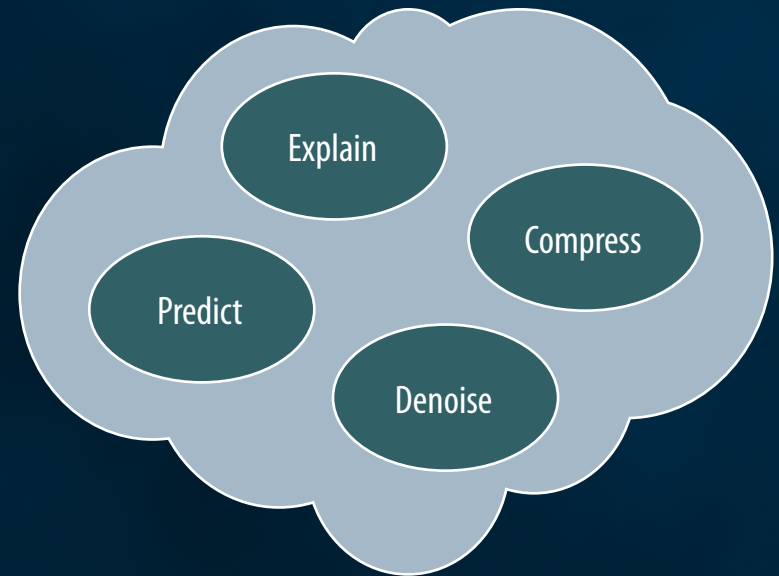


Minimum Message Length and Kolmogorov Complexity

C. S. Wallace and D. L. Dowe

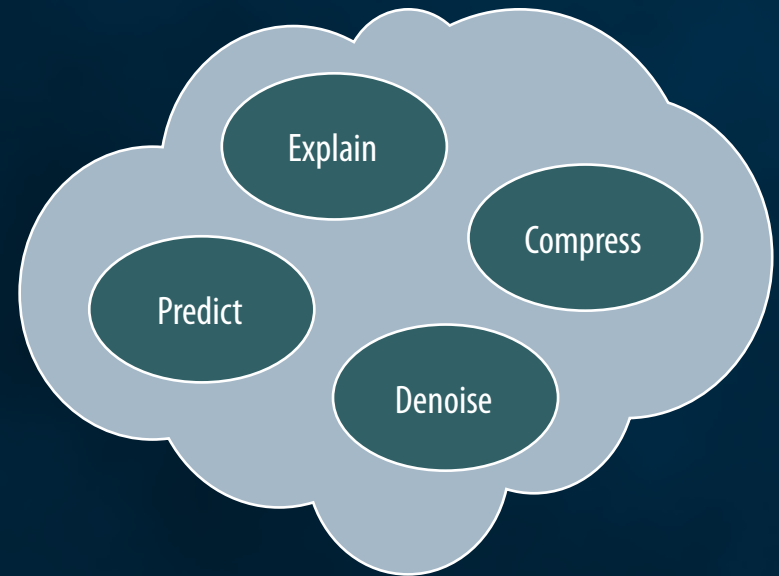
Data

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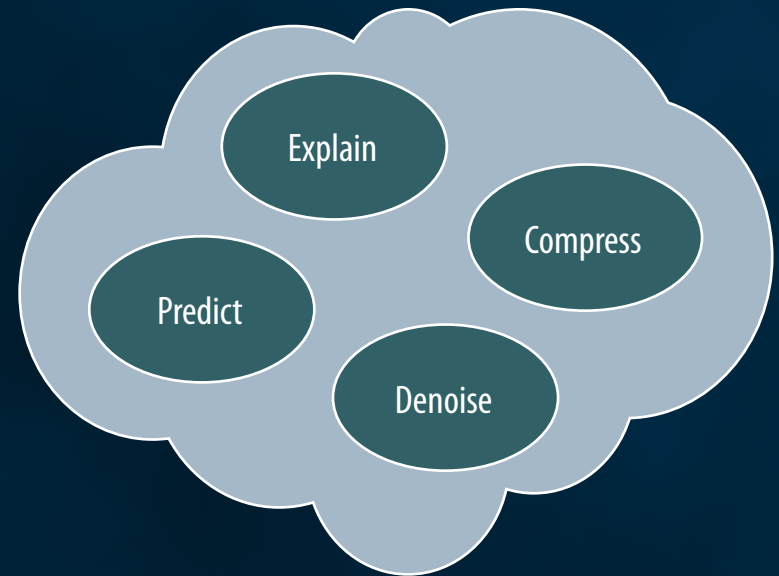
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Induction



