Active Learning for Level Set Estimation

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Problem

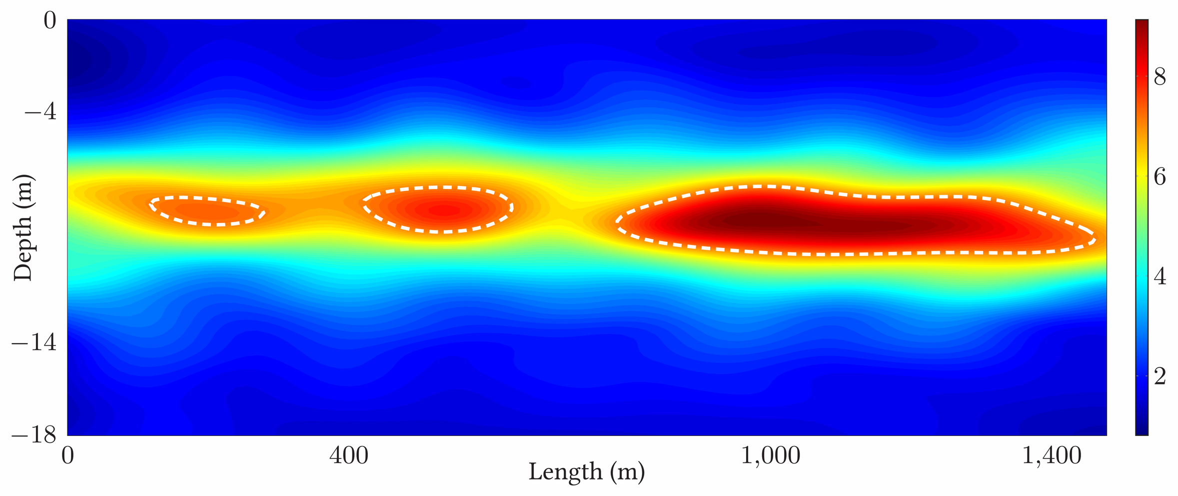
We would like to determine the regions where the value of some unknown function lies above or below a given threshold level.

The above can be posed as a classification problem (into super- and sublevel sets) with *sequential* measurements, which are assumed to be *expensive* and *noisy*.

Example applications

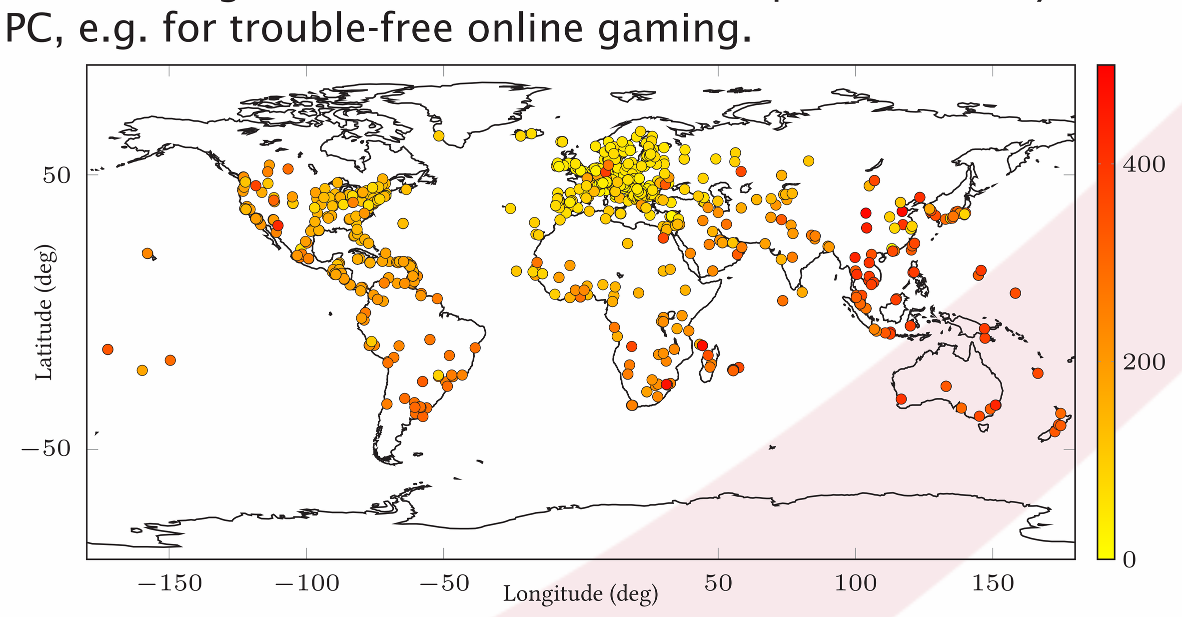
Environmental monitoring

Estimate regions of a lake transect where chlorophyll/algal concentration is "abnormally high".



