

DAGs

Tools for reasoning about causes

$$P(y|\hat{x}) = \sum_z P(y|\hat{x}, z)P(z|\hat{x})$$

Using total probability law

$$\begin{cases} G_x = \overset{\circ}{x} \rightarrow \overset{\circ}{z} \\ = \sum_z P(y|\hat{x}, z)P(z|x) \end{cases}$$

$z \perp\!\!\!\perp x$ so rule (II) yields $P(z|\hat{x}) = P(z|x)$

$$\begin{cases} G_{xz} = \overset{\circ}{x} \rightarrow \overset{\circ}{z} \\ = \sum_z P(y|\hat{x}, z)P(z|x) \end{cases}$$

$$P(y|\hat{x}, z) = P(y|\hat{x}, \hat{z})$$

$$= \sum_z P(y|\hat{x}, \hat{z})P(z|x)$$

$$\begin{cases} G_{xz} = \overset{\circ}{x} \rightarrow \overset{\circ}{z} \\ x \perp\!\!\!\perp y, \text{ rule (II)} \end{cases}$$

The Marko fallacy



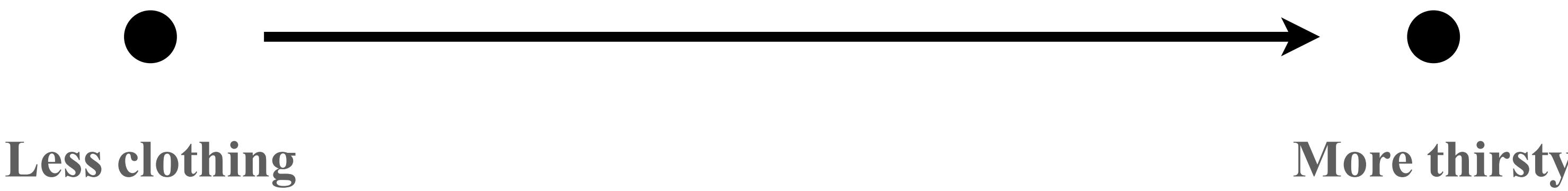
The fundamental problem of observational research:

seeing X ≠ doing X

Baby's first DAG: Marko's causal model



(D)irected (A)cyclic (G)raph



(D)irected (A)cyclic (G)raph

Technical term for some dots joined up with lines

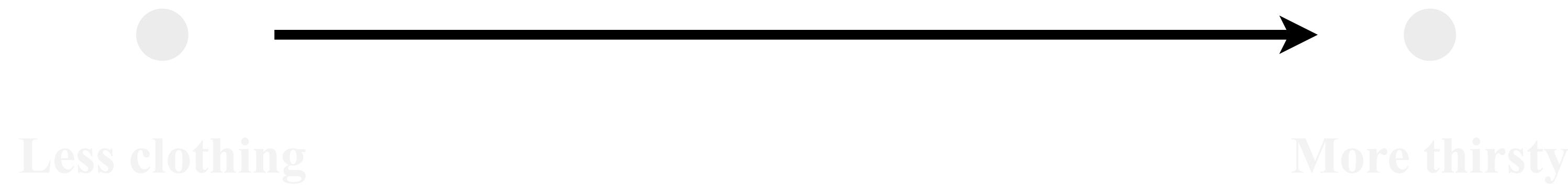


Less clothing

More thirsty

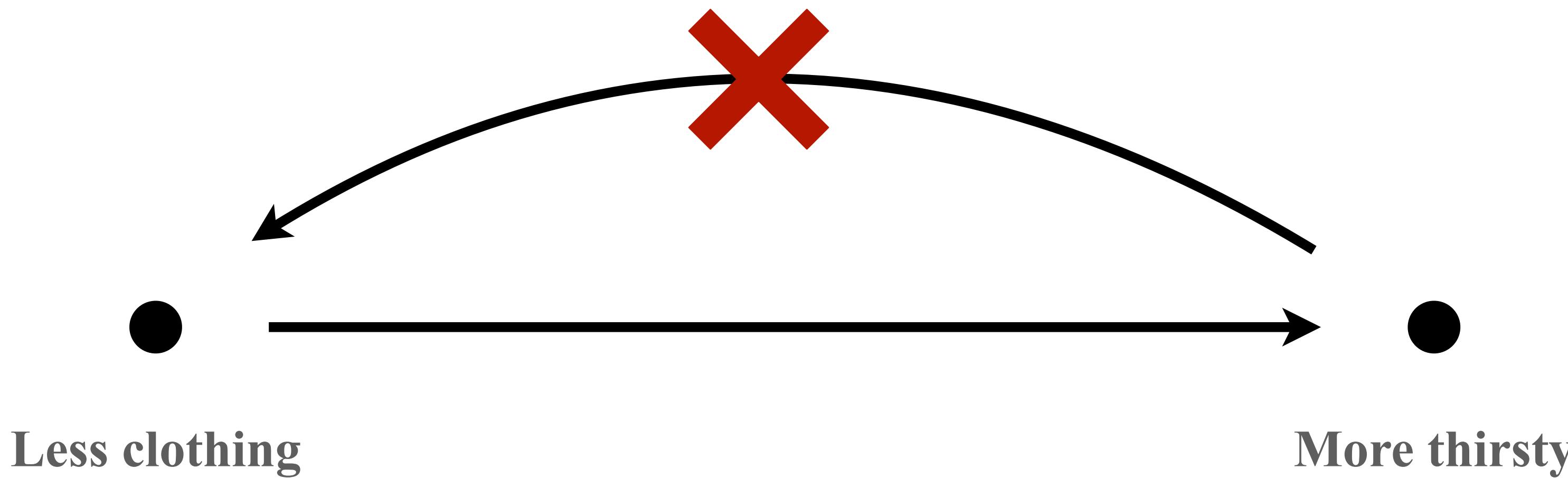
(D)irected **(A)cyclic** **(G)raph**

The lines have direction



(D)irected (A)cyclic (G)raph

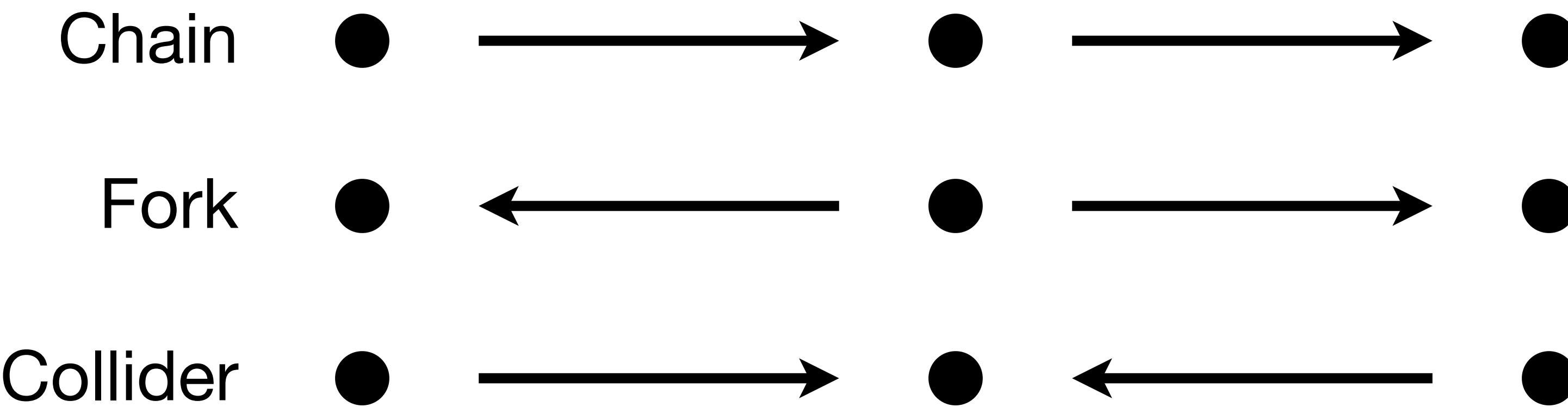
No cycles: you can't go backward, a thing cannot be its own cause



An arrow in the graph is a statement
about a thought experiment

DAG anatomy

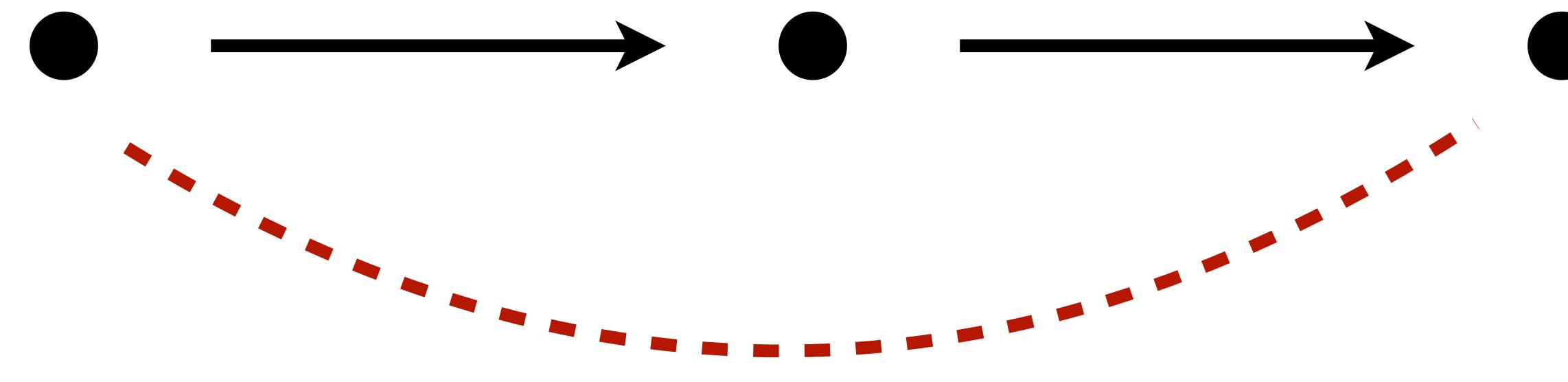
Three basic junctions



Chain: A causes B causes C

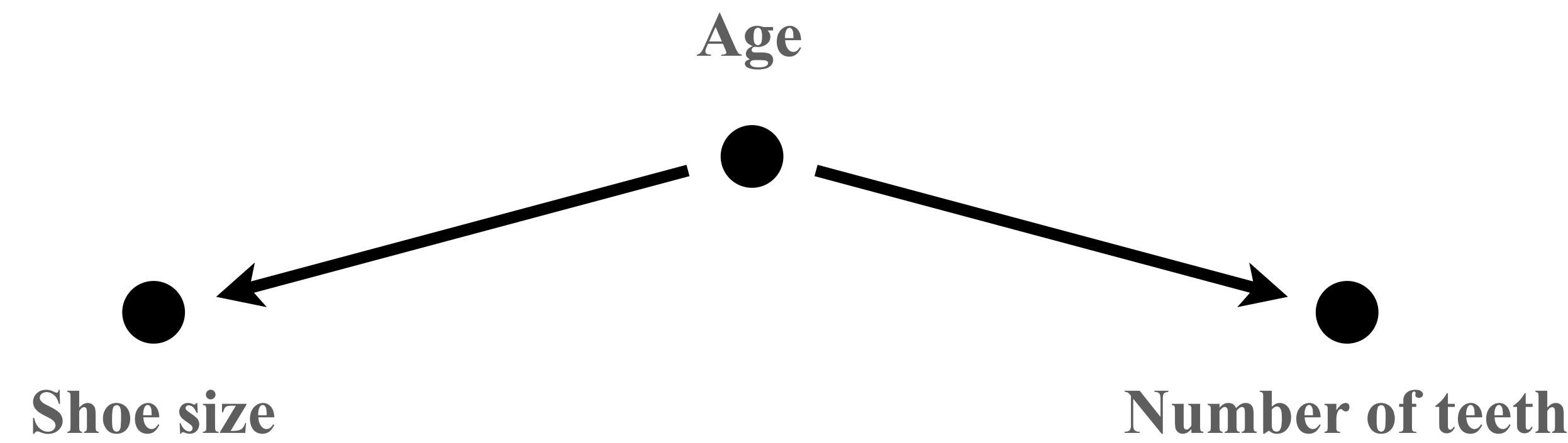


Chain: A causes B causes C

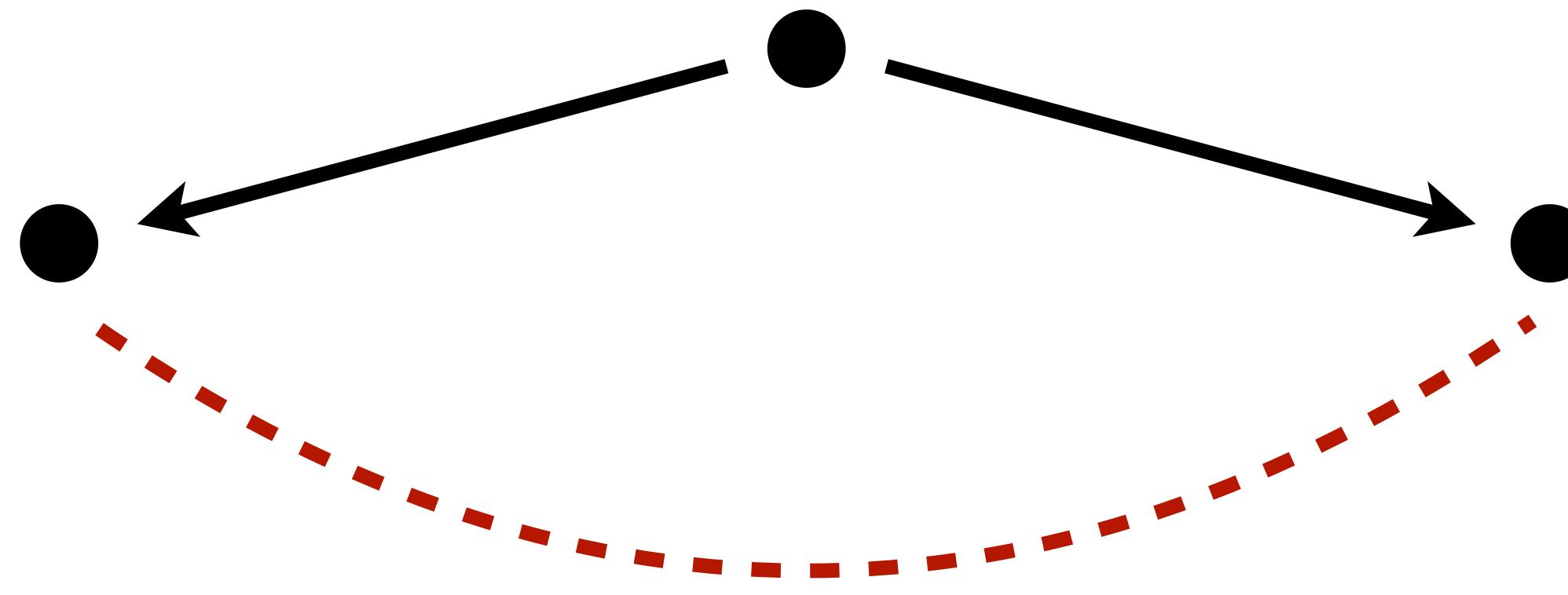


**Transmits information:
an alarm indicates fire, fire indicates alarm**

Fork: your typical confounder

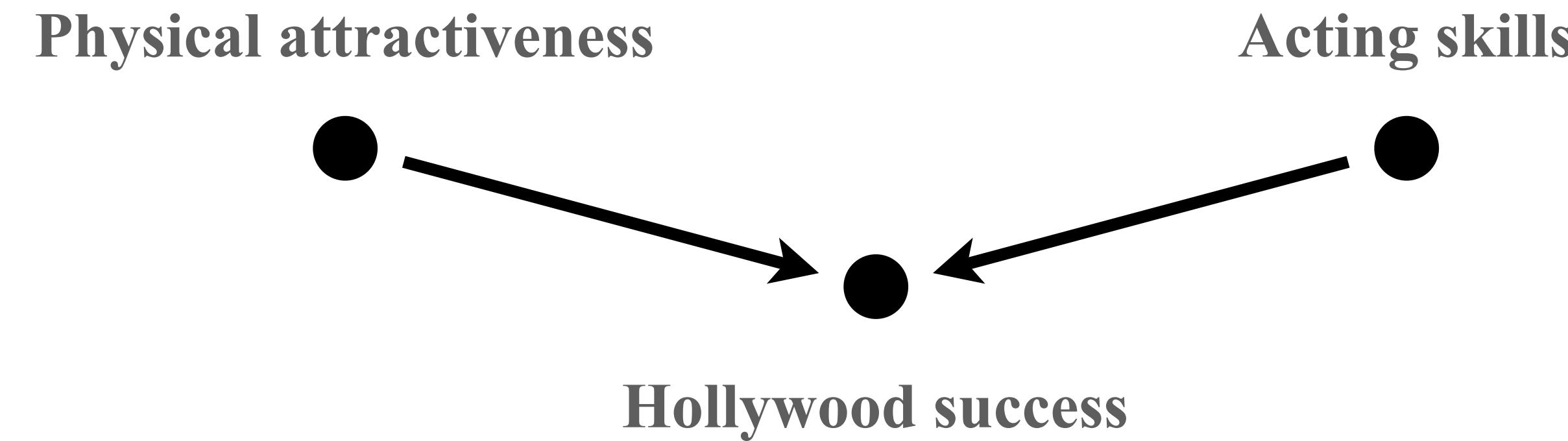


Fork: your typical confounder

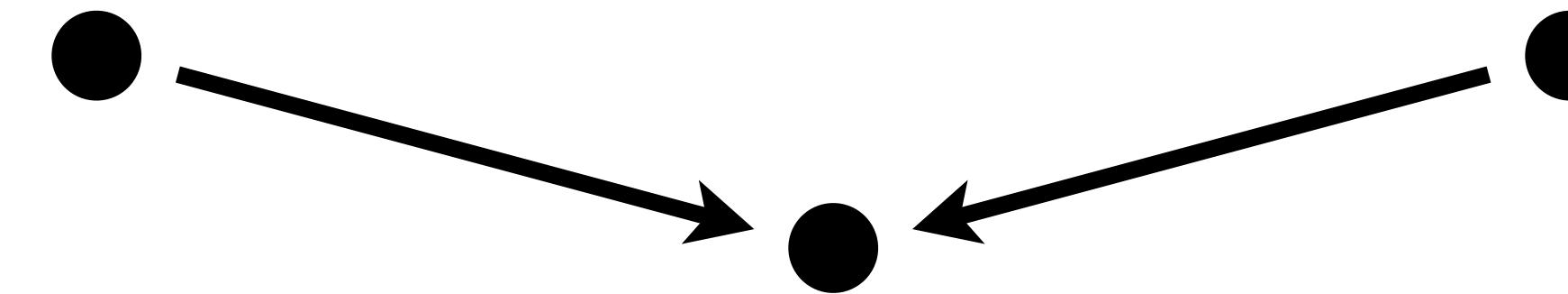


**Transmits information:
Few teeth indicates small shoes,
small shoes indicates few teeth**

Collider: a very interesting source of bias



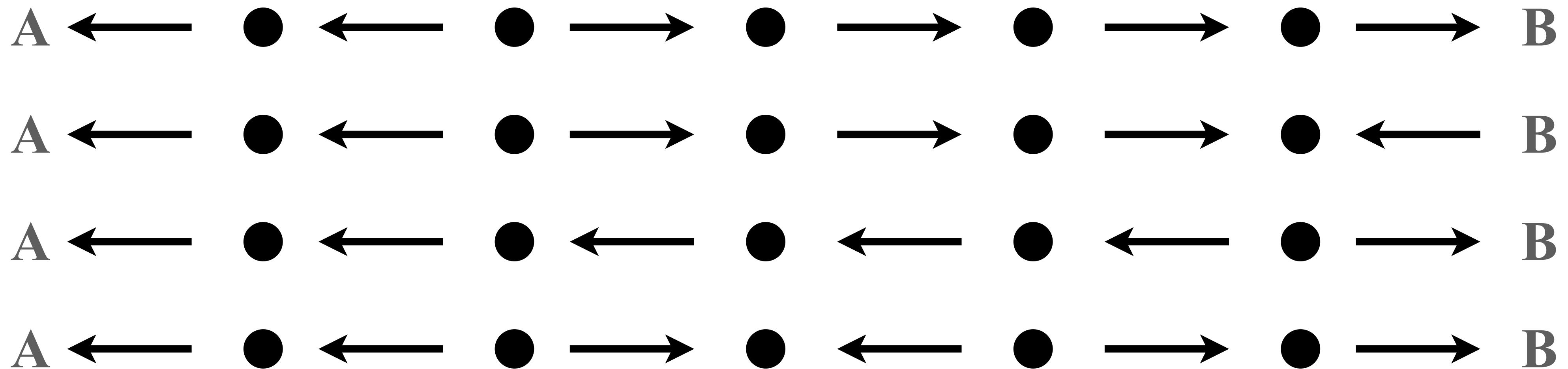
Collider: a very interesting source of bias



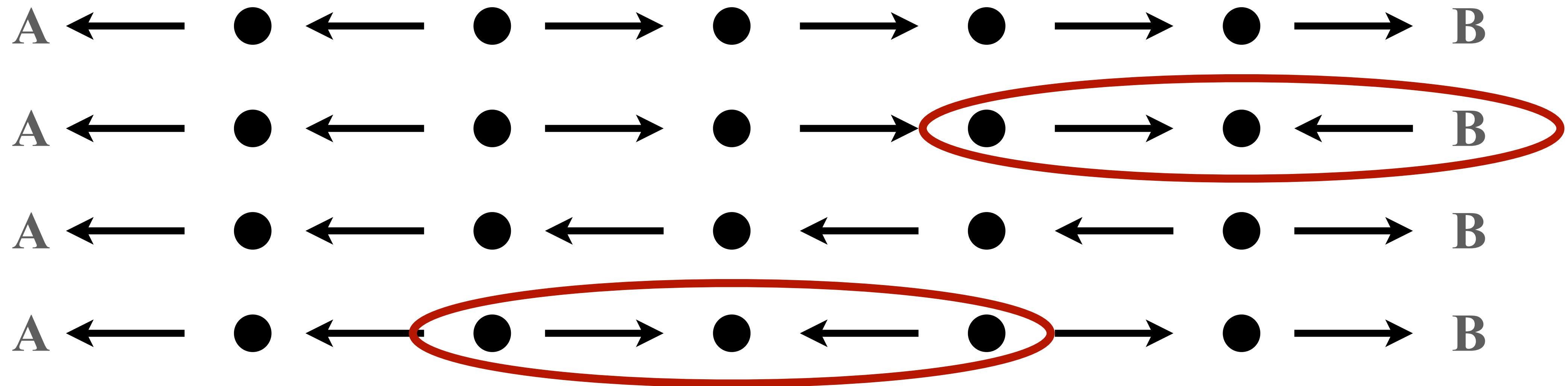
Does not transmit information:

**acting skills doesn't suggest attractiveness
or vice versa**

Puzzles: does information flow between A and B?



Puzzles: does information flow between A and B?



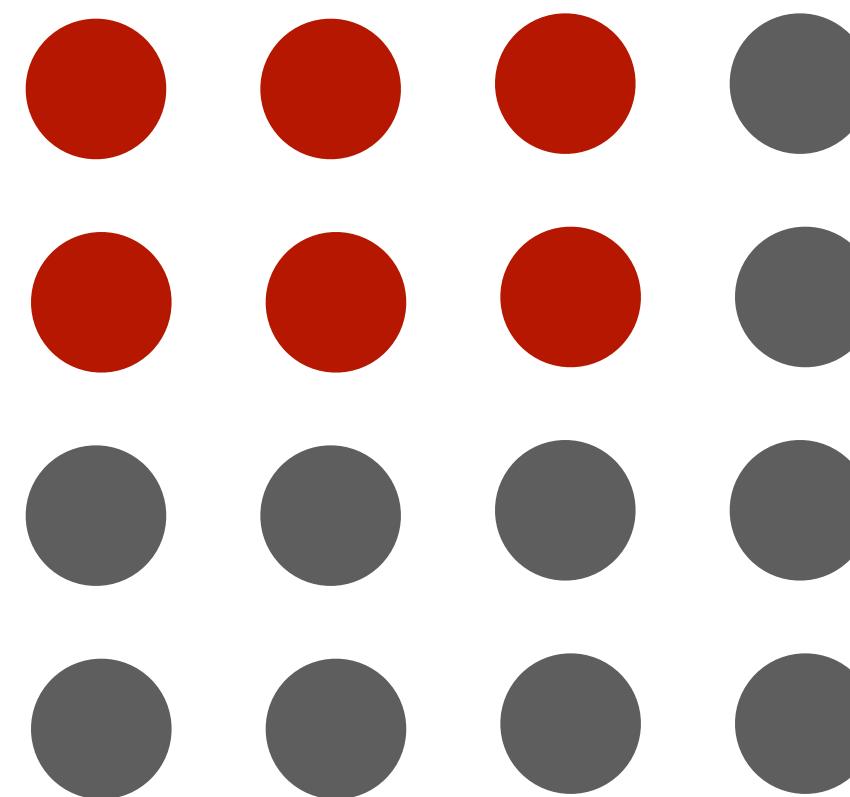
Probability intermission

Probability as counting of outcomes

$$\frac{\# \text{ interesting outcomes}}{\# \text{ possible outcomes}}$$

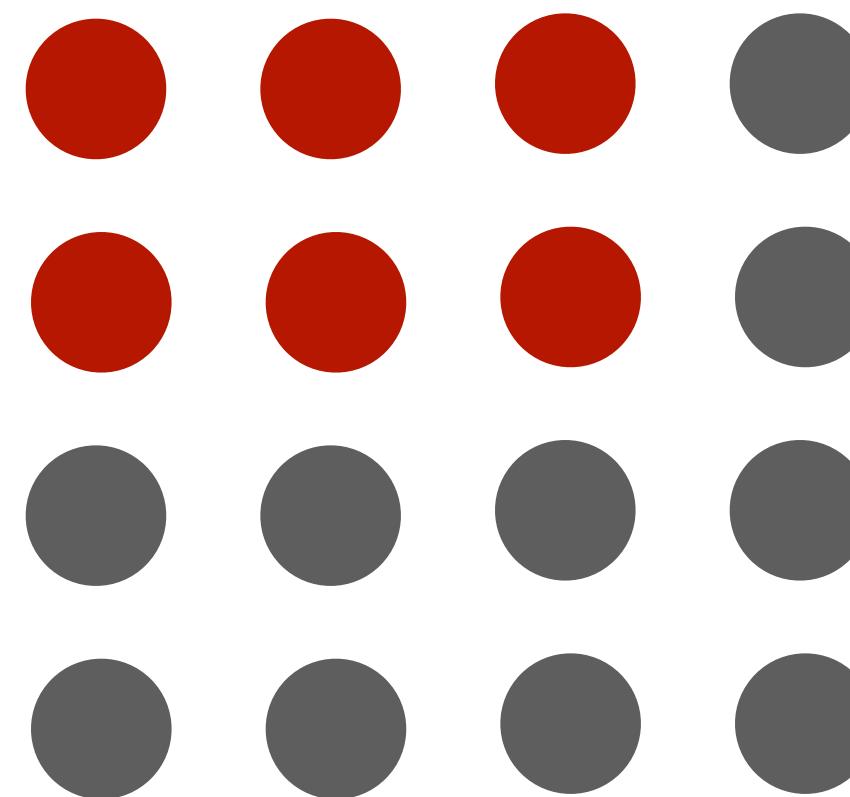
Probability as counting of outcomes

$$\frac{\# \text{ interesting outcomes}}{\# \text{ possible outcomes}}$$



Probability as counting of outcomes

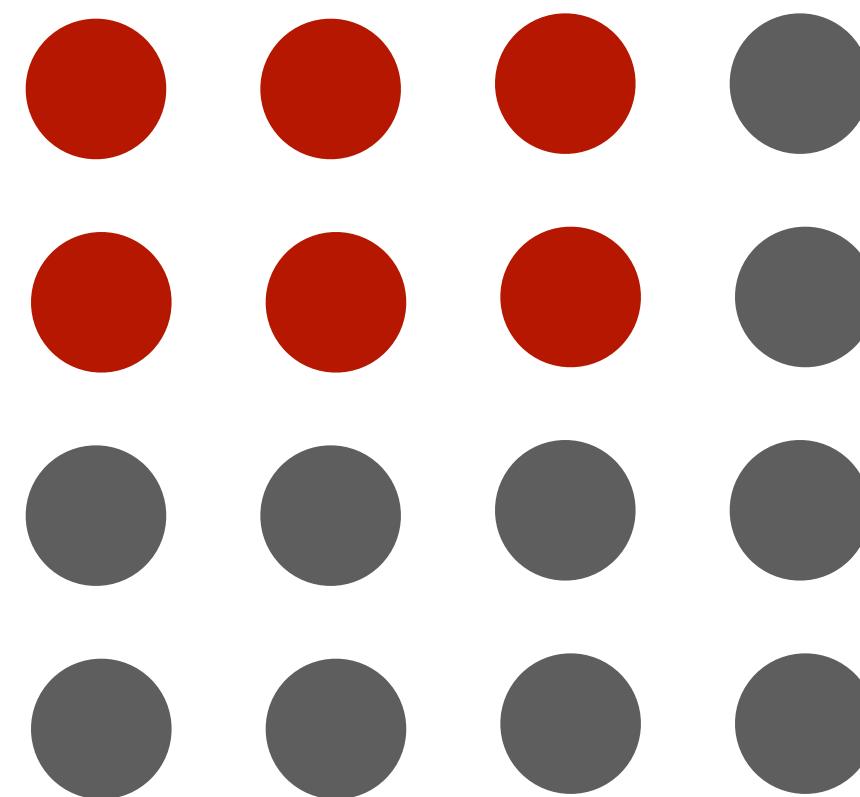
$$\frac{\# \text{ interesting outcomes}}{\# \text{ possible outcomes}}$$



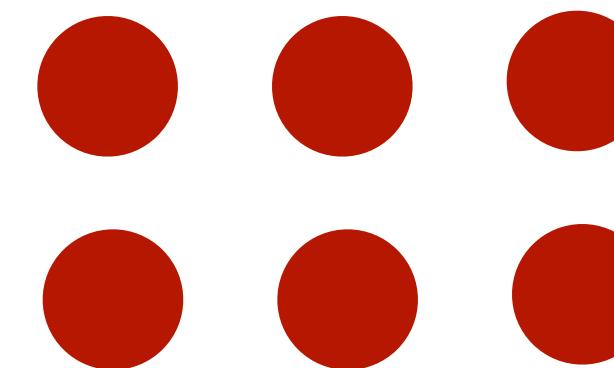
$$P(\bullet) =$$

Probability as counting of outcomes

$$\frac{\# \text{ interesting outcomes}}{\# \text{ possible outcomes}}$$

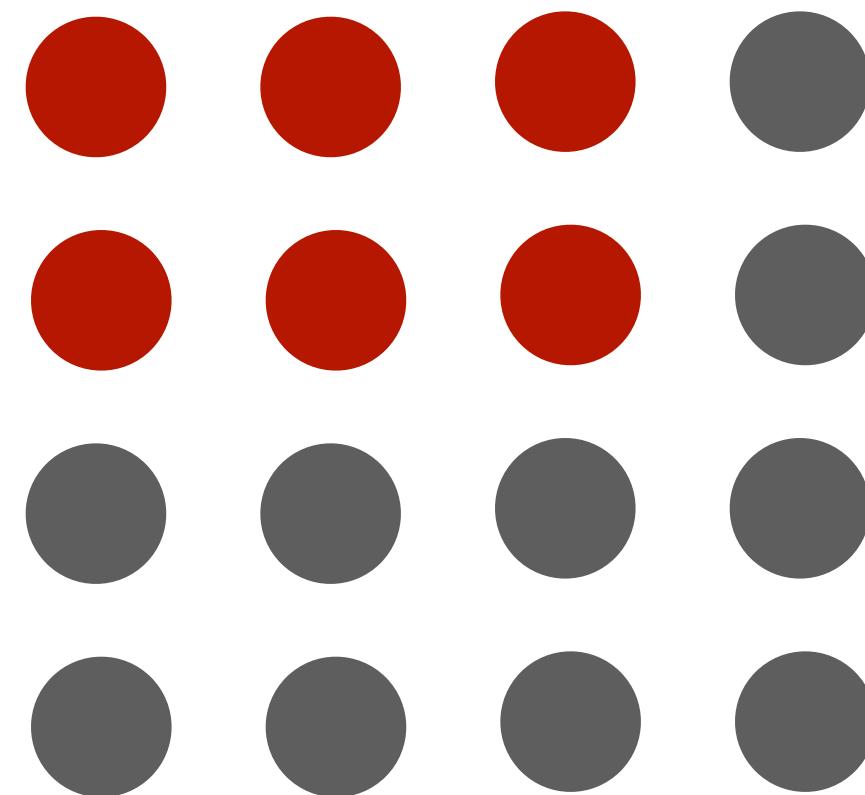


$$P(\bullet) =$$

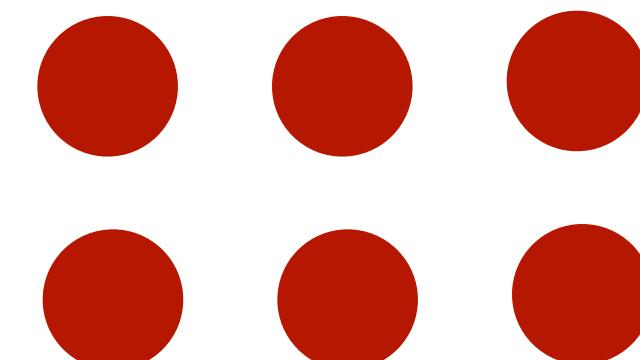


Probability as counting of outcomes

$$\frac{\# \text{ interesting outcomes}}{\# \text{ possible outcomes}}$$

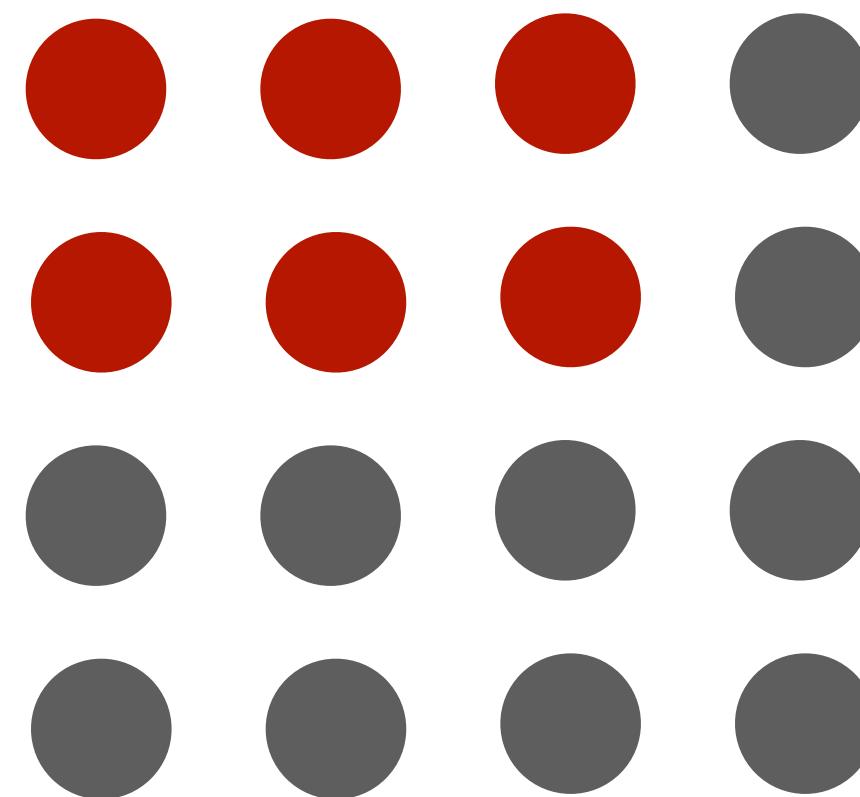


$$P(\text{red}) = \frac{\# \text{ red outcomes}}{\# \text{ possible outcomes}}$$



Probability as counting of outcomes

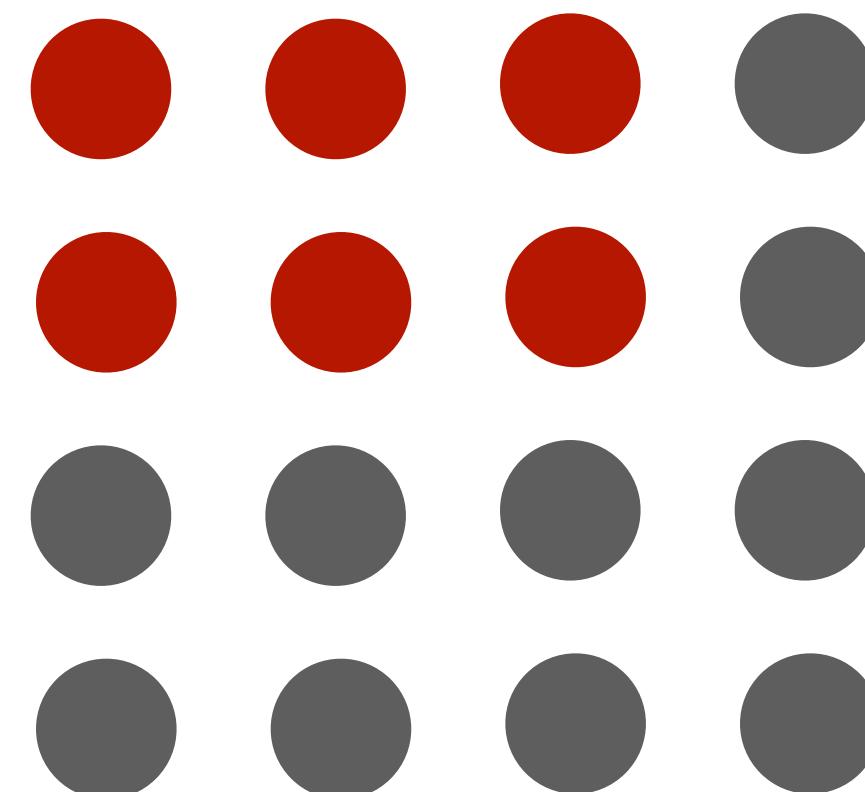
$$\frac{\# \text{ interesting outcomes}}{\# \text{ possible outcomes}}$$



$$P(\text{red}) = \frac{\text{# red circles}}{\text{# possible outcomes}}$$
A 4x4 grid of circles, all of which are red.

Probability as counting of outcomes

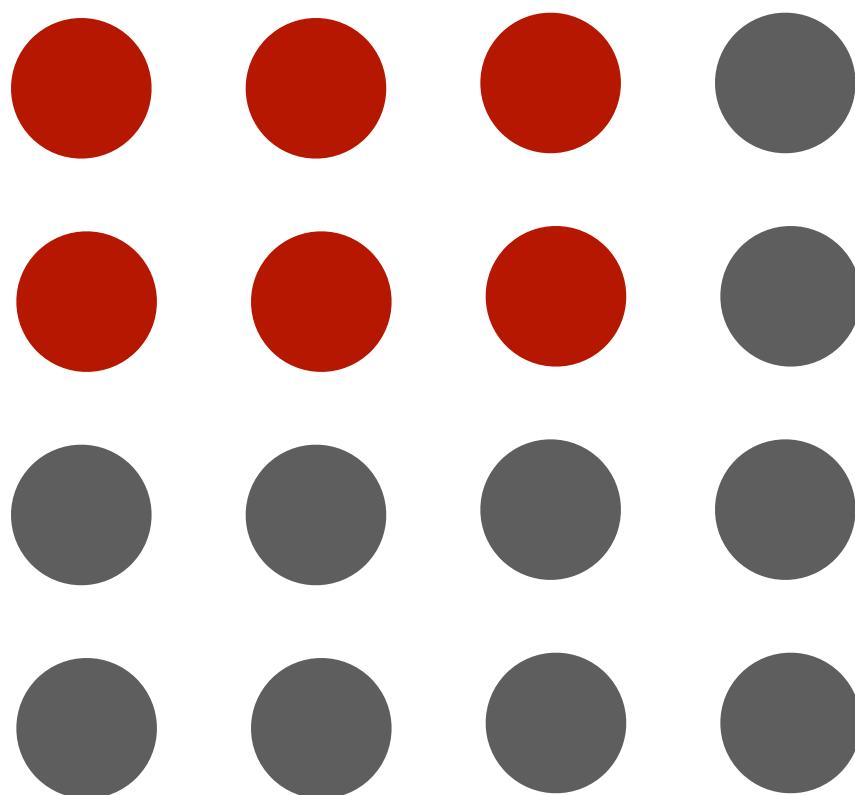
$$\frac{\# \text{ interesting outcomes}}{\# \text{ possible outcomes}}$$



$$P(\text{red}) = \frac{\# \text{ red outcomes}}{\# \text{ possible outcomes}} = \frac{6}{16}$$
A 4x4 grid of circles, all of which are red.

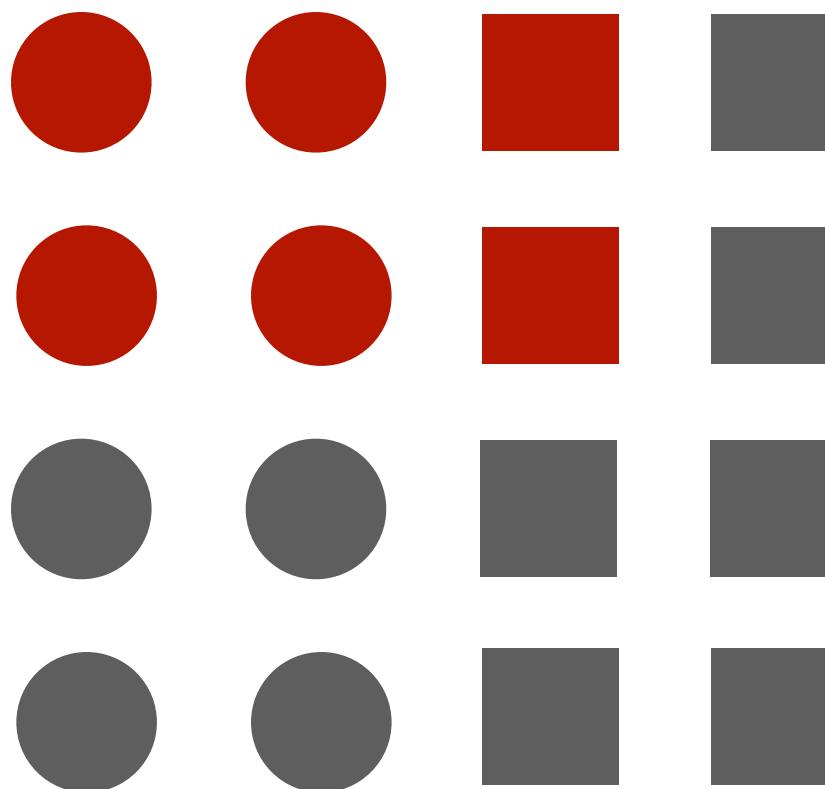
Conditional probability

Counting outcomes in a specific subset



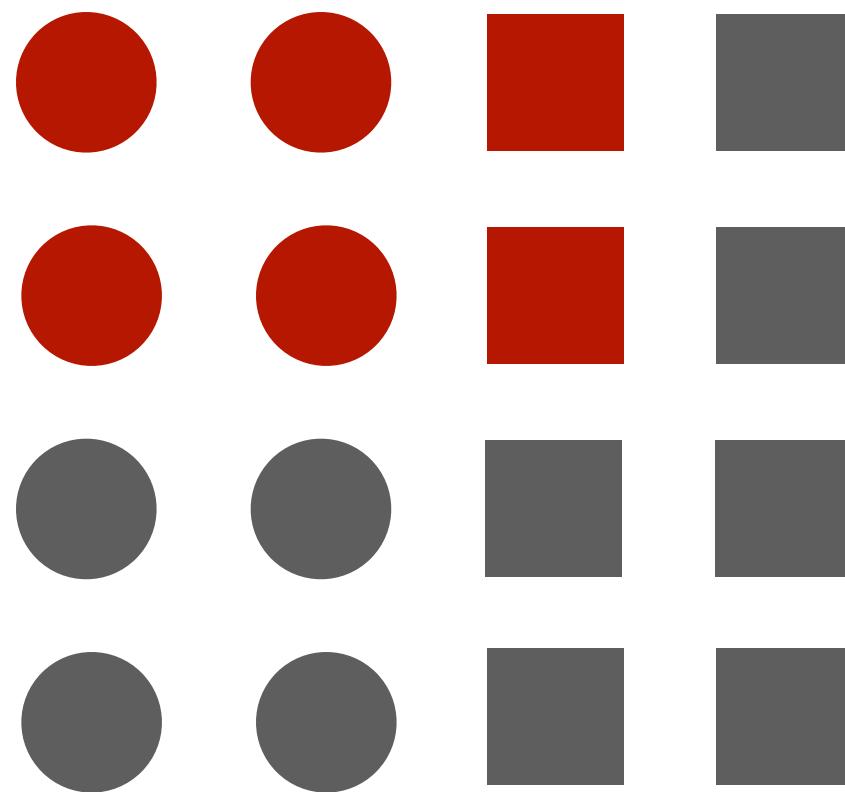
Conditional probability

Counting outcomes in a specific subset



Conditional probability

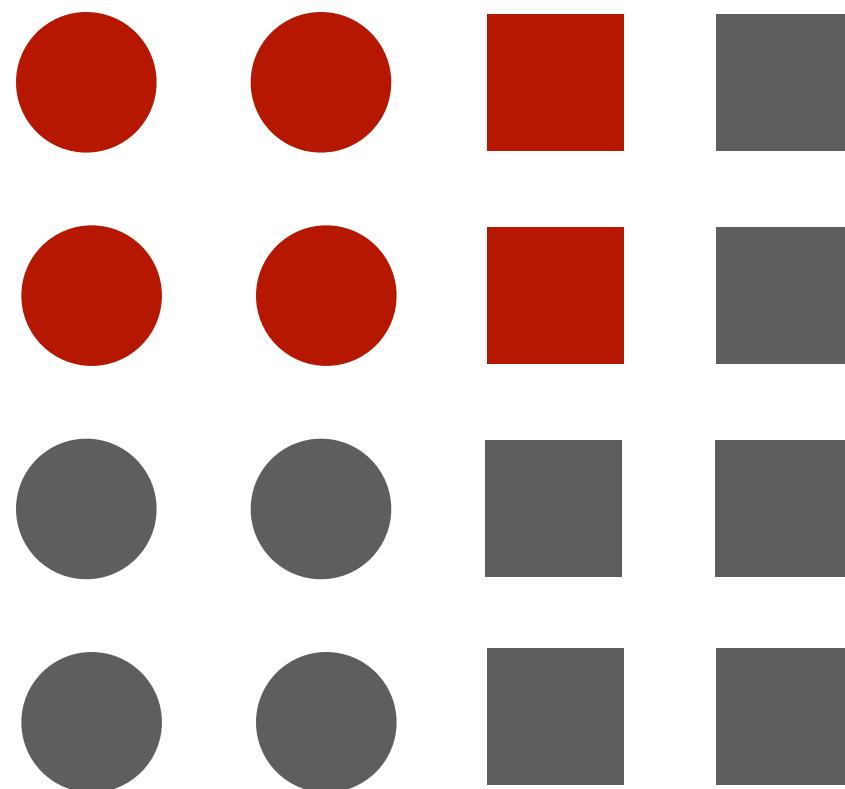
Counting outcomes in a specific subset



P(

Conditional probability

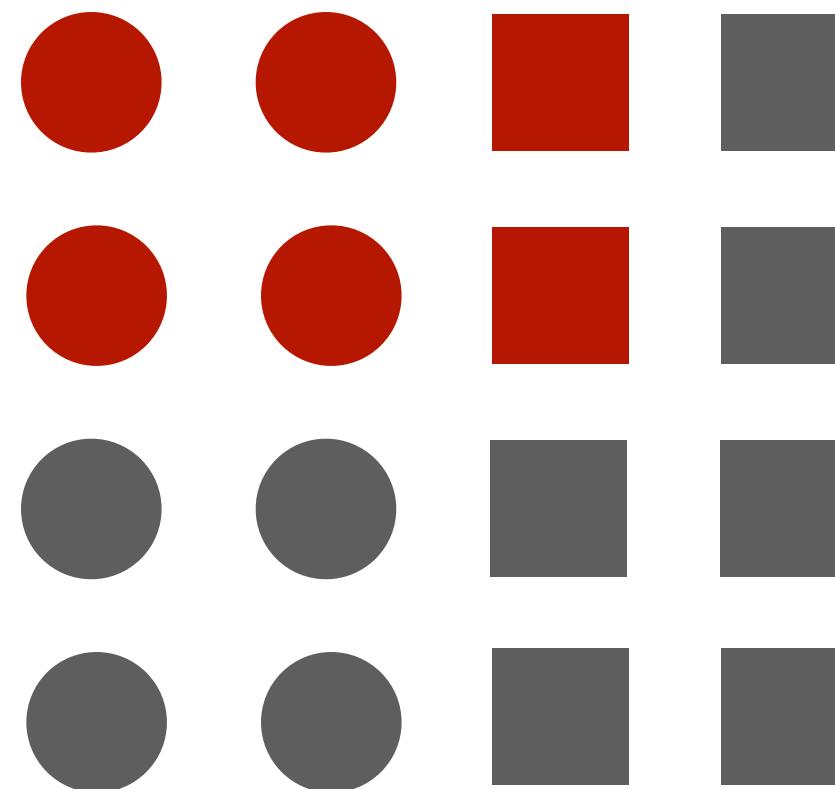
Counting outcomes in a specific subset



$$P(\text{ } \mid \square) =$$

Conditional probability

Counting outcomes in a specific subset

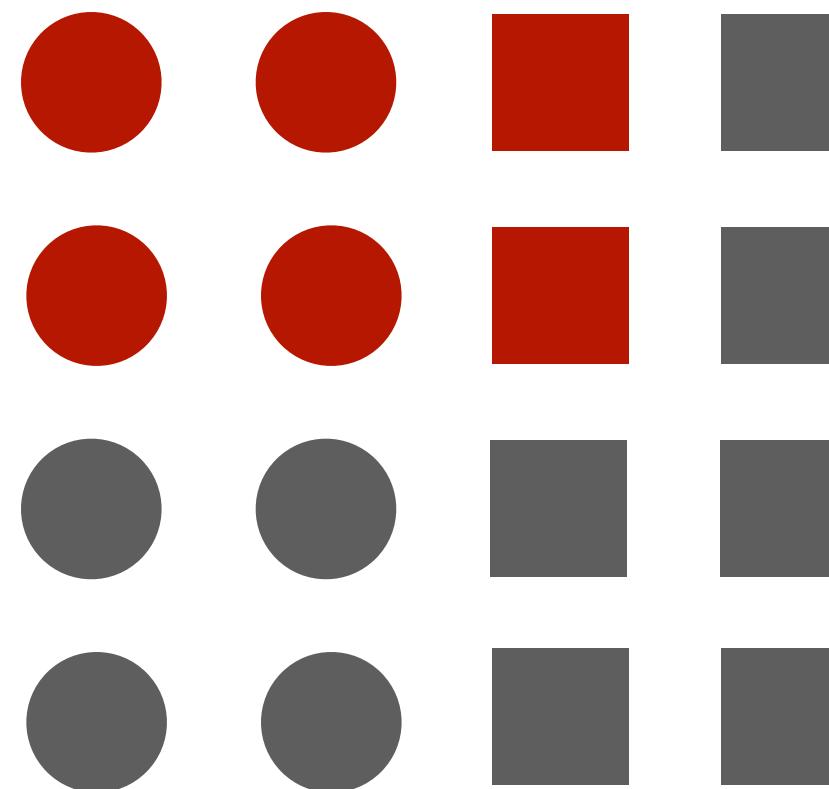


$$P(\text{Red} \mid \text{Square}) =$$

“Probability of red color given square shape” or “probability of red color conditional on square shape”

