Active Learning for Level Set Estimation

Alkis Gotovos, Nathalie Casati, Gregory Hitz and Andreas Krause

ETH zürich

Problem

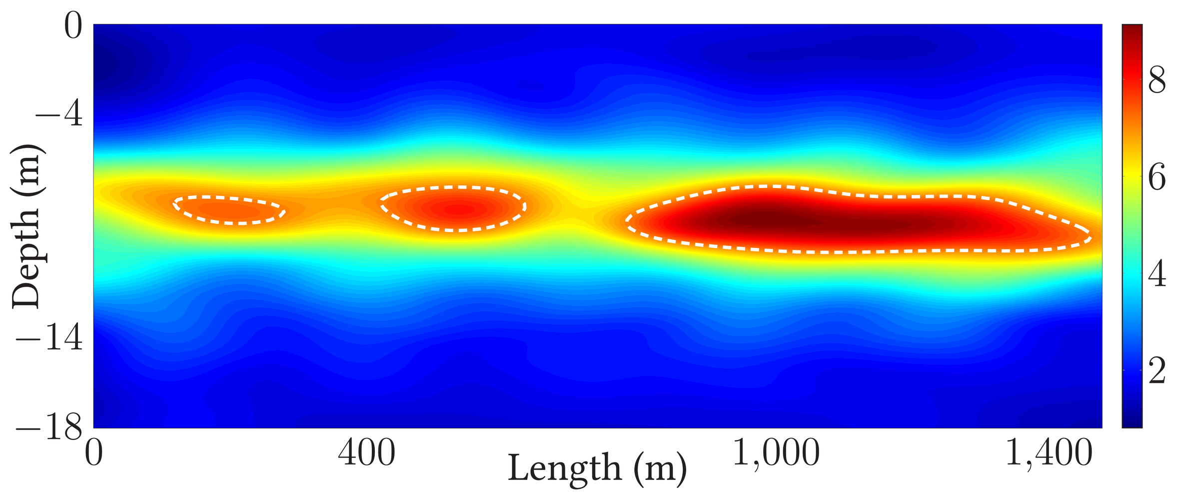
Determine the regions where the value of some unknown function lies above or below a given threshold level.

Pose as a classification problem (into super- and sublevel sets) with *sequential* measurements, which are assumed to be *expensive* and *noisy*.