Geolocating internet latency

Estimate regions of the world with "acceptable" latency to our PC, e.g. for trouble-free online gaming.

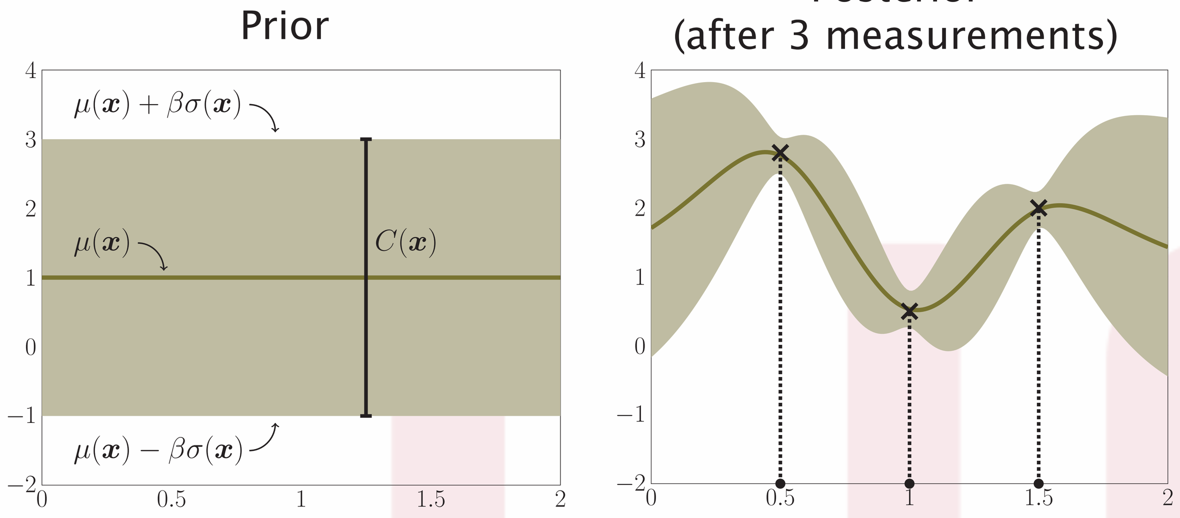


Gaussian processes

Estimation

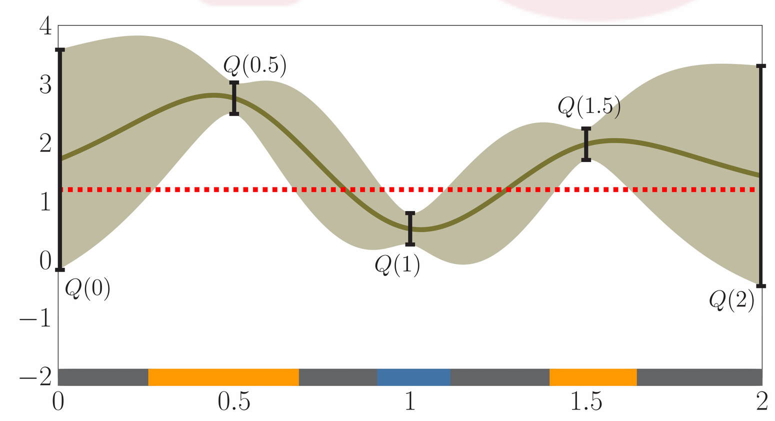
Given some measurements, GPs provide *mean and variance* estimates of the unknown function, allowing us to construct *confidence intervals* at each point.

Prior | Posterior (after 3 measurements)



Classification

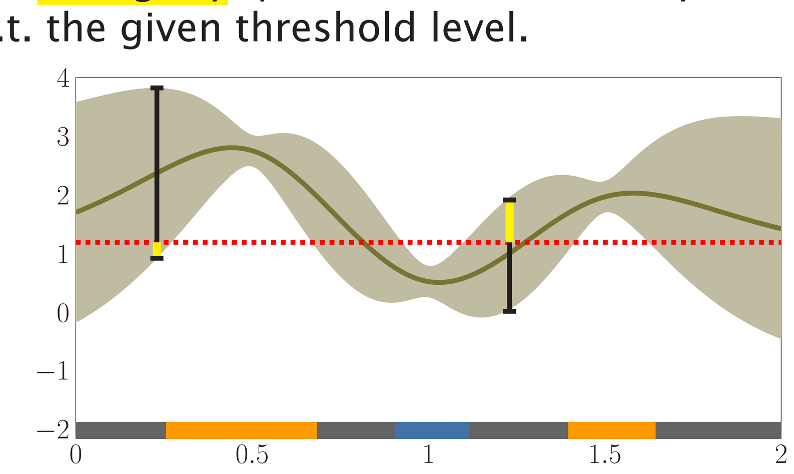
For each point, we use the GP-derived confidence intervals to either classify it into the super- or sublevel sets, or leave it unclassified.