Conditional probability

Counting outcomes in a specific subset

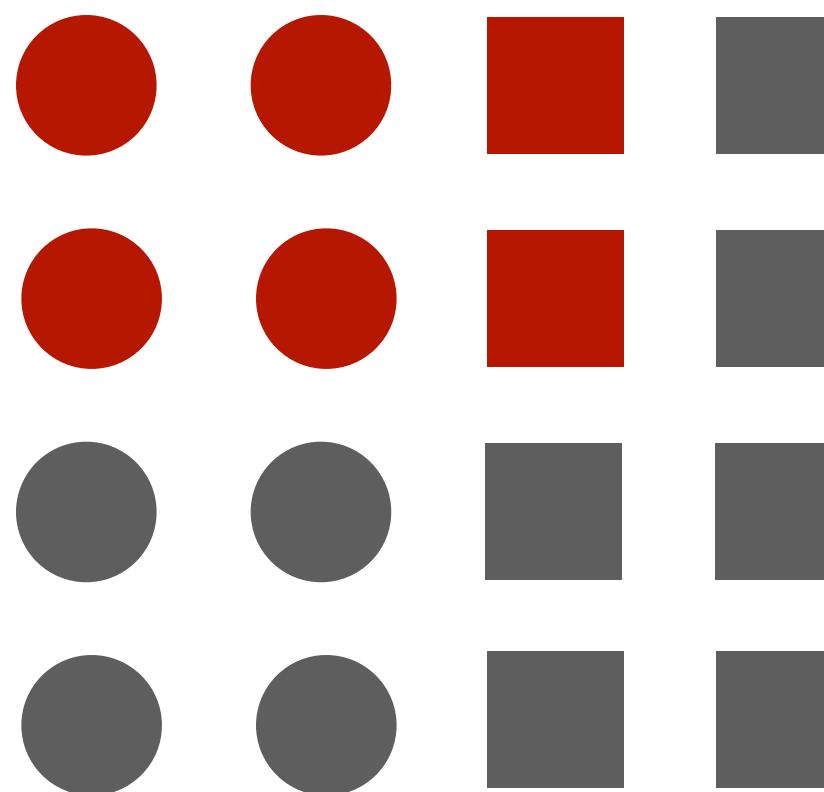


$$P(\text{Red} \mid \text{Square}) = \underline{\hspace{2cm}}$$

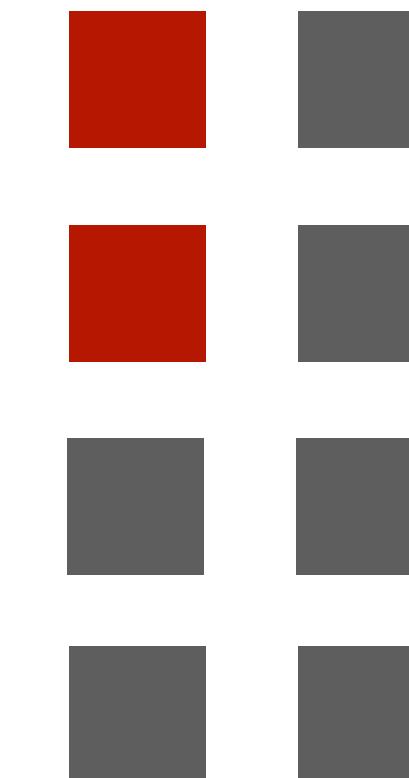
“Probability of red color given square shape” or “probability of red color conditional on square shape”

Conditional probability

Counting outcomes in a specific subset



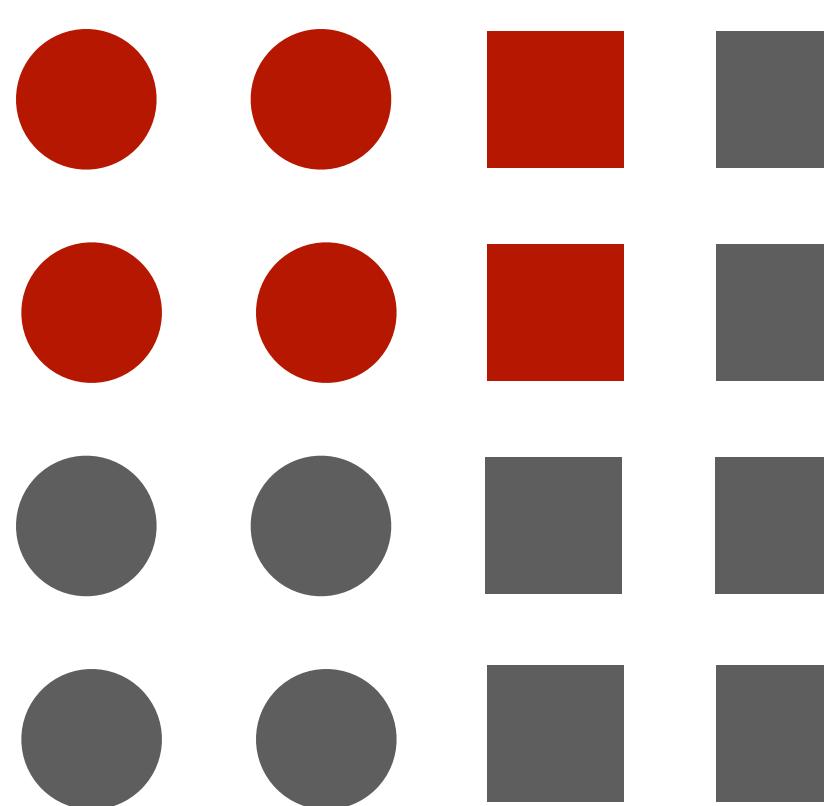
$$P(\text{Red} \mid \text{Square}) = \frac{\text{Number of Red Squares}}{\text{Total Number of Squares}}$$



“Probability of red color given square shape” or “probability of red color conditional on square shape”

Conditional probability

Counting outcomes in a specific subset



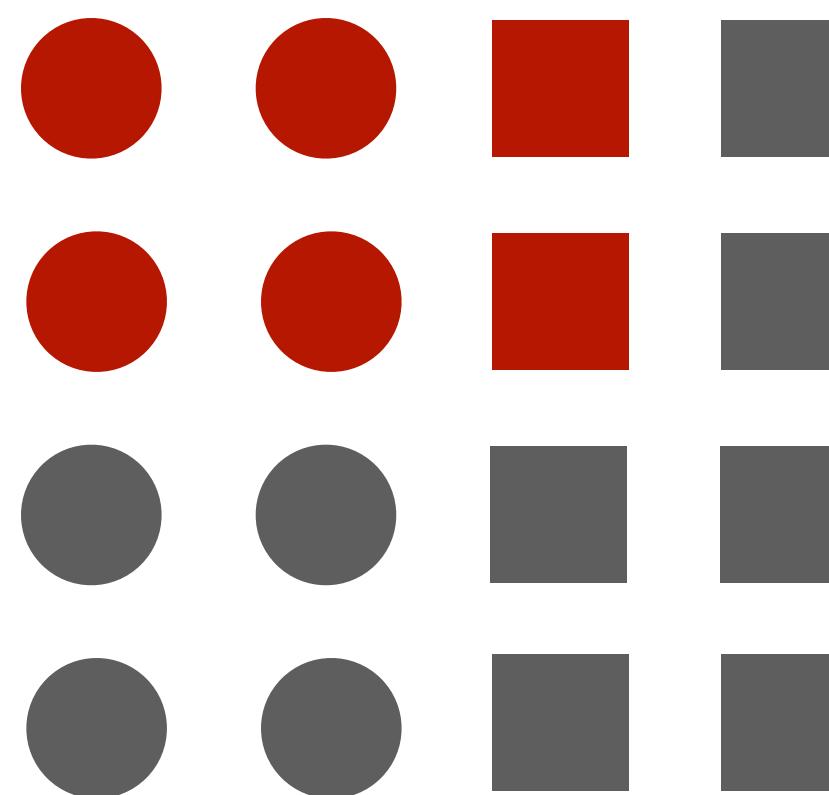
$$P(\text{Red} \mid \text{Square}) = \frac{\text{Number of Red Squares}}{\text{Total Number of Squares}}$$

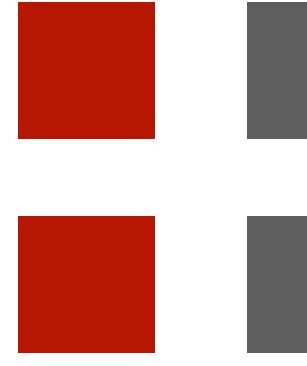
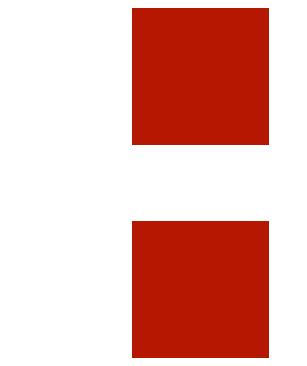
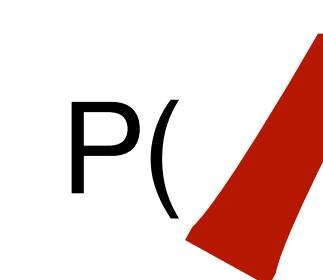
The equation shows the probability of a shape being red given that it is a square. The numerator is represented by four red squares arranged in a 2x2 pattern. The denominator is represented by eight squares in total, arranged in a 4x2 pattern.

“Probability of red color given square shape” or “probability of red color conditional on square shape”

Conditional probability

Counting outcomes in a specific subset

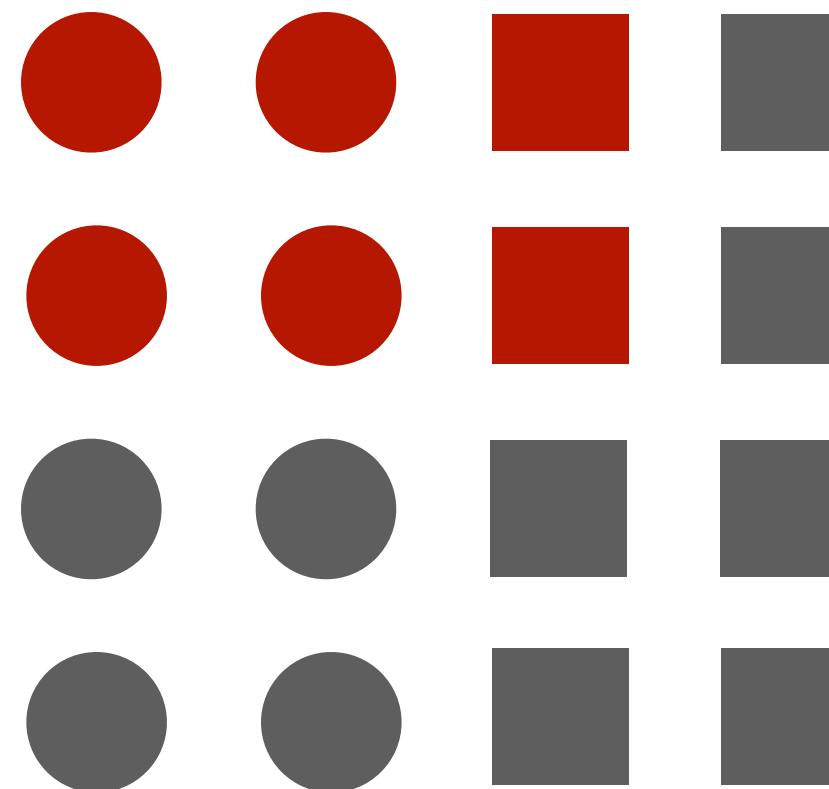


$$P(\text{Red} \mid \text{Square}) = \frac{\text{Number of Red Squares}}{\text{Total Number of Squares}} = 2/8$$


“Probability of red color given square shape” or “probability of red color conditional on square shape”

Conditional probability

Counting outcomes in a specific subset



$$P(\text{Red} \mid \text{Square}) = \frac{\text{Number of Red Squares}}{\text{Total Number of Squares}} = \frac{2}{8} \neq \frac{6}{16}$$

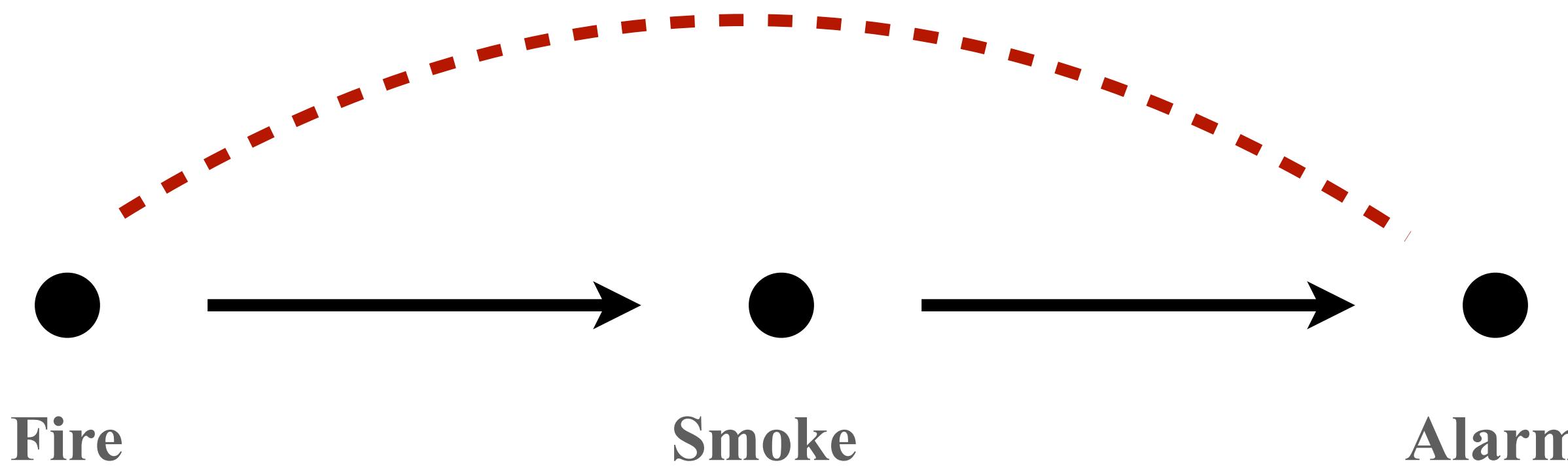
A red highlighter icon pointing to the first term in the probability formula, $P(\text{Red})$.

“Probability of red color given square shape” or “probability of red color conditional on square shape”

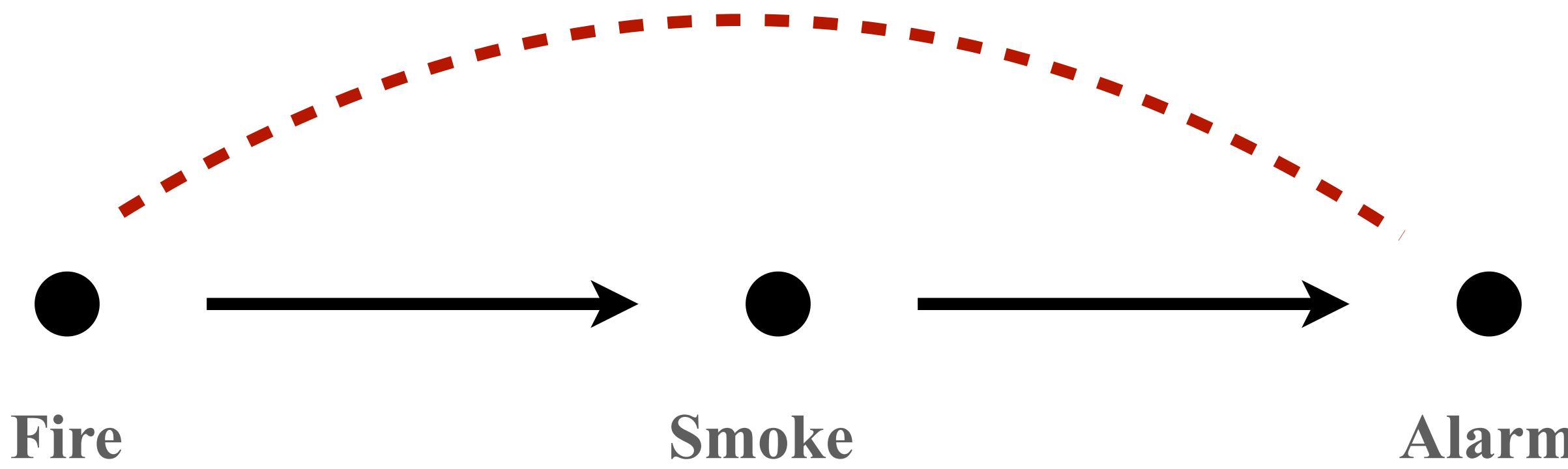
**Conditioning alters the flow of
information in the 3 basic junctions**

NB: the three data sets I use in this section are fake – I generated them with a computer to illustrate my points

Transmits info



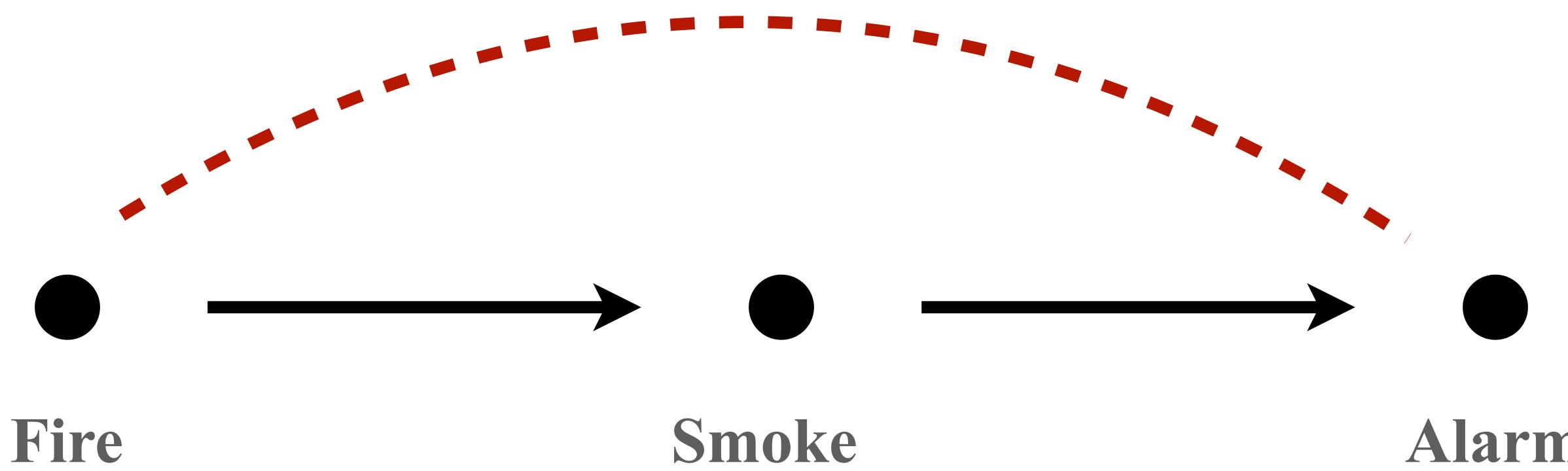
Transmits info



No alarm Alarm

No fire	9338	153
Fire	9	500

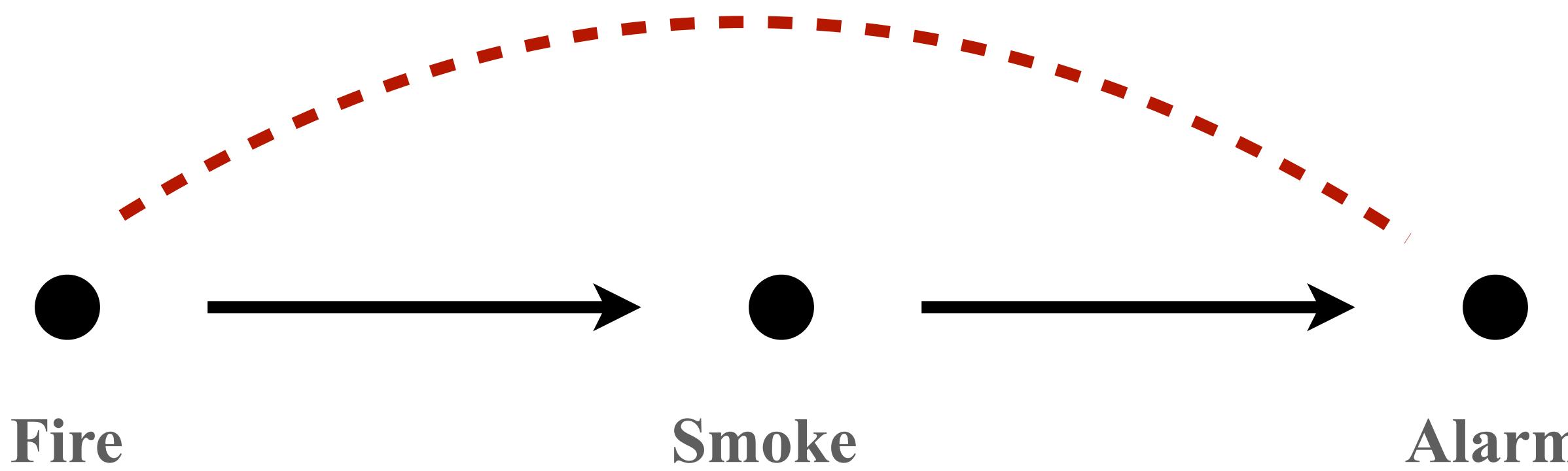
Transmits info



	No alarm	Alarm
No fire	9338	153
Fire	9	500

$$P(\text{ fire }) = \frac{\text{No fire}}{\text{No fire} + \text{Fire}}$$

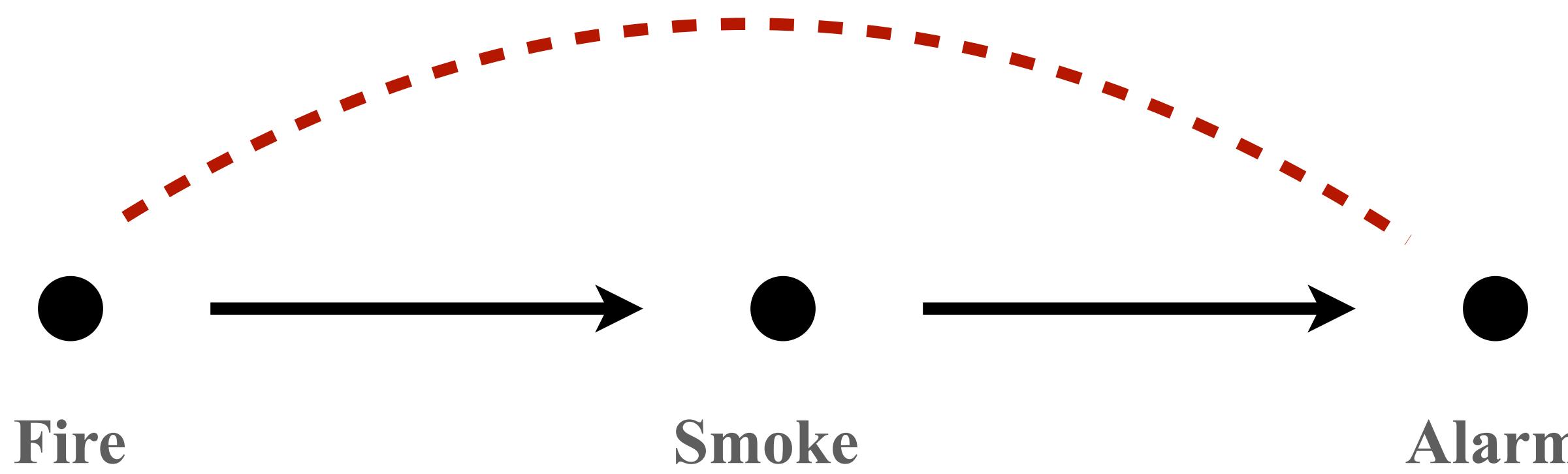
Transmits info



	No alarm	Alarm	
No fire	9338	153	
Fire	9	500	

$P(\text{ fire }) = \frac{9 + 500}{9338 + 153 + 9 + 500}$

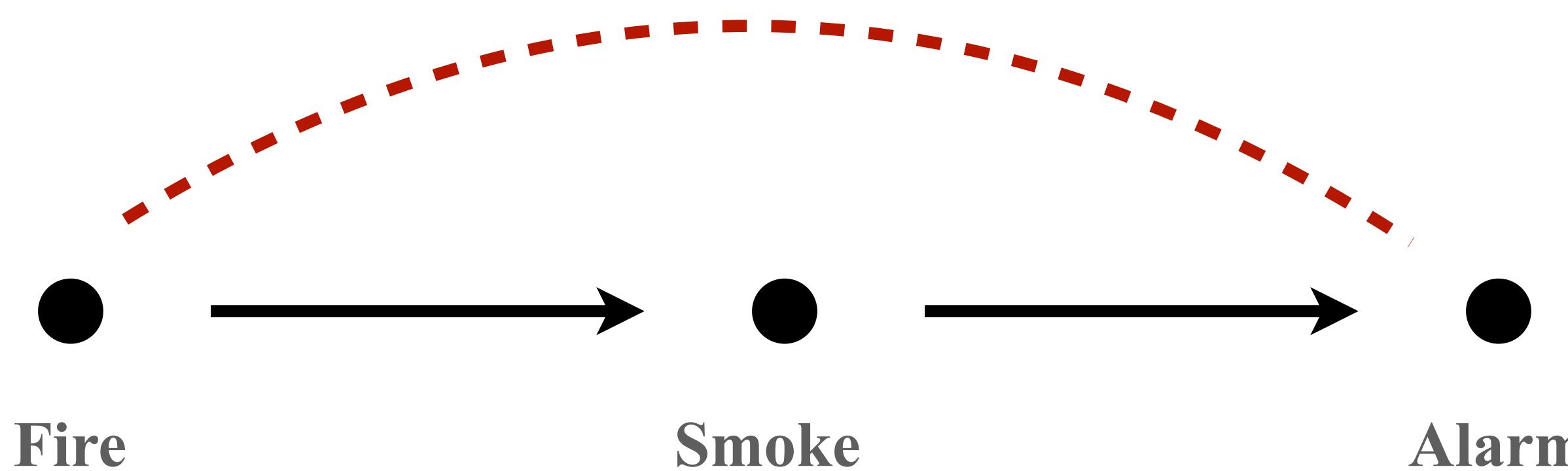
Transmits info



	No alarm	Alarm
No fire	9338	153
Fire	9	500

$$P(\text{ fire }) = \frac{9 + 500}{9 + 500 + 9338 + 153} =$$

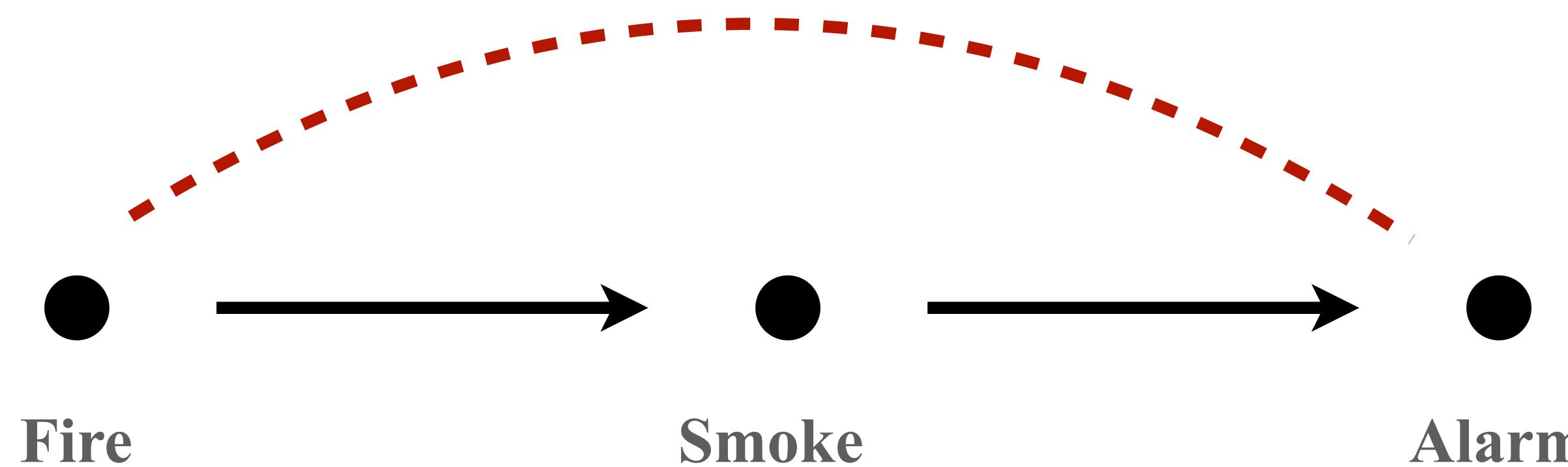
Transmits info



	No alarm	Alarm
No fire	9338	153
Fire	9	500

$$P(\text{ fire }) = \frac{9 + 500}{9 + 500 + 9338 + 153} = \frac{509}{10\,000} \approx 0.05$$

Transmits info

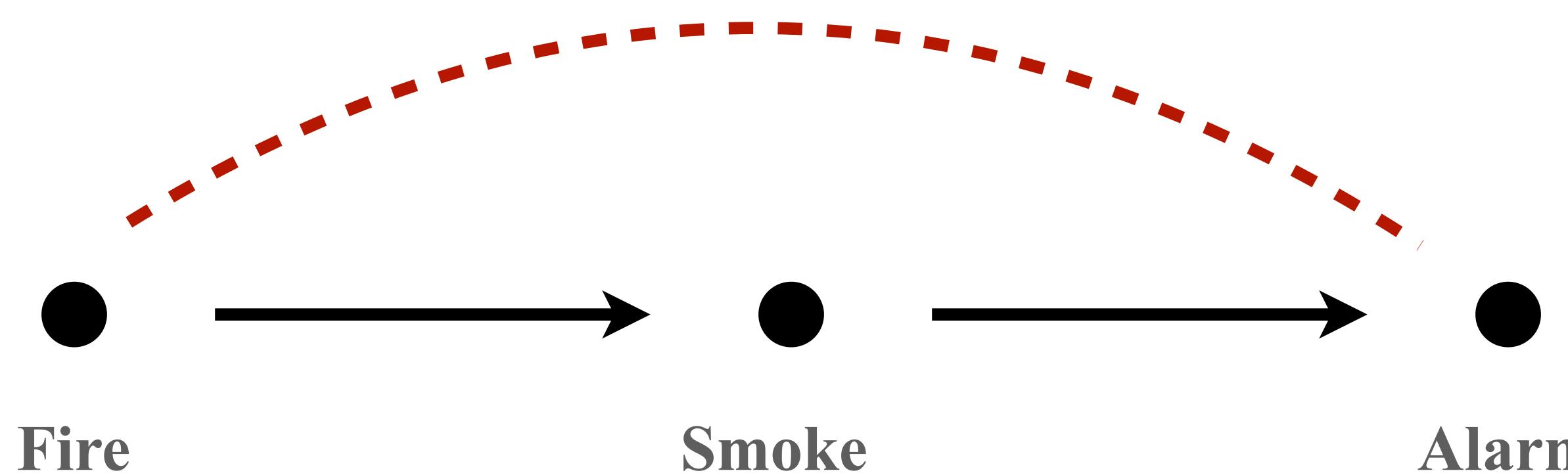


	No alarm	Alarm
No fire	9338	153
Fire	9	500

$$P(\text{ fire }) = \frac{9 + 500}{9 + 500 + 9338 + 153} = \frac{509}{10\,000} \approx 0.05$$

$$P(\text{ fire } | \text{ alarm }) = \underline{\hspace{2cm}}$$

Transmits info

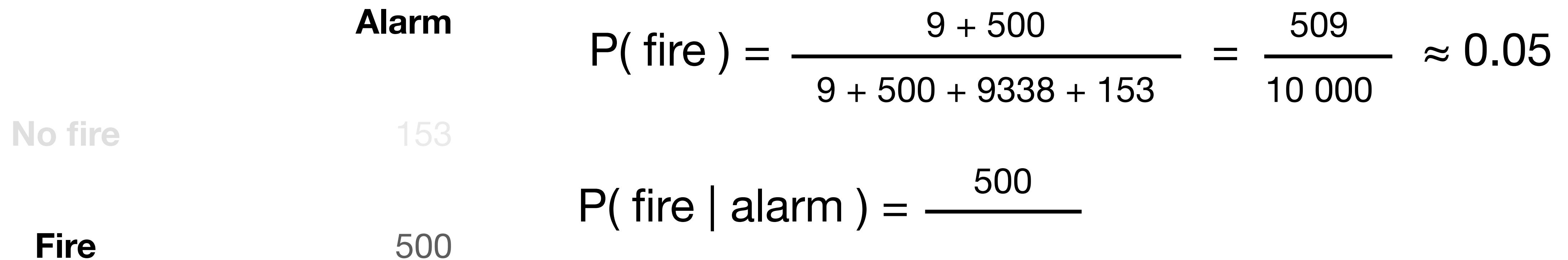
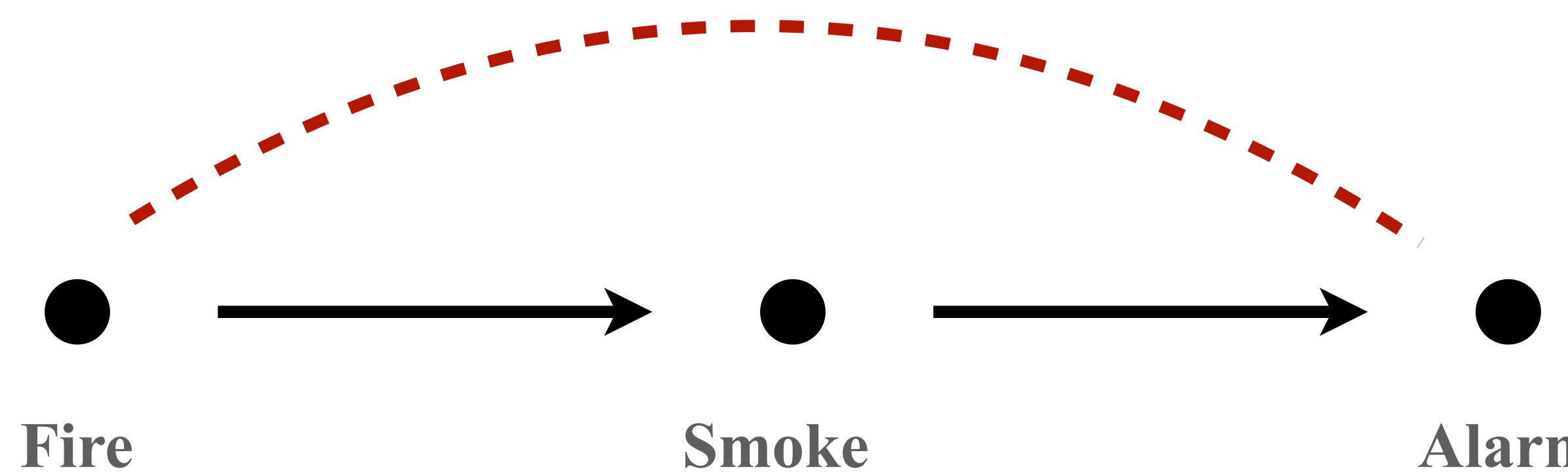


	Alarm	
No fire	153	
Fire	500	

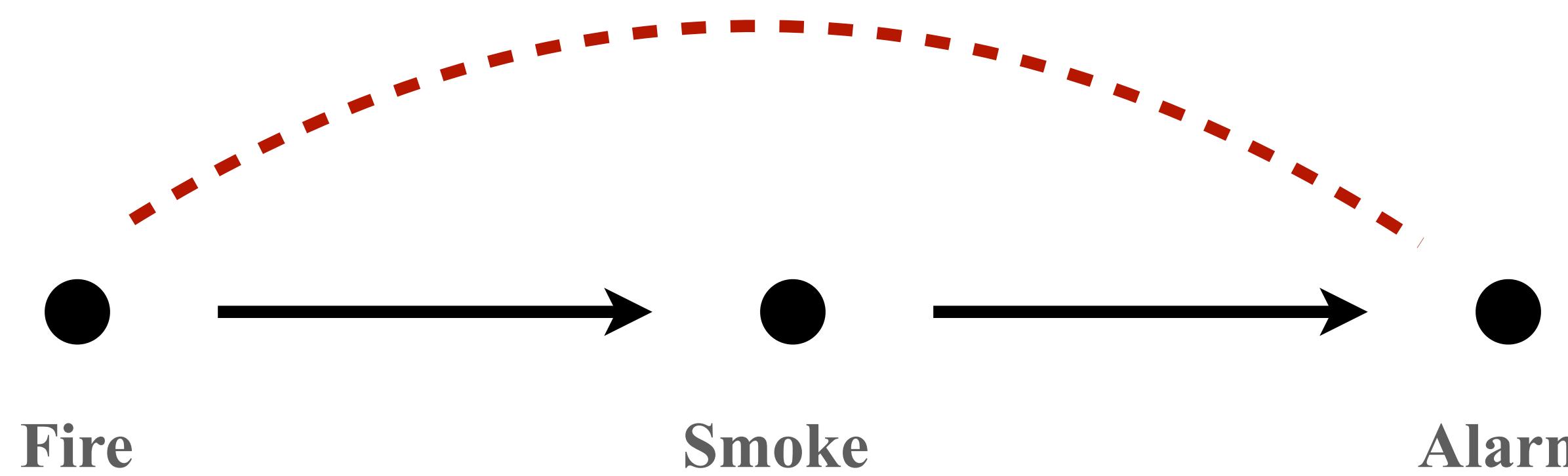
$P(\text{ fire }) = \frac{9 + 500}{9 + 500 + 9338 + 153} = \frac{509}{10\,000} \approx 0.05$

$P(\text{ fire } | \text{ alarm }) = \underline{\hspace{2cm}}$

Transmits info



Transmits info

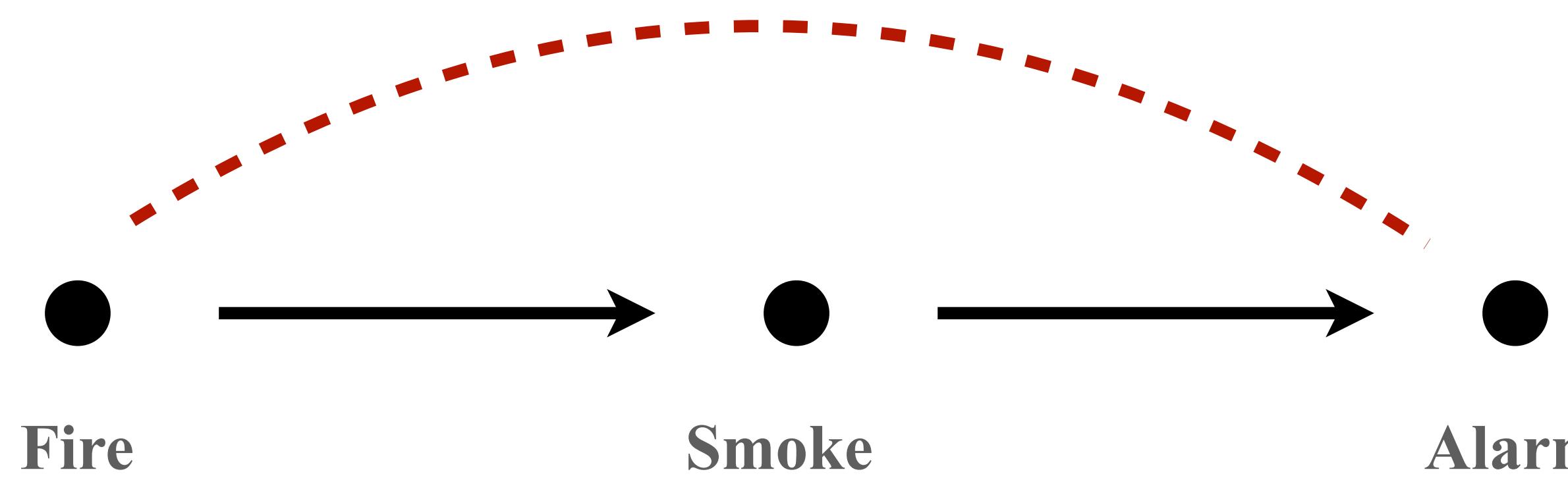


	Alarm	
No fire	153	
Fire	500	

$P(\text{ fire }) = \frac{9 + 500}{9 + 500 + 9338 + 153} = \frac{509}{10\,000} \approx 0.05$

$P(\text{ fire } | \text{ alarm }) = \frac{500}{500 + 153}$

Transmits info

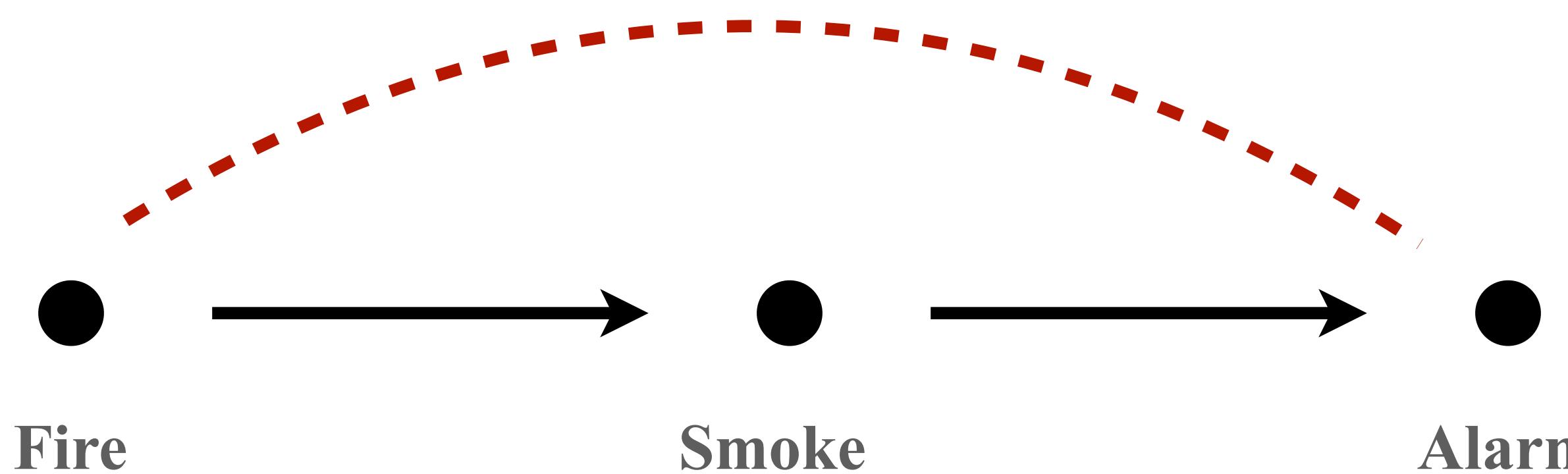


	Alarm	
No fire	153	
Fire	500	

$P(\text{ fire }) = \frac{9 + 500}{9 + 500 + 9338 + 153} = \frac{509}{10\,000} \approx 0.05$

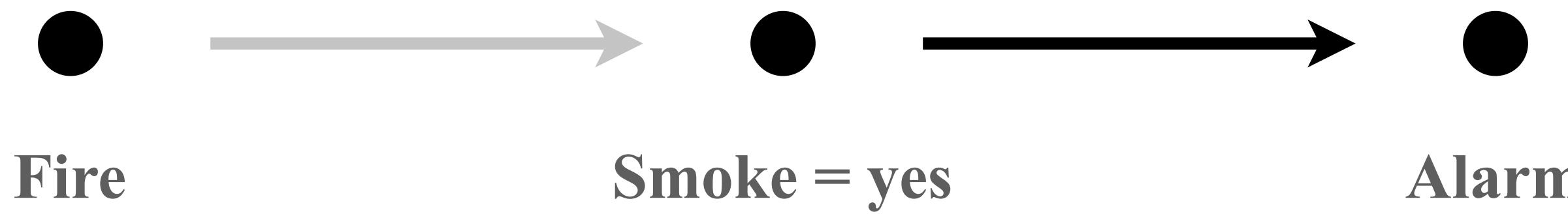
$P(\text{ fire } | \text{ alarm }) = \frac{500}{500 + 153} \approx 0.77$

Transmits info



POINT: hearing the alarm gives us information about the likelihood of any ongoing fires

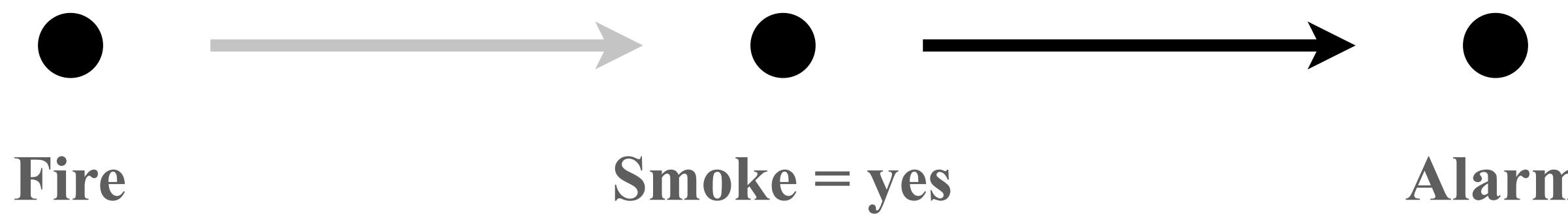
No longer transmits info



No alarm Alarm

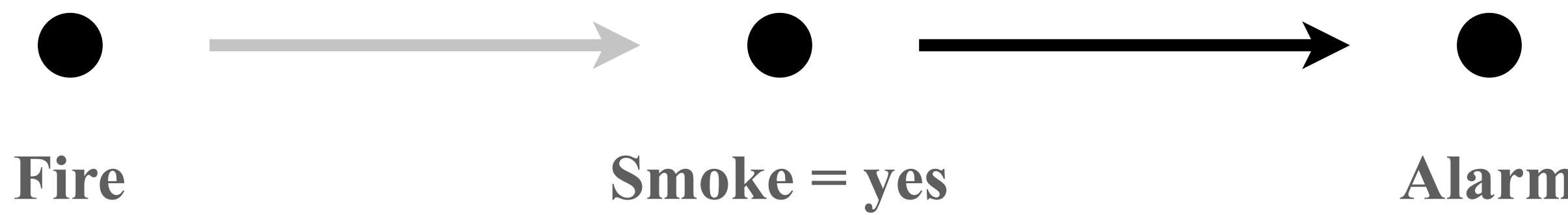
No fire	9338	153
Fire	9	500

No longer transmits info



	No alarm	Alarm	No alarm	Alarm
No fire	1	92	9337	61
Fire	6	500	3	0
Smoke = yes		Smoke = no		

