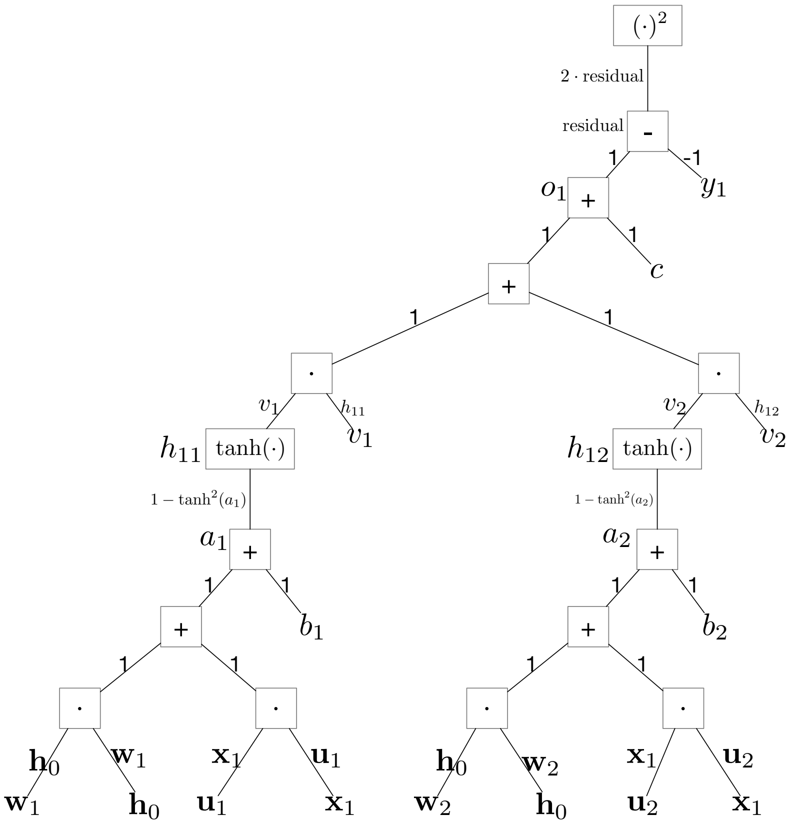
COMP7015 Artificial Intelligence **Semester 1, 2022/23**

Sample Solution to Written Assignment 2

Instructor: Dr. Kejing Yin Dec. 1, 2022

Problem 1: Formulating a Search Problem

Q1: The computational graph is as follows.



where $\mathbf{w}_n$ and $\mathbf{u}_n$ are the n-th rows of matrices, $\mathbf{W}$ and $\mathbf{U}$, respectively.

Q2: The model parameters to be learned are $\mathbf{b}$, $\mathbf{W}$, $\mathbf{U}$, $\mathbf{v}$, and c.

Q3:

(1) The forward pass:

$$\begin{aligned}
h_{11} &= \tanh\left(\mathbf{w}_1^\top \mathbf{h}_0 + \mathbf{u}_1^\top \mathbf{x}_1 + b_1\right) = \tanh(1.7) = 0.9354 \\
h_{12} &= \tanh\left(\mathbf{w}_2^\top \mathbf{h}_0 + \mathbf{u}_2^\top \mathbf{x}_1 + b_2\right) = \tanh(-0.2) = -0.1974 \\
o_1 &= (h_{11}v_1 + h_{12}v_2) + c = 1.0098 \\
\text{TrainLoss} &= (o_1 - y_1)^2 = 0.2403
\end{aligned}$$

(2) The backward pass:

