UNIVERSITEIT VAN AMSTERDAM

MSc Artificial Intelligence
Master Thesis

Knowledge Generation

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by
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Contents

Abstract

We generate Knowledge! [1]

This thesis applies the genius idea of having a VAE learn latent features of the raw data from KGs. Building on successful models of prior work, a linear, a convolutional and a architecture combining both methods are tested on the two most popular KGs. To best possible train out models on sparse graph representation, we implement a permutation invariant loss function. We compare the performance of our models to sate of the art link predictors, as well as link predictors including the variational module. The results compare ??? The model x outperfoms the others, indicating that convolutions are [/not] necessary. Interpolation of the latent space shows that the model learns features??? When generating subgraphs with up to x nodes, we see ??? Finally we filter generated triples for predicates which imply the subject or object to be entity of the class location. x out of these triples adhere to this axiom. Thus we can say that VAE's are to a certain extend able to capture the underlying semantics of a KG.

1 Introduction

Here comes a beautiful introduction. Promise!

1.1 Motivation

Computer vision reached a point, where semantics, entities and relations can be inferred in an simple image. Would it not be fantastic to be able to apply this to text too? Could we train a model to learn the semantics of a KG?

1.2 Expected Contribution

This thesis is aimed to be at fundamental research and provide insight, into if further research in this direction would be meaningful.

1.3 Research Question

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How well can a VAE learn the underlying semantics of a KG?
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2 Related Work

This section presents previous work which inspired and layed the fundamentals for this thesis. Relevant papers to three topics related to this thesis will be presented. We will present their method and results in the fields of relational graph convolutions, graph encoders, and embedding based link prediction.

2.1 Relational Graph Convolutions

We define a graph as $G = (V, E)$ with a set of nodes V and a set of edges E. The set of edges, with each edge connecting node x and y, is defined by $\{(x,y) \mid (x,y) \in V^2 \land x \neq y\}$ while the constrain $x \neq y$ disallows self-connections or self-loops, which is optional depending on the graphs function. Moreover, nodes and edges can have describing features, which contribute additional information about the nodes and their connection. Using graph convolutions, we make use of these properties holding spectral information about their neighboring nodes and relations. The two main standards of evaluate the performance of a neural network on graphs, are node classification and link prediction. Node classification is a classification problem where the model provides a probability distribution over all classes for each node. During link prediction de model scores a set of one

real and corrupted triples and aims to score the highest on the real triple. A more in-depth explanation follows later on in chapter 4.

In Thomas Kipf's and Max Welling's first paper on graph convolutions [2] a novel Graph Convolution Network (GCN) for semi-supervised classification is introduced. This method acts directly on the graph structure and shows to be linearly scalable with the number of nodes. While the authors compare different propagation models for the graph convolutions, their propagating rule using a first-order approximation of spectral graph convolutions, outperforms all other implementations. This so called renormalization trick normalized the adjacency matrix and adds it to an identity matrix of same size, what keeps the eigenvalues in a range between $[0, 2]$ what again leads to a stable training, avoiding numerical instabilities and vanishing gradients during learning. Additionally the feature information of neighboring nodes is propagates in every layer what shows improvement in comparison to earlier methods, where only label information is aggregated. Kipf and Welling perform node classification on the three citation-network datasets, Citeseer, Cora and Pubmed as well as on the KG dataset NELL. In all classification tasks, their results outperform other recently proposed methods in this field and proves computational more efficient than its competition. More details on the implementation of graph convolutions can be found in the next chapter.

In their publication *Modeling Relational Data with Graph Convolutional Networks* Schlichtkrull, Kipf, Bloem, v.d. Berg, Titov and Welling propose a relational graph convolutional network (RGCN) and evaluate it on link prediction on the FB15K-237 and WN18 dataset and node classification on the AIFB, MUTAG, BGS and AM datasets [3]. While the RGCN with its encoder properties, it is used by it self as node classifier, yet for link prediction it is coupled with a DistMult model acting as decoder which scores triples encoded by the RGCN see 2.1. More details on the DistMult can be found in 2.3.
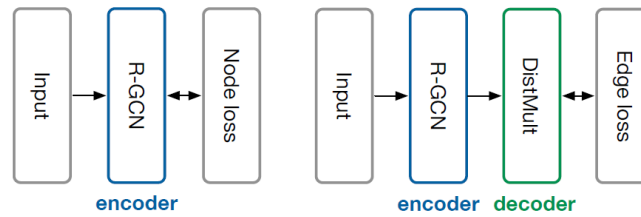

Figure 1: RGCN pipeline for node classification and link prediction experiments. The first pipeline only uses an encoder to classify the input nodes. The second pipeline additionally uses a decoder to score the input and predict the correct link, Source [3].

