COLLEGE OF ... MONTHLY EXAM

Academic year: 2021/2022
Day and Date: Thurs 24/2/2022
Examiner: Somayah Saeed Binghouth
Time allowed: 1.30

Exam Semester: First
Level: First
Department: IT
Subject: Differential Calculus

Q1: Choose the correct answer: 10 marks

(a) $\lim_{x\to 9} \frac{x-9}{\sqrt{x}-3}$ = (0 , 6 , ∞ , -∞).

(b) If $f(x) = \cot\frac{1}{x}$ Then $f'(x) =$
($-\frac{1}{x^2}\csc^2\frac{1}{x}$, $\frac{1}{x^2}\csc^2\frac{1}{x}$, $\frac{1}{x}\sec^2 x$).

(c) The domain of $f = \sqrt{2x-6}$ is ([3,∞[,]3, ∞[, [6,∞[, [-3, 3]).

(d) $\lim_{x\to 4^-} \frac{1}{(x-4)^3}$ = (4 , -∞ , ∞ , 0).

(e) Which of these functions is continuous at R :

($f(x) = \frac{1}{x}$, $f(x) = \frac{1}{\sqrt{x}}$, $f(x) = \sqrt{x}$, $f(x) = x$).

Q2: Find the Limits: (8 marks)

(a) $\lim_{x\to 1} f(x)$, $f(x) = \begin{cases} 3-x & x<1 \\ 4 & x=1 \\ x^2+1 & x>1 \end{cases}$

(b) $\lim_{x\to -\infty} \frac{2x^2-3}{4x^3+5x}$

Q3: Answer following questions: (10 marks)

(a) Find the numbers at which f is discontinuous: $f(x) = \frac{5}{x^3+2x^2+x}$.

(b) By using sandwich theorem , prove that : $\lim_{x\to 0} x^2 \sin\frac{1}{x^2} = 0$
(Hint: $-1 \le \sin t \le 1$)

Q4: Find $f'(x)$: (7 marks)

(a) $f(x) = x^2 + \frac{1}{x^2}$

(b) $f(x) = \log_5(2x^2 + x)$.

35
35 Excellent

-قسم: تقنية معلومات (IT)

Q1: Choose the correct answer: 10

(a) $= 6$ (b) $= \frac{1}{x^2} \csc^2 \frac{1}{x}$ (c) $= [3, \infty[$

(d) $= -\infty$ (e) $= f(x) = x$

Q2: Find the Limits: 8

(a) نوجد النهاية من اليمين واليسار

① $\lim_{x \to 1^+} (x^2+1) = (1)^2 + 1 = 1+1 = 2$

② $\lim_{x \to 1^-} (3-x) = 3-1 = 2$

$\therefore \lim_{x \to 1^+} (x^2+1) = \lim_{x \to 1^-} (3-x) = 2$

$\therefore \lim_{x \to 1} f(x) = 2$ (exist)

4

(b) $\lim_{x \to -\infty} \frac{\frac{2x^2}{x^3} - \frac{3}{x^3}}{\frac{4x^3}{x^3} + \frac{5x}{x^3}} = \lim_{x \to -\infty} \frac{\frac{2}{x} - \frac{3}{x^3}}{4 + \frac{5}{x^2}} = \frac{\frac{2}{-\infty} - \frac{3}{-\infty}}{4 + \frac{5}{\infty}}$

$= \frac{0-0}{4-0} = \frac{0}{4} = 0$

4

10

Q3: Answer following questions:

(a) f is a rational function كسرية
f is continuous at every real numbers exept {when denominator = 0}

$x^3 + 2x^2 + x = 0 \Rightarrow x(x^2+2x+1) = 0 \Rightarrow x(x+1)^2 = 0$
$\Rightarrow x = 0$
$(x+1)^2 = 0$ / $\sqrt{\ }$ 5
$x+1 = 0$ / -1
$\Rightarrow x = -1$

$\therefore$ f is not continuous at $\{0, -1\}$

(b) $-1 \leq \sin t \leq 1 \implies -1 \leq \sin \frac{1}{x^2} \leq 1 \quad / * x^2$

$\implies -x^2 \leq x^2 \sin \frac{1}{x^2} \leq x^2$

5

$\lim_{x \to 0} (-x^2) = \underline{0}$, $\lim_{x \to 0} (x^2) = \underline{0}$

by sandwich theorem $\lim_{x \to 0} x^2 \sin \frac{1}{x^2} = \underline{0}$

Q4: find $f'(x)$: $\boxed{7}$

(a) $f'(x) = 2x + \left(\frac{-2\cancel{x}}{x^{\cancel{4}}}\right) = 2x - \frac{2}{x^3}$ 3½

(b) $f'(x) = \frac{1}{(2x^2+x) \ln 5} * (4x+1) = \frac{4x+1}{(2x^2+x) \ln 5}$

3½

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