$$
\begin{aligned}
\frac{\partial\,\text{TrainLoss}}{\partial c} =& 2\cdot(o_1-y_1)\cdot 1\cdot 1 = -0.9804\\
\frac{\partial\,\text{TrainLoss}}{\partial v_1} =& 2\cdot(o_1-y_1)\cdot 1\cdot 1\cdot 1\cdot h_{11} = -0.9171\\
\frac{\partial\,\text{TrainLoss}}{\partial v_2} =& 2\cdot(o_1-y_1)\cdot 1\cdot 1\cdot 1\cdot h_{12} = 0.1935\\
\frac{\partial\,\text{TrainLoss}}{\partial b_1} =& 2\cdot(o_1-y_1)\cdot 1\cdot 1\cdot 1\cdot v_1\cdot(1-\tanh^2(a_1))\cdot 1 = -0.0613\\
\frac{\partial\,\text{TrainLoss}}{\partial b_2} =& 2\cdot(o_1-y_1)\cdot 1\cdot 1\cdot 1\cdot v_2\cdot(1-\tanh^2(a_2))\cdot 1 = -0.7538\\
\frac{\partial\,\text{TrainLoss}}{\partial \mathbf{w}_1} =& 2\cdot(o_1-y_1)\cdot 1\cdot 1\cdot 1\cdot v_1\cdot(1-\tanh^2(a_1))\cdot 1\cdot 1\cdot \mathbf{h}_0 = \mathbf{0}\\
\frac{\partial\,\text{TrainLoss}}{\partial \mathbf{w}_2} =& 2\cdot(o_1-y_1)\cdot 1\cdot 1\cdot 1\cdot v_2\cdot(1-\tanh^2(a_2))\cdot 1\cdot 1\cdot \mathbf{h}_0 = \mathbf{0}\\
\frac{\partial\,\text{TrainLoss}}{\partial \mathbf{u}_1} =& 2\cdot(o_1-y_1)\cdot 1\cdot 1\cdot 1\cdot v_1\cdot(1-\tanh^2(a_1))\cdot 1\cdot 1\cdot \mathbf{x}_1 = [-0.0613, 0.0613, -0.0613]\\
\frac{\partial\,\text{TrainLoss}}{\partial \mathbf{u}_2} =& 2\cdot(o_1-y_1)\cdot 1\cdot 1\cdot 1\cdot v_2\cdot(1-\tanh^2(a_2))\cdot 1\cdot 1\cdot \mathbf{x}_1 = [-0.7538, 0.7538, -0.7538]
\end{aligned}
$$

Therefore, the gradient with respect to the model parameters are:

$$
\begin{aligned}
\nabla_{\mathbf{b}}\,\text{TrainLoss} =& [-0.0613, -0.7538]^\top\\
\nabla_{\mathbf{W}}\,\text{TrainLoss} =& \begin{bmatrix} 0 & 0\\ 0 & 0\end{bmatrix}\\
\nabla_{\mathbf{U}}\,\text{TrainLoss} =& \begin{bmatrix} -0.0613 & 0.0613 & -0.0613\\ -0.7538 & 0.7538 & -0.7538\end{bmatrix}\\
\nabla_{\mathbf{v}}\,\text{TrainLoss} =& [-0.9171, 0.1935]^\top\\
\nabla_{c}\,\text{TrainLoss} =& -0.9804
\end{aligned}
$$

Problem 2: Naïve Bayes Classifier

(1) The prior probabilities are given by: $p(\text{ripe=yes}) = 8/17 = 0.47$ and $p(\text{ripe=no}) = 9/17 = 0.53$.
(2) The conditional probability of each feature is given by:

$$
\begin{aligned}
p(\text{color=green}|\text{ripe=yes}) &= 3/8 = 0.375,\\
p(\text{color=green}|\text{ripe=no}) &= 3/9 = 0.333,\\
p(\text{root=slightly curly}|\text{ripe=yes}) &= 3/8 = 0.375,\\
p(\text{root=slightly curly}|\text{ripe=no}) &= 4/9 = 0.444,\\
p(\text{texture=clear}|\text{ripe=yes}) &= 7/8 = 0.875,\\
p(\text{texture=clear}|\text{ripe=no}) &= 2/9 = 0.222,\\
p(\text{surface=hard}|\text{ripe=yes}) &= 6/8 = 0.75,\\
p(\text{surface=hard}|\text{ripe=no}) &= 6/9 = 0.67,
\end{aligned}
$$

(3) Hence, for the person with sore throat but no fever, we have

$$p(\text{ripe=yes})p(\text{c=green}|\text{ripe=yes})p(\text{r=slightly curly}|\text{ripe=yes})p(\text{t=clear}|\text{ripe=yes})p(\text{s=hard}|\text{ripe=yes}) \approx 0.043$$