The RGCN works on graphs stored as dense triples, creating a hidden state for head and tail of each triple. A novel message passing network is layer-wise propagated with the hidden states of the entities. As regularization the authors propose a *basis-* and *blockwise* decomposition. while the first aims at an effective weight sharing between different relation types, the second can be seen as a sparsity constraint on the relation type's weight. The model outperforms embedding based model on the link prediction task on the FB15K-237 dataset and scores competitive on the WN18 dataset. In the node classification task, the model sets state of the art results on the datasets AIFB and AM, while scoring competitive on the rest. The authors conclude, that the model has difficulties encoding higher-degree hub nodes on datasets with many entities and low amount of classes.

2.2 Graph VAE

We have seen how graph convolutional neural networks can be combined to a encoder-decoder architecture, resulting in a generative model suitable for unsupervised learning. We will present three recent publications with different methods and usecases of a graph generative model, in particular a VAE.

In yet another publication of Kipf and Welling introduce the Variational Graph Autoencoder (VGAE), a framework for unsupervised learning on graph-structured data [4]. This generative model uses a GCN as encoder and a simple inner product module as decoder. Similar to th GCN the VGAE incorporates node features, what significantly improves its performance on link prediction tasks compared to related models. The VGAE uses a two-layer GCN to encode the mean and the logvariance of for the stochastic module to sample the latent space representation. The activation of the inner product of this latent vector yields then the reconstruction of the adjacency matrix. Figure 2.2 shows how the model learns to represent the underlying data structure by grouping the latent representations of the datapoints according to their class, without these labels being

provided to the model during training.

Figure 2: Colorized vizualization of the latent representation of the VGAE trained on the Core citation network with colors differentiating document classes and gray links indicating citations. This shows that the model implies and featurizes the document classes, without them being provided during training. Source [4]

The VGAE with added features outperforms state of the art models in the task of link prediction on the datasets Cora, Citeseer and Pubmed. The authors point out, that a Gaussian prior might be a poor choice combined with the inner-product decoder.

While citation networks represent basic graph structures, there has also been done work on more complex KGs. Simonovsky and Komodakis introduce the GraphVAE, a generative model which outputs a probabilistic fully-connected graph of a predefined maximum size in a one-shot approach [5]. The model includes a standard graph matching algorithm to align the predicted graph to the ground truth. In contrast to the previously presented publications, the input to this model is a threefold and sparse graph, defined as $G = (A, E, F)$ with A being the adjacency matrix, E the edge attribute matrix and F the node attribute matrix, with E ad F being one-hot encoded. Considering that this method lays the foundation for this thesis, we will adopt this notation for our own methods in 4. Figure 2.2 shows the architecture of the GraphVAE. The encoder is a feed forward network with edge-conditioned graph convolutions. After the convolutions the result is flattened and conditioned on the node labels y. A simple neural network then encodes the stochastic latent space, as we know it from previous works. The decoder is again conditioned on the node labels y and in form of a fully-connected neural network reconstructs the graph prediction. The threefold decoder output is matched with the target using graph matching algorithm, which we will discuss further in 3.3. The best matching permutation is then used for the reconstruction loss term of the GraphVAE. Notable is, that the size of the target and prediction graph do not necessarily have to match. While this approach seems promising, the maximum graph size is limited to a node count of 100 by computational memory requirements.

The model is trained on the QM9 dataset, containing the graph structure of 134k organic molecules with experiments on latent space dimension in the range of $[20, 80]$. On the free generation task, about 50% of the generated molecules are chemically valid and thereof remarkably 60% are not included in the trainings dataset. When testing the model for robustness, it showed little disturbance when adding Gaussian noise to the input graph G. The authors conclude that the problem of generating graphs from a continuous embedding was addressed successfully and that the GraphVAE performs better on small molecules, thus worse on larger graphs.

- Belli recurrent VAE

- GraphVAE paper

- Variational Graph Auto-Encoders

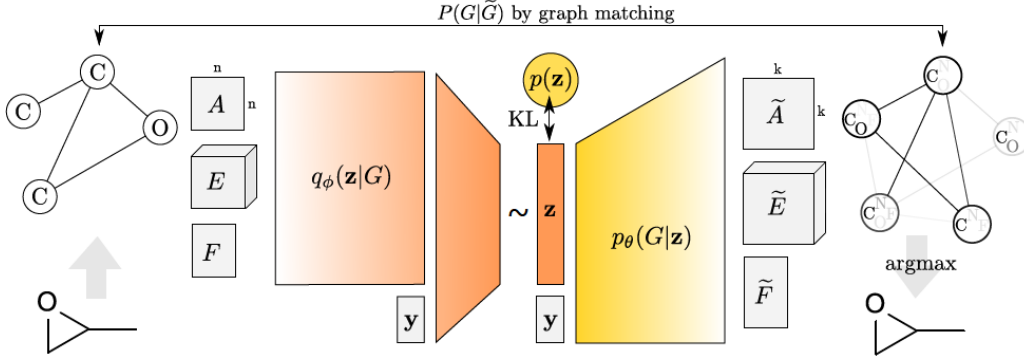model architecture: gCN variational sampling + decoder

Figure 3: Model architecture of the GraphVAE. The target graph with n nodes is encoded and conditioned on the node labels y. The KL divergence ensures a Gaussian prior to the decoder, which reconstructs the latent representation to a graph with k nodes. Target and prediction graphs are matched and permuted before the reconstruction loss. To sample, the argmax is taken directly from the prediction. Source [5].

incorporates node features outperforms other models on citation networks The model architecture presented in the GraphVAE paper is the starting point of our model.