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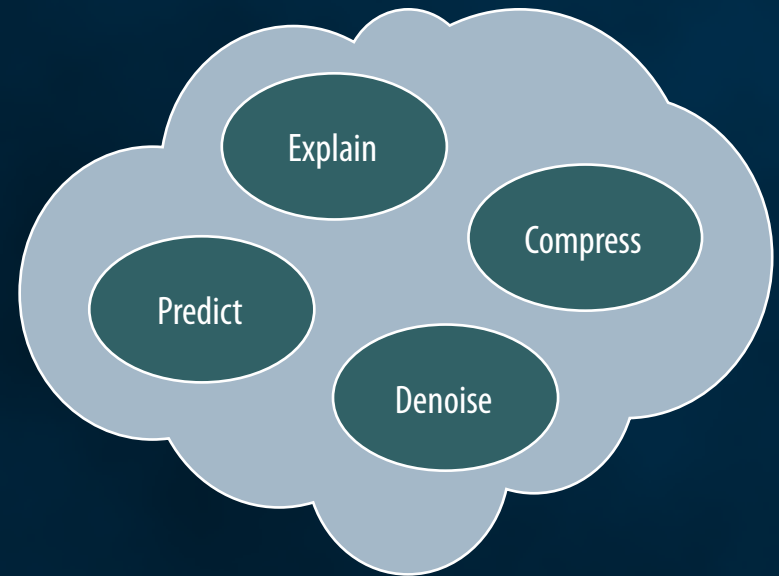
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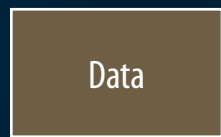
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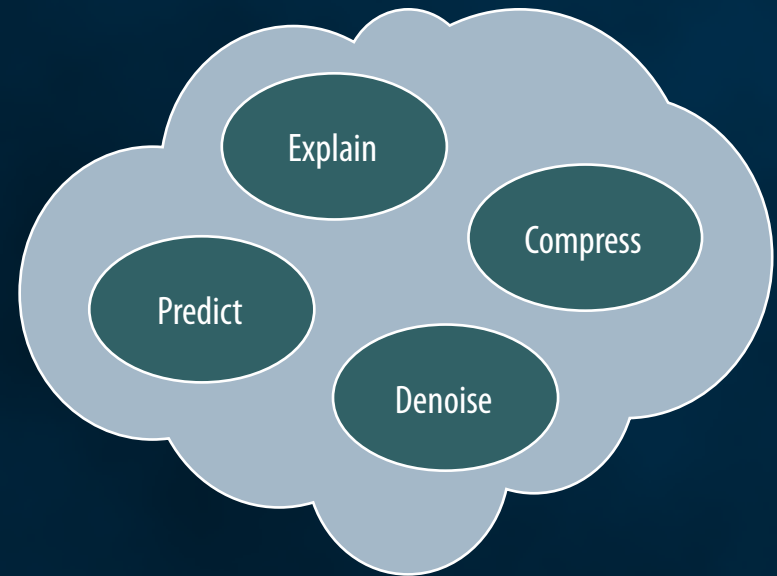
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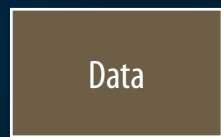
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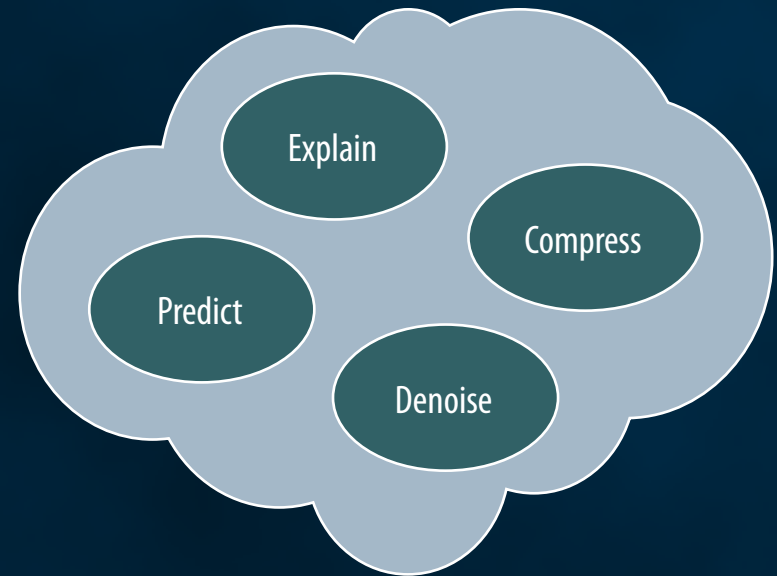
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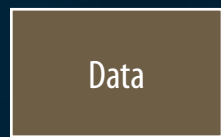
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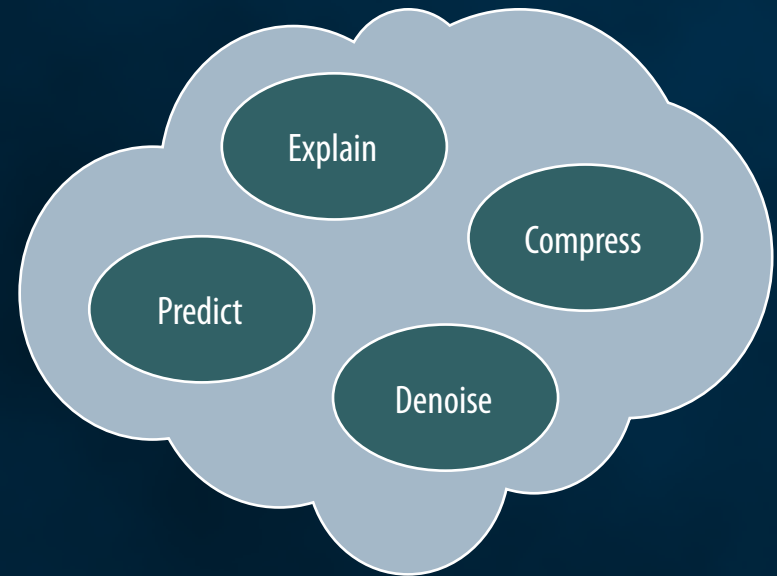
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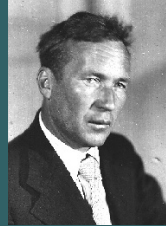
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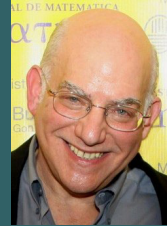
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Kolmogorov complexity

Quantify complexity of
binary strings via Turing
Machines (early '60s)



A. Kolmogorov



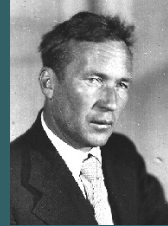
G. Chaitin



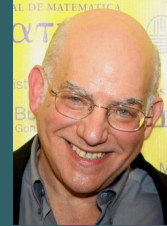
P. Martin-Löf

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Universal induction

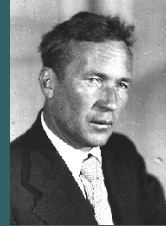
Define algorithmic probability via Turing Machines and use it for induction (early '60s)



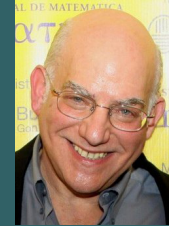
R. Solomonoff

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MML/MDL

Infer a hypothesis about the data via two-part coding (late '60s and '70s)



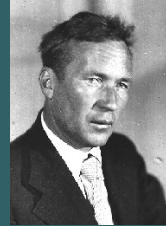
C. Wallace



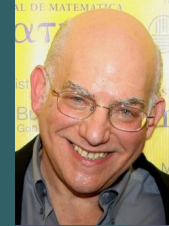
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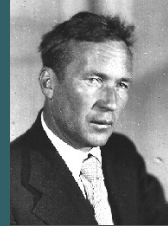
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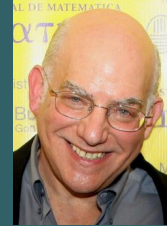
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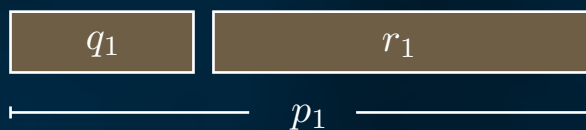
Data string



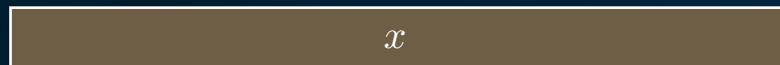
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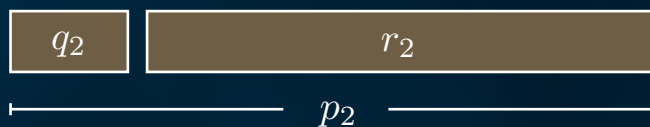
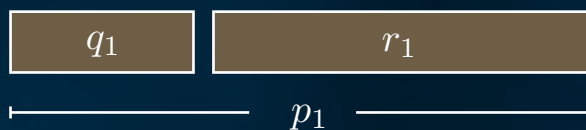
Encode x using a *two-part* scheme



