COMP 7990
Principles and Practices of Data Analytics

Lecture 1: Data Preprocessing and Linear Regression

Dr. Eric Lu Zhang

What is data analytics and data mining?

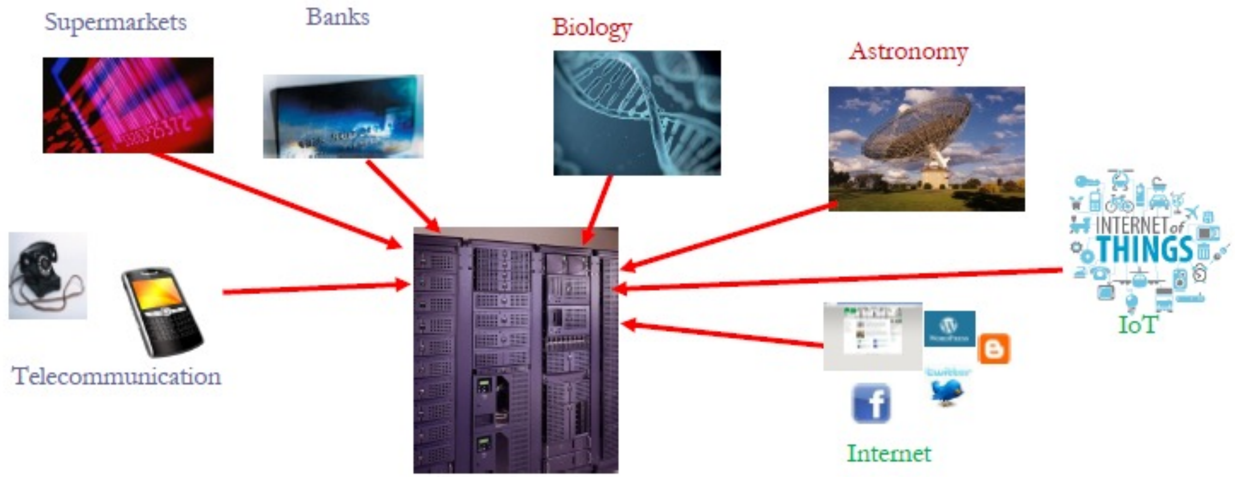
- Data Analytics: Entire process of data collection, inspecting, cleansing, transforming and modeling, interpretation and reporting.

- Data mining (knowledge discovery from data)
  - Extract of interesting (non-trivial, implicit, previously unknown potential useful) pattern or knowledge from huge amount of data.

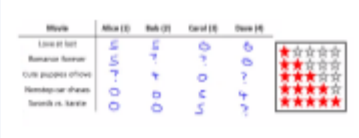
- Data analytics is a more general concept than data mining.

Why we need data mining?

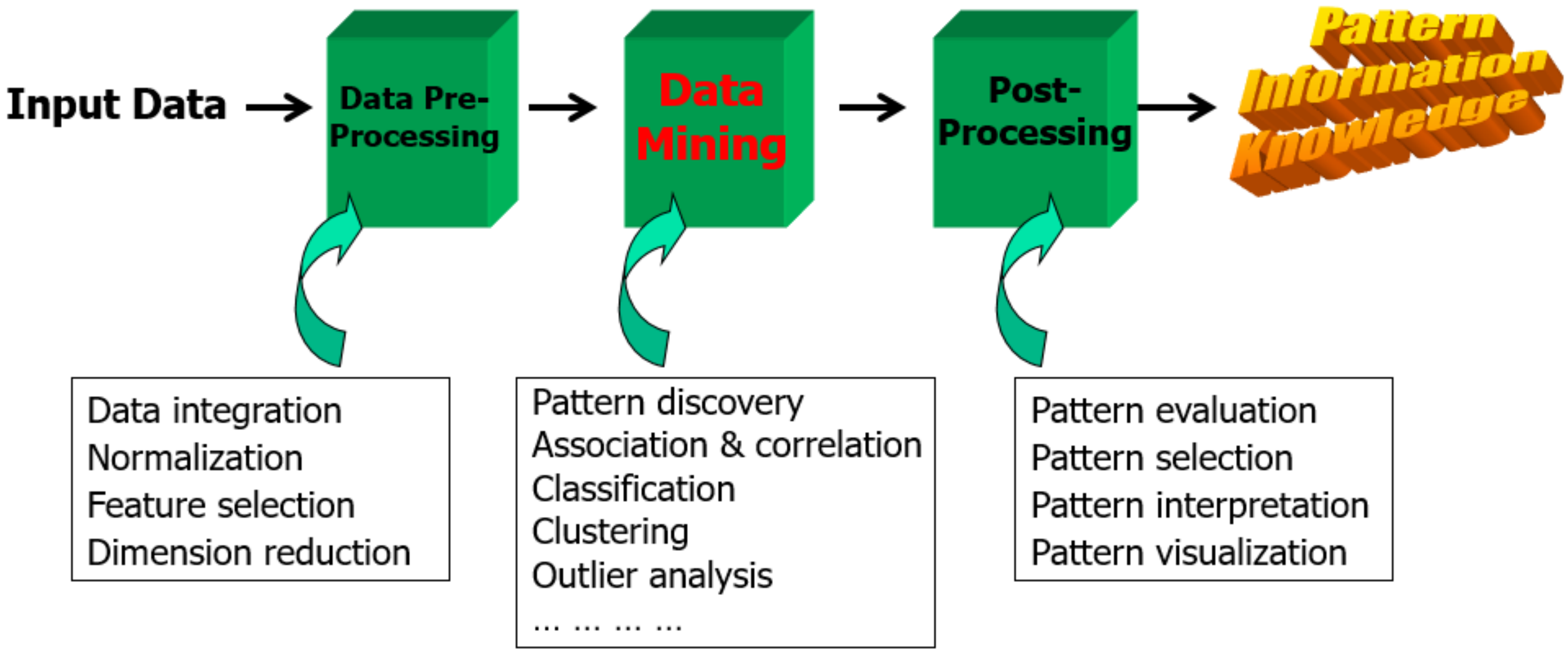
- Huge amounts of data are collected from different domains
- “We are drowning in information but starving for knowledge”-John Naibett
- The amount and the complexity of the collected data does not allow for manual analysis: we need automated analysis of massive data.



Real-life Applications of Using Data Mining

Input Data		Methods	Output
	images	Classification	Is it a banana (or an apple)?
	Movie ratings	Recommendation System	Recommend which movies to which users?
	News articles	Clustering	What are the topics people discussed about in the news today?
	English and Chinese sentences	Classification	Translation

Knowledge Discovery in Database (KDD): From machine learning perspective



Outline for Data Preprocessing and Data Mining

- **Data Preprocessing**
- **Supervised learning**

❖Regression

1. Linear regression with one variable
2. Linear Regression with multiple variables

❖Classification

1. Perceptron
2. Artificial Neural Network
3. K Nearest Neighbor
4. Support Vector Machine

- **Unsupervised learning**

1. K-means Clustering
2. Hierarchical Clustering

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❖ Classification

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- **Unsupervised learning**

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Outline for Data Preprocessing

- Why we need data preprocessing
- What is data preprocessing
- How to do data preprocessing
  - Data Clean
  - Data Transformation

Why we need preprocess the data?

- Real world data is dirty
  - Incomplete: lacking attribute values
    - e.g., Occupation = “ ”
  - Noisy: containing errors or outliers
    - E.g., Salary = “-10”
  - Inconsistent: containing discrepancies in codes or names
    - E.g., Age = “40”, Birthday = ’03/02/1990’
    - E.g., Was rating “1, 2, 3”, now rating “A, B, C”
    - E.g, discrepancy between duplicate records

ID	Birthday	Age	Salary	Occupation
1001	11/01/1986	32	-10	Engineer
1002	03/02/1990	40	30k	Manager
1003	01/01/1980	39	40k	

Noisy value

Missing value

Inconsistent value

Why Data is Dirty?

- Incomplete data may come from
  - “Not applicable” data value
    - E.g., annual income is not applicable to children
  - People do not want to disclose the information
    - E.g., age, birthday
  - Human/hardware/software problems
    - E.g., data was accidently deleted
- Noisy Data may come from
  - Faulty data collection instruments
  - Human/Computer errors
- Inconsistent data may come from
  - Different data sources

Why Data Preprocessing is important

- No quality data, no quality results
  - Quality decisions must be based on quality data
    - E.g., missing data or incorrect data may cause incorrect or even misleading statistics

- **Garbage In, Garbage Out**

- In general, data pre-processing consumes more than 60% of a data analytics project effort.

Outline

- Why we need data preprocessing
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- How to data preprocessing
  - Data Clean
  - Data Transformation

Typical tasks in data preprocessing

- Data Cleaning
  - Fill in missing values
  - Smooth noisy data
  - Identify and remove outliers

- Data transformation and data discretization
  - Feature type conversion
  - Normalization
    - Scaling attribute values to fall within a specific range (e.g., [0, 1])
  - Attribute construction

Data Cleaning: Handling Incomplete (Missing) Values

- Data is not always available
  - E.g, many data samples do not have recorded value for several attributes, such as customer age, customer income in sales data

- Missing data may be due to
  - Equipment malfunction
  - Data is not entered
  - Certain data may not be considered at the time of data collection

Missing Data Example (Titantic Data)

- Titantic Data

Missing values

PassengerId	Survived	Pclass	Sex	Age	SibSp	Parch	Ticket	Fare	Cabin	Embarked
1	0	3	male	22	1	0	A/5 21171	7.25		S
2	1	1	female	38	1	0	PC 17599	71.2833	C85	C
3	1	3	female	26	0	0	STON/O2. 3101282	7.925		S
4	1	1	female	35	1	0	113803	53.1	C123	S
5	0	3	male	35	0	0	373450	8.05		S
6	0	3	male		0	0	330877	8.4583		Q

Description for each feature contained in this dataset:

- Survival: Survival 0 = No, 1 = Yes
- Pclass: A proxy for economic status (1 = 1st class, 2 = 2nd class, 3 = 3rd class)
- SibSp: number of siblings / spouses aboard the Titanic
- Parch: number of parents / children aboard the Titanic
- Ticket: Ticket number
- Fare: Passenger fare
- Cabin: Cabin number
- embarked: Port of Embarkation C = Cherbourg, Q = Queenstown, S = Southampton

Outline

- Why we need data preprocessing

- What is data preprocessing

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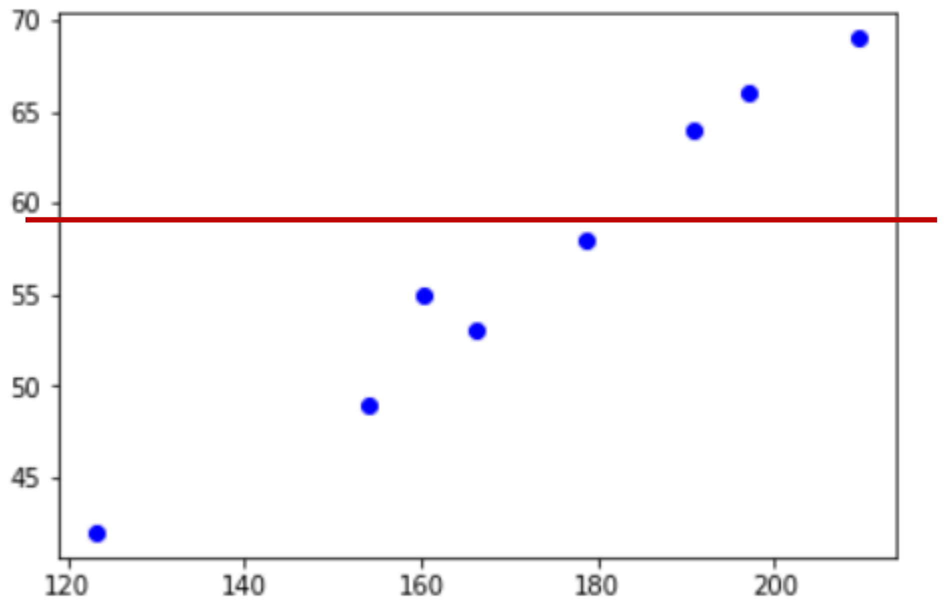
How to handle missing data?

- Ignore the data sample with missing values
  - Not a good solution, especially when data is scarce
- Ignore attributes with missing values
  - Use only attributes (features) with all values
  - May leave out important features
- Fill in it by
  - A global constant: e.g., “unknown”
  - Attribute mean/Median/mode
  - Predict the missing value (data imputation)
    - Estimate gender based on first name (name gender)
    - Estimate age based on first name (name popularity)
    - Build a predictive model based on other attributes/features

Example of handling missing value by mean

height	weight
123.20	42.0
138.60	NaN
154.00	49.0
160.16	55.0
166.32	53.0
178.64	58.0
187.88	NaN
190.96	64.0
197.12	66.0
209.44	69.0

Mean value of weight



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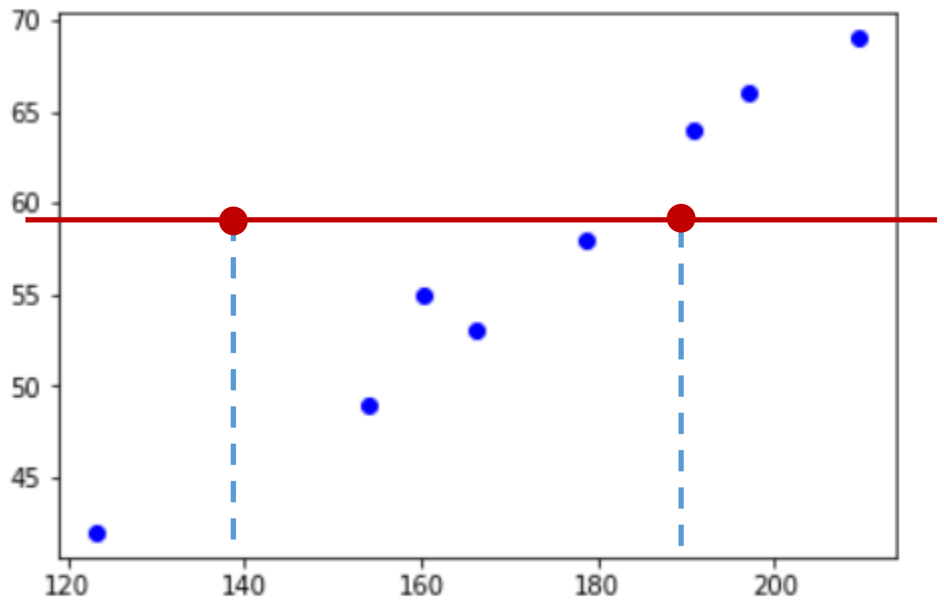
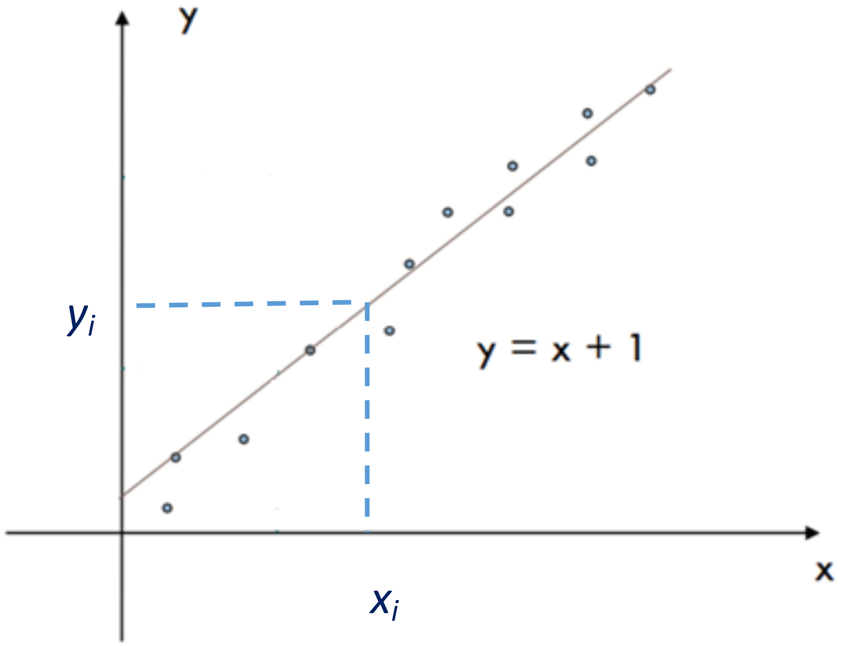
57.0

Mean value of weight



Handing missing value by prediction model

- Replace missing value by predicted values by a prediction model (e.g., a regression model)
- Requires attribute dependencies
- Can be used for handling missing data and noisy data.
- Regression models will be discussed in deep later.



