Computer Simulations

Faraday's Cage

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1. Exercise description

Program should be able to model a two-dimensional Faraday's cage from top view. Value of potential is fixed for edges of calculated domain, and "cage" is modeled as a few grounded rods (which are actually points in top view). Program evaluates potential distribution within the domain for "cages" that vary in parameters (placement, number of rods, distance between them) and boundary conditions (of first and second kind).

2. Theory

Domain is modeled to be 0V everywhere at starting point (not including edges, that are set for chosen voltage). Then according to Poisson equation(1), potential evolution is calculated (every point voltage in next iteration is calculated by comparison of four "neighbour points" from one iteration ago, and calculation of average). When program achieves desired accuracy, presented plot is actual potential distribution for given case.

$$\frac{\partial^2}{\partial x^2}V(x,y) + \frac{\partial^2}{\partial y^2}V(x,y) = f(x,y) \quad (1)$$

Main equation calculating next potential value for specific point (at x,y coordinate) is presented below(2):

$$V_{x,y}^{n+1} = \frac{V_{(x-1,y)}^{n} + V_{(x+1,y)}^{n} + V_{(x,y-1)}^{n} + V_{(x,y+1)}^{n}}{4} \quad (2)$$

It is worth to note that diagonal "neighbor points" do not communicate at all. Example calculation is presented on Fig. 1.

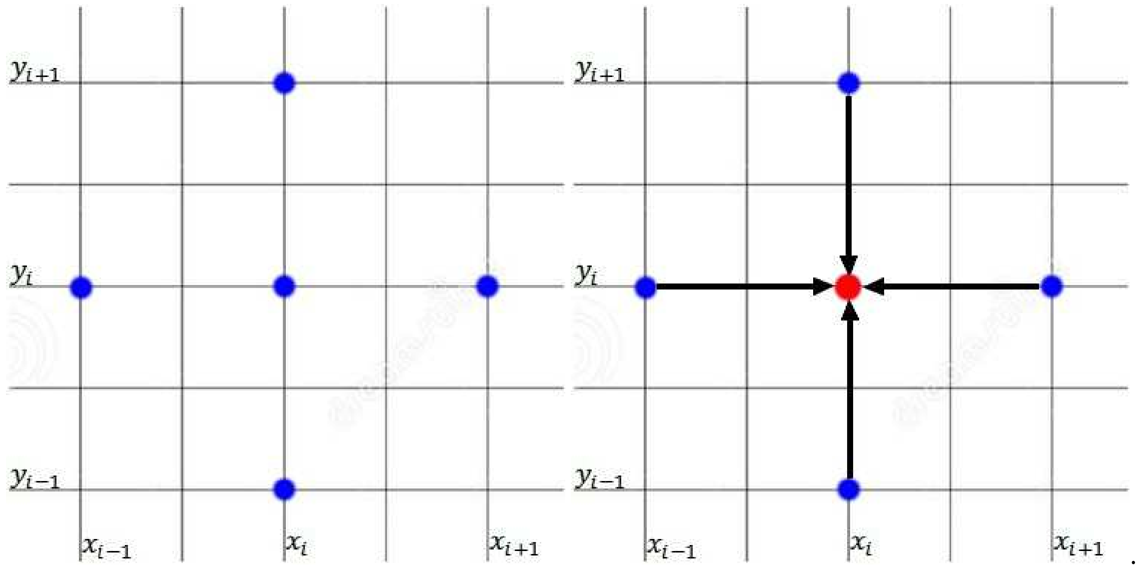

Fig. 1. Time on the left is *t=n*, while on the right it is *t=n+1*. Value of middle point is evaluated by neighbor points.

3. First and second kind boundary conditions.

Boundary conditions of first kind assume that boundary has constant voltage ($V_b = const$). That means, every iteration edge values has to be overwritten, and implies that behind the edges we have infinite capacity "voltage" reservoirs, which may be the case in real-life problems considering potential distribution, but rather not for temperature distribution (only in few simplified models). Boundary conditions of second kind state that potential derivative is constant (that is electric field intensity). To apply it to boundaries properly, it is crucial to insert proper formula that will recalculate (depending on two closest points to boundary) value for each iteration to imitate second boundary conditions.

