

ECE 351 - Lab 6 Prelab

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Tasks

Task 1

$$y''(t) + 10y'(t) + 24y(t) = x''(t) + 6x'(t) + 12x(t) = 0; y'(0^-) = y(0^-) = x'(0^-) = x(0^-) = 0$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s^2 + 6s + 12}{s^2 + 10s + 24}$$

Task 2

$$y_{step}(t) = L^{-1}\{H(s) * \frac{1}{s}\} = L^{-1}\{\frac{s^2 + 6s + 12}{s(s^2 + 10s + 24)}\}$$

$$y_{step}(t) = \frac{1}{2}(1 - e^{-4t} + 2e^{-6t})u(t)$$

$$y'' + 10y' + 24y = x'' + 6x' + 12x, \quad y'(0^-) = y(0^-) = x'(0^-) = x(0^-) = 0$$

$$s^2 Y(s) + 10s Y(s) + 24Y(s) = s^2 X(s) + 6s X(s) + 12X(s)$$

$$Y(s)(s^2 + 10s + 24) = X(s)(s^2 + 6s + 12) \quad H(s) = \frac{Y(s)}{X(s)} = \frac{s^2 + 6s + 12}{s^2 + 10s + 24}$$

$$y_{\text{step}}(t) = \mathcal{L}^{-1}\{H(s) \cdot \frac{1}{s}\} = \mathcal{L}^{-1}\left\{\frac{s^2 + 6s + 12}{s(s^2 + 10s + 24)}\right\} = \mathcal{L}^{-1}\left\{\frac{s^2 + 6s + 12}{s(s+4)(s+6)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+6}\right\} \quad A = \frac{s^2 + 6s + 12}{(s+4)(s+6)} \Big|_{s=0} = \frac{1}{2}$$

$$B = \frac{s^2 + 6s + 12}{s(s+6)} \Big|_{s=-4} = -\frac{1}{2} \quad C = \frac{s^2 + 6s + 12}{s(s+4)} \Big|_{s=-6} = 1$$

$$y_{\text{step}}(t) = \mathcal{L}^{-1}\left\{\frac{1}{2} \frac{1}{s} - \frac{1}{2} \frac{1}{s+4} + \frac{1}{s+6}\right\} = \frac{1}{2}(1 - e^{-4t} + 2e^{-6t})u(t)$$