

I. For 3 variables x, y, z , they satisfy the equality $x + y + z = 0$. Calculate the angle between vector $v = (x, y, z)$ and vector $w = (z, x, y)$.

II. Suppose $Q^T = Q^{-1}$.

(1) Show that the columns q_1, \dots, q_n are unit vectors: $\|q_i\|^2 = 1$.

(2) Show that every two columns of Q are perpendicular: $q_i^T q_j = 0$.

(3) Find a 2 by 2 example (that $Q^T = Q^{-1}$) with first entry $q_{11} = \cos\theta$.

III. These flags have rank 2. Find the singular value decomposition of

$A_{\text{Sweden}}, A_{\text{Finland}}, B_{\text{Benin}}$.



Sweden



Finland



Benin

$$A_{\text{Sweden}} = A_{\text{Finland}} = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 1 & 2 & 1 & 1 \end{bmatrix}$$

$$B_{\text{Benin}} = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 3 \end{bmatrix}$$

IV. Suppose A_0 is a 5 by 10 matrix with average grades for 5 courses over 10 years.

- (1) How would you create the centered matrix A and the sample covariance matrix S ?
- (2) When you find the leading eigenvector of S , what does it tell you?