Homework 1 Basic Probability

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1 Solution

1. Committees (3pt)

   (a) 1pt If you pick the committee (uniformly) at random, what is the probability that everyone in the group is female?

   Posibilities of a all female group:

   $$N1 = \binom{8}{6}$$

   Total posibilities:

   $$N\Omega = \binom{14}{6}$$

   Probability of a all female group:

   $$P = \frac{N1}{N2}$$

   (b) 2pt Suppose the committee should be balanced for gender (i.e. 3 men and 3 women) and that two of the men refuse to serve together. How many different committees are possible?
   Posibilities for men:

   $$N1 = \binom{6}{3}$$

   Posibilities for women:

   $$N1 = \binom{8}{3}$$

Posibilities of the two men in the same comitee:

$$N3 = (6 - 2)$$

Total of posibiities:

$$N\Omega = \binom{14}{6}$$

Final Probaility of the desired comitee omposition:

$$P = \frac{N1 + N2 - N3}{N\Omega}$$

2. (a) 3pt Before we can make a necklace (see part b), we need to pick our beads. You pick your 7 beads (uniformly) at random. What is the probability that they have the desired composition?

   Different shape compositions:

$$N1 = \frac{7!}{2!2!1!}$$

   Taking in account the colors:

$$N2 = N1 * 6$$

   Combinations with the same color in a pair:

$$N3 = \binom{5}{3} * 6$$

   Total combinations:

$$N\Omega = \binom{30}{7}$$

   Probability of desired composition:

$$P = \frac{N2 - N3}{N\Omega}$$

   (b) How many necklaces can you make from all possible groups of beads with the right composition? Note again that all beads are unique and that a necklace remains the same when you flip it around.

   Every desired composition has faculty 3 different options of forming a necklace. Because of the symmetry the two different color options are not taken in account. The middle stone is fix since there is only one bead has no pair:

$$N = (N2 - N3) * 3!$$

3. Counting functions (3pt)

    (a) Let X and Y be two finite sets. How many functions are there from X to Y ?

        X and y define a two dimensional space. The amount of functions from X to Y is:

$$Y^X$$

    (b) How many functions are there?

        Again defines a space that can be written as:

$$2^n * 2^n$$

        The amount of functions is:

$$(2^n)^{2^n}$$

    (c) question c
        X is the Union of A and B:

$$X = A \cup B$$

        The amount of functions is:

$$X^n = (A \cup B)$$

References

Schulz, Schaffner, Basic Probability and Statistics, Feb. 2018