

Exercise on Classification

Given the following dataset where $x = \{x_0, x_1, x_2\}$ is the input vector and y is the true label:

x_0	x_1	x_2	y
1	1	-1	1
1	1	1	1
1	-1	1	1
1	-1	-1	-1

The initial values of the weight vector \mathbf{w} are set to be $[-1, -1, -1]^T$.

- 1) Run through the perceptron algorithm by filling in the following table until the weight vector \mathbf{w} converges.

Iteration #	\mathbf{w} (old)	\mathbf{x}	y	$y(\mathbf{w}^T \mathbf{x})$	Update (Y/N)	$y\mathbf{x}$	\mathbf{w} (new)
1							
2							
3							
4							
5							

- 2) Write down the equation of the decision boundary.
- 3) Plot the decision boundary of the perceptron.
- 4) Assume that the initial values of the weight vector \mathbf{w} are $[0.3, 0.5, 0.5]$. Repeat 1-3.

Iteration #	\mathbf{w} (old)	\mathbf{x}	y	$y(\mathbf{w}^T \mathbf{x})$	Update (Y/N)	$y\mathbf{x}$	\mathbf{w} (new)
1							
2							
3							
4							
5							

- 5) Compare the two decision boundaries obtained and discuss which is better.

Suggested Solution:

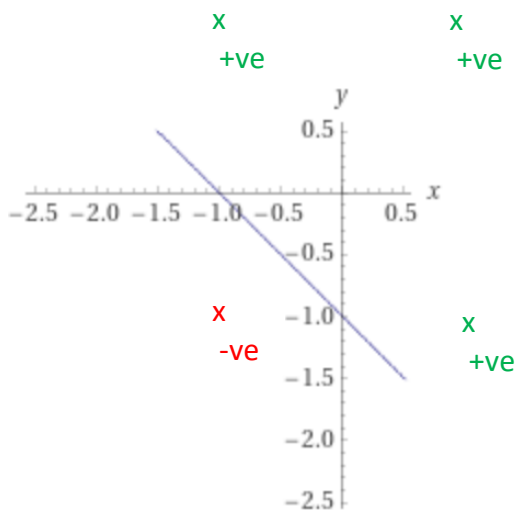
1)

Iteration #	\mathbf{w} (old)	\mathbf{x}	y	$y(\mathbf{w}^T \mathbf{x})$	Update (Y/N)	$y\mathbf{x}$	\mathbf{w} (new)
1	[-1, -1, -1]	[1, 1, -1]	1	-1	Y	[1, 1, -1]	[0, 0, -2]
2	[0, 0, -2]	[1, 1, 1]	1	-2	Y	[1, 1, 1]	[1, 1, -1]
3	[1, 1, -1]	[1, -1, 1]	1	-1	Y	[1, -1, 1]	[2, 0, 0]
4	[2, 0, 0]	[1, -1, -1]	-1	-2	Y	[-1, 1, 1]	[1, 1, 1]
5	[1, 1, 1]	[1, 1, -1]	1	1	N	-	[1, 1, 1]
6	[1, 1, 1]	[1, 1, 1]	1	3	N	-	[1, 1, 1]
7	[1, 1, 1]	[1, -1, 1]	1	1	N	-	[1, 1, 1]
8	[1, 1, 1]	[1, -1, -1]	-1	1	N	-	[1, 1, 1]
9	Converged!						

2)

Equation: $1 + x_1 + x_2 = 0$

3)



4)

Iteration #	\mathbf{w} (old)	\mathbf{x}	y	$y(\mathbf{w}^T \mathbf{x})$	Update (Y/N)	$y\mathbf{x}$	\mathbf{w} (new)
1	[0.3, 0.5, 0.5]	[1, 1, -1]	1	0.3	N	-	[0.3, 0.5, 0.5]
2	[0.3, 0.5, 0.5]	[1, 1, 1]	1	1.3	N	-	[0.3, 0.5, 0.5]
3	[0.3, 0.5, 0.5]	[1, -1, 1]	1	0.3	N	-	[0.3, 0.5, 0.5]
4	[0.3, 0.5, 0.5]	[1, -1, -1]	-1	0.7	N	-	[0.3, 0.5, 0.5]
5	Converged!						

Equation: $0.3 + 0.5 x_1 + 0.5 x_2 = 0$

