09-F

Arrangement

Geometric Transform

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Geometric Transform $\mathcal{E}$

- $\mathcal{E}$ maps $\boxed{\text{points in } \mathcal{E}^d}$ into $\boxed{\text{hyperplanes in } \mathcal{E}^{d+1}}$ //lifting

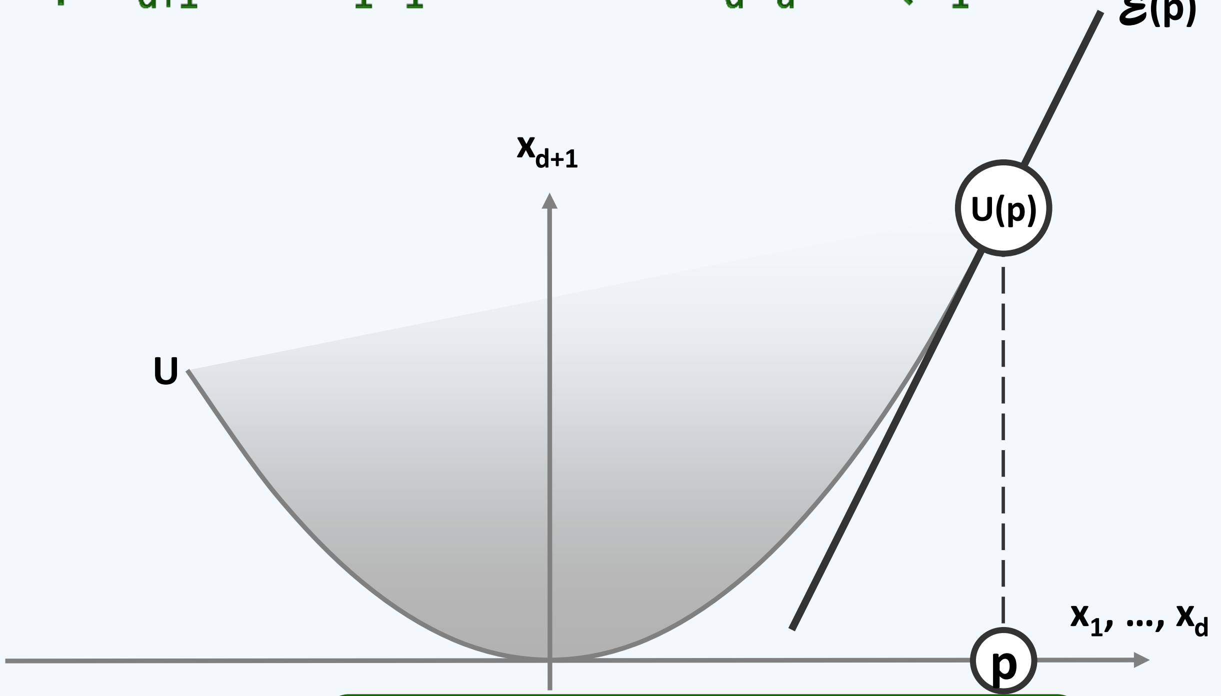
- For any point $p = (\pi_1, \ldots, \pi_d)^T \in \mathcal{E}^d$, hyperplane $\mathcal{E}(p)$

  $= \{ x = (x_1, \ldots, x_d, x_{d+1})^T \in \mathcal{E}^{d+1} \mid x_{d+1} = 2\pi_1 x_1 + \ldots + 2\pi_d x_d - (\pi_1^2 + \ldots + \pi_d^2) \}$

