Evolution of quasiparticle excitations with enhanced electron correlations in superconducting AFe_2As_2 (A = K, Rb, and Cs)

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In the heavily hole-doped iron-based superconductors AFe_2As_2 (A = K, Rb, and Cs), the electron effective mass increases rapidly with alkali-ion radius. To study the superconducting gap structure in this series, we measure the in-plane London penetration depth $\lambda_{ab}(T)$ in clean crystals of AFe_2As_2 down to low temperature $T \sim 0.1$ K. In KFe_2As_2, the superfluid stiffness $\rho_s^{ab}(T) = \lambda_{ab}^2(0)/\lambda_{ab}^2(T)$ at low temperatures can be accounted for by the strongly band-dependent multiple gaps reported recently. Although the $\lambda_{ab}(T)$ in all systems exhibits similar nonexponential temperature dependence indicating nodes or small minima in the gap, we find that the quasiparticle excitations at low temperatures show a systematic suppression with increasing alkali-ion radius. A possible origin of such evolution of low-energy excitations is discussed in terms of the momentum-dependent effect of enhanced quasiparticle mass near a quantum critical point.

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I. INTRODUCTION

An important question on the mechanism of high-temperature superconductivity in iron pnictides concerns the relationship between the quantum criticality and superconductivity. In the vicinity of the superconducting phase, an antiferromagnetic order with a close-by tetragonal-orthorhombic structural transition is frequently found, and anomalous normal-state properties near the endpoint of the order have been detected in $BaFe_2As_2$-based superconductors, implying the importance of quantum critical fluctuations [1–7]. These anomalies are mostly associated with the enhancement of electron correlations, which manifest themselves as a divergently increasing electron effective mass m^* approaching a quantum critical point (QCP) [3].

On the other hand, in the high doping regime of the $Ba_{1-x}K_xFe_2As_2$ system, an increasing trend of m^* has been found with hole doping x; the Sommerfeld coefficient of the electronic specific heat γ, a measure of m^*, reaches $\sim$100 mJ/mol K^2 in KFe_2As_2 ($x = 1$) [8–10], and even exceeds the γ of the quantum critical concentration $z = 0.3$ in the $BaFe_2(As_{1-z}P_z)_2$ system [3]. Such a very large mass in KFe_2As_2, comparable to moderately heavy-fermion materials, is also confirmed by the large Fermi-liquid coefficient of the resistivity [11], quantum oscillations [12], angle-resolved photoemission [13], and optical conductivity [14]. It has been theoretically suggested that these strongly enhanced electron correlations for the $x = 1$ system, where the number N of $3d$ electrons per Fe site is 5.5, are related to the possible proximity to the Mottness of $N = 5$ (half-filled for all $3d$ orbitals) [15], which is much more pronounced than the $N = 6$ case of $BaFe_2As_2$ [16,17]. It has also been found that the isovalent substitution for K by larger alkali ions Rb or Cs results in even larger γ values up to $\sim$180 mJ/mol K^2 [18,19]. This suggests that the negative chemical pressure effect in AFe_2As_2 (A = K, Rb, and Cs) [see Fig. 1(a)] effectively reduces the bandwidth with increasing alkali-ion radius, which leads to an approach to a QCP of another antiferromagnetically ordered phase [20].

One of the key issues is how the quantum critical fluctuations affect the superconducting properties [21–27]. It has been demonstrated both experimentally [21,27] and theoretically [23–25] that the enhanced electron correlations lead to an enhancement of the London penetration depth $\lambda_{ab}(0)$ in the superconducting phase. It has also been proposed [22,25] that the low-energy quasiparticle excitations in nodal superconductors near an antiferromagnetic QCP are significantly affected in a nontrivial way, giving rise to a reduction in the contribution of thermally excited quasiparticles to the penetration depth with an unusual power-law exponent close to 1.5 in the temperature dependence of $\lambda_{ab}(T)$. This dependence distinctly differs from the T-linear dependence expected for line nodes of the superconducting energy gap $\Delta(\boldsymbol{k})$. In this viewpoint, clarifying the effect of the possible QCP on $\lambda_{ab}(T)$ in AFe_2As_2 (A = K, Rb, and Cs) is fundamentally important. Here, we report on such measurements of $\lambda_{ab}(T)$ down to low temperatures by using very clean single crystals. We show that the low-temperature $\lambda_{ab}(T)$ for all systems exhibits the nonexponential temperature dependence, which can be explained by nodes or small minima in the superconducting gap. We also find a systematic change of $\rho_s^{ab}(T)$ at low temperatures with increasing alkali-ion radius, from K (Ref. [11]) to Cs, from which we discuss the possible importance of the momentum-dependent renormalization of quasiparticle mass considering the multigap effect. This may support the presence of quantum critical fluctuations in $CsFe_2As_2$, strongly affecting quasiparticle excitations as observed in other quantum critical superconductors.