When in program "sec_kind=" flag will triggered (%t), second kind boundary conditions will be applied to top and bottom border. Left and right borders are still first kind. "Visual" difference is presented on plot below, where first and second kind boundary conditions are compared.

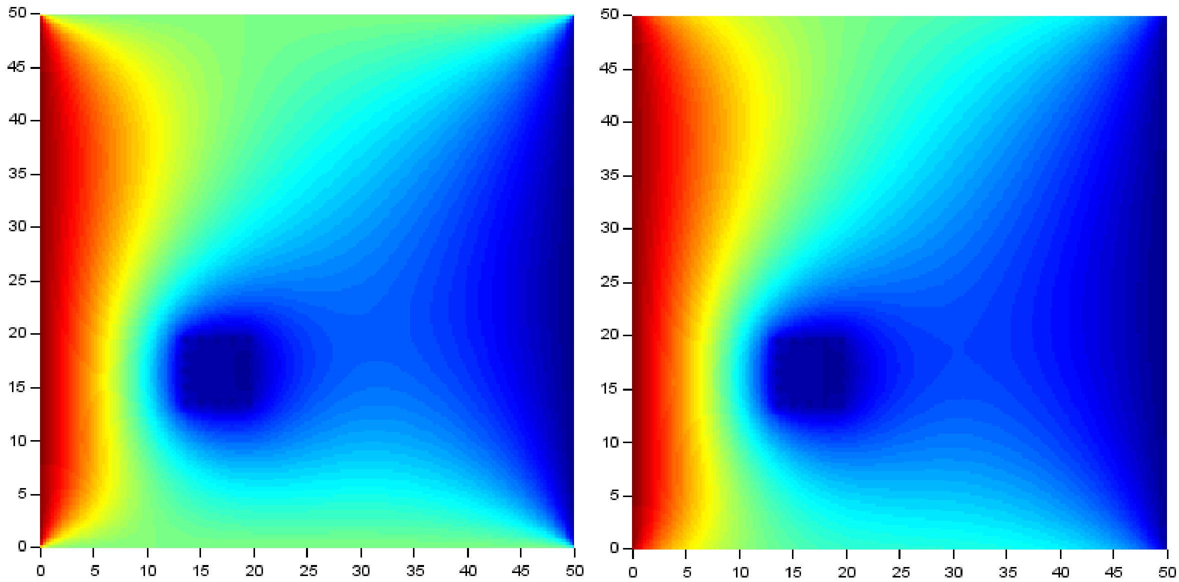

Fig. 2. Potential distribution in first kind boundary conditions (left) and second kind (right). Both cases are perfectly the same (excluding boundaries).

