02-F-04

Geometric Intersection

BO Algorithm: Analysis

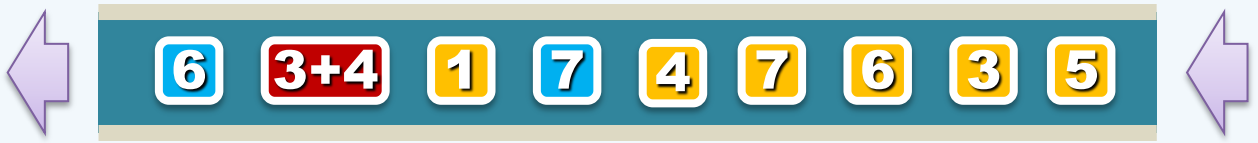
- Complexity of Event Queue

Junhui DENG

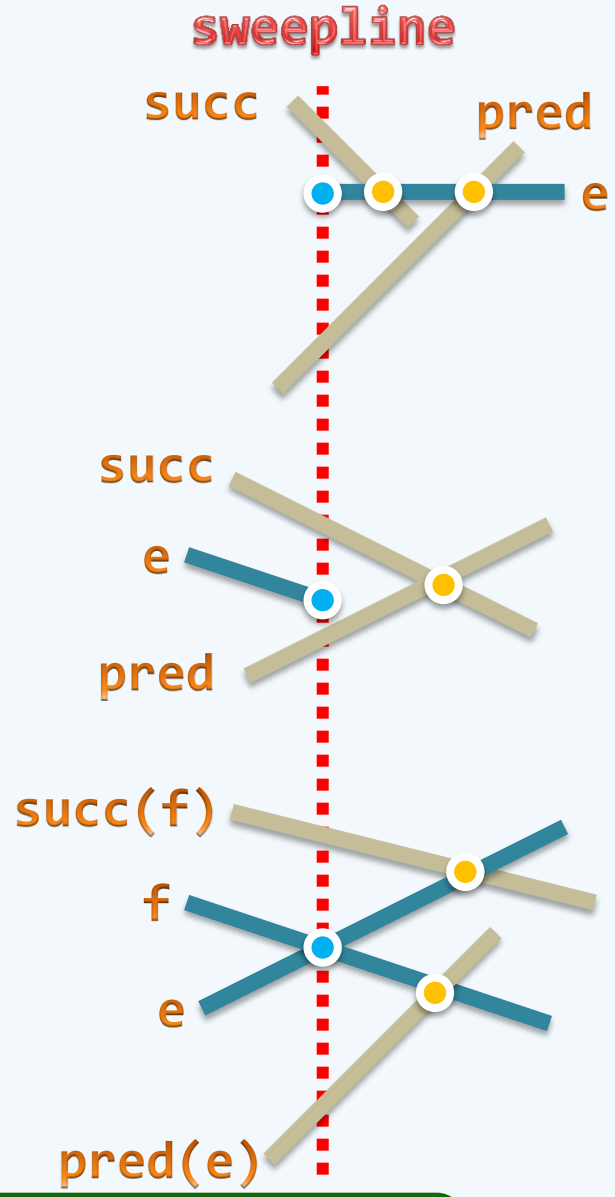
deng@tsinghua.edu.cn

Single Operation

- The time cost of the algorithm is dominated by those spent on **updating** the data structures



- Since $\mathcal{E}$/$\mathcal{L}$ is implemented as **PQ**/**BBST**, each operation (query/insertion/deletion) can be done in $O(\log S)$ time where S is the size of $\mathcal{E}$/$\mathcal{L}$
- What's the maximum **size** of $\mathcal{E}$/$\mathcal{L}$?



Size of $\mathcal{E}$

- As we have seen, $\mathcal{E}$ consists of events to be processed
- All events are classified into 3 types:
  - n **left** endpoints (blue)
  - n **right** endpoints (yellow)
  - I **intersection** points (red)

- $\mathcal{E}$ would contain no more than **2n + I** events at any time
- So the time for each operation is

$$O(\log(2n + I)) = O(\log(2n + n^2)) = \boxed{O(\log n)}$$

Maximum Size of $\mathcal{E}$

- Is it possible for $\mathcal{E}$ to have a size up to $\Omega(2n + I) = \Omega(n^2)$ at a time?
- By how many will $|\mathcal{E}|$ increase after each event?

  No more than 1!

  - left-endpoint:

    $\leq -1 + 2 = 1$

  - right-endpoint:

    $\leq -1 + 1 = 0$

  - intersection:

    $\leq -1 + 2 = 1$

- Besides the updating operations,

  how much time is required to initialize $\mathcal{E}$?