Example applications

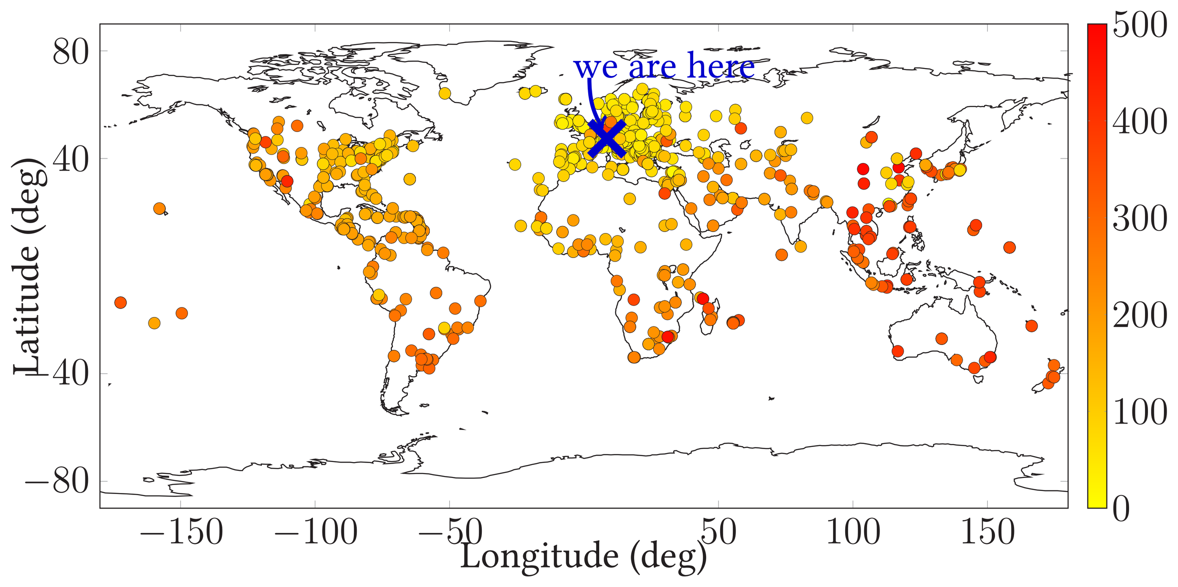
Environmental monitoring

Estimate regions of (a vertical transect of) Lake Zurich where chlorophyll/algal concentration is "abnormally high".



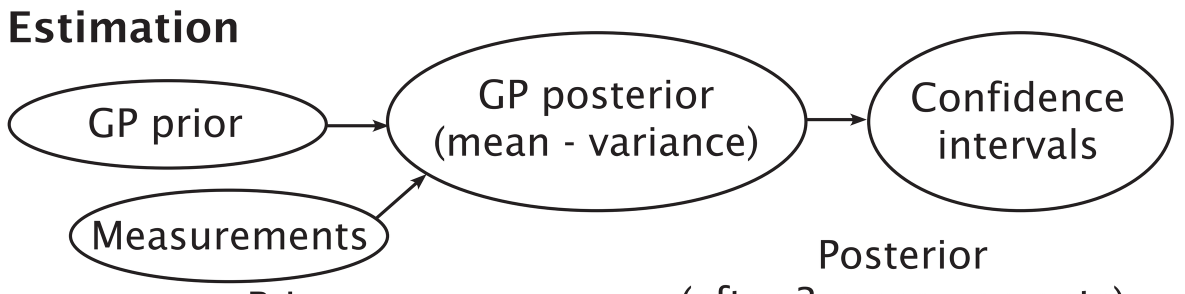
Geolocating internet latency

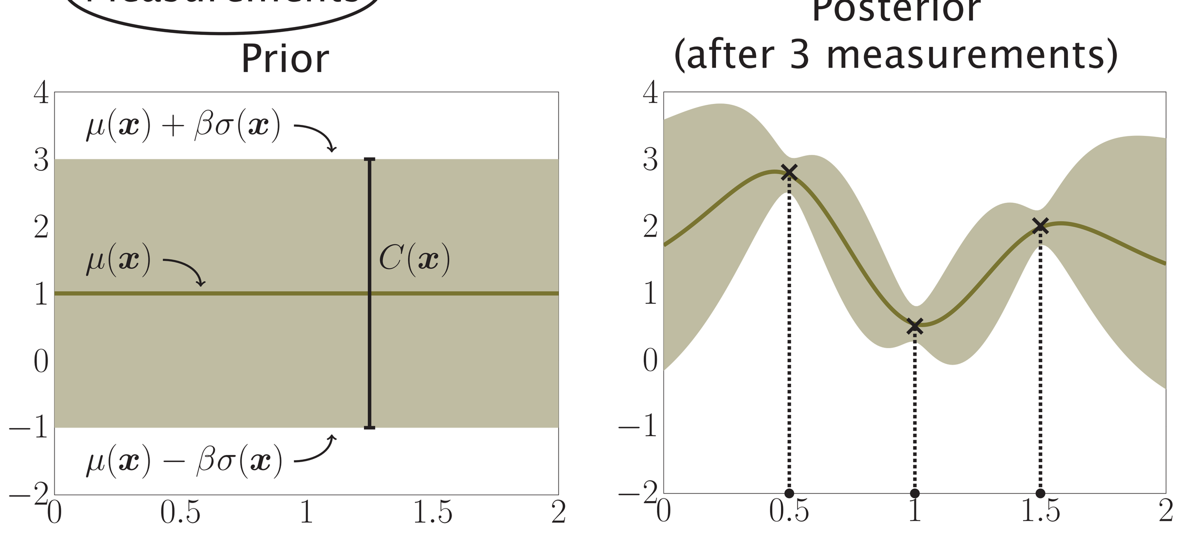
Estimate regions of the world with "acceptable" latency to our PC, e.g. for trouble-free online gaming.