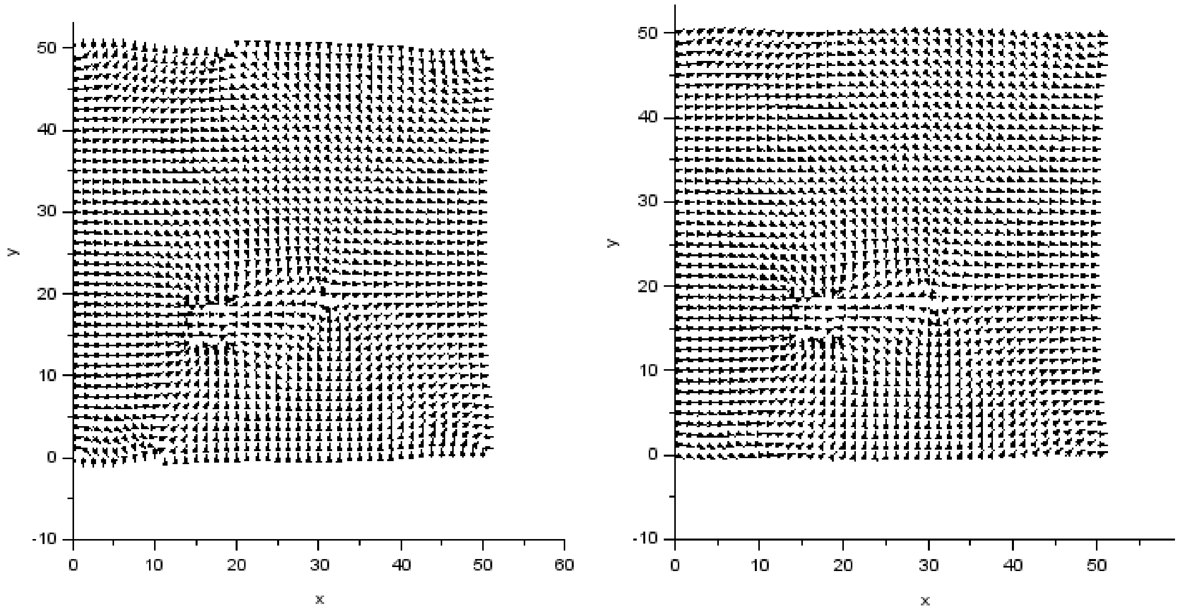

Fig. 3. Electric field intensity in first kind boundary conditions (left) and second kind (right). Both cases are perfectly the same (excluding boundaries).

4. Program

To run program, some values must be input. They are listed below:

- Domain width and height,
- Values of potential on each border,
- Number of rods, that make up Faraday's Cage (number must be a power of two (4,8,16,32)),
- Space between rods,

Program produces plot of potential distribution. It is possible (with changing flag ani=%t) to see animation exposing process of calculation. Few examples for square domain (50x50) are shown below with different parameters, their description is as follows:

VL - Left border voltage
VR - Right border voltage
VB - Bottom border voltage
VT - Top border voltage
NoR - Number of rods
DbR - Distance between the rods
1st/2nd - First / second boundary conditions

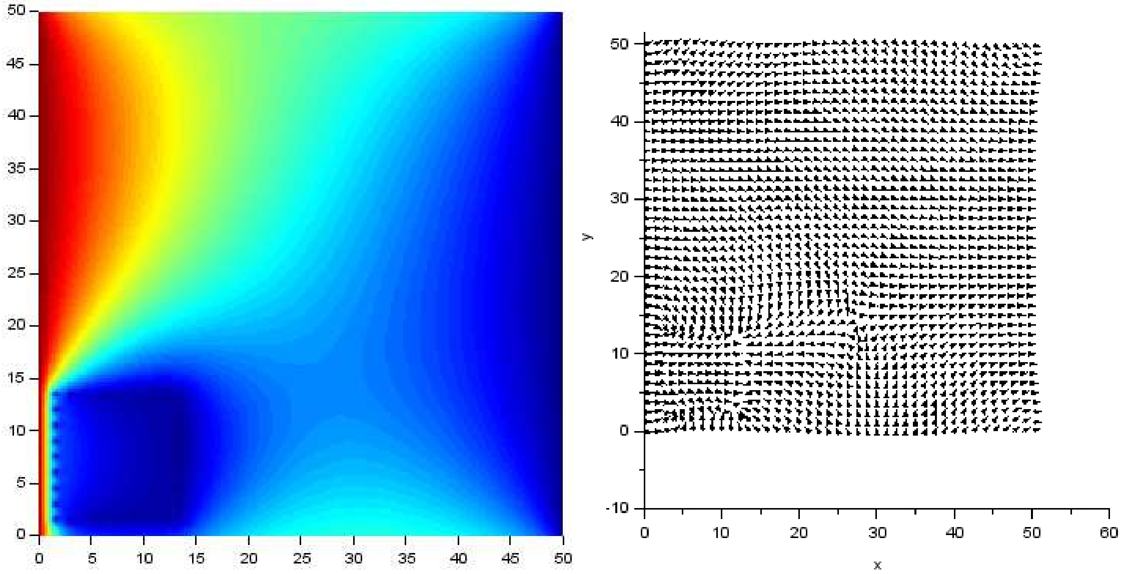

Fig. 4 - VL = 40, VR = 0, VB = 20, VT = 20, NoR = 32, DbR = 5, 2nd.

