Minimum Message Length and Kolmogorov Complexity

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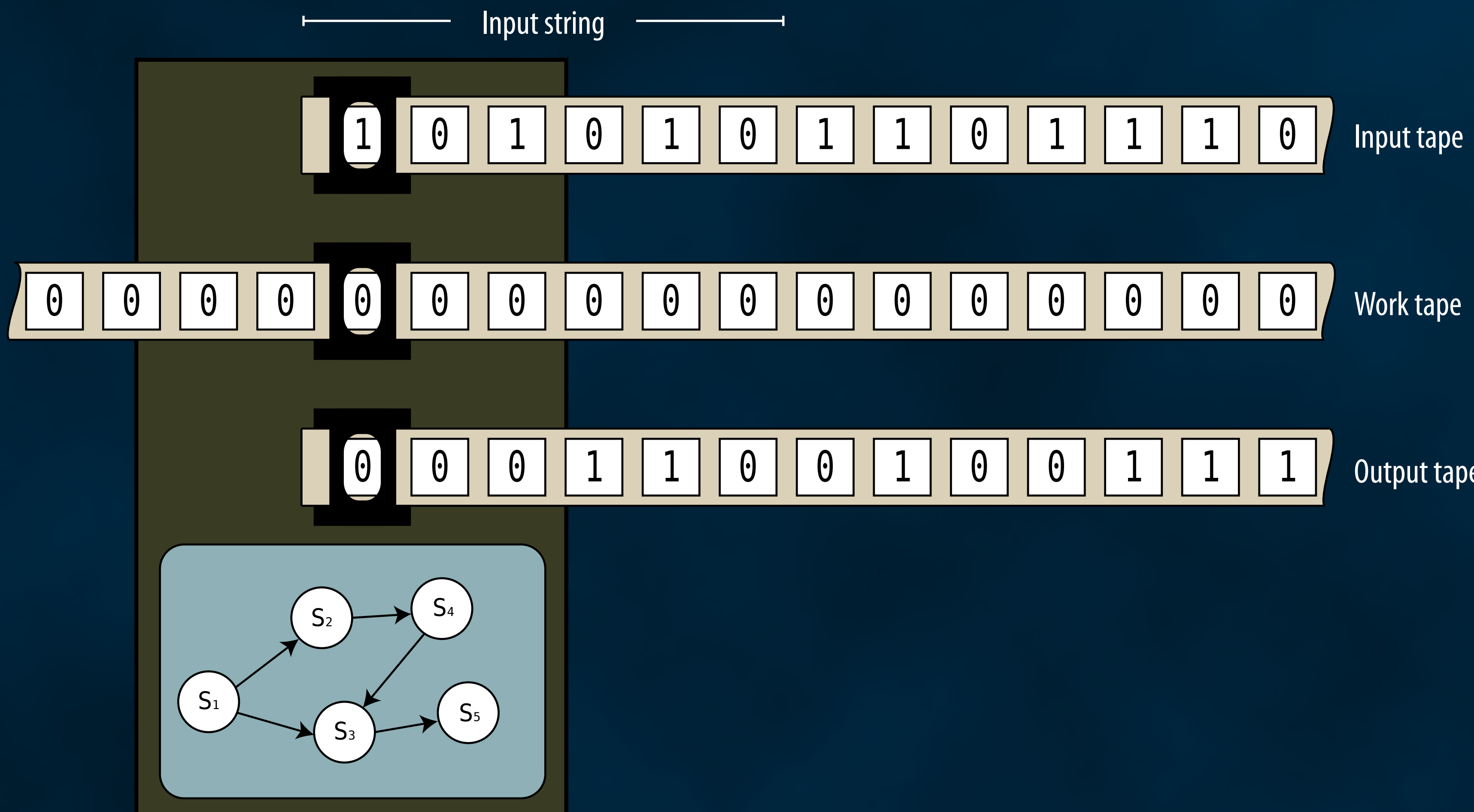
Presentation by Alkis Gkotovos