Example of handling missing value by regression

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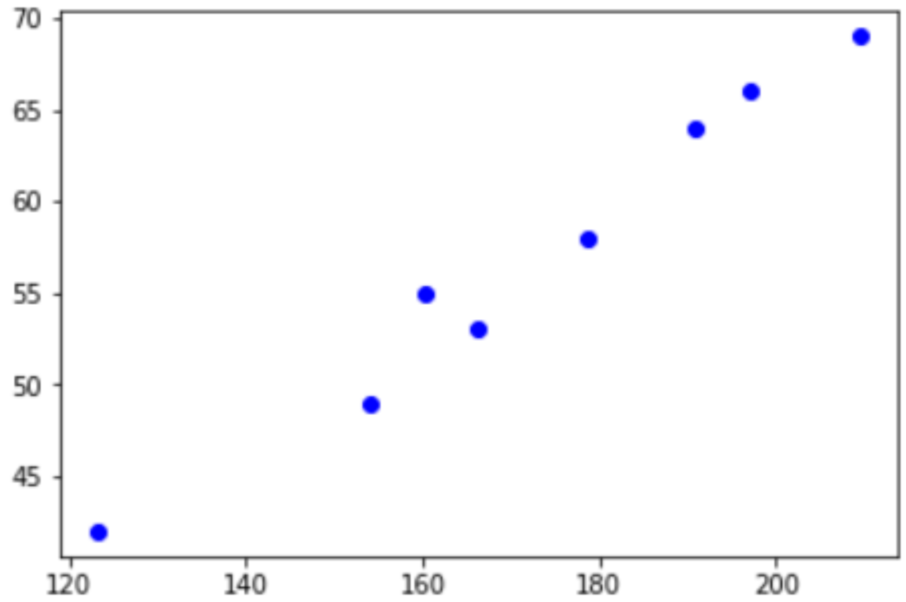

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197.12	66.0
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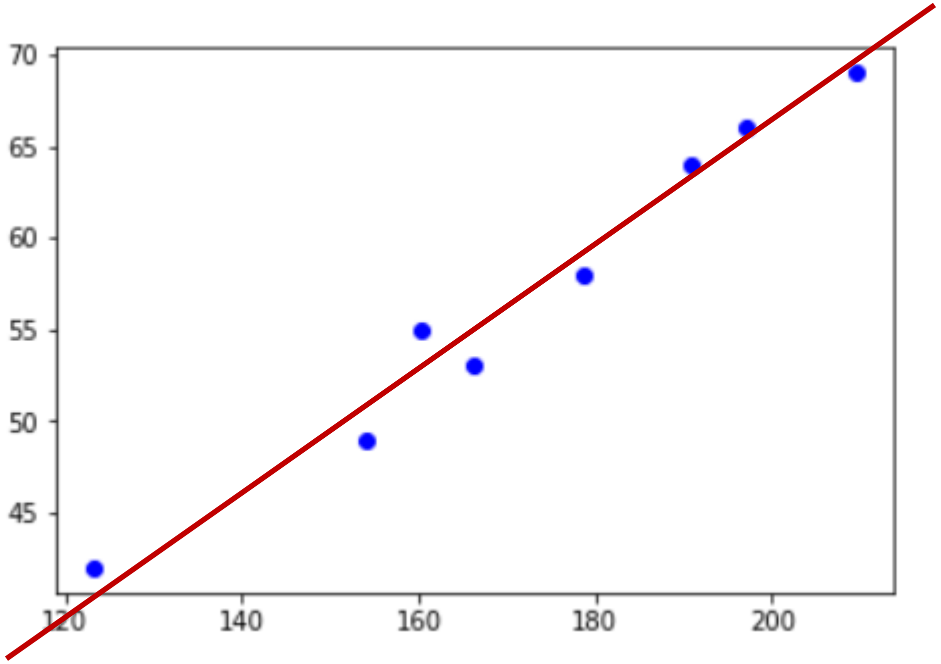
weight = 0.33*height + 0.6



Example of handling missing value by regression

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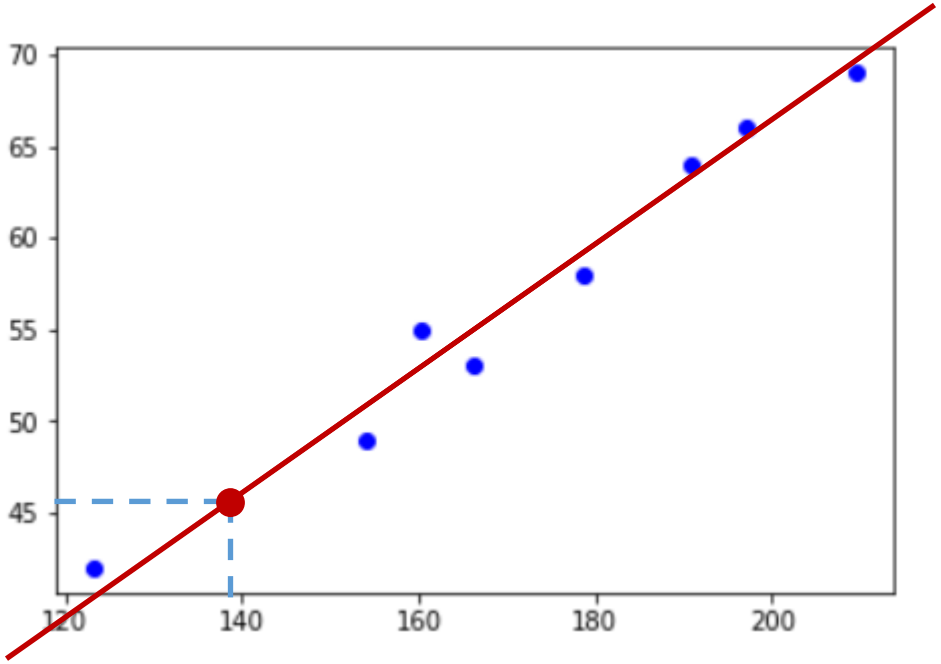
197.12	66.0
209.44	69.0

46.34

weight = 0.33*height + 0.6



Example of handling missing value by regression

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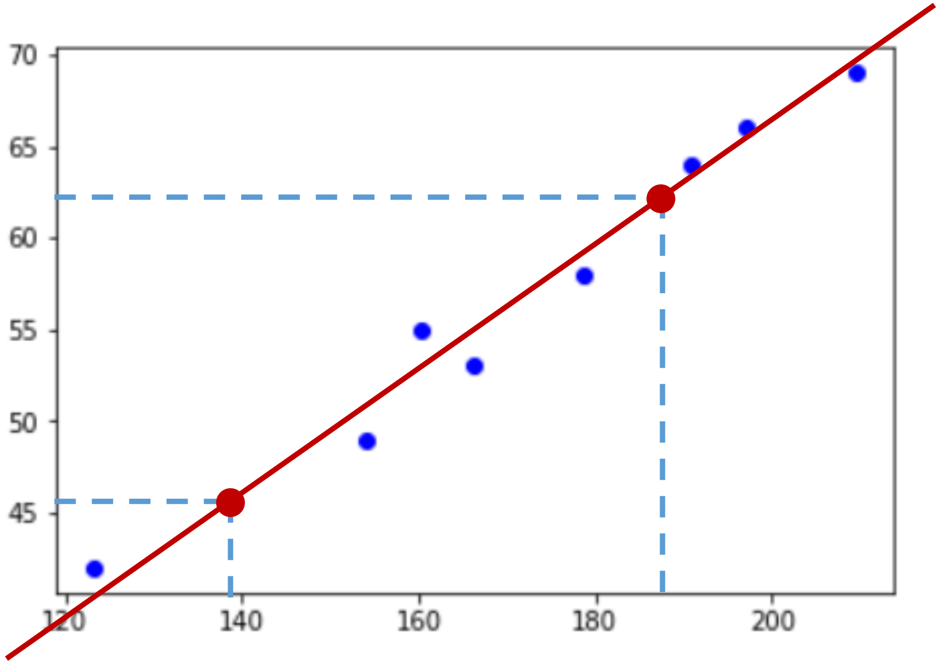
154.00	49.0
160.16	55.0
166.32	53.0
178.64	58.0
187.88	NaN
190.96	64.0
197.12	66.0
209.44	69.0

46.34

62.60

weight = 0.33*height + 0.6



Data Cleaning: Handling Noisy Data

- Noise: random error or variance in a measured variable

- Incorrect attribute values may be due to
  - Errors in data collection devices
  - Wrong input
  - Technology limitation

How to Handle Noisy Data

- Binning
  - First sort data and partition into bins
  - Smooth by bin mean/median/boundaries
- Regression
  - Smooth by fitting the data into regression functions
- Clustering
  - Detect and remove outliers

Simple Discretization Methods: Binning

- Equal-width (distance) Partitioning
  - Divides the range into *N* intervals of equal size: uniform grid
  - Suppose *min* and *max* are the lowest and highest values of the attribute, the width of intervals should be: w = (*max* – *min*)/*N*
  - The most straight-forward method
  - Outliers may dominate presentation
  - Skewed data is not handled well

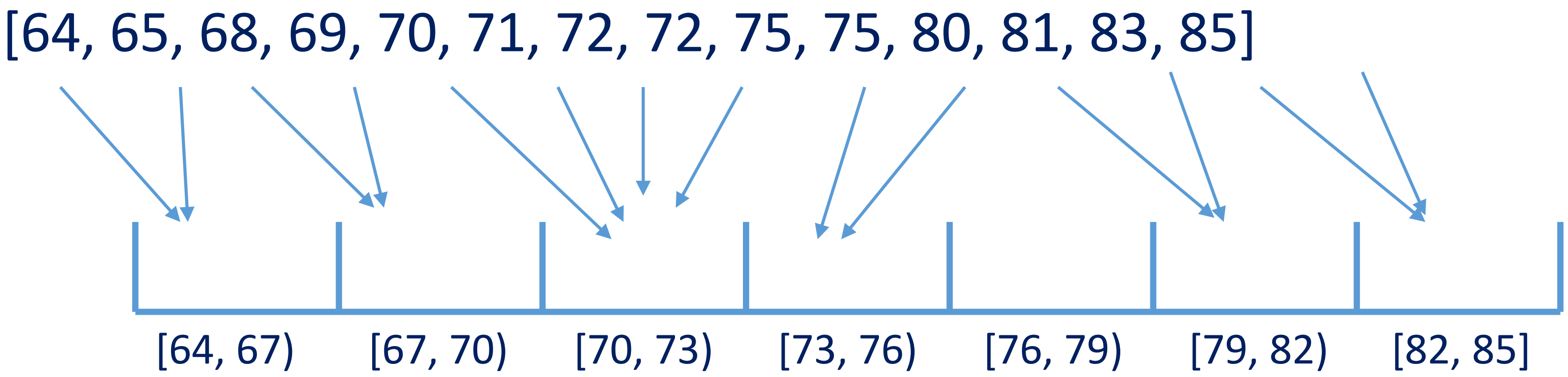
- Equal-depth (frequency) partitioning
  - Divides the range into *N* intervals, each containing approximately same the number of samples
  - Skewed data is also handled well

Example of Equal-width Binning for data smoothing

- Suppose we have the following values for temperature and we want to divided them into 7 bins

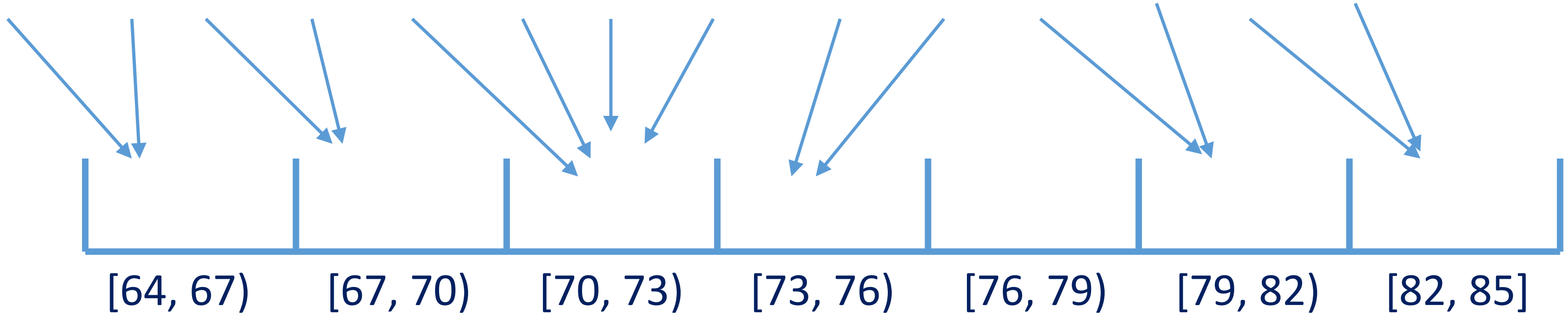
  [64, 65, 68, 69, 70, 71, 72, 72, 75, 75, 80, 81, 83, 85]

- Partition data into bins
  - Compute the width w = (85-64)/7 = 3



Example of Equal-width Binning for data smoothing

[64, 65, 68, 69, 70, 71, 72, 72, 75, 75, 80, 81, 83, 85]



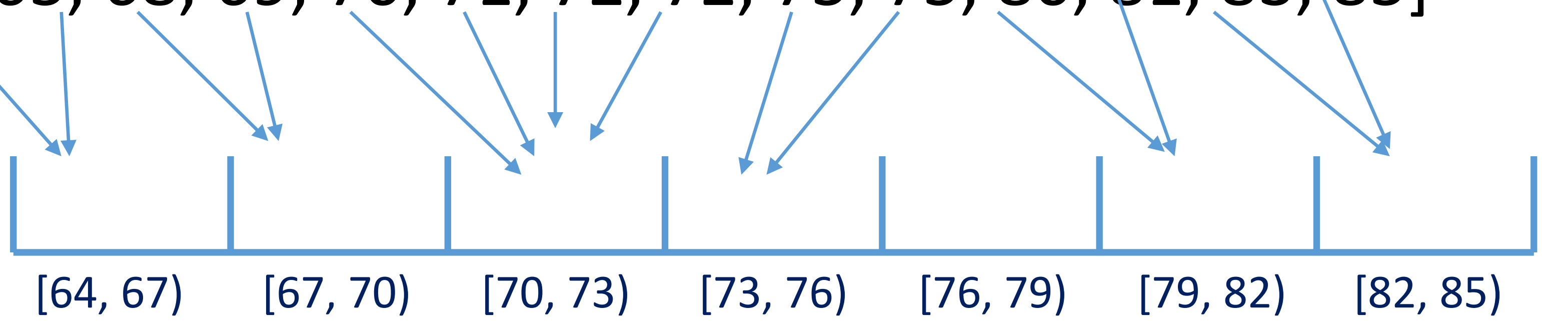
- Smoothing by bin means
  - Each value in a bin is replaced by the mean value of the bin

  [64.5, 64.5, 68.5, 68.5, 71.25, 71.25, 71.25, 71.25, 75, 75, 80.5, 80.5, 84, 84]

  - Similarly, smoothing by bin medians can be used, in which each bin value is replaced by the bin median.

Example of Equal-width Binning for data smoothing

[64, 65, 68, 69, 70, 71, 72, 72, 75, 75, 80, 81, 83, 85]



- Smoothing by bin boundaries
  - Bin boundaries are the minimum and maximum values in a given bin.
  - Each bin value then is replaced by the closest boundary value

[64, 65, 68, 69, 70, 70, 72, 72, 75, 75, 80, 81, 83, 85]

In general, the larger the width, the greater the effect of the smoothing

Example of Equal-width Binning

- Advantage
  - Simple and easy to implement
  - Produce a reasonable abstraction of data

- Disadvantage
  - Where does *N* come from?
  - Sensitive to outliers

Example of Equal-depth Binning

- Divides the range into *N* intervals, each containing **approximately same the number of samples**
- E.g., we have the following values for prices and we want to divided them into 3 bins using Equal-depth binning

  [4, 8, 15, 21, 21, 24, 25, 28, 34]

- Partition into 3 bins (equal frequency)

  [4, 8, 15, 21, 21, 24, 25, 28, 34]

- Smooth by bin means

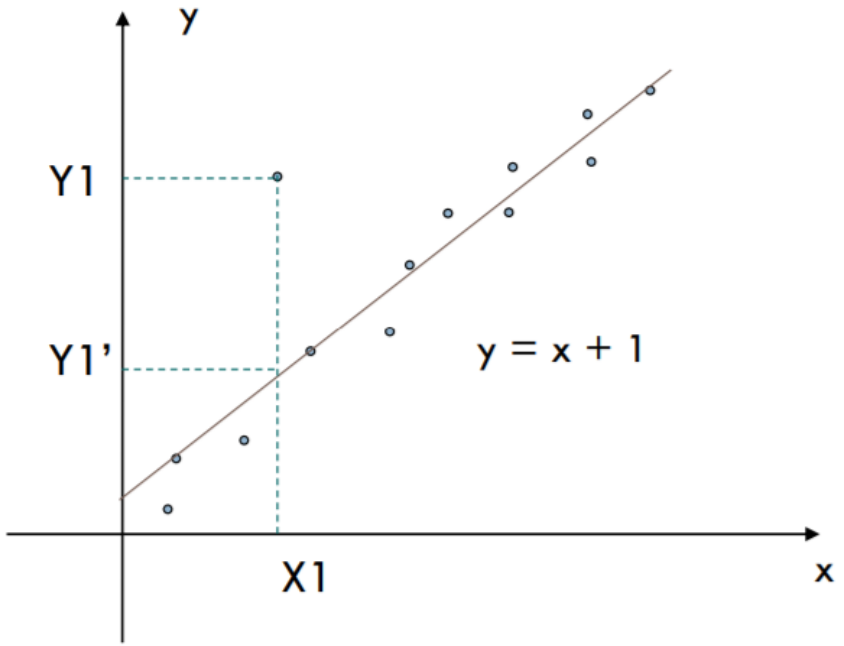
  [9, 9, 9, 22, 22, 22, 29, 29, 29]

- Smooth by bin boundaries

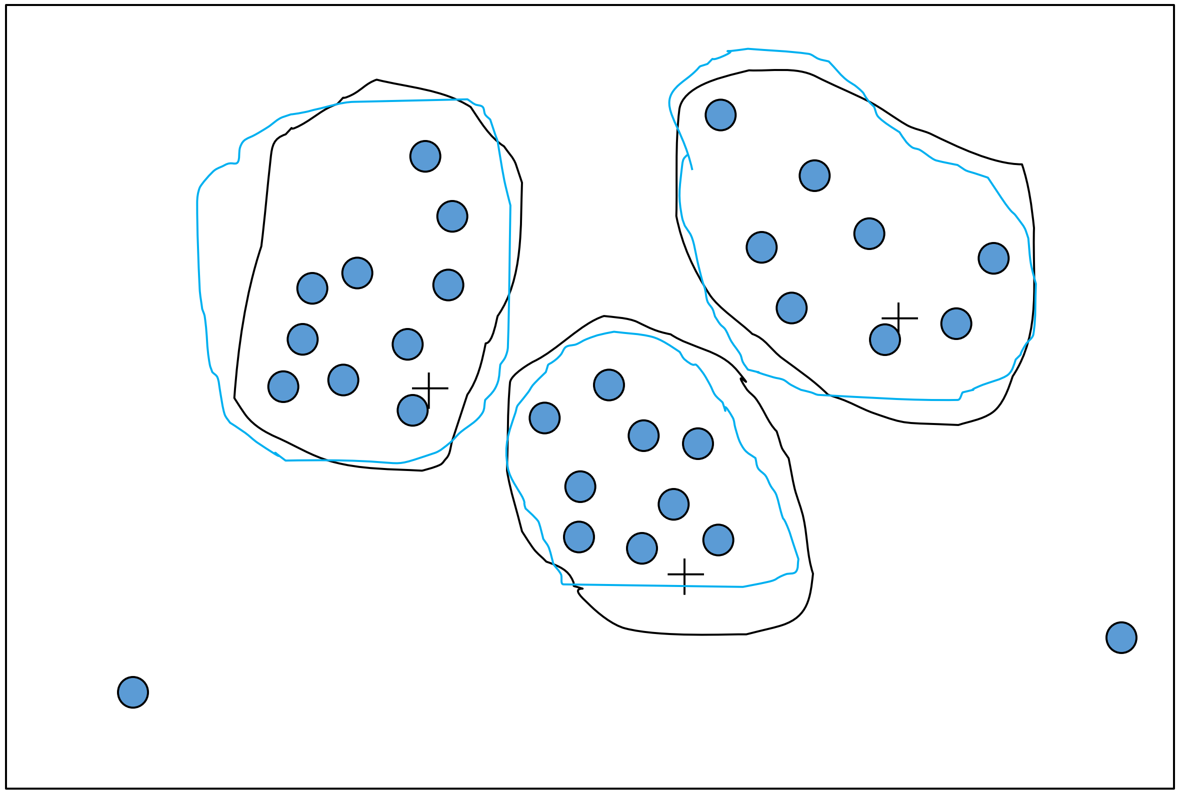
  [4, 4, 15, 21, 21, 24, 25, 25, 34]

Handling Data Noisy by Regression Analysis

- Data smoothing can also be done by regression analysis.



