This diagram (on the following page) shows the interaction of the Marlin prover and verifier. It is similar to the diagrams in the paper (Figure 5 in Section 5 and Figure 7 in Appendix E, in the latest ePrint version), but with two changes: it shows not just the AHP but also the use of the polynomial commitments (the cryptography layer); and it aims to be fully up-to-date with the recent optimizations to the codebase. This diagram, together with the diagrams in the paper, can act as a "bridge" between the codebase and the theory that the paper describes.

1 Glossary of notation

$\mathbb{F}$	the finite field over which the R1CS instance is defined
x	public input
w	secret witness
H	variable domain
K_M	matrix domain for matrix M
K	$\arg\max_{K_M} \|K_M\|$
X	domain sized for input (not including witness)
$v_D(X)$	vanishing polynomial over domain D
$s_{D_1,D_2}(X)$	"selector" polynomial over domains $D_1 \supseteq D_2$, defined as $\frac{\|D_2\| v_{D_1}}{\|D_1\| v_{D_2}}$
$u_D(X,Y)$	bivariate derivative of vanishing polynomials over domain D
A, B, C	R1CS instance matrices
A^*, B^*, C^*	shifted transpose of A, B, C matries given by $M^*_{a,b} := M_{b,a} \cdot u_H(b,b)\ \forall a, b \in H$ (optimization from Fractal, explained in Claim 6.7 of that paper)
$\mathsf{row}_M, \mathsf{col}_M, \mathsf{val}_M$	LDEs of (respectively) row positions, column positions, and values of non-zero elements of matrix M^*
rowcol_M	LDE of the element-wise product of row and col, given separately for efficiency (namely to allow this product to be part of a *linear* combination)
$\mathcal{P}$	prover
$\mathcal{V}$	verifier
$\mathcal{V}^p$	$\mathcal{V}$ with "oracle" access to polynomial p (via commitments provided by the indexer, later opened as necessary by $\mathcal{P}$)
b	bound on the number of queries
$r_M(X,Y)$	an intermediate polynomial defined by $r_M(X,Y) = M^*(Y,X)$

2 Diagram

$\mathcal{P}(\mathbb{F}, H, K, A, B, C, x, w)$ $\qquad$ $\mathcal{V}^{\mathsf{row},\mathsf{col},\mathsf{rowcol},\mathsf{val}_{A^*},\mathsf{val}_{B^*},\mathsf{val}_{C^*}}(\mathbb{F}, H, K, x)$

$z := (x, w), z_A := Az, z_B := Bz$
sample $\hat{w}(X) \in \mathbb{F}^{<|w|+\mathsf{b}}[X]$ and $\hat{z}_A(X), \hat{z}_B(X) \in \mathbb{F}^{<|H|+\mathsf{b}}[X]$
sample mask poly $m(X) \in \mathbb{F}^{<3|H|+2\mathsf{b}-2}[X]$ such that $\sum_{\kappa \in H} m(\kappa) = 0$

$$\xrightarrow{\quad \text{commitments } \mathsf{cm}_{\hat{w}}, \mathsf{cm}_{\hat{z}_A}, \mathsf{cm}_{\hat{z}_B}, \mathsf{cm}_m \quad}$$

$\eta_A, \eta_B, \eta_C \leftarrow \mathbb{F}$
$\alpha \leftarrow \mathbb{F} \setminus H$

$$\xleftarrow{\quad \eta_A, \eta_B, \eta_C, \alpha \in \mathbb{F} \quad}$$

compute $t(X) := \sum_M \eta_M r_M(\alpha, X)$

sumcheck for $m(X) + u_H(\alpha, X)\left(\sum_M \eta_M \hat{z}_M(X)\right) - t(X)\hat{z}(X)$ **over** H

let $\hat{z}_C(X) := \hat{z}_A(X) \cdot \hat{z}_B(X)$
find $g_1(X) \in \mathbb{F}^{|H|-1}[X]$ and $h_1(X)$ such that
$m(X) + u_H(\alpha, X)(\sum_M \eta_M \hat{z}_M(X)) - t(X)\hat{z}(X) = h_1(X)v_H(X) + Xg_1(X) \qquad (*)$

$$\xrightarrow{\quad \text{commitments } \mathsf{cm}_{g_1}, \mathsf{cm}_{h_1} \quad}$$