No longer transmits info



No alarm Alarm

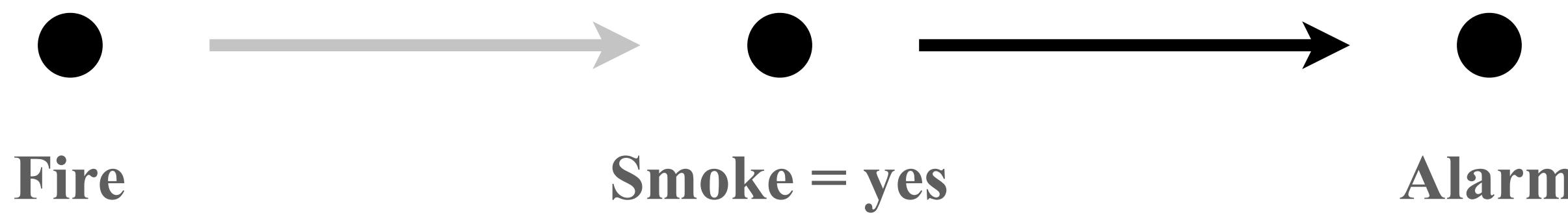
$P(\text{ fire } | \text{ smoke }) =$

No fire	1	92
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Fire	6	500
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Smoke = yes

No longer transmits info



No alarm Alarm

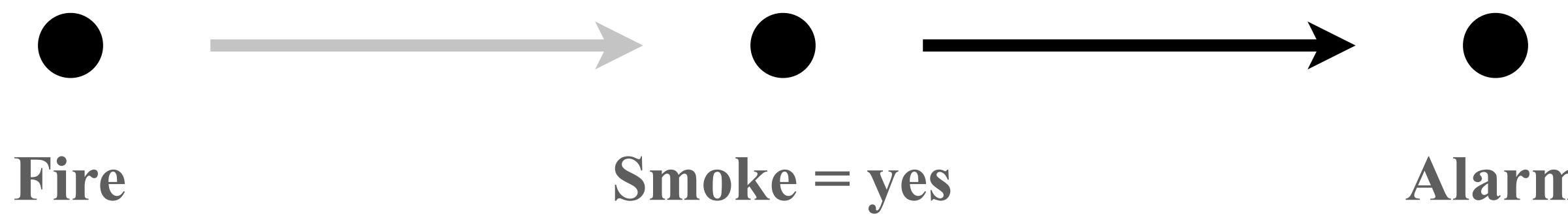
$$P(\text{ fire } | \text{ smoke }) = 506/599$$

No fire	1	92
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Fire	6	500
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Smoke = yes

No longer transmits info



No alarm Alarm

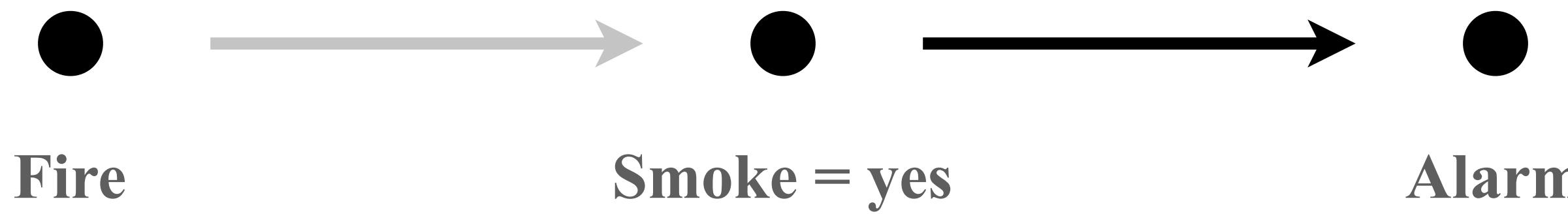
$$P(\text{ fire } | \text{ smoke }) = 506/599 \approx 0.84$$

No fire 1 92

Fire 6 500

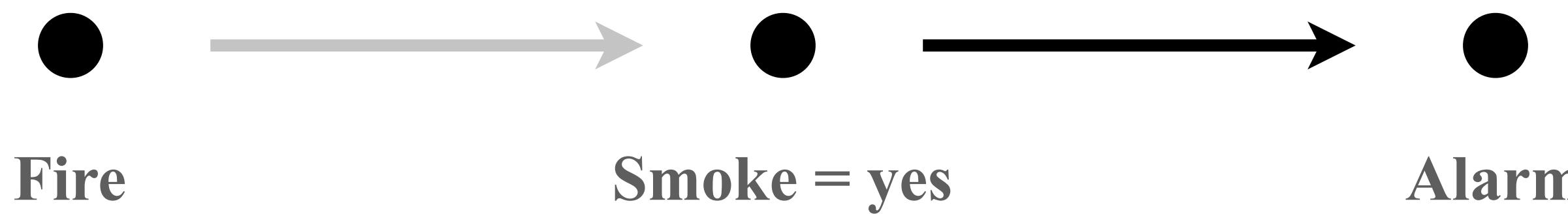
Smoke = yes

No longer transmits info



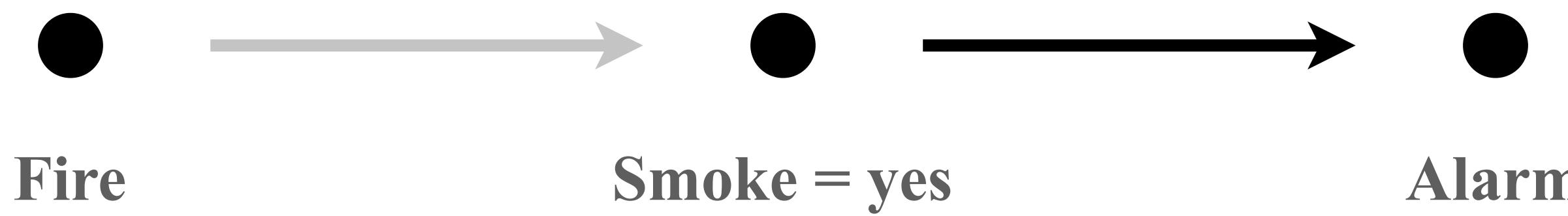
	No alarm	Alarm	$P(\text{ fire} \mid \text{smoke}) = 506/599 \approx 0.84$
No fire	1	92	$P(\text{ fire} \mid \text{alarm, smoke}) =$
Fire	6	500	
Smoke = yes			

No longer transmits info



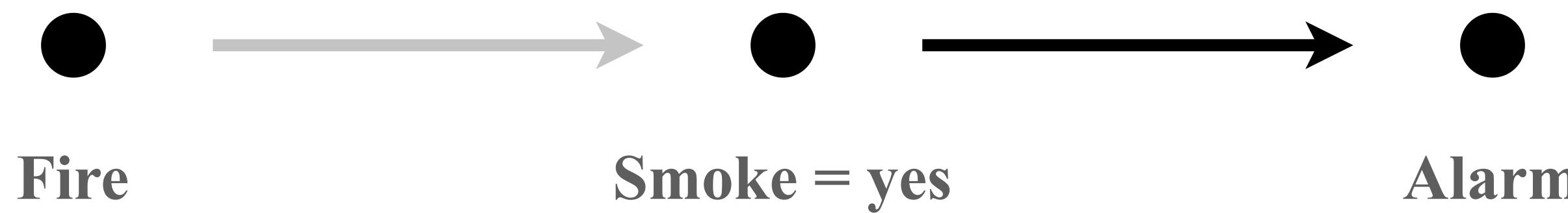
	No alarm	Alarm	$P(\text{ fire } \text{ smoke }) = 506/599 \approx 0.84$
No fire		92	$P(\text{ fire } \text{ alarm, smoke }) =$
Fire	500		
Smoke = yes			

No longer transmits info



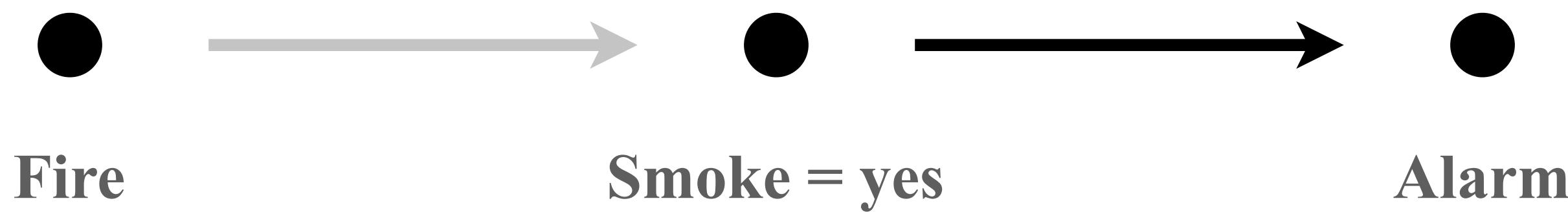
	No alarm	Alarm	$P(\text{ fire} \mid \text{smoke}) = 506/599 \approx 0.84$
No fire	92		$P(\text{ fire} \mid \text{alarm, smoke}) = 500/592$
Fire	500		
Smoke = yes			

No longer transmits info



	No alarm	Alarm	$P(\text{ fire} \mid \text{smoke}) = 506/599 \approx 0.84$
No fire		92	$P(\text{ fire} \mid \text{alarm, smoke}) = 500/592 \approx 0.84$
Fire	500		
Smoke = yes			

No longer transmits info



No alarm

Alarm

$$P(\text{fire} | \text{smoke}) = 506/599 \approx 0.84$$

Conditional on smoke, the alarm gives no extra information!

No fire

1

92

Fire

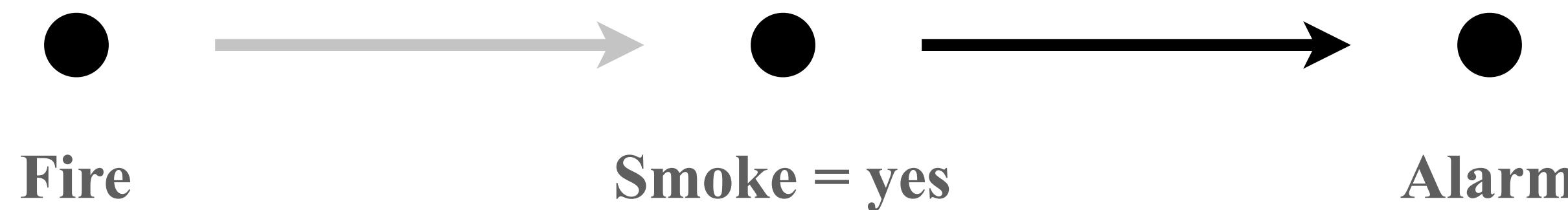
6

500

Smoke = yes

$$P(\text{fire} | \text{alarm, smoke}) = 500/592 \approx 0.84$$

No longer transmits info



$$\text{No alarm} \quad \text{Alarm} \quad P(\text{fire} | \text{smoke}) = 506/599 \approx 0.84$$

Conditional on smoke, the alarm gives no extra information!

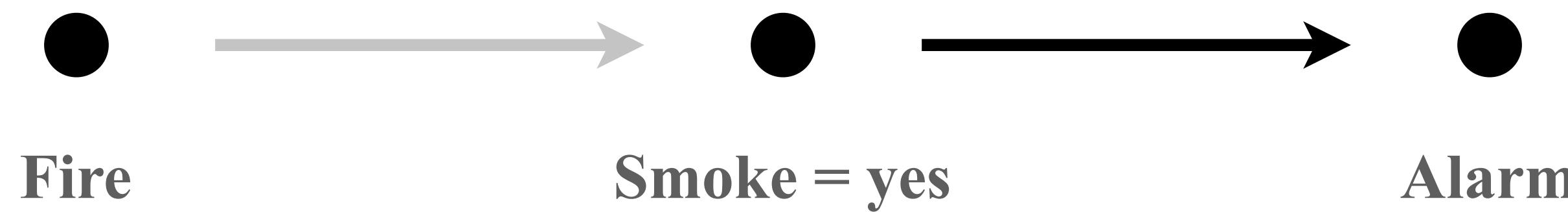
No fire 1 92

Fire 6 500

If I knew nothing about fire, smoke, or alarms, and analyzed these data haphazardly, I would conclude that smoke alarms are useless

Smoke = yes

No longer transmits info



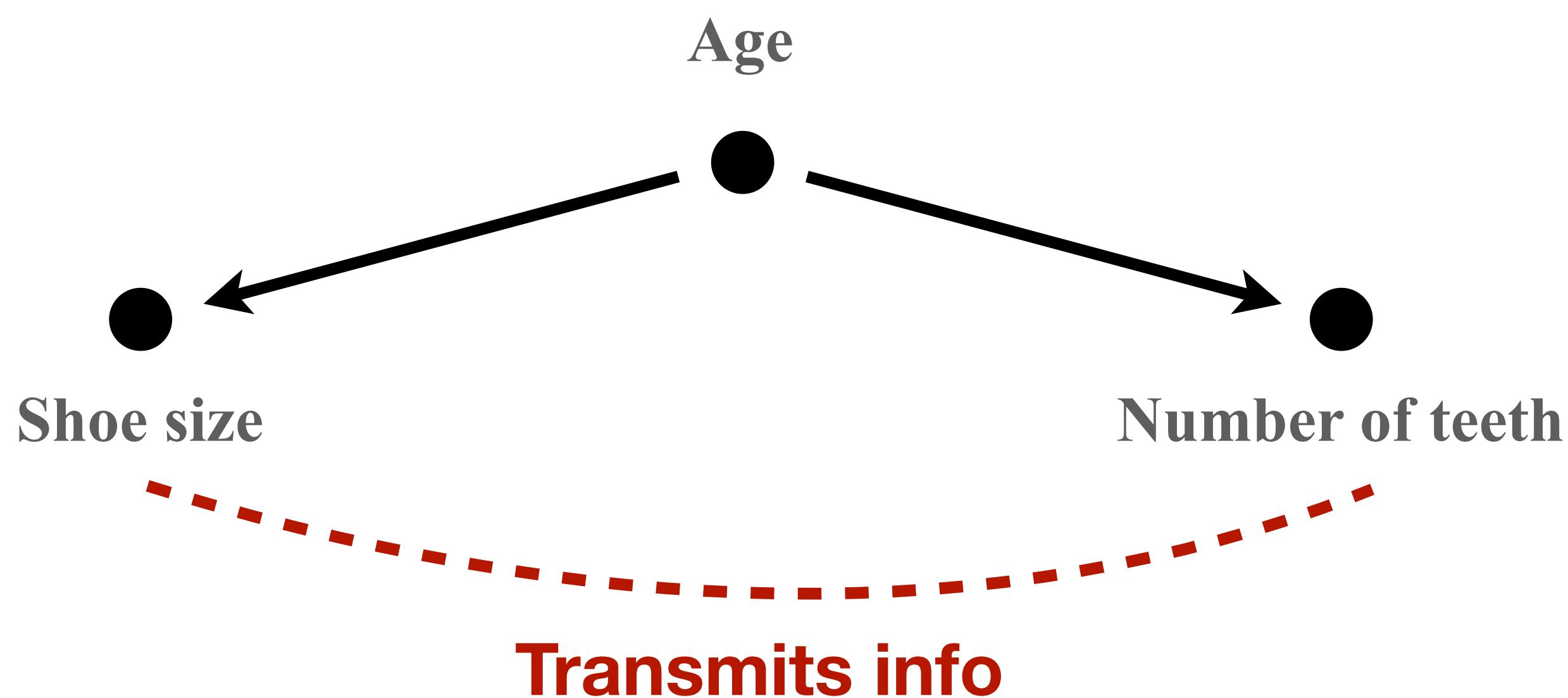
$$\begin{array}{cc} \text{No alarm} & \text{Alarm} \\ \text{No fire} & \end{array} \quad P(\text{fire} | \text{smoke}) = 506/599 \approx 0.84$$

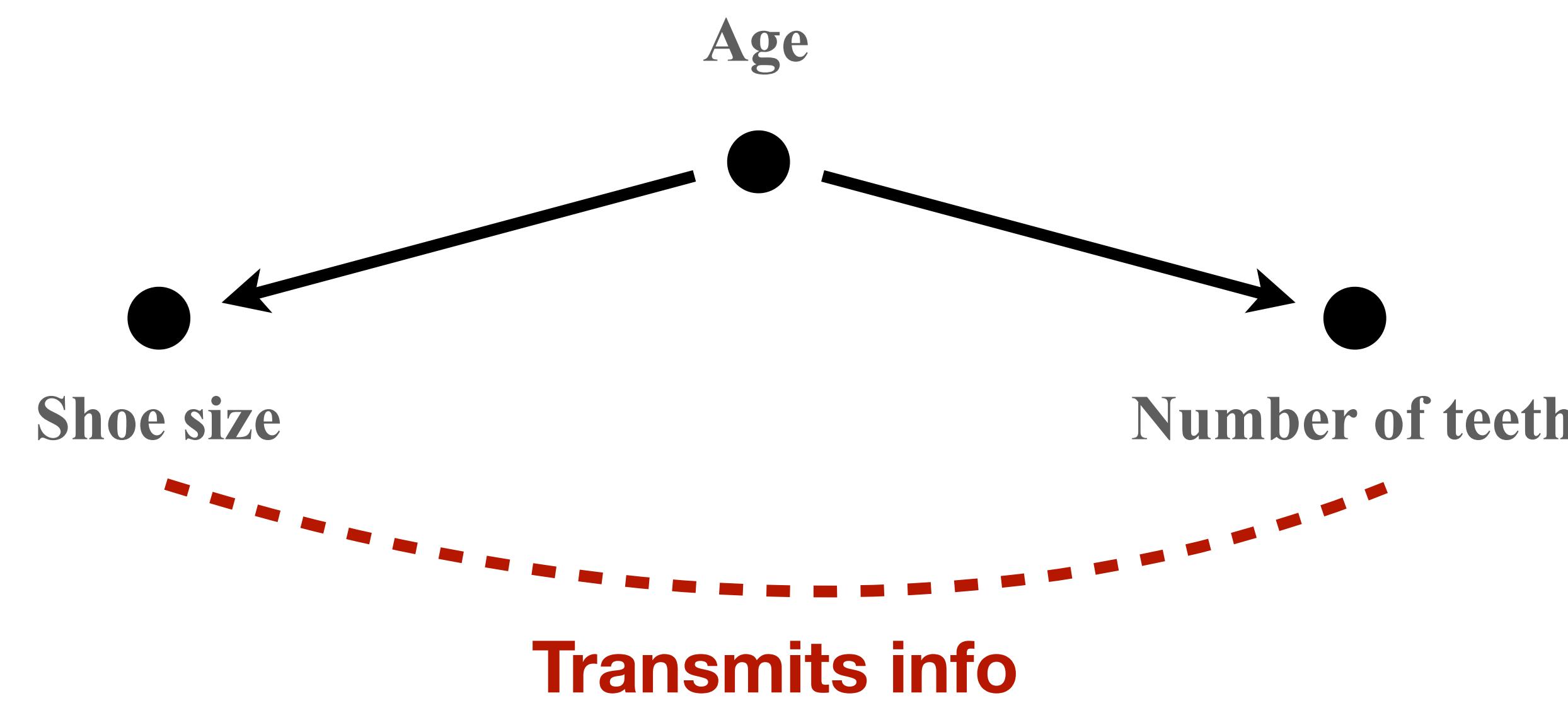
Conditional on smoke, the alarm gives no extra information!

$$\begin{array}{cc} \text{No fire} & \end{array}$$

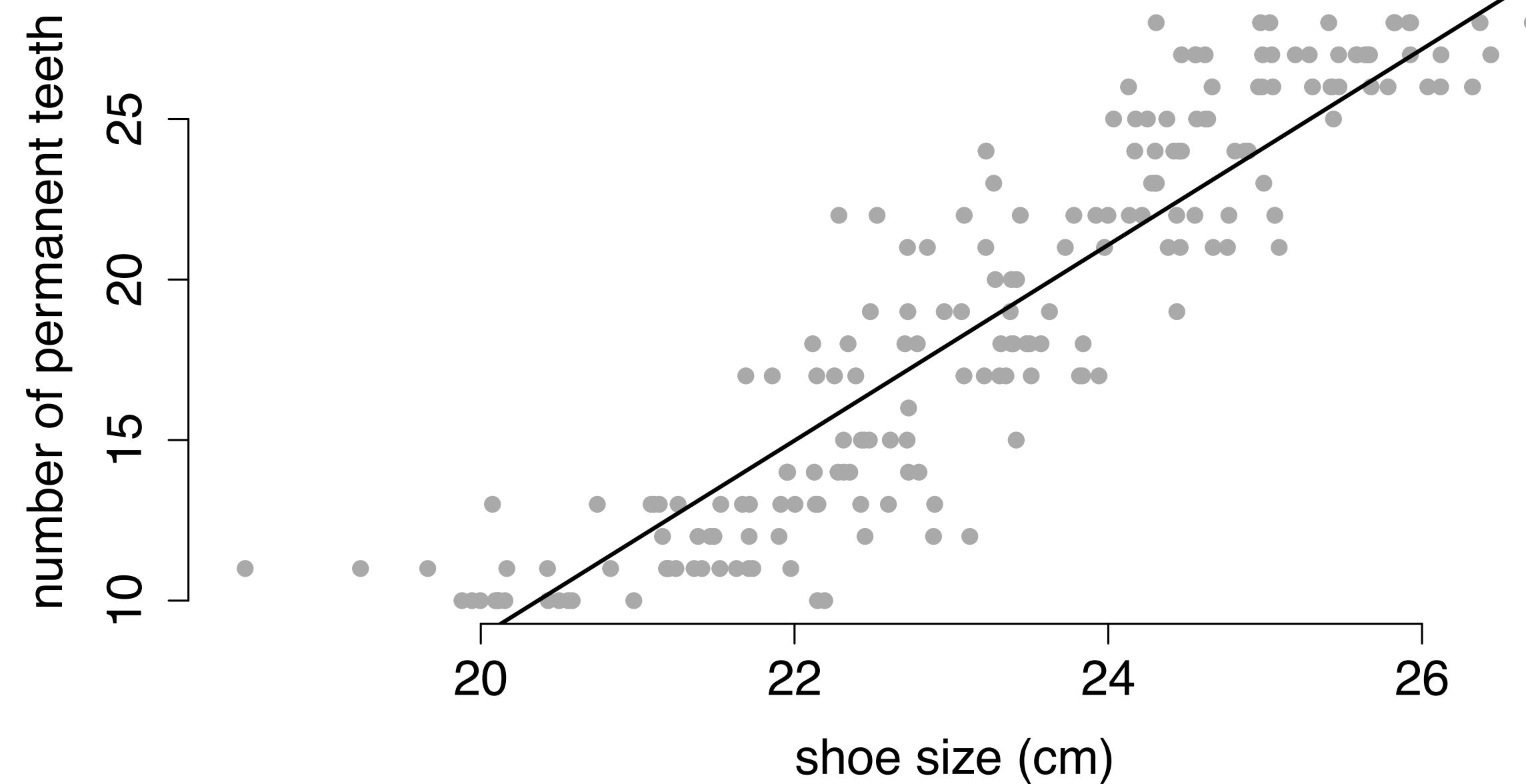
If I knew nothing about fire, smoke, or alarms, and analyzed these data haphazardly, I would conclude that smoke alarms are useless

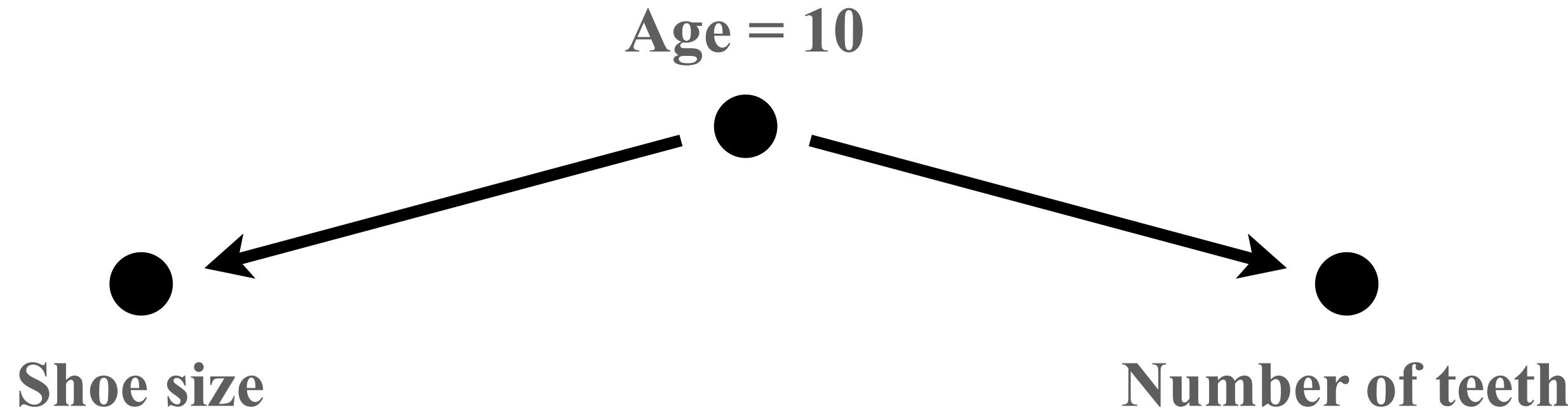
“Conditioning on a mediator” – adjusting away the effect of interest



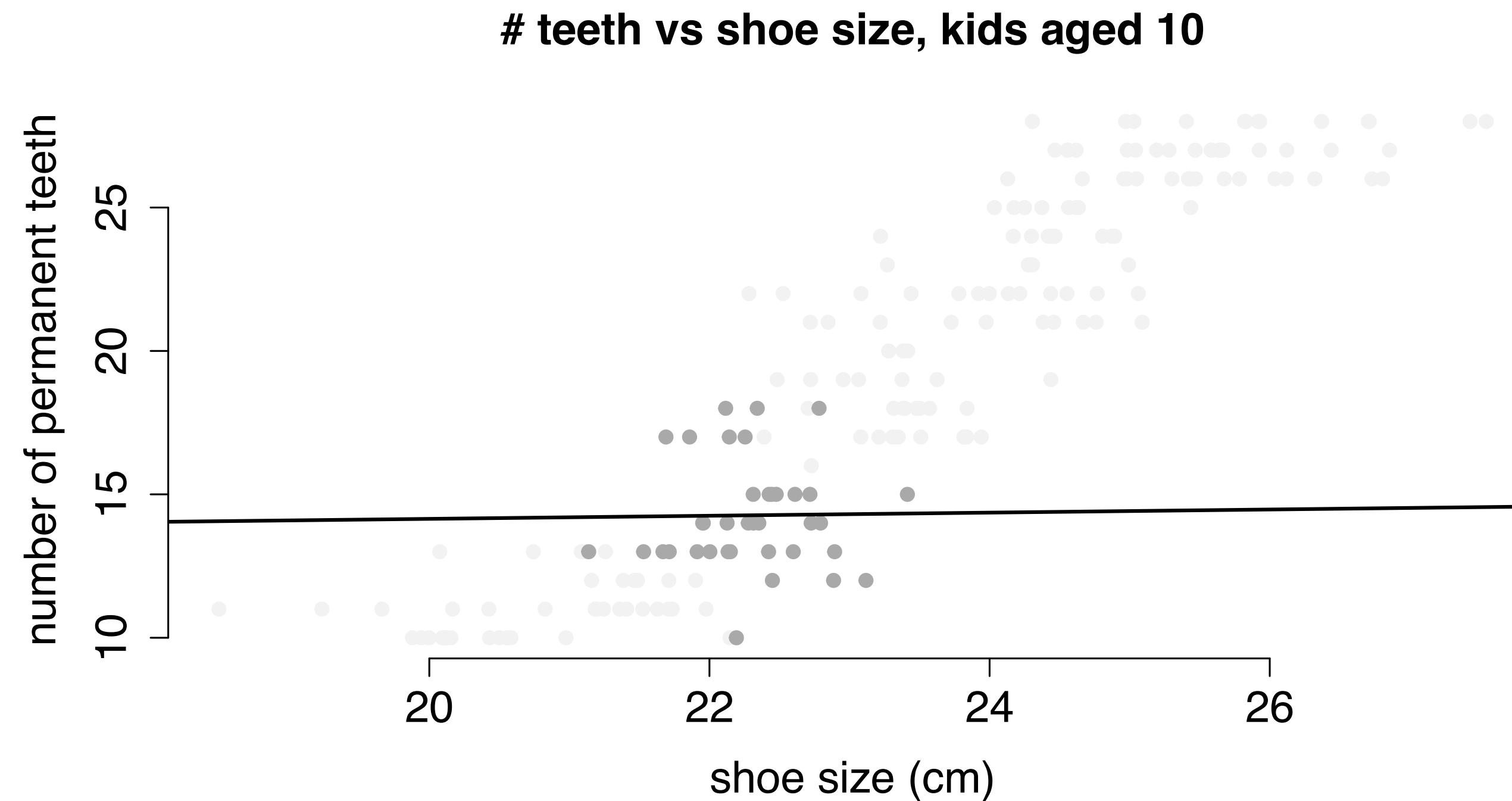


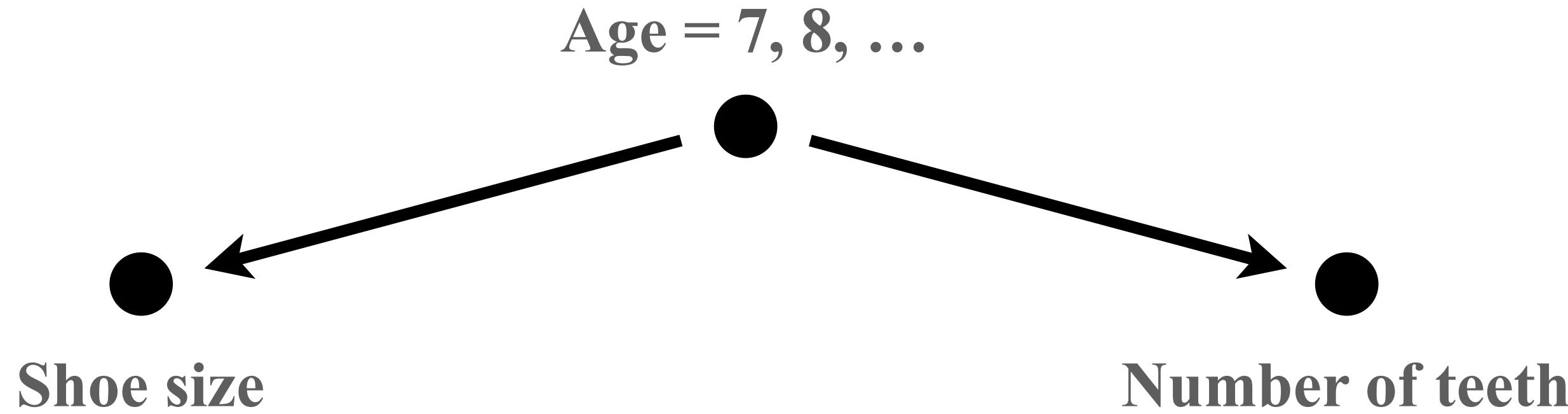
teeth vs shoe size, kids aged 7–15





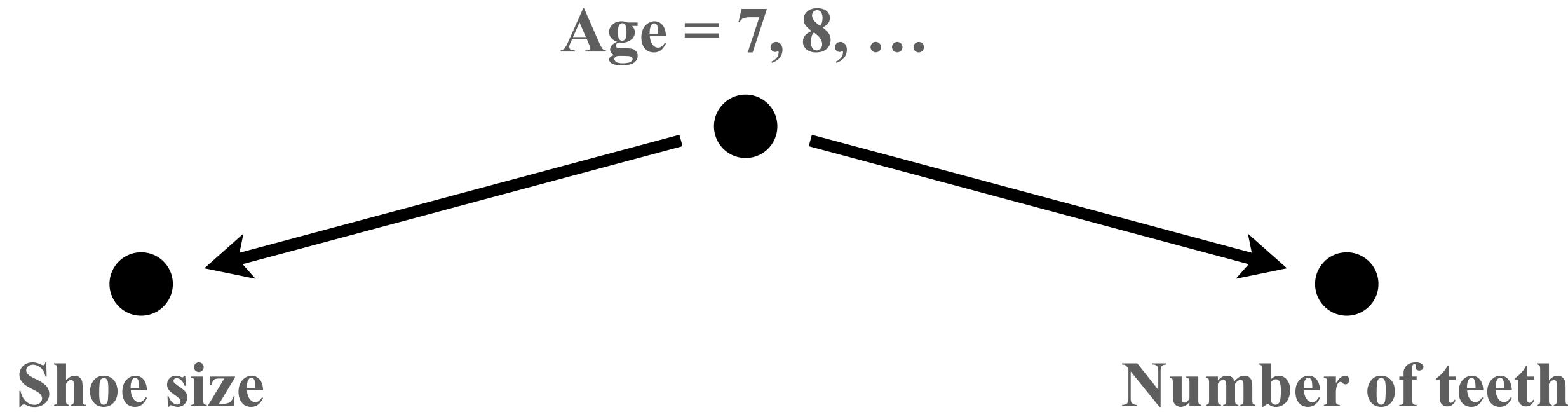
Does not transmit info





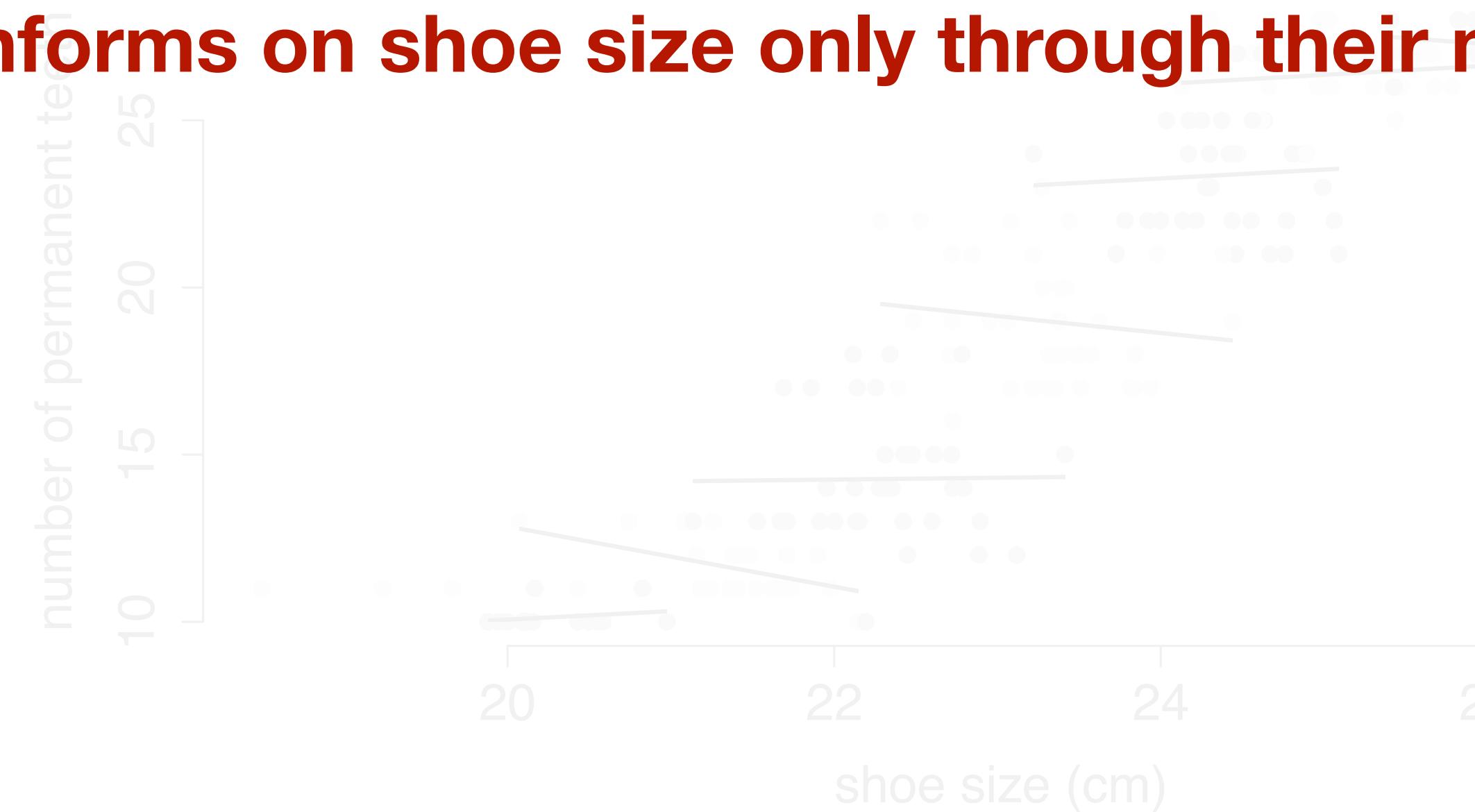
Does not transmit info

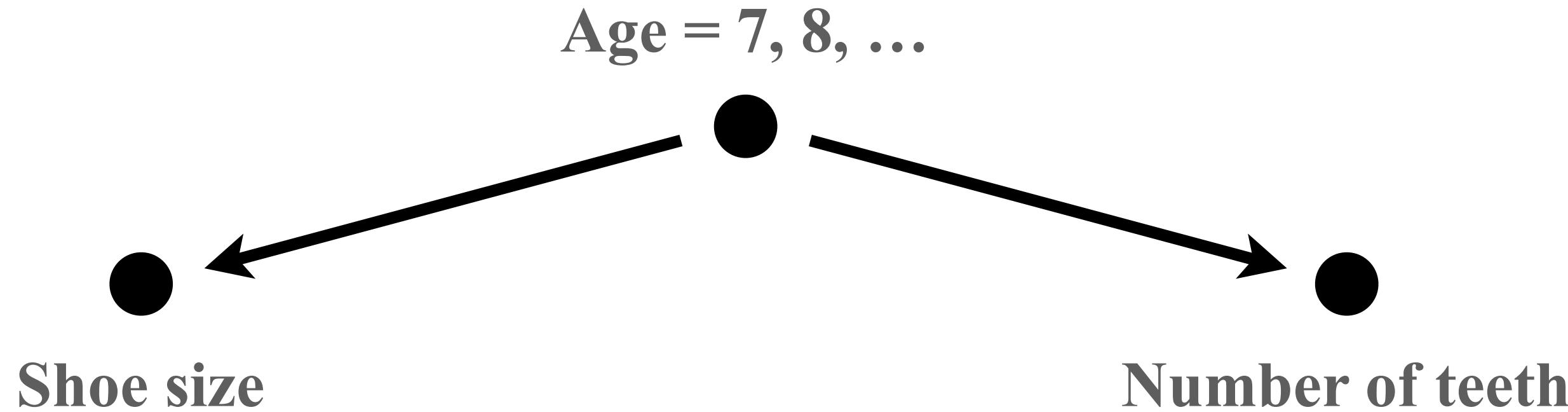




teeth vs shoe size,
regressions conditional on age

Number of teeth informs on shoe size only through their mutual association with age



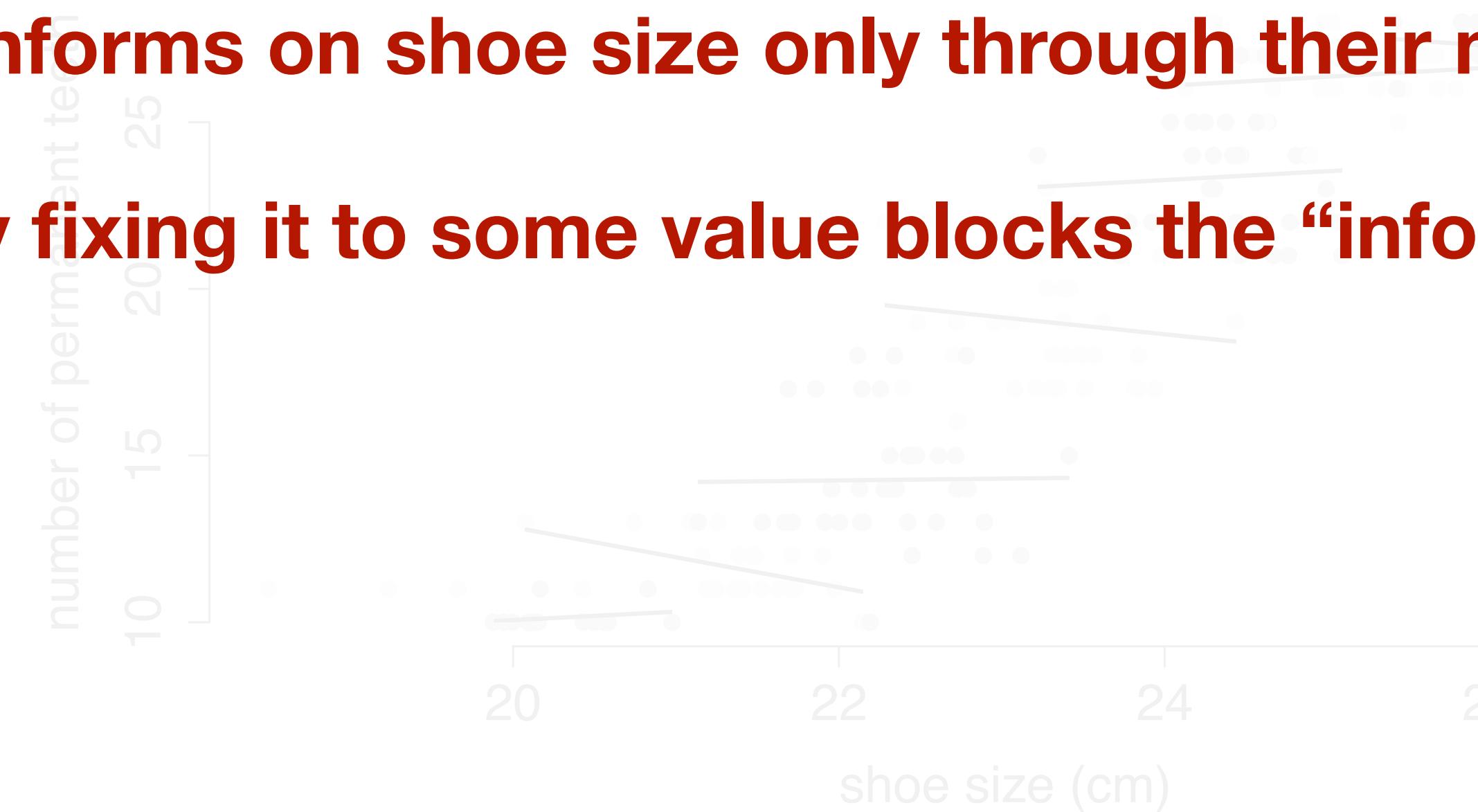


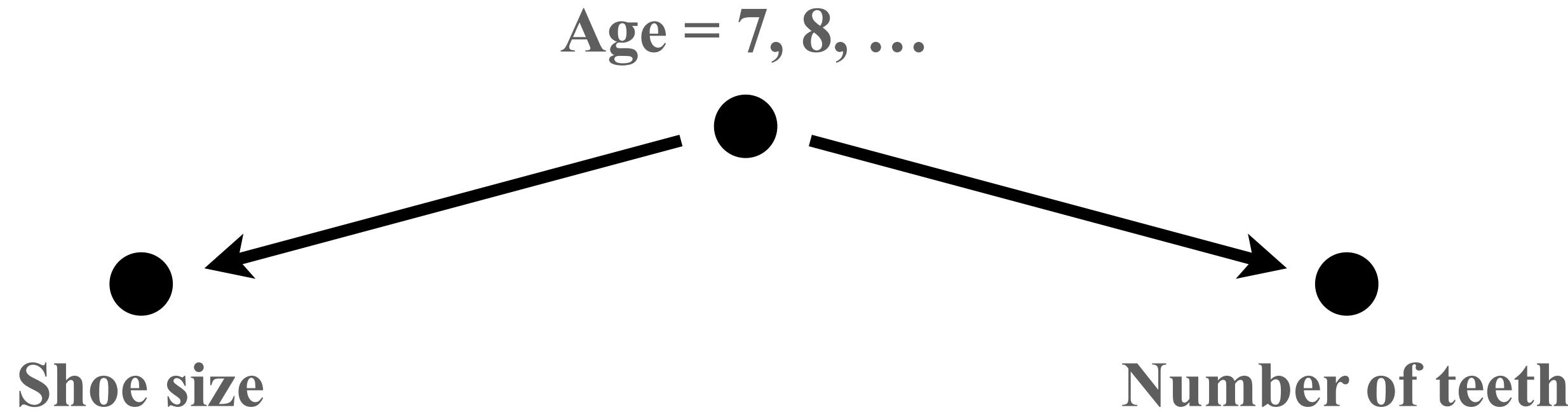
Does not transmit info

teeth vs shoe size,
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Number of teeth informs on shoe size only through their mutual association with age

Controlling age by fixing it to some value blocks the “information flow”





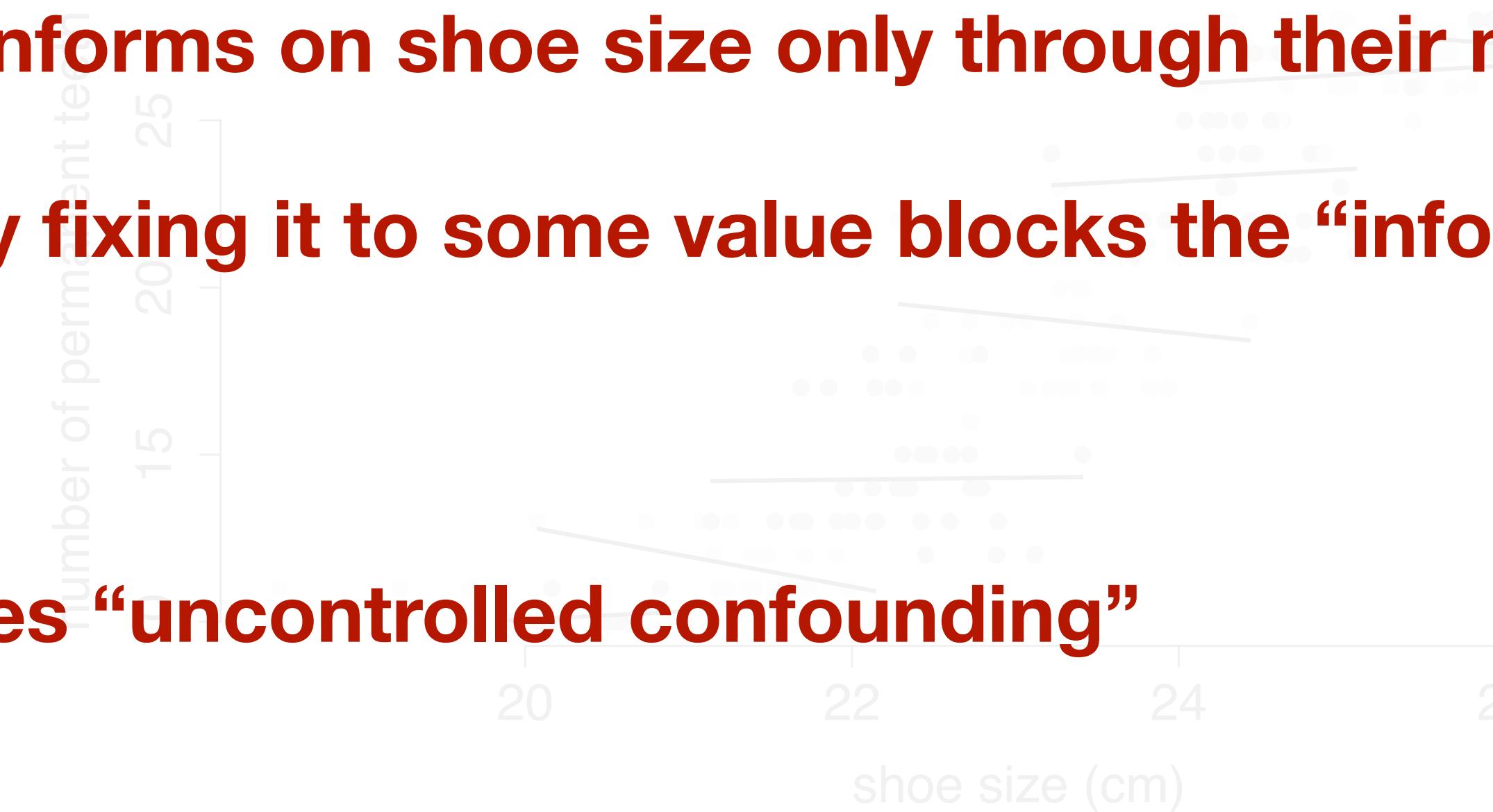
Does not transmit info

teeth vs shoe size,
regressions conditional on age

Number of teeth informs on shoe size only through their mutual association with age

Controlling age by fixing it to some value blocks the “information flow”