Handling Data Noisy by Clustering Analysis



- Outliers may be detected by clustering analysis.
- Outliers may need to be removed from the data.
- Clustering algorithms will be discussed in depth in future lectures.

Data Transformation

- Data Transformation
  - A function that maps the entire set of values of a given attribute to a new set of replacement values s.t. each old value can be identified with one of the new values

- Methods
  - Feature Type Conversion
  - Normalization
  - Feature construction

Feature Type Conversion

- Some tools can only deal with nominal values but other only deal with numeric values.
- Features have to be converted to satisfy the requirements of different tools
  - Numeric -> Nominal
    - Binning
  - Nominal -> numeric
    - One hot encoding
  - Ordinal -> numeric (order matters)
    - A -> 4.0
    - A- -> 3.7
    - B+ -> 3.3
    - B -> 3

Nominal to Numeric (one-hot encoding)

ID	Color
1	Red
2	Green
3	Blue

ID	Color_red	Color_green	Color_Blue
1	1	0	0
2	0	1	0
3	0	0	1

- One of the ways to encode the nominal variable to numeric is **one-hot encoding**
- With one-hot encoding, a nominal feature becomes a vector whose size is the number of possible choices for that features

Normalization: motivation

- Data have attribute values

- Can we compare these attribute values?

- E.g., considering the following two records, which one is more similar to (5.9 ft, 50kg)
  - (4.6 ft, 55 kg)
  - (5.6 ft, 56 kg)

- We need to normalize data to makes different attributes comparable.

Normalization

- For distance-based methods, normalization helps to prevent that attributes with large ranges out-weight attributes with small ranges

- Scale the attribute values to a small specified range

- Normalization Methods
  - Normalization by decimal scaling
  - Min-Max normalization (normalized by range)

Normalization: Decimal Scaling

- Decimal Scaling
  - The values of an attribute are normalized by moving the decimal point.
  - The number of decimal points moved depends on the maximum absolute value of the attribute.
  - Decimal scaling maps a value x_i to x_i' by

$$x_i' = \frac{x_i}{10^j}$$

j is the smallest integer such that $\max(|x_i|) < 1$

- E.g., suppose that the recorded values of an attribute range from -986 to 917. The maximum absolute value is 986. To normalize by decimal scaling, we therefore divide each value by 1,000 (i.e., $j = 3$) so that −986 normalizes to −0.986 and 917 normalizes to 0.917.

Normalization: min-max Normalization

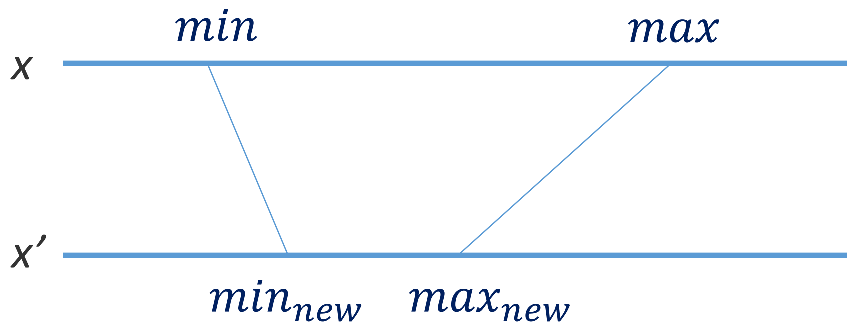
- Min-max normalization
  - Performs a linear transformation on the original data.
  - Suppose *min*, *max* are the minimum and maximum values of an attribute and we want to normalize the attribute value to $[min_{new}, max_{new}]$, min-max normalization maps a value x_i to x_i' by

$$x_i{}' = \frac{(x_i - min)}{max - min}(max_{new} - min_{new}) + min_{new}$$

- E.g., suppose that the minimum and maximum values for the feature income are \$12,000 and \$98,000. We would like to map income to the range [0.0, 1.0]. By min-max normalization, what is the mapped value for \$73,600?

$$\frac{(73{,}600 - 12{,}000)}{98{,}000 - 12{,}000}(1.0 - 0.0) + 0.0 = 0.716$$

Deriving the formula for min-max Normalization



Find a linear transform
$x' = a*x + b$

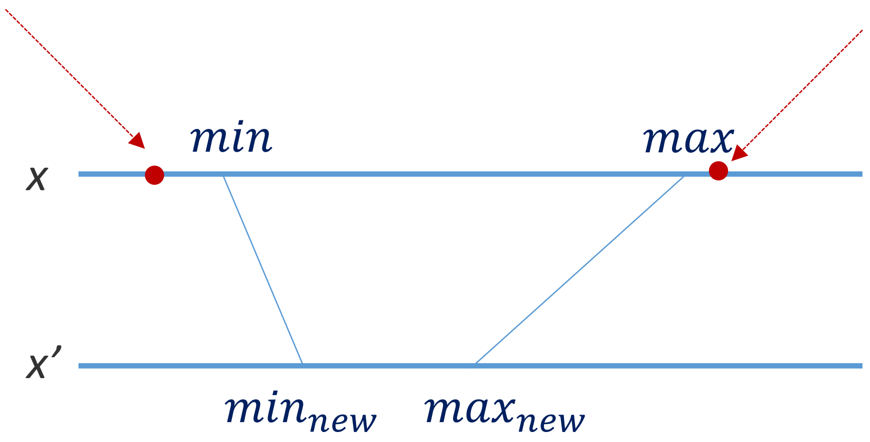
- We know min is mapped to min_{new} and max is mapped to max_{new}
  - $\text{min}_{\text{new}} = a*\text{min} + b$
  - $\text{max}_{\text{new}} = a*\text{max} + b$
  - Therefore, $a = (\text{max}_{\text{new}} - \text{min}_{\text{new}})/(\text{max}-\text{min})$;

    $b = \text{min}_{\text{new}} - (\text{max}_{\text{new}} - \text{min}_{\text{new}})*\text{min}/(\text{max}-\text{min})$;

$$x_i' = \frac{(x_i - min)}{max - min}(max_{new} - min_{new}) + min_{new}$$

We take it from here

Min-max normalization problem



- Min-max normalization will encounter an “out-of-bounds” error if a future input value is fall outside of the original data range.

- In some cases, we may do not know the minimum and maximum values of an attribute.

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Unsupervised learning

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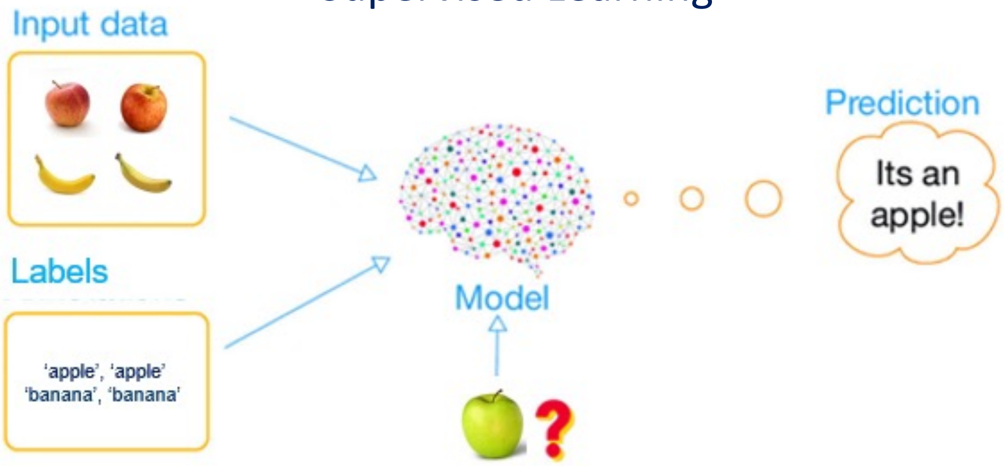
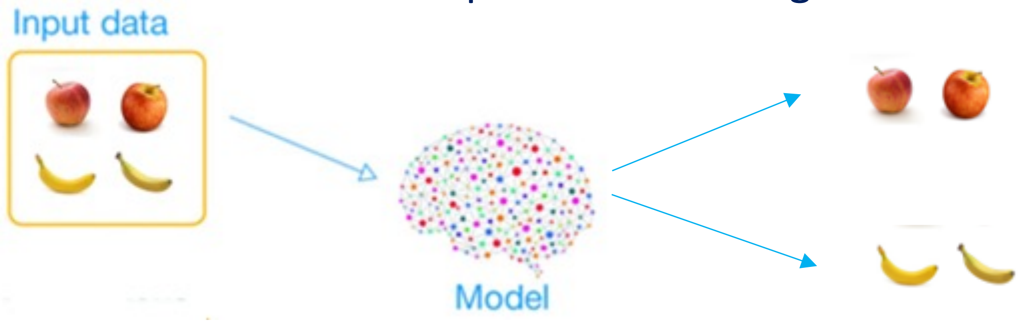
Different Types of Learning Tasks

- Supervised Learning
  - Data with labels
- Unsupervised Learning
  - Data without labels

Supervised Learning



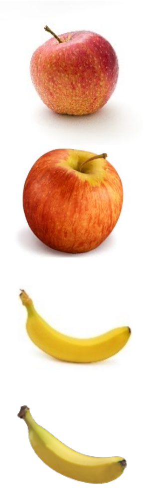
Unsupervised Learning



An Example of Classification Problem

- Learn to recognize apple or banana

Training Data with Class Label



Label

apple

apple

banana

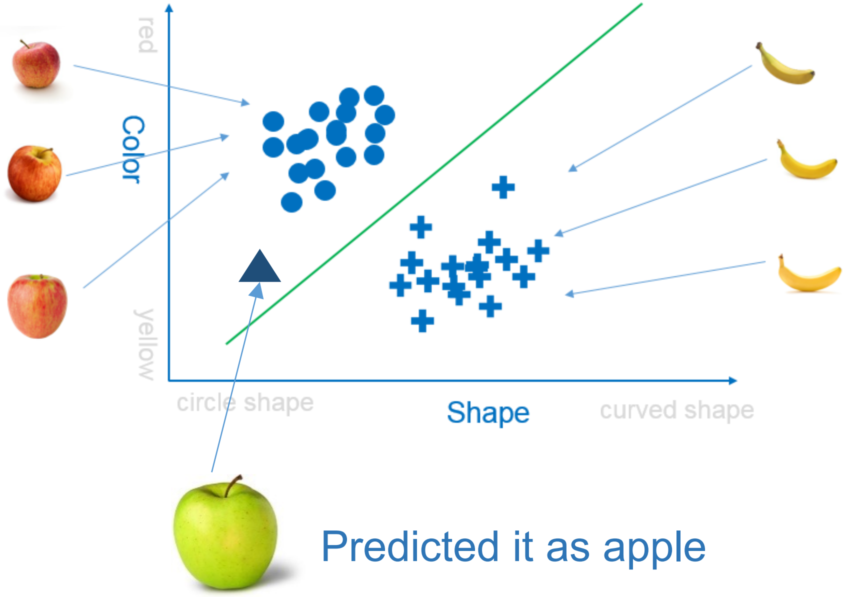
banana

… …

Feature Engineering

color(x_1)	shape (x_2)	y
0.8	0.1	1
0.75	0.12	1
0.12	0.6	0
0.2	0.8	0
…	…	…
…	…	…

Model Building



Predicted it as apple

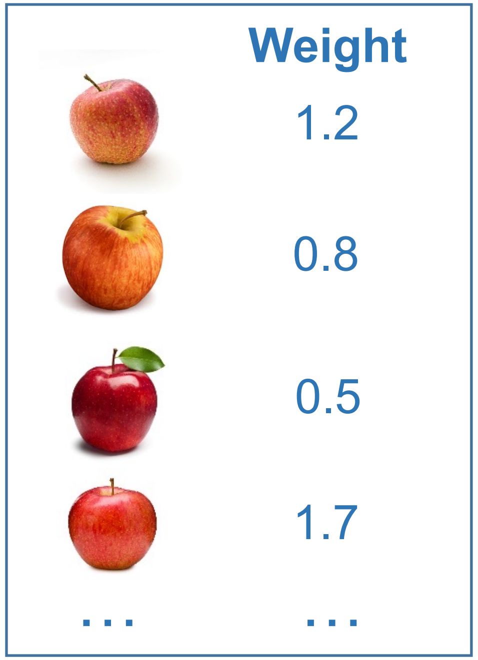
Test Data

To predict it is apple or banana

An Example of Regression Problem

- Learn to predict apple or ban

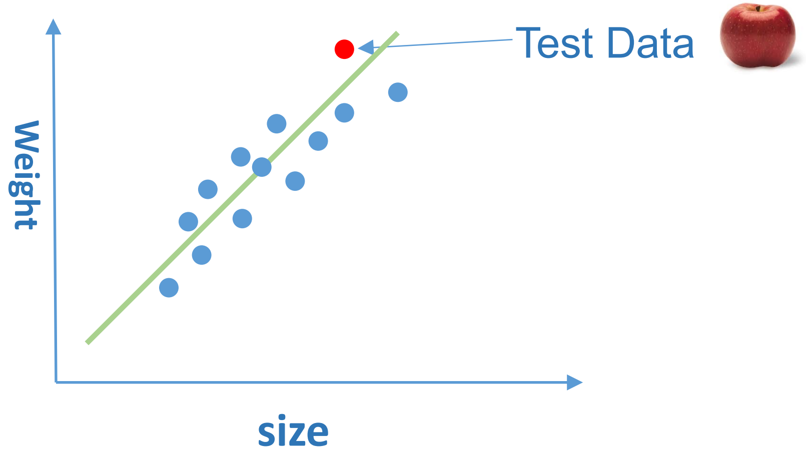
Training Data with Weight



Feature Engineering

size(x_1)	weight (x_2)
0.2	0.6
0.4	1.2
0.6	1.8
0.8	2.4
…	…
…	…

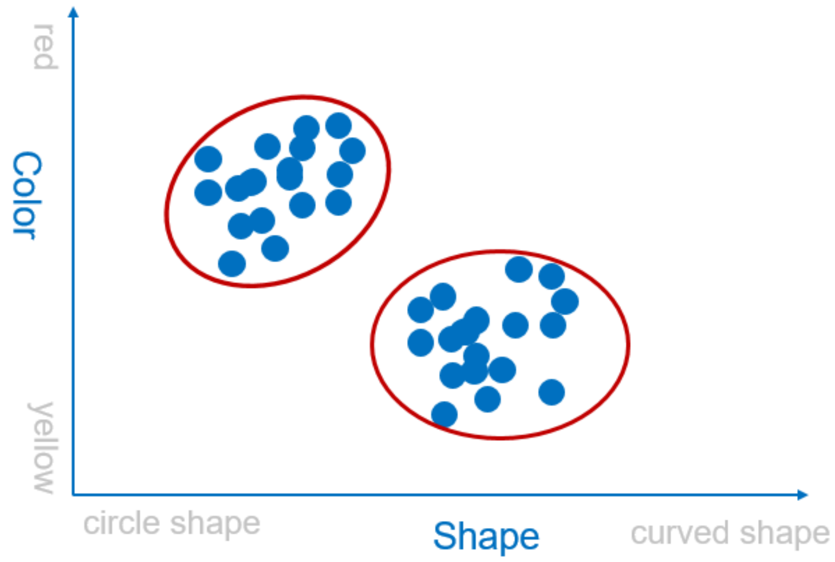
Model Building



Test Data  To predict its weight (quantitative feature)

An example for Clustering

- Clustering
  - Only the feature representation of instance is available.
  - No Label information
  - The goal is to discover groups of similar instance from data



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Model representation

- Example: We want to find out the relationships between housing prices and size in US. The known data are draw using ×.
- The model representing these data is shown using the blue line.

Model representation

Training set of housing prices

m = Number of training examples
(The number of ×)

- (x,y): one training example
- $(x^{(i)},y^{(i)})$: the *i*-th training example

x = "input" variable

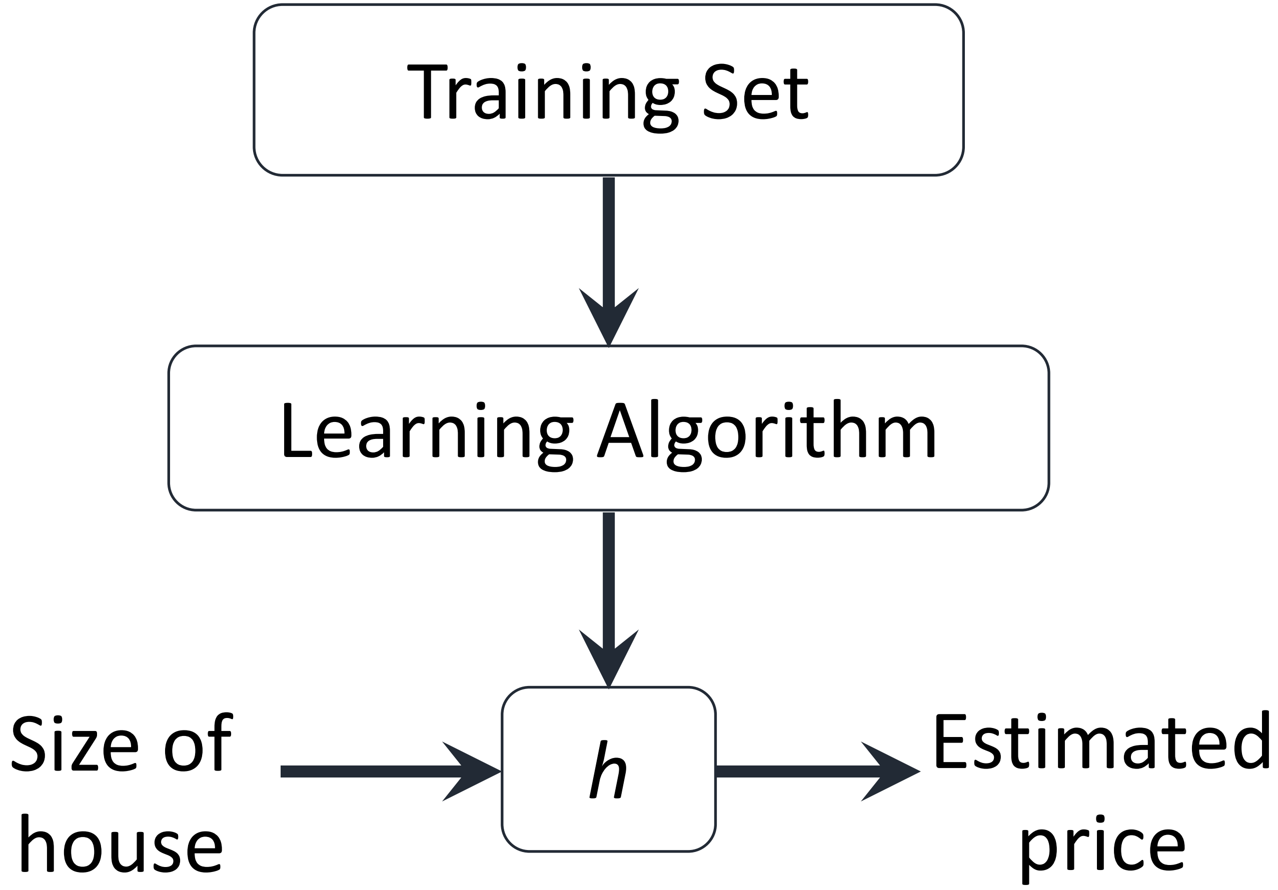
- $x^{(1)}=2104$, $x^{(2)}=1416$,...

y = "output" variable

- $y^{(1)}=460$,$y^{(2)}=232$,...