- Geometrically,

  $\mathcal{E}(p)$ is tangent to U

  at the projection of p onto U



Distance to p

❖ Let p be a fix point in $\mathcal{E}^d$

❖ For any $x \in \mathcal{E}^d$, $d(x, p) = ?$

//often asked by proximity queries

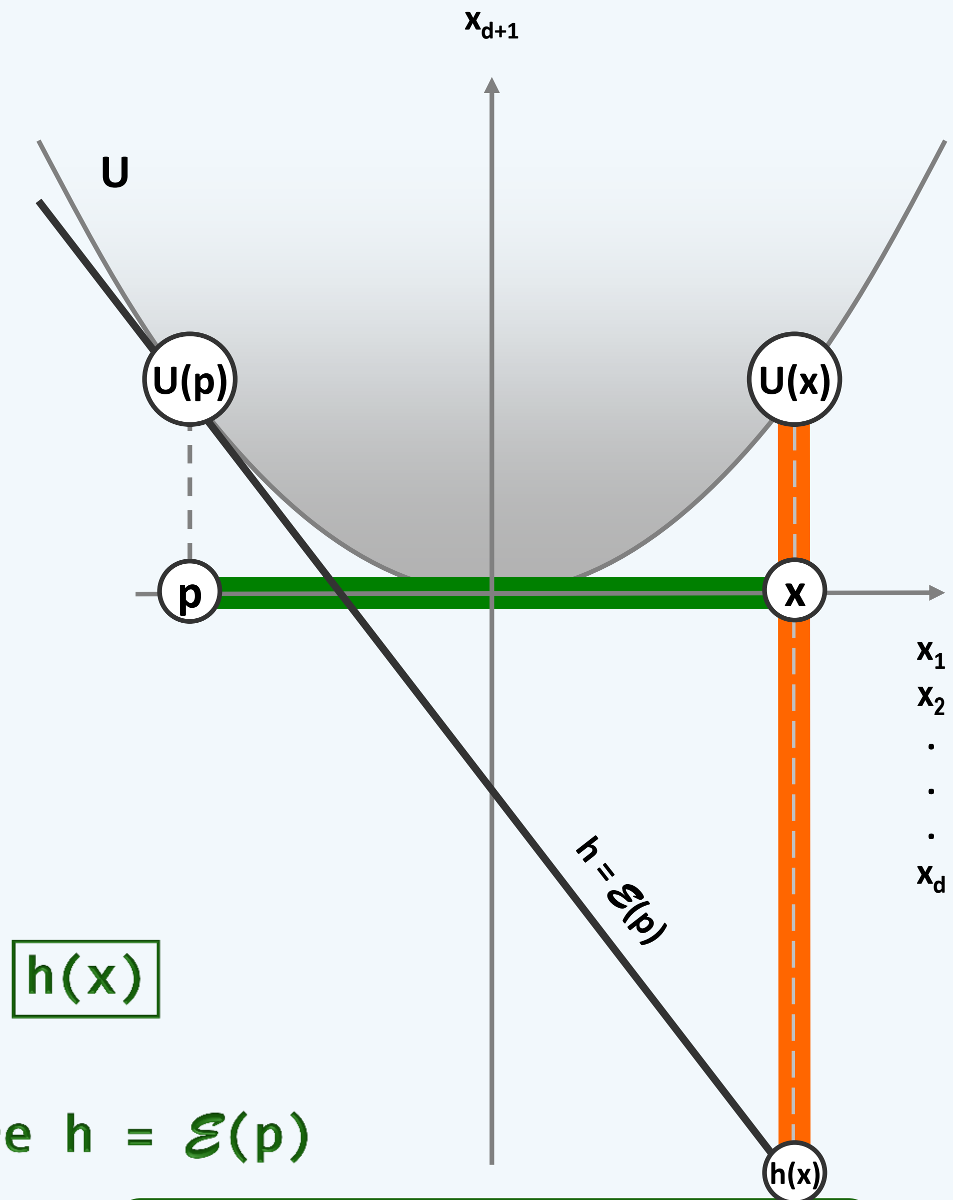
❖ Think of geometric transform…

❖ Denote the vertical projection of x

- onto U as $\boxed{U(x)}$

- onto a non-vertical hyperplane h as $\boxed{h(x)}$

❖ Claim: $d^2(x, p) = d(\ U(x), h(x)\ )$, where $h = \mathcal{E}(p)$



Bisector

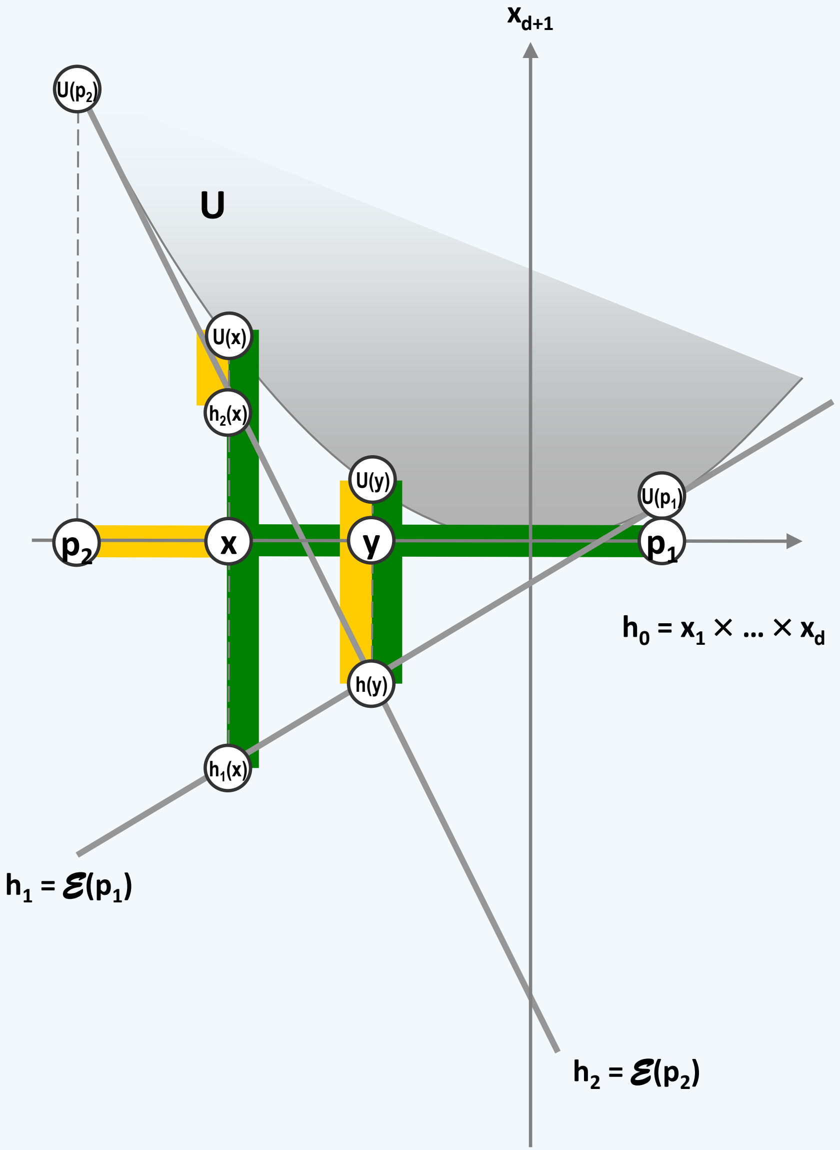
- Let h_0 be the $\mathcal{E}^d$ embedded in $\mathcal{E}^{d+1}$
- For any 2 points p_1 and p_2 in h_0

  the vertical projection

  of $\mathcal{E}(p_1) \cap \mathcal{E}(p_2)$ onto h_0

  is the bisector

  between p_1 and p_2



Upper Envelope vs. Voronoi Diagram

❖ For P a finite set of points in $\mathcal{E}^d$, let

- H = $\mathcal{E}$(P) = { $\mathcal{E}$(p) | p $\in$ P }, and
- UE(P) be the $\boxed{\text{upper envelope}}$ / $\boxed{\text{topmost cell}}$ of the arrangement A(H)

i.e. UE(P) = $\cap_{p\in P}\, \mathcal{E}(P)^+$

❖ The vertical projection of the facets of UE(P) onto h_0 is VD(P)

//How about Delaunay triangulation then?

❖ Therefore, to compute a k-D Voronoi diagram, it suffices to

- $\boxed{\text{transform}}$ the set P of sites to a set H of hyperplanes in $\mathcal{E}^{k+1}$,
- $\boxed{\text{compute}}$ the upper envelope UE($\mathcal{A}$(H)), and
- $\boxed{\text{project}}$ it back vertically onto h_0