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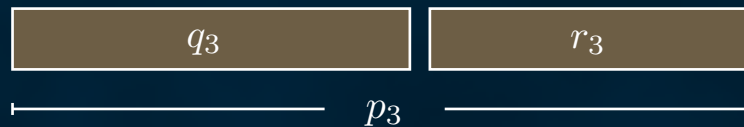
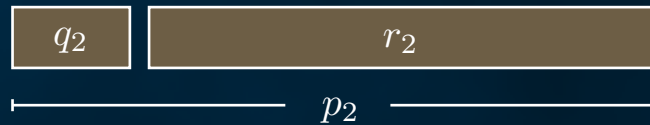
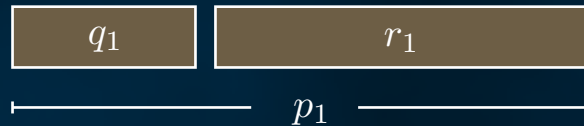
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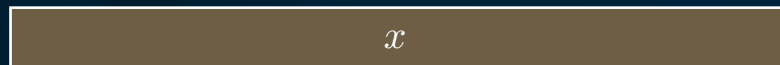
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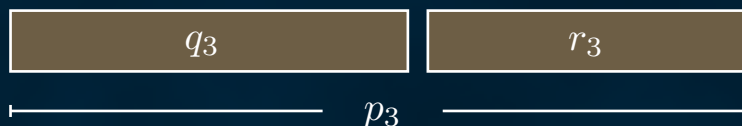
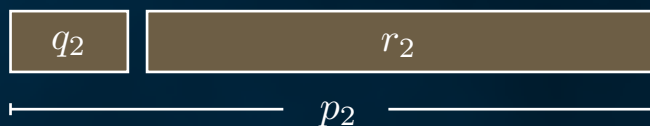
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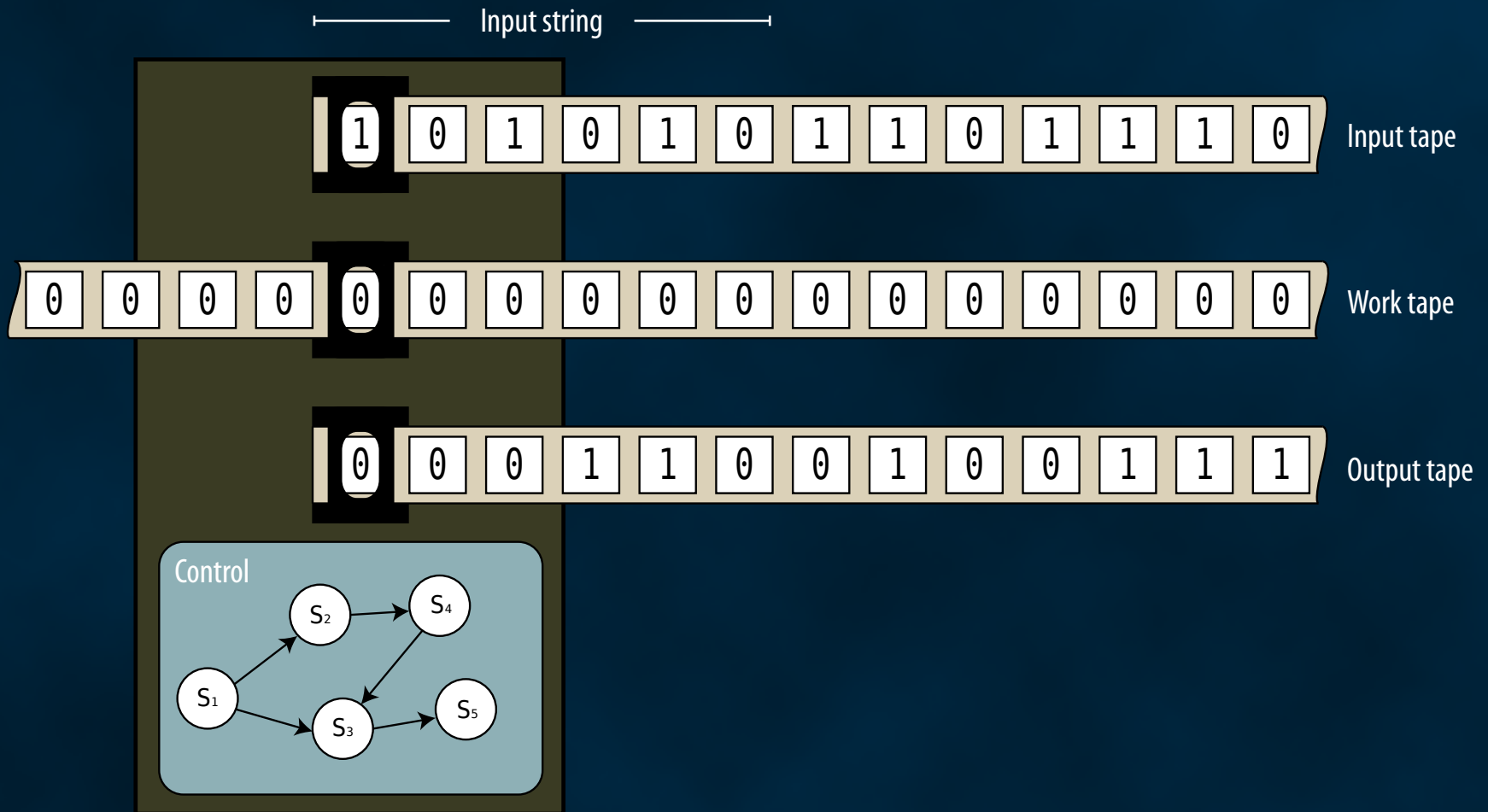


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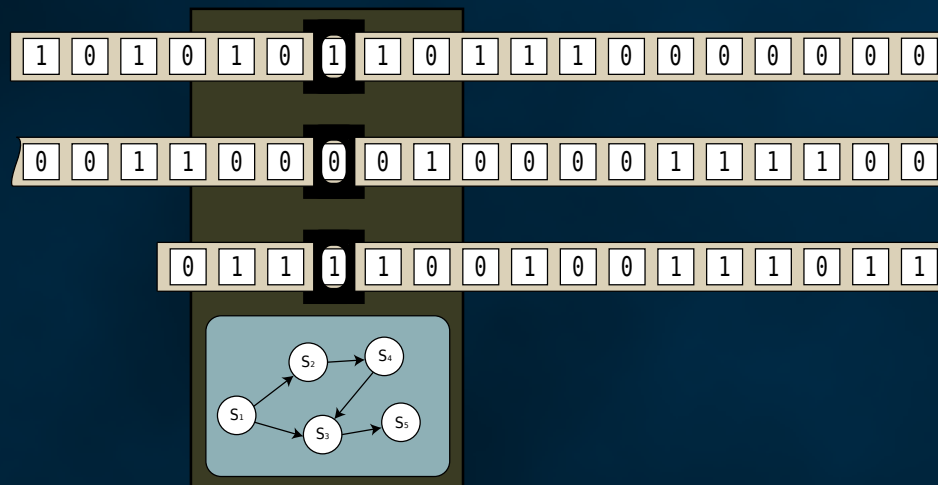