II. EXPERIMENTAL METHODS

Single crystals of AFe_2As_2 were synthesized in alumina crucibles by a self-flux method [28]. In these crystals, quantum oscillations have been observed in magnetostriction measurements for A = K [29], Rb, and Cs [20]. The mean free path is found to be much longer than the coherence length [29],

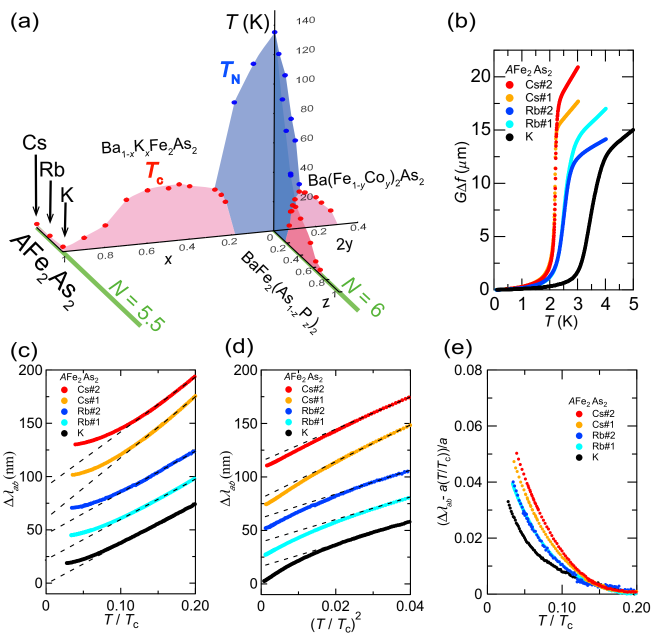

FIG. 1. (a) Schematic phase diagrams of $BaFe_2As_2$-based superconductors with Co, K, and P substitutions [1], combined with that of AFe_2As_2 (A = K, Rb, and Cs). Only antiferromagnetic T_N (blue circles) and superconducting transition temperatures T_c (red circles) are shown for clarity. The isovalent AFe_2As_2 and $BaFe_2(As_{1-z}P_z)_2$ systems correspond to $3d$ electron numbers per Fe atom of $N = 5.5$ and 6.0, respectively (green lines). (b) Temperature dependence of the frequency shift multiplied by the geometric factor G in AFe_2As_2 single crystals. The data for A = K are taken from Ref. [11]. (c) and (d) Low-temperature change of the penetration depth $\Delta\lambda_{ab}$ below $T/T_c = 0.20$ plotted against T/T_c (c) and $(T/T_c)^2$ (d). The data are vertically shifted for clarity. Dashed lines are the guides to the eyes. (e) The deviation from the T-linear dependence at low temperatures (see text).

indicating that the samples are in the clean limit. In this study, the penetration depth measurements were performed for A = Rb and Cs using a self-resonant tunnel-diode oscillator with the resonant frequency f of ∼13 MHz in a dilution refrigerator, and we compare the results with the previously reported data for A = K taken in a similar setup [11]. A small ac magnetic field is applied along the c axis of the sample so that the supercurrent flows in the ab plane. The shift of the resonant frequency Δf is directly proportional to the change in the in-plane magnetic penetration depth, $\Delta\lambda_{ab}(T) = G\Delta f(T)$, in the superconducting state below T_c. The geometric factor G is determined from the geometry of samples and the coil [30]. The samples are cleaved on all sides to avoid degradation due to the reaction with air and coated by Apiezon grease. The samples were mounted in the cryostat within 15 minutes after cleavage to minimize the exposure to air. The typical lateral size of crystals is $\sim 500 \times 500\ \mu m^2$ and the thickness is less than 50 μm.