2.3   Embedding Based Link Prediction

Present RASCAL and one or two more.

Embeddings

TransE represents entities in in low-dimensional embedding. The relationships between entities are represented by the vector between two entities [6]. (How are different relation between the same entities represented?)

OntoUSP

This method learns a hierarchical structure to better represent the relations between entities in embedding space.

3   Background

In this section we will go over related work and relevant background information for our model and experiments. The depth of the explanation is adopted to the expected prior knowledge of the reader. The reader is supposed to know the basics of machine learning and deep learning, including probability theory and basic knowledge on neural networks and their different architectures. Basic principles such as forward pass, backpropagation and convolutions are expected to be understood. Further the use and functionality of deep learning modules such as the model, the optimizer and the terms target and prediction should be known. This also includes being familiar with the training and testing pipeline of a model in deep learning.

Should all these boxes be checked, then we can expect to get a deeper understanding of the magic behind the VAE and its differences to a normal autoencoder. After that we will present how convolutional layers can be used on graphs. Of course where there is a layer there is a model, thus we are presenting the graph convolutional network (GCN). Closing the circle we show how we can adopt the VAE to graph convolutions. Wrapping things up, we present the state of the art algorithms for graph matching, which will be util to allow permutation invariance when matching prediction and target graph [7].

3.1   Knowledge Graph

Knowledge graph has become a popular key phrase. Yet, the term is so broad, that it can have various definitions. In this thesis we are going to focus on KGs in the context of relational machine learning.

A KG is a database and as all other databases it is used to store data. The main difference to tabular databases is that KGs store data in a relational fashion. A standard KG structure, introduced by the semantic-web community, is the Resource Description Framework (RDF). It is a so called schema-based approach, meaning that every entity has a unique identifier and all possible relations are stored in a vocabulary. The opposite schema-free approach is used in OpenIE models for information extraction. Here any type of triple can be extracted, eg. (Michelangelo, painted, Sixtine Chapel). Where as a triple in RDF format from Freebase, one of the largest open-source KGs, has the form

$$(/m/02mjmr, /people/person/born-in, /m/03gh4)$$

$$(s, r, o)$$

with:

- Subject $s : /m/02mjmr$ Barack Obama

- Relation/Predicate $r : /people/person/born-in$

- Object $o : /m/03gh4$ Hawaii

For all triples s and o are part of a set of so called entities, while r is part of a set of relations. This is enough to define a basic KG.

Schema-based KG can include type hierarchies and type constrains. Classes group entities of the same type together, based on common criteria, e.g all names of people can be grouped in the class 'person'. Hierarchies define the inheriting structure of classes and subclasses. Picking up our previous example, 'child' would be a subclass of 'person' and inherit its properties. At the same time the class of an entity can be key to a relation with type constrain, since some relations can only be used in conjunction with entities fulfilling the constraining type criteria.

These schema based rules of a KG are defined in its ontology. Here properties of classes, subclasses and constrains for relations and many more are defined. Again, we have to differentiate between KGs with open-world or closed-world assumption. In a closed-world assumption all constrains must be sufficiently satisfied before a triple is accepted as valid. This leads to a huge ontology and makes it difficult to expand the KG. On the other hand open-world KGs such as Freebase, accept every triple as valid, as long as it does not violate a constrain. This again leads inevitably to inconsistencies within the KG, yet it is the preferred approach for large KGs. In context of this thesis we refer to the ontology as semantics of a KG, we research if our model can capture the implied closed-world semantics of an open-world KG [8].

Lastly, we point out one major difference between KGs, namely their representation. RDF KGs are represented as set of triples, consisting of a unique combination of numeric indices. Each index linking to the corresponding entry in the entity and relation vocabulary. This is called dense representation and benefits from fast computation due to an optimized use of memory.

In contrary the dense representation of a triple is the sparse representation. Here a binary square matrix also called the adjacency matrix, indicated a link between two entities. To identify the node, each node in the adjacency matrix has a one-hot encoded entity-vocabulary vector. All one-hot encoded vectors are concatenated to a node attribute matrix. In simple cases, like citation networks this is a sufficient representation. In the case of Freebase, we need an additional edge-attribute matrix, which indicates the relation of each link. The main benefits of this method are the representation of subsets of triples, also subgraphs, with more than one relation and the computational possibility to perform graph convolutions.