Pick the hypothesis that results in the minimum encoding length

$$l(p_i) = l(q_i) + l(r_i) = -\log_2(p_q(q_i)) - \log_2(p_x(x \mid q_i))$$

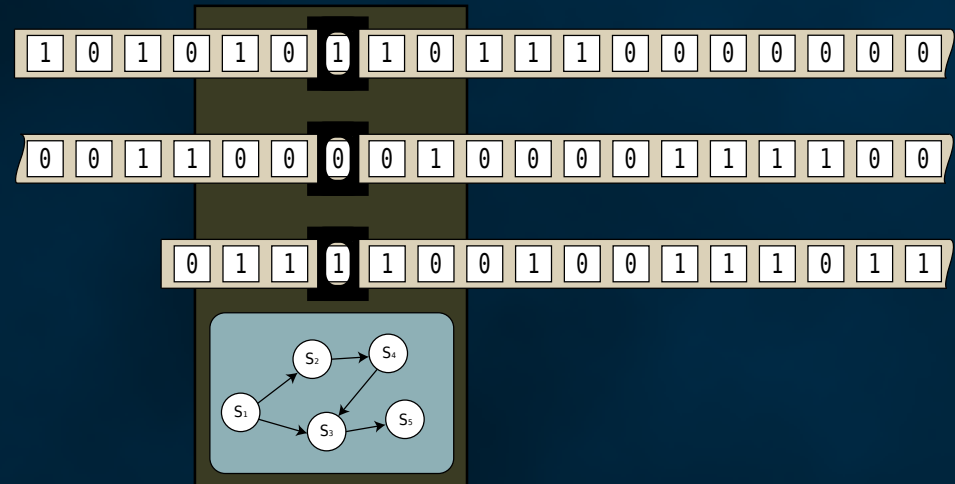


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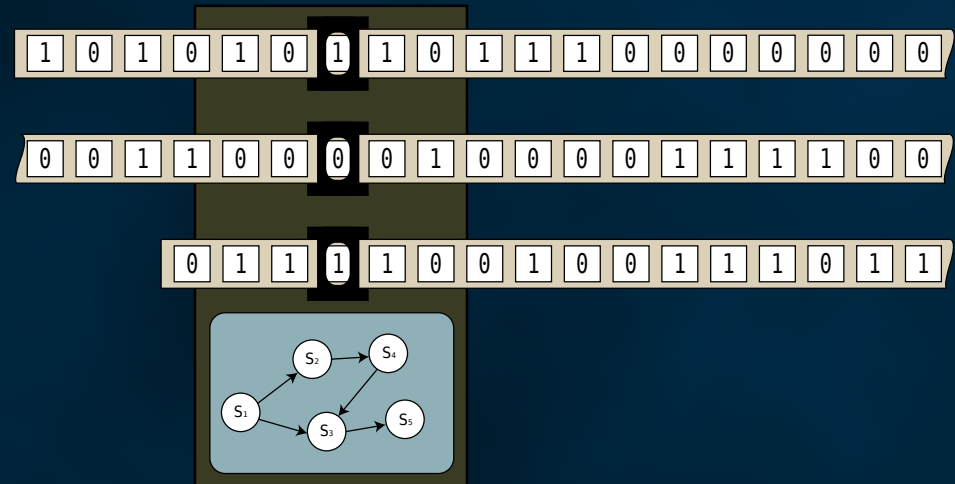
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- T reads all of the input p