III. RESULTS

Figure 1(b) shows the temperature dependence of the resonant frequency shift Δf multiplied by G. The superconducting transition temperatures T_c defined by the midpoint of the transition are 3.4, 2.5, and 2.2 K for A = K, Rb, and Cs, respectively, consistent with the previous studies [18,31–34]. In the normal state above T_c, $G\Delta f$ is determined by the skin depth $\delta(T)$, which is related to the dc resistivity $\rho(T)$ through $\delta(T) = \sqrt{2\rho(T)/\mu\omega}$. Here, μ is the permeability and $\omega = 2\pi f$ is the angular frequency of the oscillator. From these relations we estimate ρ values just above T_c as ∼0.8, ∼1.0, and ∼1.2 $\mu\Omega$ cm for the K, Rb, and Cs cases, respectively. These low resistivity values confirm the high quality of our samples.

In Figs. 1(c) and 1(d), the change in the penetration depth $\Delta\lambda_{ab}(T) = \lambda_{ab}(T) - \lambda_{ab}(0)$ at low temperatures is plotted against T/T_c and $(T/T_c)^2$, respectively, for all the measured samples. Two crystals are measured for both Rb and Cs systems and they exhibit almost identical T dependence in each case. For all systems, $\Delta\lambda_{ab}(T)$ exhibits a nonexponential T dependence. If we approximate the low-temperature data by the power-law temperature dependence, we obtain a small exponent less than 2, which is clearly demonstrated by the convex curvature against $(T/T_c)^2$ [Fig. 1(d)]. Such small exponents $\alpha < 2$ in a power-law analysis found for all three compounds, which can hardly be explained by the pair-breaking scattering effect for fully opened gaps [30], immediately imply a robust presence of low-energy quasiparticle excitations. This is consistent with the nodal gap structure inferred from the observation of a residual term κ/T for $T \to 0$ K of the thermal conductivity κ in AFe_2As_2 [19,33,35–37], although measurements at lower temperature below ∼0.1 K would be required for the ultimate determination of the presence of nodes or very small minima in $\Delta(\boldsymbol{k})$ [38,39].

A closer look at the data shows a systematic change of the $\Delta\lambda_{ab}(T)$ at low-T. To see this more clearly, we subtract the T-linear dependence aT/T_c, which is asymptotic to the high-temperature data [dashed lines in Fig. 1(c)], and the relative deviations $(\Delta\lambda_{ab}(T) - aT/T_c)/a$ plotted in Fig. 1(e) show a systematic increase in the order of K, Rb, and Cs. Here the coefficient a is determined by the fitting to the T-linear dependence in the range of $0.15 < T/T_c < 0.25$. We note that in this analysis the errors in G-factor estimates are canceled out. Thus the observed increase in the relative deviations from the T-linear behavior indicates a reduction of the low-energy quasiparticle excitations. To discuss the origin of this tendency, we first examine the evolution of electronic structure in this series. The band-structure calculations are performed within the generalized gradient approximation (GGA-PBE [40]) for the experimentally obtained lattice parameters by using the WIEN2K package [41]. As shown in Fig. 2(a), the results indicate very similar structures for all three systems, which are consistent with the local density approximation results for A = K (Ref. [12]) and Cs [42]. The Fermi surfaces of the K, Rb, and Cs systems all have the same topology, which is confirmed by the quantum oscillation experiments mentioned above [20]. Furthermore, T_c of these materials shows a universal V-shaped dependence on hydrostatic pressure [32], suggesting that the superconducting gap structure is also essentially similar.

