**COMP7180 Assignment 1**

**Note:**

1. **Instruction of assignment submission:**
   1. **write all your answers in a Microsoft Word document.**
   2. **hand written answers or importing relevant pictures is not allowed, otherwise, corresponding problems will be given Zero Mark;**
   3. **name your document using the following format: “COMP7180\_A1\_StudentID\_Surname\_Givenname.doc”; and**
   4. **submit the document on Moodle.**
2. **The submission deadline is 5pm, Nov. 8, 2022.**
3. **This is an individual work. Plagiarism is strictly forbidden. Students who plagiarized and who were plagiarized will be given Zero Mark.**

**Problem 1 (20 marks)**

Suppose

Is a convex function on S? Prove your conclusion.

The Hessian matrix of is

Assume we have a vector , and , then we have:

The equation above can be re-write as:

Because is not always tenable, we can say that is not a convex function on S.

**Problem 2 (20 marks)**

Use the definition of the eigenvalue/eigenvector to prove the following statements:

1. **(10 marks)** Show that if 5 is an eigenvalue of an *n×n* matrix , then 25 is an eigenvalue of .

Assume that λ is the eigenvalue of an n×n matrix , then we have  
  
x is the eigenvector of matrix A, then we have  
  
Because , we have . Thus we can say that the eigenvalue of is .

**(10 marks)** Let be an invertible matrix with an eigenvalue 3. Show that 1/3 is an eigenvalue of .

Assume that λ is the eigenvalue of an n×n matrix A, then we have  
  
x is the eigenvector of matrix A, then we have  
  
According to the definition of invertible matrix, we have:

So we can get this equation:

According to the definition of eigenvalue, we can say 1/ is the eigenvalue of

**Problem 3 (20 marks)**

Consider the following design matrix, representing four sample points .

We want to represent the data in only one dimension, so we turn to principal component analysis (PCA).

1. **(10 marks)** Compute all the principal component directions of X, and state which one the PCA algorithm would choose if you request just one principal component.

We can calculate the mean vector of XT:

Then we have:

Then calculate:

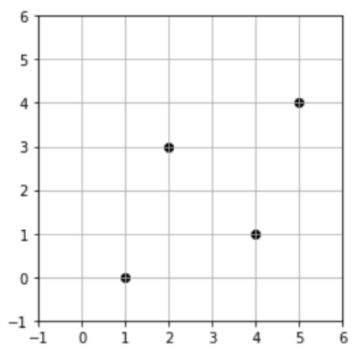
Then calculate:

The eigenvalue = 1 and 4, then calculate the eigenvector:

We choose as the only one principal component.

1. **(10 marks)** The plot below depicts the sample points from X. We want a one-dimensional representation of the data, so draw the principal component direction (as a line) and the projections of all four sample points onto the principal direction.

Label each projected point with its principal coordinate value. Give the principal coordinate values exactly.



**Problem 4 (20 marks)**

Determine if b is a linear combination of the vectors formed from the columns of the matrix A. Prove your conclusion.

Assume 3 variables c1, c2 and c3. Then we have:

We can get the data into the equation, thus we have three equations:

We can solve these equations and get c1 = 245/33, c2 = -41/33, c3 = -2/11. Because c1, c2 and c3 are not equal to 0, we can prove that b is a linear combination of the vectors formed from the columns

**Problem 5 (20 marks)**

Let for some vector space . Show that {} are linearly independent if and only if {} are linearly independent.

We can assume two variables a and b, then if v and w are linearly independent, we can get:

If (v+w) and (v-w) are linearly independent, there must have a = b = 0, the equation above can be re-write as:

Because v and w are linearly independent, we have these equations:

So we can get a = b = 0, (v+w) and (v-w) are linearly independent.

To prove that v and w are linearly independent, we can assume two variables n and m. If (v+w) and (v-w) are linearly independent, we have:

This equation holds if and only if n = m = 0, then we can re-write this equation as:

Because n = m = 0, we have n + m = 0 and n - m = 0. Thus we can prove that v and w are linearly independent.