Minimum Message Length and Kolmogorov Complexity

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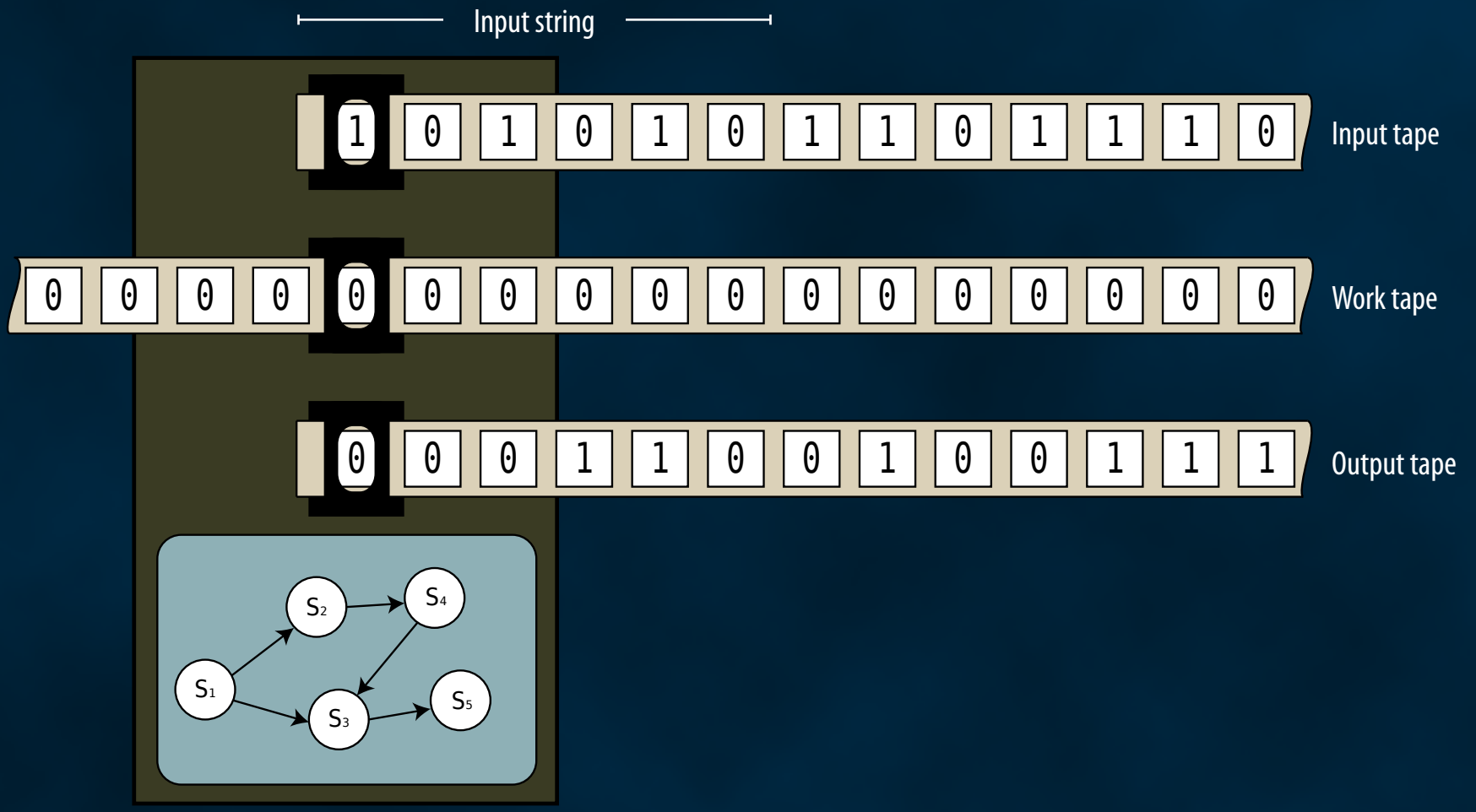
Presentation by Alkis Gkotovos