Measurement selection

To obtain informative measurements w.r.t. the problem at hand, at each iteration we select the most *ambiguous* point among the yet unclassified to be measured.

Intuitively, ambiguity quantifies our difficulty in classifying a point w.r.t. the given threshold level.



The LSE algorithm

Given a set of points (e.g. fine grid of the unknown function's domain) and a threshold level h, our proposed Level Set Estimation (LSE) algorithm iteratively *samples* and *classifies* based on GP-derived confidence intervals.

while there exist unclassified points
- **for each** unclassified point
  - **if** its confidence interval lies above h then
    - classify into superlevel set
  - **else if** its confidence interval lies below h then
    - classify into sublevel set
  - **else**
    - leave unclassified

  CLASSIFY
- select max. ambiguity (yet unclassified) point
- obtain a noisy measurement at that point

  SAMPLE
- perform GP inference and update conf. intervals

Fine print

- We enforce monotonically "shrinking" confidence intervals
- We relax classification by an accuracy parameter ϵ

Sample complexity bound

Theorem

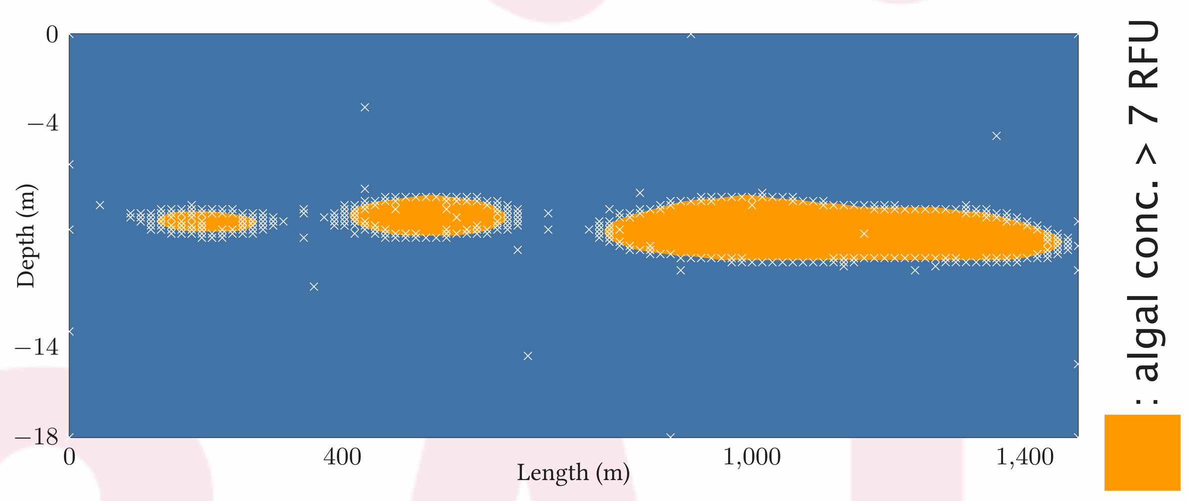
For any $h \in \mathbb{R}$, $\delta \in (0,1)$, and $\epsilon > 0$, if $\beta_t = 2\log(|D|\pi^2 t^2/(6\delta))$, LSE terminates after at most T iterations, where T is the smallest positive integer satisfying

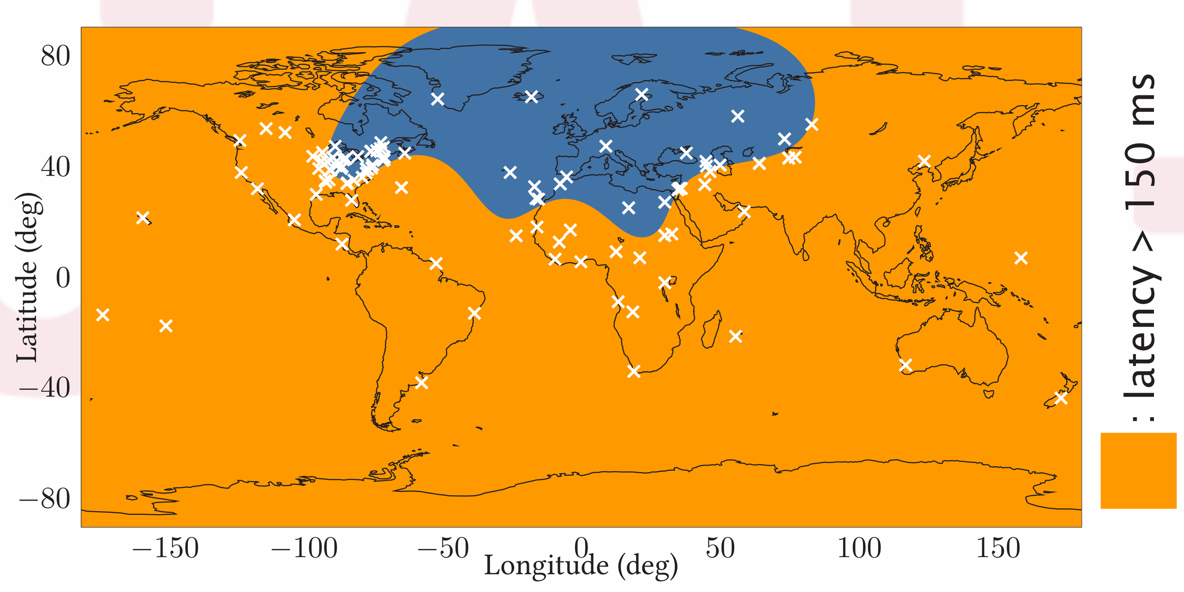
$$\frac{T}{\beta_T \gamma_T} \geq \frac{C_1}{4\epsilon^2},$$

