Voronoi Diagram

Variations

04-N

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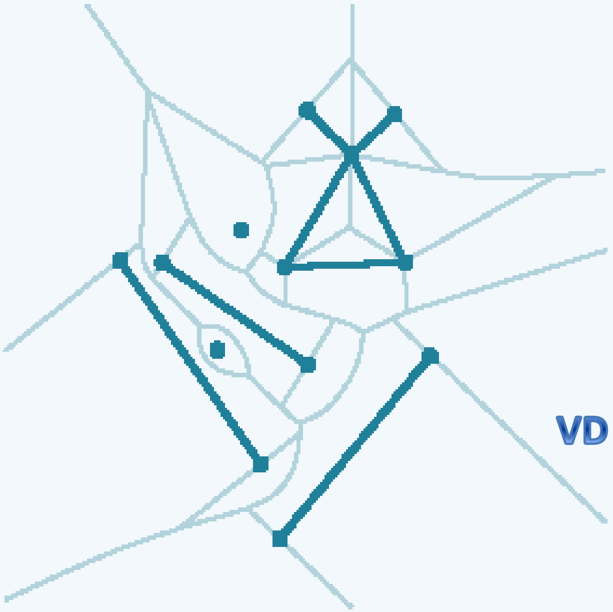
Geometric Primitives

❖ Construct Vor(S) for S a set of geometric primitives

of different dimensions and shapes:

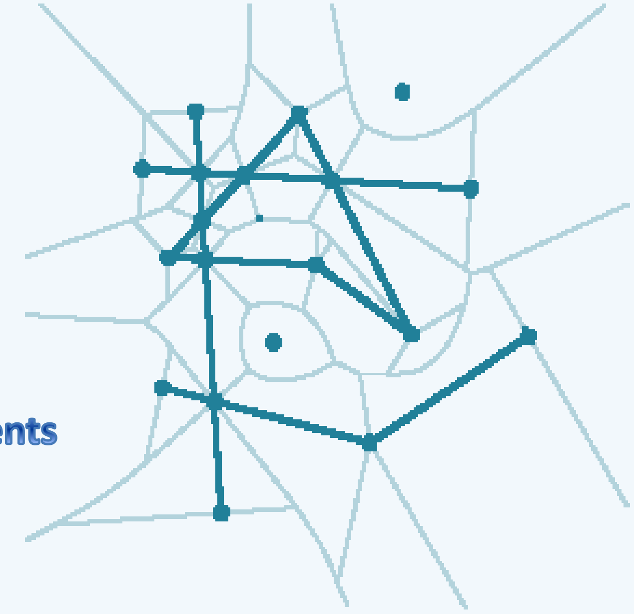
points, segments, rays, lines, arcs,

solid volumes (e.g. polygons, disks), ...