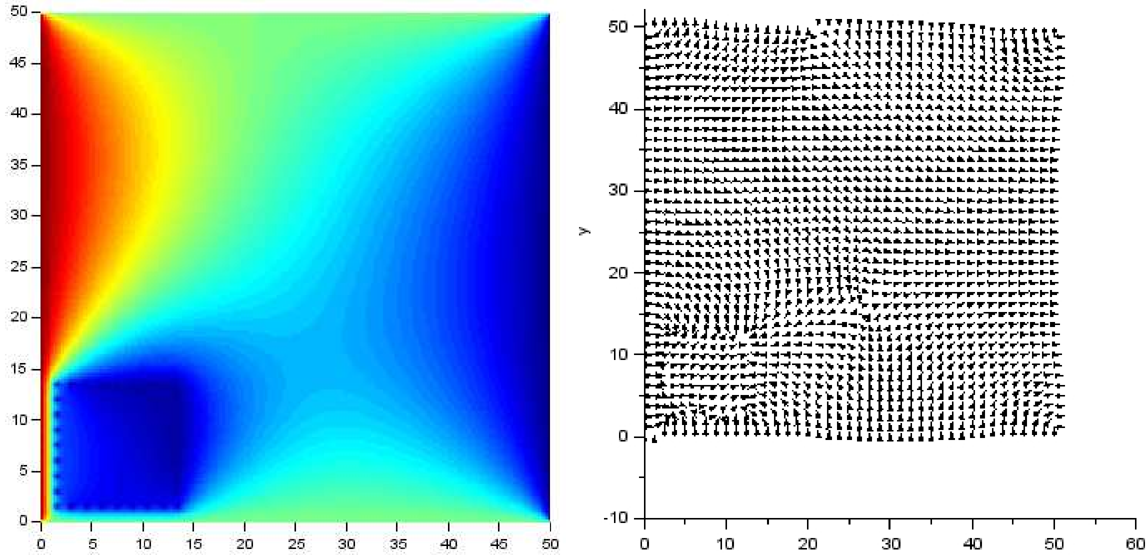

Fig. 5 - VL = 40, VR = 0, VB = 20, VT = 20, NoR = 32, DbR = 5, 1st.

