Exercise 1: Matrix Computation

Given the $n \times k$ matrix $\mathbf{A}$ and the $k \times n$ matrix $\mathbf{B}$:

1. Use an example to show that $\mathbf{AB} \neq \mathbf{BA}$ even if $n = k$.

2. When $n \neq k$, do we have $\text{tr}(\mathbf{AB}) = \text{tr}(\mathbf{BA})$? Prove your conclusion.

3. What is the relationship between eigenvalues of $\mathbf{AB}$ and eigenvalues of $\mathbf{BA}$? What is the relationship between eigenvectors of $\mathbf{AB}$ and eigenvectors of $\mathbf{BA}$?