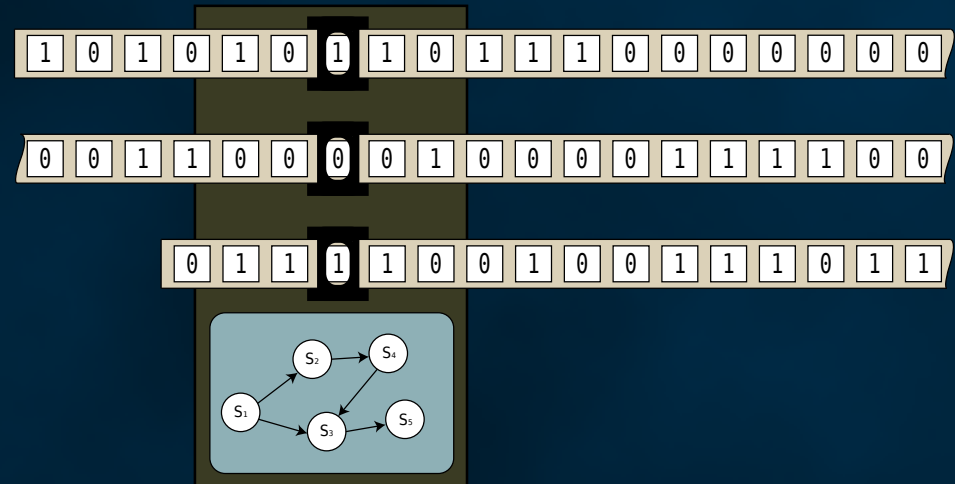
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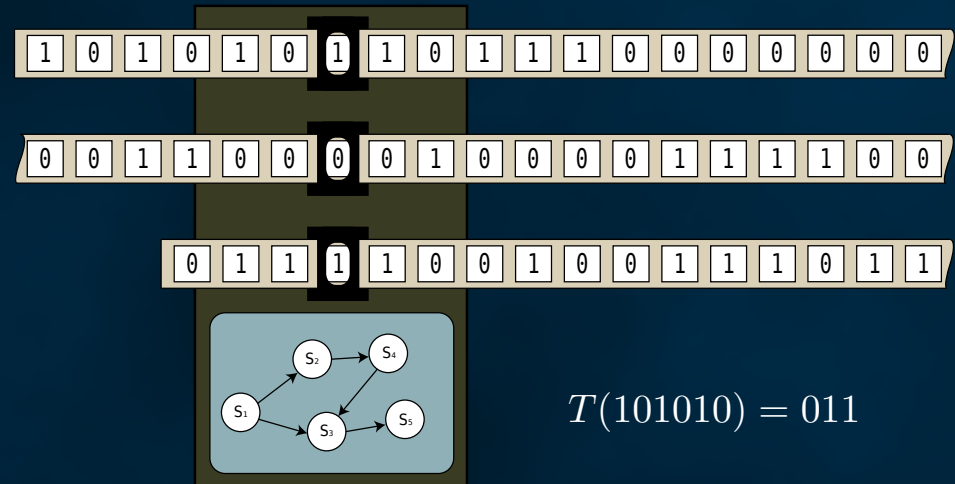
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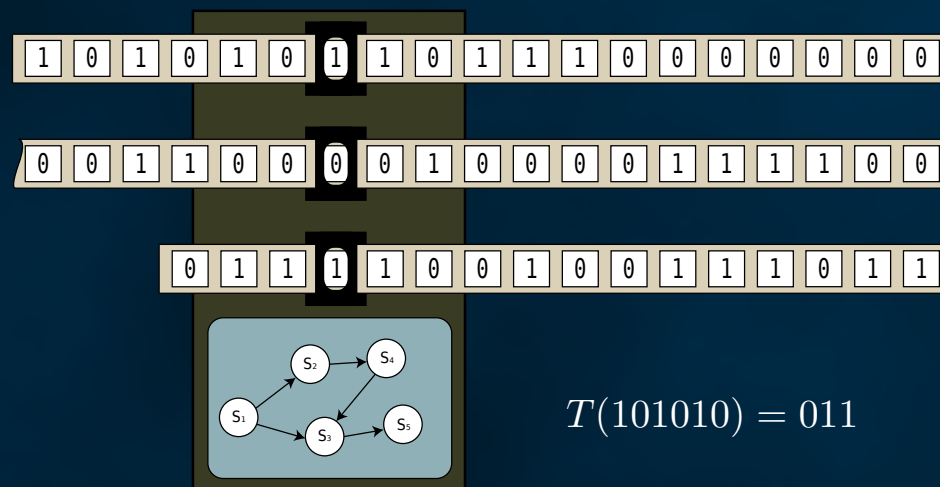
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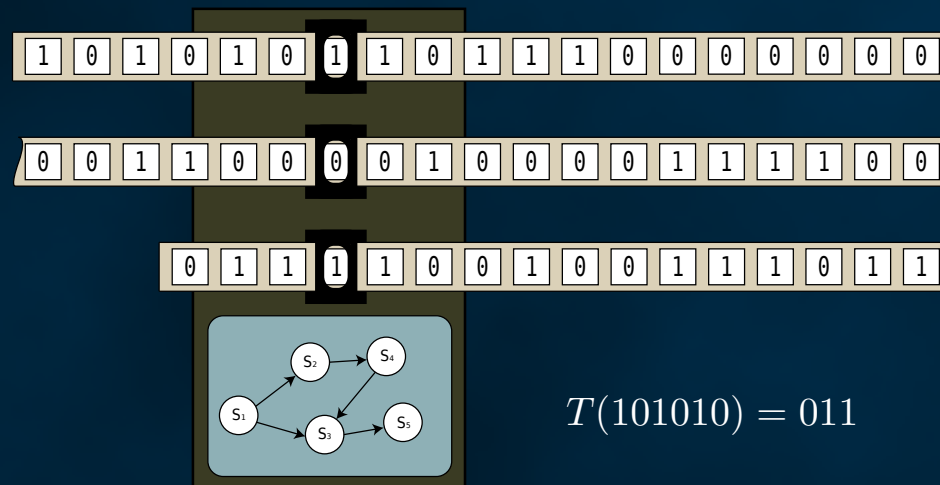


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T is a decoder of a prefix code (why prefix?)

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T is a decoder of a prefix code (why prefix?)

Universal (prefix) TM = TM that can emulate any other (prefix) TM, e.g. $T(\langle i, p \rangle) = T_i(p)$

(Prefix) Kolmogorov complexity of x = Length of the shortest input string required to output x

$$K_T(x) = \min\{l(p) \mid T(p) = x\}$$

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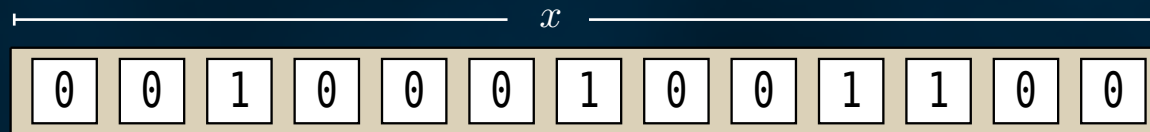
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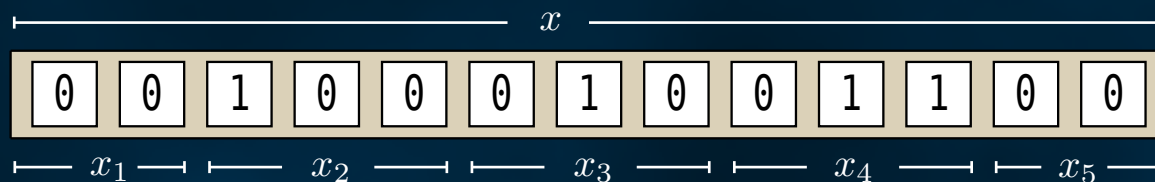
Why don't we normalize?

(Hereafter: “probability” \longleftrightarrow semimeasure)

Data string x is a representation of observational data from a real world phenomenon



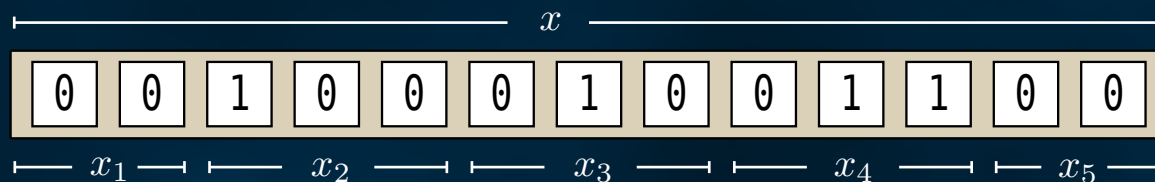
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$$L = \{00, 100, 010, 011\}$$

- “Sentences” $x_i \in L$, where L is a prefix-free set (data “language”)