3.2 Graph VAE – one shot method

3.2.1 VAE

The VAE as first presented by [9] is an unsupervised generative model in form of an autoencoder, consisting of an encoder and a decoder. Its architecture differs from a common autoencoder by having a stochastic module between encoder and decoder. The encoder can be represented as recognition model with the probability $p_{\boldsymbol{\theta}}(\mathbf{z} \mid x)$ with x being the variable we want to inference and z being the latent representation given an observed

value of x. The encoder parameters are represented by θ. Similarly, we denote the decoder as $p_{\boldsymbol{\theta}}(\mathbf{x} \mid z)$, which given a latent representation z produces a probability distribution for the possible values, corresponding to the input of x. This will be the base architecture of all our models in this thesis.

The main contribution of the VAE is the so called reparameterization trick. When sampling from the latent prior distribution, we get a stochastic module inside our model, which can not be backpropagated and makes learning impossible. By placing the stochastic module outside the model, we can again backpropagate. We use the predicted latent space as mean and variance for a Gaussian normal distribution, from which we then sample ϵ, which acts as external parameter and does not need to be updated.
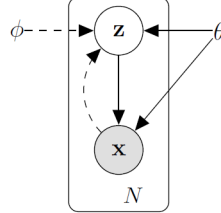

Figure 4: Representation of the VAE as bayesian network, with solid lines denoting the generator $p_{\boldsymbol{\theta}}(z)p_{\boldsymbol{\theta}}(\mathbf{x} \mid z)$ and the dashed lines the posterior approximation $q_{\phi}(\mathbf{z} \mid \mathbf{x})$ [9].

In 4 we see, that the true posterior $p_{\boldsymbol{\theta}}(\mathbf{z} \mid \mathbf{x})$ is intractable. Thus, we make the assumption that the prior to the decoder is Gaussian with an approximately diagonal covariance, which gives us the approximated posterior.

$$\log q_{\phi}\left(\mathbf{z} \mid \mathbf{x}^{(i)}\right) = \log \mathcal{N}\left(\mathbf{z}; \boldsymbol{\mu}^{(i)}, \boldsymbol{\sigma}^{2(i)}\mathbf{I}\right) \tag{1}$$

Now variational inference can be performed, which allows both θ the generative and ϕ the variational parameters to be learned jointly. Using the Monte Carlo estimation of $q_{\phi}(\mathbf{z} \mid \mathbf{x})$ we get the so called estimated lower bound (ELBO):

$$y_k(\mathbf{x}, \mathbf{w}) = \sigma\left(\sum_{j=0}^{M} w_{kj}^{(2)} h\left(\sum_{i=0}^{D} w_{ji}^{(1)} x_i\right)\right) \tag{2}$$

We denote the first term the regularization term, as it forces the model into using a Gaussian normal prior. The second term represents the reconstruction loss, matching the prediction with the target.

While we use a discrete input space, the output space is a continuous probability. To generate final result, the prediction is used as binomial probability distribution, from which we then sample. Once training of a VAE is completed, the Decoder can be used on its own to generate new samples by using latent input signals [9].

3.2.2 MLP

The Multi-Layer Perceptron (MLP) is the vanilla basemodel of all neural networks. Its properties as universal approximator has been discovered and widely studied since 1989. The innovation it brought to existing models was the hidden layer between the input and the output.

The mathematical definition of the MLP is rather simple. It takes linear input vector of the form $x_1, ..., x_D$ which is multiplied by the weight matrix $\mathbf{w}^{(1)}$ and then transformed using a non-linear activation function $h()$. Due to its simple derivative, mostly the rectified linear unit (ReLU) function is used. This results in the hidden layer, consisting of hidden units. The hidden units get multiplied with the second weight matrix, denoted $\mathbf{w}^{(2)}$ and finally transformed by a sigmoid function $\sigma()$, which produces the output.Grouping all weight

and bias parameter together we get the following equation for the MLP:

$$y_k(\mathbf{x}, \mathbf{w}) = \sigma \left(\sum_{j=1}^{M} w_{kj}^{(2)} h \left(\sum_{i=1}^{D} w_{ji}^{(1)} x_i + w_{j0}^{(1)} \right) + w_{k0}^{(2)} \right) \tag{3}$$

for $j = 1, \ldots, M$ and $k = 1, \ldots, K$, with M being the total number of hidden units and K of the output.

Since the sigmoid function returns a probability distribution for all classes, the MLP can have the function of a classifier. Instead of the initial sigmoid function, it was found to also produce good results for multi label classification transforming the output with a softmax function instead. Images or higher dimensional tensors can be processed by flattening them to a one dimensional tensor. This makes the MLP a flexible and easy to implement model [10].