In superconductors with line nodes in the gap, the penetration depth shows a T-linear dependence at low temperatures, but impurity scattering can induce a T^2 dependence of the penetration depth at low temperatures [43,44]. In anisotropic

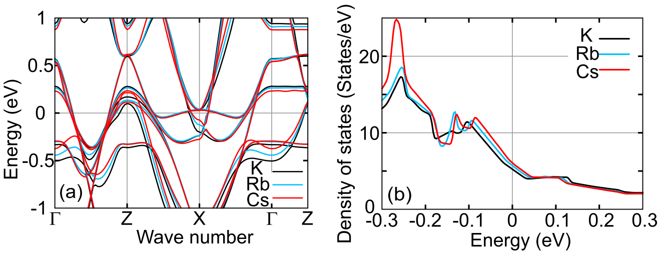

FIG. 2. (a) Band structures of $A\mathrm{Fe_2As_2}$ (A = K, Rb, and Cs) calculated within GGA-PBE. (b) Density of states as a function of energy near the Fermi level.

s-wave superconductors with accidental nodes, such disorder effects sometimes lift the nodes leading to a fully gapped state [45]. These effects, however, are limited to the correspondingly low temperature scale and will not affect the higher-T behavior, which is determined by the intrinsic spectrum of quasiparticle excitations. To describe the disorder effect in superconductors with line nodes, the empirical formula $\Delta\lambda_{ab}(T) \propto (T/T_c)^2/\{(T/T_c) + (T^*/T_c)\}$ that interpolates T-linear and T^2 dependencies has been widely used, where T^* is a measure of the impurity scattering rate [43]. If we apply this to our data in a wide temperature range up to $T/T_c = 0.2$, the result gives considerably large values of T^*/T_c; the data in $\mathrm{CsFe_2As_2}$ can be approximated by the above equation with $T^*/T_c \sim 0.9$, which is unphysically large for clean samples with low residual resistivity [22]. A similar temperature dependence may also come from a possible nonlocality effect near the nodes [46], but in this case $T^* \sim \{\xi(0)/\lambda_{ab}(0)\}\max(\Delta(\boldsymbol{k}))$ should be much smaller than T_c (where ξ is the coherence length), which is also incompatible with the present results. Therefore we conclude that the deviation from the T-linear dependence observed for the Cs system is an intrinsic property of the low-energy quasiparticles excited near the nodes or minima of $\Delta(\boldsymbol{k})$.

To make a quantitative analysis, we use the value of $\lambda_{ab}(0) \sim 203(10)$ nm reported in the small-angle neutron scattering measurements for K [47] and $\lambda_{ab}(0) \sim 267(5)$ nm obtained in the μSR measurement for Rb [31]. As there is no report on $\lambda_{ab}(0)$ for Cs so far, we roughly estimate $\lambda_{ab}(0) \sim 305(30)$ nm on the assumption that $\lambda_{ab}^2(0)$ is proportional to γ [3]. By using these values, we obtain the full temperature dependence of $\lambda_{ab}(T)$ and the normalized superfluid stiffness $\rho_s^{ab}(T) = \lambda_{ab}^2(0)/\lambda_{ab}^2(T)$ [Fig. 3(a)]. The overall T dependence of ρ_s^{ab} is quite similar in all samples, consistent with the very similar electronic and gap structures in these materials except for the effective mass, which is mainly reflected by the difference of the $\lambda_{ab}(0)$ values. The strong curvature found in $\rho_s^{ab}(T)$ near T_c indicates the importance of the multiband effect [30]. Indeed, recent ARPES and specific-heat measurements have reported highly band-dependent gap structures in $\mathrm{KFe_2As_2}$ [38,48].

