AN INTRODUCTION TO BAYESIAN INFERENCE

By Ruth Angus

8 reasons to use BAYESIAN INFERENCE

You can use Bayesian inference if <u>any</u> of these are true

8 reasons to use BAYESIAN INFERENCE

You can use Bayesian inference if any of these are true

1. You want to infer the properties of a physical process by fitting a model to data
2. You want to be explicit about your prior assumptions
3. You take a probabilistic view of the world
4. Your problem is complicated
5. Your problem is hierarchical/structured/layered
6. You would like to compare alternative models
7. You're working at the edge
8. You find frequentist statistics hard

JOINT, MARGINAL and CONDITIONAL PROBABILITIES

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"THE JOINT PROBABILITY OF BAGELS AND CREAM CHEESE"

p(X = 🍩, Y = 🥛🧀)

JOINT, MARGINAL and CONDITIONAL PROBABILITIES

"THE MARGINAL PROBABILITY OF CREAM CHEESE"

p(Y = 🥛🧀)

JOINT, MARGINAL and CONDITIONAL PROBABILITIES

"THE PROBABILITY OF CREAM CHEESE, GIVEN BAGELS"

p(Y = 🥛🧀 | X = 🍩)

OR

"THE PROBABILITY OF CREAM CHEESE, CONDITIONED ON X=🍩"

X

Y		
 	5	1
	3	7

Y \ X	🍩	🍞
🥛🧀	5	1
🥑	3	7

JOINT

1. p(X = 🍞, Y = 🥑) = 7/16
2. p(X = 🍩, Y = 🥛🧀) = 5/16

MARGINAL

3. p(Y = 🥑) = 10/16 = 5/8
4. p(X = 🍩) = 8/16 = 1/2

CONDITIONAL

5. p(X = 🍩| Y = 🥑) = 3/10
6. p(Y = 🥛🧀 | X = 🍩) = 5/8

THE SUM RULE

(MARGINALIZATION)

How did you calculate p(Y = 🥑)?

$$p(Y = \text{🥑}) = p(X = \text{🍩}, Y = \text{🥑}) + p(X = \text{🍞}, Y = \text{🥑})$$

$$p(Y = \text{🥑}) = \sum_{\text{breads}, i=1}^{L} p(X = \text{bread}_i, Y = \text{🥑})$$

(L = number of breads)

$$p(Y=Y_j) = \sum_{i=1}^{L} p(X=X_i, Y=Y_j)$$

THE SUM RULE

(MARGINALIZATION)

How did you calculate p(Y = 🥑)?

$$p(Y = \text{🥑}) = p(X = \text{🍩}, Y = \text{🥑}) + p(X = \text{🍞}, Y = \text{🥑})$$

$$p(Y = \text{🥑}) = \sum_{\text{breads}, i=1}^{L} p(X = \text{bread}_i, Y = \text{🥑})$$

(L = number of breads)

$$p(X=X_i) = \sum_{j=1}^{K} p(X=X_i, Y=Y_j)$$

(K = number of spreads)

THE SUM RULE

(MARGINALIZATION)

$$p(X=x_i) = \sum_{j=1}^{K} p(X=x_i, Y=y_j)$$

THE PRODUCT RULE

Going from a joint to a conditional distribution

How did you calculate p(X = 🍩| Y = 🥑) ?

Y \ X	🍩	🍞
🥛 🧀	5	1
🥑	3	7

THE PRODUCT RULE

Going from a joint to a conditional distribution

How did you calculate $p(X=X_i \mid Y=Y_j)$? $=n_{ij}/r_j$

		X	
		X_1	X_2
Y	Y_1	n_{11}	n_{12}
	Y_2	n_{21}	n_{22}

Q: write down $P(Y=Y_j|X=X_i)$

$=n_{ij}/c_i$

THE PRODUCT RULE

Going from a joint to a conditional distribution

$$p(X=x_i, Y=y_j) = \frac{n_{ij}}{N} = \frac{n_{ij}}{c_i} \frac{c_i}{N}$$

$$p(X=x_i, Y=y_j) = p(Y=y_j|X=x_i)\, p(X=x_i)$$

THE PRODUCT RULE

Going from a joint to a conditional distribution

$$p(X=x_i\,,\,Y=y_j) = p(Y=y_j|X=x_i)\,p(X=x_i)$$

LET'S SIMPLIFY THIS NOTATION

$$p(X=x_i, Y=y_j) = p(Y=y_j|X=x_i)\, p(X=x_i)$$

LET'S SIMPLIFY THIS NOTATION

$$p(X,Y) = p(Y|X)\, p(X)$$

NOW WE CAN DERIVE BAYES' RULE

USING THE PRODUCT RULE

$$p(X,Y) = p(Y|X)\, p(X)$$

DERIVE BAYES' RULE

$$p(X|Y) = \frac{p(Y|X)\, p(X)}{p(Y)}$$

BAYES' RULE

$$p(X|Y) = \frac{p(Y|X)\, p(X)}{p(Y)}$$

BAYES' RULE

$$\underbrace{p(\vartheta|D)}_{\text{posterior}} = \frac{\overbrace{p(D|\vartheta)}^{\text{likelihood}}\ \underbrace{p(\vartheta)}_{\text{prior}}}{\underbrace{p(D)}_{\text{evidence}}}$$

