COMP7180 Assignment 2

Note:

1. **Instruction of assignment submission:**
   a) **write all your answers in a Microsoft Word document;**
   b) **name your document using the following format: "COMP7180_A2_StudentID_Surname_Givenname.doc"; and**
   c) **submit the document on Moodle.**
2. **The submission deadline is 5pm, Jan. 4, 2020.**
3. **This is an individual work. Plagiarism is strictly forbidden. Students who plagiarized and who were plagiarized will be given Zero Mark.**

Problem 1 (16 Marks)

Give TWO examples in Artificial Intelligence, Machine Learning, or Data Analytics that differentiation and optimization play an important role. Explain the role of differentiation and optimization in these two examples in detail.

Problem 2 (36 Marks)

Answer the following questions.

2.1 Assume that *t* hours past midnight, the temperature in Hong Kong was

$C(t) = -\frac{1}{6}t^2 + 4t + 10$ degree Celsius.

(a) What was the temperature at 2:00 P.M.? Write down the detailed calculation procedure.

(6 marks)

(b) By how much the temperature increases or decreases between 6:00 and 9:00 PM? Write down the detailed calculation procedure.

(6 marks)

2.2 Find the following limits. Write down the detailed calculation procedure.

(a) $\lim_{x\to -1} \frac{x^4-1}{x^2-1}$

(3 marks)

(b) $\lim_{x\to 2} \frac{x^3-8}{x^2-4}$

(3 marks)

(c) $\lim_{x\to +\infty} \frac{2x^2-x+4}{x^2-4}$

(3 marks)

(d) $\lim_{x \to +\infty} \frac{x^2+1}{x-1}$

(3 marks)

2.3 The Gross National Product (GNP) of the country A was $N(t) = t^2 + 5t + 106$ billion dollars t years after 2010.

(a) At what rate was country A's GNP changing with respect to time in 2018? Write down the detailed calculation procedure.

(6 marks)

(b) At what percentage rate was country A's GNP changing with respect to time in 2018? Write down the detailed calculation procedure.

(6 marks)

Problem 3 (12 Marks)

Given a dataset of (x, y): $\{(1,1), (4,2), (7,3), (10,5)\}$. Assume that x and y are linearly correlated with each other: $y = a + bx$. Use least squares method to find a and b. Write down the detailed calculation procedure.

Problem 4 (36 Marks)

Answer the following questions.

4.1 If C is a convex set, then for any $x_1, x_2, \dots, x_n \in C$ and $0 < \alpha_1, \alpha_2, \dots, \alpha_n < 1$ satisfying $\alpha_1 + \alpha_2 + \dots \alpha_n = 1$, prove that $\alpha_1 x_1 + \alpha_2 x_2 + \dots \alpha_n x_n \in C$.

(12 marks)

4.2 Is the function $f(x) = -\frac{1}{3}x^2 + 8x - 3$ convex? Justify your answer using the definition of convexity.

(6 marks)

4.3 Assume that the functions f and g are both convex functions of a single variable x. Given $a_1, a_2 > 0$, is the function h defined by $h(x) = a_1 f(x) + a_2 g(x)$ convex? Justify your answer using the definition of convexity.

(6 marks)

4.4 If $f_1(x), f_2(x), \dots, f_n(x)$ are convex functions, prove that $f(x) = \max\{f_1(x), f_2(x), \dots, f_n(x)\}$ is a convex function.

(12 marks)