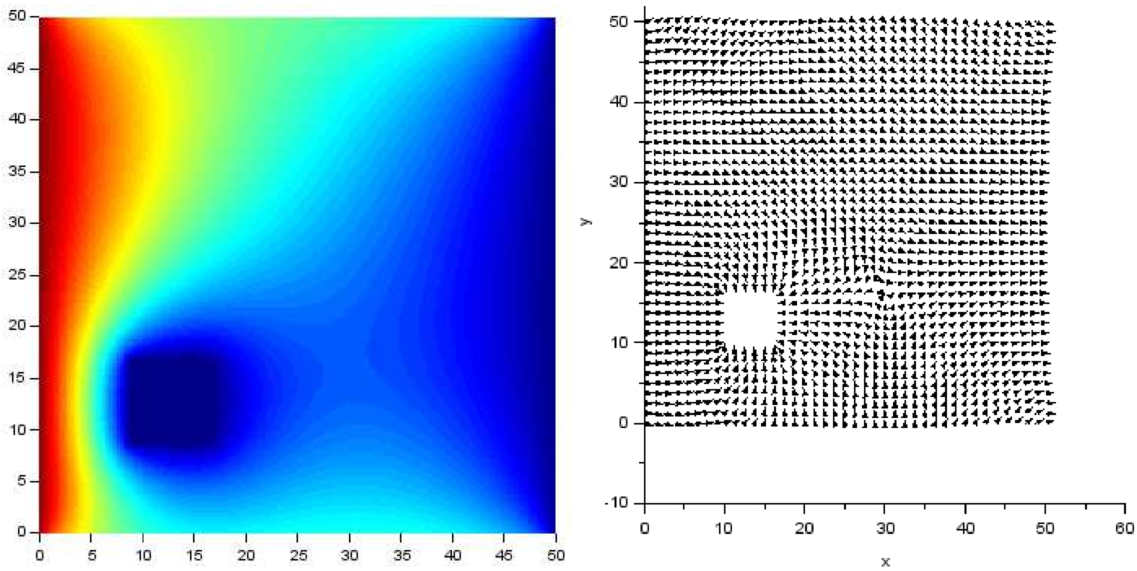

Fig. 6 - VL = 40, VR = 0, VB = 20, VT = 20, NoR = 128, DbR = 0, 2nd.

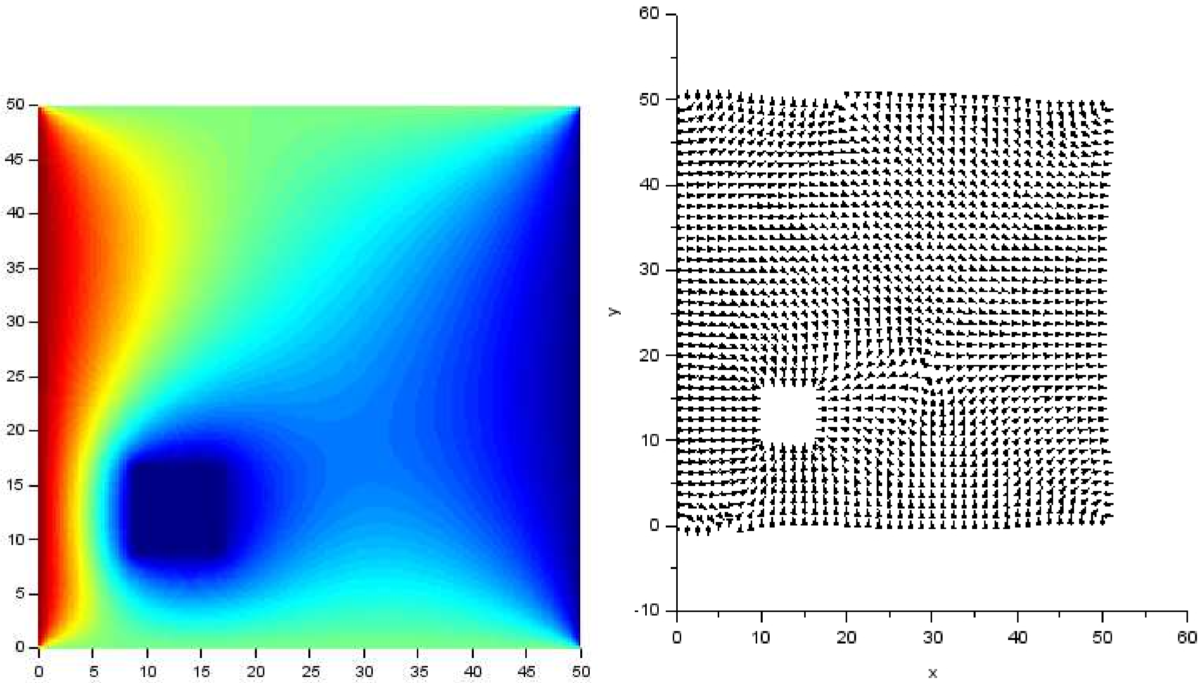

Fig. 7 - VL = 40, VR = 0, VB = 20, VT = 20, NoR = 128, DbR = 0, 1st.