evidence

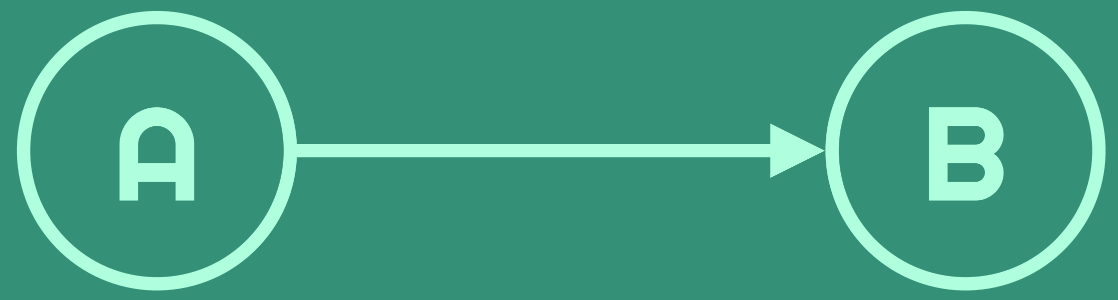
or

fully marginalized likelihood

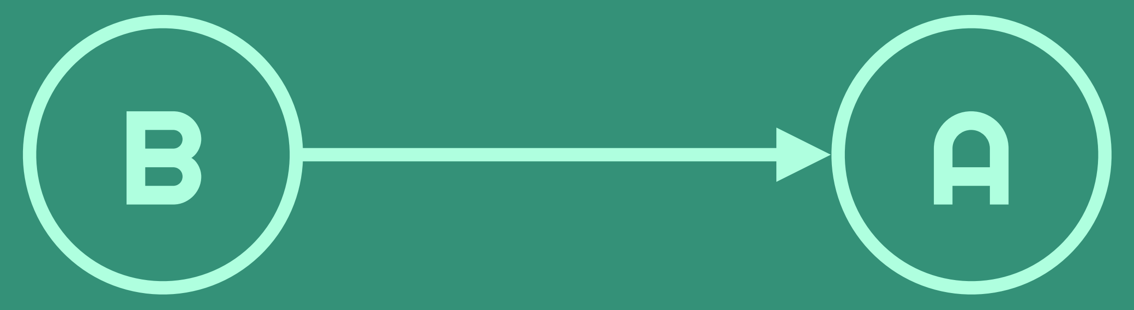
BAYES' RULE

$$\underbrace{p(\vartheta|D)}_{\text{posterior}} = \frac{\overbrace{p(D|\vartheta)}^{\text{likelihood}}\ \underbrace{p(\vartheta)}_{\text{prior}}}{\underbrace{\int p(D|\vartheta)\ p(\vartheta)d\vartheta}_{\substack{\text{evidence} \\ \text{or} \\ \text{fully marginalized likelihood}}}}$$

A BRIEF ASIDE:
PROBABILISTIC GRAPHICAL MODELS
[PGMs]

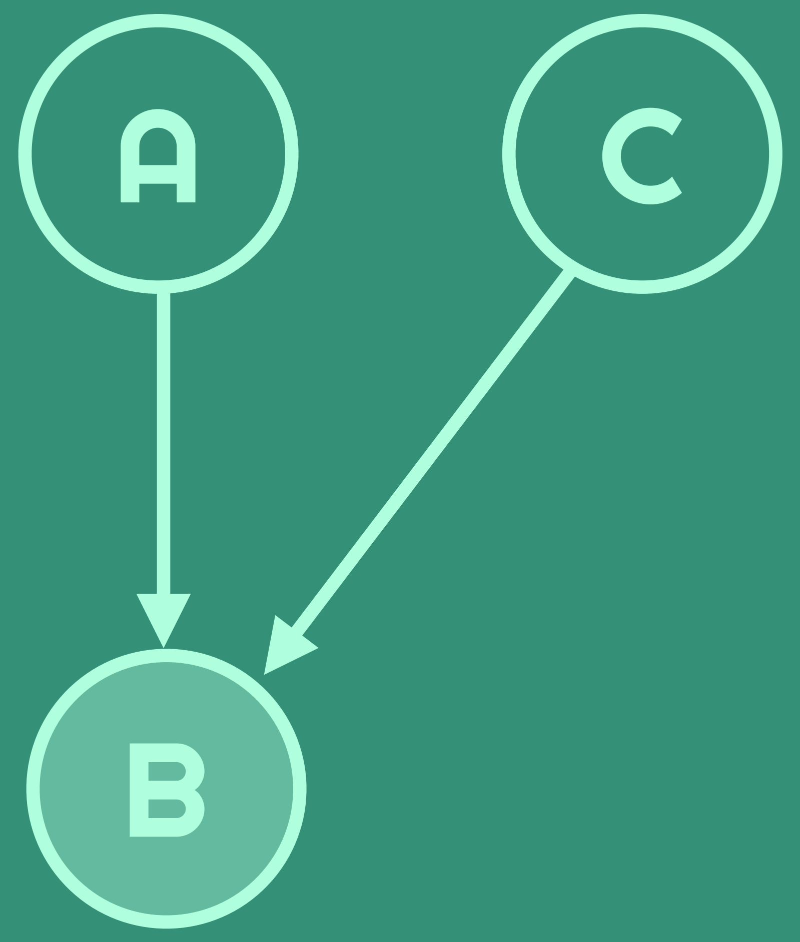


$$p(A,B) = p(A)\,p(B|A)$$



$$p(A,B) = p(B)\, p(A|B)$$

$$p(A,B) = p(A)\,p(B)$$



$$p(A, B, C) = p(A)\, p(C)\, p(B|A,C)$$

A GOOD REFERENCE:

Pattern recognition and machine learning, Bishop

GO TO NOTEBOOK

for an example of Bayesian inference

github.com/ruthangus/Probabilistic-inference-tutorial/