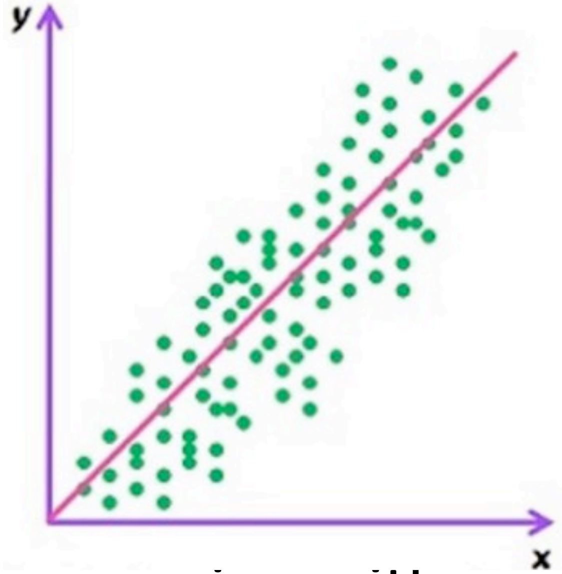
Size in feet^2 (x)	Price ($) in 1000's (y)
2104	460
1416	232
1534	315
852	178
…	…

Model representation



How do we represent *h* ?

$$h_\theta(x) = \theta_0 + \theta_1 x$$



Linear regression with one variable.
Univariate linear regression.

Cost Function

Hypothesis:

$$h_\theta(x) = \theta_0 + \theta_1 x$$

How to calculate θ_0 and θ_1?

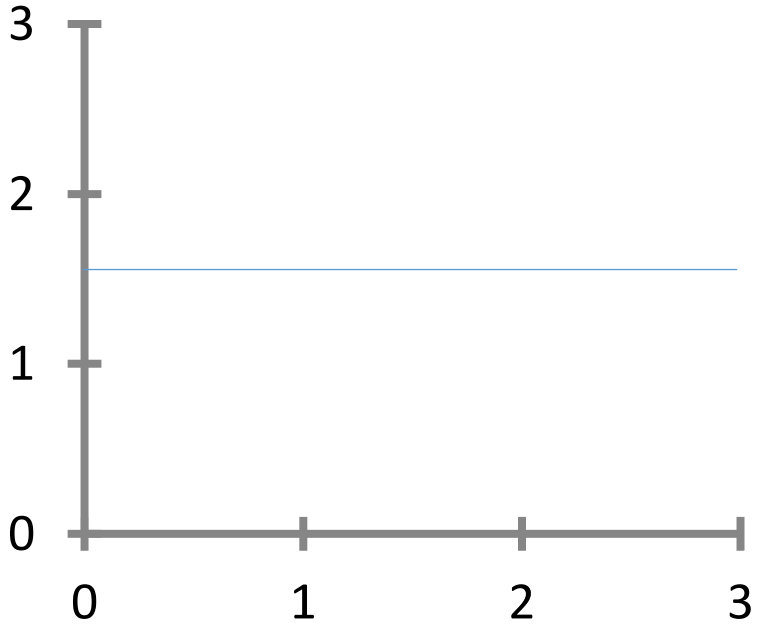
Training Set (m=42)

Size in feet^2 (x)	Price ($) in 1000's (y)
2104	460
1416	232
1534	315
852	178
…	…

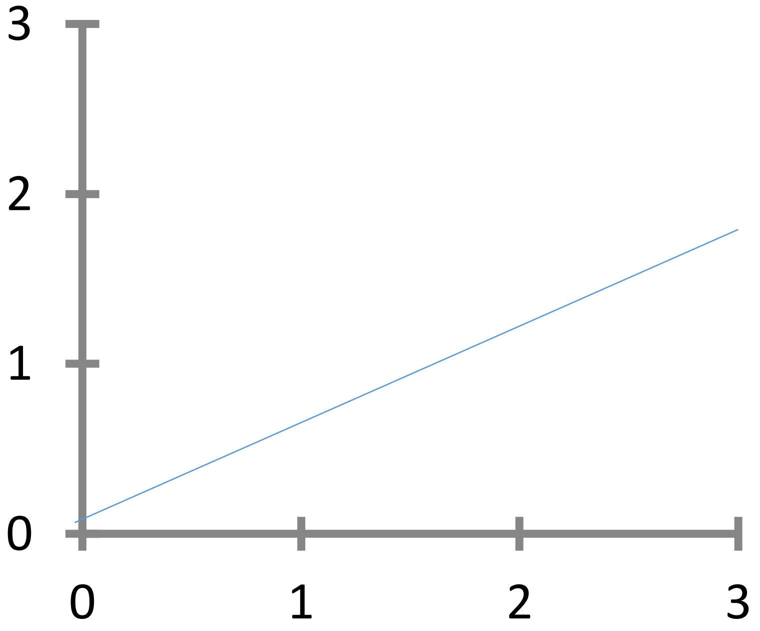
Cost Function

$$h_\theta(x) = \theta_0 + \theta_1 x$$

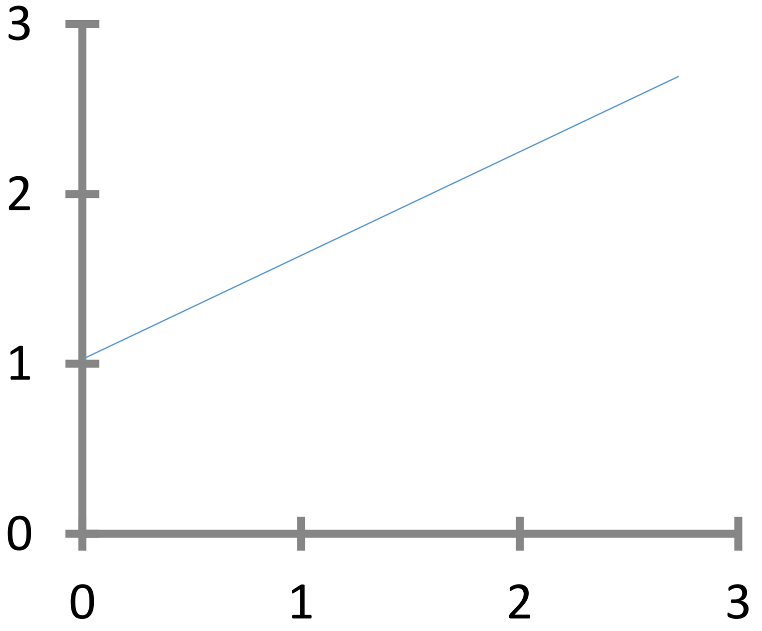
Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ is close to y for our training examples



$\theta_0 = 1.5$
$\theta_1 = 0$

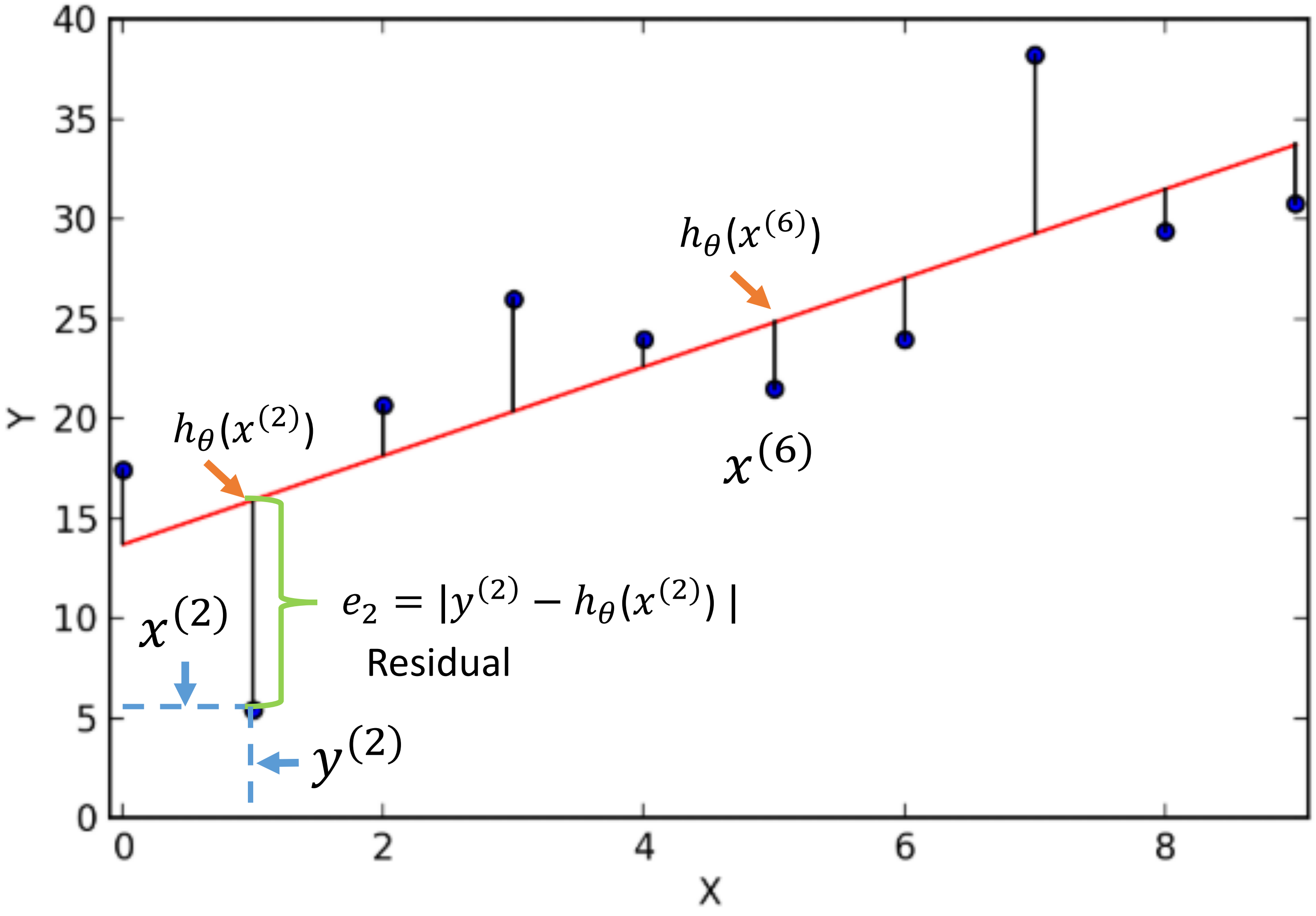


$\theta_0 = 0$
$\theta_1 = 0.5$



$\theta_0 = 1$
$\theta_1 = 0.5$

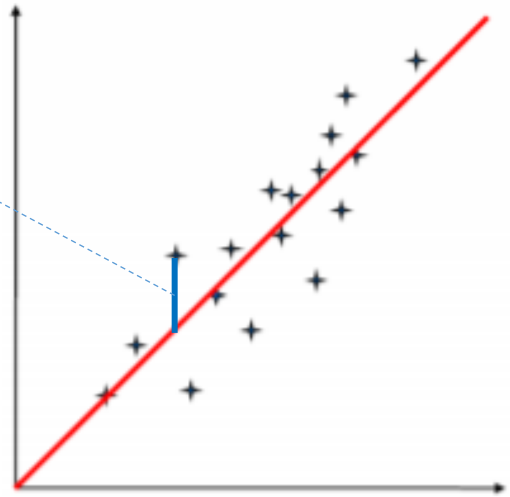
Residual



40
35
30
25
20
15
10
5
0
Y
0
2
4
6
8
X
$h_\theta(x^{(6)})$
$h_\theta(x^{(2)})$
$x^{(6)}$
$e_2 = |y^{(2)} - h_\theta(x^{(2)})\,|$
Residual
$x^{(2)}$
$y^{(2)}$

Cost function

$e^{(i)} = |y^{(i)} - (\theta_1 * x^{(i)} + \theta_0)|$



- Error for a data sample $(x^{(i)}, y^{(i)})$ is $e^{(i)} = |y_i - \left(\theta_1 * x^{(i)} + \theta_0\right)|$
- The **sum of squared errors** is:

$$SSE = \sum_{i=1}^{m} (y^{(i)} - (\theta_1 * x^{(i)} + \theta_0))^2 = \sum_{i=1}^{m} (h_\theta\left(x^{(i)}\right) - y^{(i)})^2$$

- The objective is the find the best fitting (i.e., minimizing the sum of squared errors)

