01-J-03

Convex Hull

Graham Scan: Analysis

- Amortization

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Monotonicity

❖ In your IDE

- set a **break point**

  at the entrance of

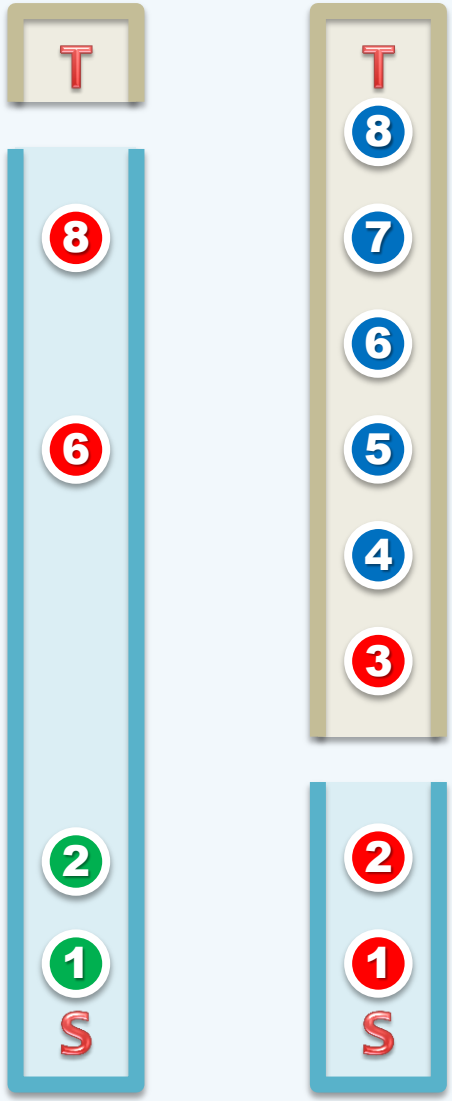
  the **while** loop and

- **watch** the expression:

  $\mathcal{A}$ = **S.size() + 2*T.size()**

❖ Claim: after each iteration, $\mathcal{A}$ deceases by **exactly 1**

```
if : S.size()++; T.size()--;      //  1 - 2
else : S.size()--;                //  -1 + 0
```



Amortization

- Before the loop starts:

  $\mathcal{A} = 2 + 2*(n-2) = 2n - 2$

- When the loop finishes:

  $\mathcal{A} \geq 3$

- It's implied that

  the `while` loop will terminate

  in no more than $2n - 5$ = $O(n)$ iterations