HONG KONG BAPTIST UNIVERSITY
SEMESTER 1 EXAMINATION, 2005-2006

Subject Code: MATH1140 Section Number: 01 Time Allowed: 2 Hour(s)

Subject Title: Computational Mathematics Total No. of Pages: 2

INSTRUCTIONS:

1. Answer ALL of the following questions.
2. The full mark for this examination is 100.
3. Calculators are allowed, but they must not be pre-programmed or have stored text.

1. (14 marks)
Do the following four matrices

$$\begin{bmatrix} 0 & 4 \\ 0 & 2 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 \\ 4 & 2 \end{bmatrix}, \quad \begin{bmatrix} 4 & 0 \\ 2 & 0 \end{bmatrix}, \quad \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix}$$

have LU factorization in which L is a unit lower triangular matrix ? Give the reason.

2. (8 marks)
A square matrix $A = [a_{ij}]$ if $a_{ij} = 0$ for $i > j+1$ and for $j > i+1$, is called a tridiagonal matrix. In a tridiagonal system of n equations with just one right-hand side, how many number of operations (addition, subtraction, multiplication and division) required to compute the solution ?

3. (10 marks)
Use the golden section search to find the minimum of the function $f(x) = e^x + 2 - \cos(x)$ on $[-3, 1]$. Continue the search until the intermediate points are within 0.5 of each other.

4. (14 marks)
Consider the following linear programming problem:

$$\text{Maximize Profit} = \$1X_1 + \$1X_2$$

subject to

$$2X_1 + X_2 \le 100,$$
$$X_1 + 2X_2 \le 100,$$
$$X_1, X_2 \ge 0.$$

(a) What is the optimal solution to this problem ?

(b) If a technical breakthrough occurred that raised the profit per unit of X_1 to \$3, would this affect the optimal solution ?

(c) Instead of an increase in profit coefficient of X_1 to \$3, suppose profit was overestimated, and should only have been \$1.25. Does this change the optimal solution ?

Subject Code: MATH1140 Section Number: 01 Time Allowed: 2 Hour(s)

Subject Title: Computational Mathematics Total No. of Pages: 2

---

5. (24 marks)
Michael is the proud owner of a 1955-sports car. On any given day, Michael never knows whether or not his car will start. Ninety percent of the time it will start if it started the previous morning, and 70% of the time it will not start if did not start the previous morning.

   (a) Construct the matrix of transition probabilities.
   (b) What is the probability that it will start tomorrow if it started today ?
   (c) What is the probability that it will start tomorrow if it did not start today.
   (d) What is the probability that it will not start five days from now if it started today.
   (e) What is the probability that it will not start five days from now if it did not start today.
   (f) Write down the algorithm of the power method ?
   (g) By using the power method, compute the probability that it will start in the long run if the matrix of transition probabilities does not change ?

6. (12 marks)
Compute the LU factorization of the following two matrices

$$A = \begin{bmatrix} 0.0001 & 1 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad A' = \begin{bmatrix} 1 & 1 \\ 0.0001 & 1 \end{bmatrix}.$$

We note that A' is obtained by interchanging the first and the second rows of A. If we need to solve a linear system $Ax = b$, what is your suggestion based on the above factorization results ?

7. (18 marks)
Suppose you know how to solve a linear system with A as the system matrix. Then show how you can solve the augmented system

$$\begin{bmatrix} A & d \\ c & \alpha \end{bmatrix} \begin{bmatrix} x \\ \beta \end{bmatrix} = \begin{bmatrix} b \\ \gamma \end{bmatrix}$$

where A is nonsingular and of size $n \times n$, c is the $1 \times n$-vector, x, d and b the $n \times 1$-vector, and α, β and γ are scalars.

Apply your result to solve

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 10 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}.$$

— END —