Gaussian processes

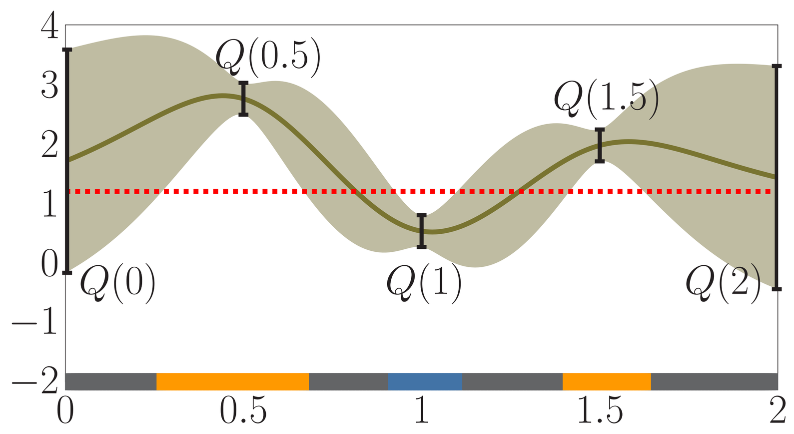
Estimation





Classification

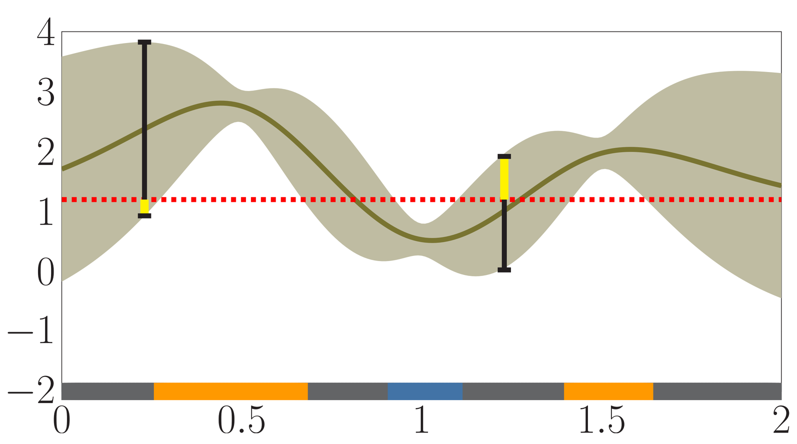
For each point, we use the GP-derived confidence intervals to either classify it into the super- or sublevel sets, or leave it unclassified.



Measurement selection

To obtain informative measurements, sample at the most *ambiguous* point among the yet unclassified.

Ambiguity $\approx$ Difficulty in classifying a point w.r.t. the given threshold level.



The LSE algorithm

We propose the Level Set Estimation (LSE) algorithm:

- Input: - Sample space D (e.g. fine grid of function domain)
  - Threshold level h
- Idea: Iteratively *sample* and *classify* based on GP-derived confidence bounds

while there exist unclassified points in D
- **for each** unclassified point
  - **if** its confidence interval lies above h **then**
    - classify into superlevel set
  - **else if** its confidence interval lies below h **then**
    - classify into sublevel set
  - **else**
    - leave unclassified

  CLASSIFY
- select max. ambiguity (yet unclassified) point
- obtain a noisy measurement at that point

  SAMPLE
- perform GP inference and update conf. intervals

Fine print

- Enforce monotonically shrinking confidence intervals
- Relax classification by an accuracy parameter ϵ