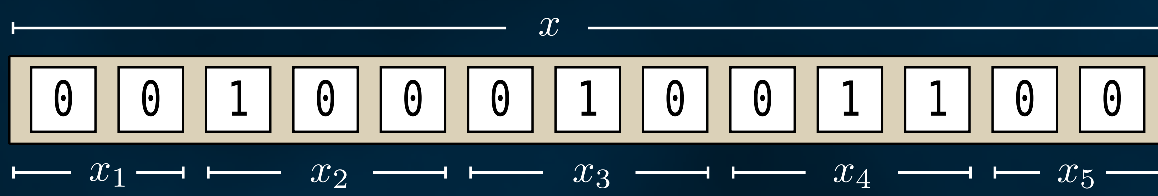
Overview

Turing Machines



Data & Hypotheses

Data string x is a representation of observational data from a real world phenomenon



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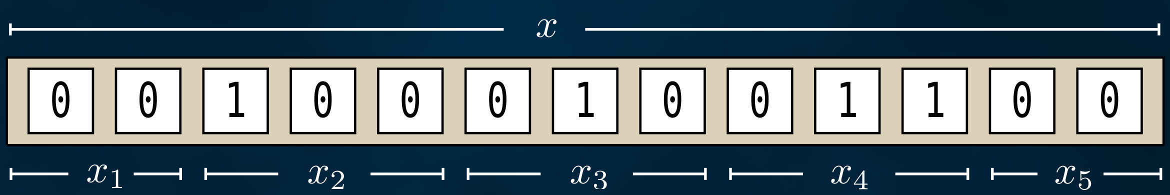
- “Sentences” $x_i \in L$, where L is a prefix-free set (data “language”)
- Distinct sentences represent distinct real-world facts
- Sentences are conditionally independent given full knowledge of the phenomenon
- Strings are invariant to sentence permutation

Hypothesis Q is a (computable) probability distribution over L

Conditional independence of sentences implies

$$x = x_1 \dots x_n \Rightarrow Q(x) = Q(x_1) \times \dots \times Q(x_n)$$



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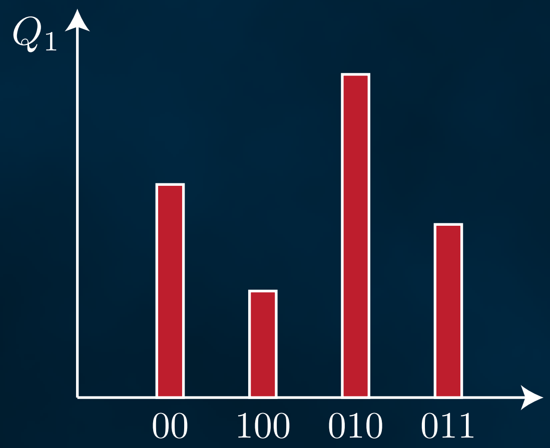

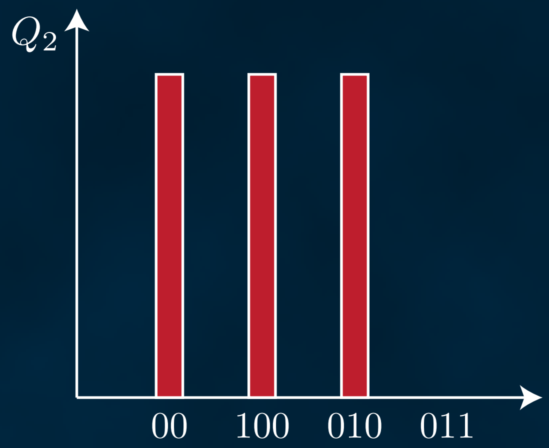

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and interpret y as hypothesis and x as data.

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Proposal: Have hypothesis be a prefix of input string p.
Force intended two-part encoding by imposing conditions on p.

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9)	No prefix of q satisfies all the above conditions	all of q is required