where $C_1 = 8/\log(1+\sigma^{-2})$.
Furthermore, with probability at least $1-\delta$, the algorithm returns an ϵ-accurate solution, that is

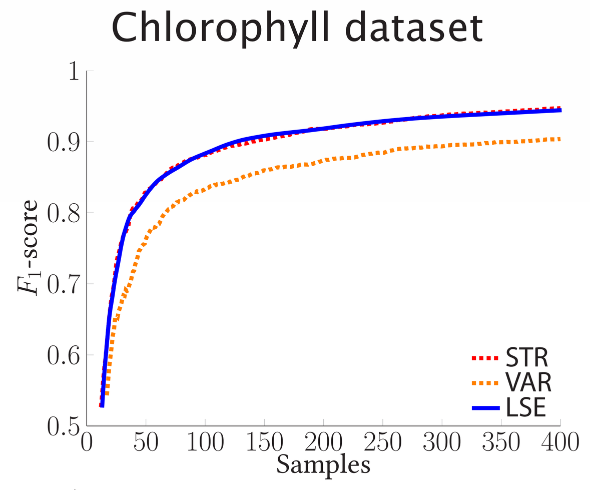
$$\Pr\left\{\max_{x \in D} \ell_h(x) \leq \epsilon\right\} \geq 1-\delta.$$

Experimental results



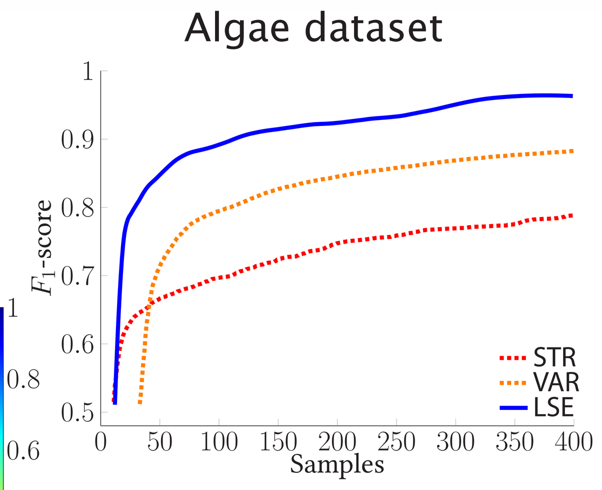


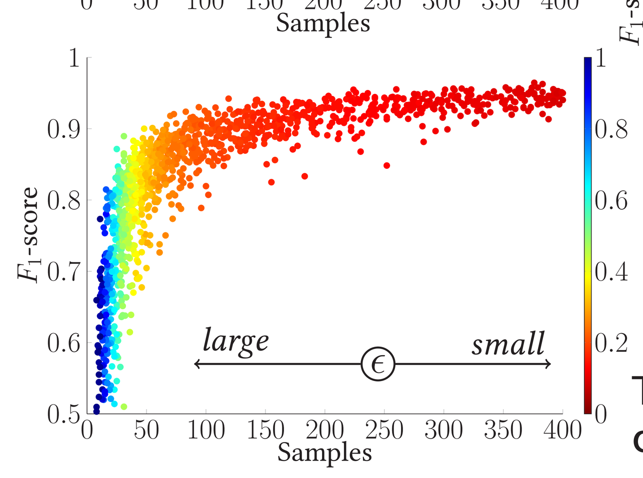
Chlorophyll dataset



Comparison to state-of-the-art "straddle" heuristic (Bryan et al., 2005) and maximum variance sampling.

Algae dataset





The trade-off effect of the accuracy parameter ϵ.