Fermi liquid vs. non-Fermi liquid systems

Diego Zocco - 211A Final Essay

November 26, 2006

Abstract

Transport and magnetic properties of metals can be described using numerous models. These models differ, in general, in the type of interactions that are taken into account. The free electron model (Fermi gas model) and the Fermi liquid theory are examples of theories of metals, with different degrees of electron-electron interactions. In this report I will review some of the properties of these two models, and their failures in the description of some particular systems, the so-called Non-Fermi liquid systems.

1 Fermi gas model

At the beginning of the last century, Drude formulated the free electron model to describe the properties of metals [1]. In this model, the valence electrons of the atoms in a metal are described as a gas of non-interacting conduction electrons. They are non-interacting in the sense that no forces are assumed to act between the particles, or between the particles (electrons) and the positively charged ions of the lattice. In this context, the electrons of this gas travel ballistically (in straight lines) until they collide elastically with one another. The Drude model is successful in calculating the resistivity of different metals at room temperature, via the well-known Ohm's law:

$$\rho = \frac{m_e}{ne^2\tau} \tag{1}$$

τ is the characteristic time between collisions, and is defined as the mean free path distance divided by the average velocity of the electrons. In Drude's time, it was reasonable to assume that the electronic velocity distribution was given in equilibrium at a temperature T by the Maxwell-Bolzmann distribution. This leads to good order of magnitude agreement with the resistivity values at room temperature, and with the Wiedermann-Franz law of thermal conductivity, but also predicts a contribution to the specific heat of a metal of $3/2K_B$ per electron that was not observed for all T. The success of the Drude's model in predicting the Wiedemann-Franz law is a consequence of the cancellation of two errors: at room temperature, the actual electronic specific heat is 100 times smaller than the Drude's prediction, but the velocity of the particles taken into account is 10 times larger.

This problem was solved by substituting the Maxwell-Bolzmann distribution by the Fermi-Dirac distribution function, which takes into account Pauli exclusion principle. Under this model, the average velocity of the electrons is replaced by the Fermi velocity, and this substitution gives us a good representation of the free electron gas in a wide range of temperatures, in good agreement with the experiments. In a free gas of N Fermi particles of spin $1/2$, the single-particle eigenstates are plane-wave states of wave vector $\mathbf{k}$ and spin projection $\sigma_z = \pm 1/2$ with energy $\epsilon_k = \frac{\hbar^2 k^2}{2m}$. The ground state is the so-called Fermi sea: all single-particles states are filled up to a limiting wave vector $k_F = (3\pi^2 N)^{1/3}$. We can define the Fermi momentum, energy and velocity by

$$p_F = \hbar k_F = \hbar(3\pi^2 N)^{1/3}, \qquad \epsilon_F = \frac{p_F^2}{2m}, \qquad v_F = \left(\frac{d\epsilon}{dp}\right)_{p_F} = \frac{p_F}{m} \tag{2}$$

and the density of states per unit energy and unit volume at the Fermi surface:

$$\left(\frac{dn}{d\epsilon}\right) = \frac{3Nm}{p_F^2} \tag{3}$$

The low temperature static properties are determined entirely by the density of states. For example, the specific heat and susceptibility can be written [2]:

$$C_V = \left(\frac{\pi^3}{3}\right) k_B^2 \left(\frac{dn}{d\epsilon}\right) T, \qquad \chi = \mu^2 \frac{dn}{d\epsilon} \tag{4}$$

The Sommerfeld theory has problems in describing the properties of some metals at low temperature. For example, it accounts reasonable well for the size of the term linear in T in the low temperature specific heat of the alcali metals (Li, Na, K, etc, all with ns^1 electronic configuration), but very poorly for transition metals such as Fe and Mn, and much worse for Bi and Sb (poor metals). In the next section we will see how the Fermi liquid theory treats this problem, introducing the concept of quasiparticle.