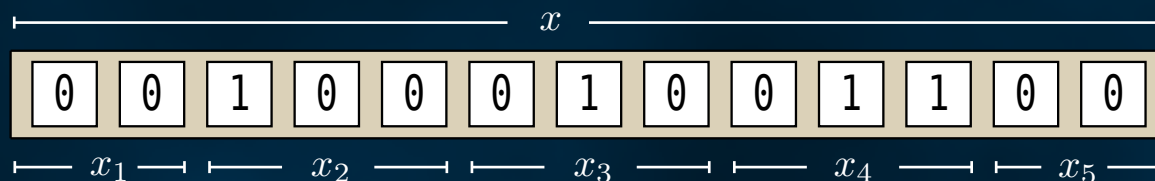
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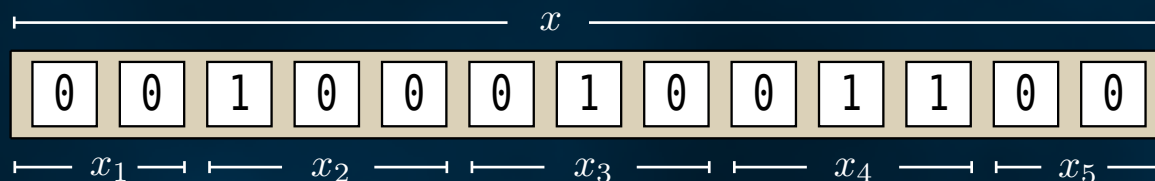
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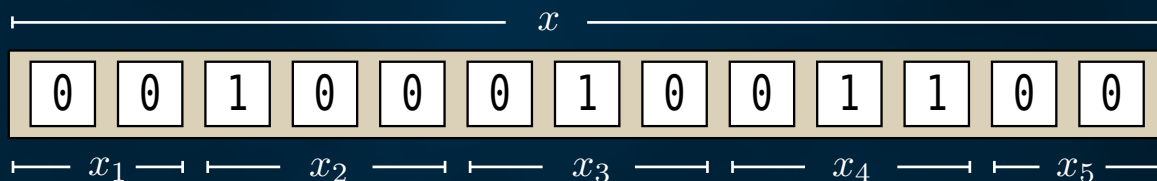
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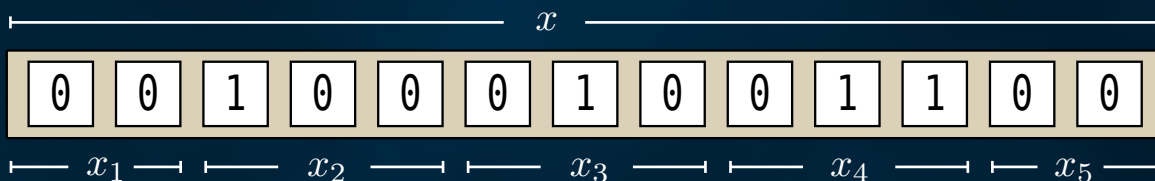


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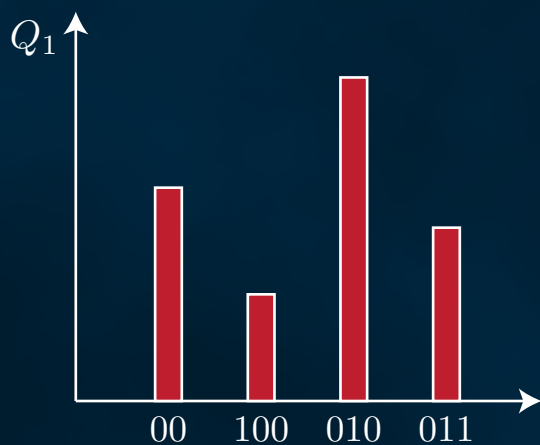
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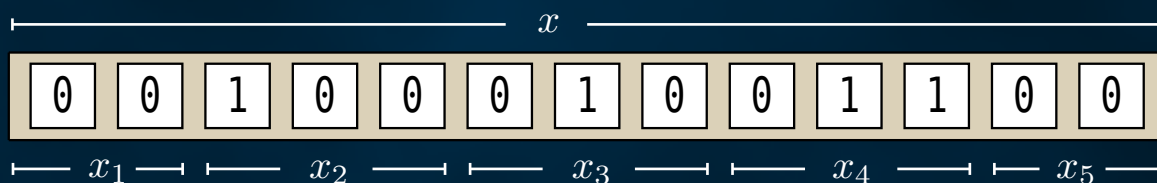
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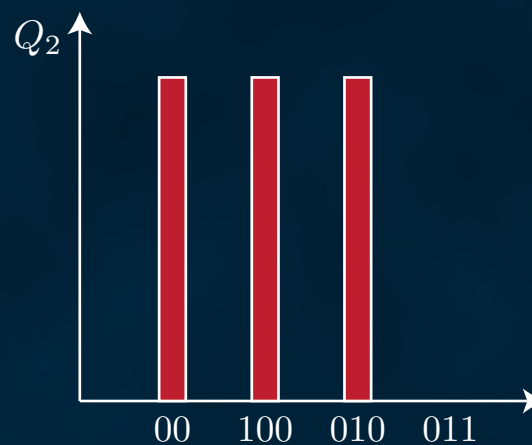
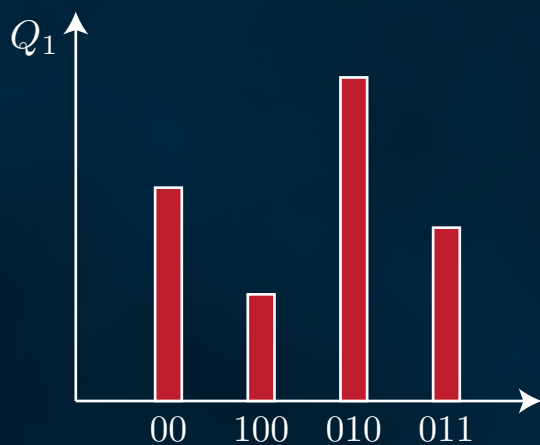
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Problem

Probability can never be 0, i.e. Popper-falsification not possible, because

$$K_T(x \mid y) < K_T(x) + O(1) \Rightarrow P_T(x \mid y) > P_T(x) + O(1)$$

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Probability can never be 0, i.e. Popper-falsification not possible, because

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- Have hypothesis be a prefix of input string p
- Force intended two-part encoding by imposing conditions on p