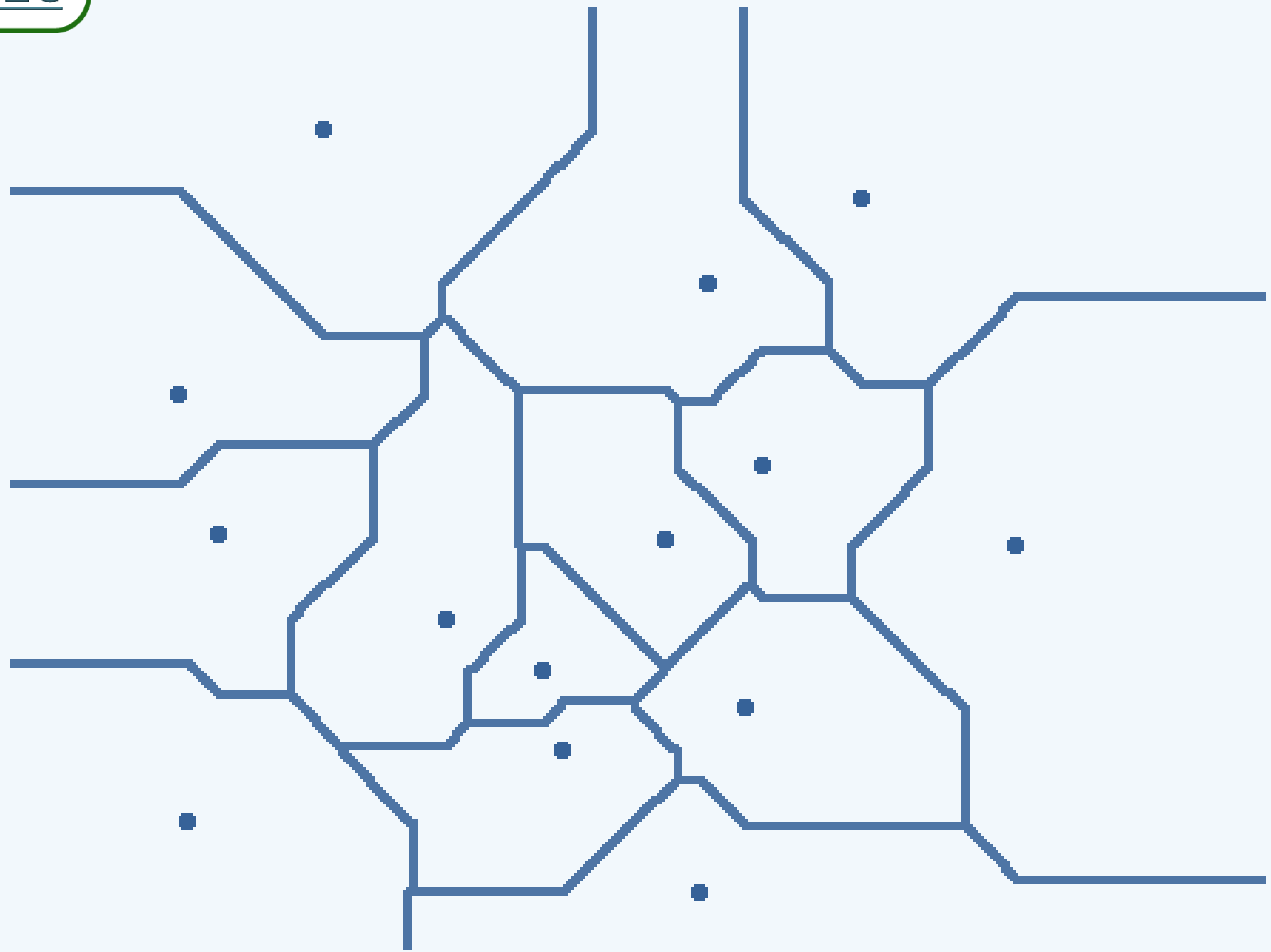
weakly intersecting sites

VD of 2 points and 8 segments



strongly intersecting sites

L_k-Metric



Voronoi diagram of 14 points in L1-metric

Higher-Order Voronoi Diagrams

❖ Let S be a set of n sites in the plane

❖ For an integer k, $1 \le k < n$,

the k^{th}-order VD of S is a partition of the plane

into (connected) cells s. t.

any point within a fixed cell has the same k closest sites

// The 1^{st}-order Voronoi diagram is the normal Voronoi diagram

❖ The maximum $\boxed{\text{complexity}}$ of the k^{th}-order VD of n sites in the plane

is $\boxed{\Theta(\ k*(n-k)\ )}$

Higher-Order Voronoi Diagrams

- The currently best algorithm for constructing 2-D k^{th}-order VD is [Be97]
- [de Berg et al]

  The k^{th}-order VD of n sites in the plane can be constructed

  in $\boxed{\mathcal{O}(\ n\log^3 n + k(n-k)\ )}$ time
- M. de Berg et. al.

  Computational Geometry: Algorithms and Applications

  Springer-Verlag, Germany, 1997
- What is the lower bound

  for constructing k^{th}-order VD (in the plane or $\mathcal{E}^d$)?

// TTBOMK, remains open ...

Furthest Point Voronoi Diagrams (FPVD)

- The $\boxed{\text{(n-1)}}$-order VD is also called

  the $\boxed{\text{furthest point Voronoi diagram}}$

- A site of S has a non-empty cell in FPVD

  $\boxed{\text{iff}}$

  it lies on CH(S)

- The FPVD of a set of n points

  can be computed

  in $\boxed{\mathcal{O}(n\log n)}$ time

The FPVD of 12 points consists of 5 cells, each for an EP

Minimum Enclosing Circle

- One of the applications of FPVD is to construct the MEC
- [MEC] // also known as the $\boxed{\text{1-circle problem}}$

  Find the minimum circle s.t. no point of S lies exterior to the circle
- [Bhattacharya & Toussaint, 1985]

  The MEC of a set of n points in $\mathcal{E}^2$ can be constructed in $\boxed{\mathcal{O}(n\log n)}$ time
- Note that the brute-force MEC algorithm runs in $\boxed{\mathcal{O}(n^4)}$ time
- On the other hand, however,

  Seidel presented an optimal and easy-to-implement MEC algorithm
- [Seidel, 1990]

  The MEC of a set of n points in $\mathcal{E}^2$ can be constructed in $\boxed{\mathcal{O}(n)}$ time!