Cost function

Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

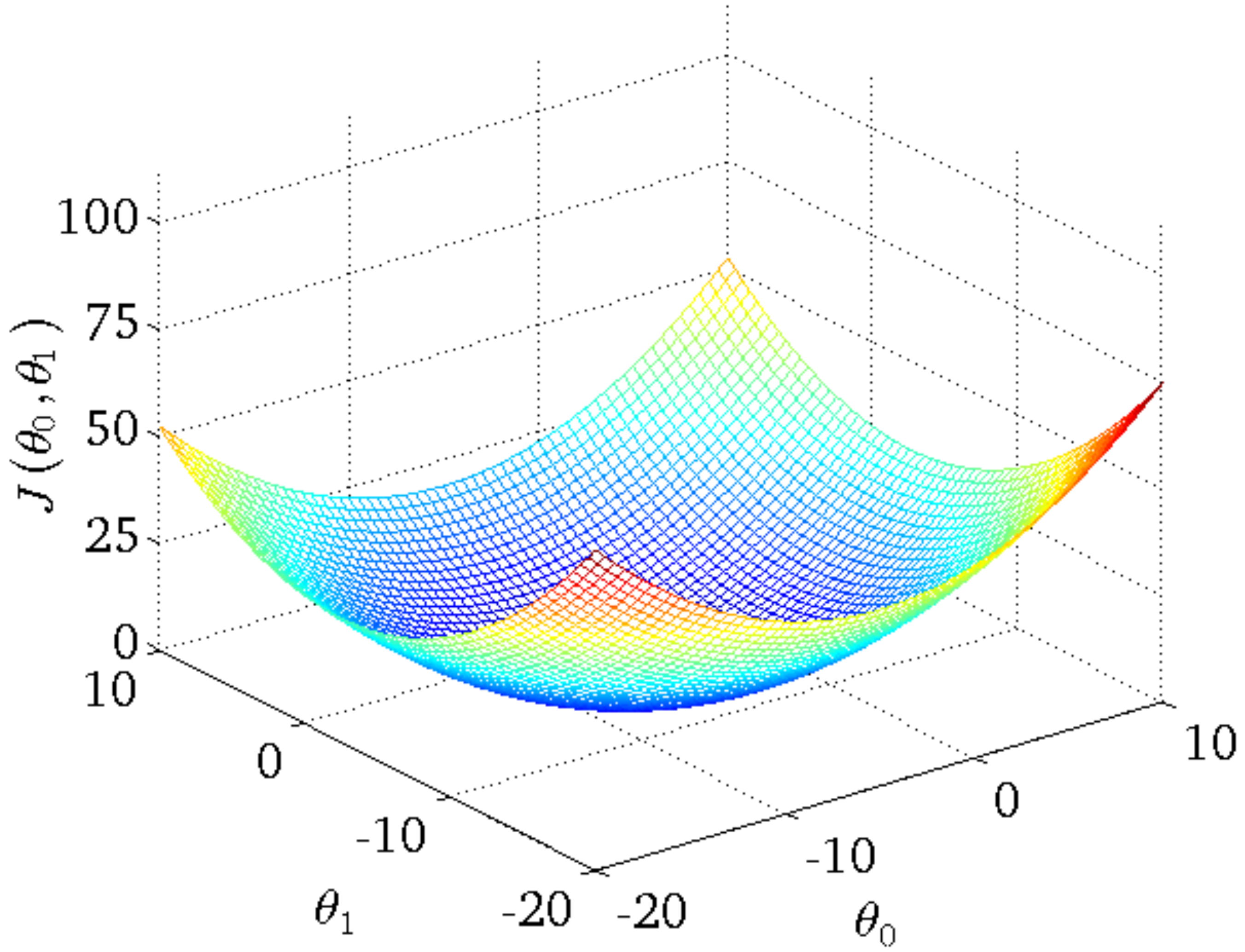
Parameters: θ_0, θ_1

Cost Function: $J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$

Goal: $\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$

Cost function

$J(\theta_0, \theta_1)$



Gradient descent

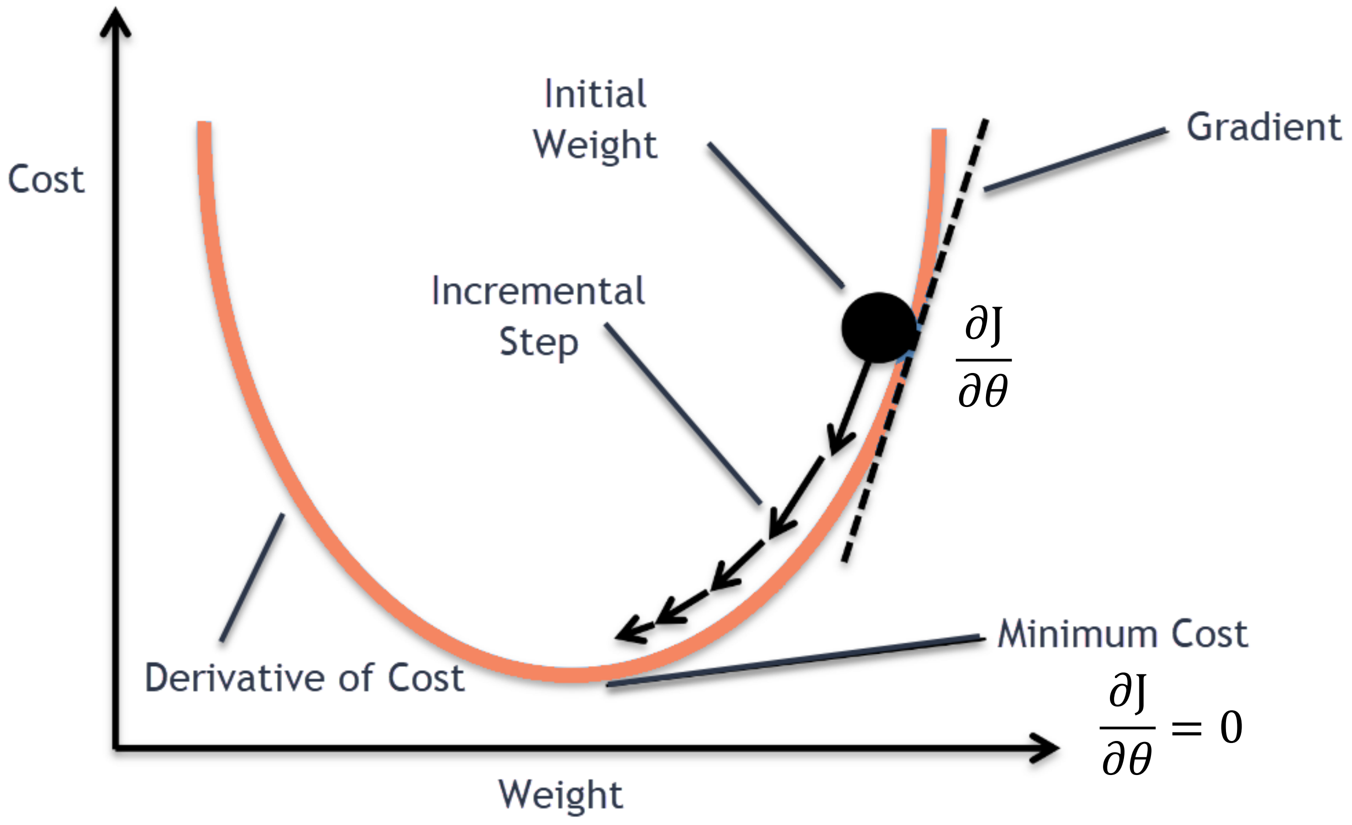
Have some function $J(\theta_0, \theta_1)$

Want $\min_{\theta_0, \theta_1} J(\theta_0, \theta_1)$

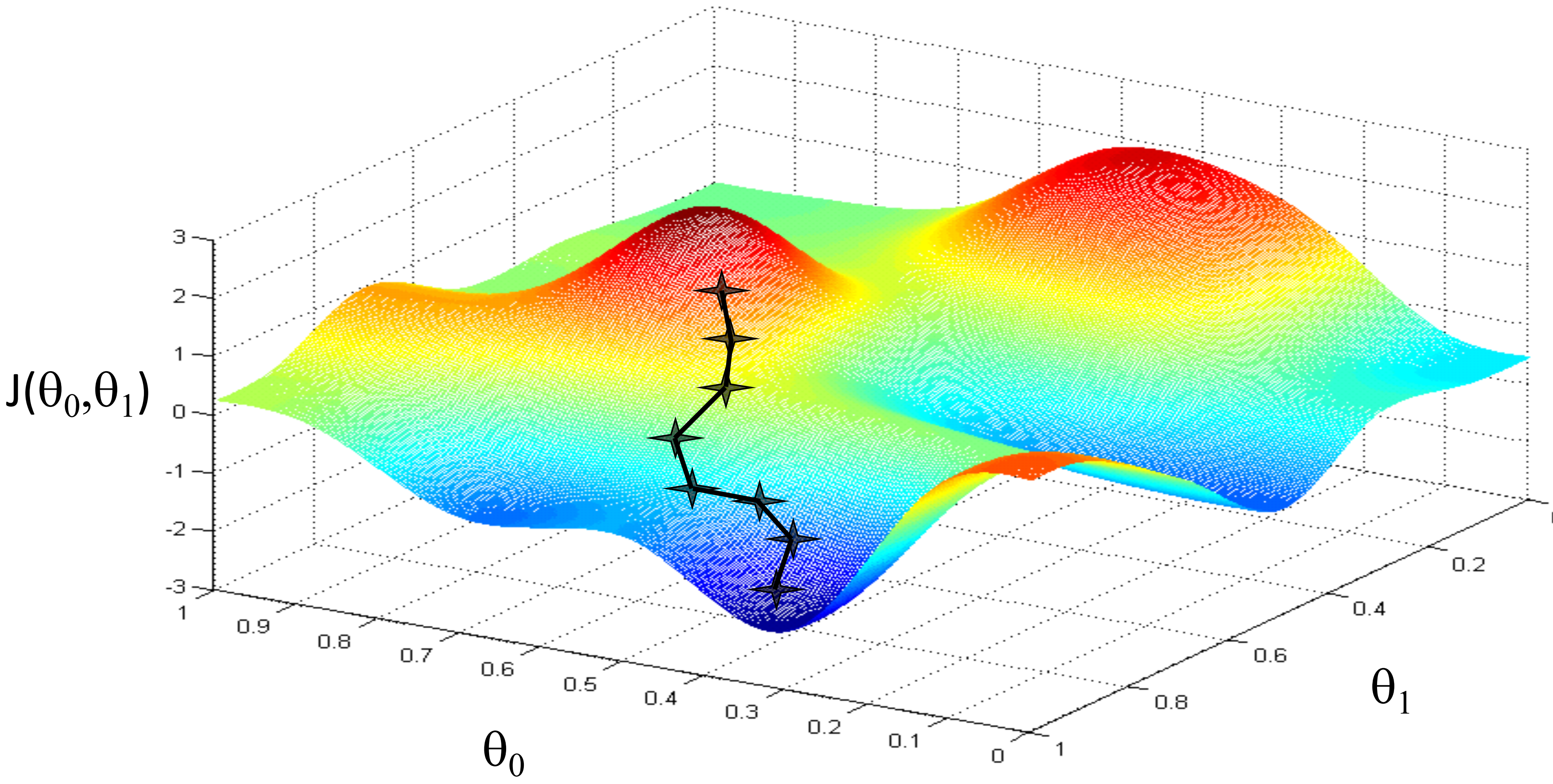
Outline:

- Start with some θ_0, θ_1
- Keep changing θ_0, θ_1 to reduce $J(\theta_0, \theta_1)$ until we hopefully end up at a minimum

Gradient descent



Gradient descent



Gradient descent

$$\text{repeat until convergence } \{$$
$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \qquad (\text{for } j = 0 \text{ and } j = 1)$$
$$\}$$

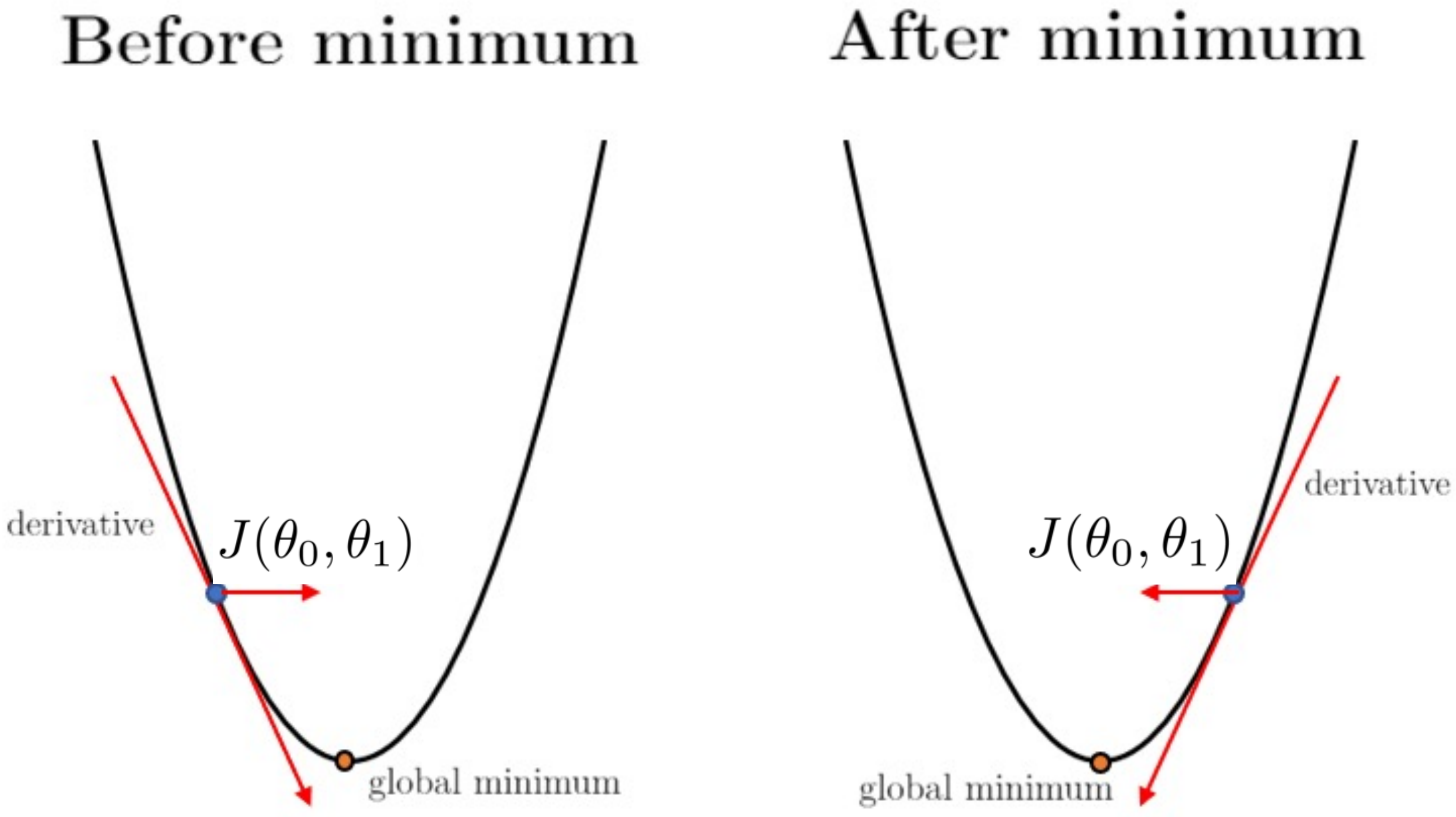
Correct: Simultaneous update

$$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$
$$\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$
$$\theta_0 := \text{temp0}$$
$$\theta_1 := \text{temp1}$$

Incorrect:

$$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$
$$\theta_0 := \text{temp0}$$
$$\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$
$$\theta_1 := \text{temp1}$$

Gradient descent

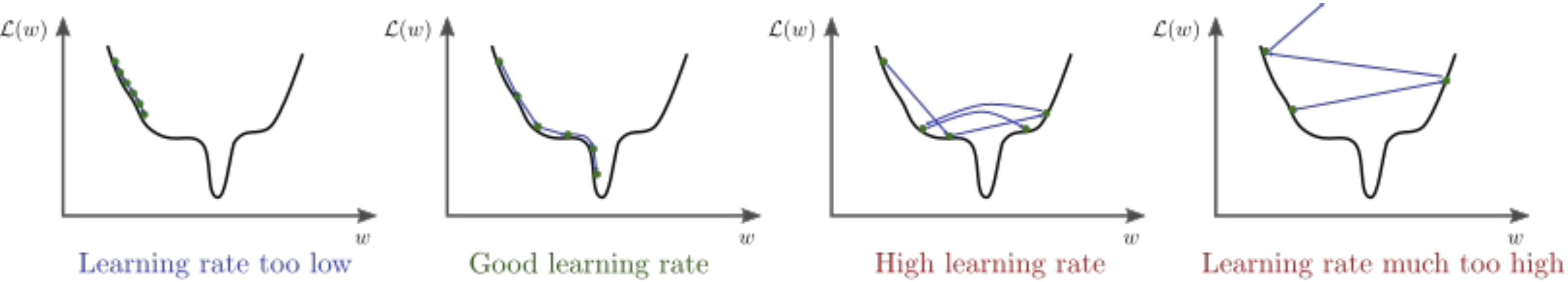


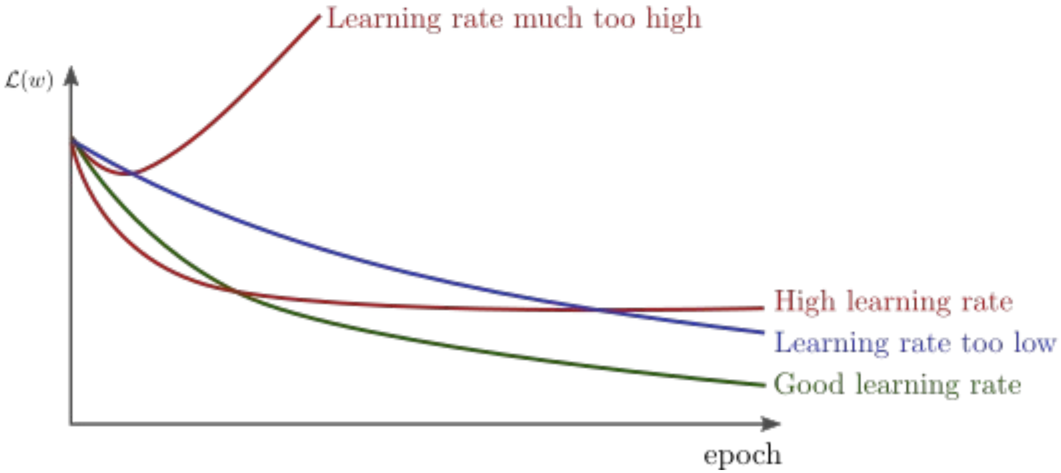
Since the derivative is negative,
if we subtract the derivative from θ
it will increase and go closer the minimum.

Since the derivative is positive,
if we subtract the derivative from θ
it will decrease and go closer the minimum.

Learning rate

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$





- If α is too small, gradient descent can be slow.
- If α is too large, gradient descent can overshoot the minimum. It may fail to converge, or even diverge.

Gradient descent on linear regression

Gradient descent algorithm

$$
\begin{aligned}
&\text{repeat until convergence } \{ \\
&\qquad \theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \\
&\qquad\qquad (\text{for } j = 1 \text{ and } j = 0) \\
&\}
\end{aligned}
$$

Linear Regression Model

$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

Gradient descent on linear regression

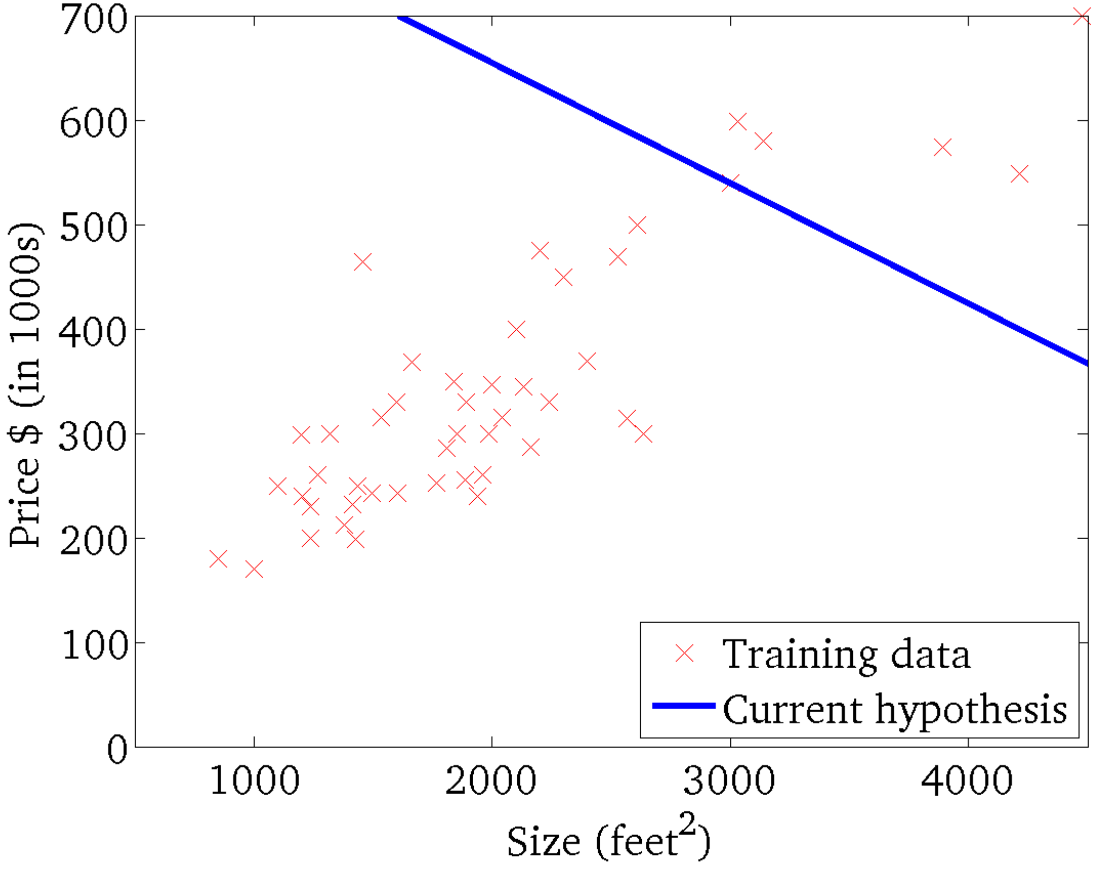
$$
\begin{aligned}
&\text{repeat until convergence } \{ \\
&\quad \left.\begin{aligned}
\theta_0 &:= \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) \\
\theta_1 &:= \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) \cdot x^{(i)}
\end{aligned}\right\} \\
&\}
\end{aligned}
$$

