Minimum Message Length and Kolmogorov Complexity

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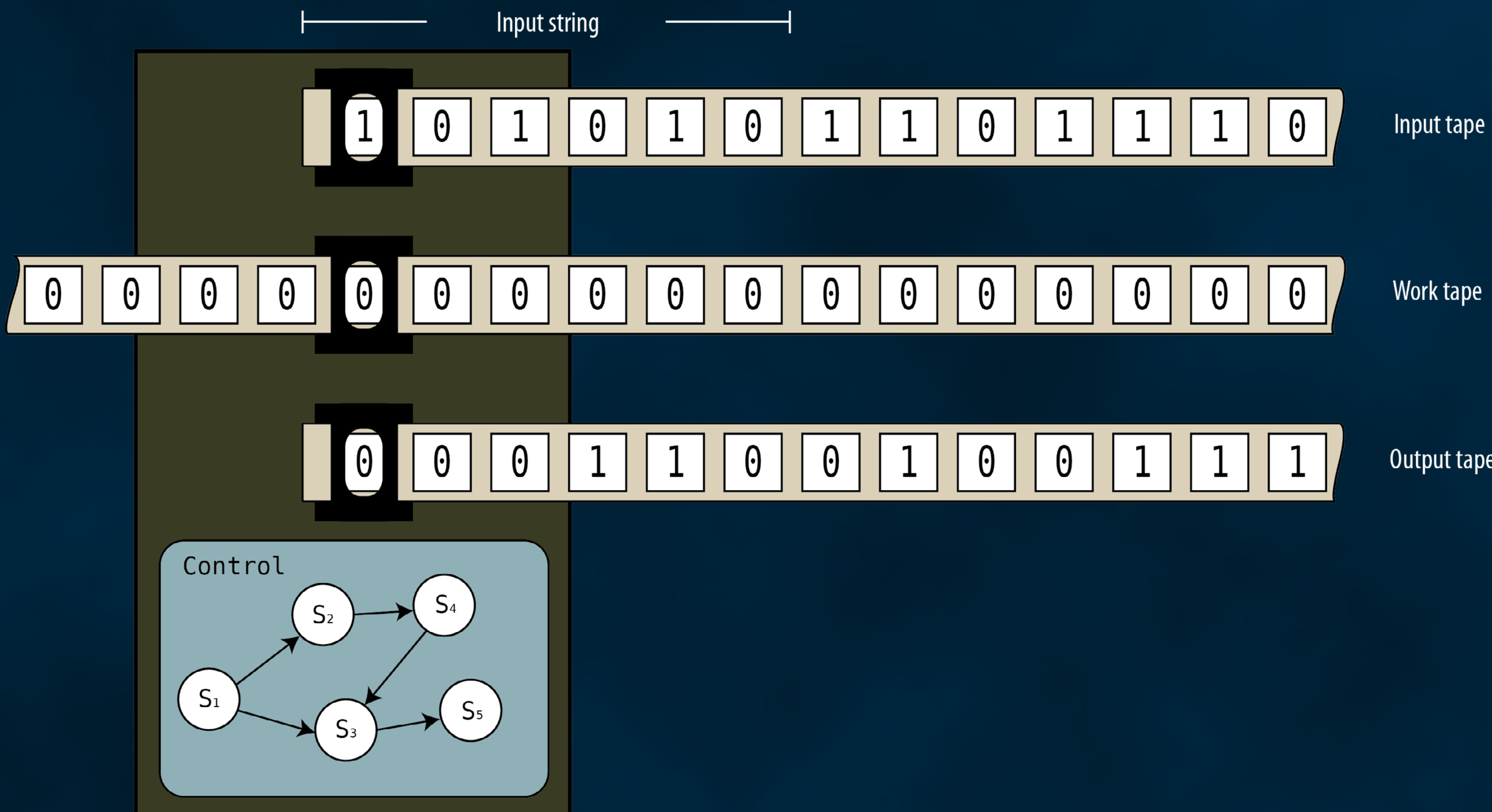
Presentation by Alkis Gkotovos

Overview

Turing Machines



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Proposal: Have hypothesis be a prefix of input string p.
Force intended two-part encoding by imposing conditions on p.

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7)	$x = x_1 \dots x_n \Rightarrow \begin{cases} r = r_1 \dots r_n \\ T_q(r_i) = x_i, \; i = 1 \dots n \end{cases}$	conditionally independent sentences
8)	$\begin{matrix} x' = x^{(1)}x^{(2)} \\ j' = j^{(1)}j^{(2)} \end{matrix} \Rightarrow \begin{matrix} T_q(j^{(1)}) = x^{(1)}, \; j^{(1)} < K_T(x^{(1)}) \\ T_q(j^{(2)}) = x^{(2)}, \; j^{(2)} < K_T(x^{(2)}) \end{matrix}$	hypothesis q is "general"

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9) No prefix of q satisfies all the above conditions — all of q is required