Sample complexity bound

Theorem

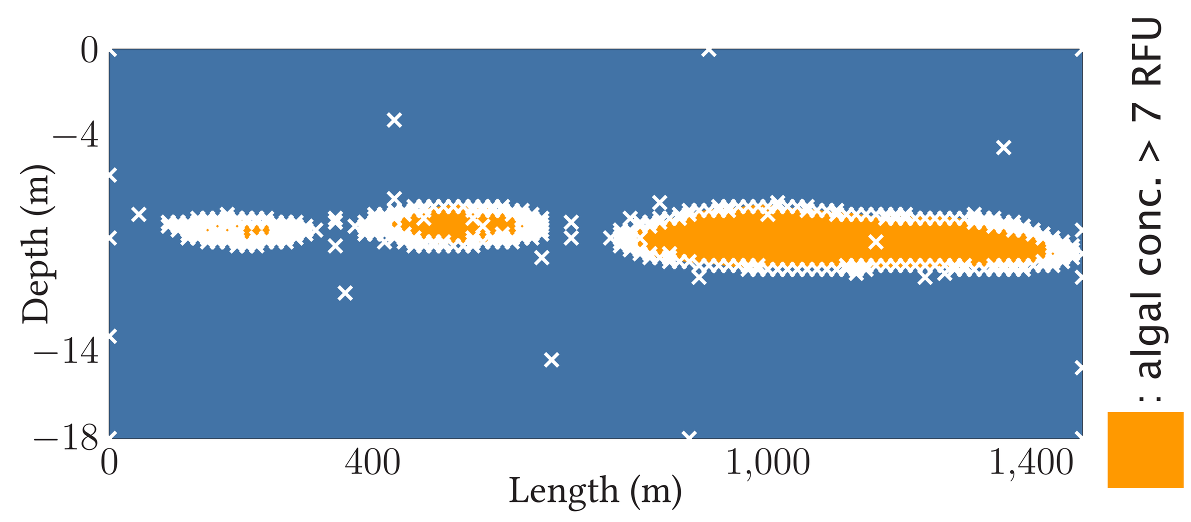
For any $h \in \mathbb{R}$, $\delta \in (0,1)$, and $\epsilon > 0$, if $\beta_t = 2\log(|D|\pi^2 t^2/(6\delta))$, LSE terminates after at most T iterations, where T is the smallest positive integer satisfying

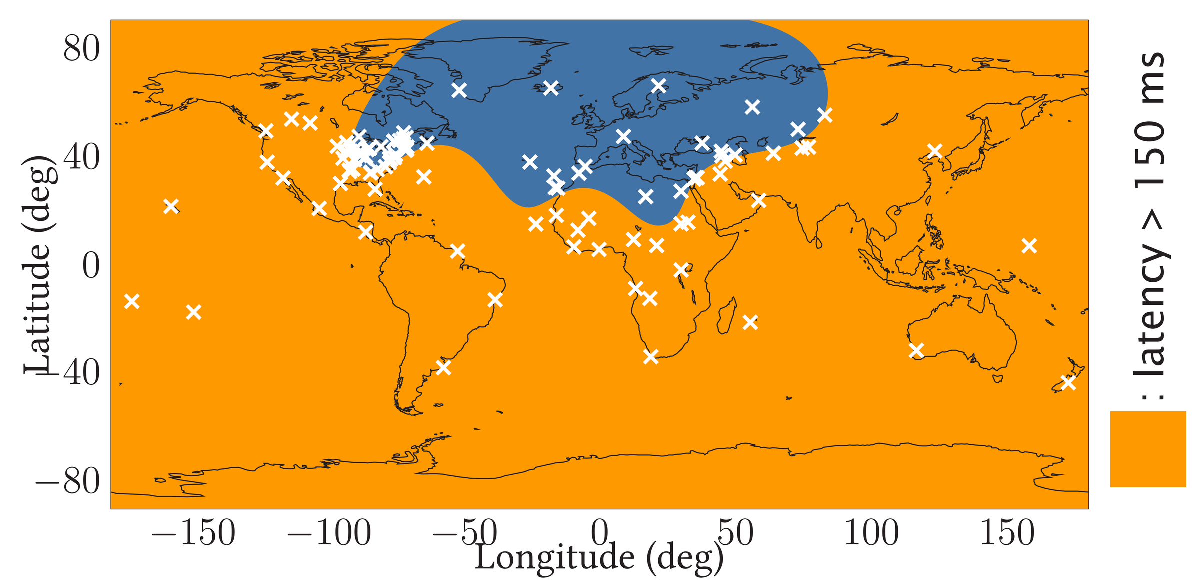
$$\frac{T}{\beta_T \gamma_T} \geq \frac{C_1}{4\epsilon^2},$$