2 Fermi liquid model

The investigation of superfluidity of Helium-II stated the question of the properties of a quantum liquid, which is a system of interacting particles whose behavior is largely conditioned by quantum effects. Landau's theory of superfluidity was at the same time the first theory of a quantum liquid [3]. It described the properties of so-called Bose liquids, i.e. of quantum systems of interacting particles which obey Bose statistics. However, besides these liquids there are others which form a much wider class: the so-called Fermi liquids. This class has spin $1/2$ excitations and obey Fermi statistics, and contains liquid ^{3}He, electrons in metals and heavy nuclei. For electrons in a metal, we imagine that each electron collects around itself a screening cloud of other electrons, thereby becoming a quasiparticle with some effective mass m^*. The number of quasiparticles is equal to the number N of free electrons in the metal; like them, the quasiparticles occupy plane wave states of momentum $\mathbf{p} = \hbar\mathbf{k}$ and spin projection $\pm 1/2$, and must obey the Pauli exclusion

principle. Consequently in the ground state the quasiparticles fill the Fermi sea up to the Fermi momentum, which is still given in terms of N by Eq. 1. Excited states are formed by taking a quasiparticle out of a filled state and putting it in an empty state. We can describe any state by specifying for each state $(\mathbf{p}, \sigma)$ the number of quasiparticles $n(\mathbf{p}, \sigma)$ in that state. The energy of a single quasiparticle state is just

$$\epsilon_{(p)} = \frac{p^2}{2m^*} \tag{5}$$

Consequently, the Fermi velocity and density of states at the Fermi surface are found from Eqs. 2 and 3 simply replacing m by m^*:

$$v_F = \frac{p_F}{m^*}, \qquad \left(\frac{dn}{d\epsilon}\right) = \frac{3Nm^*}{p_F^2} \tag{6}$$

We can see from the above equations that there is a one-to-one mapping of the quasiparticles with the single particles of the free electron gas. In the Fermi liquid theory, Landau makes another assumption: the interaction of the quasiparticles can be taken into account as a self-consistent field of surrounding particles, which leads to the fact that the energy of the system does not equal the sum of energies of the N quasiparticles, but it is a functional of its distribution function. This is reflected in the form that Landau defines the energy of the quasiparticle:

$$\epsilon = \epsilon_0(\mathbf{p}, \sigma) + \delta_{(p,\sigma)}^{meanfield} \tag{7}$$

where $\epsilon_0(\mathbf{p}, \sigma)$ is the energy of the quasiparticles at T = 0, and $\delta_{(p,\sigma)}^{meanfield}$ is the mean-field effect of the interactions with other quasiparticles, defined as

$$\delta\epsilon_{(p,\sigma)}^{meanfield} = \frac{1}{2} S_{\sigma'} \int f(\mathbf{p}, \sigma; \mathbf{p}', \sigma') \delta n(\mathbf{p}, \sigma) 2 \frac{dp_x dp_y dp_z}{(2\pi\hbar)^3} \tag{8}$$

The function $f(\mathbf{p},\sigma;\mathbf{p}',\sigma')$ is related to the scattering amplitude of two quasiparticles, and verifies time reversal symmetry, i.e. $f(\mathbf{p},\sigma;\mathbf{p}',\sigma') = f(\mathbf{p}',\sigma';\mathbf{p},\sigma)$.

$\delta n(\mathbf{p},\sigma) = n - n_{Fermi}$, and accounts for small deviations of the density of states from the equilibrium value n_{Fermi}. In this context, the quasiparticles are associated with low-energy excitations of the interacting system of electrons with a long lifetime near the Fermi energy, and hence it was created to explain the low-temperature $(T \ll T_{Fermi})$ properties of a Fermi system. The total energy of the system is given by

$$E = E_0 + \sum_{p,\sigma} \int \delta\epsilon(\mathbf{p},\sigma)\delta n(\mathbf{p},\sigma) \tag{9}$$

which shows explicitly that the total energy is not just the sum of the energies of each quasiparticle. The predictions for the temperature dependence of the magnetic susceptibility χ [3, 4], specific heat C [3] and electrical resistivity ρ [5] at low temperatures ($T \sim 1 - 10K$, $T \ll T_{Fermi}$) in the Fermi liquid scenario are given by

