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Arrangement

Duality: Lower Envelope

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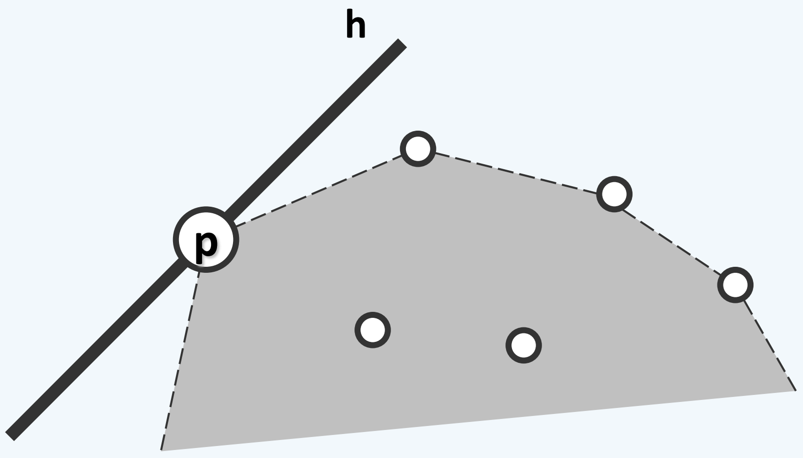
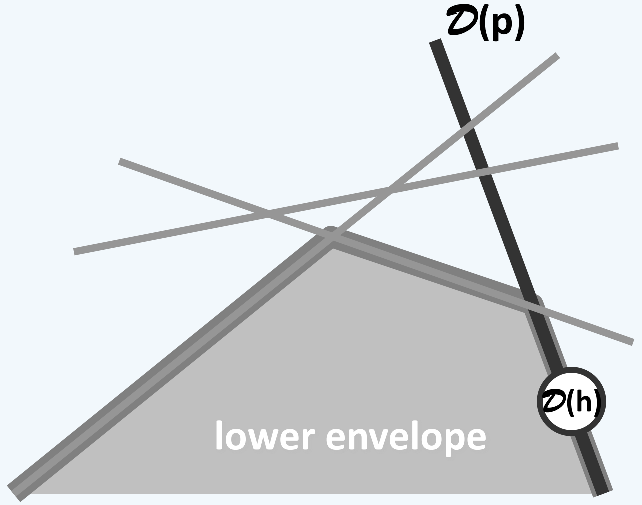
Lower Envelope vs. Upper Hull

- Given a set P of points in $\mathcal{E}^d$,
  compute its upper hull UH(P)



- p $\in$ P is a vertex of UH(P) iff
  there exists a hyperplane h thru p
  s.t. P\{p} lies $\boxed{\text{below}}$ h
  //Or, equivalently, in the dual space ...

- There exists a point $\mathcal{D}$(h) in H = $\mathcal{D}$(p)
  s.t. H\{$\mathcal{D}$(p)} lies $\boxed{\text{above}}$ $\mathcal{D}$(h)
  //i.e. $\mathcal{D}$(p) contributes a facet
  to the $\boxed{\text{lower envelope}}$ of $\mathcal{A}$(H)



Lower Envelope vs. Upper Hull

- Planar case ...

- So, to compute UH(P) in $\mathcal{E}^d$, it suffices to
  - transform P to an arrangement $\mathcal{D}$(P) of hyperplanes,
  - compute LE($\mathcal{D}$(P)) using, say,
    the hyperplane intersection algorithm, and
  - transform LE($\mathcal{D}$(P)) back to the primal space