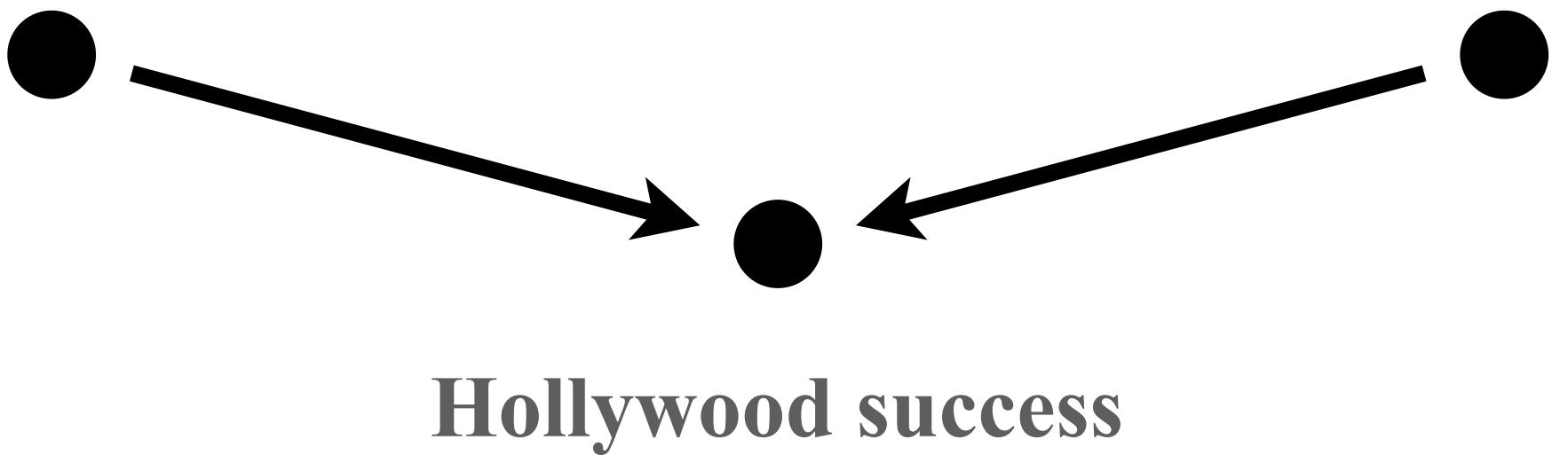
Ignoring age leaves “uncontrolled confounding”

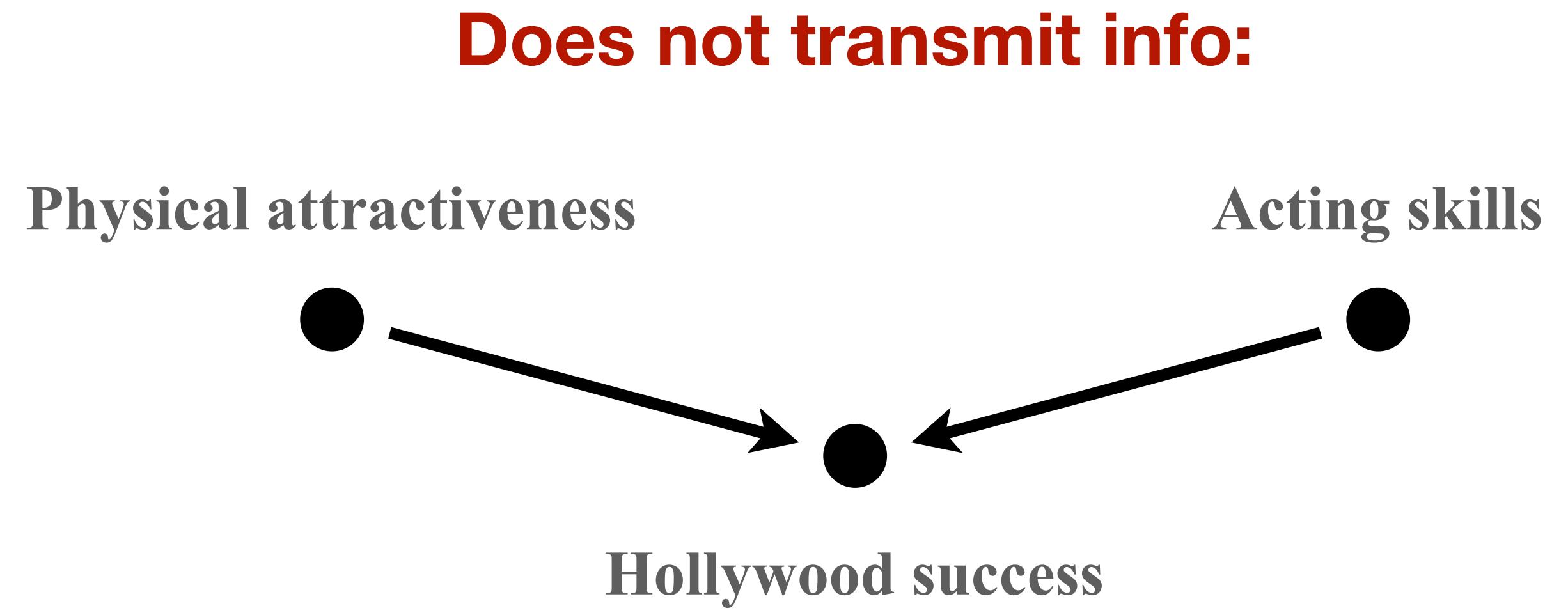


Does not transmit info:

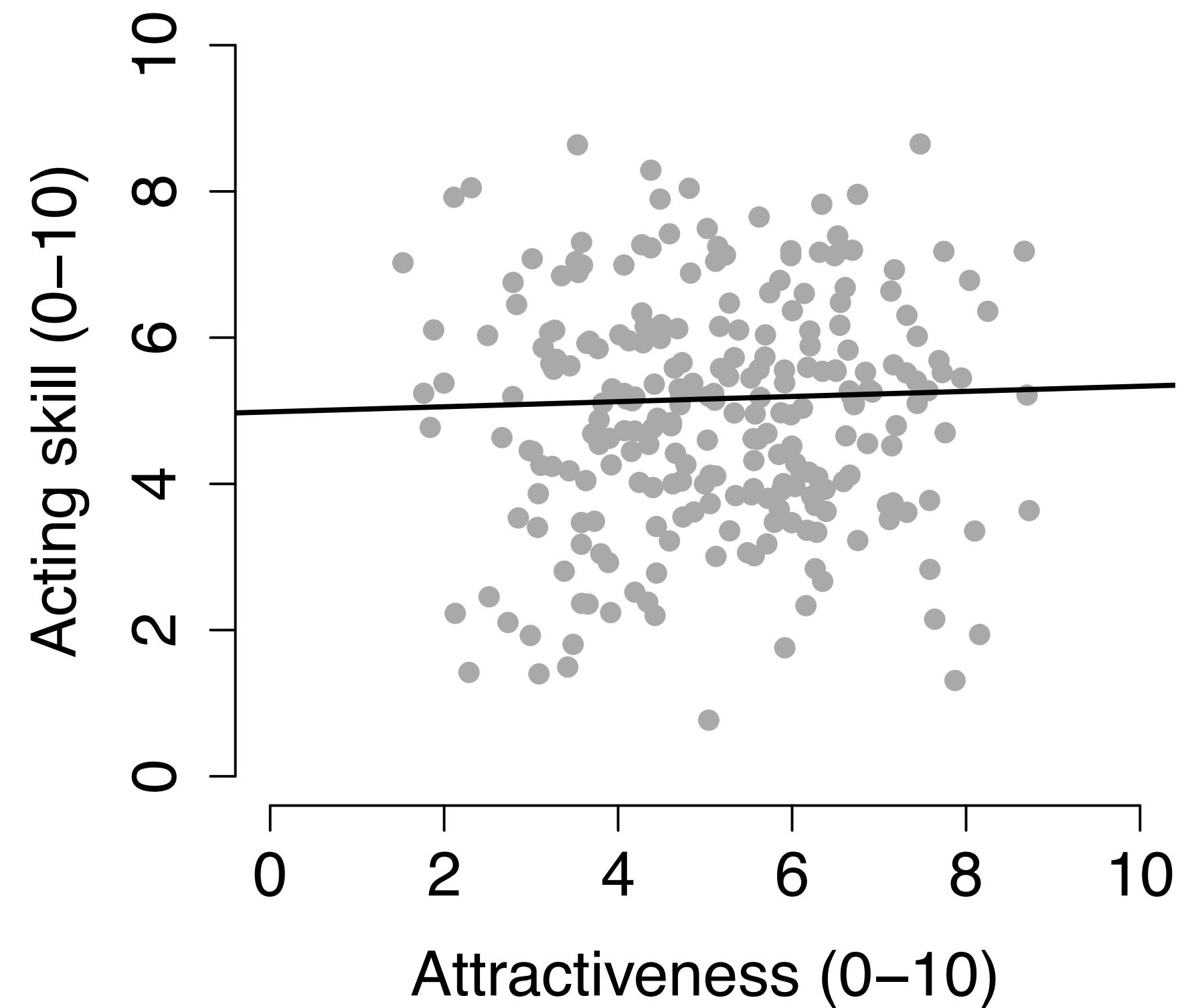
Physical attractiveness

Acting skills





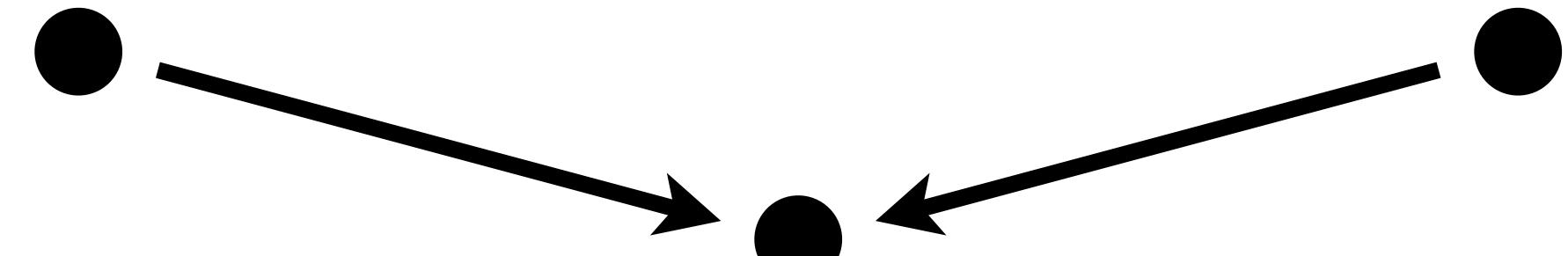
**Acting skill vs. attractiveness
in the general population**



Transmits info

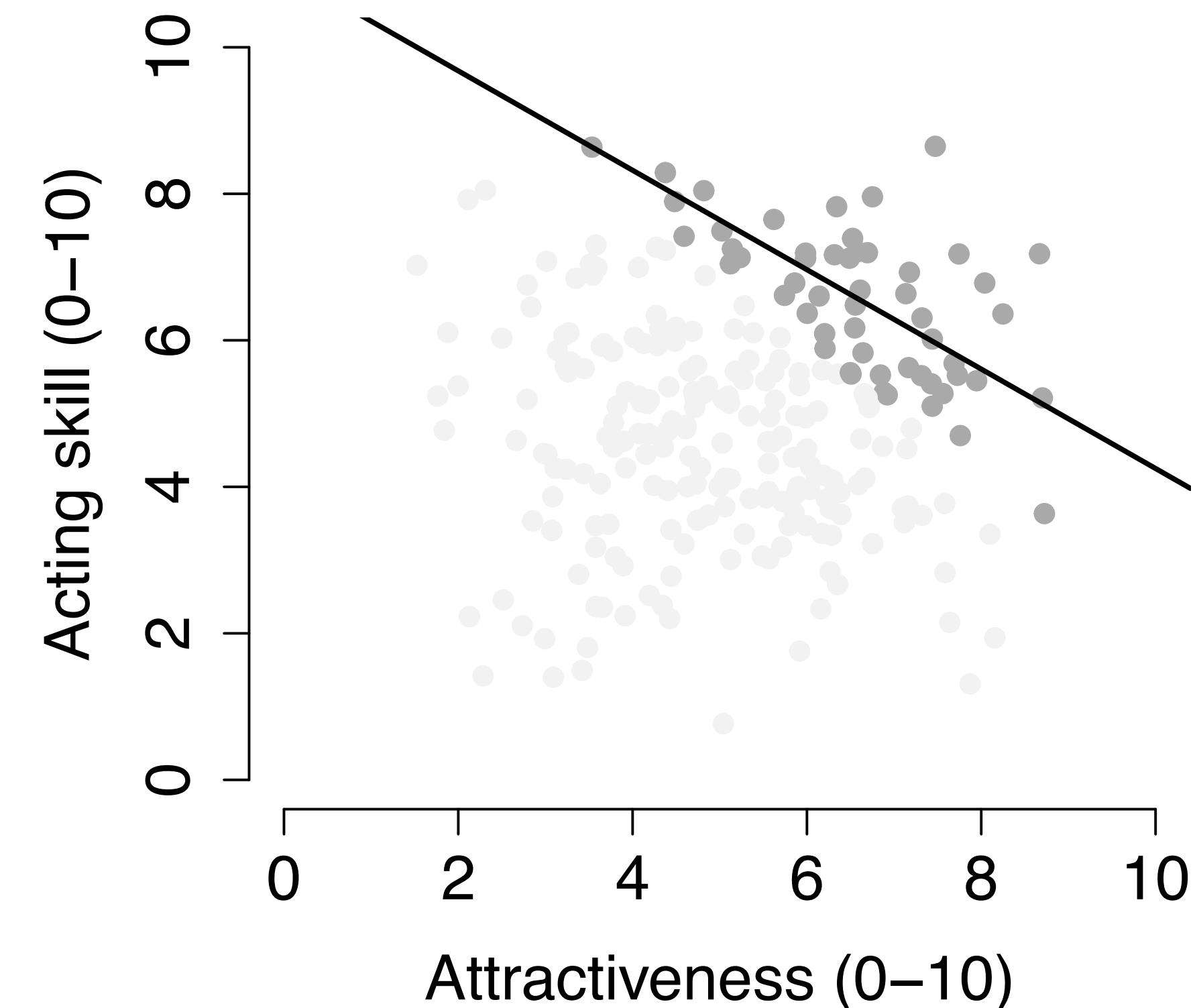
Physical attractiveness

Acting skills



Hollywood success = Yes

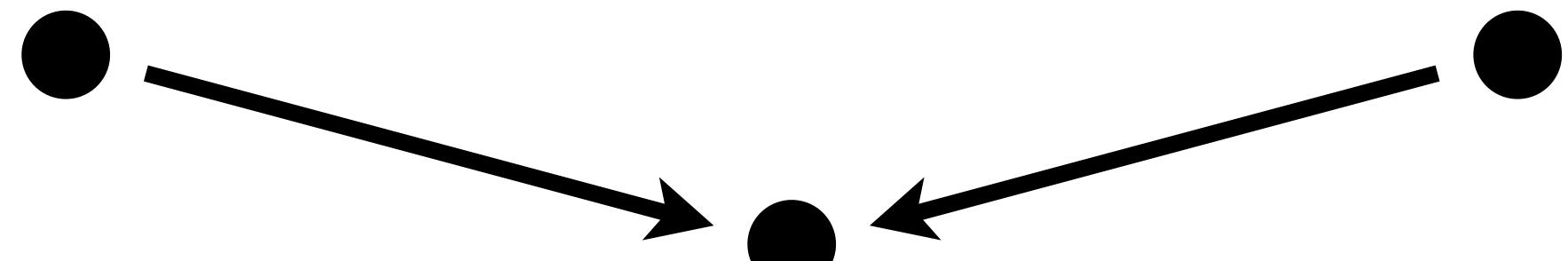
Acting skill vs. attractiveness among Hollywood stars



Transmits info

Physical attractiveness

Acting skills



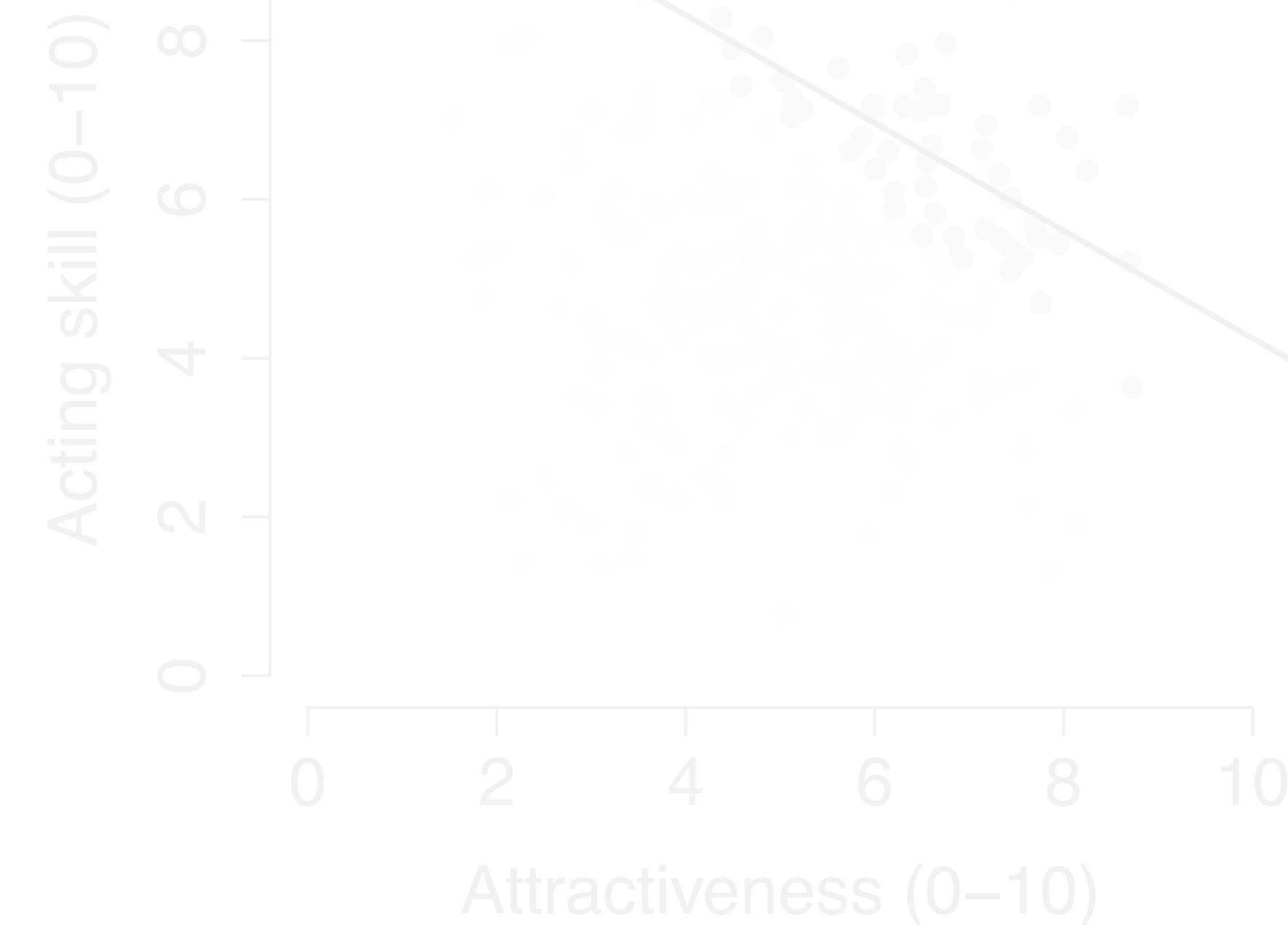
Hollywood success = Yes

Acting skill and attractiveness not related among ordinary people

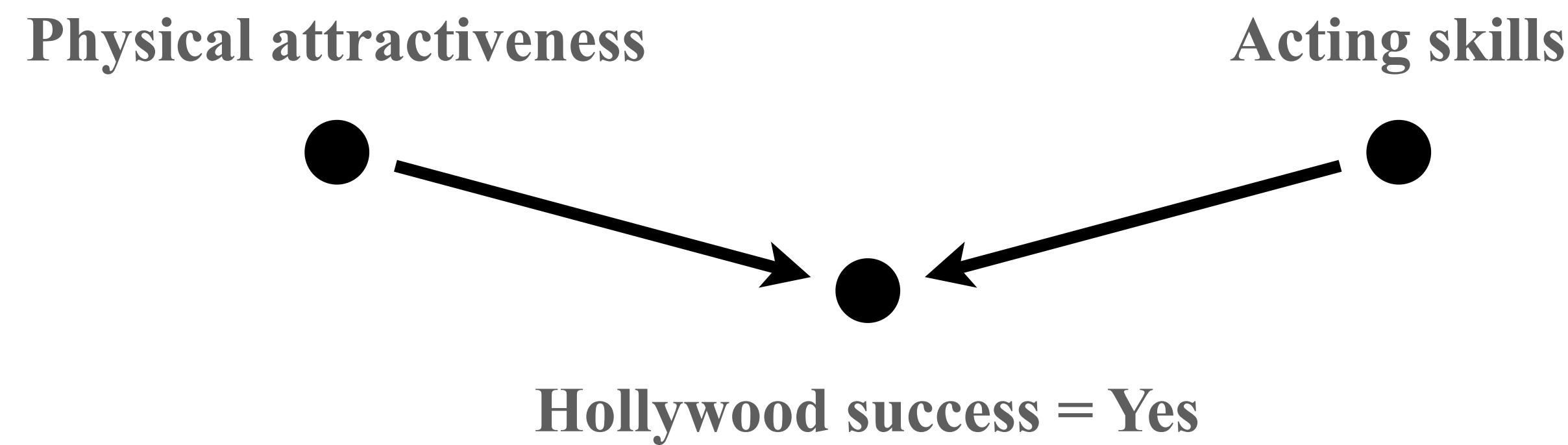
Acting skill vs. attractiveness

Hollywood stars

Ordinary people



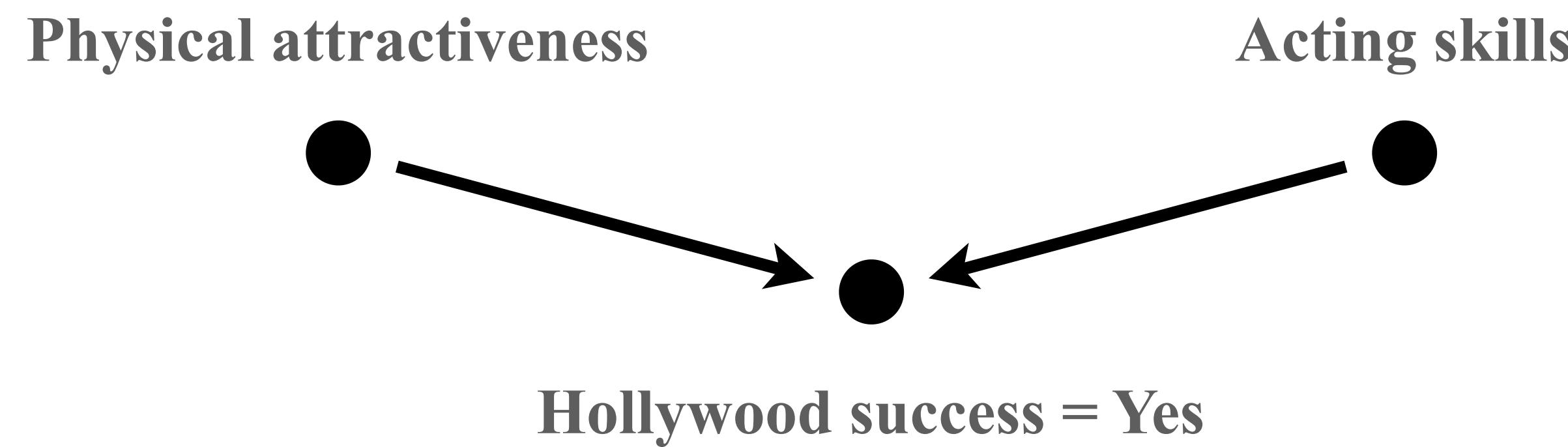
Transmits info



Acting skill and attractiveness not related among ordinary people



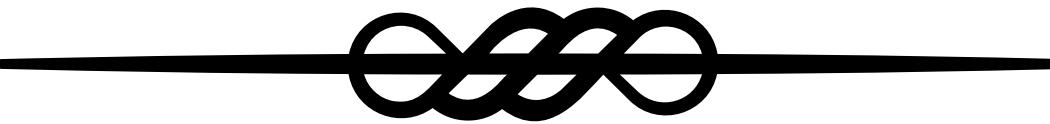
Transmits info



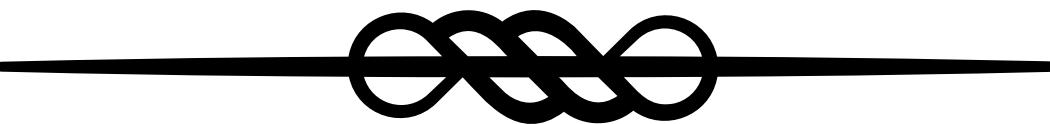
Acting skill and attractiveness not related among ordinary people

Very high attractiveness or very high acting skill alone enough to ensure success

Conditioning on collider called “collider bias:” introduces an effect where there should be none



break probably



The Marko fallacy: confounding

Lessons so far

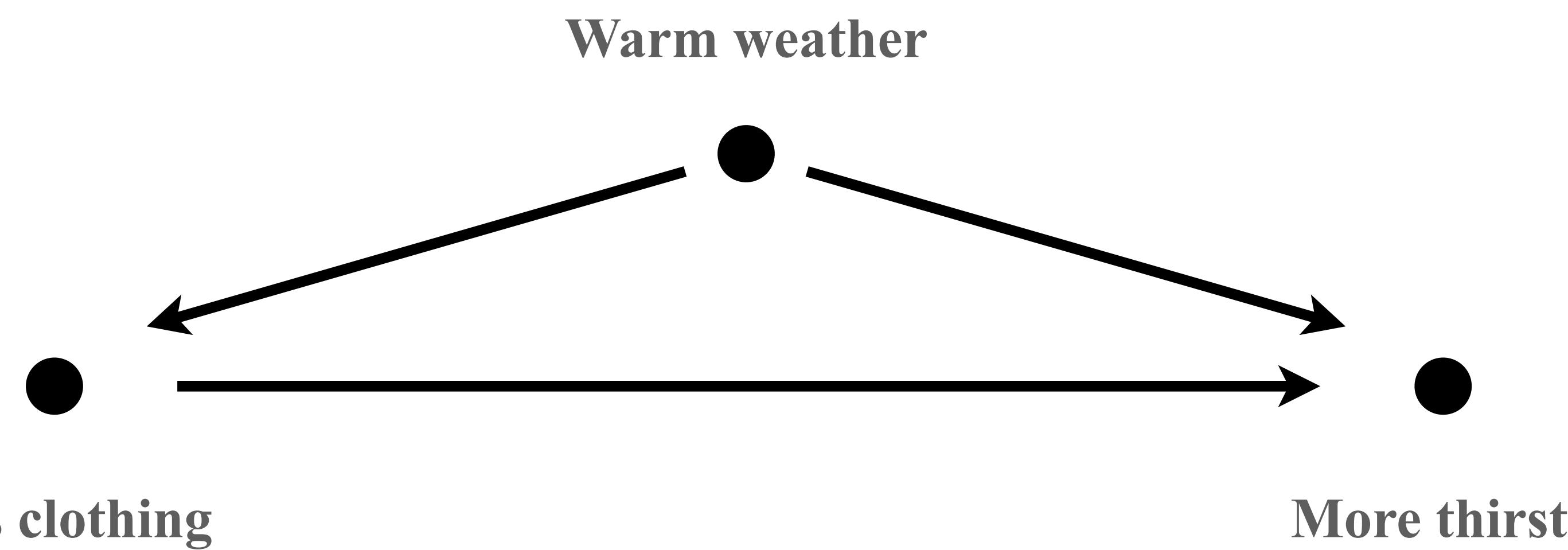
- The three basic junctures are like “information pipes”
- The chain and the fork are open, information flows through them
- The collider is closed, information does not flow through
- Conditioning on the middle node of a chain or fork closes the pipe
- Conditioning on the middle node of a collider opens the pipe

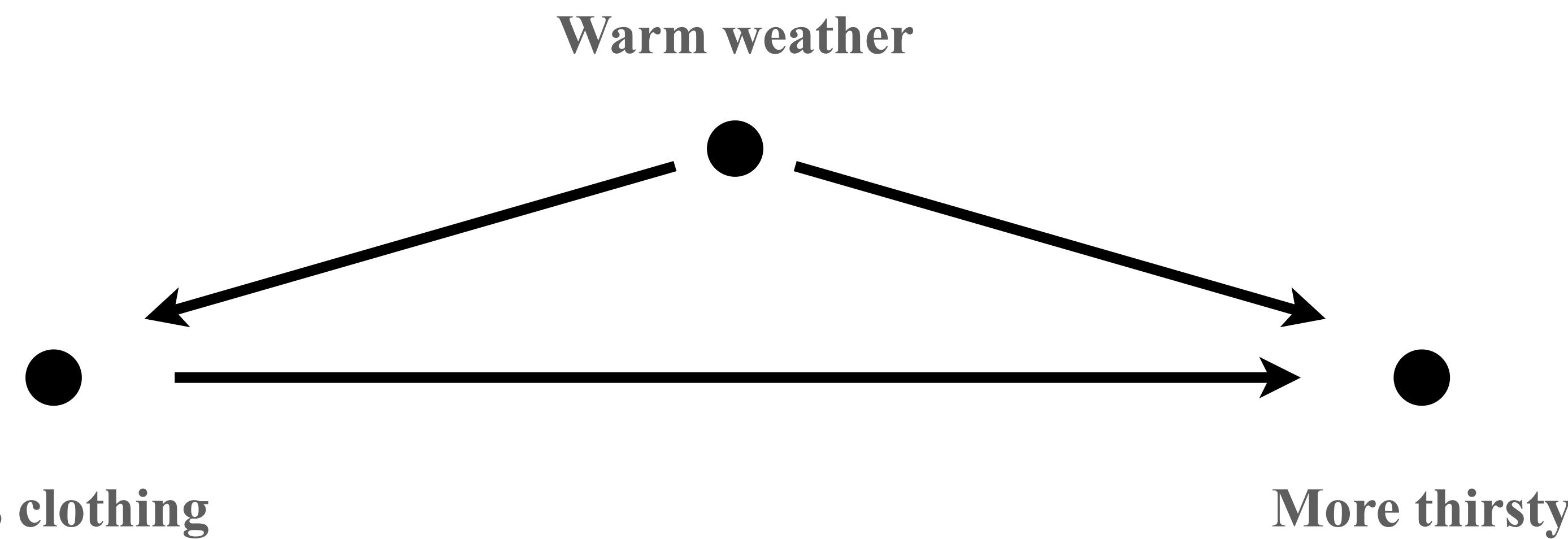
Please suggest a more sane model by applying one of the basic junctions to this mess



Less clothing

More thirsty

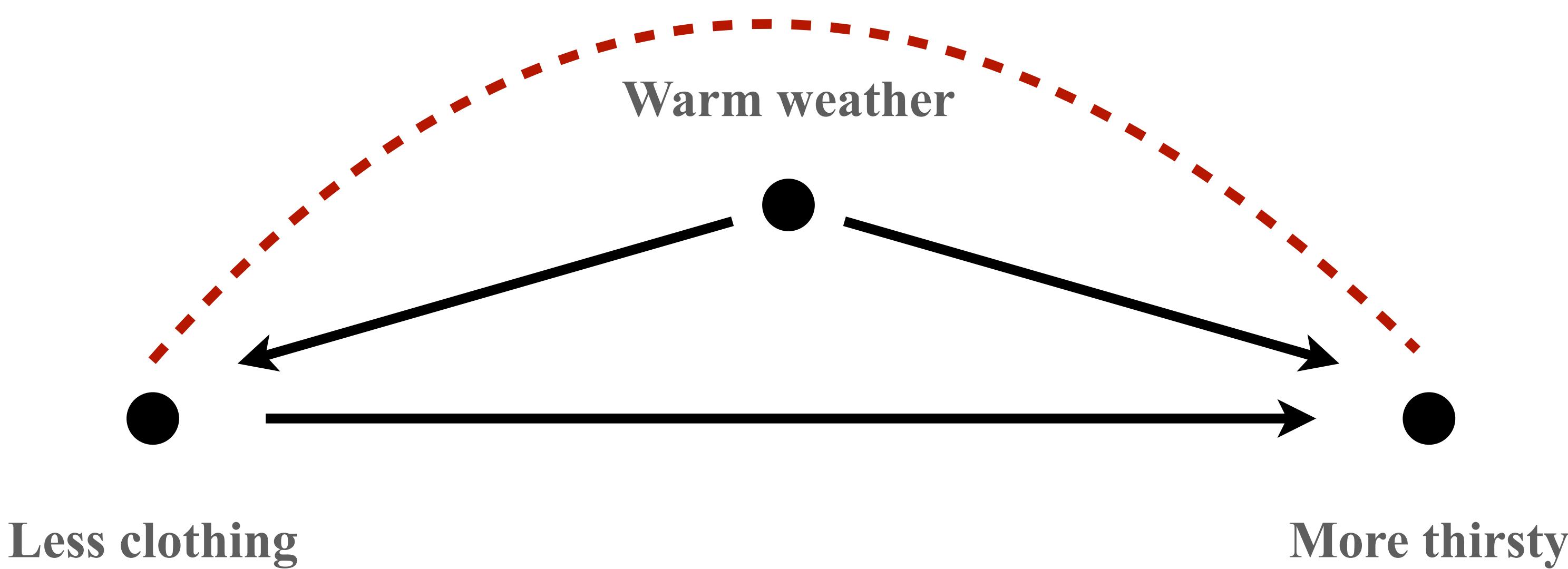




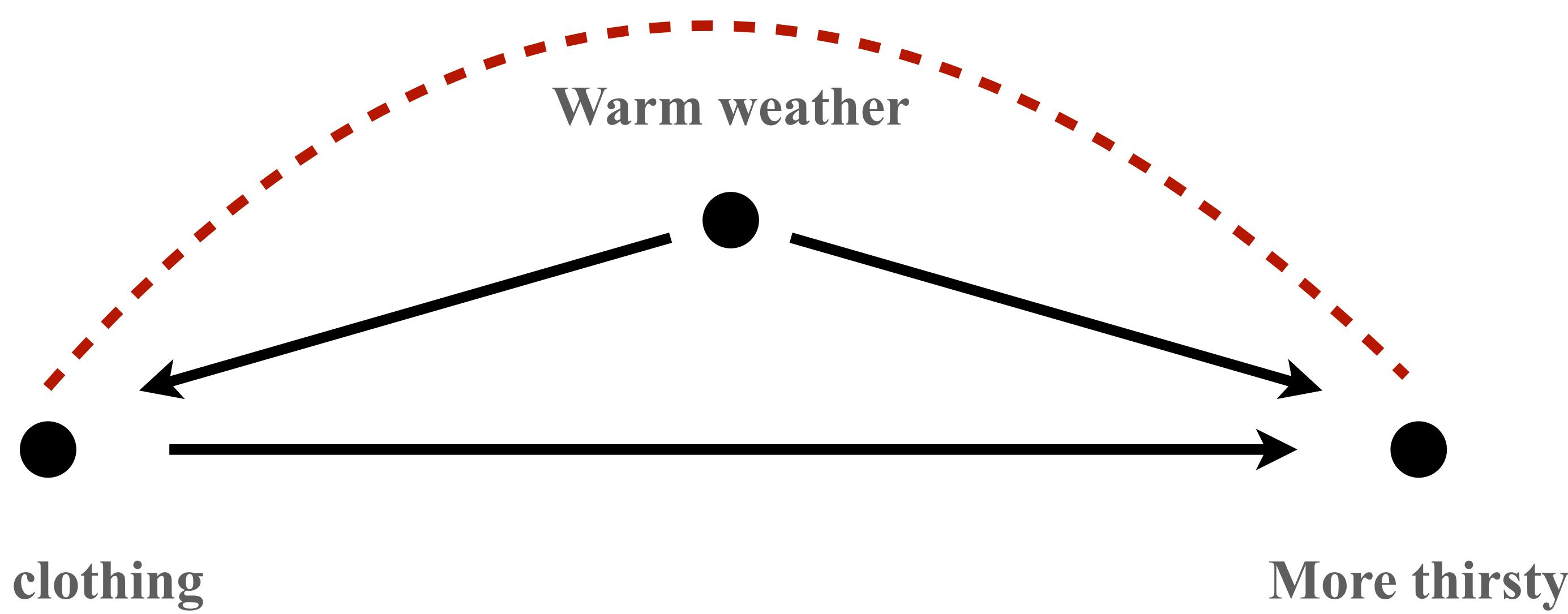
Note: leaving an arrow in is a weaker assumption than removing it.

**assuming an effect might be anything (including 0)
VS assuming that it is 0 exactly**

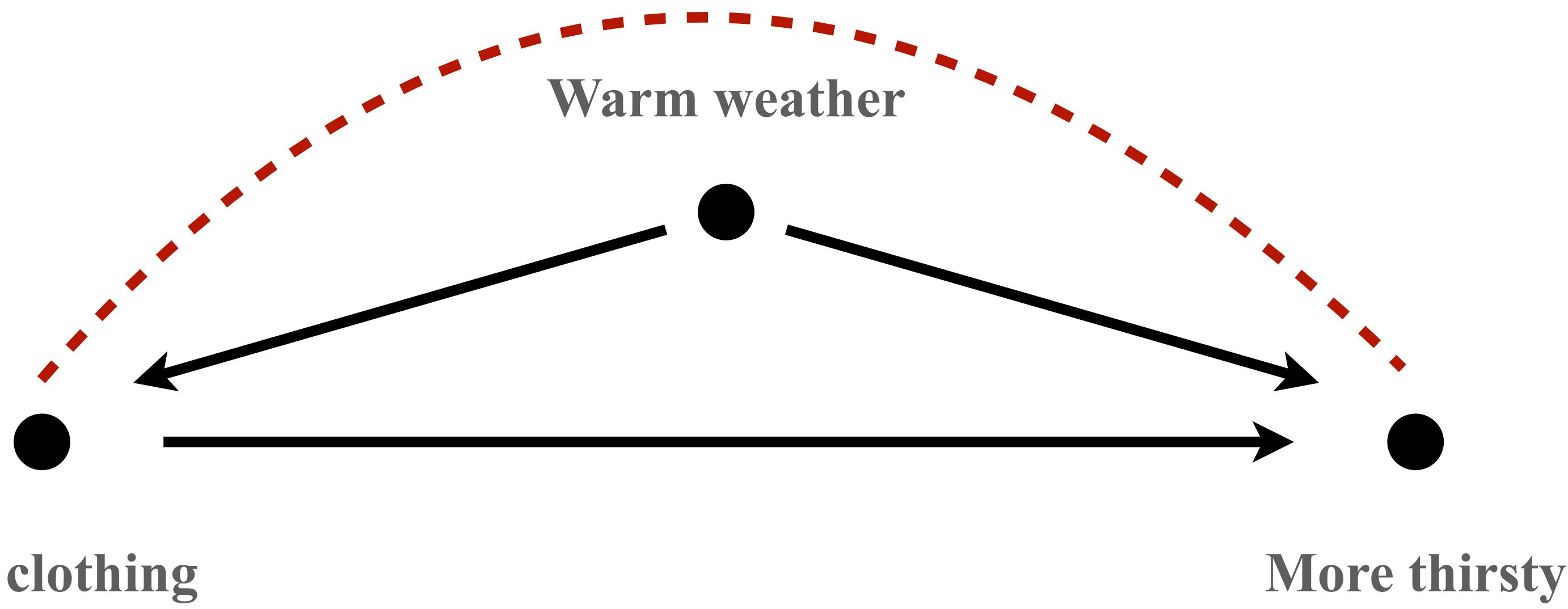
Transmits info



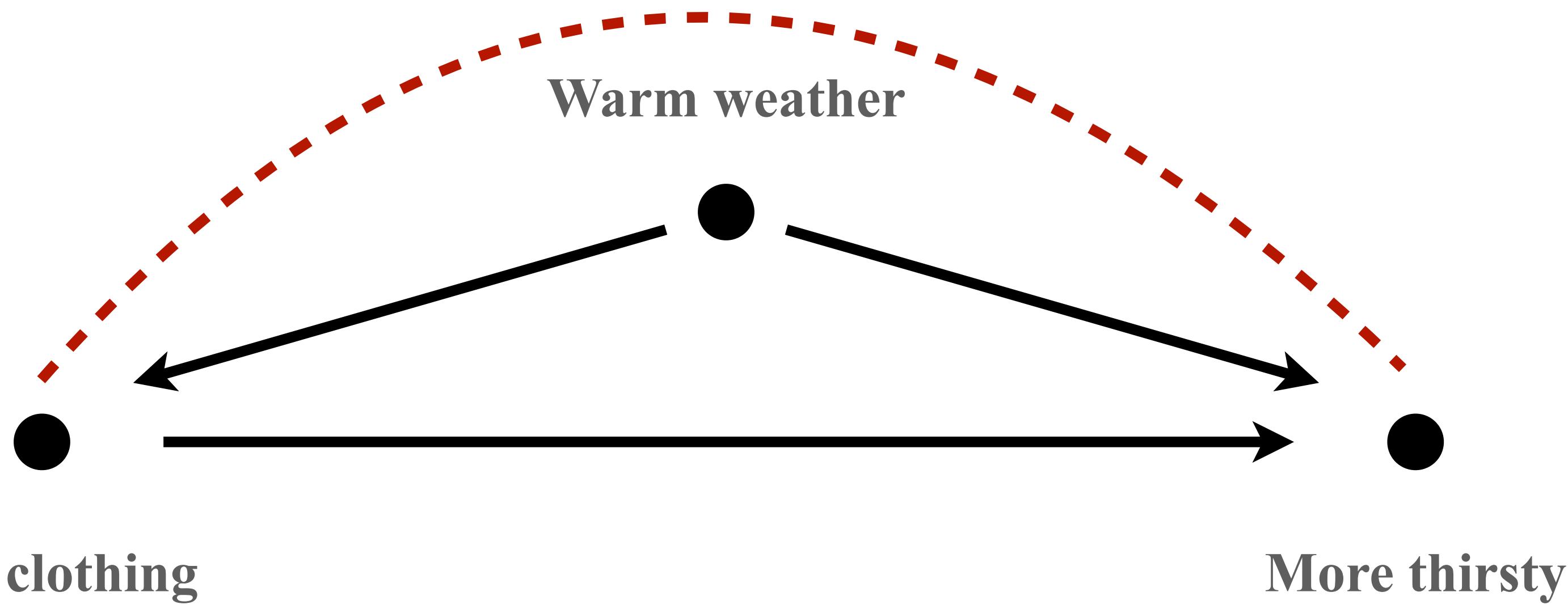
Transmits info



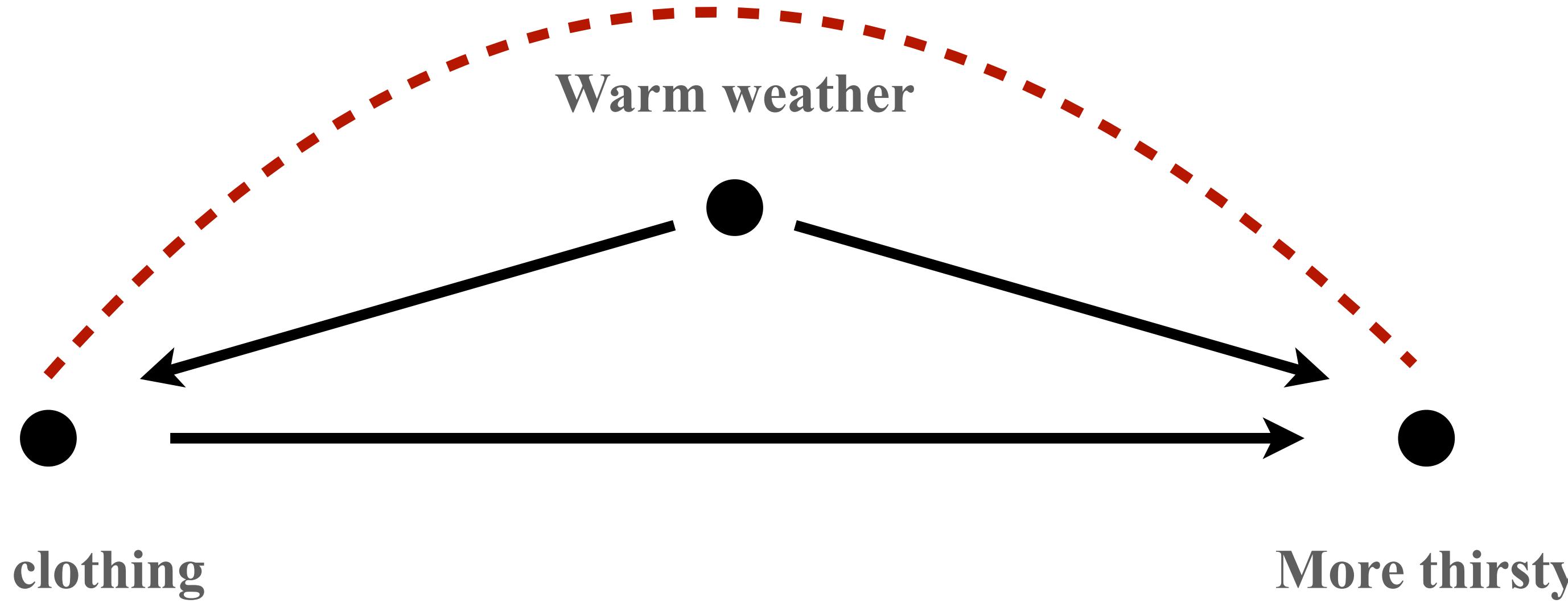
to confound: to mix or confuse



Confounding shows up as an open “back-door path” between exposure and outcome:

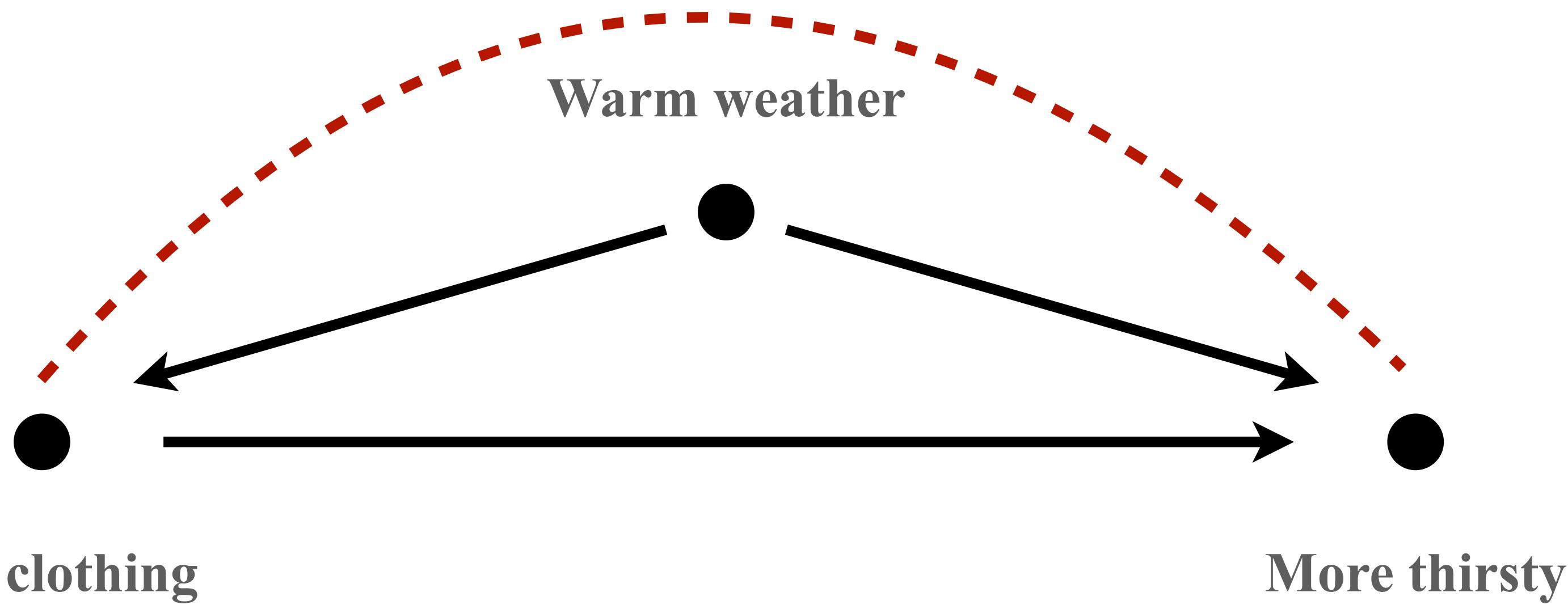


Confounding shows up as an open “back-door path” between exposure and outcome:



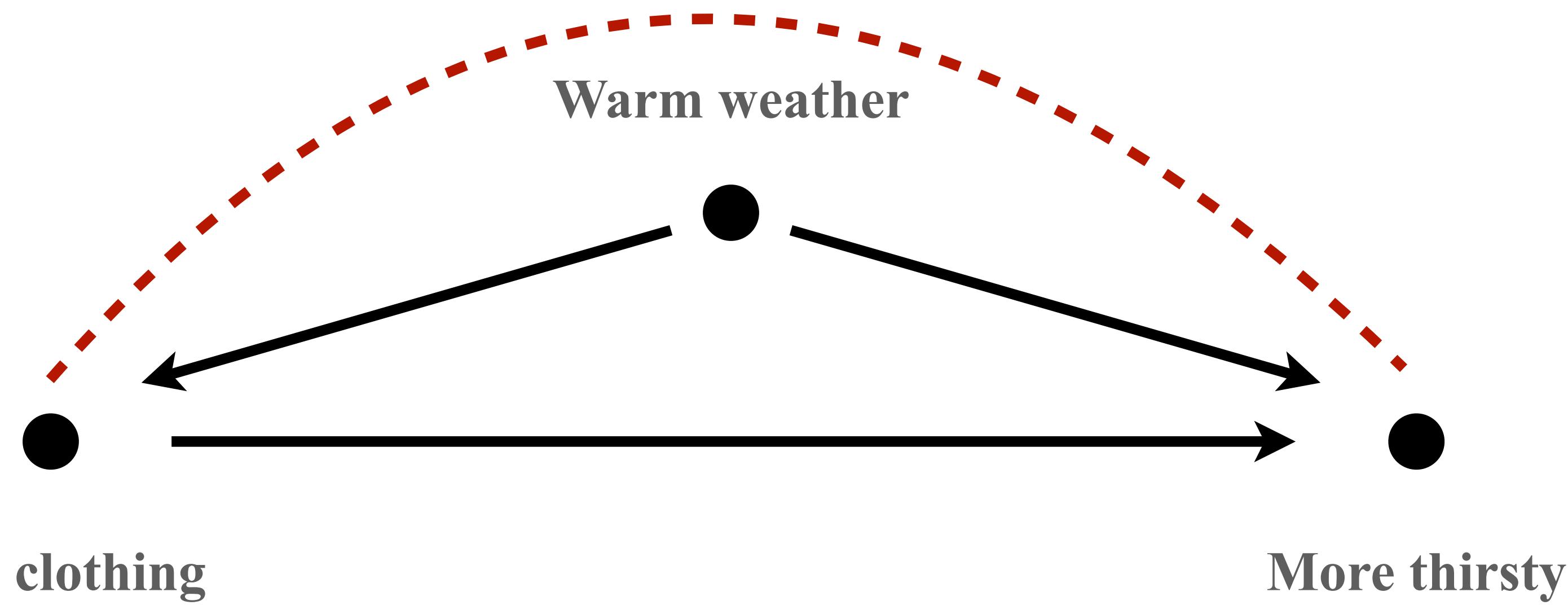
Confounding shows up as an open “back-door path” between exposure and outcome:

- **open:** no collider along the path



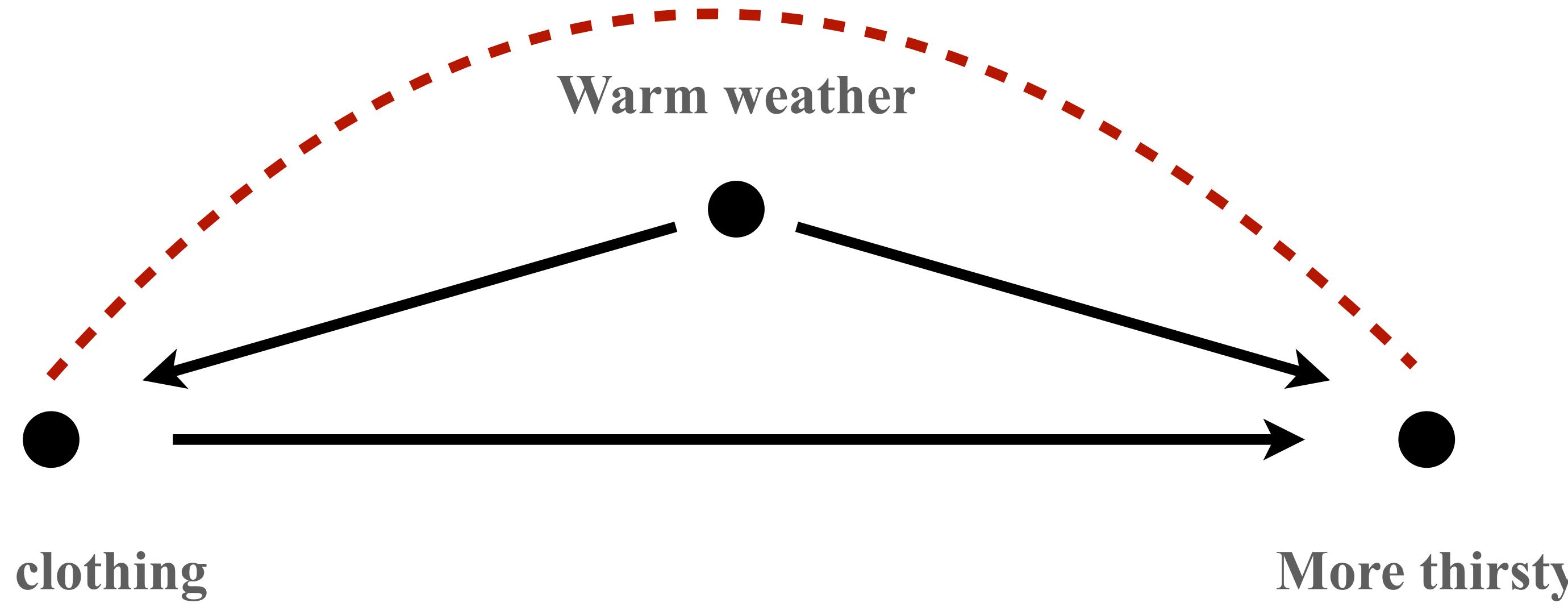
Confounding shows up as an open “back-door path” between exposure and outcome:

- **open:** no collider along the path
- **back-door:** an arrow goes *into* the exposure



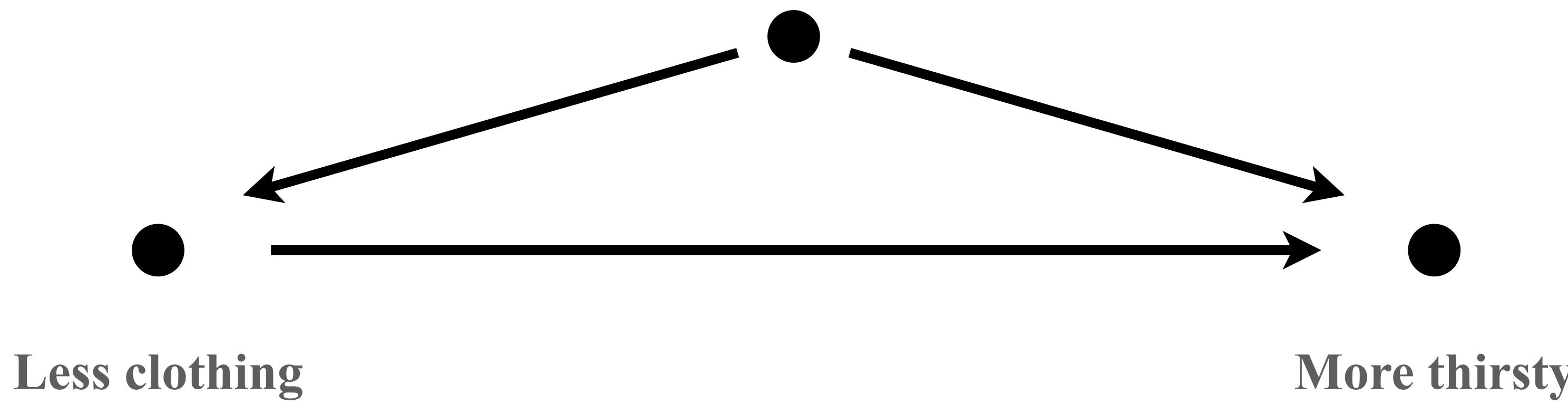
Confounding shows up as an open “back-door path” between exposure and outcome:

- **open**: no collider along the path
- **back-door**: an arrow goes *into* the exposure
- **The problem**: there is a mixing of the presumed causal relation along the direct path and the purely associational relation through the back-door path

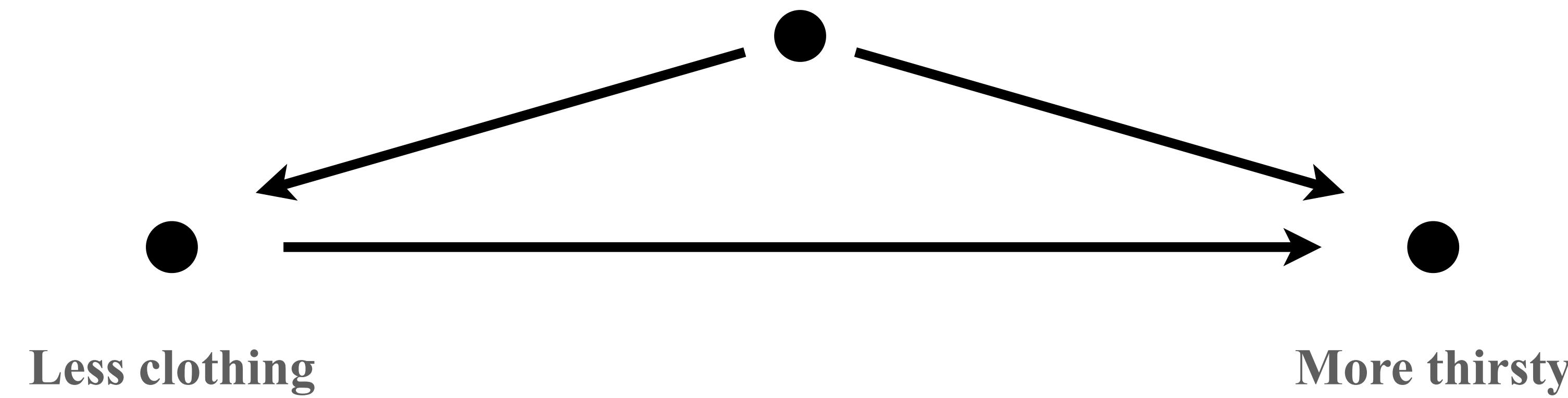


Please to tell me what I can do about this situation.

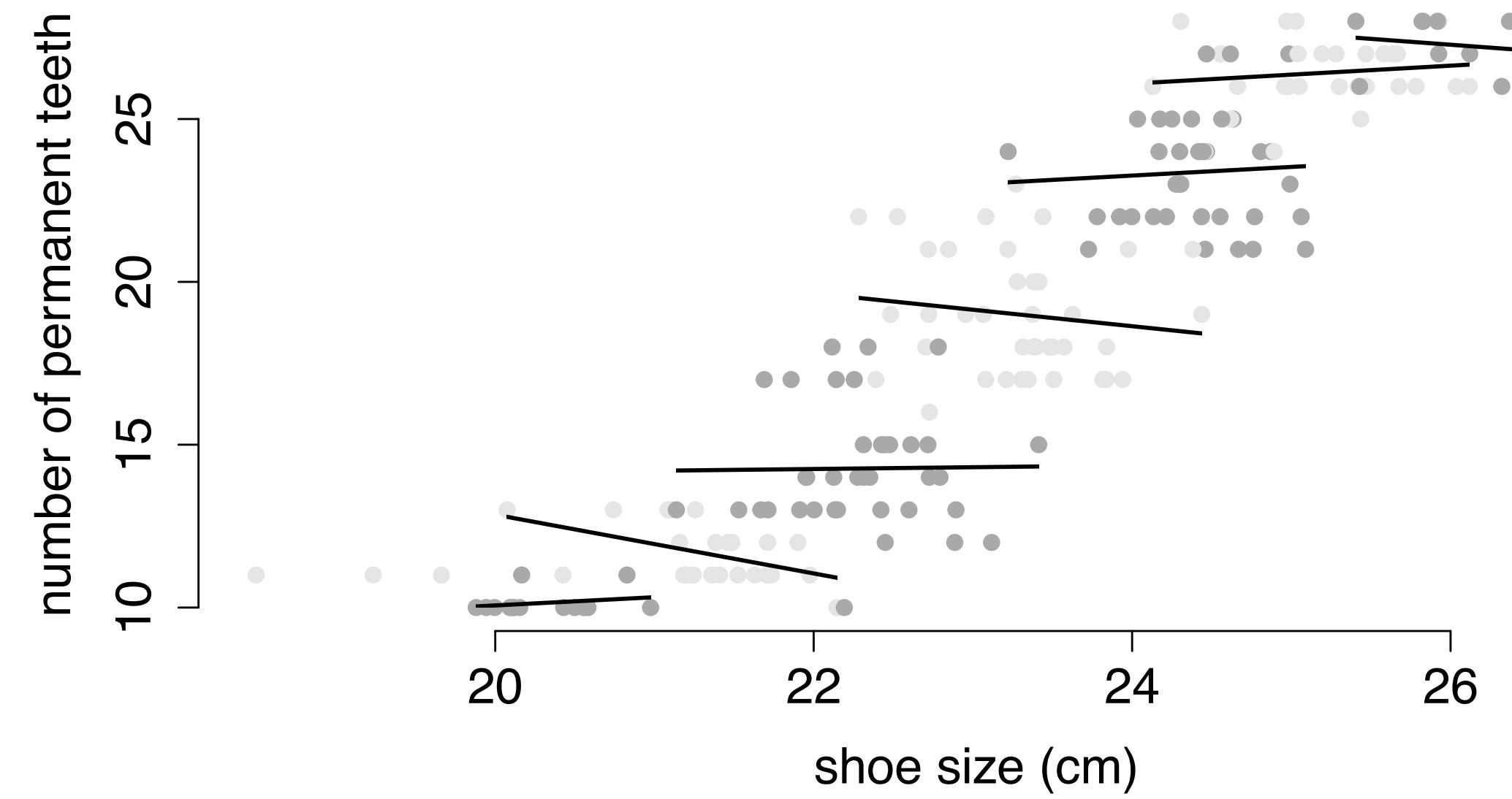
Temperature = x, y, z, ...



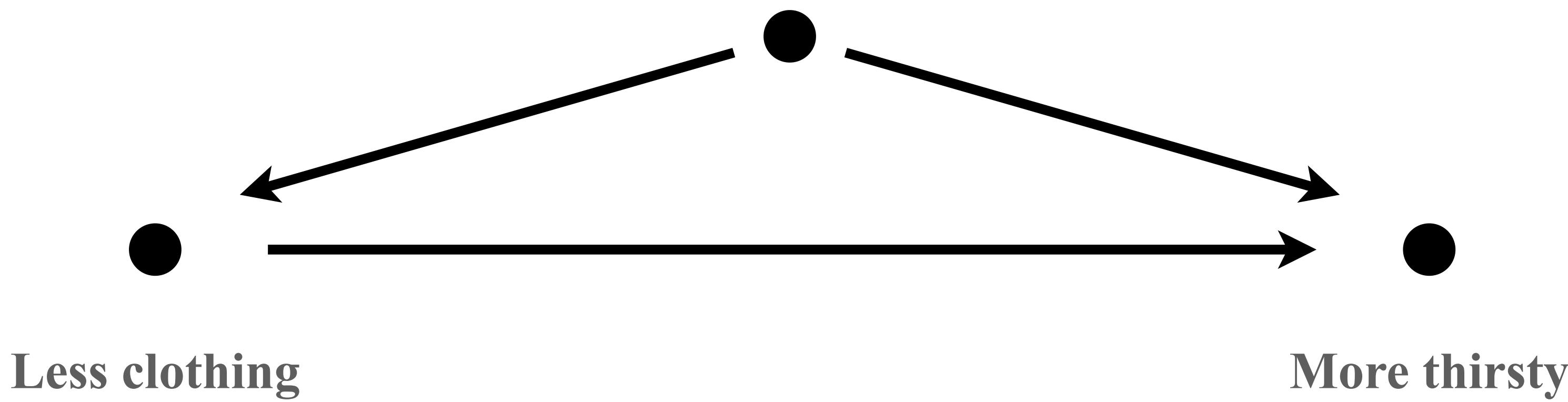
Temperature = x, y, z, ...



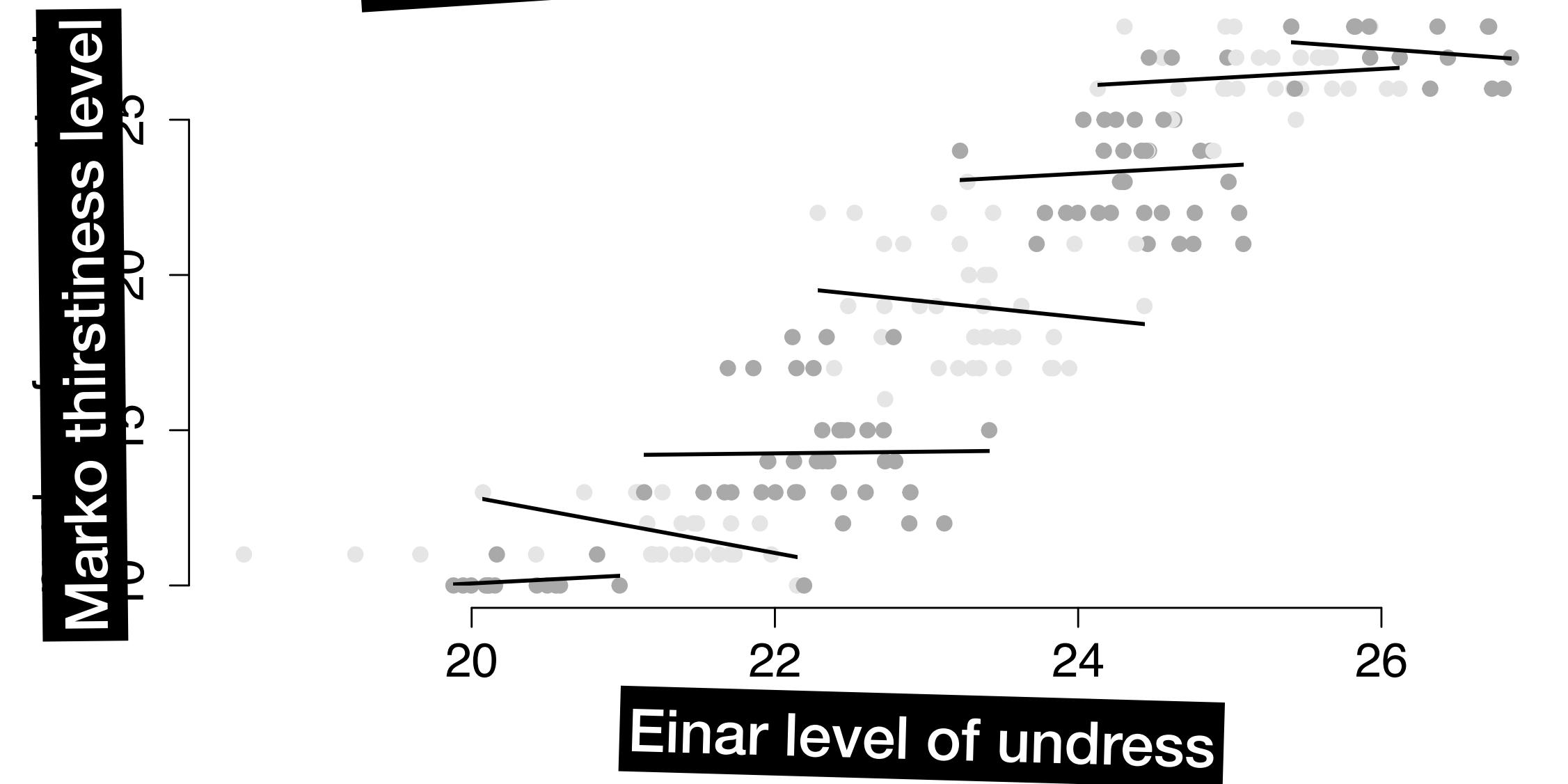
teeth vs shoe size,
regressions conditional on age



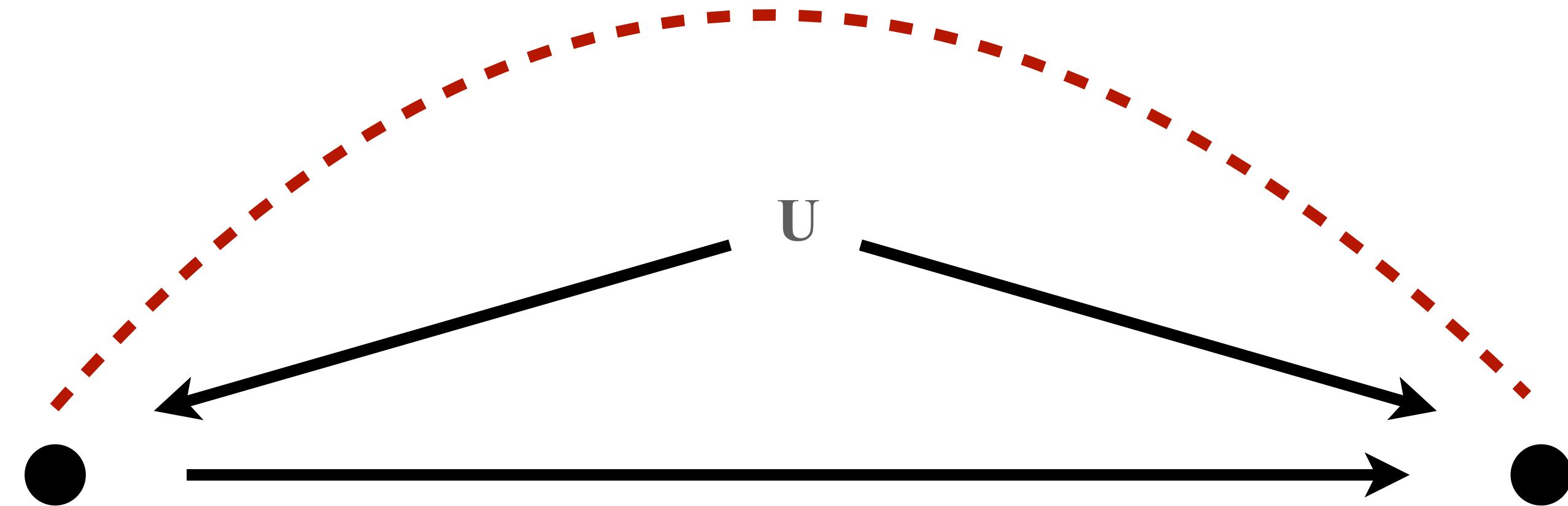
Temperature = x, y, z, ...



Thirst level as a function of state of undress,
regressions conditional on temperature
and age



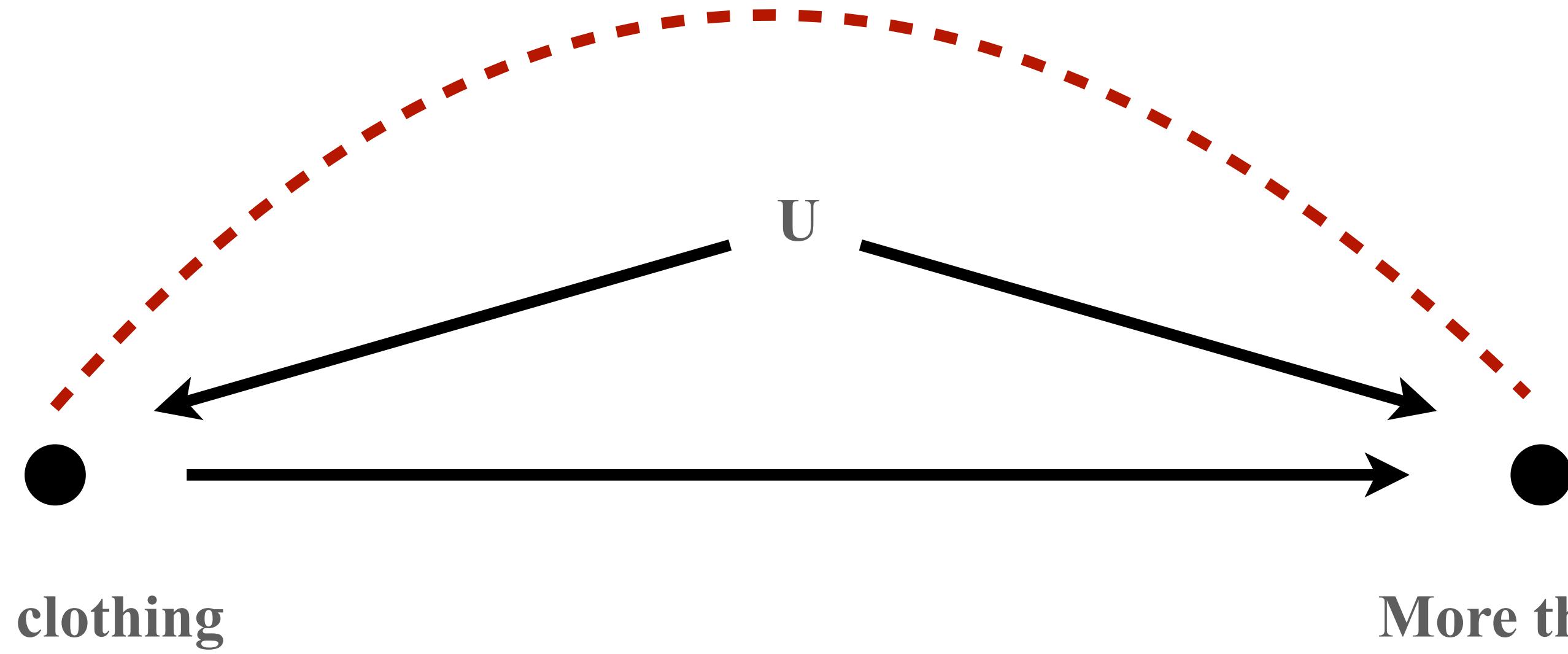
**Comparing 25 degree days with one
another: comparing like with like**



Less clothing

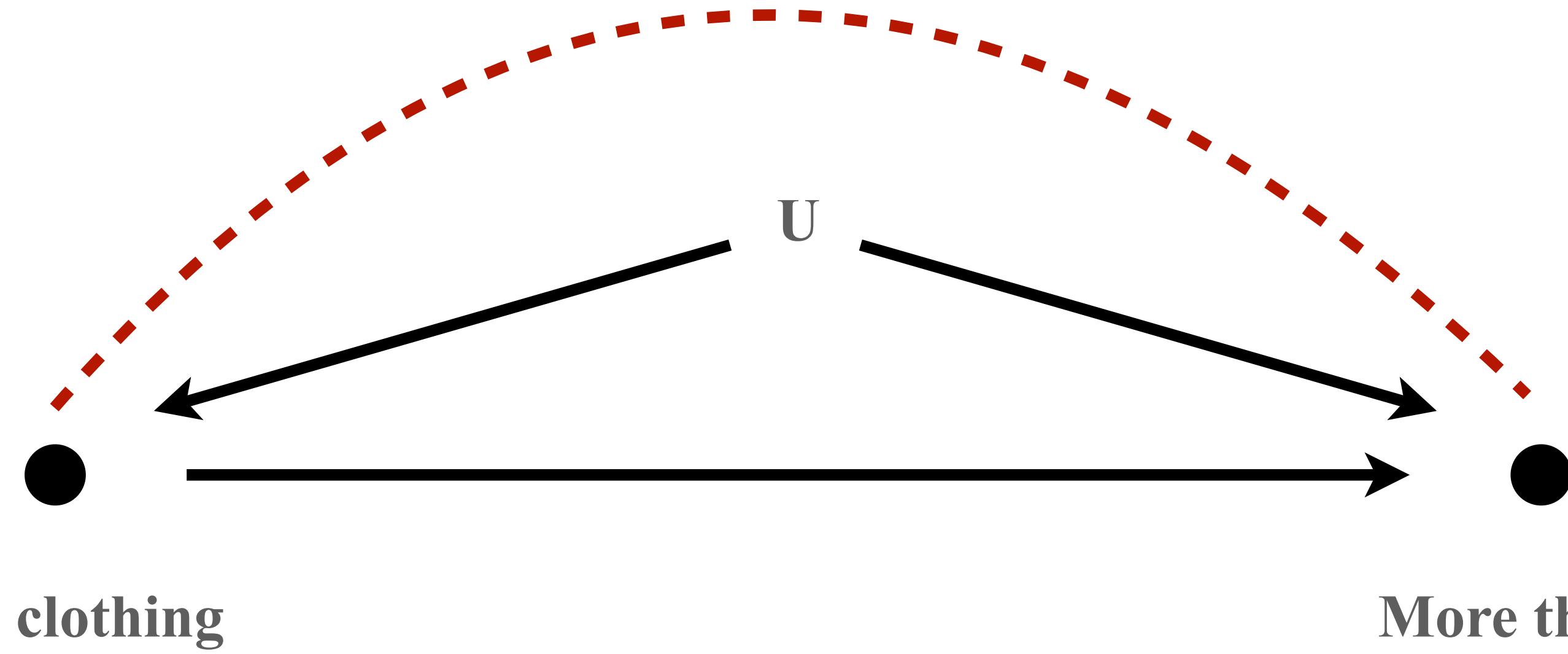
More thirsty

Temperature UNOBSERVED: what do we do???



Temperature UNOBSERVED: what do we do???

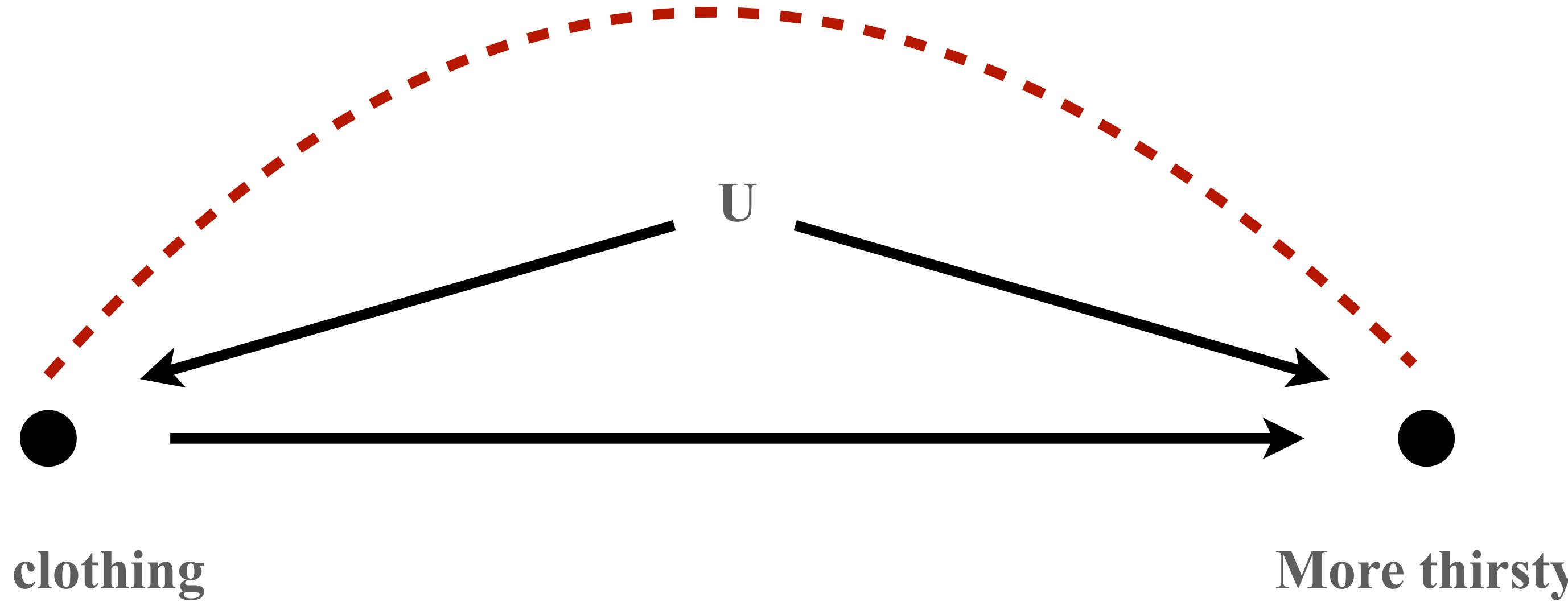
Not much to do: the effect is **non-identifiable** (get more data or do an actual experiment)



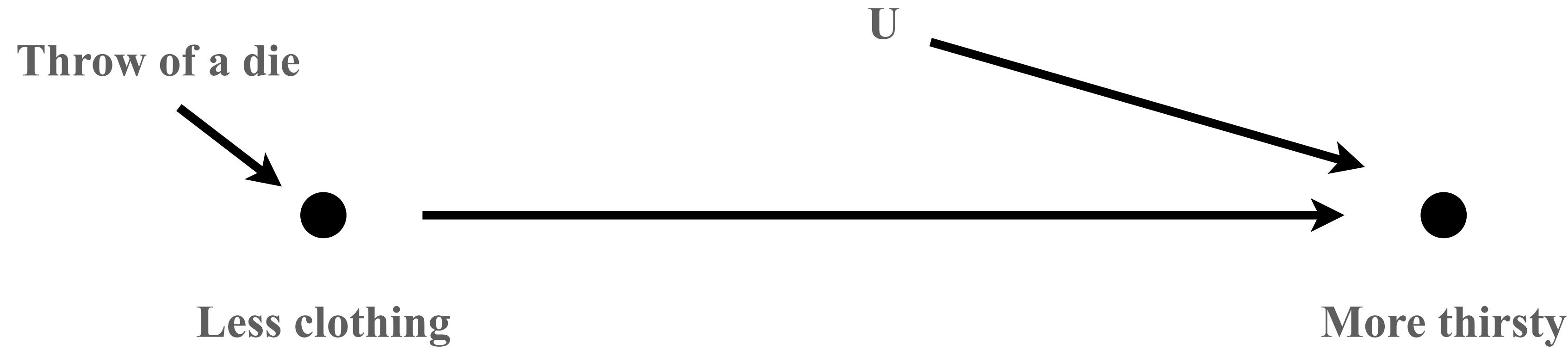
Temperature UNOBSERVED: what do we???

Not much to do: the effect is **non-identifiable** (get more data or do an actual experiment)

An effect is **identifiable** if it is possible to close all back-door paths without opening new ones



Why randomization is so good



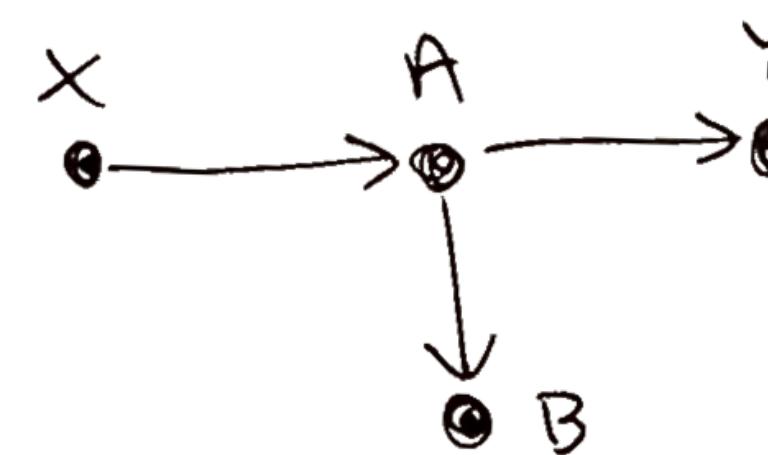
Why randomization is so good

Point: For the price of making some causal assumptions, the rule about closing back-door paths tells us exactly what to adjust for

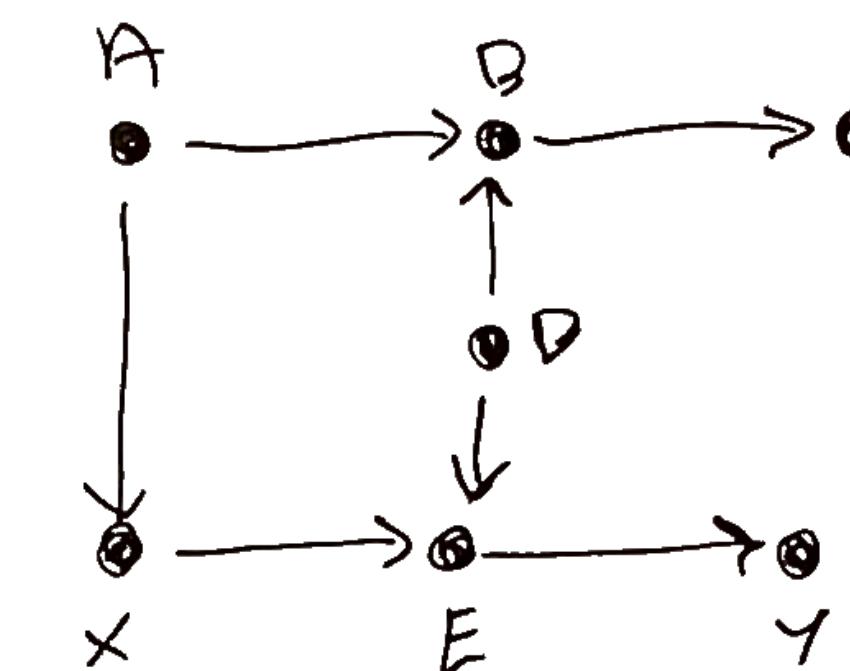
Point: For the price of making some causal assumptions, the rule about closing back-door paths tells us exactly what to adjust for

(Also you need to have made the right measurements)

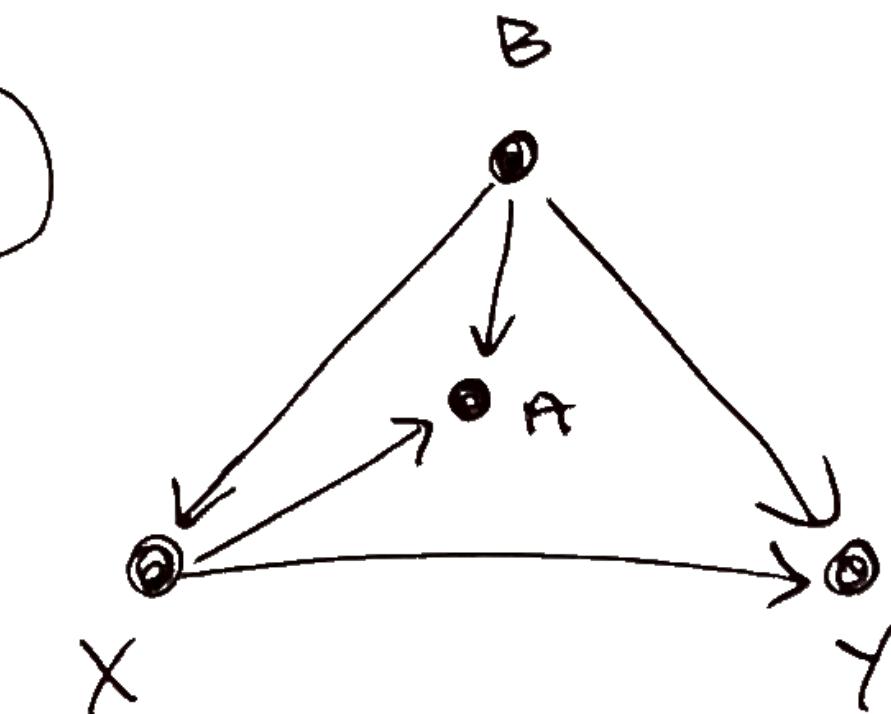
1.



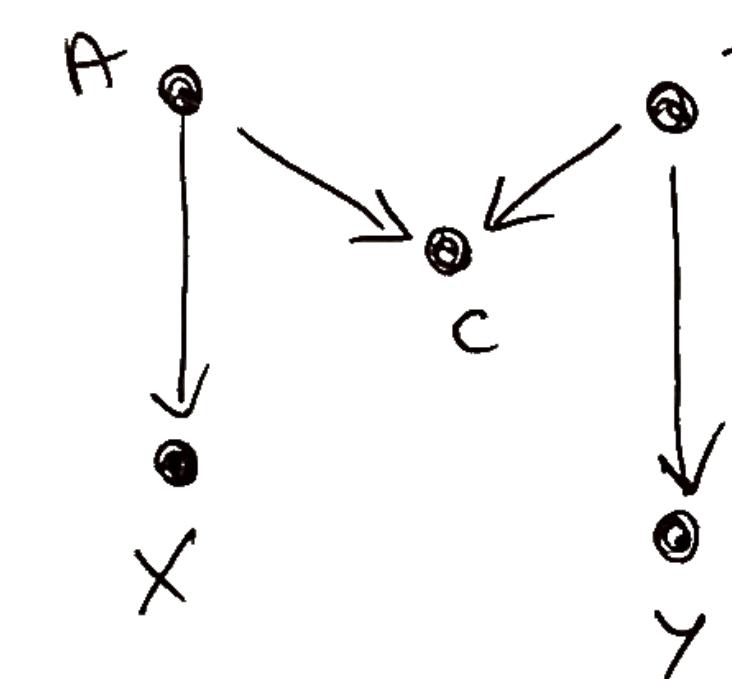
2.



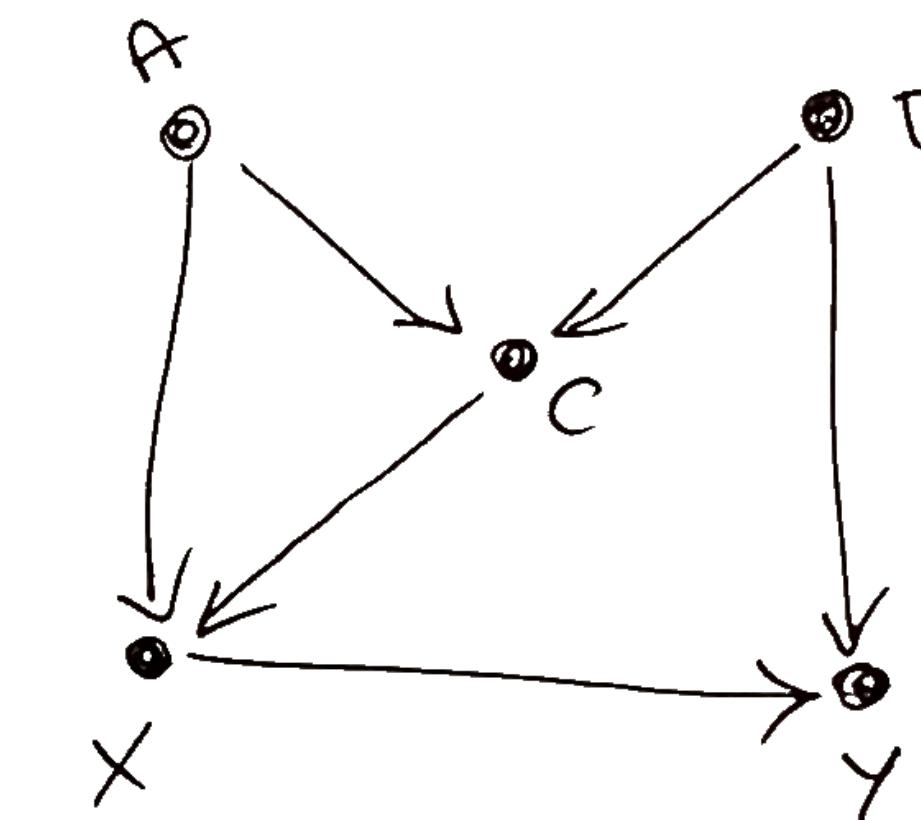
3.



4.

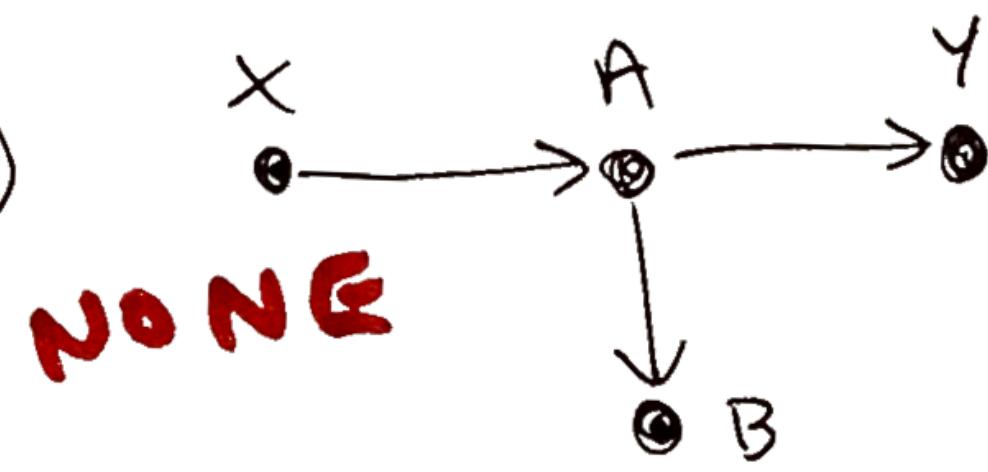


5.



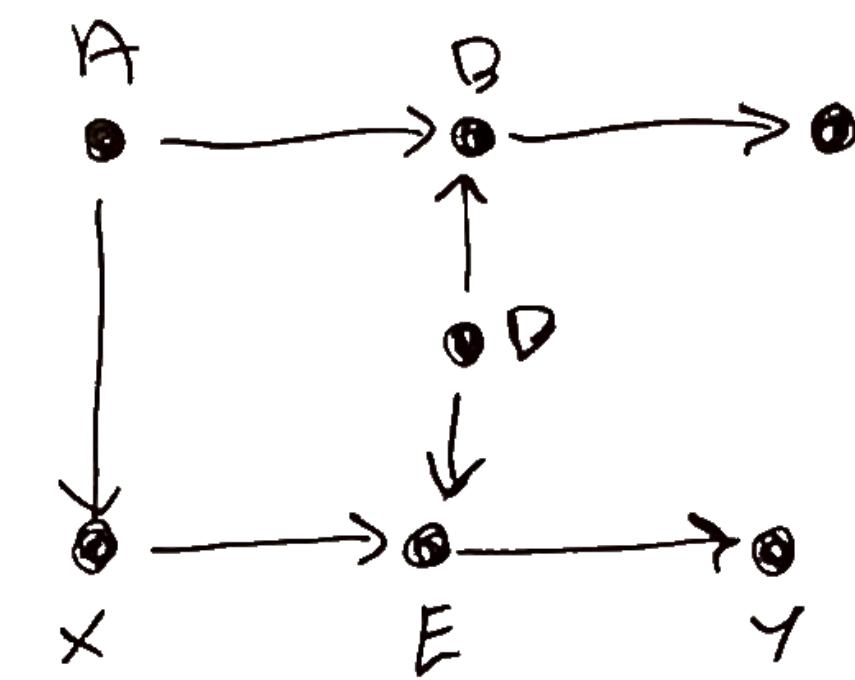
Puzzles: what should we adjust for to close the back-door paths between x and y?

①.

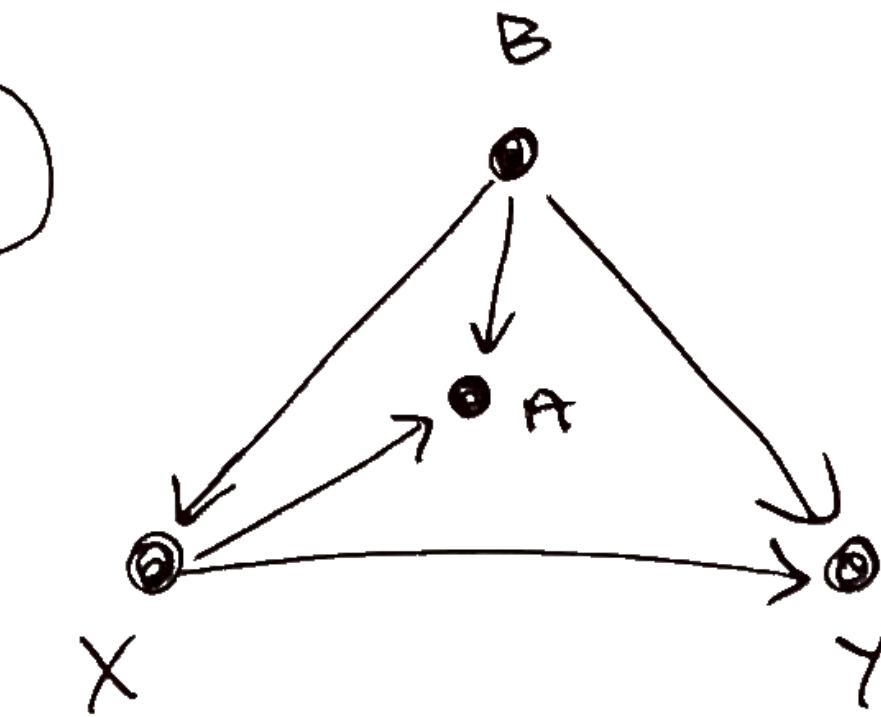


NON E

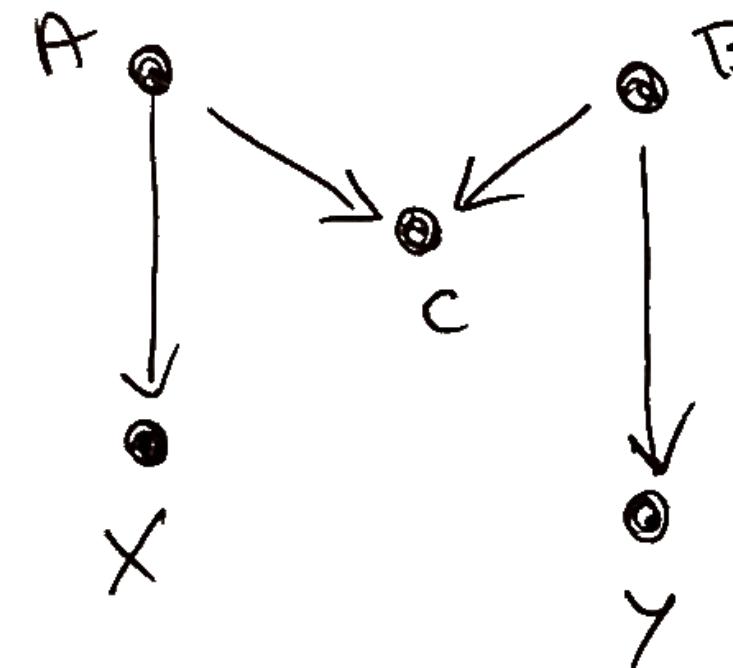
②.



