ME18
Introduction to Engineering Computation
Fall 2017

Design Project

Subject: Design of an Application for the Deflection Analysis of Plates
Teams: Work in teams of 1-3, one application and project report per team.
Due Dates: Phase 1 Due 11/21/17, Phase 2 Due 12/1/17

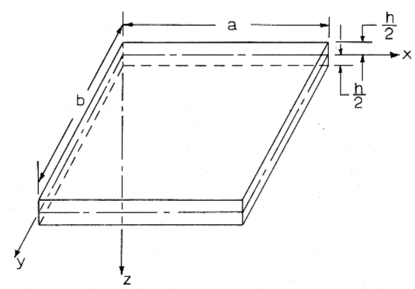
Background

Plates are structural components that have a thickness that is much smaller than the other dimensions. Although plates are often subjected to different types of loads, the lateral deflection of thin plates subjected to lateral loads is an important problem in structural analysis. The analysis of plates began in the early 1800s with the research of Cauchy, Poisson, Navier, Lagrange and Kirchoff and continued in the 1900s with the work of Timoshenko and others. The classical solutions for lateral deflections of plates developed by these early researchers were extremely significant in the development of the field of solid mechanics.

Over the past several decades, the development of computational mechanics has led to the widespread use of the finite element method in structural analysis including the deflection of plates. For problems with simple geometries such as rectangular and circular plates of constant thickness, the classical solutions for deflection are still relevant and can be quite useful if programmed in an application that allows for quick iterations through different loads and plate dimensions.

<u>Rectangular Plates</u>

The deflection of a simply supported rectangular plate of dimensions *a* and *b* subjected to a concentrated load *P* located at the point (x_0, y_0), is given by the classical solution



$$w_c(x,y) = \frac{4P}{\pi^4 abD}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin\frac{m\pi x_0}{a}\sin\frac{n\pi y_0}{b}}{\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

where

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

and *E* is the modulus of elasticity and ν is Poisson's ratio of the plate material, and *h* is the plate thickness. In this double series, *m* and *n* are positive integers (*m* = 1, 2, 3, 4, 5, ... and *n* = 1, 2, 3, 4, 5, …).

The deflection of a simply supported rectangular plate of dimensions *a* and *b* subjected to a uniformly distributed load q_0 over the entire surface is given by the classical solution

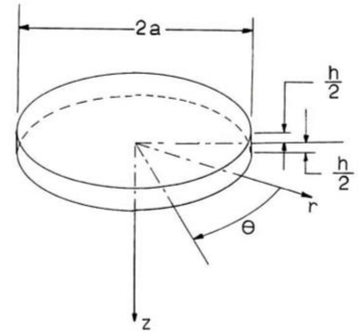
$$w_d(x,y) = \frac{16q_0}{\pi^6 D}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)}{mn\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}$$

where in this double series, *m* and *n* are odd integers (*m* = 1, 3, 5, … and *n* = 1, 3, 5, ...).

In both cases, the series converges quite rapidly and satisfactory approximations are obtained by taking only the first few terms in the series. Using *m* = *n* =10 terms is sufficient for this application.

Circular Plates

The deflection of a simply supported circular plate with a diameter $2a$ subjected to a concentrated load P located at the center, is given by the classical solution



$$w_c(r) = \frac{P}{16\pi D}\left[\frac{3+\nu}{1+\nu}(a^2 - r^2) + 2r^2 \log\frac{r}{a}\right]$$

The deflection of a simply supported circular plate with a diameter $2a$ subjected to a uniformly distributed load q_0 over the entire surface is given by the classical solution

$$w_d(r) = \frac{q_{0(a^2-r^2)}}{64D}\left[\frac{5+\nu}{1+\nu}a^2 - r^2\right]$$

Reference: Timoshenko and Woinowsky-Krieger, *Theory of Plates and Shells, 2nd ed.*

Combined Loading - Principle of Superposition

Using the principle of superposition, the deflection of a particular plate subjected to a concentrated load and a distributed load simultaneously can be computed by taking the sum of the deflections for each load condition:

$$w(x, y) = w_c(x, y) + w_d(x, y)$$

There are several assumptions that are made when using superposition. It is assumed that the plate has linearly elastic material properties and undergoes linear deflection, and all deflections are small with no shear deformation.