update θ_0 and θ_1 simultaneously

Gradient descent on linear regression

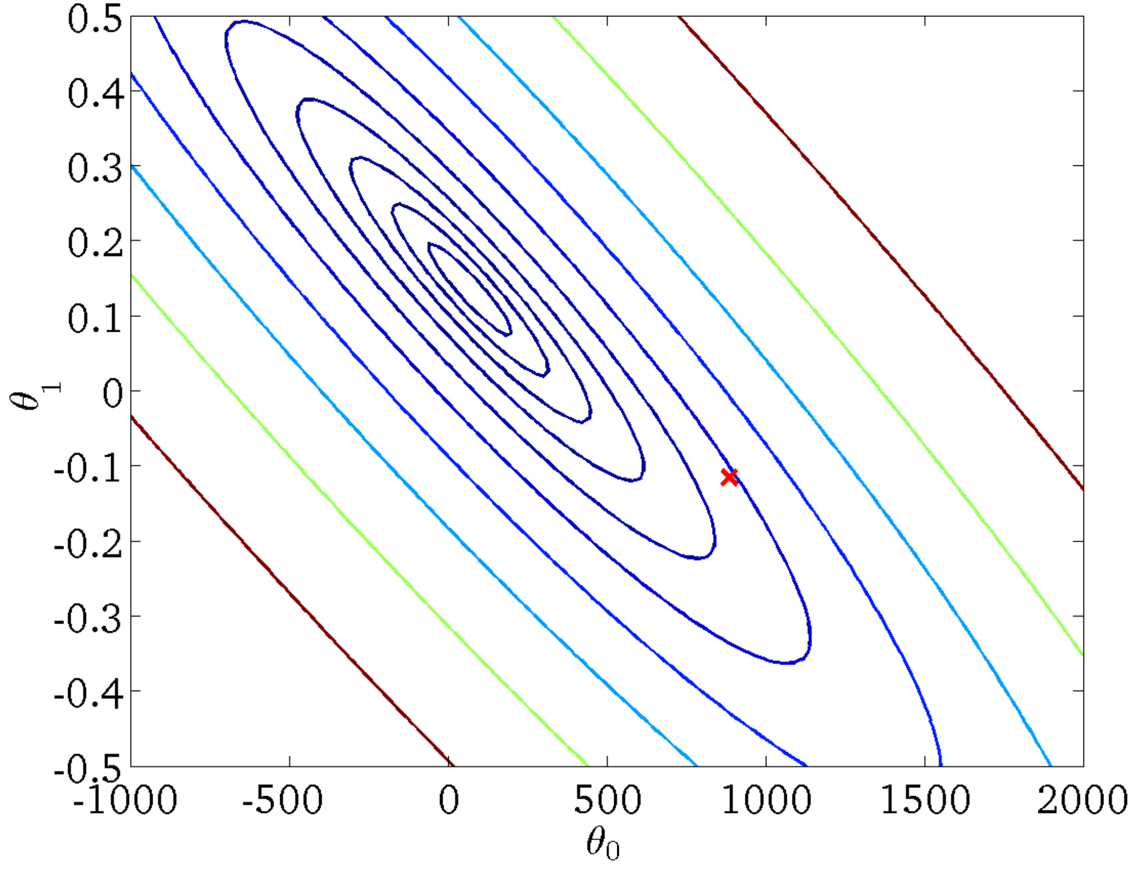
$$h_\theta(x)$$

(for fixed θ_0, θ_1, this is a function of x)



$$J(\theta_0, \theta_1)$$

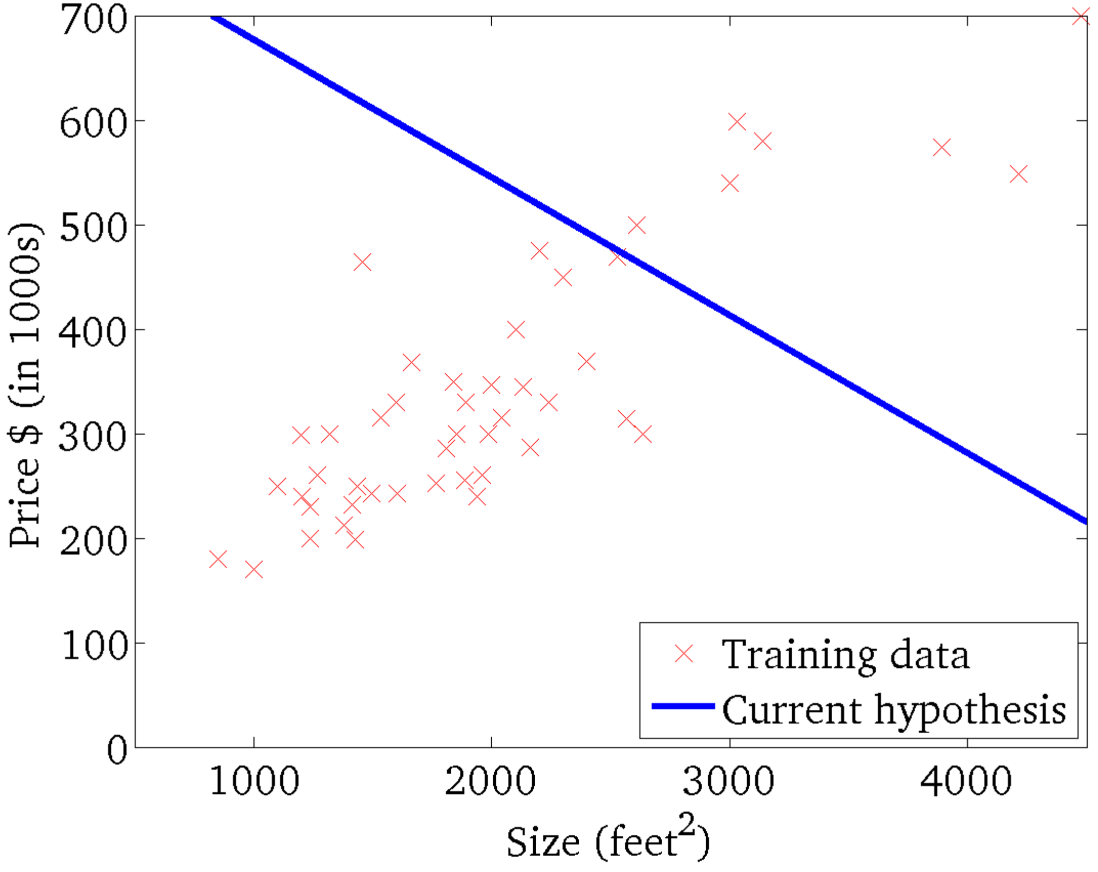
(function of the parameters θ_0, θ_1)



Gradient descent on linear regression

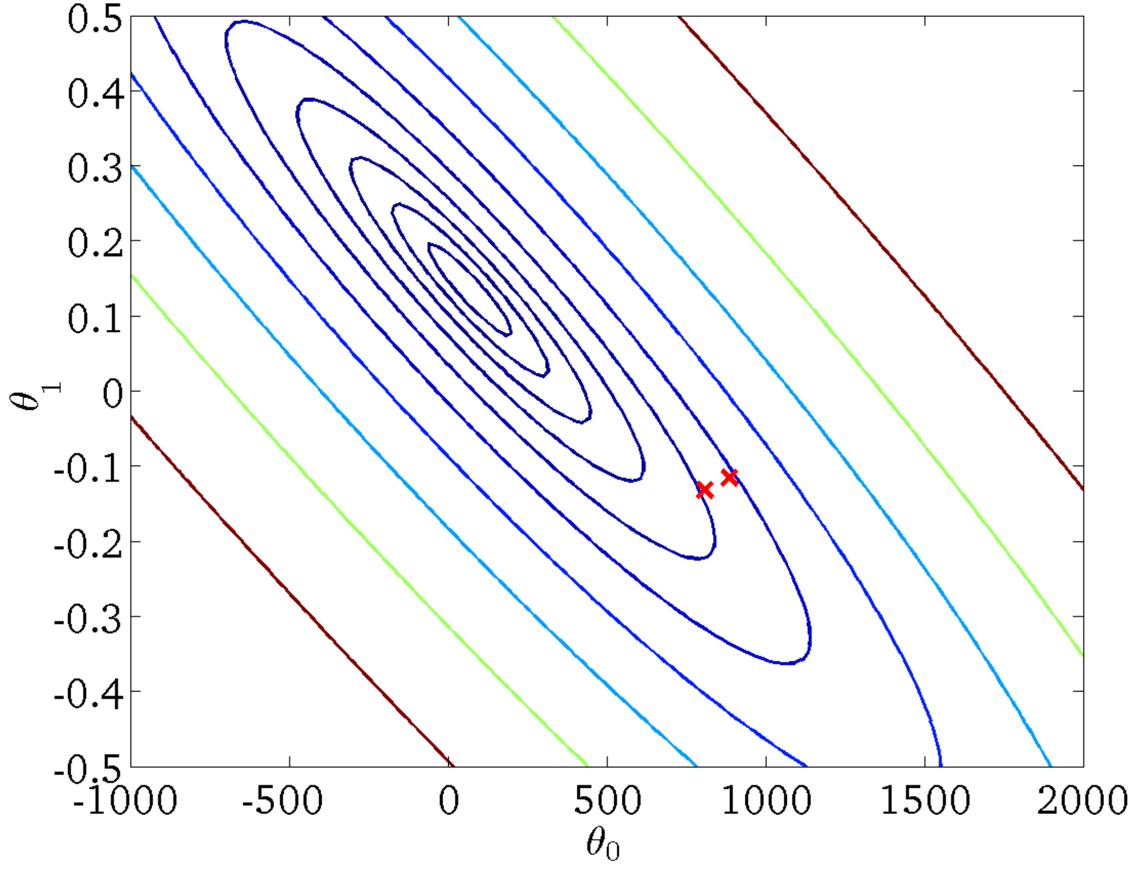
$$h_\theta(x)$$

(for fixed θ_0, θ_1, this is a function of x)



$$J(\theta_0, \theta_1)$$

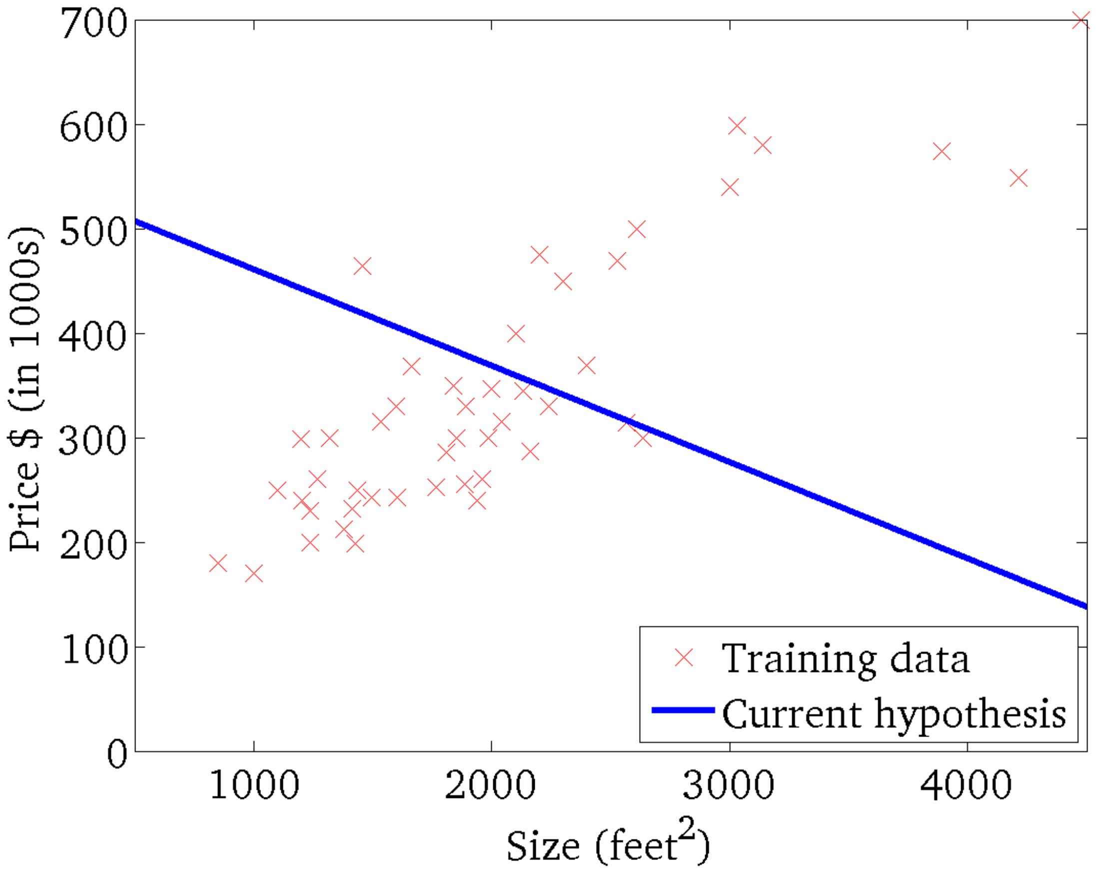
(function of the parameters θ_0, θ_1)



Gradient descent on linear regression

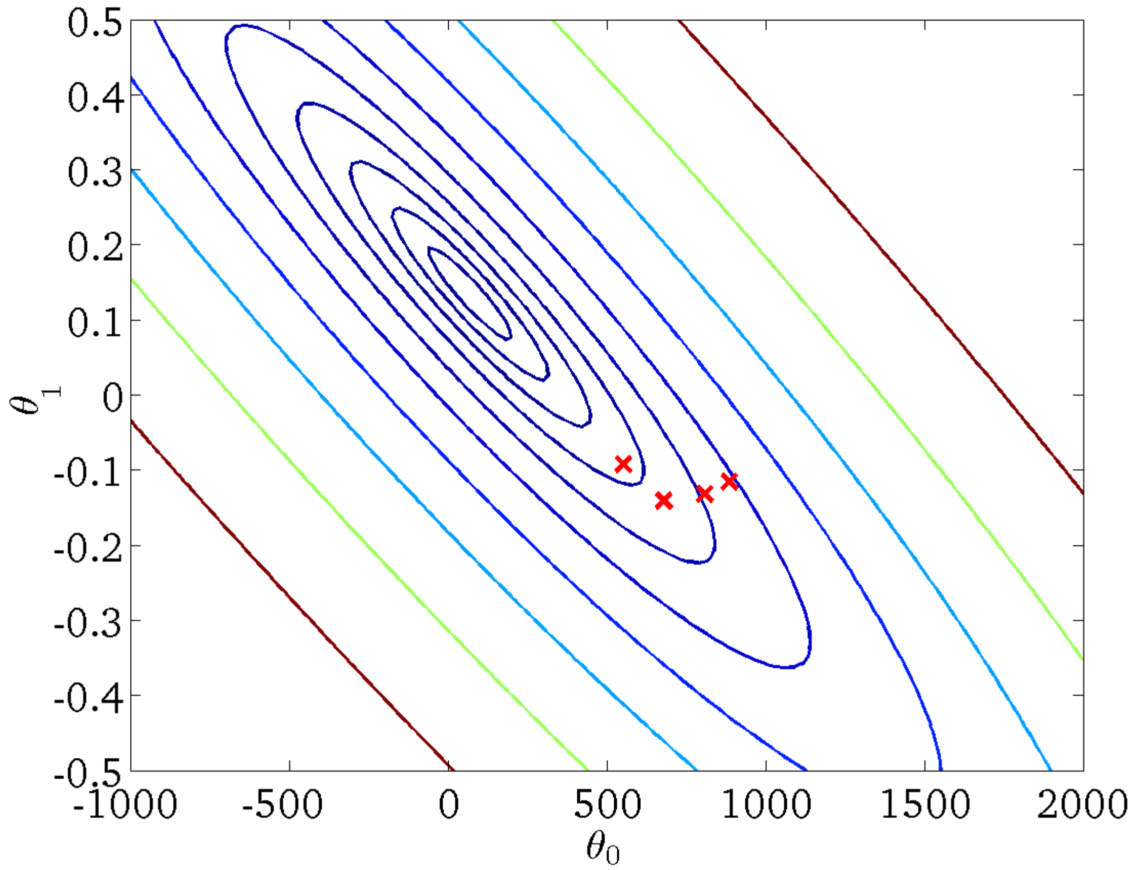
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

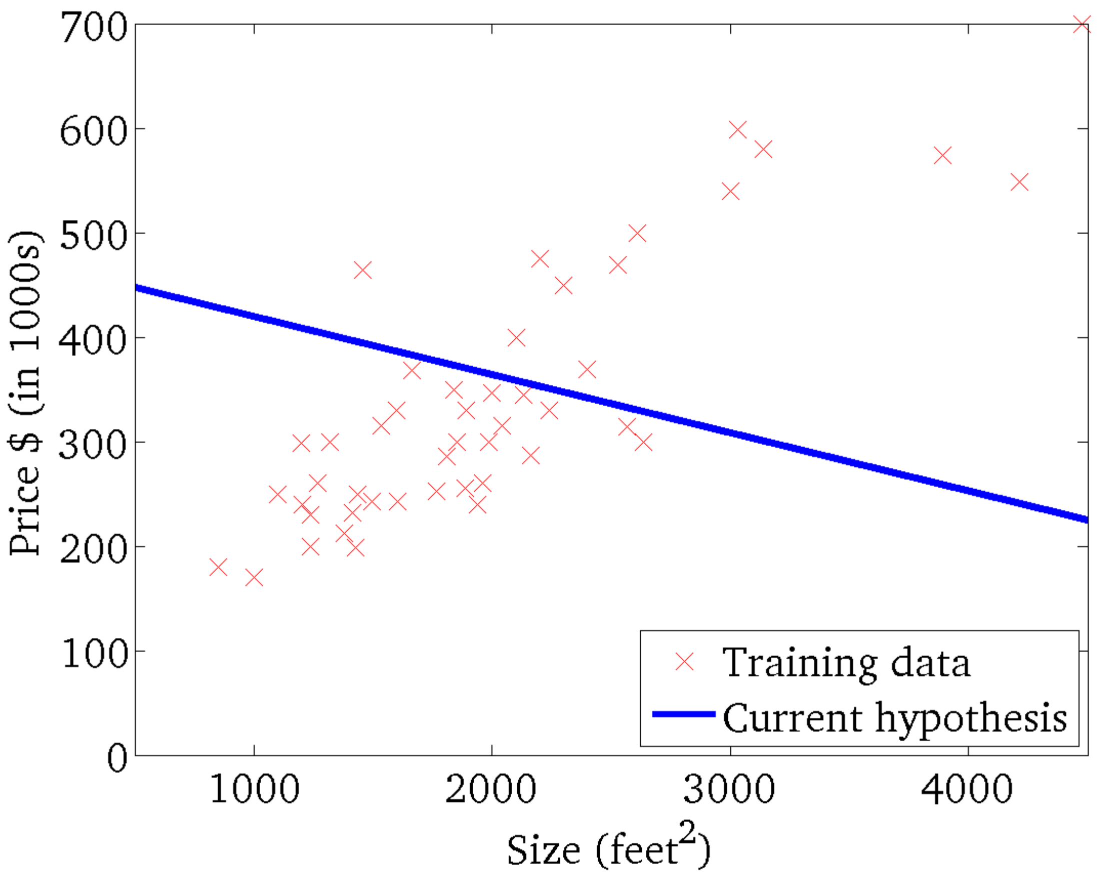
(function of the parameters θ_0, θ_1)