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TWO-PART ENCODING

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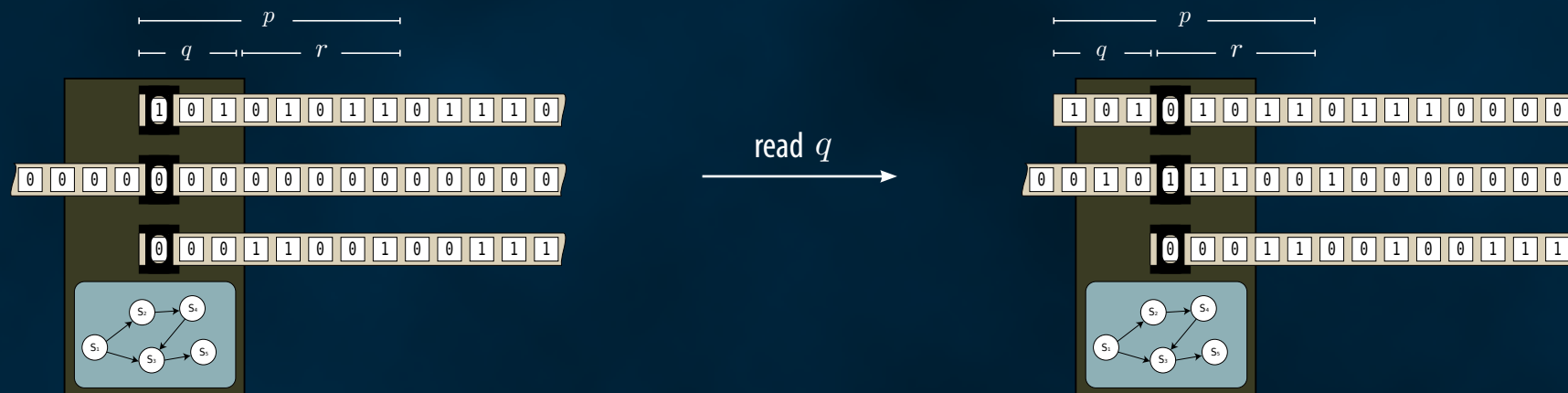
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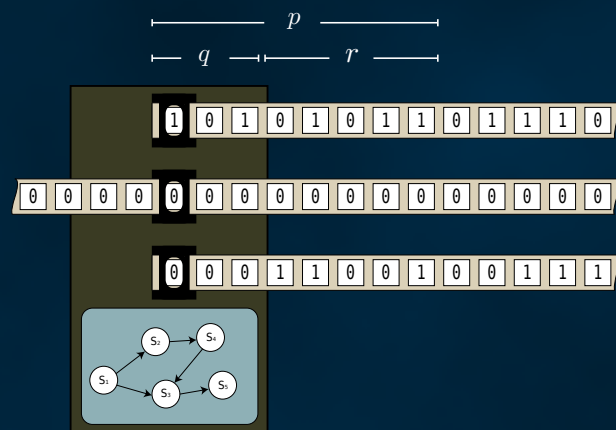
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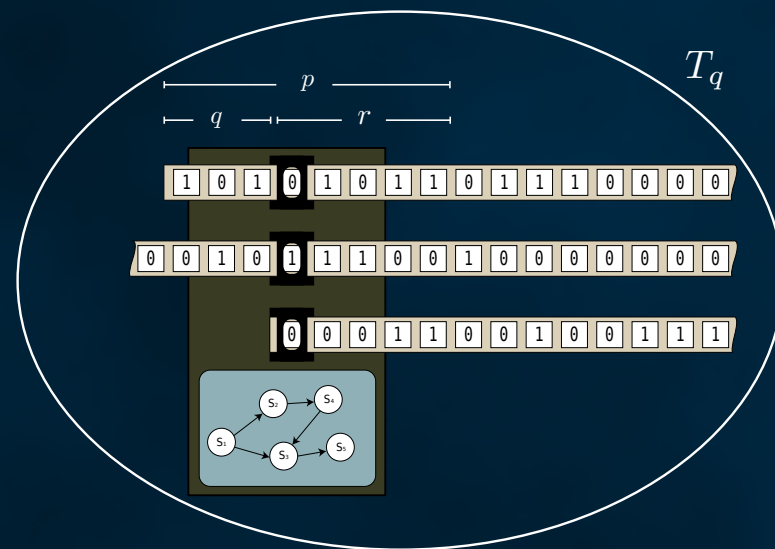
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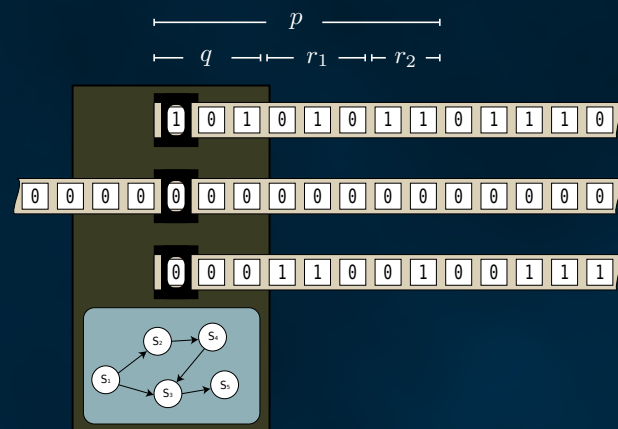
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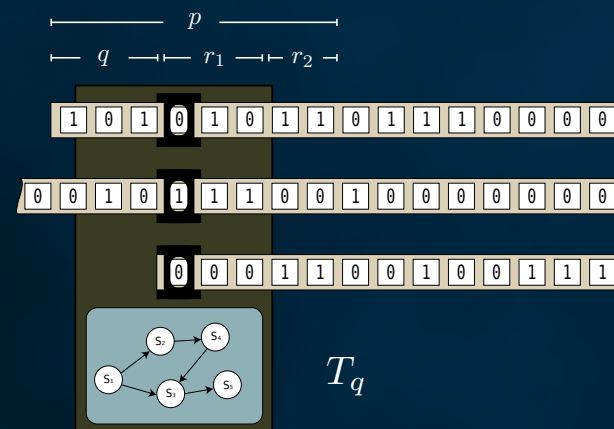
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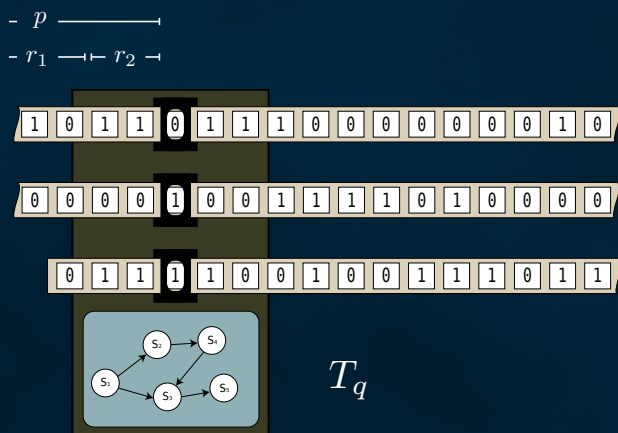
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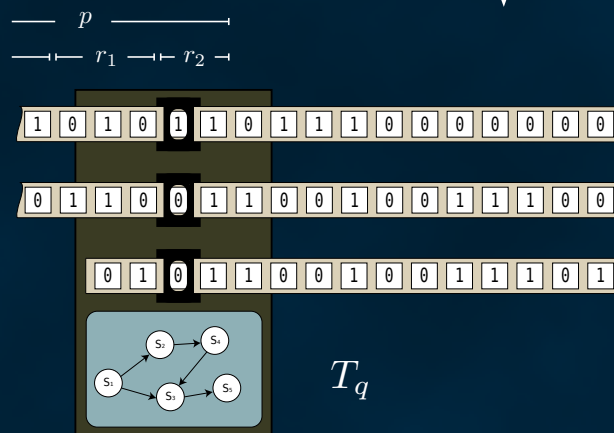
read q



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hypothesis q is "general"