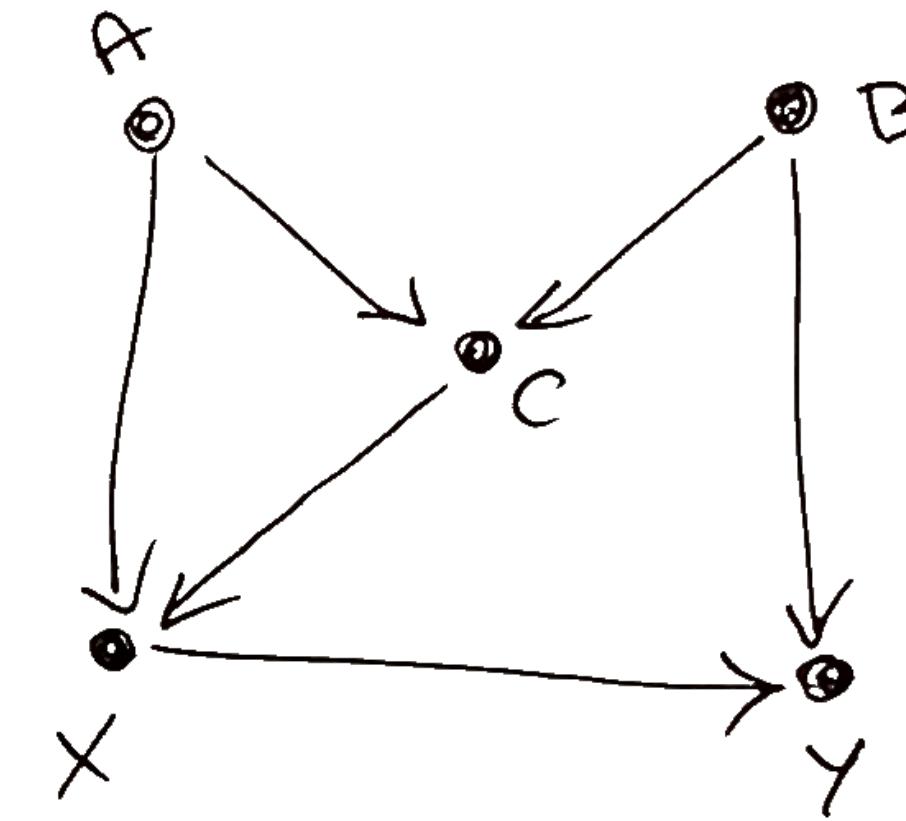
③.



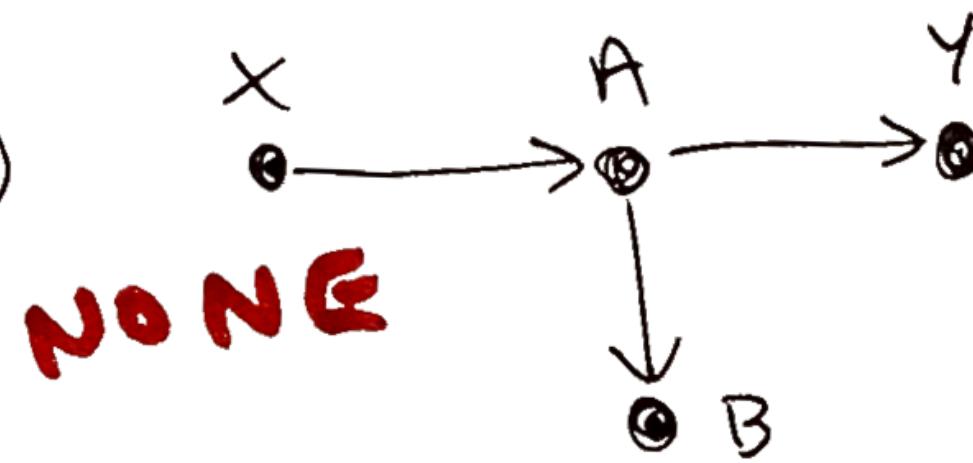
④.



⑤.

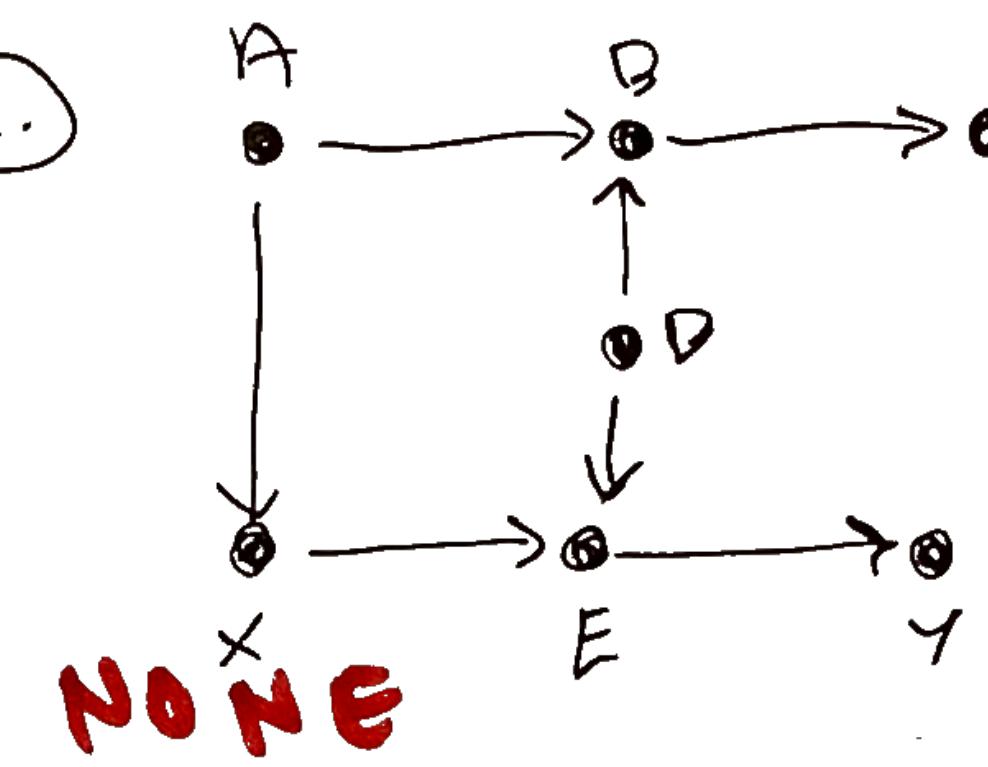


①.



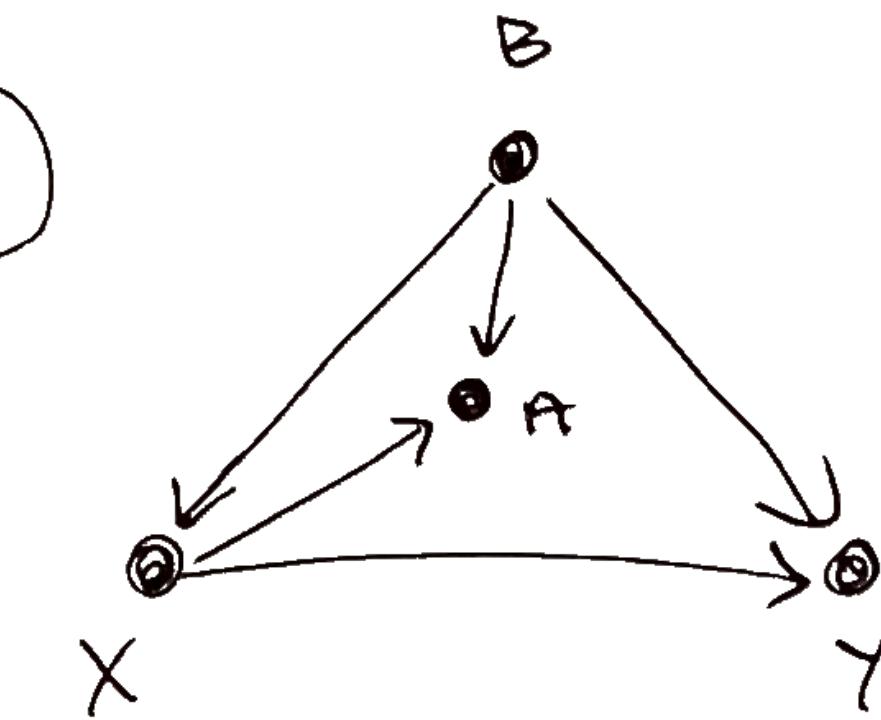
NON E

②.

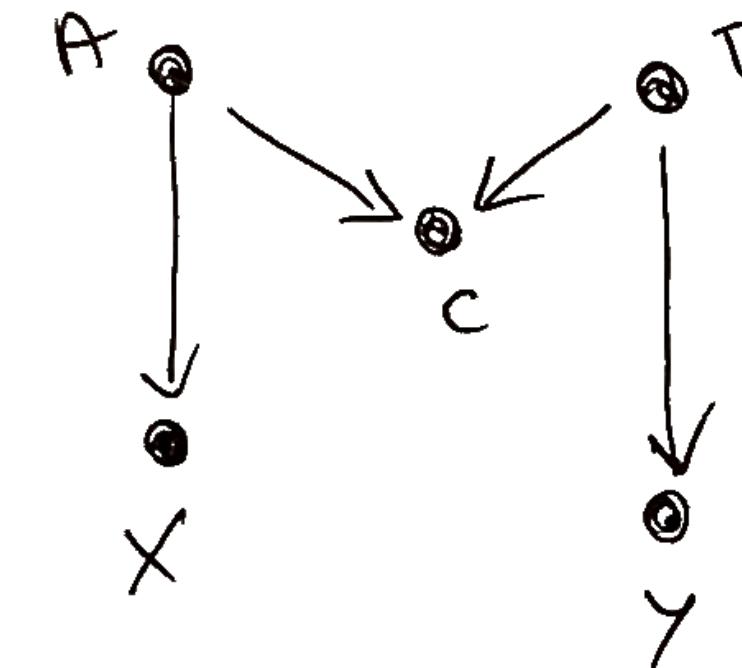


NON E

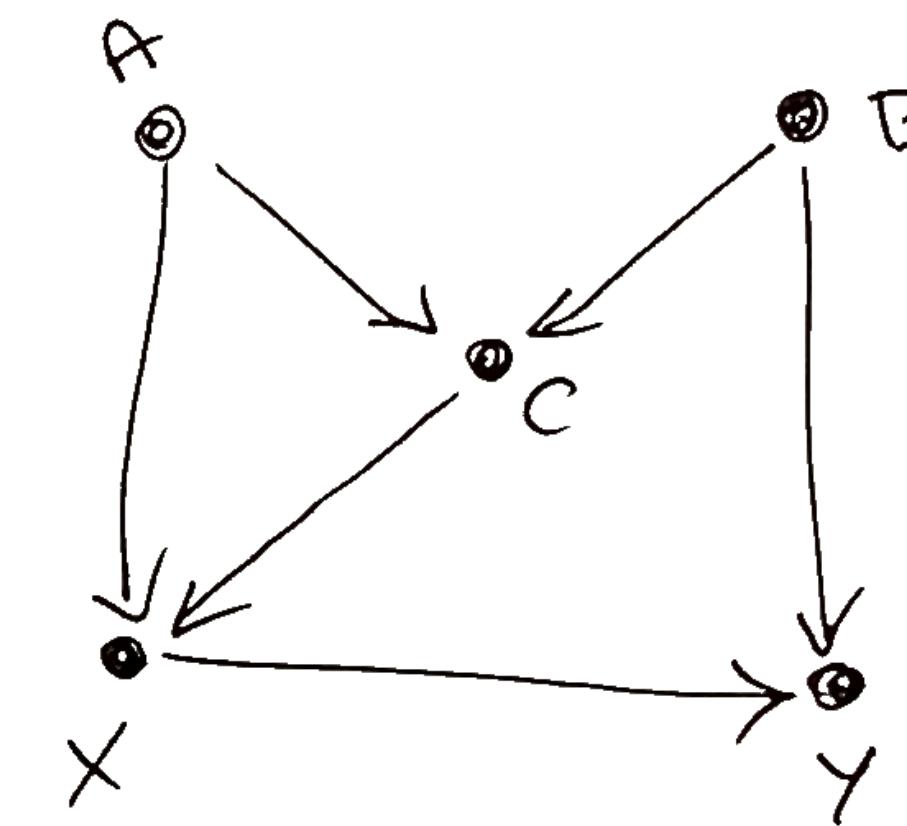
③.



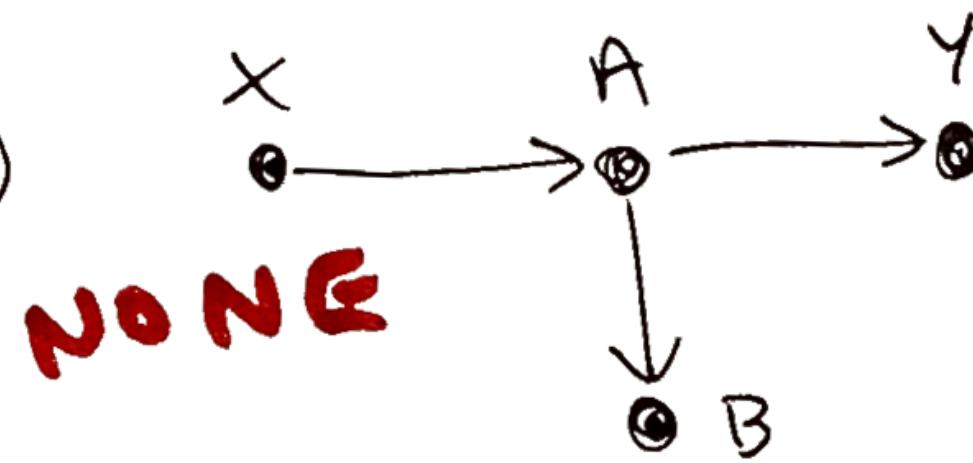
④.



⑤.

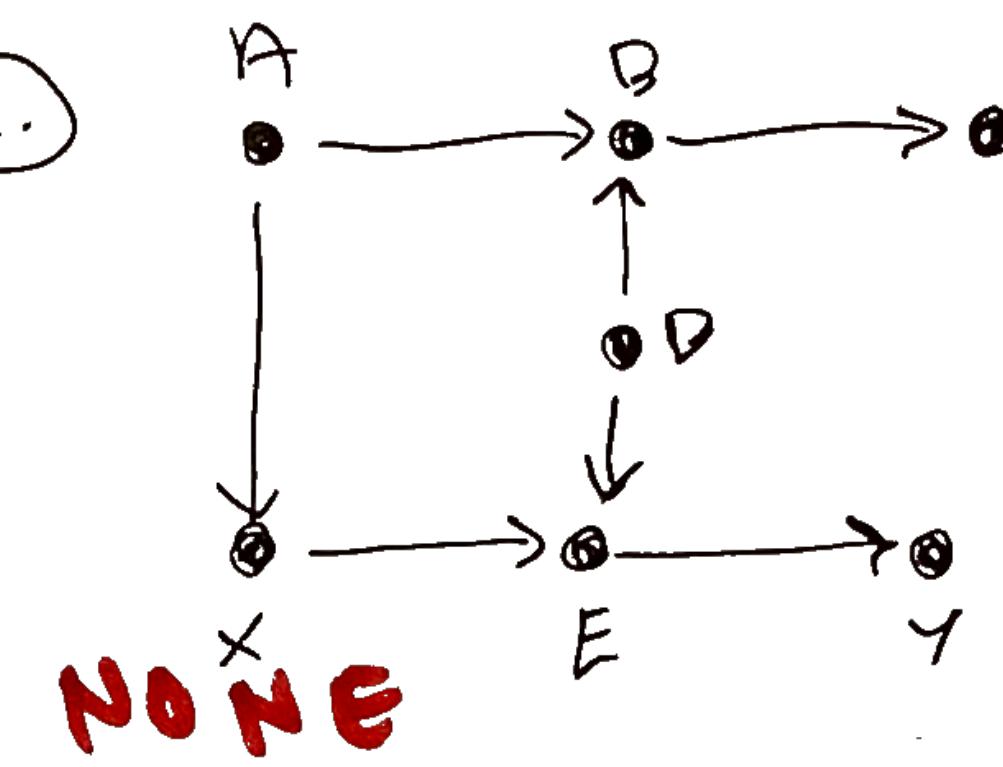


①.



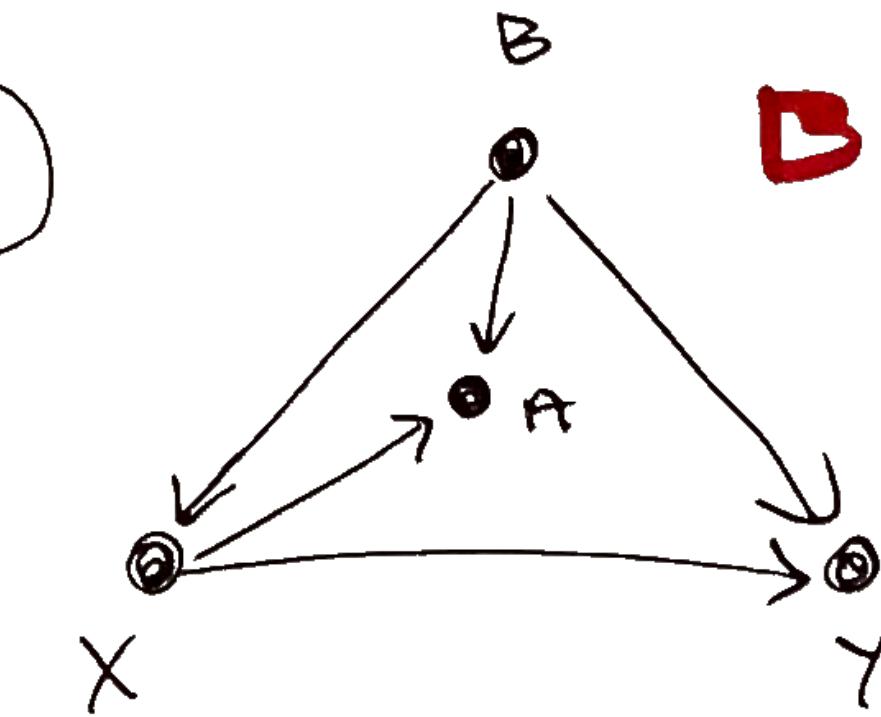
NON E

②.

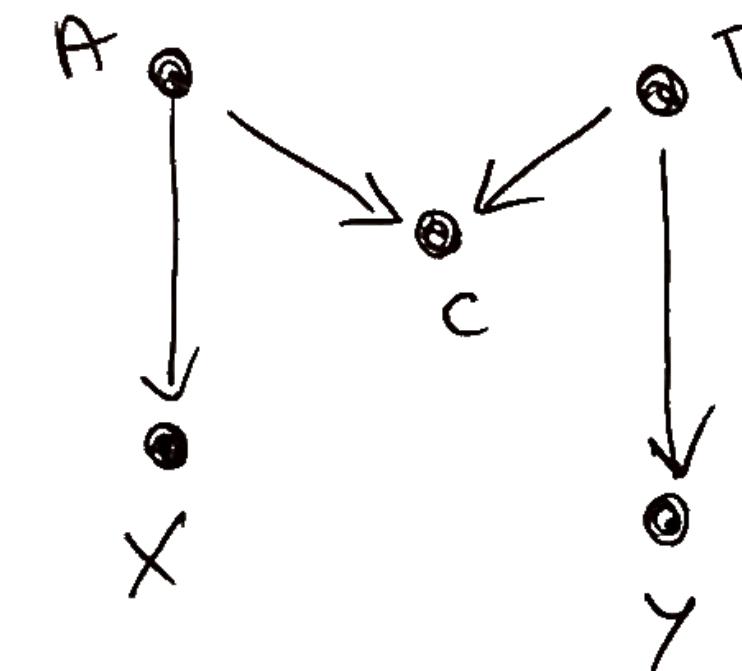


NON E

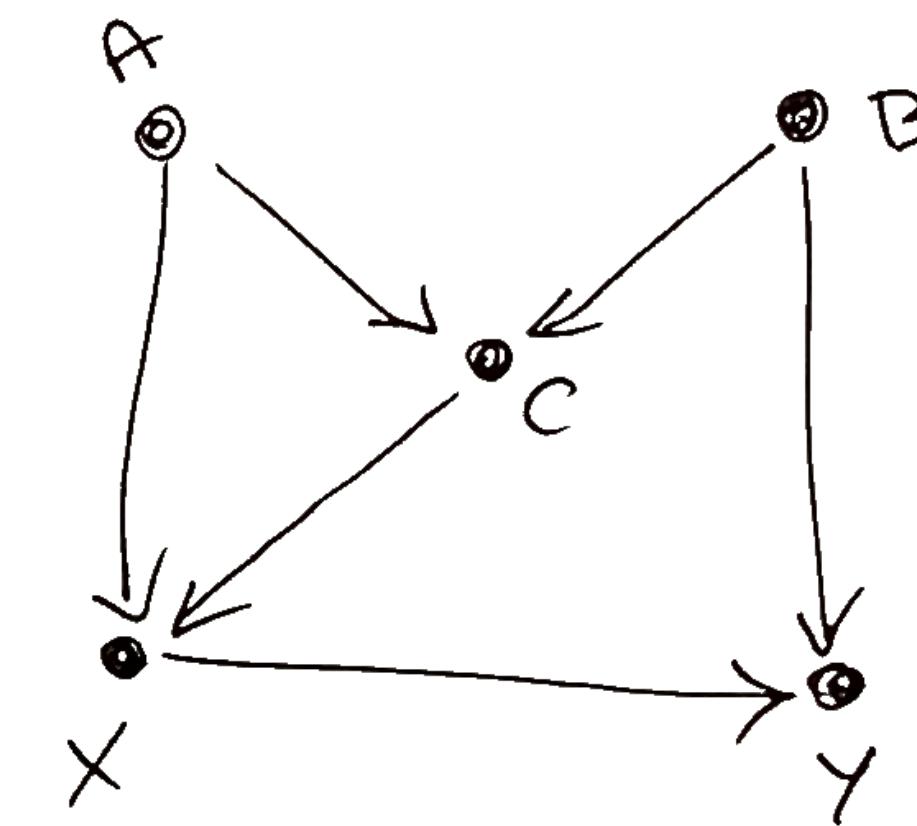
③.



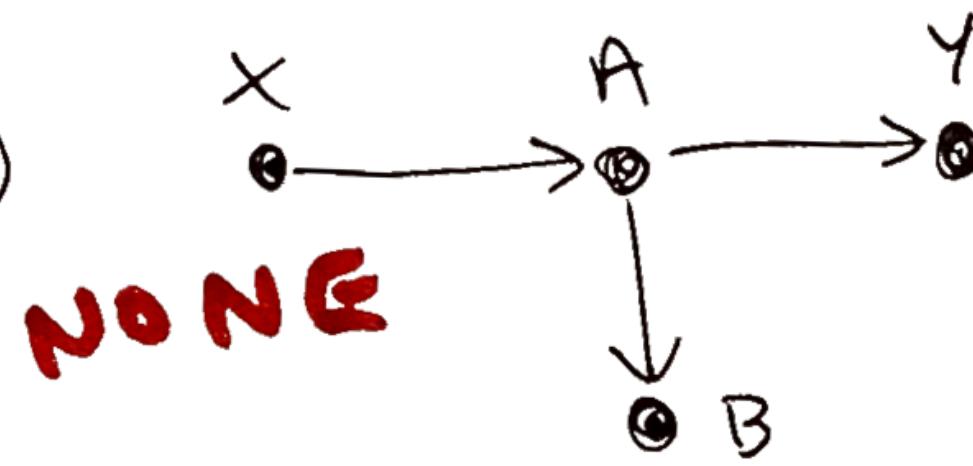
④.



⑤.

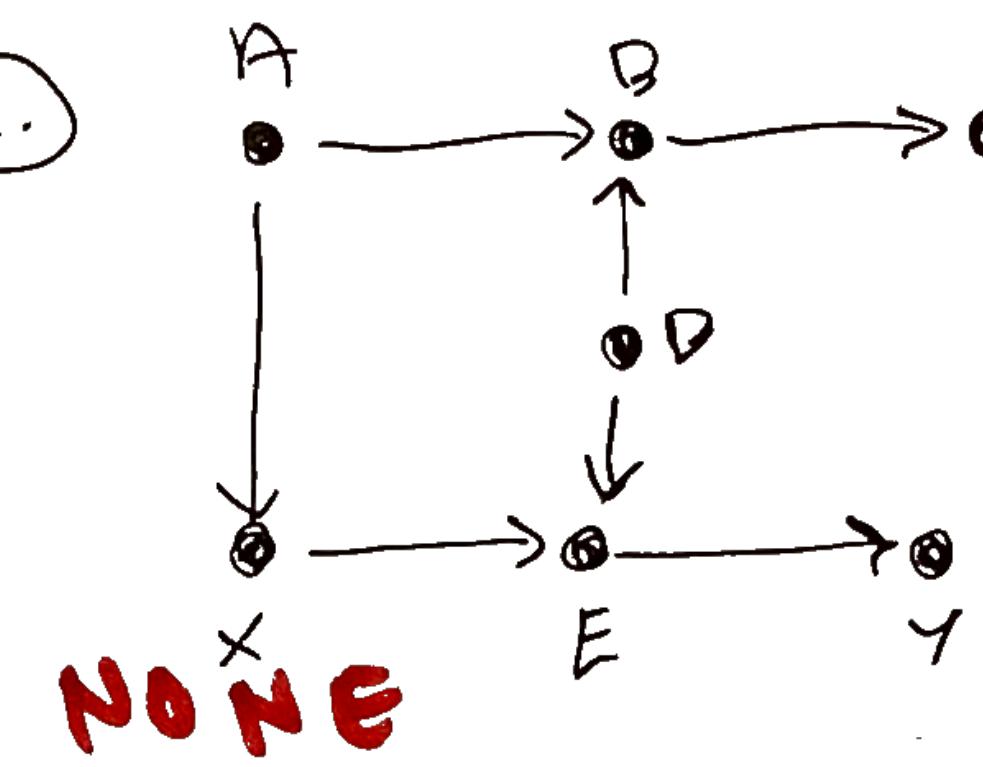


①.



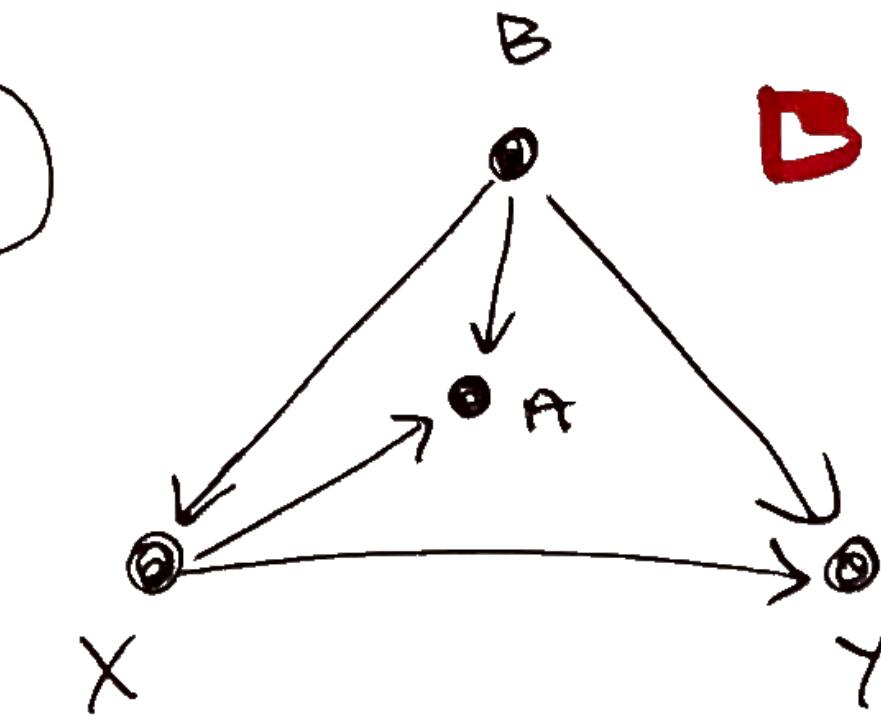
NONE

②.



NONE

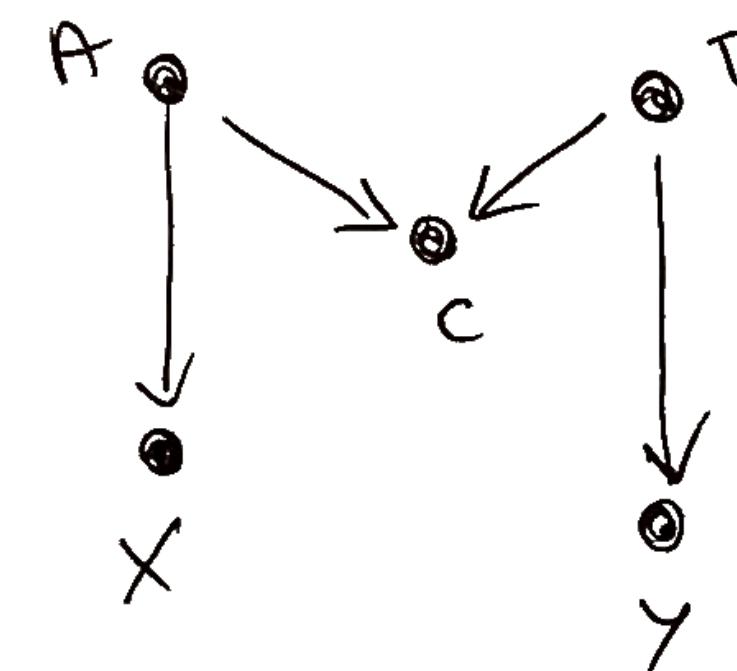
③.



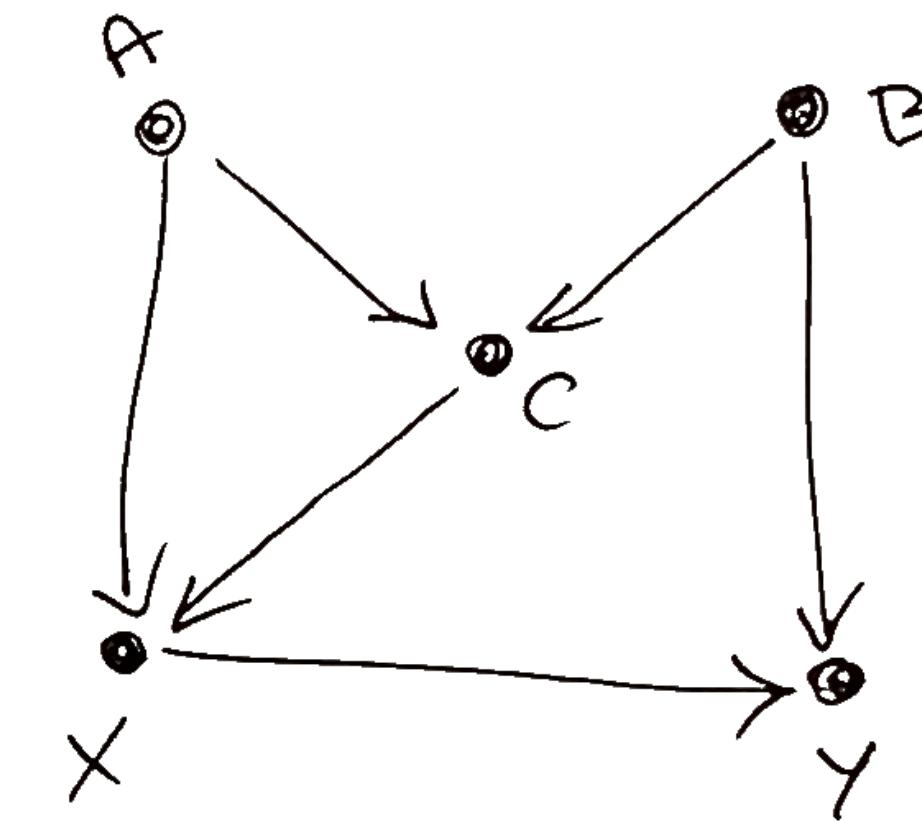
D

④.

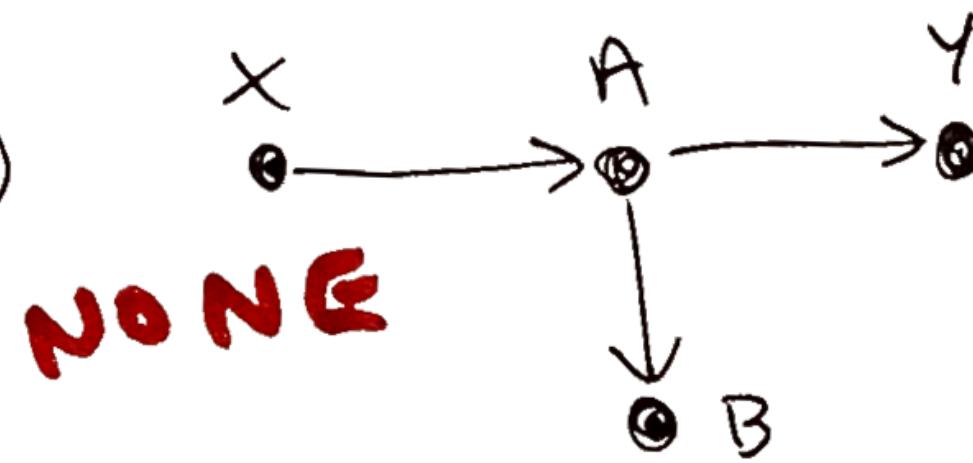
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⑤.

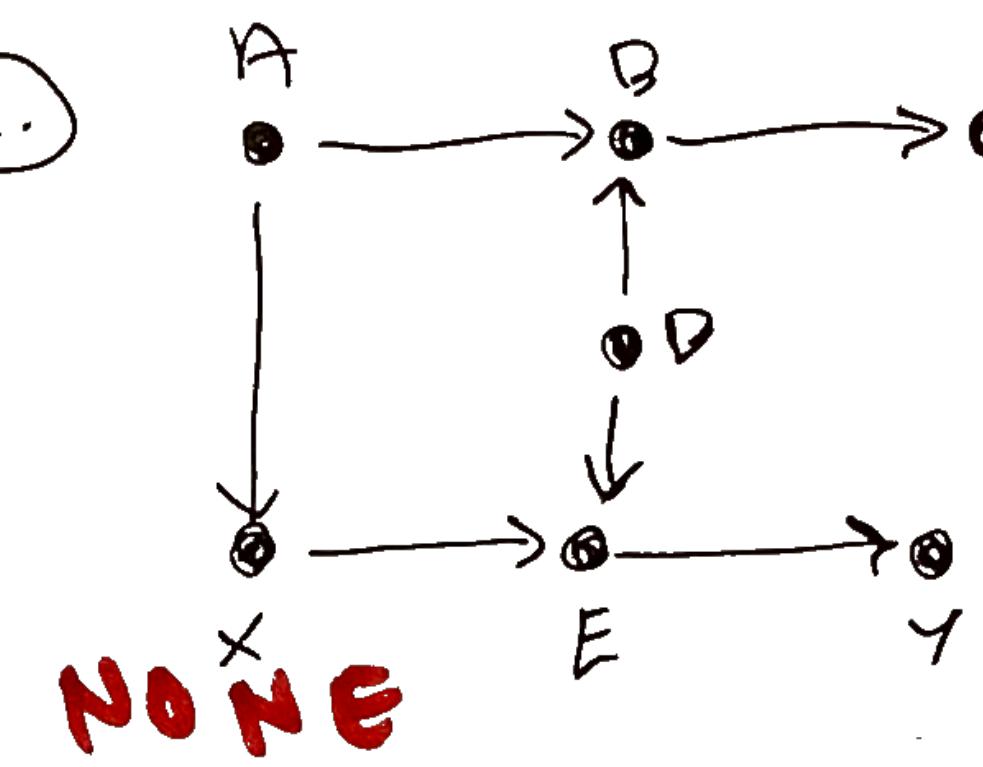


①.



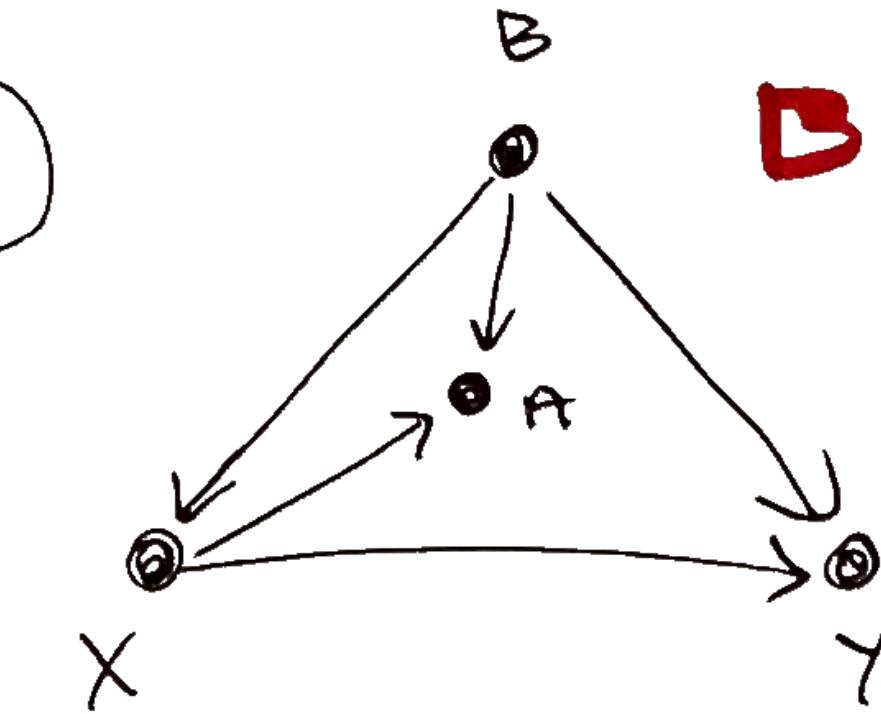
NONE

②.



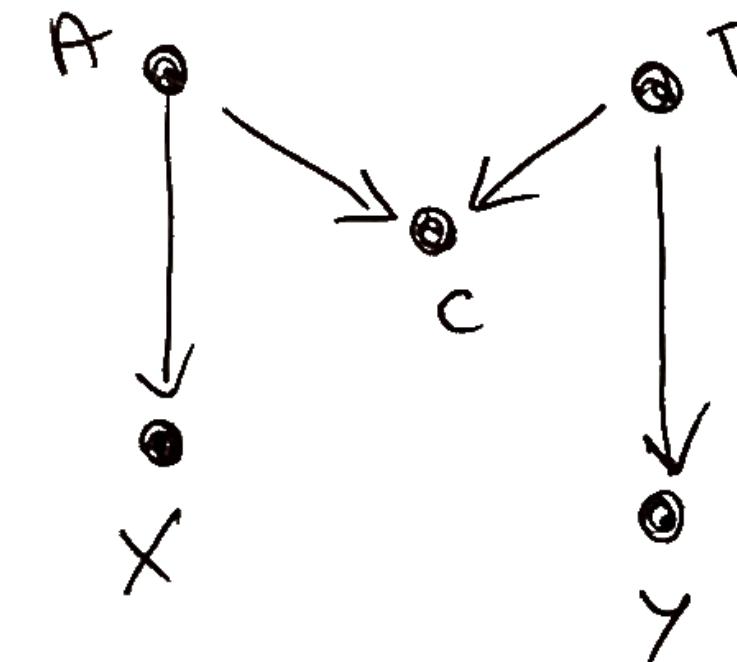
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③.



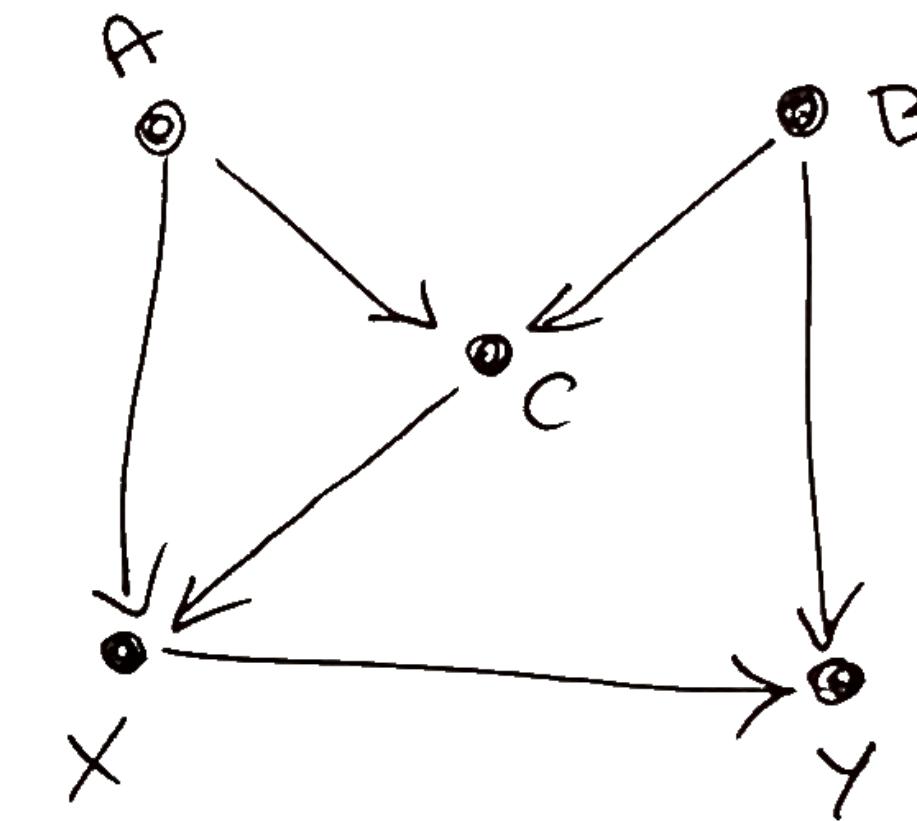
④.

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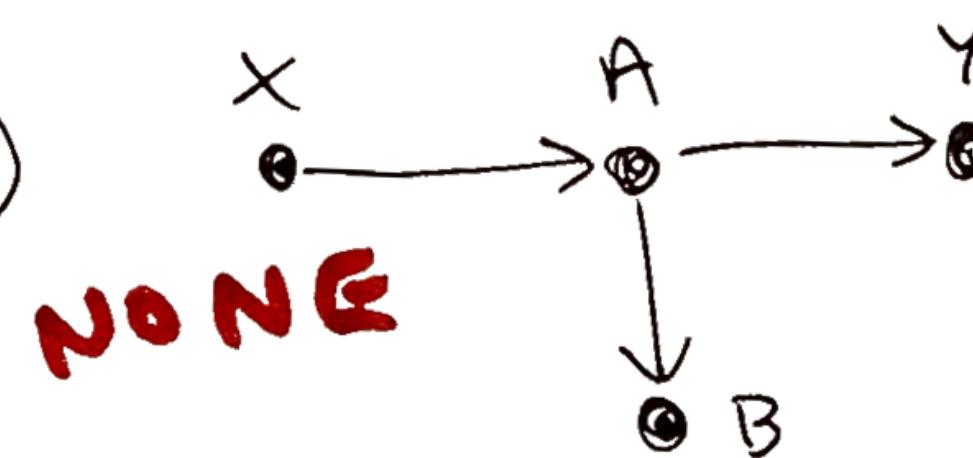


⑤.

B
- OR -
C + A

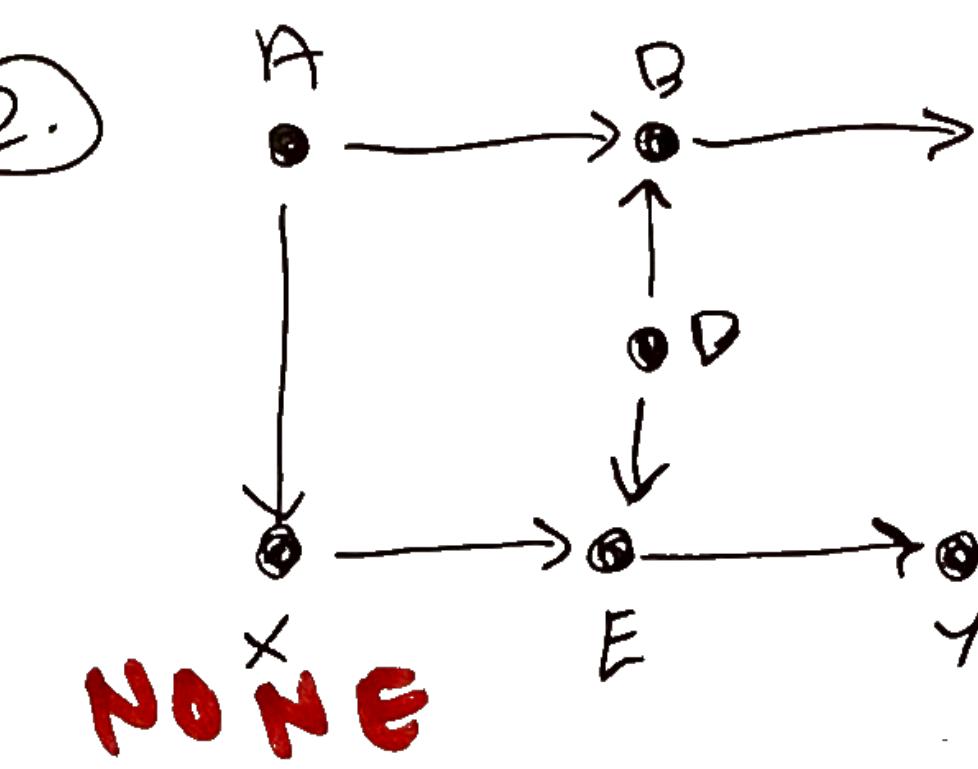


①.



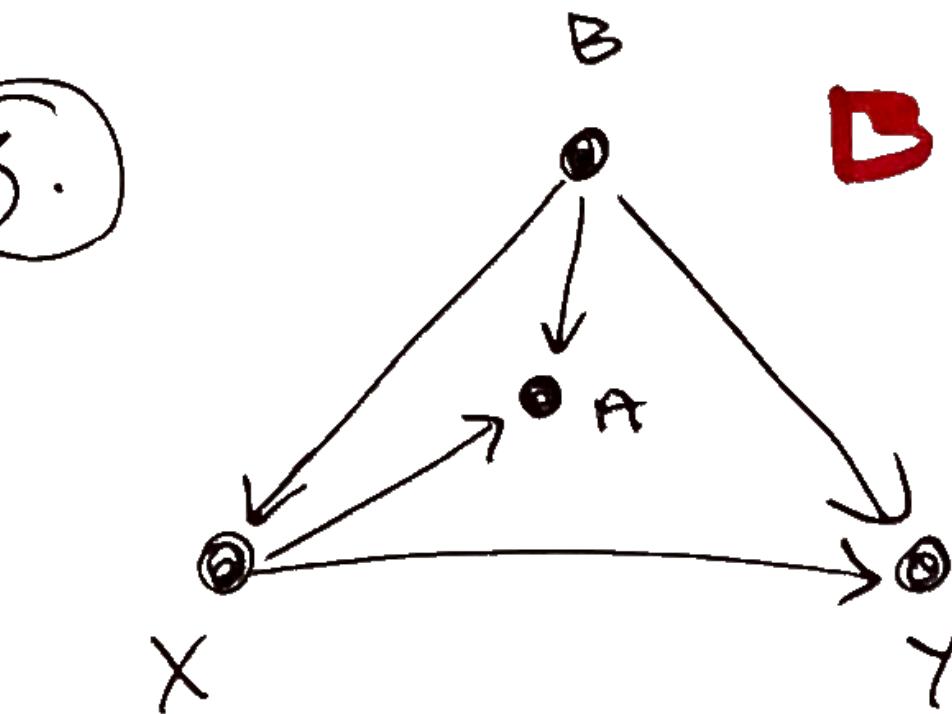
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②.