IV. DISCUSSION

In contrast to the overall similarity, $\rho_s^{ab}(T)$ at low temperatures shows systematic differences for different systems. Compared with the data for K, the Rb and Cs data show more flattened curvatures [Fig. 3(b)–3(d)], indicating a significant change in the amount of quasiparticle excitations at low energies. The most pronounced change with the alkali-ion radius R_{ion} in this series is the strong enhancement of the experimentally determined Sommerfeld coefficient γ_{exp}, as shown in Fig. 4(a). This R_{ion} dependence of γ_{exp} is much more rapid than that of the band-structure calculations γ_{calc}, which changes by only $\sim$15% as can be seen in a small increase in the density of states at the Fermi level [Fig. 2(b)]. This indicates that the mass renormalization factor averaged over the Fermi surface, $1/Z = \gamma_{\mathrm{exp}}/\gamma_{\mathrm{calc}}$, is strongly enhanced with increasing R_{ion} [Fig. 4(b)], which is consistent with the analysis of quantum oscillations [20]. The value of $1/Z$ in $\mathrm{CsFe_2As_2}$ reaches $\sim$13, which is larger than the estimated value of $\sim$10 for the quantum critical concentration $z = 0.3$ in the $\mathrm{BaFe_2(As_{1-z}P_z)_2}$ system [3]. This result suggests that negative chemical pressure brings the $A\mathrm{Fe_2As_2}$ system toward a QCP. To see how the low-energy quasiparticle excitations change with R_{ion}, we evaluate the $(\Delta\lambda_{ab}(T) - aT/T_c)/a$ at a constant reduced temperature $T/T_c = 0.05$ to quantify the deviation from the T-linear dependence. The trend shown in Fig. 4(b) highlights a close link between the mass enhancement and the deviation from the T-linear dependence. Considering the fact that such a reduced contribution of quasiparticle excitations to the penetration depth has been reported for several unconventional superconductors in the vicinity of QCPs [22], we infer that this anomaly might have a common origin associated with quantum critical fluctuations.

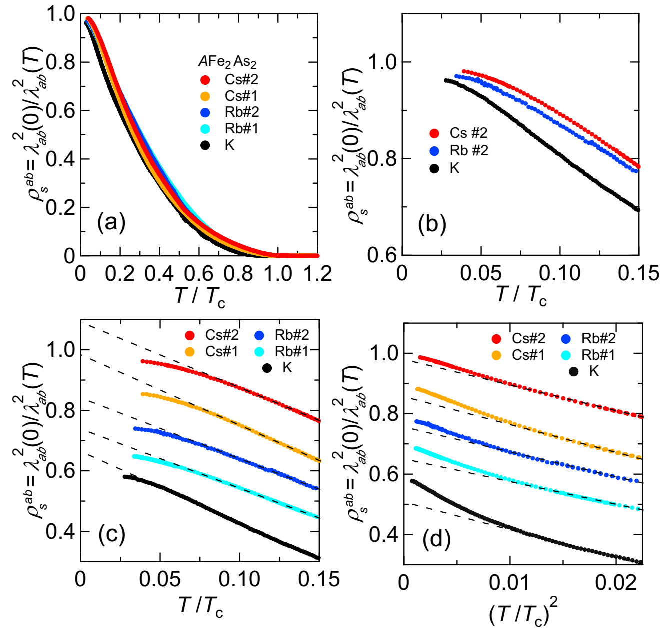

FIG. 3. (a) Superfluid stiffness $\rho_s^{ab}(T) = \lambda_{ab}^2(0)/\lambda_{ab}^2(T)^2$ obtained by using the reported values of $\lambda_{ab}(0) = 203$ [47], 267 nm [31], and the estimate of $\lambda_{ab}(0) = 305$ nm (see text) for A = K, Rb, and Cs, respectively. (b) Comparisons of $\rho_s^{ab}(T)$ at low temperatures $T/T_c < 0.15$ for A = K, Rb, and Cs. (c) and (d) Low-T part of $\rho_s^{ab}(T)$ below $T/T_c = 0.15$ plotted against T/T_c for K, Rb, and Cs (c), and $(T/T_c)^{2.0}$ (d). The data are vertically shifted for clarity. Dashed lines are the guides for the eyes.

It has been proposed [22] that the unusual T dependence of the superfluid stiffness might arise from a strong momentum

