02-C-02

Geometric Intersection

Segment Intersection Reporting

- Hardness

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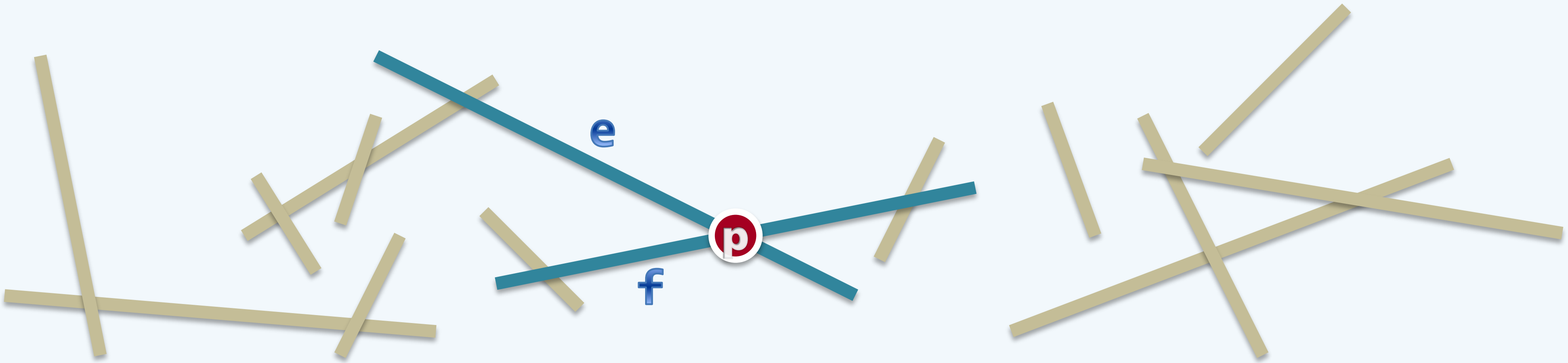
Lower Bound

❖ Claim: The worst-case optimal algorithm reports all the intersections among n segments in the plane in $\boxed{\Omega(n\log n + I)}$ time, where I is the number of intersections



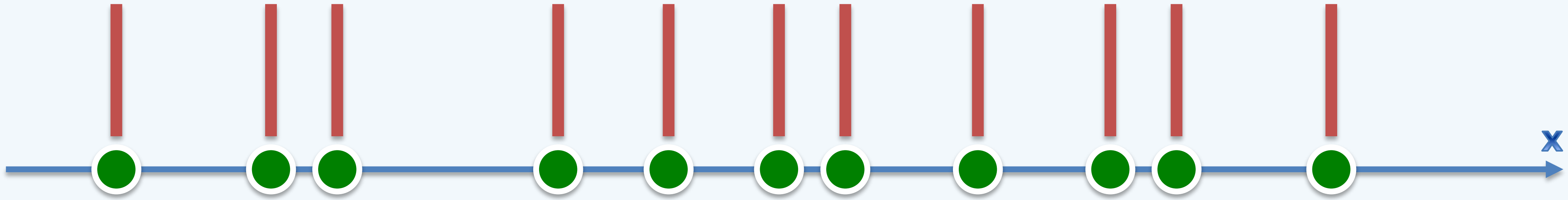
❖ Why?

Reduction

- Since $\Omega(I)$ time is needed to enumerate and output the I intersections

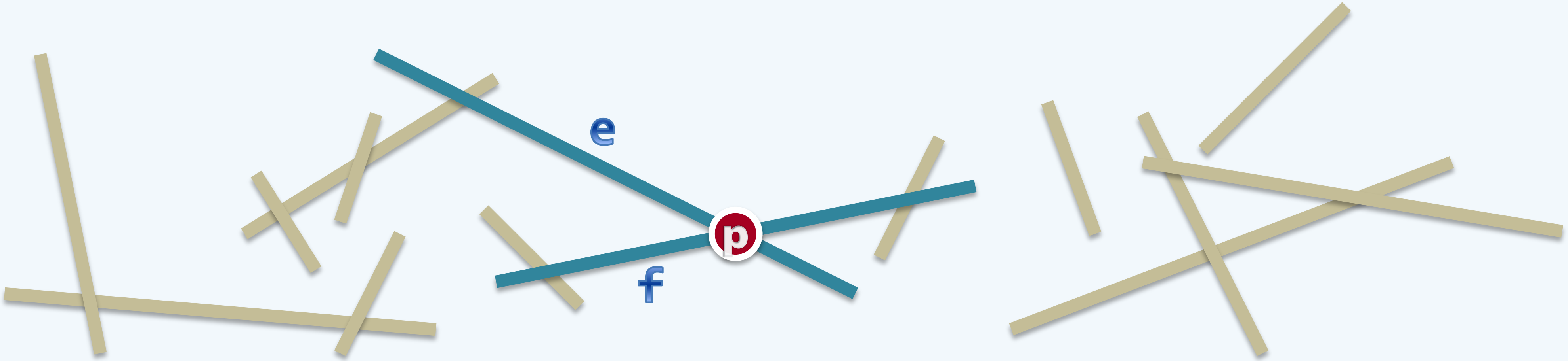
  it suffices to prove that $\boxed{\text{SID}}$ requires $\Omega(n\log n)$ time

  by a reduction from ...



Optimal Algorithms

- Is $\boxed{\Omega(n\log n + I)}$ a $\boxed{\text{tight}}$ bound for $\boxed{\text{SIR}}$?



- In other words,

  is there an $\boxed{\text{output-sensitive}}$ algorithm solving $\boxed{\text{SIR}}$

  in $\boxed{\mathcal{O}(n\log n + I)}$ time?

Optimal Algorithms

- Optimal algorithm for $\boxed{\text{SIR}}$

  Chazelle & Edelsbrunner, 1992 $\boxed{\mathcal{O}(n\log n + I)}$ time + $\boxed{\mathcal{O}(n + I)}$ space

  Balaban, 1995 $\boxed{\mathcal{O}(n\log n + I)}$ time + $\boxed{\mathcal{O}(n)}$ space

- Optimal algorithm for $\boxed{\text{SID}}$

  Shamos & Hoey, 1976 $\boxed{\mathcal{O}(n\log n)}$ time

- This section will discuss Bentley & Ottmann's algorithm which takes the $\boxed{\text{plane-sweep}}$ paradigm and solves 2D $\boxed{\text{SIR}}$ in $\boxed{\mathcal{O}((n + I)*\log n)}$ time and using $\boxed{\mathcal{O}(n + I)}$ space