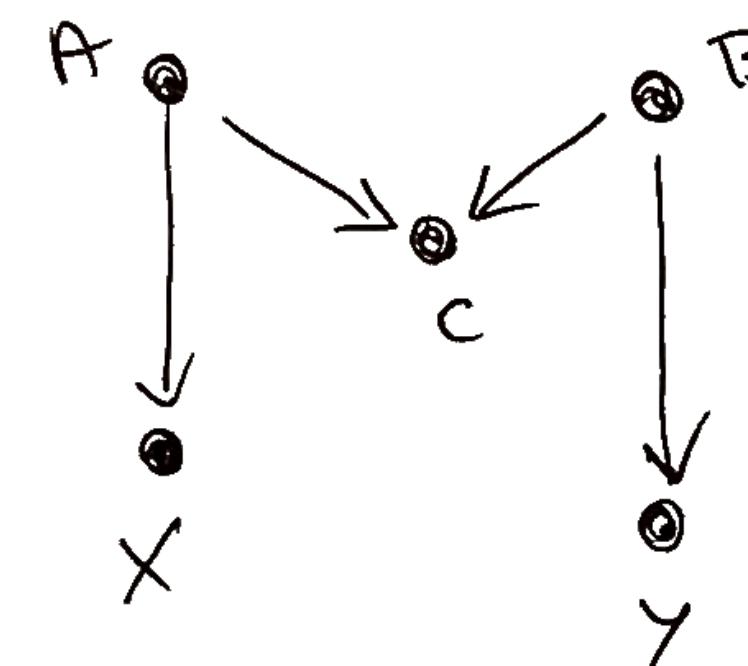
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③.



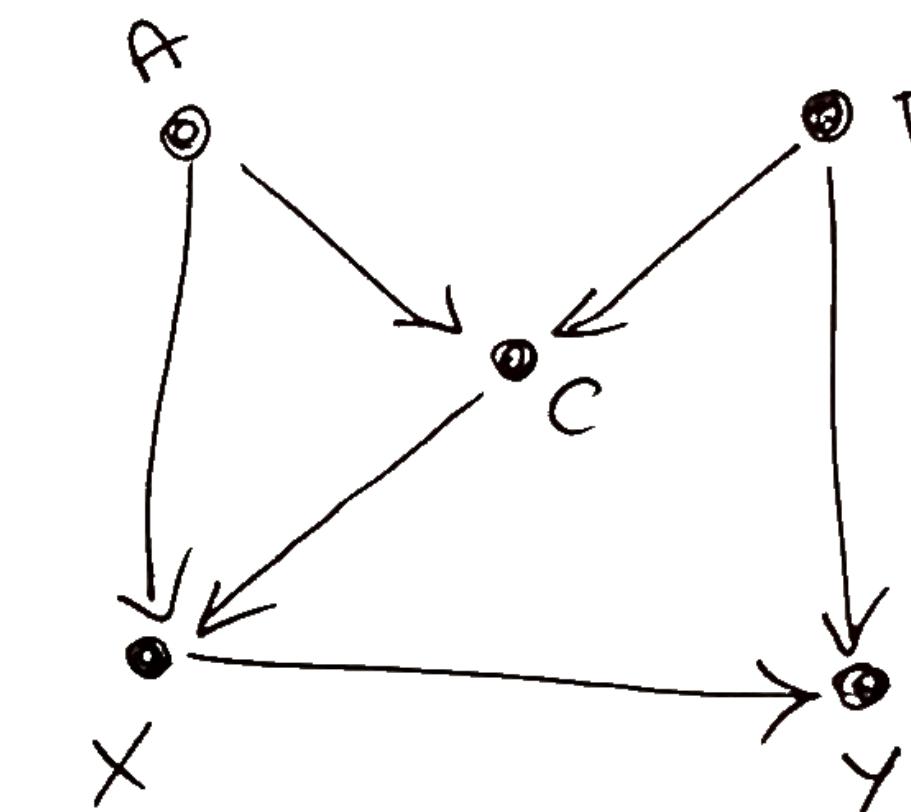
B

④.
NONE



⑤.

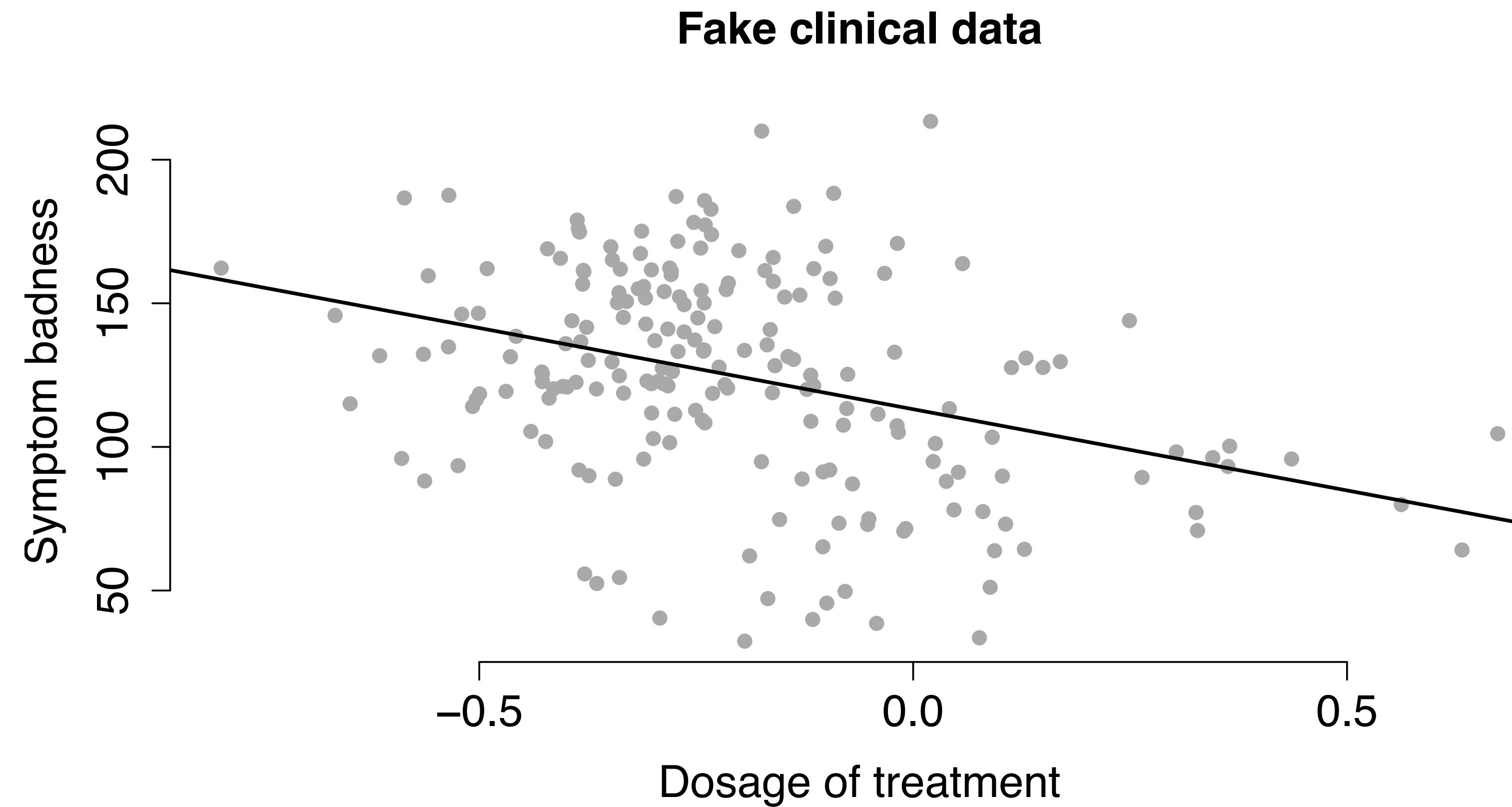
B
- or -
C + A



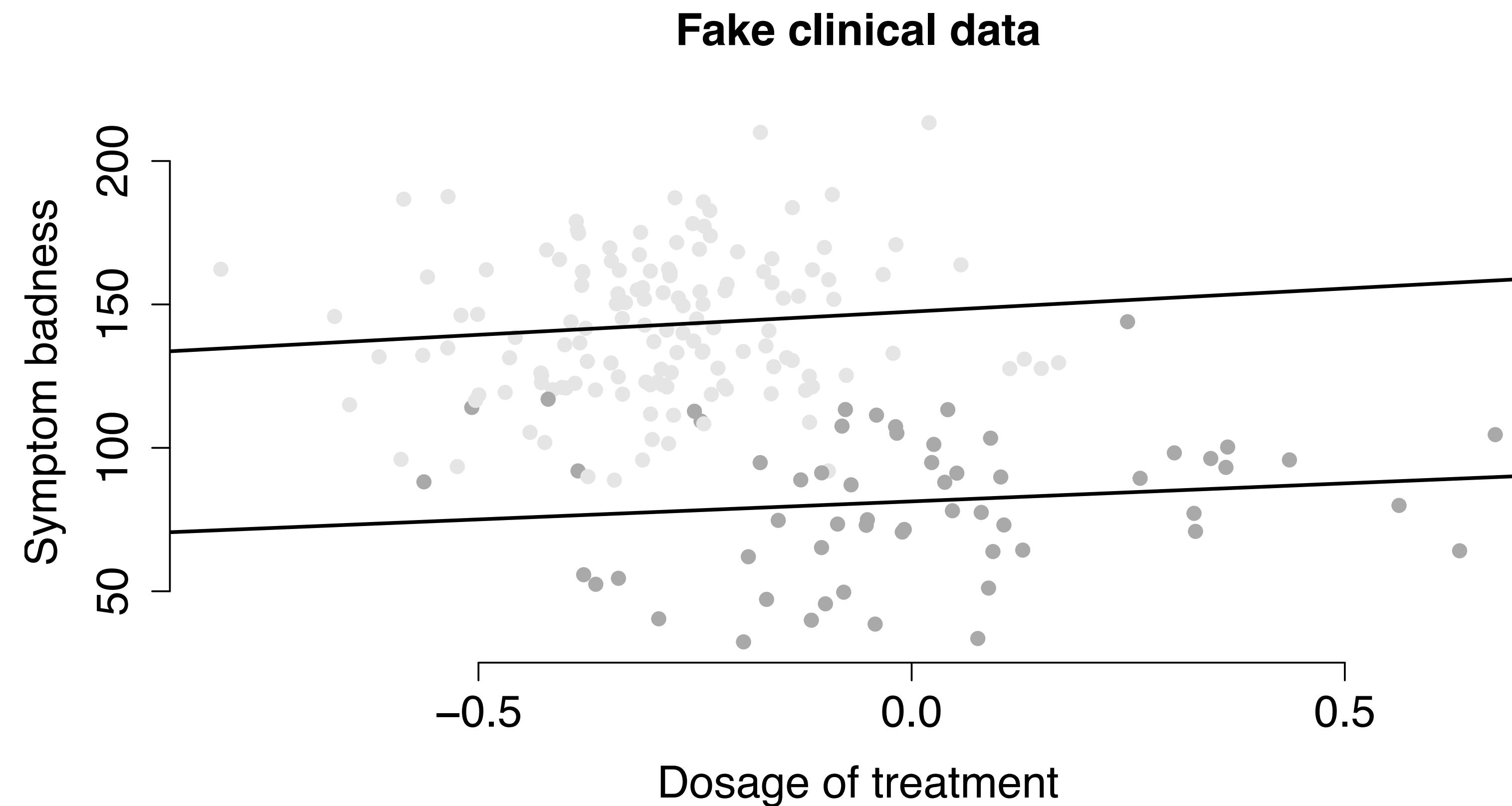
A+B+C also works but this set is not *minimal*,
since we can remove something from it and
still have a valid adjustment set

A warning

The analysis and interpretation of your data depends on the assumed causal model

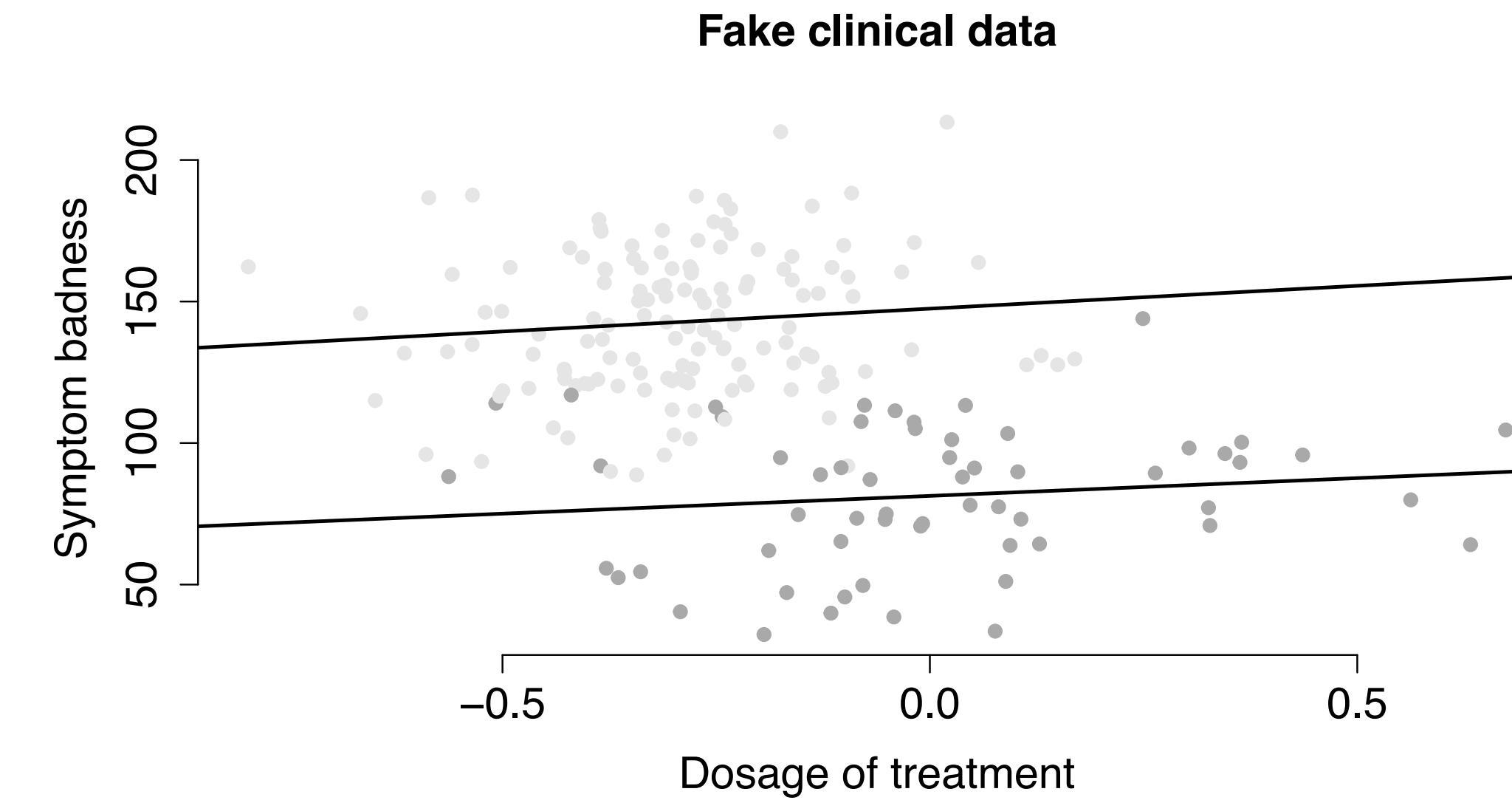
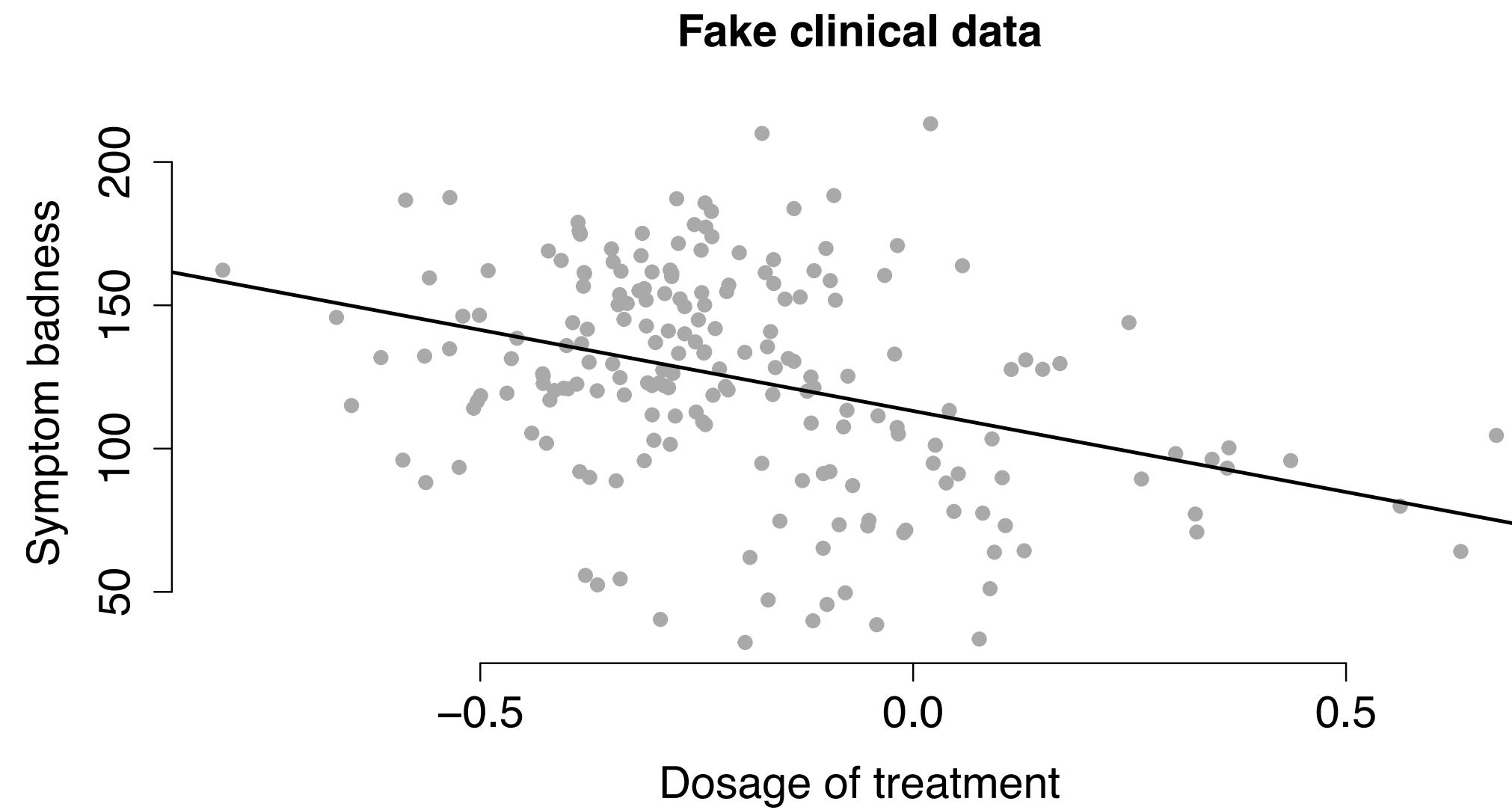


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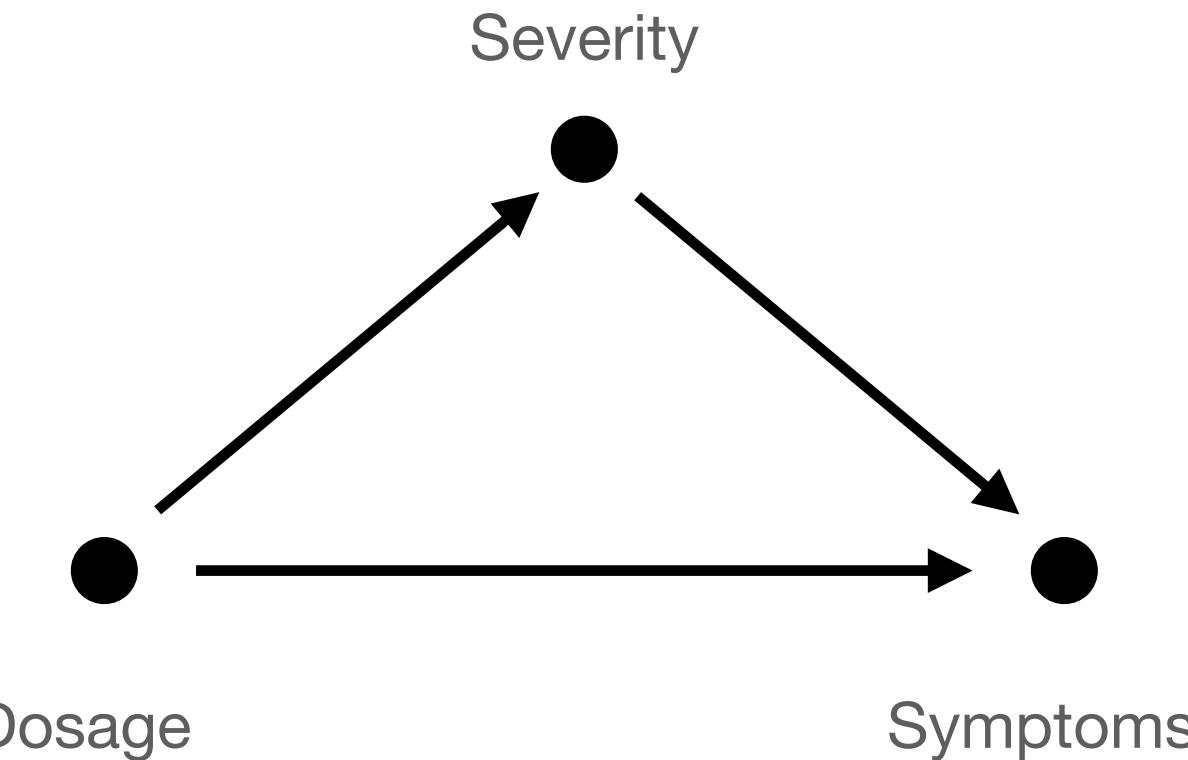


Effect reverses if we condition on disease severity

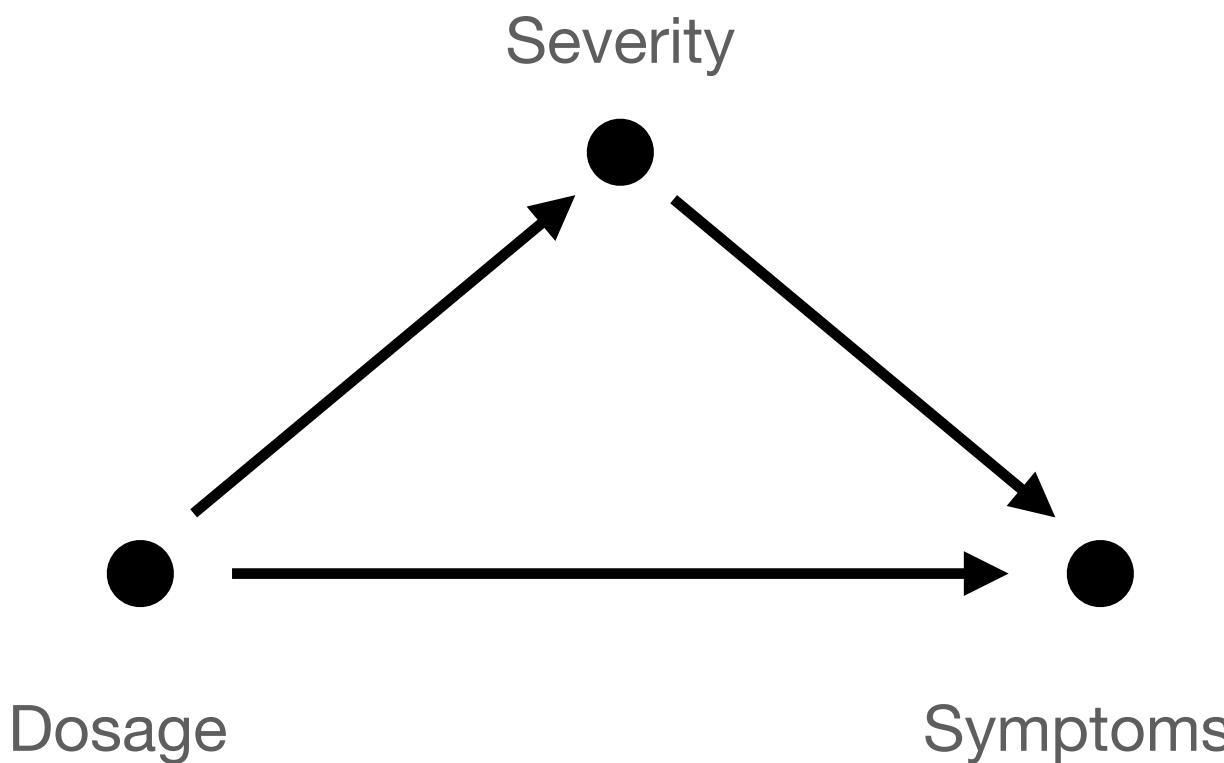
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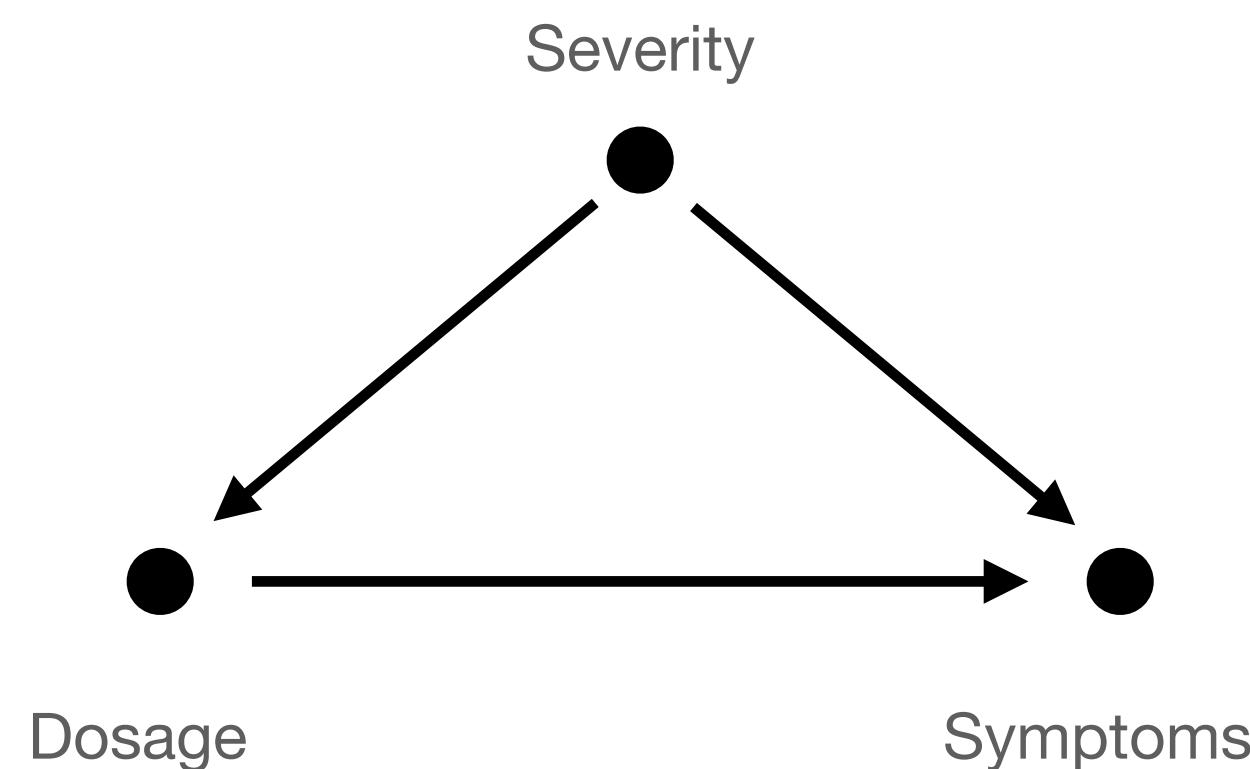
“Simpson’s paradox”: which is it, is the treatment good or bad???



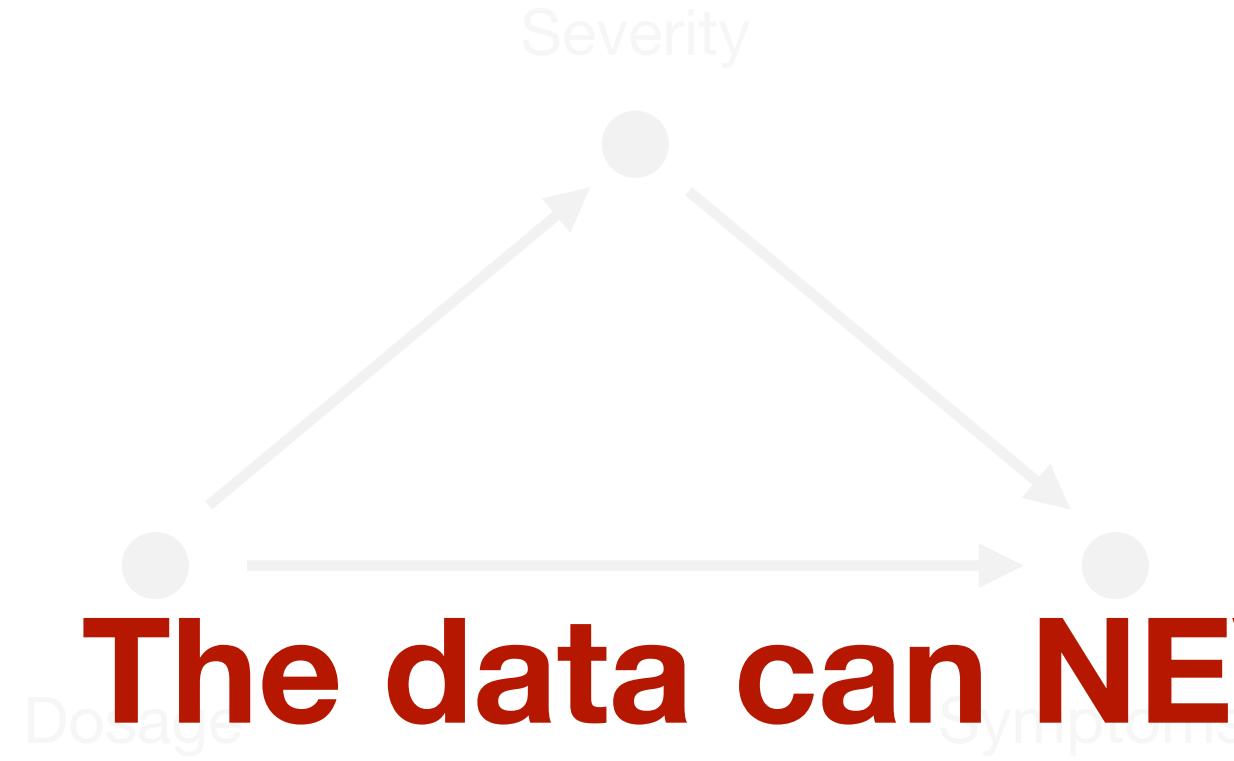
Drug acts by reducing both symptoms and disease severity.
No back-door path: don't adjust.



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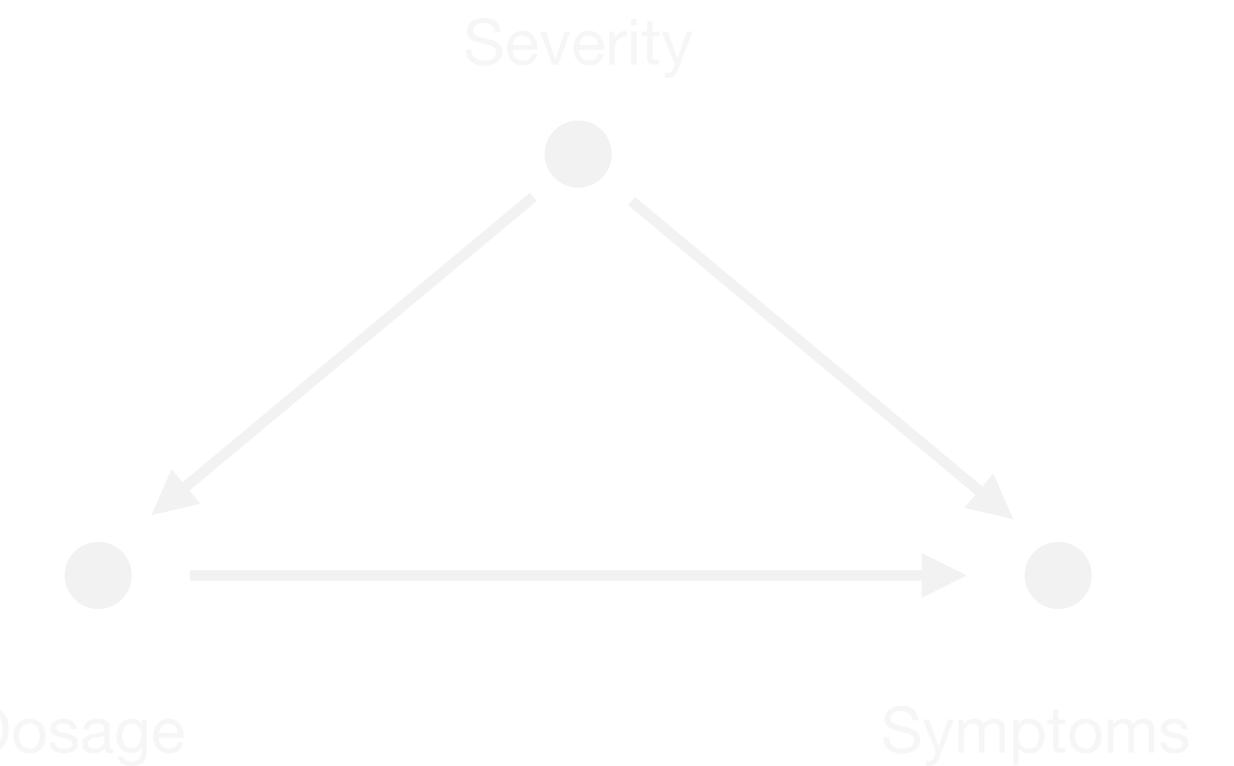


Drug acts on symptoms. Different doses given to severe and non-severe cases. Back-door path present: adjust.

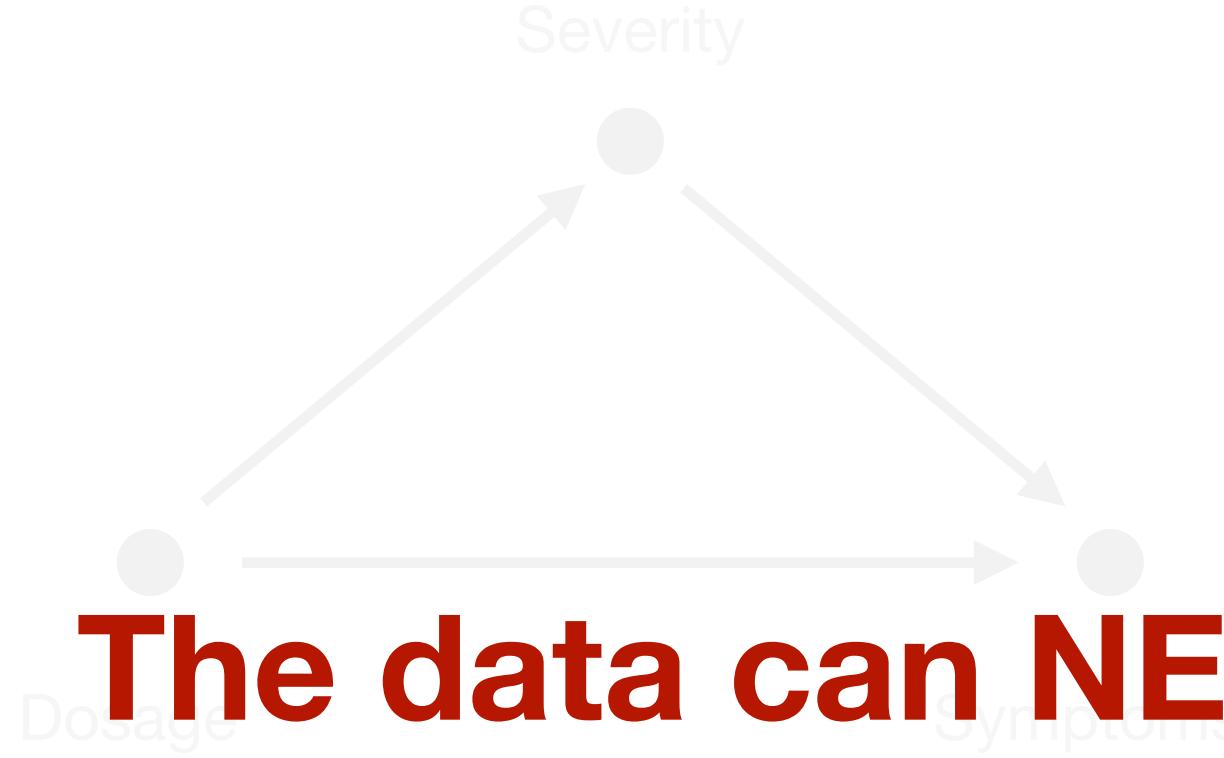


Drug acts by reducing both symptoms and disease severity.
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The data can NEVER tell you which is right.



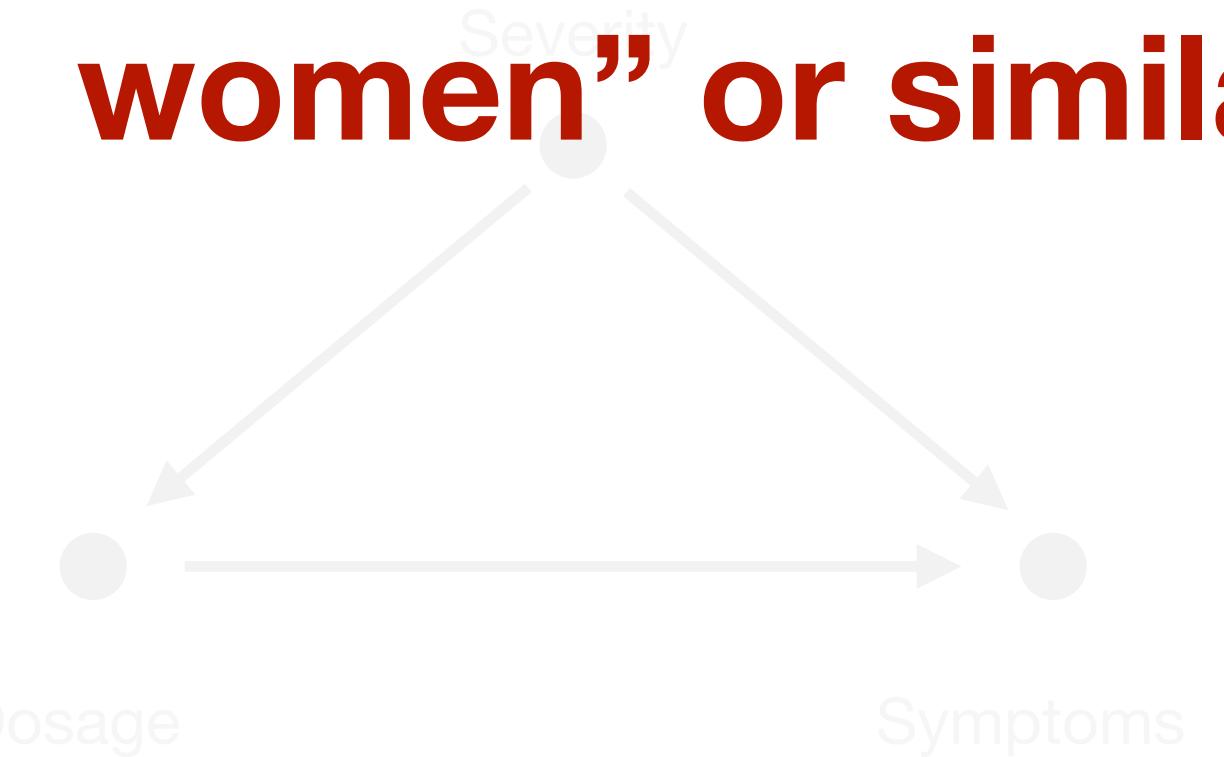
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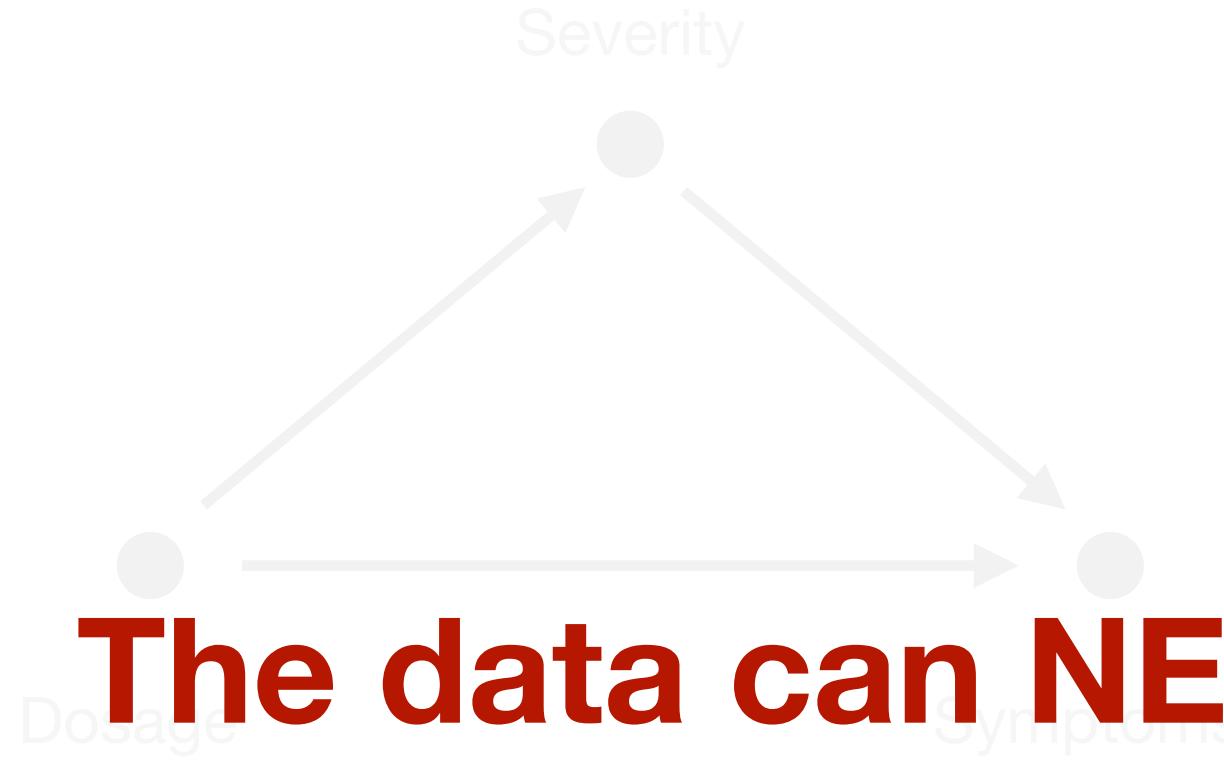
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Be very skeptical of claims like “we found an effect but only for women” or similar.



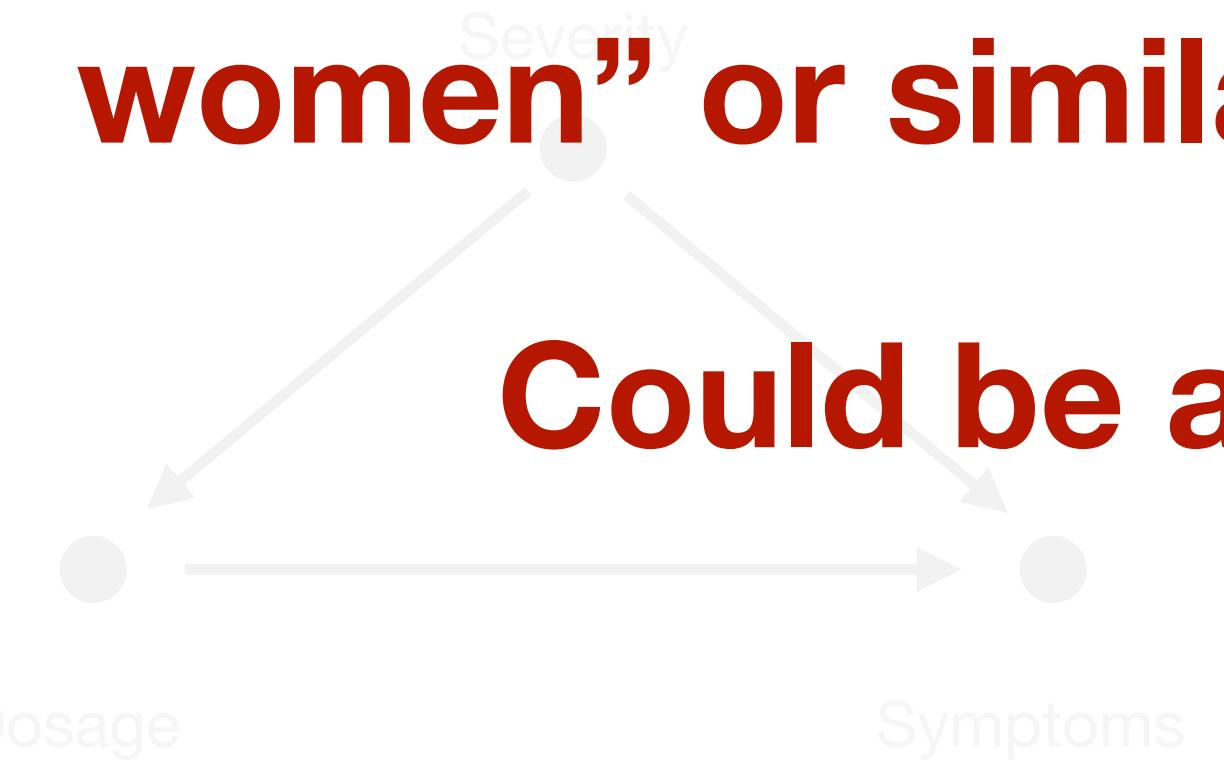
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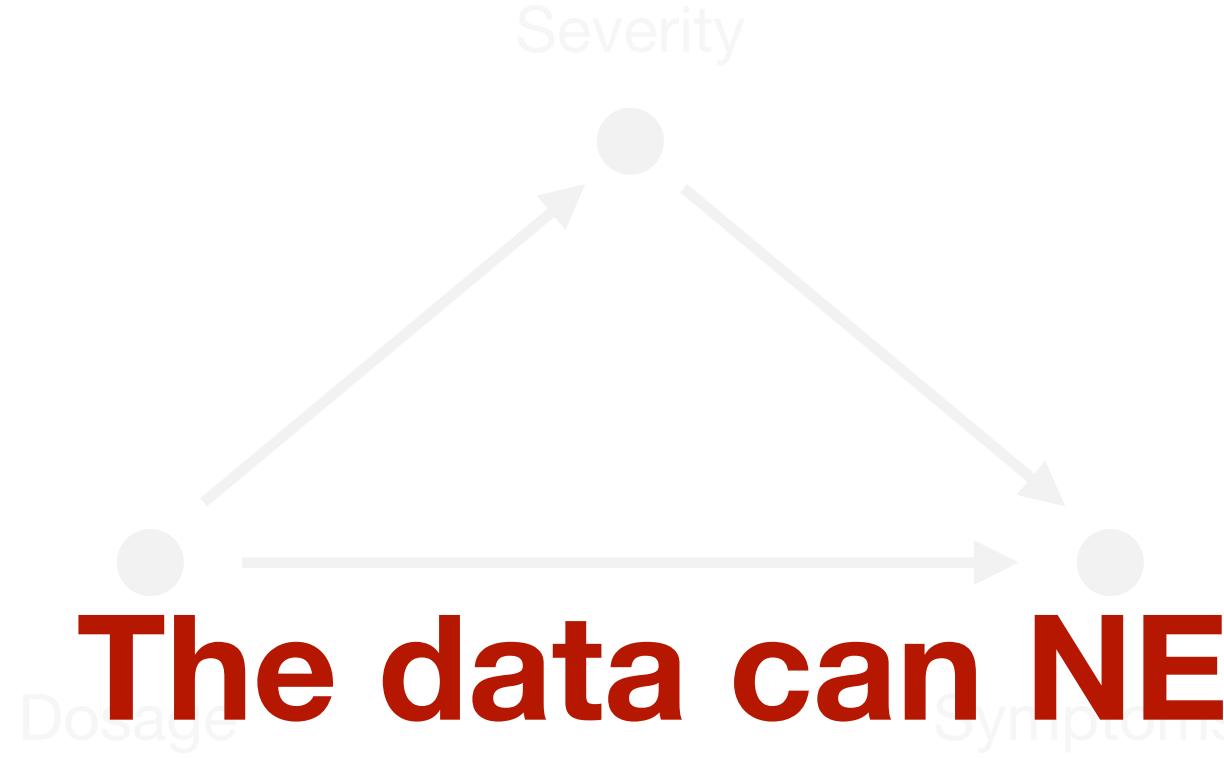
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Drug acts on symptoms. Different doses given to severe and non-severe patients. Back-door path: adjust.

