pycse - Python Computations in Science and Engineeringcomputations

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2013-01-21 Mon

Contents

1 Plane poiseuelle flow solved by finite difference

Matlab post

Adapted from http://www.physics.arizona.edu/r̃estrepo/475B/Notes/sourcehtml/node24.html

We want to solve a linear boundary value problem of the form: y" = p(x)y' + q(x)y + r(x) with boundary conditions y(x1) = alpha and y(x2) = beta.

For this example, we resolve the plane poiseuille flow problem we previously solved in Post 878 with the builtin solver bvp5c, and in Post 1036 by the shooting method. An advantage of the approach we use here is we do not have to rewrite the second order ODE as a set of coupled first order ODEs, nor do we have to provide guesses for the solution.

we want to solve u" = 1/mu*DPDX with u(0)=0 and u(0.1)=0. for this problem we let the plate separation be d=0.1, the viscosity $\mu = 1$, and $\frac{\Delta P}{\Delta x} = -100$.

The idea behind the finite difference method is to approximate the derivatives by finite differences on a grid. See here for details. By discretizing the ODE, we arrive at a set of linear algebra equations of the form $Ay = b$, where A and b are defined as follows.

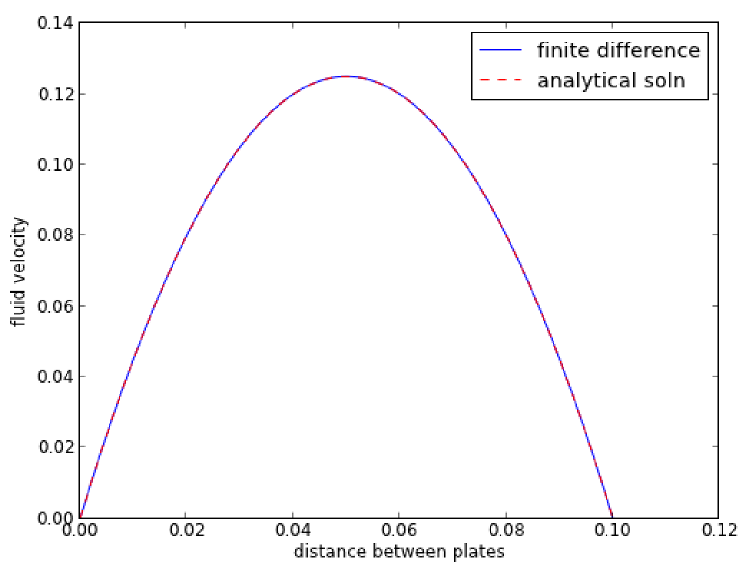
$$A = \left[\begin{array}{ccccc} 2 + h^2 q_1 & -1 + \frac{h}{2} p_1 & 0 & 0 & 0 \\ -1 - \frac{h}{2} p_2 & 2 + h^2 q_2 & -1 + \frac{h}{2} p_2 & 0 & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & -1 - \frac{h}{2} p_{N-1} & 1 + h^2 q_{N-1} & -1 + \frac{h}{2} p_{N-1} \\ 0 & 0 & 0 & -1 - \frac{h}{2} p_N & 2 + h^2 q_N \end{array}\right]$$

$$y = \begin{bmatrix} y_i \\ \vdots \\ y_N \end{bmatrix}$$

$$b = \begin{bmatrix} -h^2 r_1 + (1 + \frac{h}{2} p_1)\alpha \\ -h^2 r_2 \\ \vdots \\ -h^2 r_{N-1} \\ -h^2 r_{N+(1-\frac{h}{2}p_N)\beta} \end{bmatrix}$$

```python
import numpy as np

# we use the notation for y'' = p(x)y' + q(x)y + r(x)
def p(x): return 0
def q(x): return 0
def r(x): return -100

#we use the notation y(x1) = alpha and y(x2) = beta

x1 = 0; alpha = 0.0
x2 = 0.1; beta = 0.0

npoints = 100

# compute interval width
h = (x2-x1)/npoints;

# preallocate and shape the b vector and A-matrix
b = np.zeros((npoints - 1, 1));
A = np.zeros((npoints - 1, npoints - 1));
X = np.zeros((npoints - 1, 1));

#now we populate the A-matrix and b vector elements
for i in range(npoints - 1):
    X[i,0] = x1 + (i + 1) * h

    # get the value of the BVP Odes at this x
    pi = p(X[i])
    qi = q(X[i])
    ri = r(X[i])

    if i == 0:
        # first boundary condition
        b[i] = -h**2 * ri + (1 + h / 2 * pi)*alpha;
    elif i == npoints - 1:
        # second boundary condition
        b[i] = -h**2 * ri + (1 - h / 2 * pi)*beta;
    else:
        b[i] = -h**2 * ri # intermediate points

    for j in range(npoints - 1):
        if j == i: # the diagonal
            A[i,j] = 2 + h**2 * qi
        elif j == i - 1: # left of the diagonal
            A[i,j] = -1 - h / 2 * pi
        elif j == i + 1: # right of the diagonal
            A[i,j] = -1 + h / 2 * pi
        else:
            A[i,j] = 0 # off the tri-diagonal

# solve the equations A*y = b for Y
```

```
Y = np.linalg.solve(A,b)

x = np.hstack([x1, X[:,0], x2])
y = np.hstack([alpha, Y[:,0], beta])

import matplotlib.pyplot as plt

plt.plot(x, y)

mu = 1
d = 0.1
x = np.linspace(0,0.1);
Pdrop = -100 # this is DeltaP/Deltax
u = -(Pdrop) * d**2 / 2.0 / mu * (x / d - (x / d)**2)
plt.plot(x,u,'r--')

plt.xlabel('distance between plates')
plt.ylabel('fluid velocity')
plt.legend(('finite difference', 'analytical soln'))
plt.savefig('images/pp-bvp-fd.png')
plt.show()
```



You can see excellent agreement here between the numerical and analytical solution.