3.2.3 Graph convolutions

Convolutional neural nets (CNN) have the advantage to be invariant to permutations of the input. Convolutional layers exploit the property of datapoints which are close to each other and thus, have a higher correlation. CNNs have shown great results in the field of images classification and object detection. Neighboring pixel contain information about each other, thus by detecting local features, the model can then merged those to high-level features, e.g a face in an image [10]. Similar conditions hold for graphs. Neighboring nodes contain information about each other and can used to infer local features.

Let us shortly go over the definition and math behind graph convolutions. Different approaches have been published on this topic, here we will present the graph convolution network (GCN) of [2]. We consider $f(X, A)$ a GCN with an undirected graph input $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $v_i \in \mathcal{V}$ is a set of N nodes and $(v_i, v_i) \in \mathcal{E}$ the set of edges. The input is a sparse graph representation, with X being a node feature matrix and $A \in R^{N \times N}$ being the adjacency matrix, defining the position of edges between nodes. In the initial case, where self-connections are not considered, the adjacency's diagonal has to be one resulting in $\vec{A} = A + I_N$. The graph forward pass through the convolutional layer l is then defined as

$$H^{(l+1)} = \sigma \left(\tilde{D}^{-\frac{1}{2}} \tilde{A} \tilde{D}^{-\frac{1}{2}} H^{(l)} W^{(l)} \right). \tag{4}$$

Here $\tilde{D}_{ii} = \sum_j \tilde{A}_{ij}$ acts as normalizing constant. $W^{(l)}$ is the layer-specific weight matrix and contains the learnable parameters. H returns then the hidden representation of the input graph.

The GCN was first introduced as node classifier, meaning it returns a probability distribution over all classes for each node in the input graph $\mathcal{V}$. Assuming that we preprocess $\vec{A}$ as $\hat{A} = \tilde{D}^{-\frac{1}{2}} \vec{A} \tilde{D}^{-\frac{1}{2}}$, the equation for a two-layer GCN for z classes is

$$Z = f(X, A) = \text{softmax} \left(\hat{A} \, \text{ReLU} \left(\hat{A} X W^{(0)} \right) W^{(1)} \right). \tag{5}$$

3.2.4 Graph VAE

Now we have understood all the principles needed for a Graph VAE. While there are many variations, not only in architecture but also in graph representation, namely sparse or dense, we will focus here on the graph VAE presented in [5]. A sparse graph model with graph convolutions.

The encoder $q_\phi(\mathbf{z} \mid \mathcal{G})$ takes a graph $\mathcal{G}$ as input, on which graph convolutions are applied. After the convolutions the hidden representation is flattened and concatenated with the node label vector y. A simple MLP encodes the mean and logvariance of the latentsapce distribution. Using the reparametrization trick we sample the latent representation.

For the decoder $p_\theta(\mathbf{x} \mid \mathcal{G})$ the latent representation is again concatenated with the node labels. The decoder architecture for this model is a reverse MLP, which outputs a flat prediction of $\mathcal{G}$, which is split and reshaped in to the sparse matrix representation.
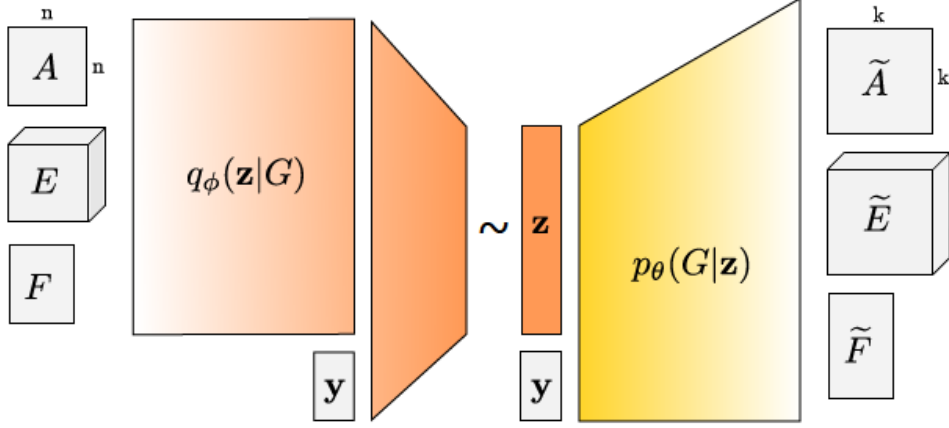
Figure 5: Illustration of the GraphVAE architecture as presented in [5]. The decoder combines graph convolutions and a MLP with concatenated labels y. The decoder reconstruct $\mathcal{G}$ from the latent representation and the concatenated node labels y.

3.2.5 One Shot vs. Recursive

The concept of the VAE has been used to generate data for various usecases. When using the VAE as generator, by sampling from the approximated posterior distribution $q_\phi(\mathbf{z})$, we can reconstruct the data in a singular run or recursive manner.

