

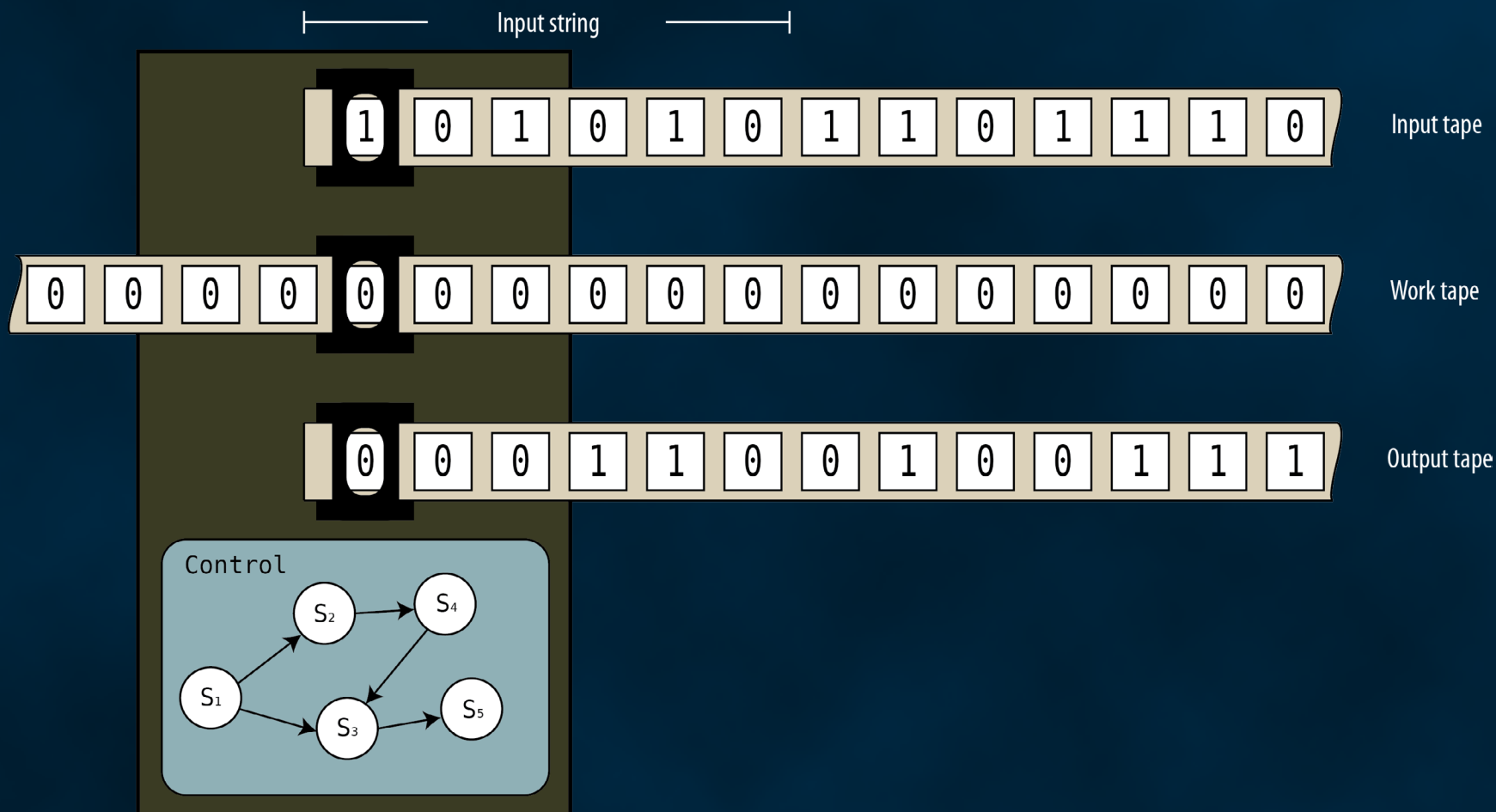
# Minimum Message Length and Kolmogorov Complexity

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# Overview

# Turing Machines





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$$9) \quad \text{No prefix of } q \text{ satisfies all the above conditions}$$

all of  $q$  is required