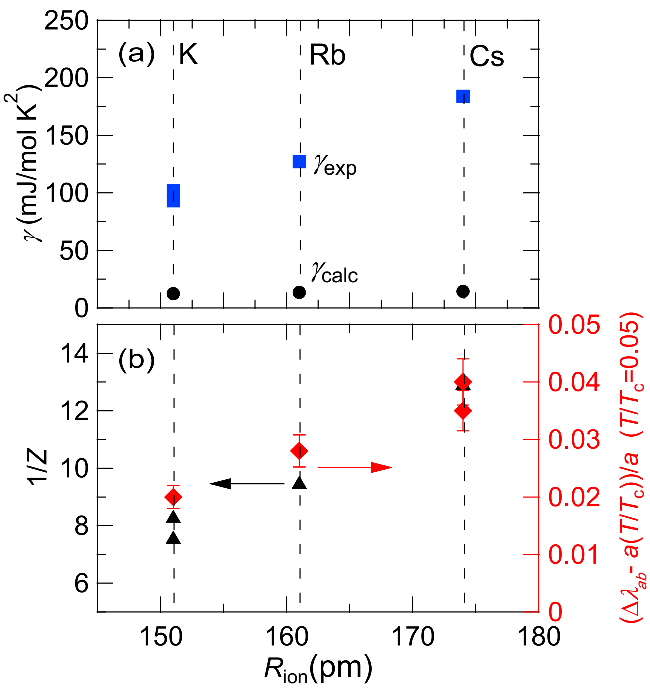

FIG. 4. (a) Sommerfeld coefficients $\gamma_{\rm exp}$ in single crystals of $A\mathrm{Fe_2As_2}$ (A = K, Rb, and Cs) [8,9,18,19], compared with the estimated $\gamma_{\rm calc}$ from the band-structure calculations, as a function of alkali-ion radius $R_{\rm ion}$ in the eight-fold coordination. (b) $R_{\rm ion}$ dependence of mass renormalization factor $1/Z$ extracted from the comparisons of experimental and calculated γ values and the value of deviation from the T-linear dependence at $T/T_c = 0.05$ expressed in Fig. 1(e).

dependence of the effective Fermi velocity $v_F^*(\boldsymbol{k})$ near the nodes of the superconducting gap $\Delta(\boldsymbol{k})$. Such a $\boldsymbol{k}$ dependence of $v_F^* \propto 1/m^*$ requires that quantum critical fluctuations responsible for the mass renormalization also have a strong momentum dependence associated with $\Delta(\boldsymbol{k})$. This can be naturally expected when the quantum critical fluctuations are quenched by opening the superconducting gap, the degree of which is determined by the gap magnitude $|\Delta(\boldsymbol{k})|$ [22]. Recent theoretical calculations for quantum critical superconductors with line nodes [25] indicate that the current vertex corrections to the T-linear penetration depth can be significant if the antiferromagnetic hot spots are located near the nodal points such as the case of electron-doped cuprates.

Now we extend this consideration to the multigap systems, where $\Delta(\boldsymbol{k})$ is very different for different bands. As the electronic and gap structures do not change significantly [20], we need another ingredient to explain the evolution of low-temperature quasiparticle excitations with $R_{\rm ion}$. Let us first examine the validity of the multigap analysis for $\mathrm{KFe_2As_2}$.

The London penetration depth in a superconductor can be calculated from the superconducting gap $\Delta(\boldsymbol{k},T)$ through the following equation [22]:

$$\lambda_{jj'}^{-2}(T) = \frac{\mu_0 e^2}{4\pi^3\hbar}\int \frac{v_{Fj}(\boldsymbol{k})v_{Fj'}^*(\boldsymbol{k})}{|\boldsymbol{v}_F(\boldsymbol{k})|}[1 - Y(\boldsymbol{k},T)]d\boldsymbol{S}, \quad (1)$$

where v_F^* (v_F) is the effective velocity in the superconducting (normal) state, and the subscripts j, j' denote the directions of

TABLE I. Multigap parameters used in the calculations of $\rho_s^{ab}(T)$. The weight w^i to ρ_s^{ab} of band i is estimated from the number of holes n and the effective mass m^* in units of the electron mass m_e [11], which are determined by the quantum oscillations in $\mathrm{KFe_2As_2}$ [12]. The gap values of each band and the angle dependence of $\Delta(\phi)$ of ζ band are assumed to be the following:

band	n	m^*/m_e	w^i	$\Delta(\phi)/k_BT_c$
α (inner)	0.17	6	0.31	0.61
ζ (middle)	0.26	13	0.23	$0.35 + 0.45\cos(4\phi)$
β (outer)	0.48	18	0.31	0.25
ε (corner)	0.09	7	0.15	1.8
total	1.0		1.0	

the current and vector potential. Here,

$$Y(\boldsymbol{k},T) = -2\int_{\Delta(\boldsymbol{k},T)}^{\infty} \frac{\partial f(E)}{\partial E}\frac{E}{\sqrt{E^2-\Delta^2(\boldsymbol{k},T)}}dE \quad (2)$$

is the Yosida function and $f(E)$ is the Fermi-Dirac function for the quasiparticle energy E. In this study, we measure the in-plane penetration depth $\lambda_{ab}(T)$ and thus consider the in-plane effective Fermi velocities.