The one shot method is the used in the popular example of the VAE generator on the MNIST dataset [9]. each sample is independent from each other.

Recursive methods take part of the generated datapoint as input for the next datapoint, thus continuously generating the sample. This has been applied to generate audio and to reproduce videogame environments [11]. In [12] variation of the GraphVAE has been used to recursively construct a vector roadmap, which has been presented in [link to related work].

For this thesis, we will use the one-shot method, predicting each datapoint independent from each other. The datapoints will be sparse subgraph with n nodes. A single triple being $n = 2$ and a subgraph representation $2 < n < 100$.

3.3 Graph Matching

In this subsection we will explain the term permutation invariance and present an algorithm to find the optimal permutation in order to match two graphs. We then derive the loss term of the graph VAE 3.2.4 applying the optimal matching matrix to the models prediction.

3.3.1 Permutation Invariance

Permutation invariance refers to the invariance of a permutation of an object. An visual example is the the image generation of numbers. If the loss function of the model would not be permutation invariant, the generated image could show a perfect replica of the input number but being translated by one pixel the loss function would penalize the model. Geometrical permutations can be translation, scale or rotation around any axis.

In the context of sparse graphs the most common, and relevant permutation for this thesis, is be the position of a link in the adjacency matrix. By altering its position with help of a permutation matrix, the link will connect different nodes. When matching graphs with more than two nodes, we can partially match the graphs by permuting only a subgraph instead of the full graph. This way also graphs of different sizes, can be matched.

A model is permutation invariant, when it includes a function to detect and match such permutations between target and prediction.

3.3.2 Max-Pool Graph matching algorithm

While there are numerous graph matching algorithms, we will focus on the max-pool algorithm, which can be effectively implement and used in the VAE setting. First presented in [13] in the context of computer vision and successfully trained to match graphs from feature points in an image. It uses a max-pooling graph matching approach, which is resilient to deformations and highly tolerant to outliers. The output is a reliable cost matrix. Notable is, that also graphs with different number of nodes can be matched.

Let us introduce a new sparse representation for a sampled subgraph, where the discrete target graph is $G = (A, E, F)$ and the continuous predicted graph $\widetilde{G} = (\widetilde{A}, \widetilde{E}, \widetilde{F})$. The A, E, F are store the discrete data for the adjacency, for node attributes and the node attribute matrix of form $A \in \{0,1\}^{n \times n'}$ with n being the number of nodes in the target graph. $E \in \{0,1\}^{n \times n' \times d_e}$ is the edge attribute matrix and a node attribute tensor of the shape $F \in \{0,1\}^{n \times d_n}$ with d_e and d_n being the size of the entity and relation dictionary. For the predicted graph with k nodes, the adjacency matrix is $\widetilde{A} \in [0,1]^{k \times k}$, the edge attribute matrix is $\widetilde{E} \in R^{k \times k \times d_e}$ and the node attribute matrix is $\widetilde{F} \in R^{k \times d_n}$.

Given these graphs the algorithm aims to find the affinity matrix $S : (i,j) \times (a,b) \to R^+$ where $i, j \in G$ and $a, b \in \widetilde{G}$. The affinity matrix expresses the similarity of all nodepairs between the two graphs and is calculated

$$S((i,j),(a,b)) = \left(E_{i,j,\cdot}^T, \widetilde{E}_{a,b,\cdot} \right) A_{i,j} \widetilde{A}_{a,b} \widetilde{A}_{a,a} \widetilde{A}_{b,b}[i \neq j \wedge a \neq b] + \left(F_{i,\cdot}^T \widetilde{F}_{a,\cdot} \right) \widetilde{A}_{a,a}[i = j \wedge a = b] \qquad (6)$$

where the square brackets define Iverson brackets [5].

The next step is to find the similarity matrix $X^* \in [0,1]^{k \times n}$. Therefore we iterate a fist-order optimization framework and get the update rule

$$\mathbf{x}_{t+1} \leftarrow \frac{1}{\|\mathbf{S}\mathbf{x}_t\|_2} \mathbf{S}\mathbf{x}_t. \qquad (7)$$

To calculate $\mathbf{Sx}$ we find the best candidate $x_{i,a}$ from the possible pairs in the affinity matrix. Heuristically, taking the argmax over all neighboring nodepair affinities yields the best result. Other options are sum-pooling or average-pooling, which do not discard potentially irrelevant information, yet have shown to perform worse. Thus, using the max-pooling approach, we can pairwise calculate

$$\mathbf{Sx}_{ia} = \mathbf{x}_{ia}\mathbf{S}_{ia;ia} + \sum_{j \in \mathcal{N}_i} \max_{b \in \mathcal{N}_a} \mathbf{x}_{jb}\mathbf{S}_{ia;jb}. \qquad (8)$$

Depending on the matrix size, the number of iterations are adjusted. The resulting similarity matrix $X*$ yields a normalized probability of matching nodepairs. In order to get a discrete translation matrix, we need to find the optimal match for each node.

