**COMP7180 Assignment 1**

**Note:**

1. **Instruction of assignment submission:**
   1. **write all your answers in a Microsoft Word document.**
   2. **hand written answers or importing relevant pictures is not allowed, otherwise, corresponding problems will be given Zero Mark;**
   3. **name your document using the following format: “COMP7180\_A1\_StudentID\_Surname\_Givenname.doc”; and**
   4. **submit the document on Moodle.**
2. **The submission deadline is 5pm, Nov. 8, 2022.**
3. **This is an individual work. Plagiarism is strictly forbidden. Students who plagiarized and who were plagiarized will be given Zero Mark.**

**Problem 1 (20 marks)**

Suppose

Is a convex function on S? Prove your conclusion.

Answer:   
For the multivariate function , if has the second-order continuous partial derivatives.

A = = , B = = and C = = -4.

And A > 0 & AC - >=0

But when =1 and = 1, A = -24+8 = -16 < 0

So the function is not a convex function

**Problem 2 (20 marks)**

Use the definition of the eigenvalue/eigenvector to prove the following statements:

1. **(10 marks)** Show that if 5 is an eigenvalue of an *n×n* matrix , then 25 is an eigenvalue of .  
   Answer:   
   Let λ is the eigenvalue of an n×n matrix , then we get  
      
   x is the eigenvector of matrix A,then:   
   Because and ==> x = so, 25 is an eigenvalue of .
2. **(10 marks)** Let be an invertible matrix with an eigenvalue 3. Show that 1/3 is an eigenvalue of .  
   Answer:   
   Let λ is the eigenvalue of an n×n matrix , then we get  
      
   x is the eigenvector of matrix A,then:   
   Then and ==> ==>   
   Because definition of eigenvalue, we can say is the eigenvalue of

**Problem 3 (20 marks)**

Consider the following design matrix, representing four sample points .

We want to represent the data in only one dimension, so we turn to principal component analysis (PCA).

**(10 marks)** Compute all the principal component directions of X, and state which one the PCA algorithm would choose if you request just one principal component.   
Answer:

S = =

S - λI = = =>

When ， When

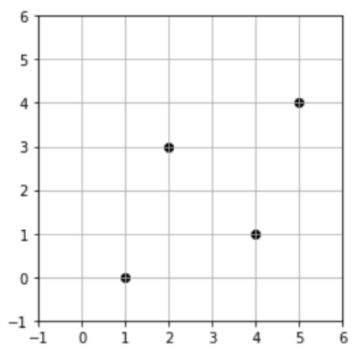
+ 1.5 = - and + 1.5 = .

Then

We choose as the only one principal component because >

1. **(10 marks)** The plot below depicts the sample points from X. We want a one-dimensional representation of the data, so draw the principal component direction (as a line) and the projections of all four sample points onto the principal direction.

Label each projected point with its principal coordinate value. Give the principal coordinate values exactly.



**Problem 4 (20 marks)**

Determine if b is a linear combination of the vectors formed from the columns of the matrix A. Prove your conclusion.

Answer:

we make the augment matrix [,,, b ]

By matrix transformation:

So, b is a linear combination of the vectors

**Problem 5 (20 marks)**

Let for some vector space . Show that {} are linearly independent if and only if {} are linearly independent.

Answer:

Let a and b constants, then if (v + w )and (v - w) are linearly independent, we get:

==> a=b=0 then we can conduct to:

==> a+b = a-b = 0

So v and w are linearly independent.

Let c and d constants, then if v and w are linearly independent, we get:

==>

And v and w are are linearly independent, so c+d = 0 and c-d=0 ==> c=d=0

Then and