Input p is an acceptable MML message encoding data string x , if

- 1) $T(p) = x$ p encodes x
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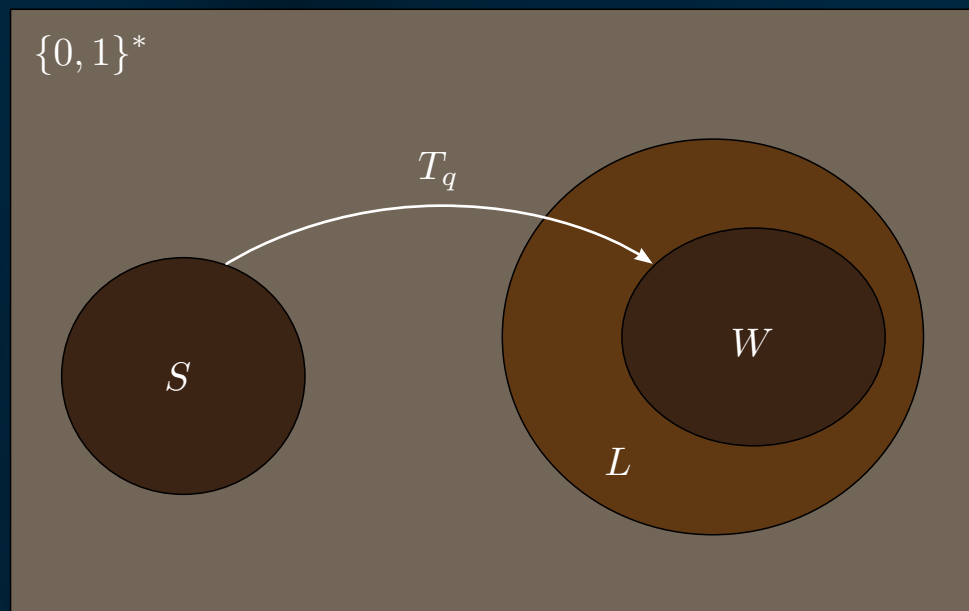
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Code words

$$S = \{r_i \in \{0, 1\}^* \mid T_q(r_i) \in L\}$$

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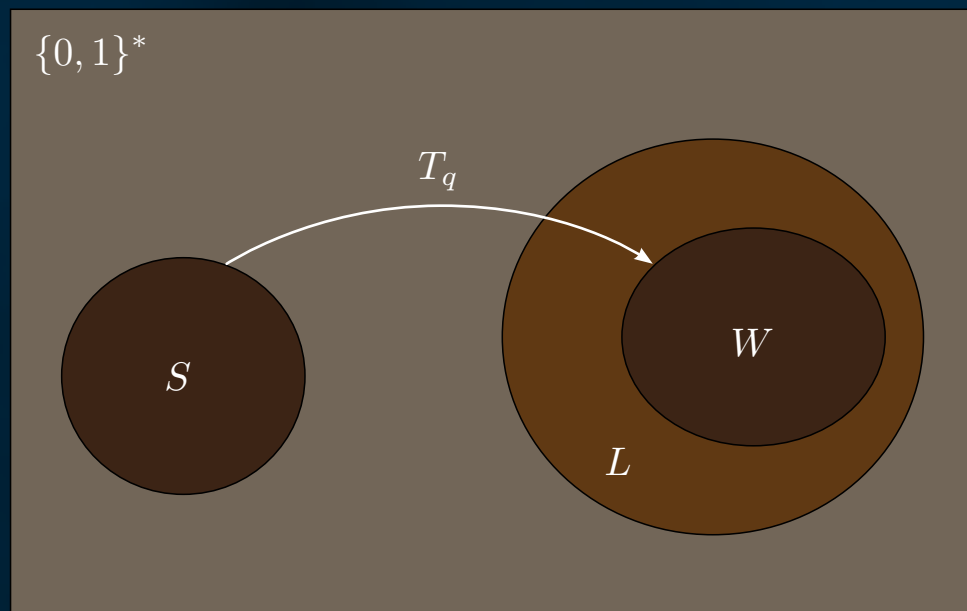
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In fact, T_q decodes a prefix code (why?)



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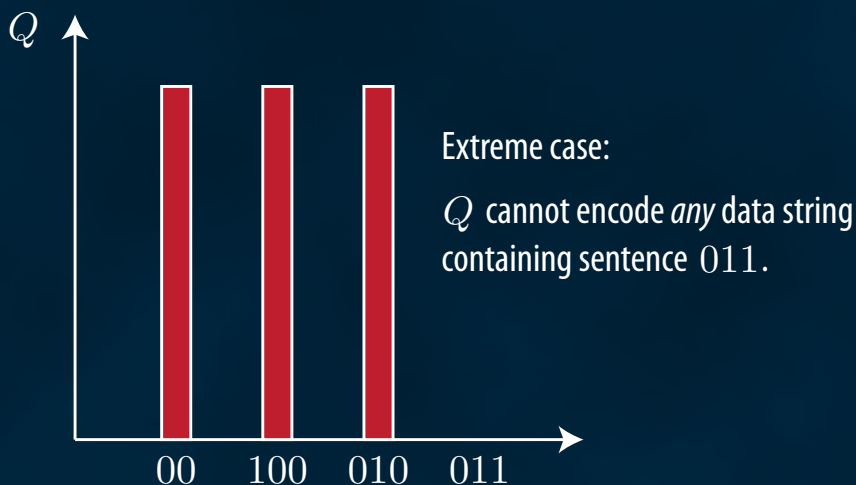
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The log posterior odds ratio of two hypotheses is

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where p_1 and p_2 are shortest input strings for their respective hypotheses

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[...] Bayes/Solomonoff is the Gold standard for prediction, but MML/MDL is (often) a good “approximation/simplification” for explanation and understanding.

M. Hutter

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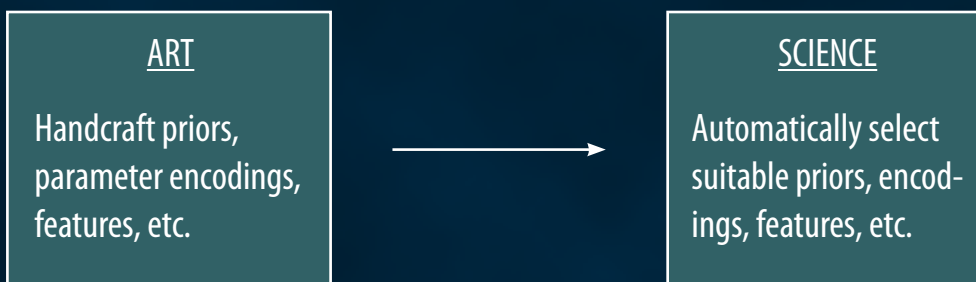
ART

Handcraft priors,
parameter encodings,
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