Gradient descent on linear regression

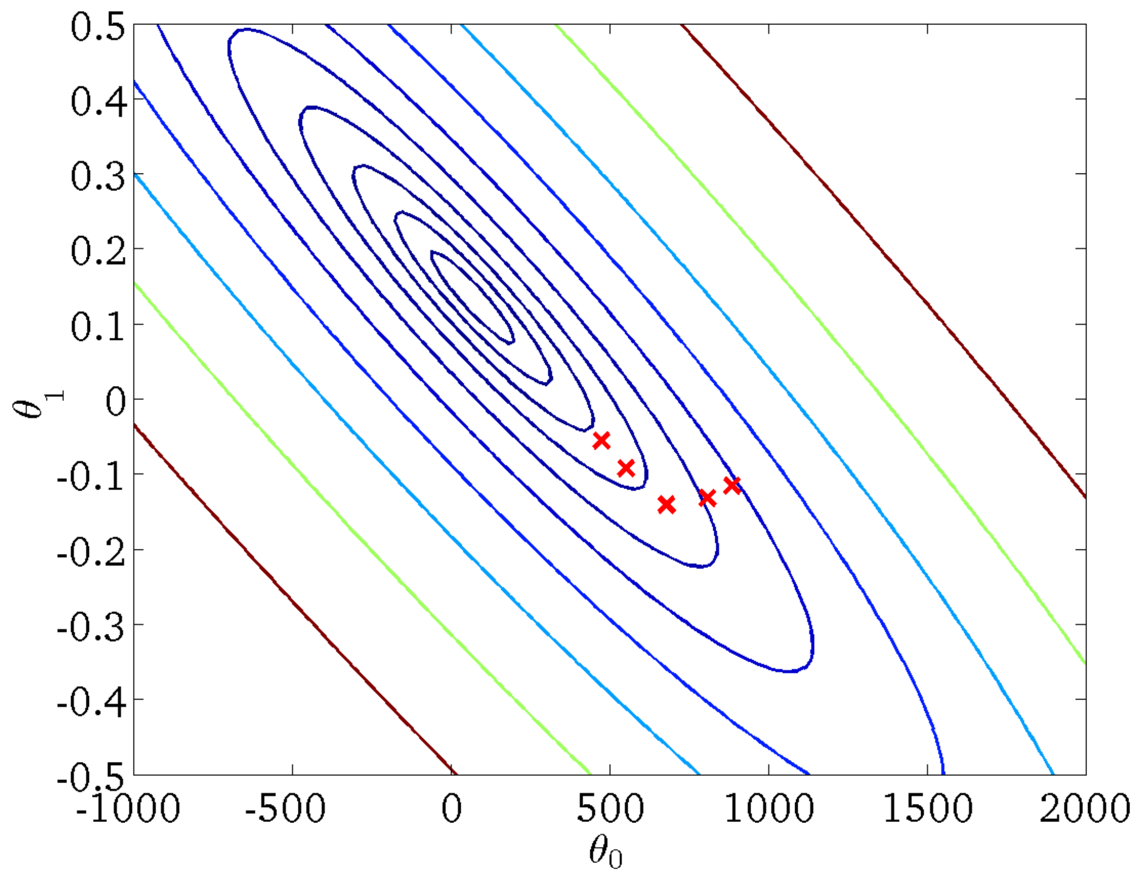
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

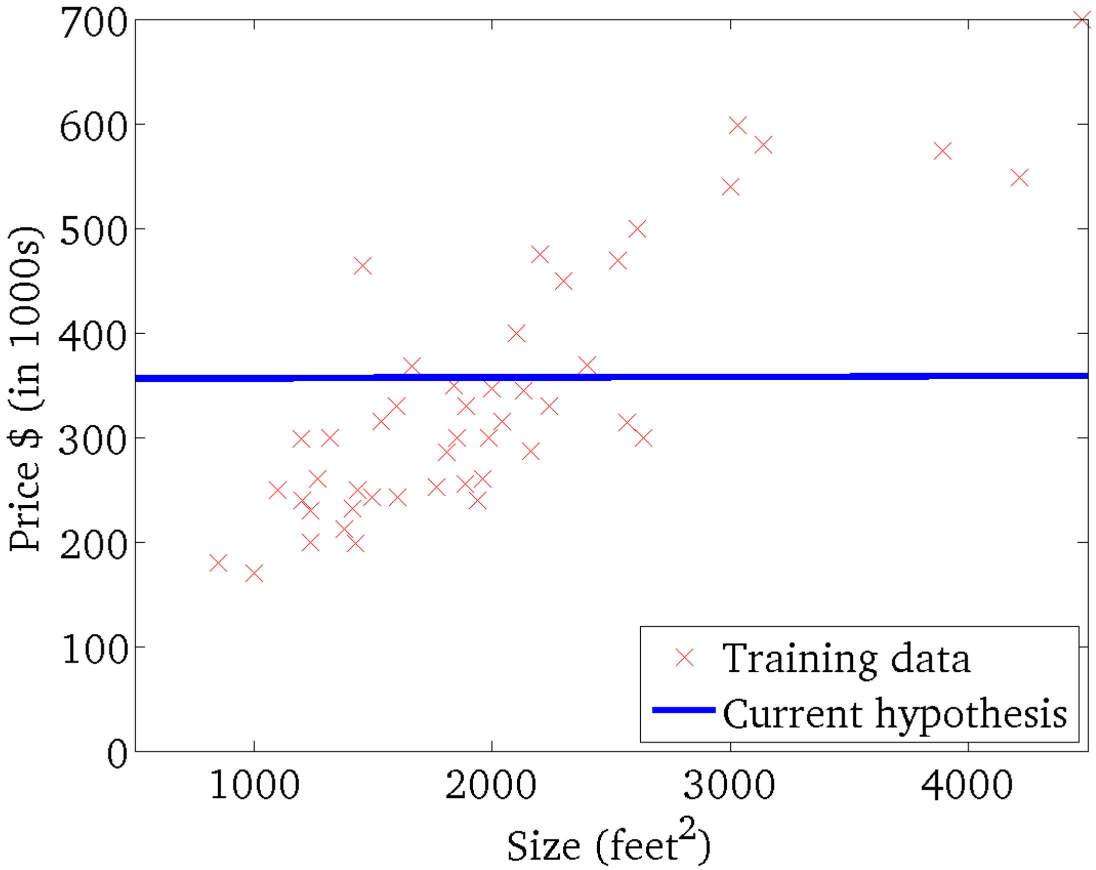
(function of the parameters θ_0, θ_1)



Gradient descent on linear regression

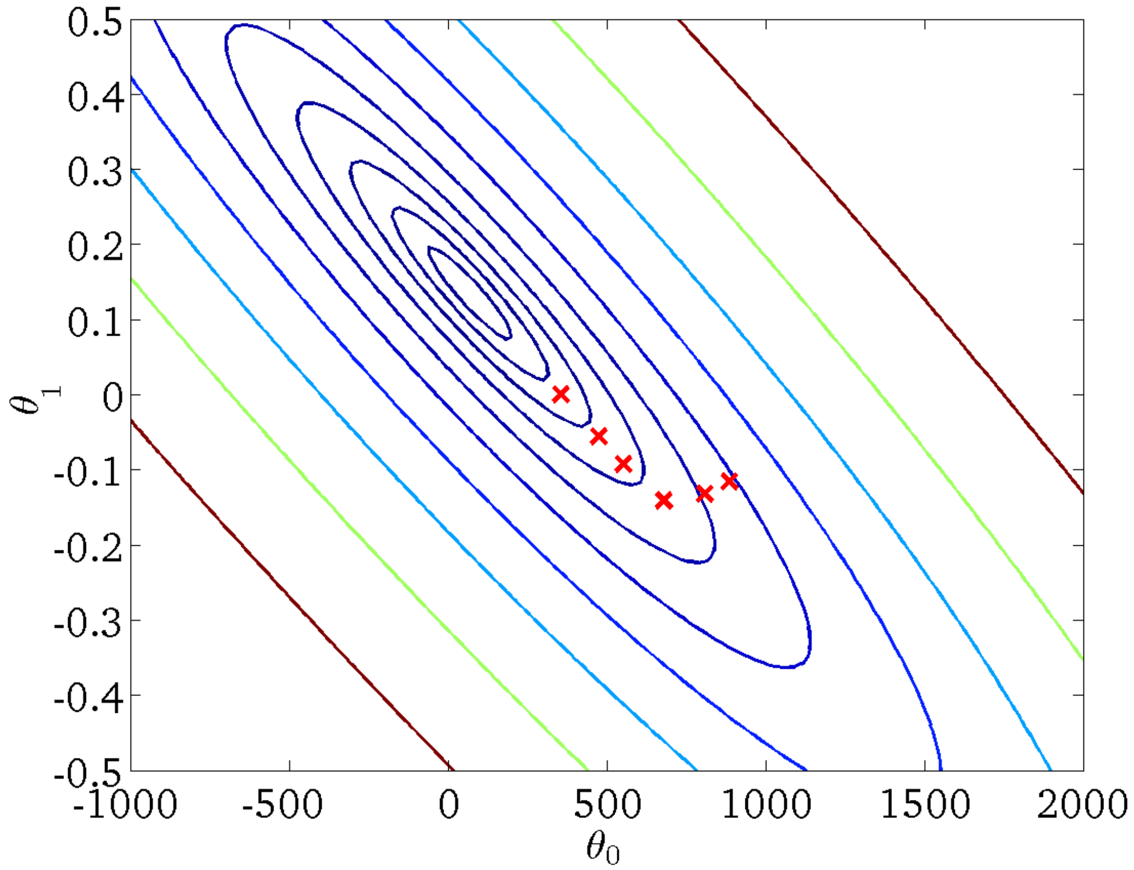
$$h_\theta(x)$$

(for fixed θ_0, θ_1, this is a function of x)



$$J(\theta_0, \theta_1)$$

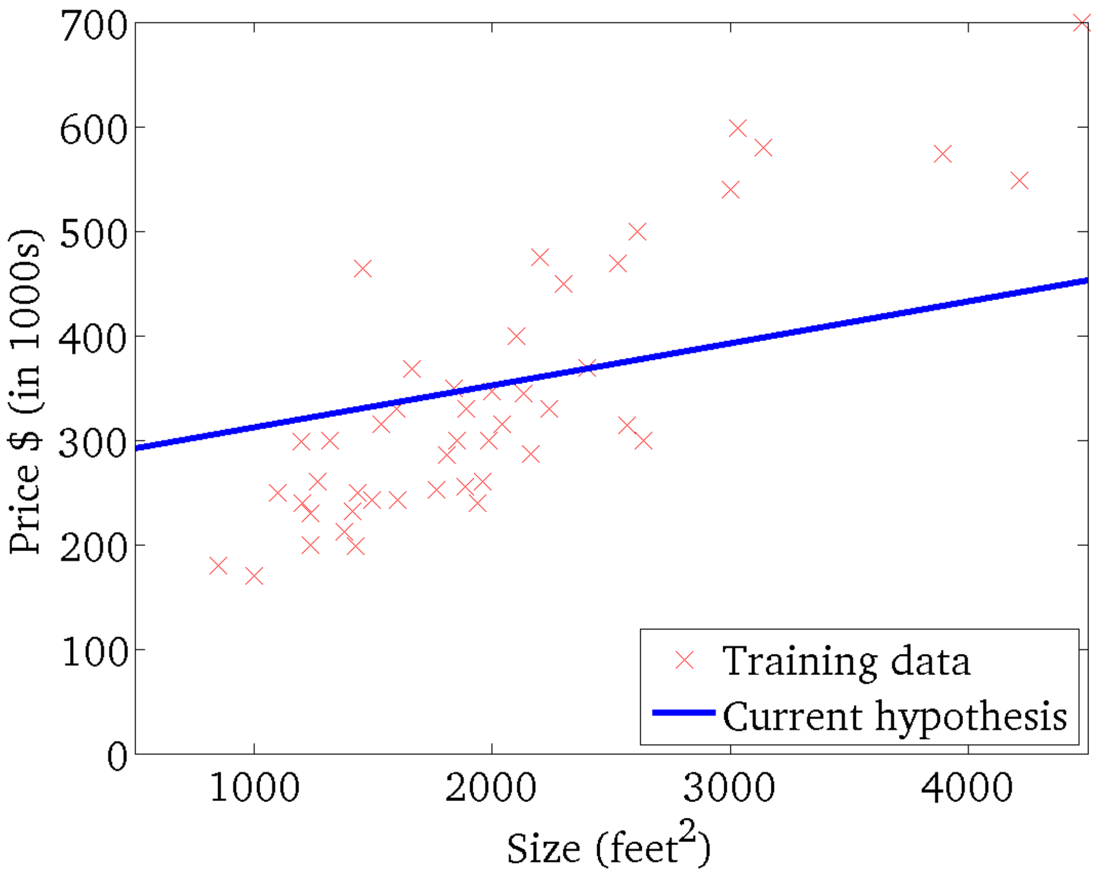
(function of the parameters θ_0, θ_1)



Gradient descent on linear regression

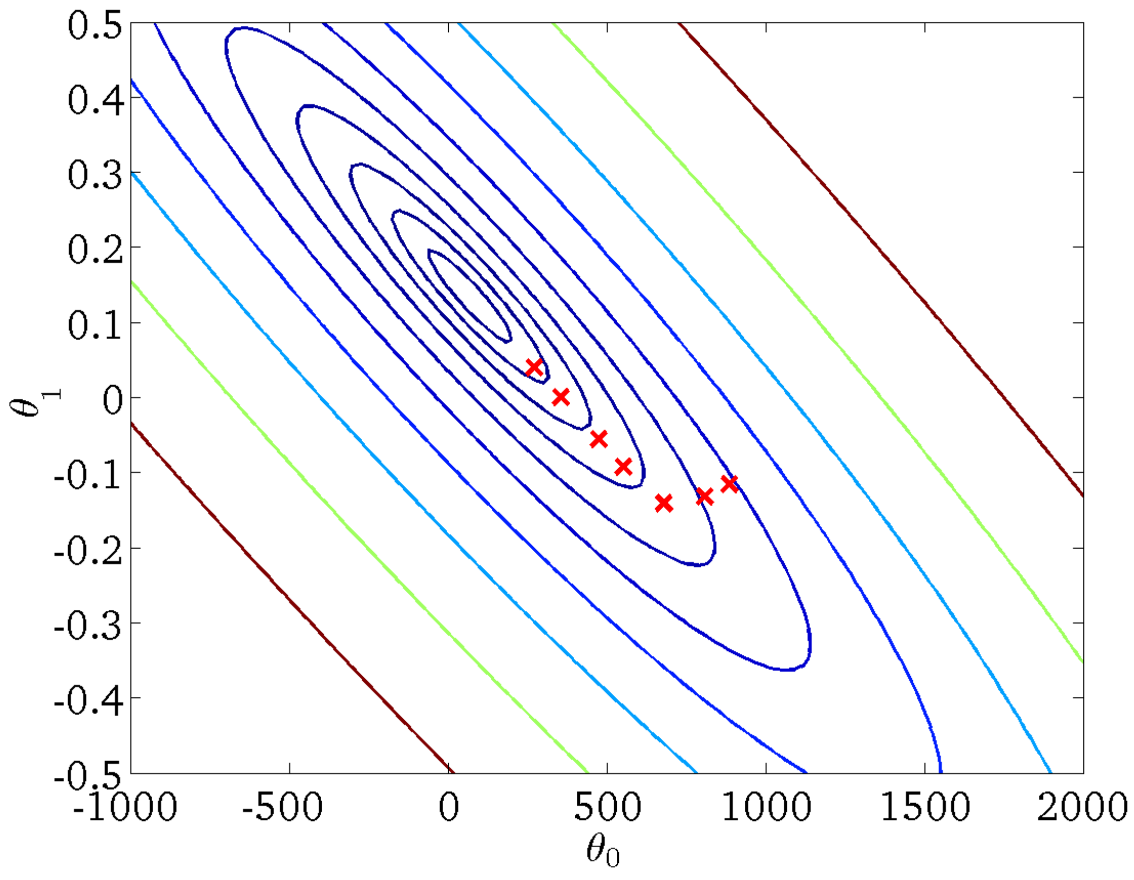
$$h_\theta(x)$$

(for fixed θ_0, θ_1, this is a function of x)



$$J(\theta_0, \theta_1)$$

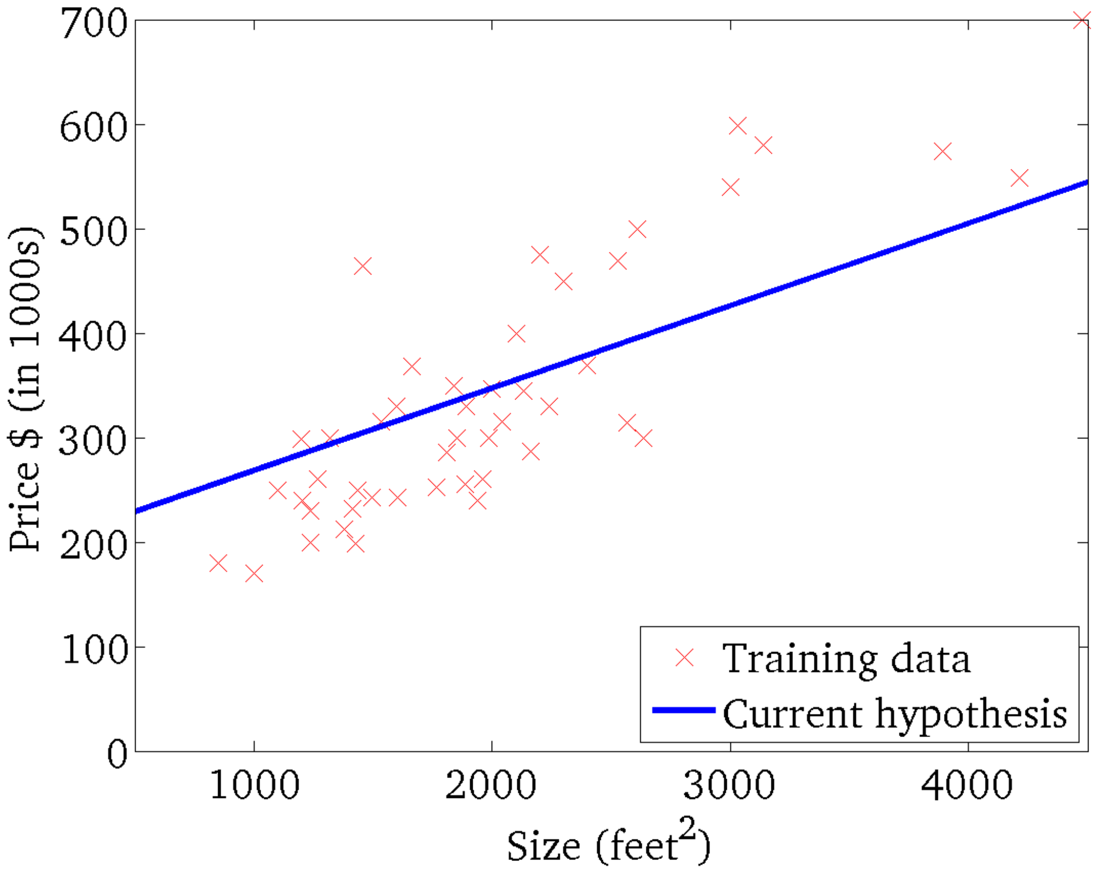
(function of the parameters θ_0, θ_1)