$$\chi = \chi_0, \qquad \chi_0 = \chi_0(m^*) \tag{10}$$

$$C = \gamma_0 T, \qquad \gamma_0 = \gamma_0(m^*) \tag{11}$$

$$\rho = \rho_0 + AT^2 \tag{12}$$

These formulae are valid for an isotropic, 3-dimensional system. For a dimensionality discussion, see [5]. The magnetic susceptibility and the specific heat have the same form as in the classic Fermi gas. The coefficients γ_0 and χ_0 are related now with the effective mass of the quasiparticles:

$$\gamma_0 = \left(\frac{\pi}{3N}\right)^{2/3} \frac{m^*}{\hbar} \tag{13}$$

The Eq. 12 is obtained from considering the scattering of two electrons lying in a thin

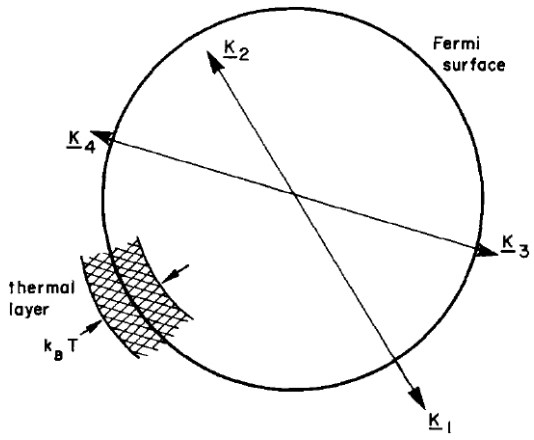

Figure 1: Schematic diagram of a normal electron-electron scattering event, in which two electrons, initially in states K_1 and K_2 near the spherical Fermi surface, are scattered into final states K_3 and K_4.

energy layer around the Fermi sphere surface (Fig. 1).

Using conservation of energy and momentum, it can be shown that the scattering rate between electrons τ_{e-e} is inversely proportional to the square of the temperature. Equations 10 and 11 have been confirmed experimentally [1]. The prediction for the electrical resistivity is very difficult to observe in most normal metals, provided that the contribution ρ_0 due to impurities in the samples at low temperatures is in general very large and the coefficient A is too small. In recent years, experiments performed on a new class of materials called heavy fermions allowed us to confirm the Fermi liquid resistivity law.

Heavy fermions materials are intermetallic compounds that usually contain a rare earth or actinide atom (U, Ce, Yb, e.g. URu_2Si_2 is a HF superconductor below $T_C = 0.8K$) that has a partially filled $4f$ or $5f$ electron shell. At low temperatures, the f-electrons strongly couple to the conduction band which results in enhancements of the effective mass m^* to values 100-1000 times larger than the electron mass. As a consequence of this,

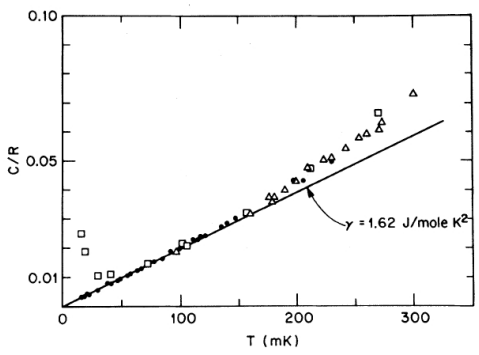

Figure 2: Specific heat of $CeAl_3$ at low temperatures. The value of γ_0 for this system is $1620\frac{mJ}{molK^2}$.

