HONG KONG BAPTIST UNIVERSITY
SEMESTER 1 EXAMINATION, 2007-2008

Course Code: MATH1140 Section Number: 01 Time Allowed: 2 Hour(s)

Course Title: Computational Mathematics Total No. of Pages: 2

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INSTRUCTIONS:

1. Answer ALL of the following questions.
2. The full mark for this examination is 100.
3. Calculators are allowed, but they must not be pre-programmed or have stored text.

1. (15 marks)
Using Gaussian elmination to solve the following problem.

$$\begin{aligned} x + 5y + 3z &= 1 \\ 2x + 7y + 4z &= 4 \\ x + 2y + z &= 3 \end{aligned}$$

2. (30 marks)
Consider the following matrix

$$A = \begin{bmatrix} -6 & 5 \\ -5 & 4 \end{bmatrix}$$

(a) Show that $\lambda = -1$ is the only eigenvalue of the matrix $A$ and $v = [\frac{1}{2}, \frac{1}{2}]^T$ is an eigenvector. (5 marks)

(b) Let $u$ be a vector satisfying $(A - \lambda)u = v$, where $v$ is the eighenvector in (a). Find the unit length of $u$ with respect to norm-1. Hence prove $u$ and $v$ are linear independent. (10 marks)

(c) Let $U = [v, u]$, show that $AU = UD$, where

$$D = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$$

(5 marks)

(d) Ues the reslult obtained in Part (c) to solve $A^{20}x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. (10 marks)

3. (10 marks)
Let $L_1$ be the line with y-intercept = -1 and slope = 2, and $L_2$ be the line with x-intercept=2 and $L_2$ passes through the point $(1, 1)$. Find the slope-intercept form of the line $L_3$ such that $L_3 \perp L_2$ and $L_3$ passes through the intersection of $L_1$ and $L_2$.

4. (25 marks)
   Fruit Computer Company manufactures three different brands of memory chips. Based on the marketing survey results, the numbers of customers using Brands X, Y, and Z in a city are represented by $\mathbf{X}_0 = (x_0, y_0, z_0)^T = (2000, 6000, 4000)^T$.

   Each month some customers may decide to keep using the same brand or switch brands.

   The probability that a user of brand X will switch to brand Y or Z is 0.1 and 0.3, respectively.

   The probability that a user of brand Y will switch to brand X or Z is 0.3 and 0.2, respectively.

   The probability that a user of brand Z will switch to brand X or Y is 0.1 and 0.3, respectively.

   (a) Construct the transition probability matrix, $P$. (5 marks)

   (b) Is the corresponding Markov chain reducible or irreducible? Also, is it periodic or aperiodic? A proper explanation is required. (10 marks)

   (d) Find the stationary distribution vector of $P$ and, in the long run, find the number of perople who use brands X, Y, and Z, respectively. (10 marks)

5. (20 marks)
   In a company, there are three levels of workers: senior technician, technician, and junior worker. Everyday, there are 98 minutes of senior technician's time available, 60 minutes of technician's time available, and 10 minutes of junior worker's time available. The company intends to produce two products: tea and juice. For the production of a unit gallon of tea, it requires 7 minutes of senior technician's time; 3 minutes of technician's time; and 1 minute of junior worker's time. For the production of a unit gallon of juice, it requires 14 minutes of senior technician's time; 5 minutes of technician's time; and 1 minute of junior worker's time. The net profit of unit gallon of tea and juice are 2 thousands and 1 thousand, respectively. Formulate a linear programming problem to find the maximum daily profit using graphical method.

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