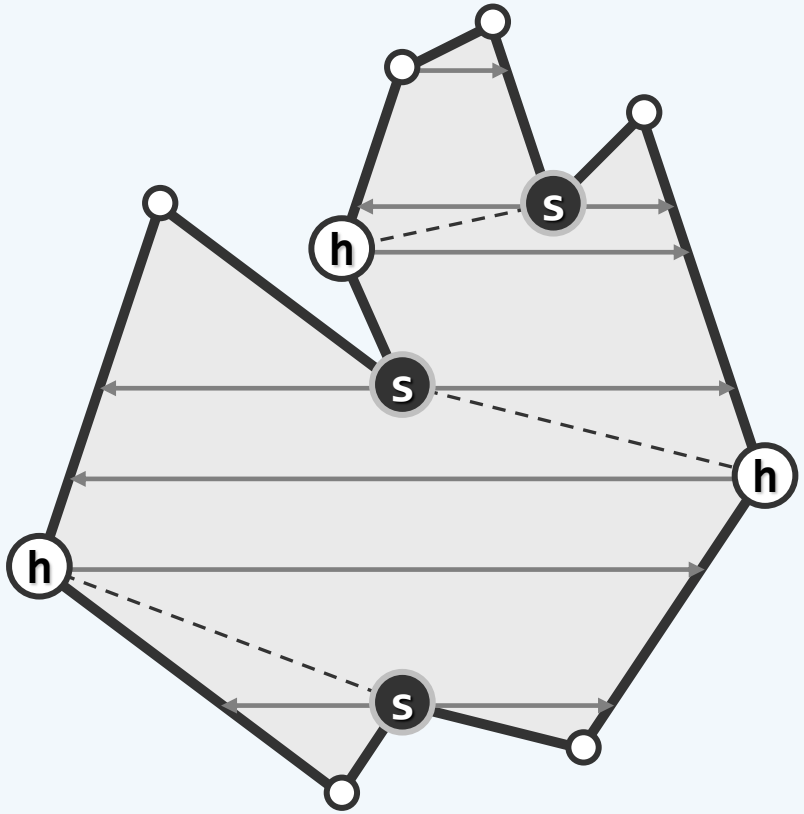
- Trapezoidalization, a key to $o(n \log n)$ triangulation algorithms,
  was first considered by
  [Chazelle-Incerpi, 1984] and
  [Fournier-Montuno, 1984]

- The trapezoidalization Trap(P) of a polygon P
  is obtained by
  drawing a horizontal line
  through every vertex,
  extended towards both directions
  to the point where it first crosses to the exterior



Monotone Partition

- The horizontal lines partition P
  into trapezoids (or triangles)
- Each trapezoid is supported
  by exactly 2 vertices of P
  on its top and bottom sides resp.
- Given its trapezoidalization,
  a polygon can be divided
  into monotone pieces by
  joining each violate vertex (s)
  with its counter vertex (h)



//Stalactite/Stalagmite

//Helper

Existence, Uniqueness & Complexity

❖ Although a simple polygon may have more than one triangulation,

it has been proved that

each simple polygon has **one and only one** trapezoidalization

❖ Each vertex of P brings **at most 2** more vertices to Trap(P)

- an upward/downward vertex brings **no** vertices to the trapezoidalization;
- an internal cusp (stalactite or stalagmite) brings **2** more vertices; and
- a general vertex brings **1** more vertex

❖ Therefore

- the trapezoidalization has no more than **3n** vertices, and
- the trapezoidalization of a simple **n**-gon has **$O(n)$** vertices

Reduction

❖ [Chazelle et al, 1984]

Triangulation $\leq_N$ Monotone Decomposition $\leq_N$ Trapezoidalization

❖ B. Chazelle & J. Incerpi

Triangulation and shape complexity

ACM Transactions on Graphics, 1984(3), 135-152

❖ It's followed immediately by that

we can triangulate a polygon in $o(n\log n)$ time if

we can trapezoidalize it in $o(n\log n)$ time