ECE 351 - Lab 11

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Contents

1 Introduction

The goal for this lab was to utilize Python's built-in functions as well as a provided function to perform analysis on a discrete system.

2 Equations

$$y[k] = 2x[k] - 40x[k-1] + 10y[k-1] - 16y[k-2]$$

3 Methodology

Using the causal discrete function listed in the Equations section with y[k] as the output and x[k] as the input, with the system at rest, we derived the expression for H(z) = $\frac{Y(z)}{X(z)}$. From the H(z) expression, we used partial fraction expansion and the inverse Z-transform to find h[k].

$$Y(z) = 2X(z) - 40z^{-1}X(z) + 10z^{-1}Y(z) - 16z^{-2}Y(z)$$

$$Y(z)(1 - 10z^{-1} + 16z^{-2} = X(z)(2 - 40z^{-1}))$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2 - 40z^{-1}}{1 - 10z^{-1} + 16z^{-2}} = \frac{2z^2 - 40z}{z^2 - 10z + 16}$$

$$\frac{H(z)}{z} = \frac{2z - 40}{(z-2)(z-8)}$$

$$A = \frac{2z - 40}{z - 8}|_{z=2} = 6, B = \frac{2z - 40}{z - 2}|_{z=8} = -4$$

$$\frac{H(z)}{z} = 6\frac{1}{z-2} - 4\frac{1}{z-8}; H(z) = 6\frac{z}{z-2} - 4\frac{z}{z-8}$$

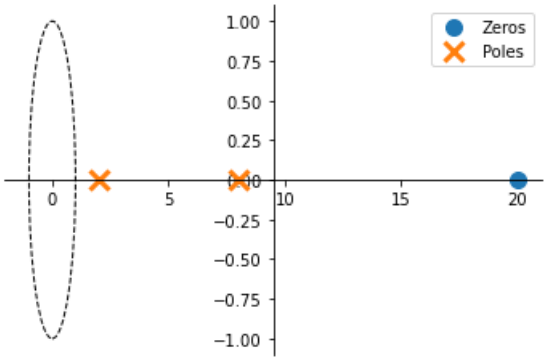
$$h[k] = Z^{-1}\{H(z)\} = (6 * 2^k u[k] - 4 * 8^k)u[k]$$

We then verified the values we obtained for A and B in our partial fraction expansion using the scipy.signal.residuez() function[1].

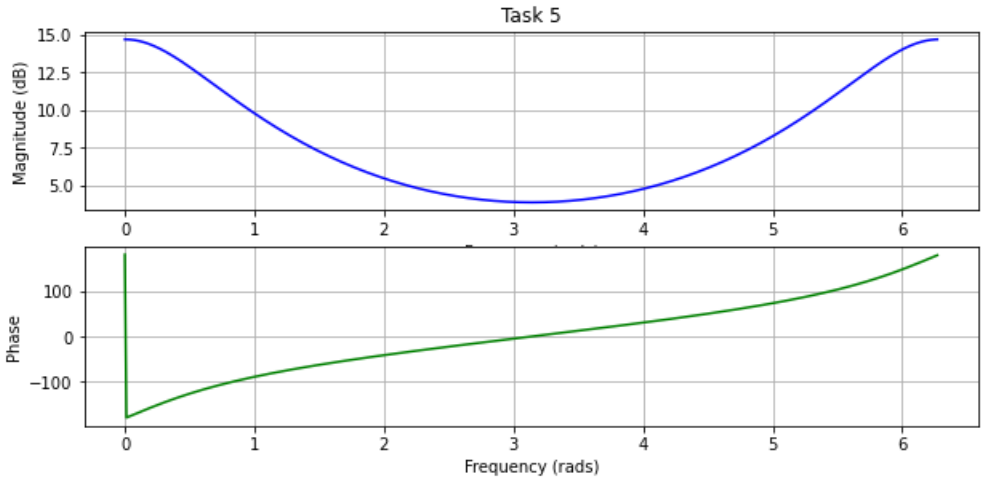
A zplane() function was provided for obtaining the pole-zero plot for $\frac{H(z)}{z}$. We afterwards plotted the magnitude and phase responses of $\frac{H(z)}{z}$ using the scipy.signal.freqz() function, with the parameter whole = True.

4 Results

The pole-zero plot for $\frac{H(z)}{z}$ is shown below, with poles at 2 and 8, and a zero at 20, which matches the equation for the system.



Below are the magnitude and phase responses of $\frac{H(z)}{z}$ plotted.



5 Error Analysis

The lab was straightforward with little room for error.

6 Questions

- H(z) is not stable. Looking at the zero-pole plot, the poles are completely outside of the unit circle, in which all stable pole values are contained. We can further verify this with examination of the equation h[k], in which the k-exponent values are 2 and 8, which will increase to infinity over time.
- The clarity of the lab tasks, expectations and deliverables was good as per usual.

7 Conclusion

This lab focused on using Z-transforms and Python tools to perform analysis on a discrete system. In addition to verifying our hand calculations, we used Python tools to graphically confirm the behavior of our discrete system; in this case, we saw that the transfer function would be unstable.

8 Appendix

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 r: [ 6. -4.]
p: [2. 8.]
k: []
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