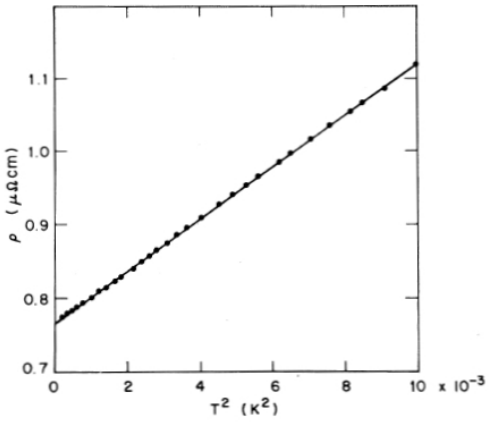

Figure 3: Electrical resistivity of $CeAl_3$ below 100 mK.

$\gamma_0(m^*)$, $\chi_0(m^*)$ and $A(m^{*2})$ are also enhanced. For example, a regular metal has

$$\gamma_0^{metal} \sim 1 - 10\frac{mJ}{molK^2} \tag{14}$$

On the other hand, for heavy fermions:

$$\gamma_0^{HF} \sim 100 - 1000\frac{mJ}{molK^2}, \qquad for\ \ URu_2Si_2,\ \ \gamma_0^{URu_2Si_2} = 180\frac{mJ}{molK^2} \tag{15}$$

Figure 2 and 3 show an example of experimental confirmation of Landau's theory in some Fermi liquid systems [6].

It is also worth to mention that there is a universal relation between the large mag-

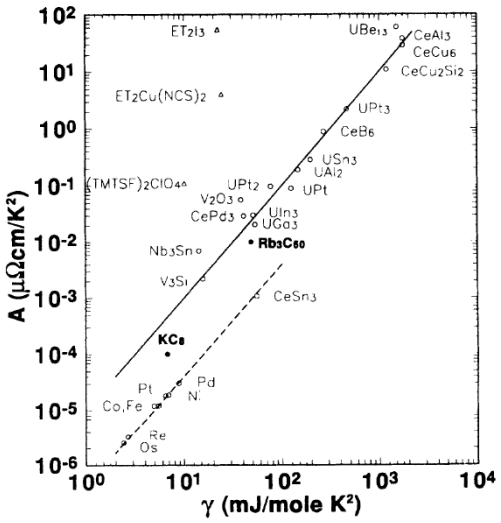

Figure 4: Resistivity coefficient A vs linear specific heat coefficient γ_0. The full line through the heavy fermion compounds corresponds to $A/\gamma_0^2 = 1{\times}10^{-5}$. The dashed line through the transition metals corresponds to $A/\gamma_0^2 = 0.4 \times 10^{-6}$.

nitude of A and the enhanced electronic specific heat coefficient γ_0 in heavy fermions materials, as was found by Kadowaki and Woods (Figure 4) [7]. They observed that most of heavy-fermion compounds obeyed the relation $A/\gamma_0^2 = 1 \times 10^{-5}\ \mu\Omega\text{cm}\ (molK/mJ)^2$.

Most heavy-fermion materials are in good agreement with Landau Fermi-liquid theory. However, a new class of compounds has emerged in recent years whose physical properties at low temperatures show remarkable deviations from equations 10 - 12. Those compounds, known as non-Fermi-liquids are discussed in the next section.

3 Non - Fermi liquid systems

In 1991, Seaman $et.al$ [8] presented measurements of specific heat, magnetic susceptibility and electrical resistivity on the $Y_{1-x}U_xPd_3$ system. These measurements strongly disagreed with the Fermi liquid model of Landau and launched the intense experimental and theoretical efforts to understand such non-Fermi-liquid (NFL) behavior in bulk d

and f-electron metals. The $Y_{1-x}U_xPd_3$ compound is a system with electron-electron interactions that are too strong to permit entry into the Fermi-liquid ground state at low temperatures. NFL behavior in f-electron systems is characterized by weak power-law or logarithmic temperature dependences of the physical properties at low temperatures $T \ll T_{onset}$ and can be summarized as follows:

$$\rho(T) \propto \left[1 - a\left(\frac{T}{T_o}\right)^n\right], \qquad where\ |a| \sim 1\ and\ n \sim 1 - 1.5 \tag{16}$$

$$\frac{C(T)}{T} \propto -\left[\frac{1}{T_o}\right] ln\left(\frac{T}{T_o}\right)\ \ or\ \ T^{-1+\lambda} \qquad (\lambda \sim 0.7 - 0.8) \tag{17}$$

$$\chi \propto \left[1 - \left(\frac{T}{T_o}\right)^{1/2}\right], \qquad -ln\left(\frac{T}{T_o}\right)\ \ or\ \ T^{-1+\lambda} \qquad (\lambda \sim 0.7 - 0.8) \tag{18}$$