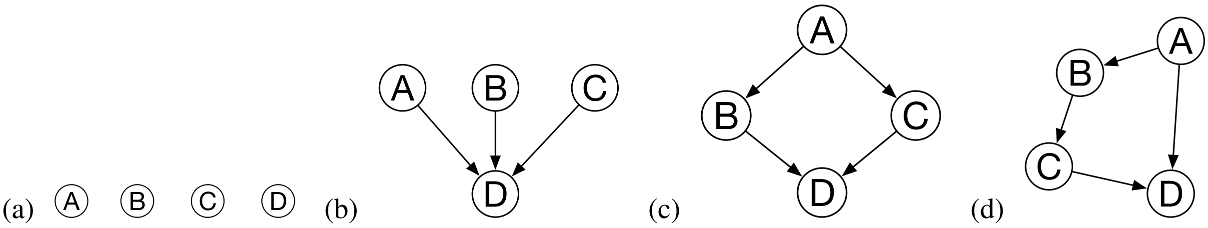
and

$$p(\text{ripe=no})p(\text{c=green}|\text{ripe=no})p(\text{r=slightly curly}|\text{ripe=no})p(\text{t=clear}|\text{ripe=no})p(\text{s=hard}|\text{ripe=no}) \approx 0.017$$

Since $0.043 > 0.017$, the naïve Bayes classifier will classify the new sample as ripe=yes.

Problem 3: Bayesian Networks

Q1: The Bayesian networks are shown as follows.



Q2: The belief propagation is applied as follows.

$$
\begin{aligned}
P(A=1,B=1) =& P(B=1|A=1)P(A=1) = 0.8 \times 0.4 = 0.32\\
P(A=1,B=0) =& P(B=0|A=1)P(A=1) = (1-0.8) \times 0.4 = 0.08
\end{aligned}
$$

$$
\begin{aligned}
P(A=1,B=1,D=1) =& P(D=1|A=1,B=1)P(A=1,B=1)\\
=& P(D=1|B=1)P(A=1,B=1) = 0.224\\
P(A=1,B=0,D=1) =& P(D=1|A=1,B=0)P(A=1,B=0)\\
=& P(D=1|B=0)P(A=1,B=0) = 0.016\\
P(A=1,D=1) =& P(A=1,B=1,D=1) + P(A=1,B=0,D=1) = 0.234\\
P(A=1,D=0) =& P(A=1,B=1,D=0) + P(A=1,B=0,D=0)\\
=& P(D=0|B=1)P(A=1,B=1) + P(D=0|B=0)P(A=1,B=0) = 0.16
\end{aligned}
$$

$$
\begin{aligned}
P(A=1,E=1) =& P(A=1,D=1,E=1) + P(A=1,D=0,E=1)\\
=& P(E=1|D=1)P(A=1,D=1) + P(E=1|D=0)P(A=1,D=0)\\
=& 0.2426
\end{aligned}
$$

Problem 4: Reinforcement Learning

The Q-table is initialized to all zeros as follows.

	MoveSouth	MoveEast	MoveNorth	MoveWest
1	0	0		
2	0			0
3		0	0	
4			0	0

(1) $s = 1$, $a =$MoveSouth, $s' = 3$, $r = 10$.

$$Q(1, \text{MoveSouth}) \leftarrow Q(s,a) + \eta\left(r + \left[\gamma \max_{a'} Q(s',a')\right] - Q(s,a)\right) = 0 + 0.5 \times (10 + 0 - 0) = 5$$

	MoveSouth	MoveEast	MoveNorth	MoveWest
1	5	0		
2	0			0
3		0	0	
4			0	0

(2) $s = 3$, $a =$MoveEast, $s' = 4$, $r = 0$.

$$Q(3, \text{MoveEast}) \leftarrow Q(s,a) + \eta\left(r + \left[\gamma \max_{a'} Q(s',a')\right] - Q(s,a)\right) = 0 + 0.5 \times (0 + 0 - 0) = 0$$

(3) $s = 4$, $a =$MoveNorth, $s' = 2$, $r = 0$.

$$Q(4, \text{MoveNorth}) \leftarrow Q(s,a) + \eta \left(r + \left[\gamma \max_{a'} Q(s',a') \right] - Q(s,a) \right) = 0 + 0.5 \times (0 + 0 - 0) = 0$$

(4) $s = 2$, $a =$MoveWest, $s' = 1$, $r = 0$.

$$Q(2, \text{MoveWest}) \leftarrow Q(s,a) + \eta \left(r + \left[\gamma \max_{a'} Q(s',a') \right] - Q(s,a) \right) = 0 + 0.5 \times (0 + 0.9 \times 5 - 0) = 2.25$$

	MoveSouth	MoveEast	MoveNorth	MoveWest
1	5	0		
2	0			2.25
3		0	0	
4			0	0

Therefore, at the end of this phase, the nonzero entries of the Q-table are:

$$Q(1, \text{MoveSouth}) = 5, \quad Q(2, \text{MoveWest}) = 2.25$$