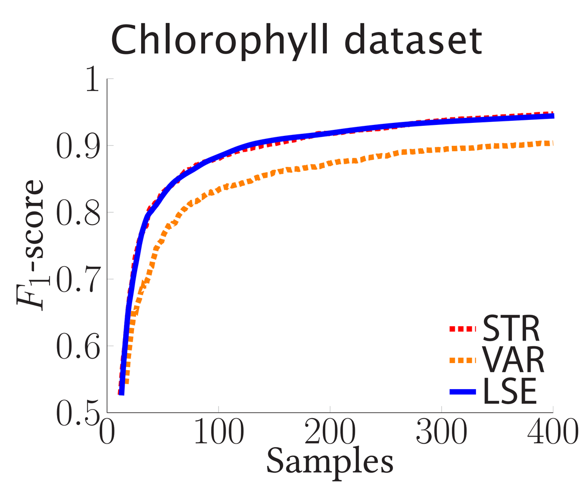
where $C_1 = 8/\log(1+\sigma^{-2})$.
Furthermore, with probability at least $1-\delta$, the algorithm returns an ϵ-accurate solution, that is

$$\Pr\left\{\max_{\boldsymbol{x}\in D} \ell_h(\boldsymbol{x}) \leq \epsilon\right\} \geq 1-\delta.$$

Experimental results

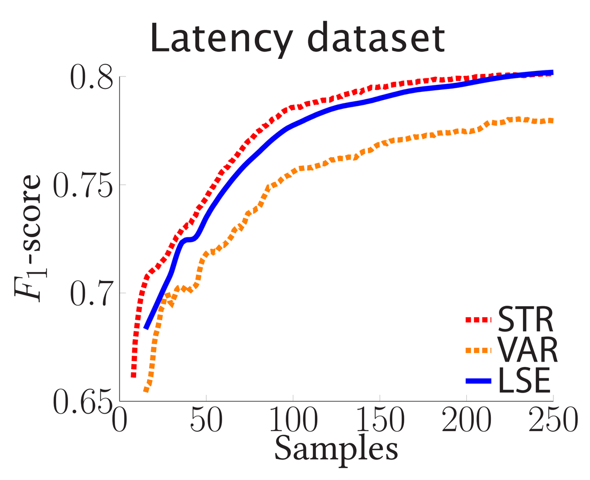


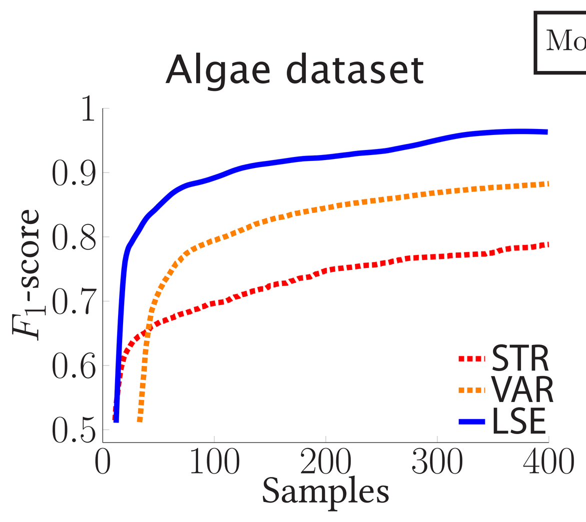




Compare LSE to:

- State-of-the-art "straddle" heuristic (Bryan *et al.*, 2005)
- Maximum variance sampling





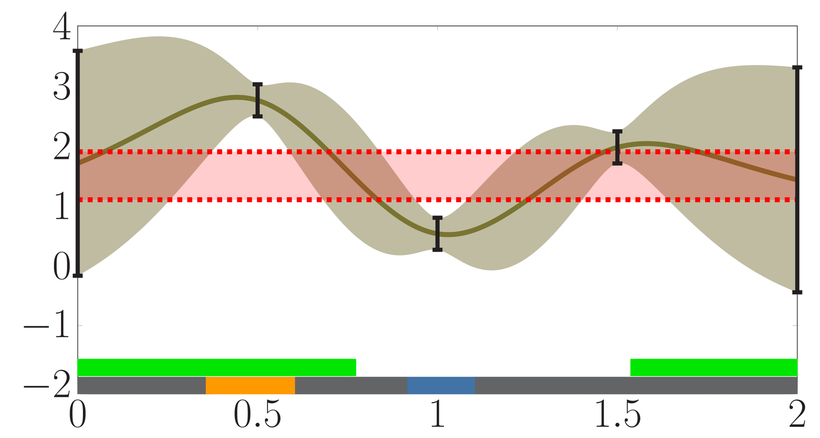
Extension 1: Implicit threshold level

What if we do not have a predefined threshold level h? (E.g. determine *relative* hotspots of algal concentration.)

Implicitly defined thr. level: $h = \omega \max f(\boldsymbol{x}),\ 0 < \omega < 1$

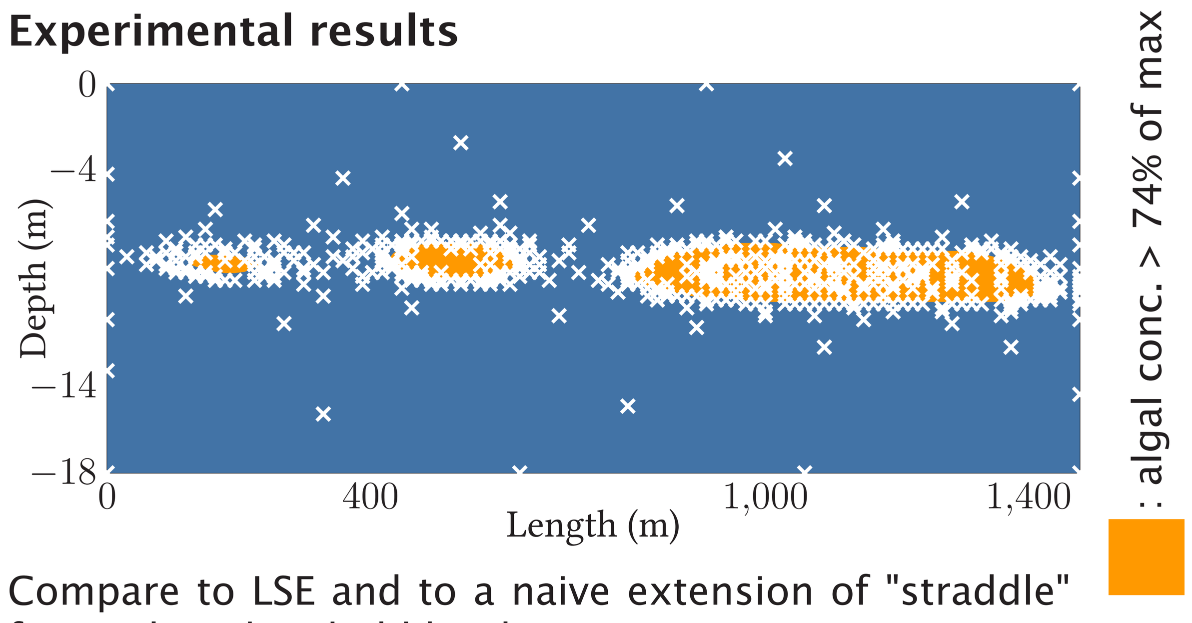
We propose the $\mathrm{LSE}_{\mathrm{imp}}$ extension of LSE:

- h is now an estimated quantity with associated uncertainty, which leads to slower classification.