Problem Statement

Develop a MATLAB application with a graphical user interface (GUI) that computes the deflection of a simply supported rectangular or circular plate subjected to a uniformly distributed load and/or a concentrated load. The application shall have the following features:

1. The graphical-user-interface (GUI) shall be user friendly, intuitive and well organized.
2. The user input controls in the GUI shall include:
    a. Radio button to specify a *circular* or *rectangular* plate.
    b. Edit boxes for the plate dimensions (a, b), the plate thickness h, the material properties (E, ν), the concentrated load value P and position (x_0, y_0 for rectangular plates), and the value of a uniformly distributed load q_0.
    c. Pushbuttons to trigger specific actions (“calculate” and “clear” buttons, etc.).
    d. Additional controls for plot specifications as described below.
3. The plate deflection shall be displayed graphically in the GUI. The deflection shall be displayed in a 3-D surface or contour plot. The user shall be able to select the type of plot to be displayed . The user shall also be able to control the mesh size (number of points) used in the analysis. For circular plates, in addition to the 3-D plots, the user shall have the option to display the deflection as a 2-D plot (deflection w vs. r).
4. The maximum deflection and its location shall be noted on the plot or somewhere else in the GUI.
5. The program should be robust in that it includes error-checking and verification of user inputs.

Development Procedure

Phase 1 (Due 11/21)

Create a user-defined function to perform the numerical analysis without the GUI. The function should accept all arguments needed to perform the plate deflection analysis and to generate the plot. Utilize the top-down approach and pseudo-code in designing the function. At this stage of development, the plot will be generated in a Figure Window. Execute this function from the Command Window or a simple script to test, debug and revise the function as needed. Be sure your function is well documented with comments in the code. Use the function to evaluate the deflection for all test cases listed in Table 1. Verify the function with hand calculations for the maximum deflection in Test Cases 1, 4 and 7 (Note that the maximum deflection in these cases occurs in the center of the plate).

Phase 2 (Due 11/31)

Once the calculation and plot generation function has been developed and verified, create the GUI. You should be able to call the plot generation function from within the GUI's "calculate" button callback function with little or no further modification. That is, the callback function for a "calculate" button could simply get and verify the needed values from the GUI, then call the function you created in Phase 1 of the project. This is an example of modular programming. To demonstrate the functionality of your GUI, repeat all the test cases listed in Table 1and Table 2. Record the output of each case with a screenshot of the GUI.

Deliverables

Prepare a one-page memo that describes the functionality of your program as well as the development and testing procedures used. Attach the screenshots (jpegs) of the results for all test cases. Also attach the calculation and plotting function, and the main GUI function (complete m-file). **Be sure all team members are listed on the memo.** Save the memo and all attachments as a single PDF file. Before submitting the file, be sure to proofread and check it for completeness.

In addition to the PDF file described above, submit all files necessary to run the application. This includes the MATLAB function m-file(s) and the GUI figure (.fig) file so that the application can be verified.

Sign-up for a group on iLearn. Be sure that all team members are listed as group members. Submit the project deliverables using the **group assignment** link so that all team members receive credit.

Table 1. Rectangular plate configurations and load conditions for test cases.

Test Case No.	1	2	3	4	5	6	7
Material	Steel	Steel	Aluminum	Steel	Steel	Aluminum	Aluminum
Dimension, a	30 in	30 in	0.15 m	8 in	20 in	10 in	10 in
Dimension, b	30 in	30 in	0.1 m	20 in	20 in	20 in	20 in
Thickness, h	0.25 in	0.25 in	0.002 m	0.25 in	0.25 in	0.125	0.125
Modulus of Elasticity, E	30.0×10^6 psi	30.0×10^6 psi	70.0×10^9 Pa	30.0×10^6 psi	30.0×10^6 psi	10.0×10^6 psi	10.0×10^6 psi
Poisson's Ratio, ν	0.3	0.3	0.334	0.3	0.3	0.3	0.3
Distributed Load, q_0				200 psi	200 psi	80 psi	40 psi
Load, P	30000 lb	3000 lb	1500 N			120 lb	120 lb
Location, x_0	15 in	5 in	0.025 m			5 in	2 in
Location, y_0	15 in	15 in	0.025 m			10 in	10 in

Table 2. Circular plate configurations and load conditions for test cases.

Test Case No.	8	9	10	11	12	13	14
Material	Steel	Steel	Aluminum	Steel	Steel	Aluminum	Aluminum
Dimension, a	24 in	24 in	0.10 m	8 in	20 in	10 in	10 in
Thickness, h	0.25 in	0.25 in	0.002 m	0.25 in	0.25 in	0.125	0.125
Modulus of Elasticity, E	30.0×10^6 psi	30.0×10^6 psi	70.0×10^9 Pa	30.0×10^6 psi	30.0×10^6 psi	10.0×10^6 psi	10.0×10^6 psi
Poisson's Ratio, ν	0.3	0.3	0.334	0.3	0.3	0.3	0.3
Distributed Load, q_0				150 psi	150 psi	80 psi	40 psi
Load, P	2500 lb	2500 lb	2000 N			120 lb	120 lb