$\beta \leftarrow \mathbb{F} \setminus H$

$$\xleftarrow{\quad \beta \in \mathbb{F} \quad}$$

for each $M \in \{A, B, C\}$, **sumcheck for** $\frac{v_H(\beta)v_H(\alpha)\mathsf{val}_{M^*}(X)}{(\beta - \mathsf{row}(X))(\alpha - \mathsf{col}(X))}$ **over** K_M

let $a_M(X) := v_H(\beta)v_H(\alpha)\mathsf{val}_{M^*}(X)$
let $b_M(X) := (\beta - \mathsf{row}_M(X))(\alpha - \mathsf{col}_M(X))$
$= \alpha\beta - \alpha\mathsf{row}_{M^*}(X) - \beta\mathsf{col}_{M^*}(X) + \mathsf{rowcol}_{M^*}(X)$ (over K_M)

find $g_M(X), h_M(X) \in \mathbb{F}^{|K_M|-1}[X]$ and $\sigma_M \in \mathbb{F}$ s.t.
$h_M(X)v_{K_M}(X) = a_M(X) - b_M(X)(Xg_M(X) + \sigma_M/|K_M|)$

$$\xrightarrow{\quad \text{commitments } \mathsf{cm}_{g_A}, \mathsf{cm}_{g_B}, \mathsf{cm}_{g_C}, \text{ and claimed sums } \sigma_A, \sigma_B, \sigma_C \quad}$$

$$\xleftarrow{\qquad}$$ $\delta_A, \delta_B, \delta_C \leftarrow \mathbb{F}$

let $h(X) := (\sum_{M \in \{A,B,C\}} \delta_M h_M s_{K,K_M})$ (mod v_K)

$$\xrightarrow{\quad \text{commitment } \mathsf{cm}_h \quad}$$

$$\xleftarrow{\qquad}$$ $\gamma \leftarrow \mathbb{F}$

$\mathcal{V}$ will need to check the following:

$$\underbrace{v_K(\gamma)h(\gamma) \quad - \sum_{M \in \{A,B,C\}} \delta_M s_{K,K_M}(a_M(\gamma) - b_M(\gamma)(\gamma g_M(\gamma) + \sigma_M/|K_M|))}_{\mathsf{inner}(\gamma)} \overset{?}{=} 0$$

Compute $\hat{x}(X) \in \mathbb{F}^{<|x|}[X]$ from input x

To verify $(*)$, $\mathcal{V}$ will compute $t := \sum_{M \in \{A,B,C\}} \eta_M \sigma_M/|K_M|$, and will need to check the following:

$$\underbrace{s(\beta) + v_H(\alpha, \beta)(\eta_A \hat{z}_A(\beta) + \eta_C \hat{z}_B(\beta)\hat{z}_A(\beta) + \eta_B \hat{z}_B(\beta)) - tv_X(\beta)\hat{w}(\beta) - t\hat{x}(\beta) - v_H(\beta)h_1(\beta) - \beta g_1(\beta)}_{\mathsf{outer}(\beta)} \overset{?}{=} 0$$

$v_{g_A} := g_A(\gamma), v_{g_B} := g_B(\gamma), v_{g_C} := g_C(\gamma)$
$v_{g_1} := g_1(\beta), v_{\hat{z}_B} := \hat{z}_B(\beta)$

$$\xrightarrow{\quad v_{g_A}, v_{g_B}, v_{g_C} v_{g_1}, v_{\hat{z}_B} \quad}$$

use cm_h, and for each $M \in \{A, B, C\}$, index commitments to $\mathsf{row}_M, \mathsf{col}_M, \mathsf{rowcol}_M, \mathsf{val}_M$, evaluation $g_M(\gamma)$, and sum σ_M
to construct virtual commitment $\mathsf{vcm}_{\mathsf{inner}}$

use commitments $\mathsf{cm}_m, \mathsf{cm}_{\hat{z}_A}, \mathsf{cm}_{\hat{w}}, \mathsf{cm}_{h_1}$ and evaluations $\hat{z}_B(\beta), g_1(\beta)$ and sums $\sigma_A, \sigma_B, \sigma_C$
to construct virtual commitment $\mathsf{vcm}_{\mathsf{outer}}$

$\xi_1, \dots, \xi_5 \leftarrow F$

$$\xleftarrow{\quad \xi_1, \dots, \xi_5 \quad}$$

use PC.Prove with randomness $\xi_1, \dots, \xi_5$ to
construct a batch opening proof π of the following:
$(\mathsf{cm}_{g_A}, \mathsf{cm}_{g_B}, \mathsf{cm}_{g_C}, \mathsf{vcm}_{\mathsf{inner}})$ at γ evaluate to $(v_{g_A}, v_{g_B}, v_{g_C}, 0) \qquad (**)$
$(\mathsf{cm}_{g_1}, \mathsf{cm}_{\hat{z}_B}, \mathsf{cm}_t, \mathsf{vcm}_{\mathsf{outer}})$ at β evaluate to $(v_{g_1}, v_{\hat{z}_B}, 0) \qquad (*)$

$$\xrightarrow{\quad \pi \quad}$$

verify π with PC.Verify, using randomness $\xi_1, \dots, \xi_5$,
evaluations $v_{g_A}, v_{g_B}, v_{g_C}, v_{g_1}, v_{\hat{z}_B}$, and
commitments $\mathsf{cm}_{g_A}, \mathsf{cm}_{g_B}, \mathsf{cm}_{g_C}, \mathsf{vcm}_{\mathsf{inner}}, \mathsf{cm}_{g_1}, \mathsf{cm}_{\hat{z}_B}, \mathsf{vcm}_{\mathsf{inner}}$