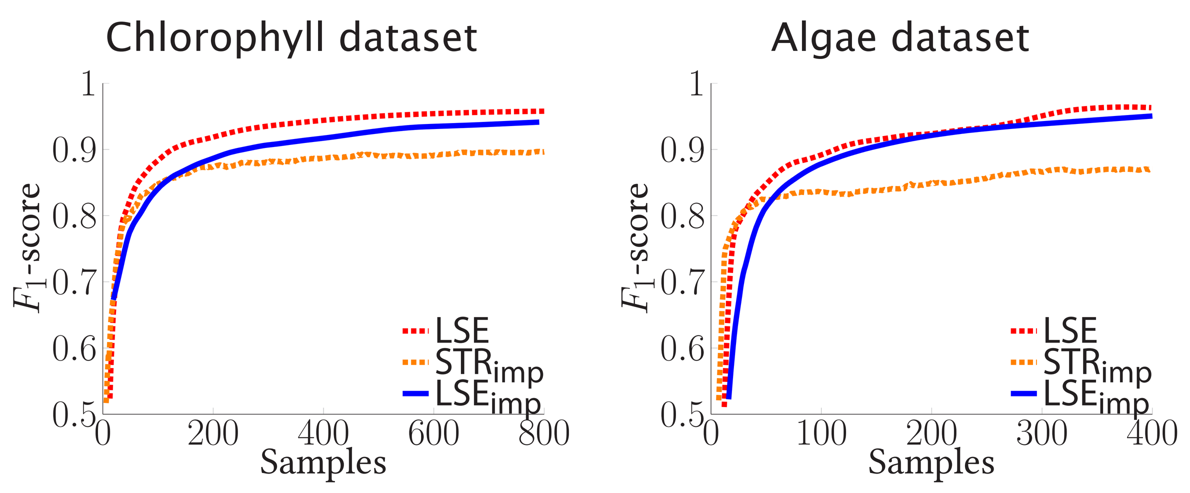


- We need to accurately estimate the function maximum, therefore we need to keep sampling at regions where the maximum may lie.
- Similar theoretical guarantees to LSE.

Experimental results



Compare to LSE and to a naive extension of "straddle" for implicit threshold levels.



Extension 2: Batch sampling

We propose the $\mathrm{LSE}_{\mathrm{batch}}$ extension of LSE for selecting a *batch* of B measurements at a time.

Why?

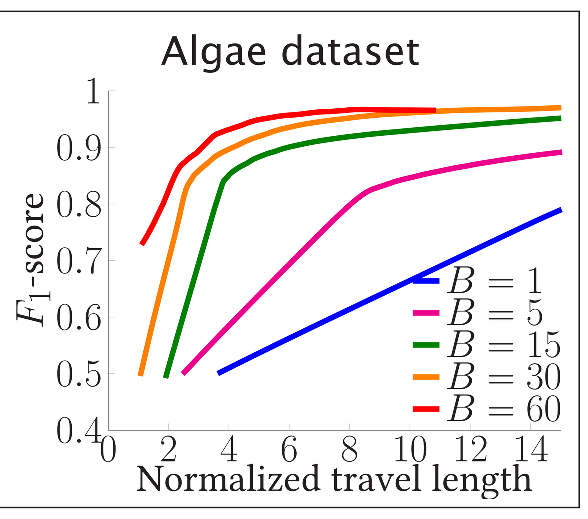
Latency geolocation

Send multiple ping requests in parallel at essentially the same cost as a single request, thus increasing sampling throughput.

Environmental monitoring

Reduce the total traveling distance by planning ahead:

- Select a batch of sampling locations
- Connect them using a Euclidean TSP path
- Traverse path and collect measurements



Extra: Proof outline of LSE bound