In several of those systems, the scaling temperature T_o can be identified with the Kondo temperature T_K [9, 10]. Figure 5 show an example of these behaviors for the $Y_{0.8}U_{0.2}Pd_3$ [11]. NFL behavior is often observed to occur near a second-order phase transition that has been supressed to zero by pressure, chemical substitution or magnetic field, known as a quantum critical point (QCP). A wide variety of f-electron materials display the suppression of a second order phase transition, including $CeRhIn_5$ (antiferromagnetism suppressed by substitution), $CeIn_3$ and $CePd_2Si_2$ (AFM suppressed by pressure), etc. QCPs are also observed in the high-T_c cuprate superconductors in which both an antiferromagnetic transition and a pseudogap are suppressed by electron or hole doping [12].

A number of models [12] have been developed to explain the origin of NFL behavior. The underlying bases for these models revolve around one or a combination of three principal themes: a) single ion effects, b) disorder, and c) vicinity to a T = 0 K phase transition (QCP). Information regarding these models can be found in the bibliography and escape from the scope of this report.

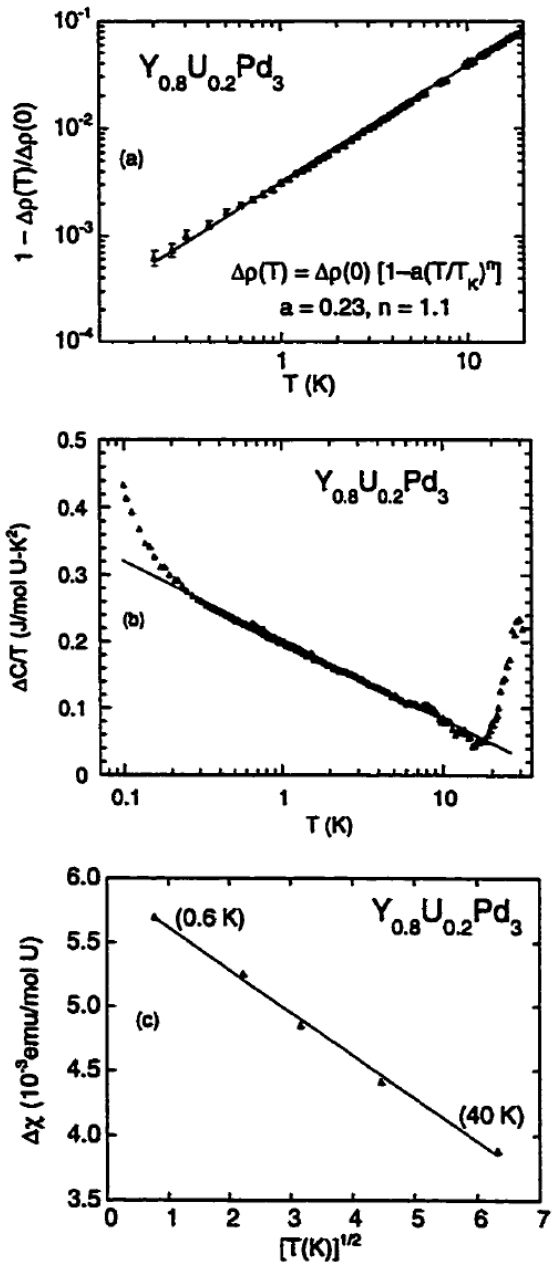

Figure 5: Low temperature physical properties for $Y_{0.8}U_{0.2}Pd_3$. The electrical resistivity ρ, specific heat divided by temperature C/T and the magnetic susceptibility χ have characteristic NFL temperature dependences