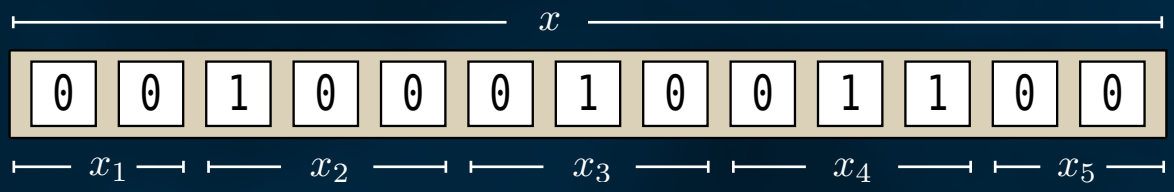
Overview

Turing Machines



Data & Hypotheses

Data string x is a representation of observational data from a real world phenomenon



$$L = \{00, 100, 010, 011\}$$

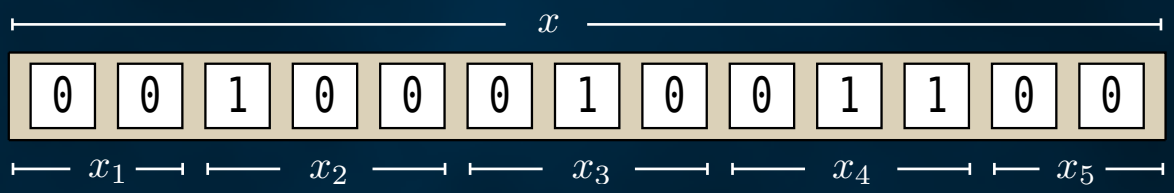
- "Sentences" $x_i \in L$, where L is a prefix-free set (data "language")
- Distinct sentences represent distinct real-world facts
- Sentences are conditionally independent given full knowledge of the phenomenon
- Strings are invariant to sentence permutation

Data & Hypotheses

Hypothesis Q is a (computable) probability distribution over L

Conditional independence of sentences implies

$$x = x_1 \dots x_n \Rightarrow Q(x) = Q(x_1) \times \dots \times Q(x_n)$$



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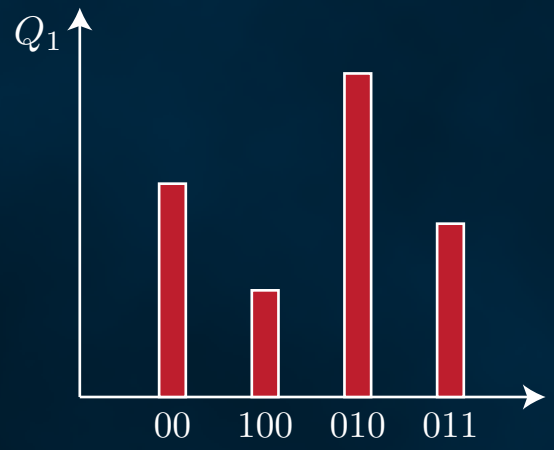

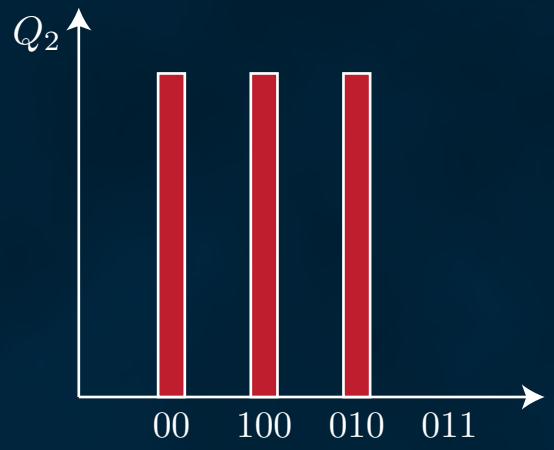

Two-part encoding

How do we acquire a hypothesis-based encoding of data in the Algorithmic Complexity framework?

Idea

- Use conditional Kolmogorov complexity

$$K_T(x \mid y) = \min\{l(p) \mid T(\langle y, p\rangle) = x\}$$

  and interpret y as hypothesis and x as data

- Corresponding conditional algorithmic probability

$$P_T(x \mid y) = 2^{-K_T(x|y)}$$

Problem

Probability can never be 0, i.e. Popper-falsification not possible, because

$$K(x \mid y) < K(x) + O(1) \Rightarrow P_K(x \mid y) > P_K(x) + O(1)$$

Why? Hypothesis y acts as "extra info", instead of assertively.

Proposal

- Have hypothesis be a prefix of input string p
- Force intended two-part encoding by imposing conditions on p

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9) No prefix of q satisfies all the above conditions — all of q is required