Could be a collider, a mediator, or a confounding factor.



Drug acts by reducing both symptoms and disease severity.
No back-door path: don't adjust.

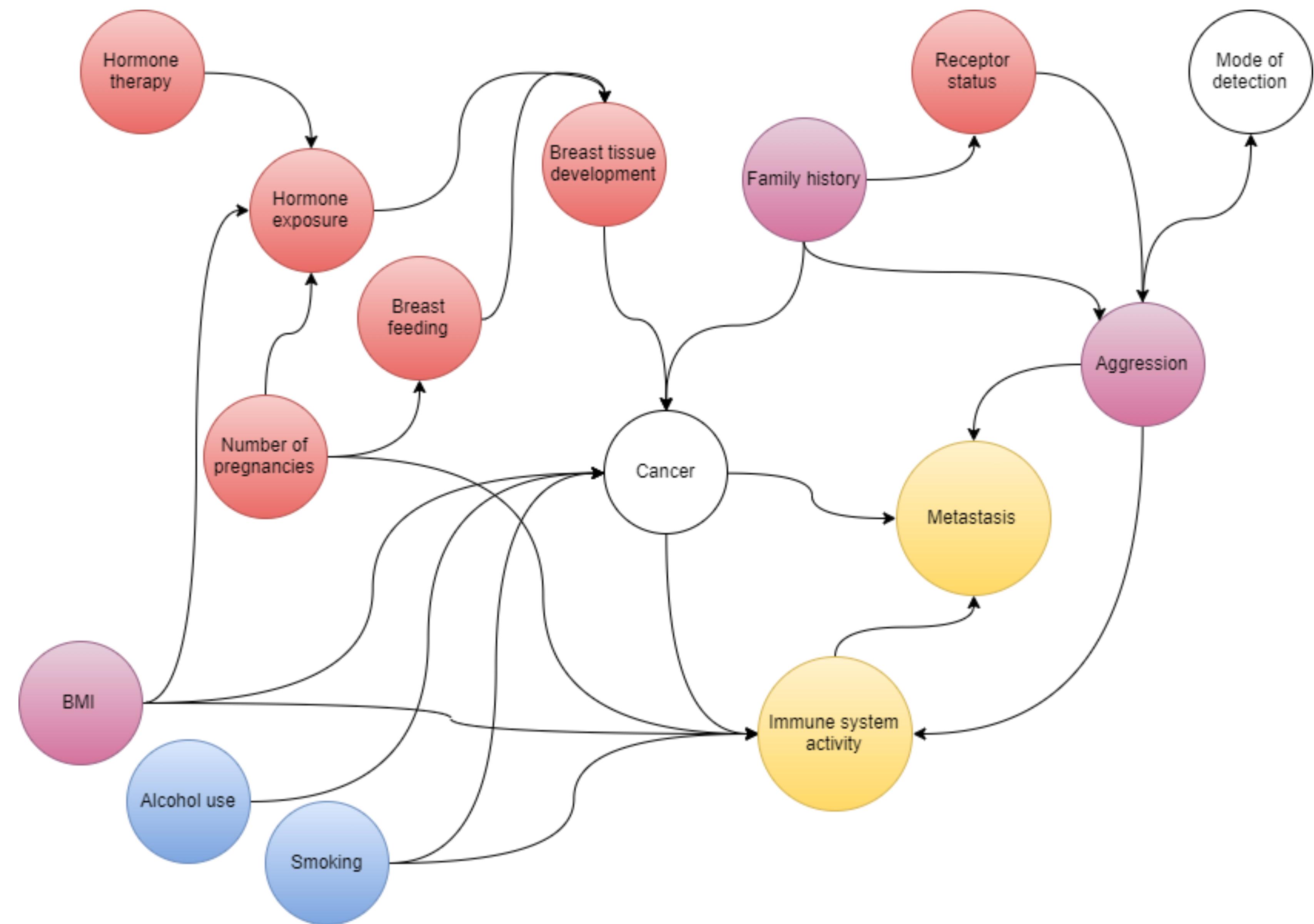
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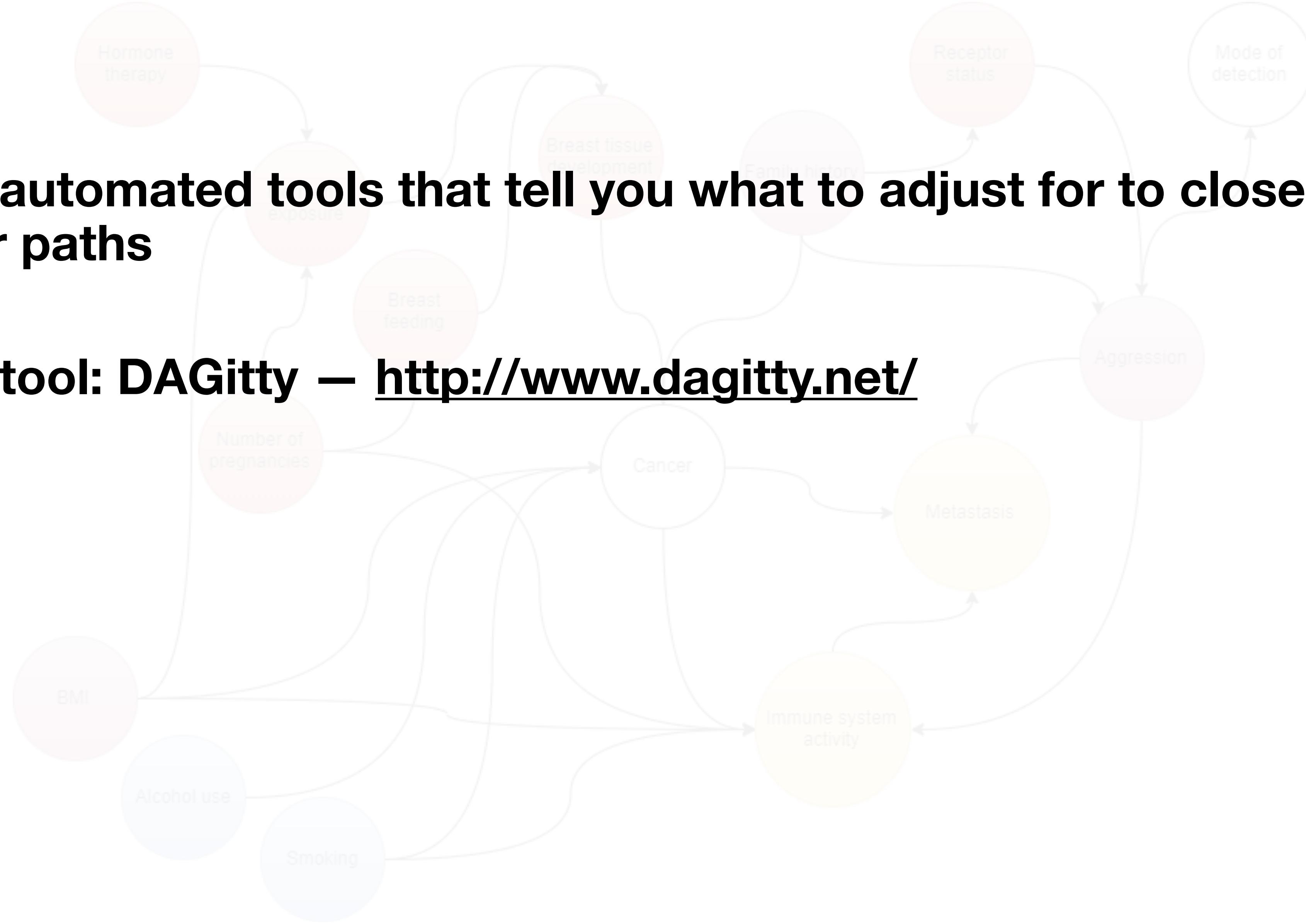
Adjustment without some kind of domain knowledge (which you can encode as a DAG) practically impossible to interpret

Real DAGs can be complicated

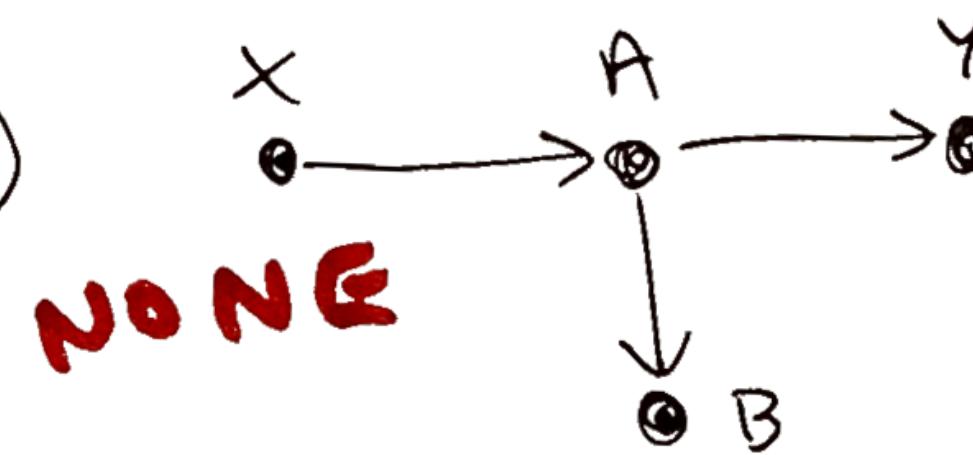


There are automated tools that tell you what to adjust for to close back-door paths

One such tool: DAGitty – <http://www.dagitty.net/>

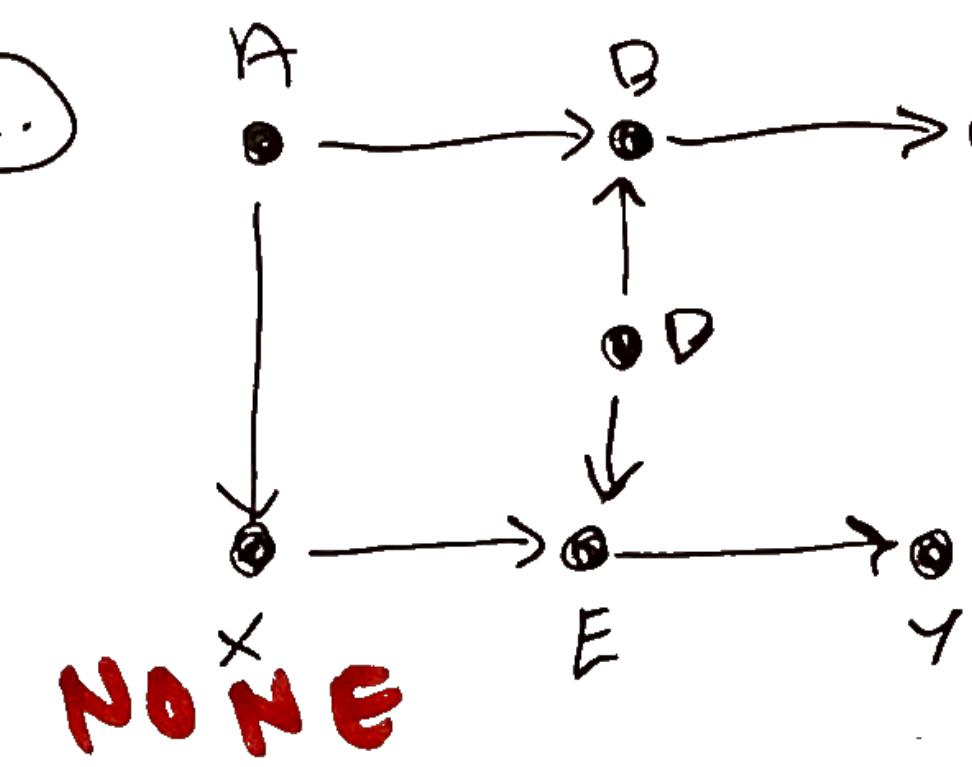


①.



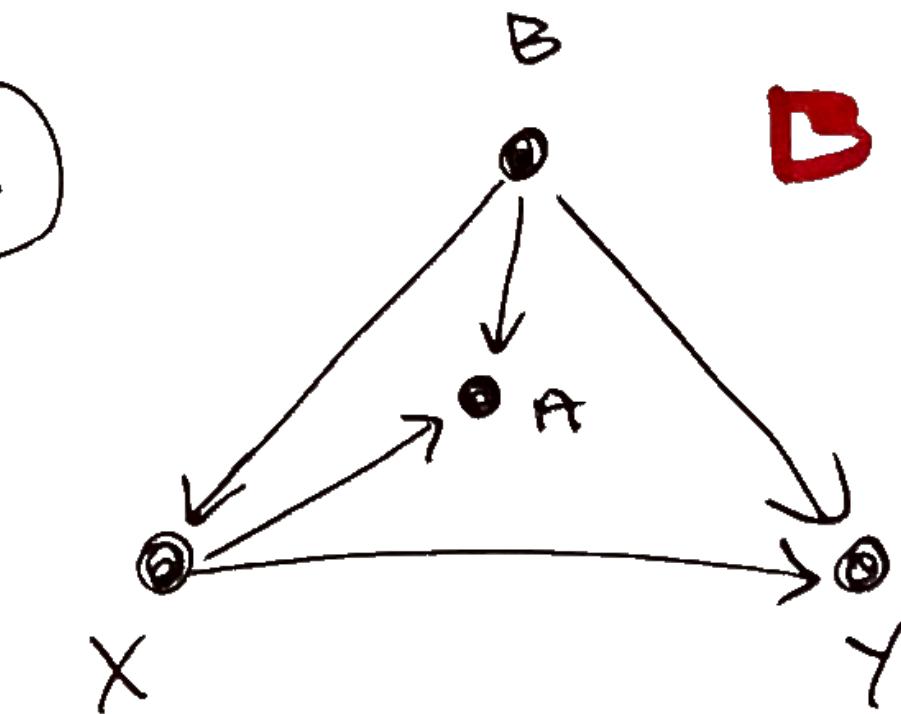
NONE

②.



NONE

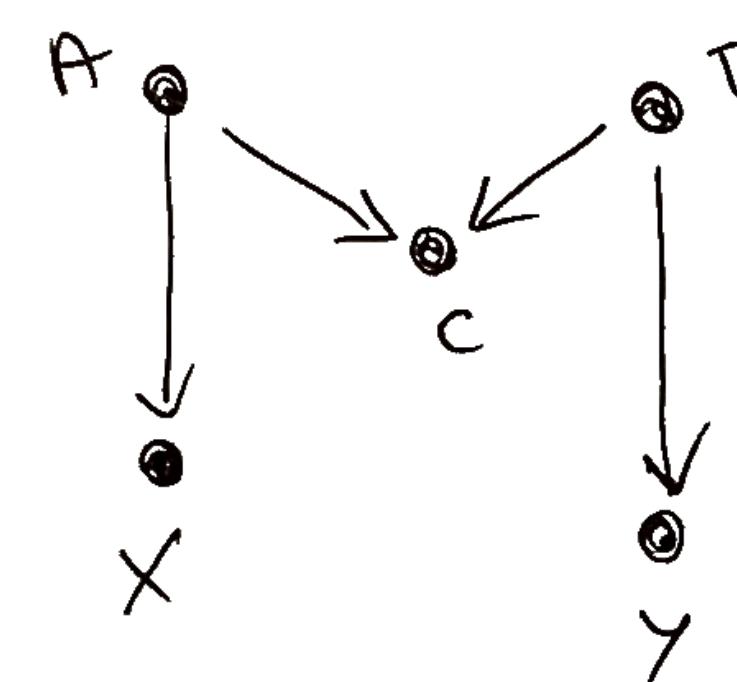
③.



B

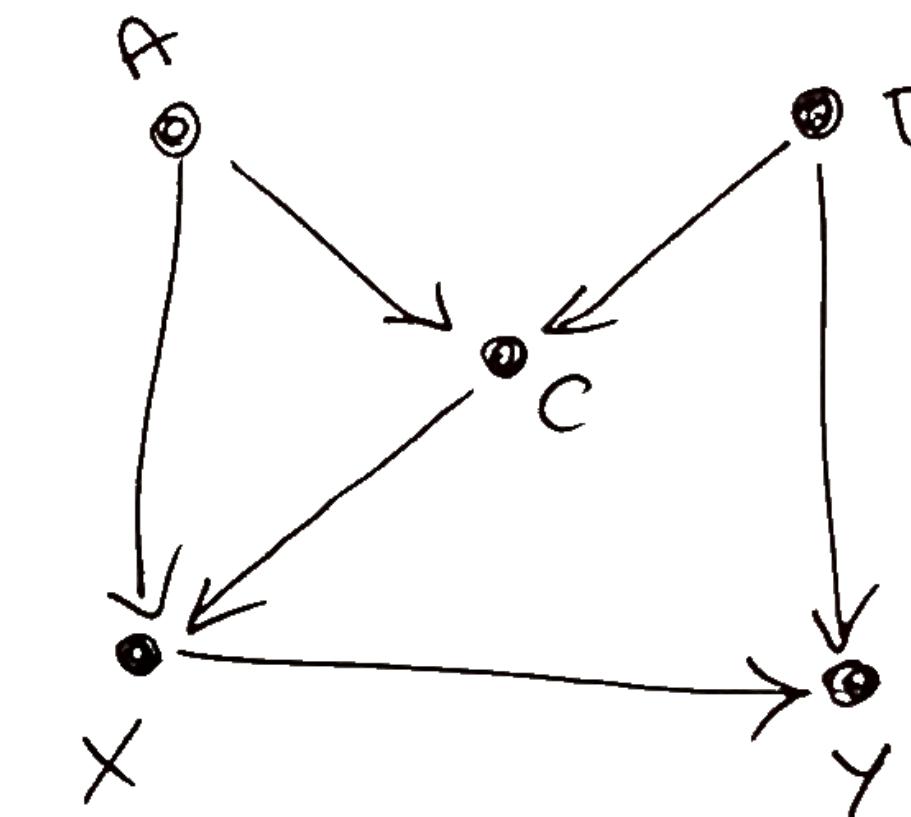
④.

NONE



⑤.

B
- or -
C + A



Go to dagitty.net, start the online browser mode. Input these DAGs and see that you get the adjustment sets you expect

Model code

```
dag {  
A [pos="-2.200,-1.520"]  
B [pos="1.400,-1.460"]  
D [outcome,pos="1.400,1.621"]  
E [exposure,pos="-  
2.200,1.597"]  
Z [pos="-0.300,-0.082"]  
A --> E  
A --> Z [pos="-0.791,-1.045"]  
B --> D  
B --> Z [pos="0.680,-0.496"]  
E --> D  
}
```

**Save your DAG by
copying this text. Can be
pasted into the same
box when you want to
continue**

Once you have a diagram, computers will help you do a lot of the analysis. Frees up time to think about the science

**Adjustment in terms of
regression models**

Adjustment in terms of regression

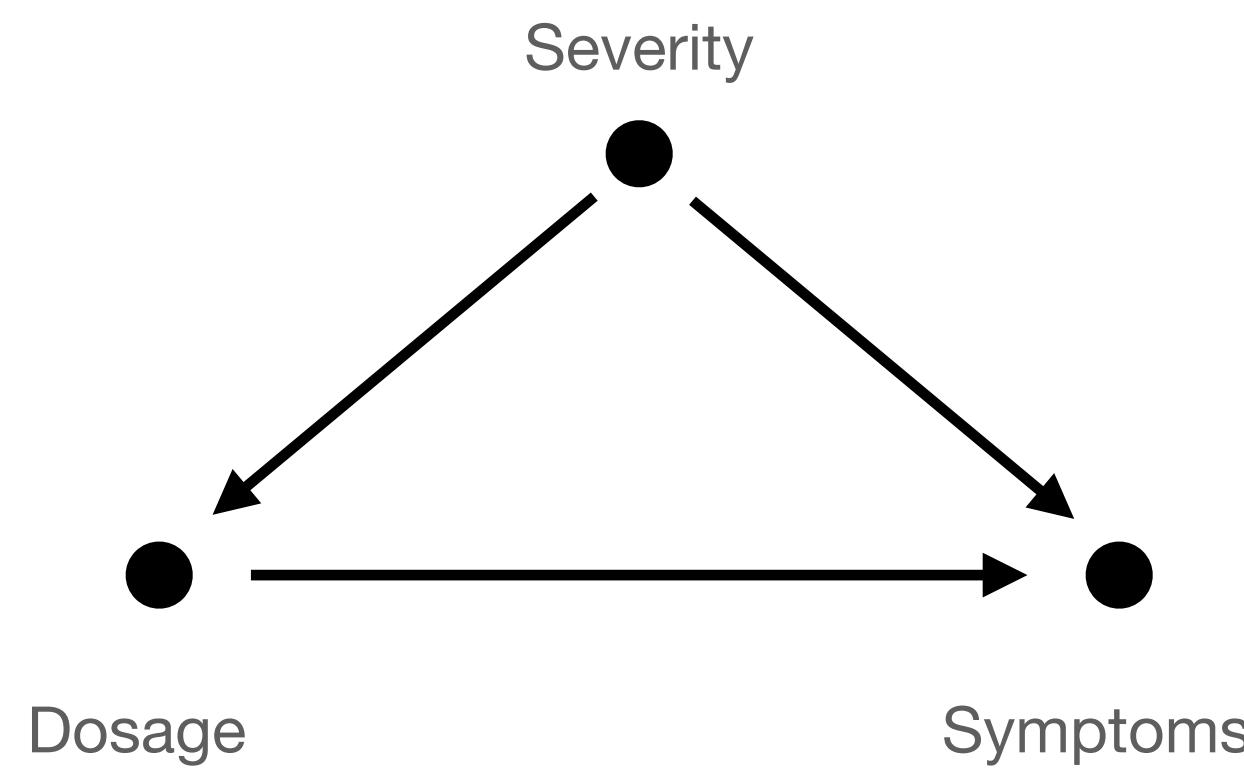
Broadly speaking: adjustment for Z to close back door paths is simply to add it as a predictor in the regression model

Big assumption: the mathematical form of the regression model is correct

Adjustment in terms of regression

Basic linear model (symptom is continuous: blood pressure maybe)

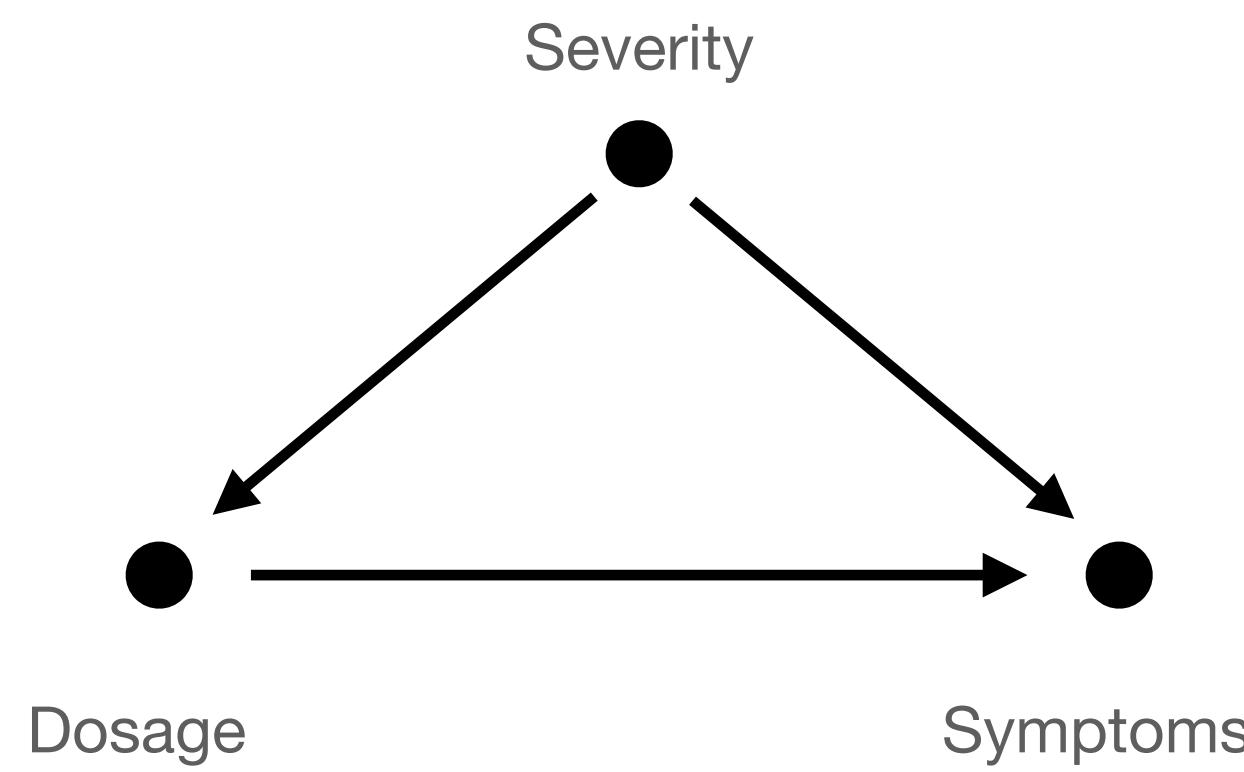
Enough to include the adjustment variable as a predictor:



Adjustment in terms of regression

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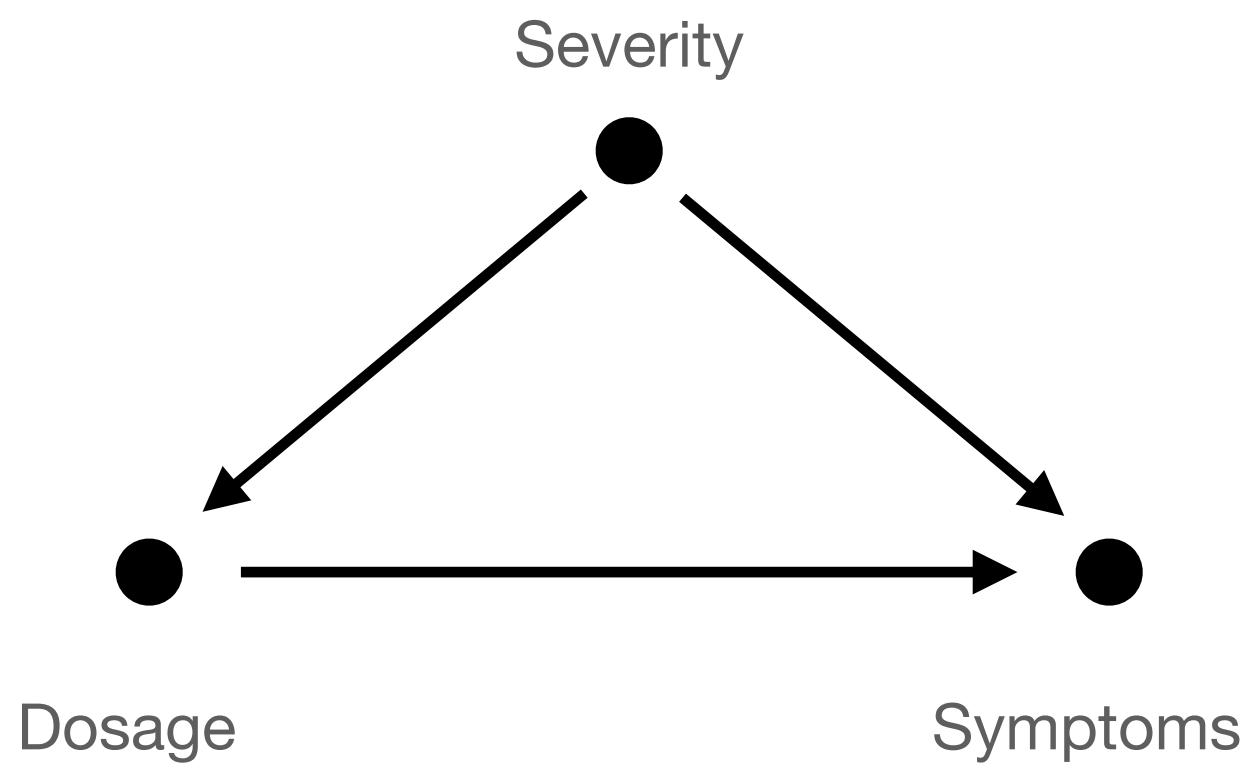


$$\text{symptom} = \beta_0 + \beta_1 \text{dosage} + \beta_2 \text{severity}$$

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Enough to include the adjustment variable as a predictor:



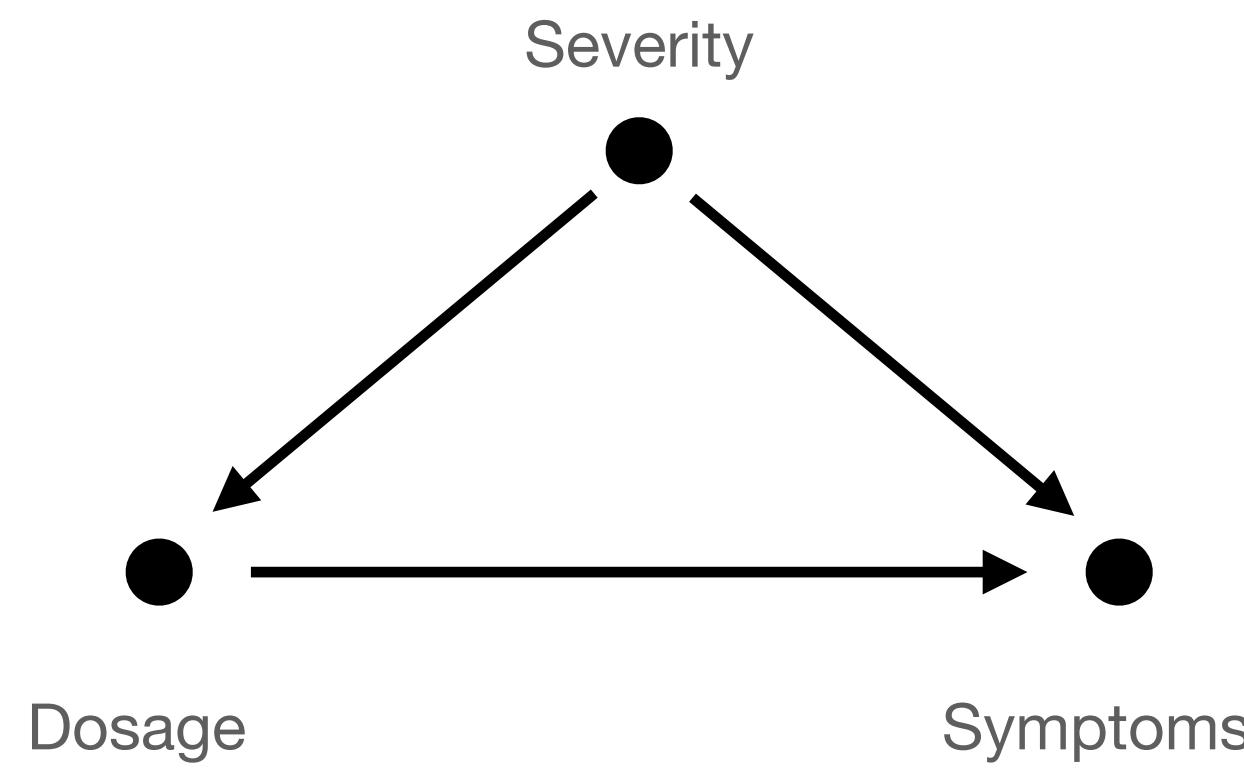
$$\text{symptom} = \beta_0 + \beta_1 \text{dosage} + \beta_2 \text{severity}$$

β_1 is an estimate of the causal effect of dosage on symptom

Adjustment in terms of regression

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Enough to include the adjustment variable as a predictor:



$$\text{symptom} = \beta_0 + \beta_1 \text{dosage} + \beta_2 \text{severity}$$

β_1 is an estimate of the causal effect of dosage on symptom

β_2 does not have a straight-forward interpretation: Simply there to “deconfound”

Adjustment in terms of regression

Average treatment effect

The β_1 of linear regression is an estimate of the **average treatment effect**:

$$\mathbb{E}[Y_1 - Y_0]$$

Adjustment in terms of regression

Average treatment effect

The β_1 of linear regression is an estimate of the **average treatment effect**:

$$\mathbb{E}[Y_1 - Y_0]$$

Which reads “average difference between giving someone the treatment and not giving them the treatment”

Adjustment in terms of regression

Logistic regression and similar (eg. binary outcome)

$$\log \frac{p}{1-p} = \beta_0 + \beta_1 x + \beta_2 z \quad p = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x + \beta_2 z)}}$$

Adjustment is still to add Z as predictor, BUT interpretation more tricky.

Adjustment in terms of regression

Logistic regression and similar (eg. binary outcome)

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Adjustment is still to add Z as predictor, BUT interpretation more tricky.

We know that e^{β_1} is an estimate of odds ratio, but it is not the “average causal” odds ratio. We say that the odds ratio is *noncollapsible*.

Adjustment in terms of regression

Logistic regression and similar (eg. binary outcome)

It's always possible to “stratify-and-average:”

Adjustment in terms of regression

Logistic regression and similar (eg. binary outcome)

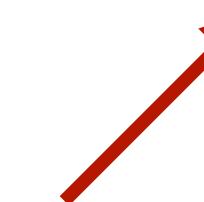
It's always possible to “stratify-and-average:”

$$P(Y_x) = \sum_z P(Y \mid X = x, Z = z)P(Z = z)$$

Adjustment in terms of regression

Logistic regression and similar (eg. binary outcome)

It's always possible to “stratify-and-average:”

$$P(Y_x) = \sum_z P(Y | X = x, Z = z)P(Z = z)$$


- i) Estimate effect in each stratum (ie. Condition on $Z = z$)

Adjustment in terms of regression

Logistic regression and similar (eg. binary outcome)

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$$P(Y_x) = \sum_z P(Y | X = x, Z = z)P(Z = z)$$

- i) Estimate effect in each stratum (ie. Condition on $Z = z$)
- ii) Weight by fraction of observations in stratum z

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Logistic regression and similar (eg. binary outcome)

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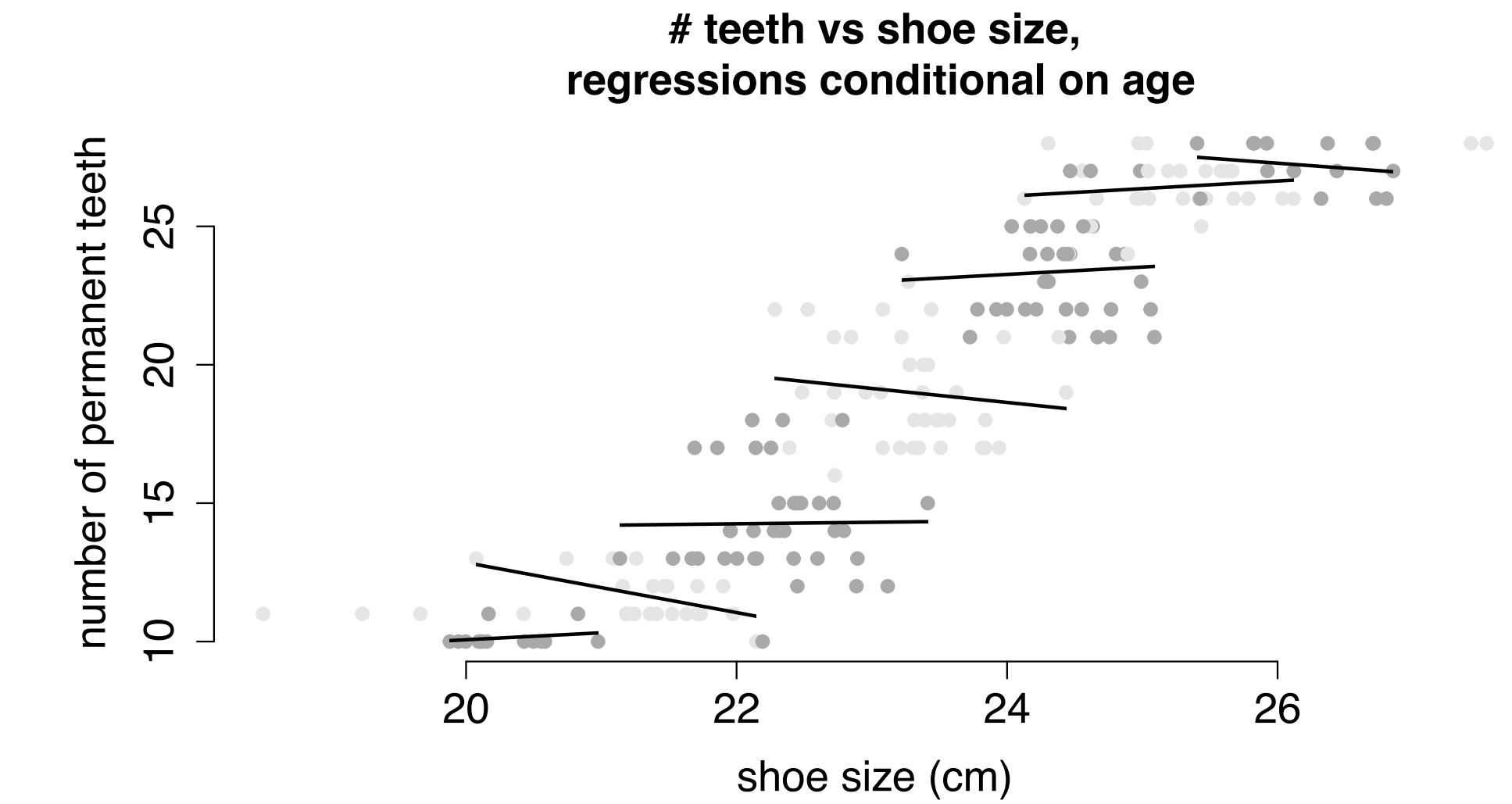
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Slightly annoying to do by hand, but there is software

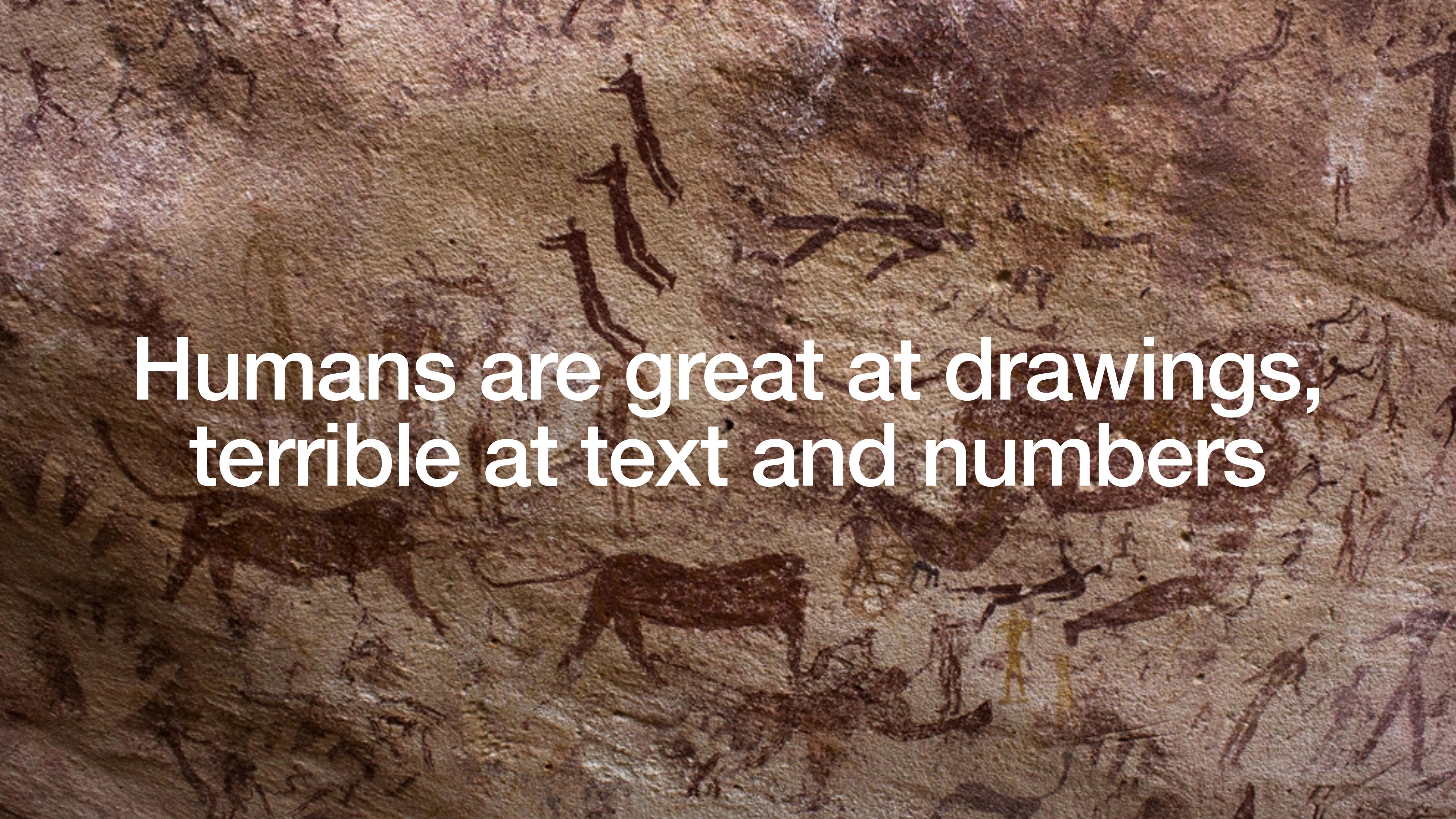
> [Eur J Epidemiol. 2016 Jun;31\(6\):563-74. doi: 10.1007/s10654-016-0157-3.](#)
Epub 2016 May 14.

Regression standardization with the R package stdReg

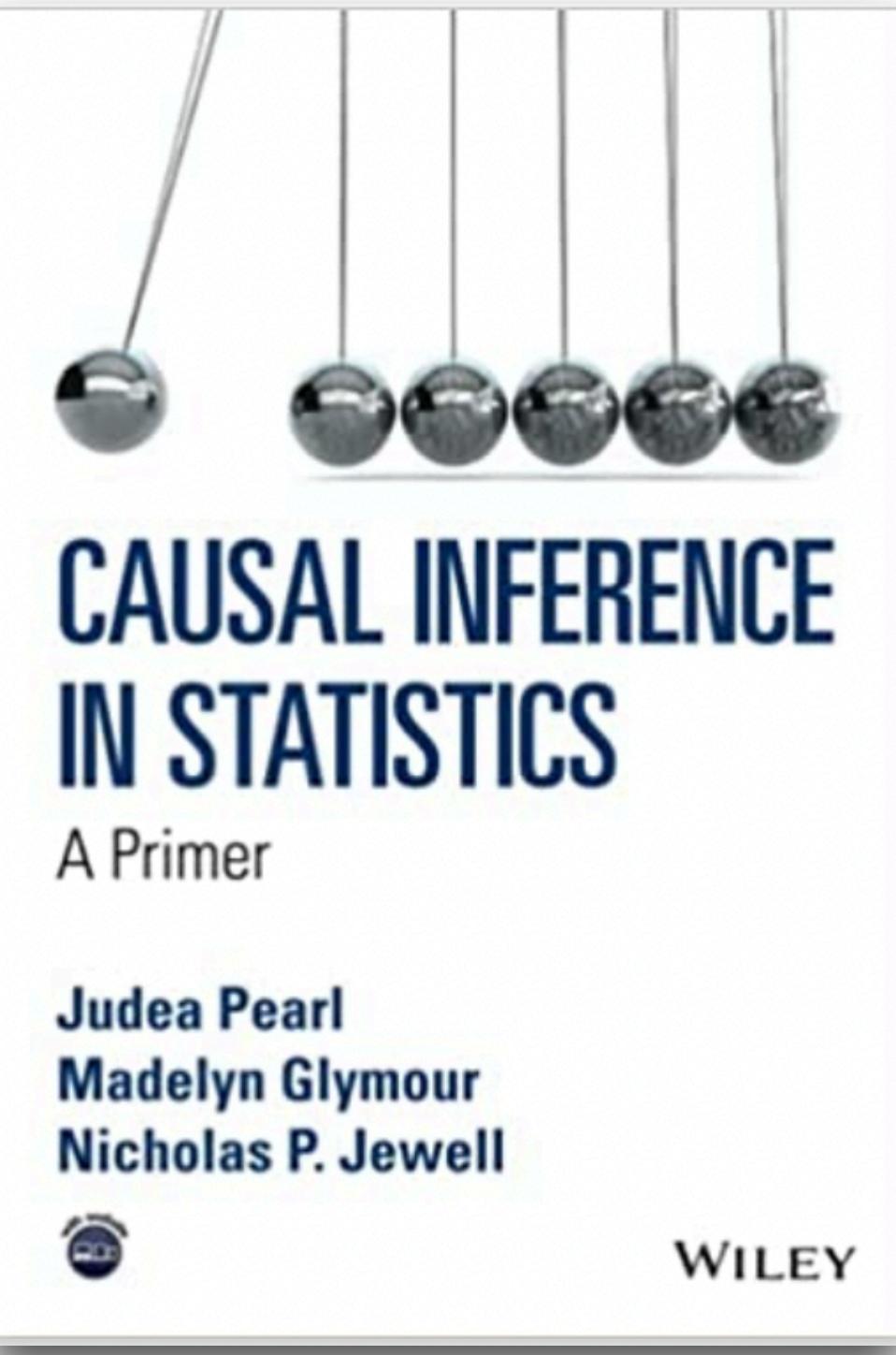
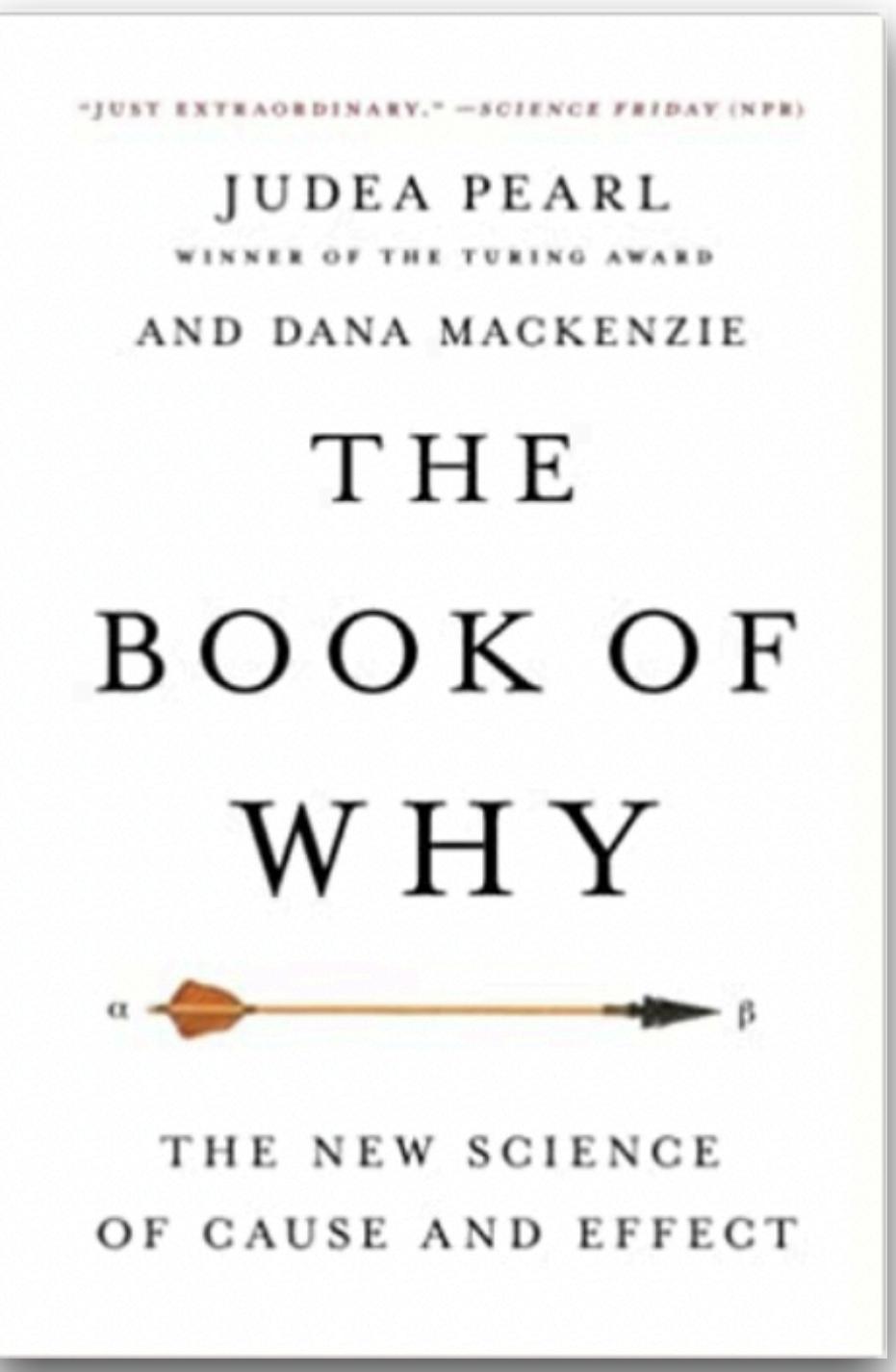


Last slides coming up!





Humans are great at drawings,
terrible at text and numbers



Thank you.