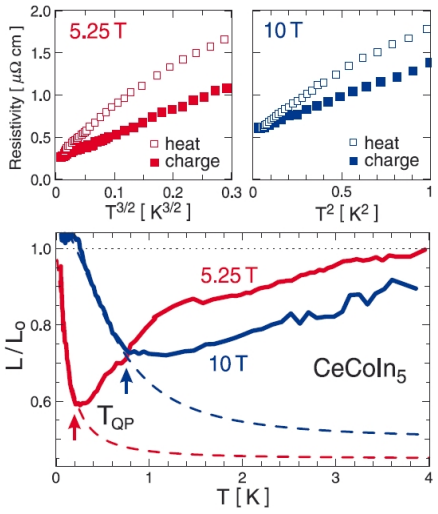

Figure 6: Upper panels: comparison of thermal and electric resistivities for two values of magnetic field, $5.25T$ and $10T$, plotted as a function of $T^{3/2}$ and T^2 respectively. Lower panel: normalized Lorentz ratio vs temperature

4 Is it really a Non-Fermi liquid system?

At this point of the discussion, we should start asking ourselves if every system that doesn't behave as a Fermi liquid is a NFL system. Let me quickly disappoint you: the answer is no. The Fermi gas model was successful in predicting the Wiedemann-Franz law for a electronic system which is characterized by fermionic particles of spin $1/2$ and charge e. Later, the Landau's Fermi liquid theory confirmed those results, which were proved with experiments carried out in many different metals. The Wiedemann-Franz law states that the ratio of thermal (κ) to electrical (σ) conductivities (the Lorentz ratio L_0) is a universal constant in the $T \to 0$ limit: $\kappa/\sigma T = L_0 \equiv \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2$. This law comes from considering $1/2$-fermionic particles with charge e. A violation of this law would imply a profound breakdown of the Fermi-liquid model, in the sense that low-lying excitations would no longer be quasiparticles of charge e obeying Fermi statistics. The Wiedemann-Franz law violation-test was performed recently by Paglione $et.al$ in a $CeCoIn_5$ system that shown an apparent NFL behavior at $T > 0$ [13]. In the Figure 6 we can see from the

resistivity measurements that this system behaves as a FL or as a NFL depending on the value of the applied magnetic field. However, the calculation of the Lorentz ratio shows that at $T \rightarrow 0$, the Lorentz ratio approximates to one for both values of magnetic fields, which is an indicator that the ground state in both cases corresponds to the one of a Fermi liquid system.

References

[1] N. W. Ashcroft and N.D. Mermin, *Solid State Physics*, 1976

[2] A. J. Leggett, Rev. Mod. Phys. **47**, 331 (1975)

[3] A. A. Abrikosov and I. M. Khalatnikov, Rep. Prog. Phys. **22**, 329 (1959)

[4] See also Ref. [1], page 661, *Susceptibility of metals: Pauli paramagnetism*

[5] M. Kaveh and N. Wiser, Advances in Physics **33**, 257 (1984)

[6] K. Andres, J. E. Graebner and H. R. Ott, Phys. Rev. Lett. **35**, 1779 (1975)

[7] K. Kadowaki and S. B. Woods, Solid State Commun. **58**, 507 (1986)

[8] C. L. Seaman $et.al$, Phys. Rev. Lett. **67**, 2882 (1991)

[9] W. J. de Hass, J. H. de Boer and G. J. van den Berg, Physica **1**, 1115 (1934)

[10] M. B. Maple $et.al$, in *The Physics of Conventional and Unconventional Superconductors* (Springer Verlag, Berlin, 2003)

[11] M. B. Maple $et.al$, J. Phys.: Condens. Matter **8**, 9773 (1996)

[12] G. R. Stewart, Rev. Mod. Phys. **73**, 797 (2001)

[13] J. Paglione $et.al$, Phys. Rev. Lett. **97**, 106606 (2006)