Minimum Enclosing Circle

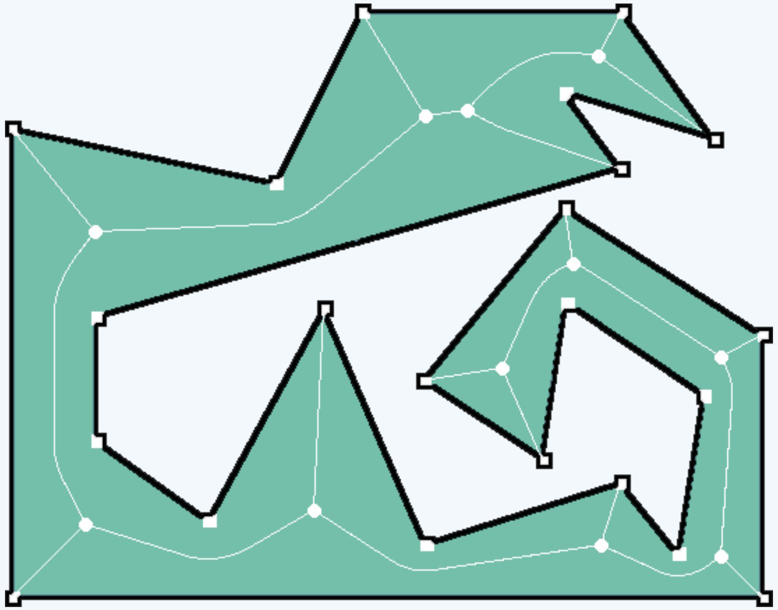
❖ Algorithm ConstructMECbyFPVD() //Bhattacharya & Toussaint

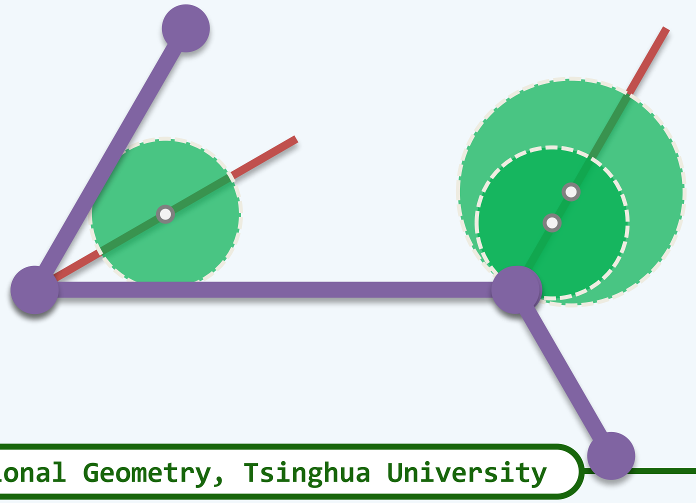
```
Compute CH(P), the convex hull of the P: CH(P)        //O(nlogn)
Compute diam(CH(P)), the diameter of CH(P)            //O(n)
If diam(CH(P)) defines the MEC then exit with the MEC //O(n)
Compute FPVD(P), the FPVD of the point set            //O(nlogn)
For every vertex in FPVD(P)                           //O(n)
    check the spanning circle, and
    exit with the smallest such circle found
```

Skeleton & Medial Axis

- A disc (or ball) B is said to be $\boxed{\text{maximal}}$ in a set A if
  - B $\subseteq$ A and
  - another disc D contains B only if D $\nsubseteq$ A
- The $\boxed{\text{skeleton}}$ of a shape A is the set
  - of centers of all maximal discs in A,

    or equivalently,
  - of centers of the discs that touch the ∂A in $\boxed{\text{two}}$ or $\boxed{\text{more}}$ points





Medial Axis Algorithms

- [1982, D. T. Lee]

  The medial axis of an $\boxed{\text{n-gon}}$ can be constructed in $\boxed{\mathcal{O}(n\log n)}$ time

- [1989, A. Aggarwal et al]

  The medial axis of a $\boxed{\text{convex}}$ polygon can be constructed in $\boxed{\text{linear}}$ time

- [1992, O. Devillers]

  A $\boxed{\text{randomized}}$ algorithm constructs the medial axis of an $\boxed{\text{n-gon}}$

  in $\boxed{\mathcal{O}(n\boxed{\log^* n})}$ time

- [1995, F. Chin et al]

  The medical axis of an n-gon can be constructed in $\boxed{\mathcal{O}(n)}$ time

VD of Subsets

- [B. Chazelle, 2002]

  Splitting a Delaunay triangulation in linear time,

  Algorithmica, 34(1): 39-46

- Given VD(P) of P a planar set of n points,

  for any subset S of P, VD(S) can be computed

  in $\boxed{\text{expected linear}}$ time

- Given S and T two planar point sets,

  if $\boxed{\text{VD(S + T)}}$ is known, then

  both $\boxed{\text{VD(S)}}$ and $\boxed{\text{VD(T)}}$ can be computed

  in $\boxed{\text{expected linear}}$ time