In multiband superconductors, the superfluid stiffness can be calculated by adding the contributions from different bands as

$$\rho_s^{ab}(T) = \frac{\lambda_{ab}^2(0)}{\lambda_{ab}^2(T)} = \sum_i w^i \rho_s^i(T), \quad (3)$$

where

$$w^i = \left(\frac{\lambda_{ab}^i(0)}{\lambda_{ab}(0)}\right)^{-2} = \lambda_{ab}^2(0)\frac{\mu_0 e^2}{4\pi^3\hbar}\int_{\text{band } i}\frac{v_{Fj}(\boldsymbol{k})v_{Fj'}^*(\boldsymbol{k})}{|\boldsymbol{v}_F(\boldsymbol{k})|}d\boldsymbol{S}. \quad (4)$$

Here, band i has the superfluid stiffness $\rho_s^i(T)$ and the penetration depth at zero temperature $\lambda_{ab}^i(0)$. The weight w^i to the total $\rho_s^{ab}(T)$ is determined by the corresponding carrier concentration divided by its effective mass.

The weight of each band can be estimated from the analysis of quantum oscillations, which yields information on the cross-sectional area and the effective mass m^* of the observed extremal orbits. Importantly, the magnitude of the effective mass averaged within one band affects only the absolute value of λ_{ab} as well as the weight w^i but does not affect the T dependence of λ_{ab} as one can see in Eq. (1).

Based on the quantum oscillation results [12], we estimate the weight w^i as listed in Table I for $\mathrm{KFe_2As_2}$ [11]. Here we ignore the δ band near the Z point, whose contribution is very small. The calculated contributions $w^i\rho_s^i(T)$ and the total $\rho_s^{ab}(T)$ are shown in Fig. 5(a), together with the experimental data of $\mathrm{KFe_2As_2}$ [11]. Here the gap function $\Delta(\boldsymbol{k})$ of each band is assumed as suggested by Hardy *et al.* [38] (see Table I). Only the ζ band does have a wave form with nodes at eight directions as found in high-resolution laser ARPES measurements [48]. For the other bands we assume constant Δ values with large differences in magnitude. As we are interested in $\rho_s^{ab}(T)$ at low temperatures, we simply assume the BCS-type T dependence of the gap which vanishes at the same T_c for all bands. This

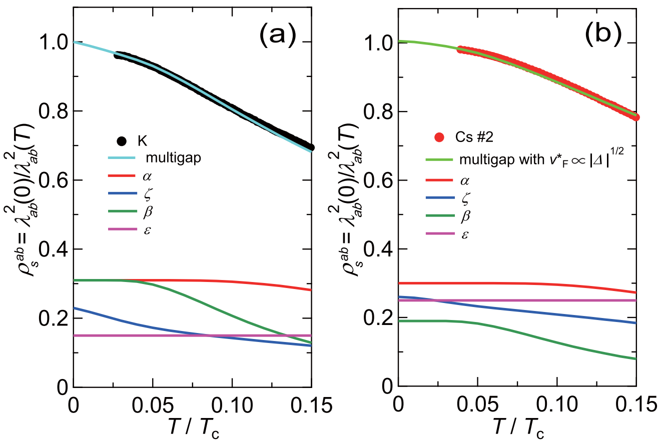

FIG. 5. (a) Low-temperature superfluid stiffness of KFe_2As_2 (circles) with the multigap fit. Contributions from different bands calculated by the parameters given in Table I are also shown (solid lines). (b) Low-temperature superfluid stiffness of $CsFe_2As_2$ (circles) with the multigap fit considering the effect of quantum criticality assuming the momentum dependence of $v_F^*(\boldsymbol{k}) \propto |\Delta(\boldsymbol{k})|^{1/2}$. The zero-temperature values of each curve correspond to the effective weight taking into account the difference of gap magnitude.

