HONG KONG BAPTIST UNIVERSITY
SEMESTER 1 FINAL EXAMINATION, 2008-2009

Course Code: MATH1550 Section No.: 01 Time Allowed: 2 Hour(s)

Course Title: **Calculus and Linear Algebra** Total Number of Pages: 2

INSTRUCTIONS:

1. Answer ALL of the following questions.
2. The full mark for this examination is 100.
3. Calculators are allowed, but they must not be pre-programmed or have stored text.

1. (30 marks)
Evaluate the following expressions:

(a) $\lim_{x\to 1}\sqrt{\dfrac{x-1}{x^2-1}}$ (5 marks)

(b) $\dfrac{d}{dx}\ln(1+x^2e^x)$ (5 marks)

(c) $\dfrac{d}{dx}\left(\dfrac{x+1}{2x^2-x-1}-\sin x\right)$ (5 marks)

(d) $\displaystyle\sum_{k=0}^{4}\frac{(-1)^k}{k^2+1}$ (5 marks)

(e) $\displaystyle\int\frac{4x^3-6x+2}{5x^4}\,dx$ (5 marks)

(f) $\displaystyle\int xe^{2x}\,dx$ (5 marks)

2. (15 marks)
Find the absolute maximum and minimum values of

$$f(x)=x^3-3x^2-9x+1$$

on the closed interval $[-2, 2]$, and state where those values occur.

3. (20 marks)

(a) Express the following system of linear equations in matrix form $A\mathbf{x}=\mathbf{b}$.

$$\begin{cases} 3x-1y+2z=6 \\ -6x+4y-5z=-3 \\ 9x+5y+6z=2 \end{cases}$$

(5 marks)

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(b) Find a and b in the given matrices so that they become an LU-factorization for the coefficient matrix A in part (a).

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & a & 0 \\ 3 & 4 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 3 & -1 & 2 \\ 0 & b & -1 \\ 0 & 0 & 4 \end{bmatrix}$$

(5 marks)

(c) Solve the equation $A\mathbf{x} = \mathbf{b}$ in part (a) by using the LU-factorization given in part (b). (10 marks)

4. (15 marks)
Let $f(x, y) = x^3 + x^2y + y^2$ for $x \geq 1$ and all real y. Find and classify all critical points of f.

5. (20 marks)
Solve the following nonlinear differential equations.

(a) $\dfrac{dy}{dx} = \dfrac{3x^2 + 1}{4y}$ and $y(0) = 1$ (10 marks)

(b) $(e^{2x} + 2xy)\dfrac{dy}{dx} + (2e^{2x} + 2xy) = 0$ and $y(2) = -1$. (10 marks)

— GOOD LUCK —