I. A department store is fencing off part of the store for children to meet and be photographed with Santa Claus. They have decided to fence off a rectangular region of fixed area 800 ft^2. Fire regulations require that there be three gaps in the fencing: 6 ft openings on the two facing sides and a 10 ft opening on the remaining wall (the fourth side of the rectangle will be against the building wall). Find the dimensions that will minimize the length of fencing used.

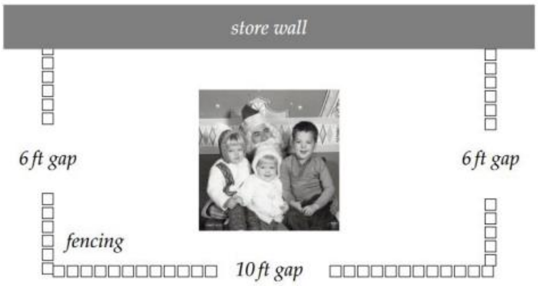

Figure 1: Picture of the store

Let x be the side with 10 ft gap and y be the side 6 ft gap. It is given that $xy = 800$ and we want to minimize

$$f = (x - 10) + 2(y - 6)$$

we can write

$$f = (x - 10) + 2(\frac{800}{x} - 6)$$

and the domain of this function is $[10, \infty]$, because we need at least 10 ft gap. To find minimal value

$$f' = 1 - \frac{1600}{x^2} = 0$$

Thus $x = 40$ and $y = 20$.

II. Show that S is a convex set.

$$S = \{(x_1, x_2) | x_2 \geq |x_1|\}$$

For any two points in S, $x^{(1)} = \begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \end{bmatrix}$ and $x^{(2)} = \begin{bmatrix} x_1^{(2)} \\ x_2^{(2)} \end{bmatrix}$, and for any $\lambda \in [0,1]$, we have,

$$\lambda x^{(1)} + (1-\lambda)x^{(2)} = \begin{bmatrix} \lambda x_1^{(1)} + (1-\lambda)x_1^{(2)} \\ \lambda x_2^{(1)} + (1-\lambda)x_2^{(2)} \end{bmatrix}$$

$$\lambda x_2^{(1)} + (1-\lambda)x_2^{(2)} \geq \lambda \left|x_1^{(1)}\right| + (1-\lambda)\left|x_1^{(2)}\right| \geq \left|\lambda x_1^{(1)} + (1-\lambda)x_1^{(2)}\right|$$

Therefore, $\lambda x^{(1)} + (1-\lambda)\, x^{(2)} \in S$, S is a convex set.