Convex Hull

Approximate Convex Hull

01-XC

Junhui DENG

deng@tsinghua.edu.cn

Thinking Approximation

- Why approximation?
  1) Exact CH algorithms usually take much more time
  2) In many applications, approximate convex hulls are good enough
- Not like the exact algorithms, approximate algorithms are $\boxed{\text{not}}$ guaranteed to compute the $\boxed{\text{precise}}$ hull
- How $\boxed{\text{better}}$ is the approximation? And how to estimate and bound the $\boxed{\text{error}}$?

Approximate Convex Hull

- Divide the plane into k vertical $\boxed{\text{strips}}$ with a $\boxed{\text{uniform}}$ width

  $(x_{max} - x_{min}) / k$

- Find the highest/lowest point inside each strip

  (what if an empty strip?)

- Connect all $\boxed{\text{highest}}$/$\boxed{\text{lowest}}$ points in turn

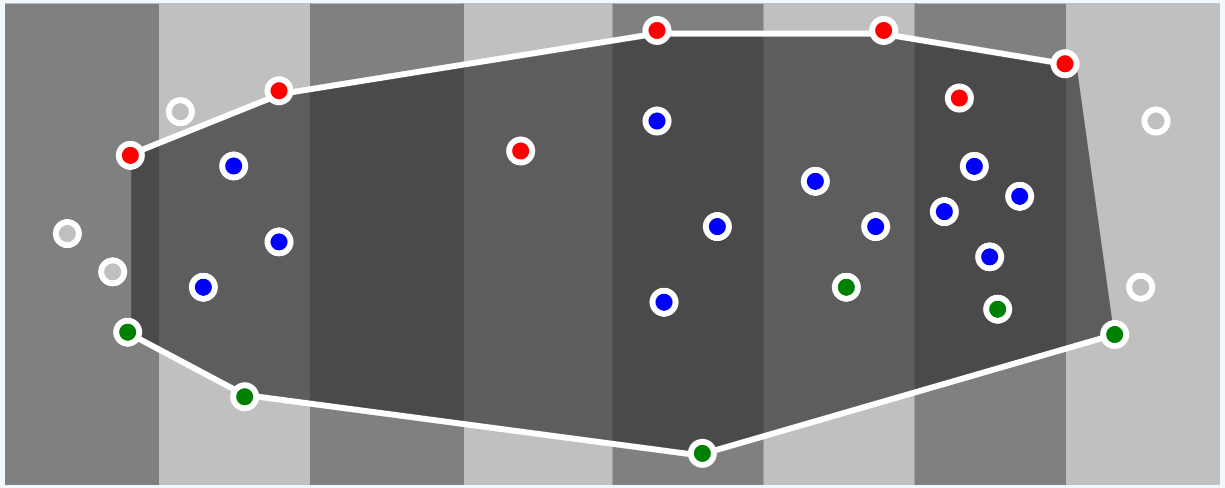
  into a chain $\boxed{\text{monotone}}$ w.r.t.

  $(0, -\infty)$ / $(0, +\infty)$

- Apply Graham scan

  to compute the upper/lower hull

- Concatenate upper & lower hulls



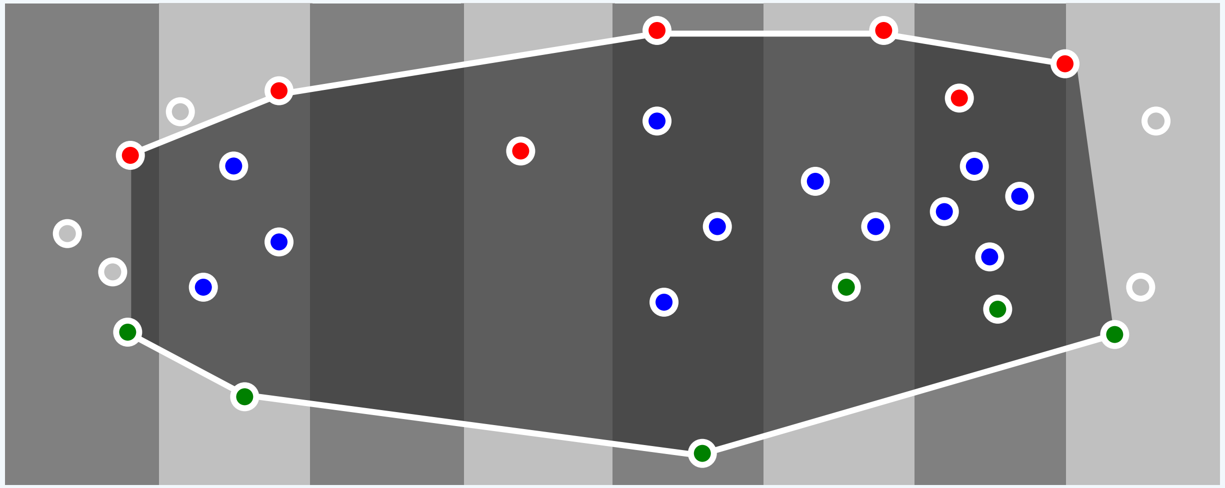
Complexity

❶ Finding the leftmost and rightmost points costs $\Theta(n)$ time

❷ Finding the $\boxed{\text{highest}}$&$\boxed{\text{lowest}}$ points in each strip costs $\Theta(n)$ time all together

❸ Graham scan costs $\Theta(k)$ time, where k is the number of strips

❖ An approximate convex hull of n points in the plane can be computed in $\Theta(n+k)$ time



Accuracy

- It is possible for some points to $\boxed{\text{escape}}$ from the constructed hull
- Luckily they won't escape $\boxed{\text{too far away}}$
- Any point of P that lies outside the approximate hull is within a distance

  $(x_{max} - x_{min}) / k$

  of the hull
- That means,

  the error of approximate hull

  is no more than the strip width