assumption will introduce some errors at high temperatures when the interband coupling is strong [30,38], but the low-temperature behavior should not be affected by the interband effect. As expected, $\rho_s^{\varepsilon}(T)$, whose gap magnitude is the largest, is very flat at low temperatures, and $\rho_s^{\zeta}(T)$ with a nodal gap has the steepest T dependence. Interestingly, $\rho_s^{\beta}(T)$ with a very small gap also shows very strong T dependence at low T. The total $\rho_s^{ab}(T)$ obtained by adding $\rho_s^{i}(T)$ with different T dependencies is in good agreement with the experimental data for KFe_2As_2, indicating that the set of gap parameters suggested in Ref. [38] can account for both the specific heat and low-temperature penetration depth in a semiquantitative manner.

Having established the validity of the multigap fit for KFe_2As_2, we now consider the T dependence found in $CsFe_2As_2$. The quantum oscillation experiments for the $A\mathrm{Fe}_2\mathrm{As}_2$ series [20] indicate that the size of the Fermi surface of each band is almost unchanged and that the effective mass of each band is nearly equally enhanced with alkali-ion radius. These features suggest that w^i does not change strongly. To model the effect of the quantum critical mass enhancement in the superconducting state, we take the same set of superconducting gaps, yet with the momentum dependence of $v_F^*(\boldsymbol{k}) \propto |\Delta(\boldsymbol{k})|^{1/2}$, which has been suggested to account for the unconventional T dependence in nodal quantum critical superconductors by Hashimoto *et al.* [22]. When we extend this model to multigap systems, we expect that it will also affect the relative weight, i.e., $w^i/w^{i'}$ can be replaced by $|\Delta^i|^{1/2}w^i/|\Delta^{i'}|^{1/2}w^{i'}$ (where $|\Delta^i|$ is the gap magnitude of band i). For the ζ band, $\rho_s^{\zeta}(T)$ also changes because the $\boldsymbol{k}$ dependence of v_F^* affects the integration in Eq. (1).

The results of $\rho_s^{i}(T)$ calculations for each band and the total $\rho_s^{ab}(T)$ for $CsFe_2As_2$ are shown in Fig. 5(b) together with the experimental data. The difference of $\rho_s^{\zeta}(T)$ for the nodal gap $\Delta(\phi)/k_BT_c = 0.35 + 0.45\cos(4\phi)$ between constant v_F^* and $v_F^*(\boldsymbol{k}) \propto |\Delta(\boldsymbol{k})|^{1/2}$ can be clearly seen. This simple calculation is in good qualitative accord with the evolution of $\rho_s(T)$ with A [Fig. 3(b)] below $T/T_c = 0.15$, although at higher temperatures, there are some deviations which are possibly due to the interband coupling effect. The essence of this momentum-dependent mass renormalization is that it can naturally reduce the contribution of quasiparticles excited at the $\boldsymbol{k}$ positions where the gap magnitude $|\Delta(\boldsymbol{k})|$ is small. Thus such a mechanism can be a possible origin of the observed positive correlation between the enhancement of effective mass and the change in quasiparticle excitations in the superconducting state near a quantum critical point.

V. CONCLUSIONS

In summary, from the penetration-depth measurements of $A\mathrm{Fe}_2\mathrm{As}_2$ (A = K, Rb, and Cs) in the heavily hole-doped regime with $3d$ electron number of $N = 5.5$, we show that the superconducting gaps have robust strong anisotropies, consistent with the previous reports of thermal conductivity measurements [19,33,35–37]. The temperature dependence of in-plane penetration depth $\Delta\lambda_{ab}(T)$ and superfluid stiffness $\rho_s^{ab}(T)$ at low temperatures show a systematic evolution with negative chemical pressure, indicating a suppression of the thermally excited quaiparticles. This evolution of low-energy excitations correlates with the enhancement of the effective mass toward a possible QCP in $A\mathrm{Fe}_2\mathrm{As}_2$. This observation suggests that quasiparticle excitations of the superconducting condensate with an anisotropic gap can be affected by the momentum and band dependencies of quantum critical antiferromagnetic fluctuations.

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