3.3.3 Hungarian algorithm

Picking up on the nodepair probabilities X^* of last chapter, we reformulate it as a linear assignment problem. Here comes into play an optimization algorithm, the so called hungarian algorithm. It original objective is to optimally assign n resources to n tasks. The cost of assigning task $i \in R^n$ to $j \in R^n$ is stored in x_{ij} of the quadratic cost matrix $x \in R^{n \times n}$. By assuming tasks and resources are simple nodes and taking $C = 1 - X^*$ we get the cost matrix C for the optimal translation. This algorithm is has a complexity of $O\left(n^4\right)$, thus is not applicable to complete KGs but only to subgraphs with limited number of nodes per graph [14].

The core of the hungarian algorithm consist of four main steps, initial reduction, optimality check, augmented search and update. The now presented algorithm is a slight alternative of the original algorithm and improves the complexity of the update step from $O(n^2)$ to $O(n)$ and thus, reduces the total complexity to $O(n^3)$. It takes as input a bipartie graph $G = (V, U, E)$ and the cost matrix $C \in R^{n \times n}$ where $V \in R^n$ and $U \in R^n$ are sets of nodes and $E \in R^{n \times n}$ the set of edges between the nodes. The algorithm's output is a discrete matching matrix M. To avoid two irrelevant pages of pseudocode, the steps of the algorithm are presented in the following short summary [15].

1. Initialization:

   (a) Initialize the empty matching matrix $M_0 = \emptyset$.
   (b) Assign α_i and β_i as follows:

   $$\forall v_i \in V, \qquad\qquad \alpha_i = 0$$
   $$\forall u_i \in U, \qquad\qquad \beta_j = \min_i (c_{ij})$$

2. Loop n times over the different stages:

   (a) Each unmatched node in V is a root node for an Hungarian tree with when completed results in an augmentation path.
   (b) Expand the Hungarian trees in the equality subgraph. Store the indices i of v_i encountered in the Hungarian tree in the set $I*$ and similar for j in u_j and the set J^*. If an augmentation path is found, skip the next step.
   (c) Update α and β to add new edges to the quality subgraph and redo the previous step.

   $$\theta = \frac{1}{2} \min_{i \in I^*, j \notin J^*} (c_{ij} - \alpha_i - \beta_j)$$
   $$\alpha_i \leftarrow \begin{cases} \alpha_i + \theta & i \in I^* \\ \alpha_i - \theta & i \notin I^* \end{cases}$$
   $$\beta_j \leftarrow \begin{cases} \beta_j - \theta & j \in J^* \\ \beta_j + \theta & j \notin J^* \end{cases}$$

   (d) Augment M_{k-1} by flipping the unmatched with the matched edges on the selected augmentation path. Thus M_k is given by $(M_{k-1} - P) \cup (P - M_{k-1})$ and P is the set of edges of the current augmentation path.

3. Output M_n of the last and n^{th} stage.

3.3.4 Graph Matching VAE Loss

Coming back to our generative model, we now explain how the loss function needs to be adjusted to work with graphs and graph matching, which results in a permutation invariant graph VAE.

The normal VAE maximizes the evidence lower-bound or, in a practical implementation, minimizes the upper-bound on negative log-likelihood. Using the notation of 3.2.1 the graph VAE loss is

$$\mathcal{L}(\phi, \theta; G) = \quad = E_{q_\phi(\mathbf{z}|G)} [-\log p_\theta(G \mid \mathbf{z})] + \text{KL} [q_\phi(\mathbf{z} \mid G) \| p(\mathbf{z})] \qquad (9)$$

The loss function $\mathcal{L}$ is a combination of reconstruction term and regularization term. The regularization term is the KL divergence between a standard normal distribution and the latent space distribution of Z. This term does not change when adopting to graphs. The reconstruction term is the binary cross entropy between prediction and target, which in the sparse graph representation are threefold with A, E, F.