Gradient descent on linear regression

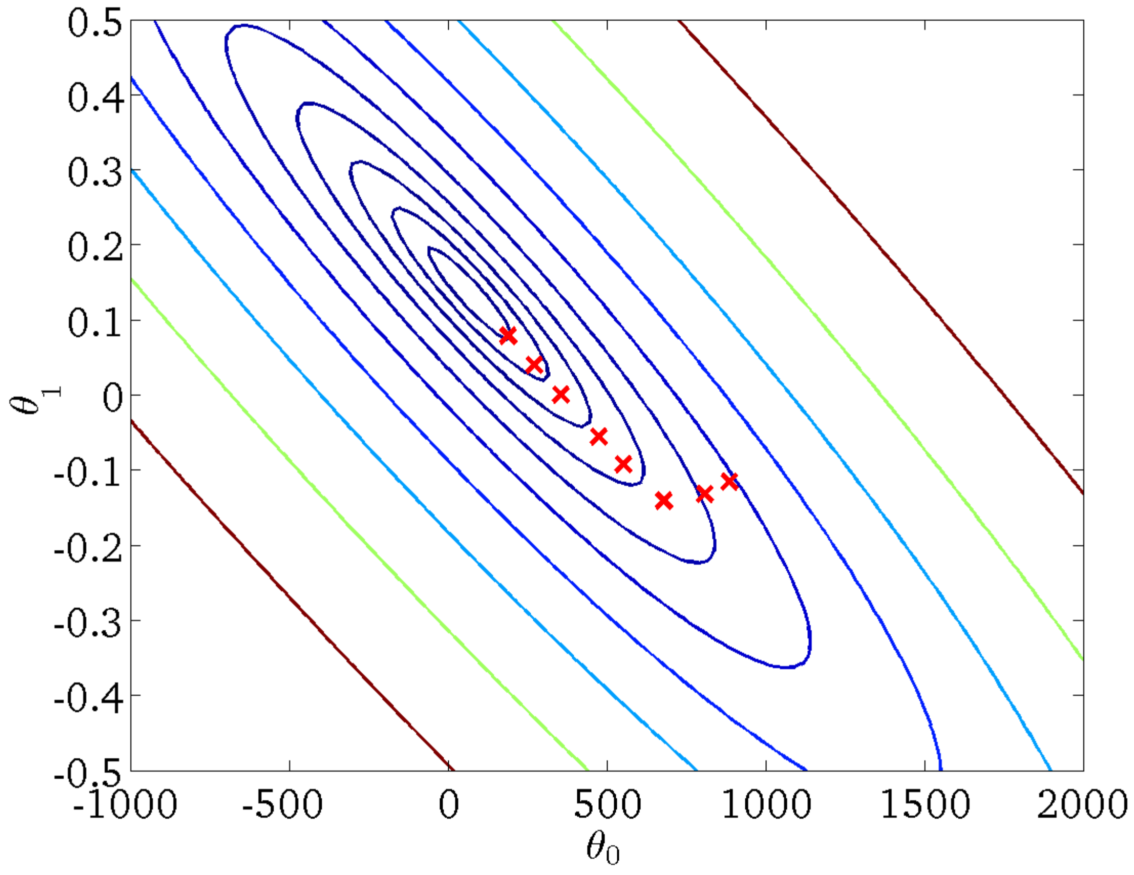
$$h_\theta(x)$$

(for fixed θ_0, θ_1, this is a function of x)



$$J(\theta_0, \theta_1)$$

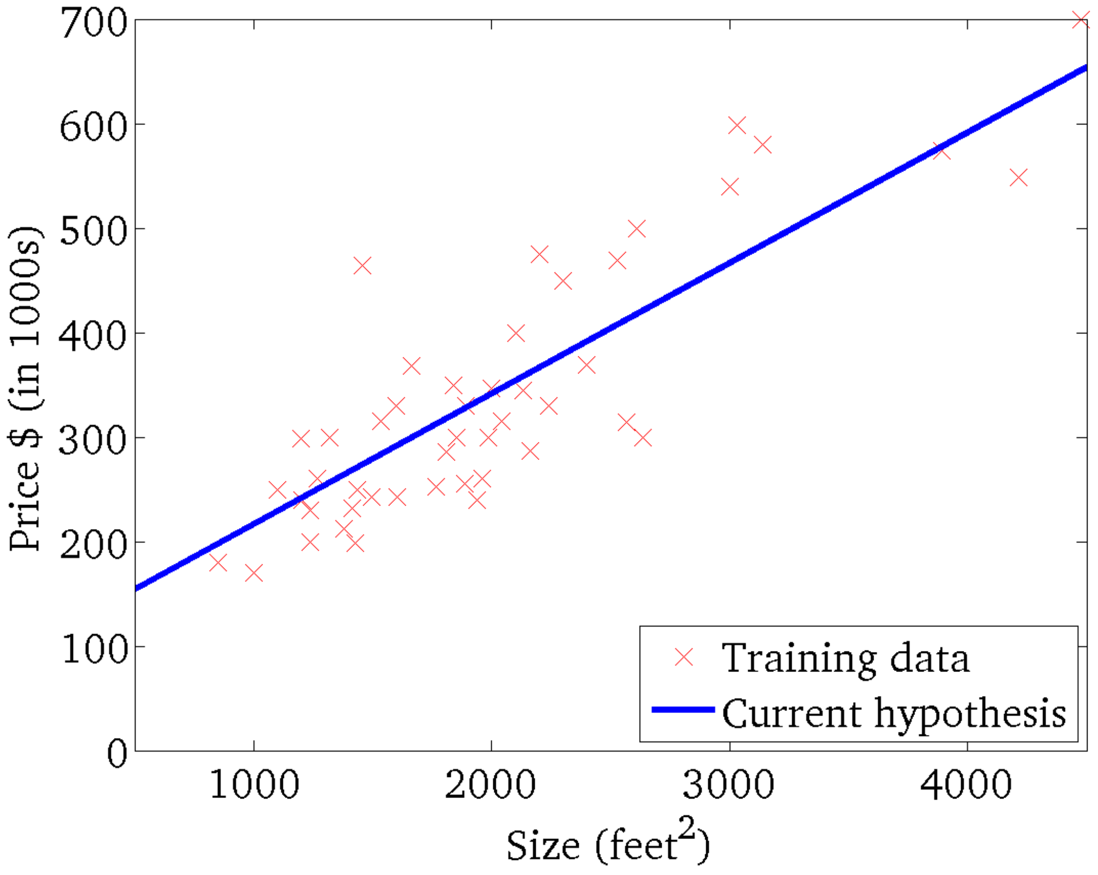
(function of the parameters θ_0, θ_1)



Gradient descent on linear regression

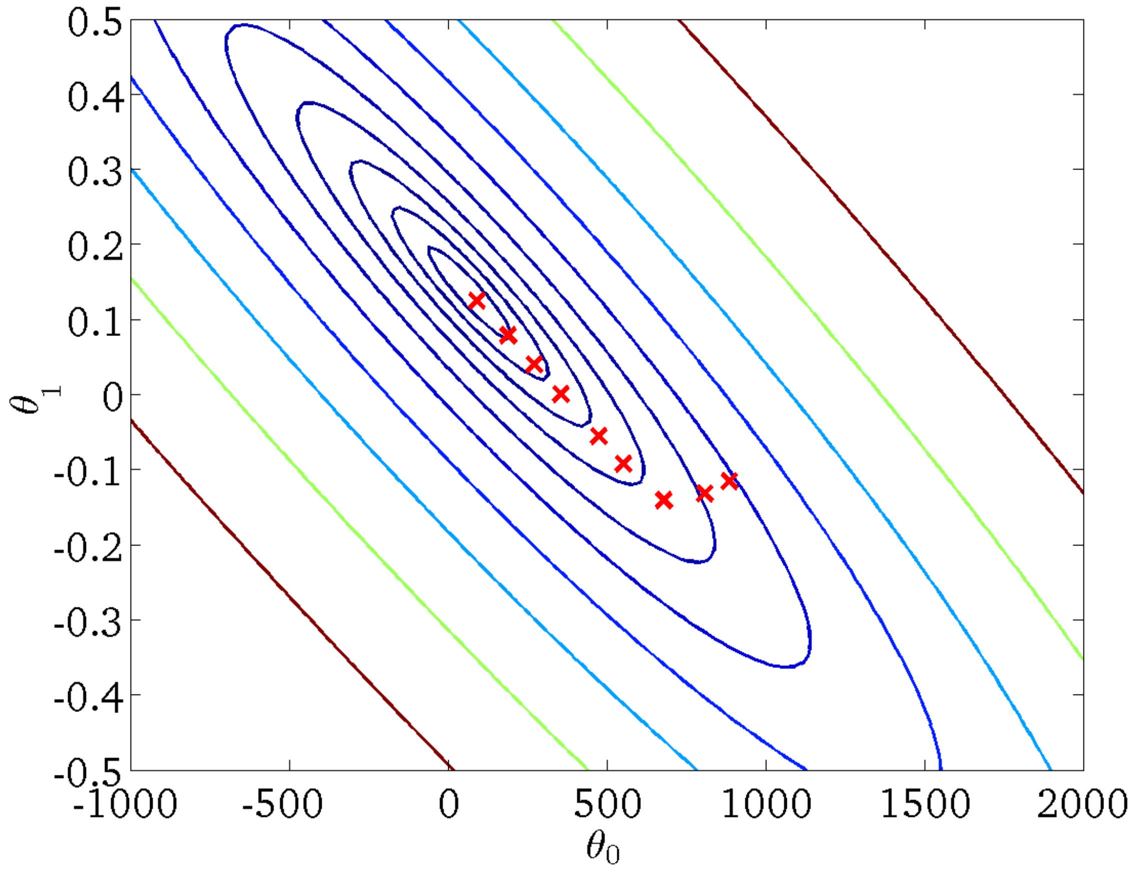
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



Derive the Solution for Linear Regression on One-Dimensional Data

$$\theta_1 = \frac{\sum(x^{(i)} - \bar{x})(y^{(i)} - \bar{y})}{\sum(x^{(i)} - \bar{x})^2}$$

$$\theta_0 = \bar{y}\text{-}\theta_1 * \bar{x}$$

Derive the Solution for Linear Regression on One-Dimensional Data (optional)

- θ_1, θ_0 are estimated by minimizing the cost function.

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

- The optimal θ_1, θ_0 must satisfy

$$\frac{\partial J}{\partial \theta_1} = 0 \quad \Rightarrow \frac{1}{m} \sum_{i=1}^{m} \left((\theta_1 x^{(i)} + \theta_0) - y^{(i)} \right) x^{(i)} = 0 \Longrightarrow \sum_{i=1}^{m} y^{(i)} x^{(i)} - \theta_1 \sum_{i=1}^{m} (x^{(i)})^2 - \theta_0 \sum_{i=1}^{m} x^{(i)} = 0$$

$$\frac{\partial J}{\partial \theta_0} = 0 \quad \Rightarrow \frac{1}{m} \sum_{i=1}^{m} \left((\theta_1 x^{(i)} + \theta_0) - y^{(i)} \right) = 0 \Longrightarrow \sum_{i=1}^{m} y^{(i)} - \theta_1 \sum_{i=1}^{m} x^{(i)} - \sum_{i=1}^{m} \theta_0 = 0 \Longrightarrow \boxed{\theta_0 = \bar{y} - \theta_1 \bar{x}}$$

$$\sum_{i=1}^{m} y^{(i)} x^{(i)} - \theta_1 \sum_{i=1}^{m} (x^{(i)})^2 - (\bar{y} - \theta_1 \bar{x}) \sum_{i=1}^{m} x^{(i)} = 0 \Longrightarrow \theta_1 = \frac{\bar{y} \sum_{i=1}^{m} x^{(i)} - \sum_{i=1}^{m} y^{(i)} x^{(i)}}{\bar{x} \sum_{i=1}^{m} x^{(i)} - \sum_{i=1}^{m} (x^{(i)})^2} = \frac{\sum_{i=1}^{m} (x^{(i)} - \bar{x})(y^{(i)} - \bar{y})}{\sum_{i=1}^{m} (x^{(i)} - \bar{x})^2}$$

Example

Student	Test score	IQ	Study hours
1	100	110	40
2	90	120	30
3	80	100	20
4	70	90	0
5	60	80	10

develop a regression equation to predict test scores (y), based on students' IQs (x).

$$y = \theta_0 + \theta_1 \mathrm{x}$$

Solution

$$\theta_1 = \frac{\sum(x^{(i)} - \bar{x})(y^{(i)} - \bar{y})}{\sum(x^{(i)} - \bar{x})^2}$$

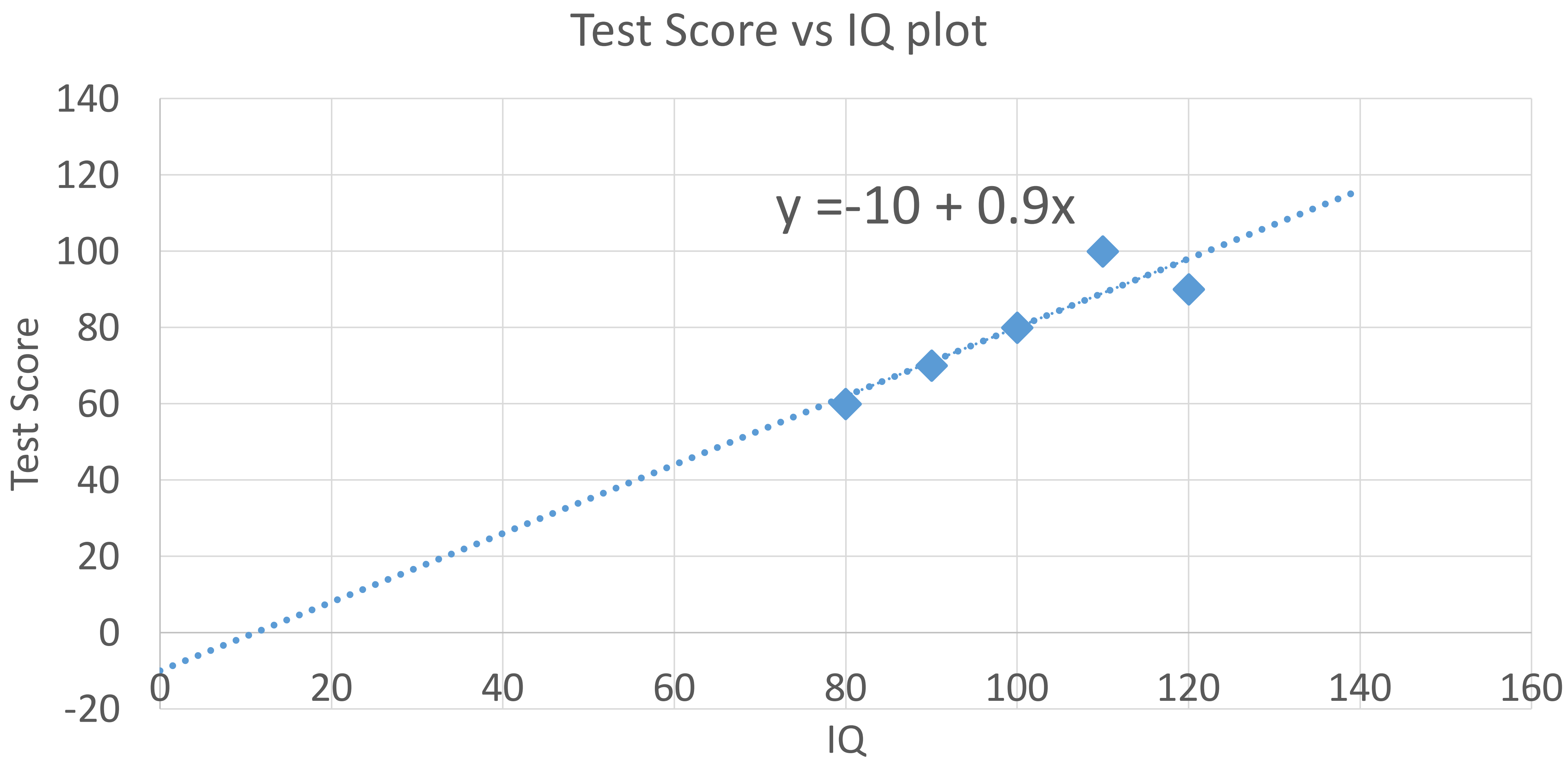
$$\bar{x} = 100, \bar{y} = 80$$

$$\theta_1 = \frac{(110-100)(100-80) + (120-100)(90-80) + (100-100)(80-80) + (70-80)(90-100) + (60-80)(80-100)}{(110-100)^2 + (120-100)^2 + (100-100)^2 + (90-100)^2 + (80-100)^2}$$

$$= \frac{200+200+100+400}{100+400+100+400} = \frac{900}{1000} = 0.9$$

$\theta_0 =$ 80-0.9*100=-10

$$y = -10 + 0.9\text{x}$$


Test Score vs IQ plot
y =-10 + 0.9x
Test Score
IQ
140
120
100
80
60
40
20
0
-20
0
20
40
60
80
100
120
140
160