The predicted output of the decoder is split in three parts and while $\tilde{A}$ is activated through sigmoid, $\tilde{E}$ and $\tilde{F}$ are activated via edge- and nodewise softmax. The graph matching permutation for A is applied to the target $A' = XAX^T$ and for E and F on the prediction, $\widetilde{F}' = X^T \widetilde{F}$ and $\widetilde{E}'_{\cdot,\cdot,l} = X^T \widetilde{E}_{\cdot,\cdot,l} X$, l being the one-hot

encoded edge attribute vector which is permuted. These permuted subgraphs are then used to calculate the maximum log-likelihood estimate [5]

$$\log p\left(A' \mid \mathbf{z}\right) = 1/k \sum_a A'_{a,a} \log \widetilde{A}_{a,a} + \left(1 - A'_{a,a}\right) \log \left(1 - \widetilde{A}_{a,a}\right) +$$
$$+ 1/k(k-1) \sum_{a \neq b} A'_{a,b} \log \widetilde{A}_{a,b} + \left(1 - A'_{a,b}\right) \log \left(1 - \widetilde{A}_{a,b}\right) \tag{10}$$

$$\log p(F \mid \mathbf{z}) = 1/n \sum_i \log F_{i,.}^T \widetilde{F}'_{i,} \tag{11}$$

$$\log p(E \mid \mathbf{z}) = 1/\left(\|A\|_1 - n\right) \sum_{i \neq j} \log E_{i,j,.}^T, \widetilde{E}'_{i,j,.} \tag{12}$$

3.4 Ranger Optimizer

Finalizing this chapter, we introduce the novel deep learning optimizer Ranger. An optimizer, which in 2020 placed itself on the top of 12 FastAI leaderboards. Ranger combines Rectified Adam (RAdam), lookahead and optionally gradient centralization. let us shortly look into the different components.

RAdam is based on the popular adam optimizer. It improves the learning by dynamically rectifying Adam's adaptive momentum. This is done by reducing the variance of the momentum, which is especially large at the beginning of the training. Thus, leading to a more stable and accelerated start [16].

The Lookahead optimizer was inspired by recent advances in the understanding of loss surfaces of deep neural networks, thus proposes an approach where, a second optimizer 'looks ahead' on a set of parallel trained weights. while the computation and memory cost are negligible, it learning improves and the variance of the main optimizer is reduced [17].

The last and most novel optimization technique, Gradient Centralization, acts directly on the gradient by normalizing it to a zero mean. Especially on convolutional neural networks, this helps regularizing the gradient and boosts learning. This method can be added to existing optimizers and can be seen as constrain of the loss function [18].

Concluding we can say that Ranger is a state of the art deep learning optimizer with accelerating and stabilizing properties, incorporating three different optimization methods, which synergize with each other. Considering that generative models are especially unstable during training, we see Ranger as a good fit for this research.

4 Methods

5 Experiments & Results

This section presents the experiments we ran. We covered link and node prediction and compared those to SOTA scores. Further we ran experiments on investigating the coherence of the reproduced graph structure. Lastly we measured the adherence of our model to the KG's underlying syntax.

5.1 Datasets and Training

For this sake of meaningful results, we chose to use the earlier presented, two most popular dataset used in this field, FB15k237 and WN18rr.

Training models on each dataset for 333 epochs, without early stopping.

5.2 Link Prediction

We used link prediction as evaluation protocol and for comparison state of the art models. For each triple in the dataset, we remove the tail and combine it with all possible entities in our dataset. EVen though this is called 'triple-corruption', also correct triples can be generated, which could appear in the trainset. These have bo the filtered out before evaluation. Link prediction on unfiltered test data is termed 'raw'. Our model then computes the ELBO loss for all corrupted triples, which are sorted and stored in ascending order. The same procedure is repeated the triple's head in place of the tail.

The metrics used to evaluate the predictions, is the mean reciprocal rank (MRR) and hits at 1,3 and 10. The MRR ??? explain what the MRR is. Hits at 1 indicates what percentage of the triples in the test set have been ranked the highest compared to their corruptions. Similar, hits at 3 and 10, give a percentage of triples ranked in the top 3 and top 10. These metrics allow a fair comparison on a dataset between models, regardless of the ranking method of each model.

5.3 Sanity Checks

Check if adj matrix adheres to edge attribute matrix.

5.4 Interpolate Latent Space

We tale two random triples and interpolate the latent space of these two triples. The interpolations result in: HERE AN EXAMPLE.

Further we go ahead and test what happens if we modify one latent dimension of a triple. HERE AN EXAPMPLE. Can the model assign logical features to latent dimensions?

5.5 Subgraph Generation

Until now our model trained on only one triple per sparse graph. What will happen if we train it on more than one triples?

5.6 Syntax coherence

Check if generated triples follow basic logic.

- Generate triples by random signals

- Filter these triples on a certain relation

- Check if the entities are part of the linked class

Since there is no preset for how to check the semantics of a KG, we will use simple basic logical criteria. The generated triples are filtered for the relation 'is capital of', thus the subject enntity should be a city and he object entity member of the class 'country', Hope this gives good results.

6   Discussion & Future Work

We will discuss certain aspects of our results and give advice on further research.

6.1 Discuss Aspect 1

Well well well

6.2 Future Work

There are many avenues to follow for future work.

Basing on the believe that the experiments were successful, we recommend:

- Improve the model (deeper)

- link the latent space to word signals e,g text

- prior not normal, e.g NF

- try a GAN

Good luck amigos!

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