Exercises 4

1) Decided whether the following values are Eigenvalues

a) $-2\,,0\,,3\,,4\ for\ \begin{bmatrix}2 & -1\\ -4 & 2\end{bmatrix}$ $\quad b)-1\,,1\,,8\,,10\quad \begin{bmatrix}5 & 4 & 2\\ 4 & 5 & 2\\ 2 & 2 & 2\end{bmatrix}$

2) Decided whether the following values are Eigenvectors

$$a)\ \begin{bmatrix}2\\ -5\end{bmatrix},\begin{bmatrix}1\\ 0\end{bmatrix}\ for\ \begin{bmatrix}2 & 2\\ 5 & -1\end{bmatrix}\qquad b)\ \begin{bmatrix}0\\ 6\\ 3\end{bmatrix},\begin{bmatrix}1\\ 2\\ 0\end{bmatrix},\begin{bmatrix}0\\ 2\\ 1/3\end{bmatrix}\ \ for\ \begin{bmatrix}4 & -1 & 6\\ 2 & 1 & 6\\ 2 & -1 & 8\end{bmatrix}$$

3) Find the Eigenvalue and Eigenvector of the matrices

$$a)\ A=\begin{bmatrix}10 & -18\\ 6 & -11\end{bmatrix}\quad Ans:\ \lambda=-2\,,1\quad X=\begin{bmatrix}3\\ 2\end{bmatrix}X=\begin{bmatrix}4\\ 2\end{bmatrix}$$

$$b)\ B=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & -3 & 3\end{bmatrix}\quad Ans:\ \lambda=1\quad X=\begin{bmatrix}2\\ 2\\ 2\end{bmatrix}$$

$$c)\ D=\begin{bmatrix}2 & 0 & 1 & -3\\ 0 & 2 & 10 & 4\\ 0 & 0 & 2 & 0\\ 0 & 0 & 0 & 3\end{bmatrix}\quad Ans:\ \lambda=\ 2\,,3\quad X=\begin{bmatrix}1\\ 0\\ 0\\ 0\end{bmatrix}\ ,\ X=\begin{bmatrix}0\\ 3\\ 0\\ 0\end{bmatrix}\ ,\qquad X=\begin{bmatrix}-3\\ 4\\ 0\\ 1\end{bmatrix}$$

$$d)\ C=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}\quad Ans\ :\ \lambda=8\,,2\ \ X=\begin{bmatrix}2\\ -1\\ 1\end{bmatrix}\ ,\qquad X=\begin{bmatrix}0\\ 3\\ 3\end{bmatrix},X=\begin{bmatrix}5\\ 5\\ -5\end{bmatrix}$$

$$e)\ A=\begin{bmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{bmatrix}\ Ans:\ \lambda=-2\,,3\,,6\quad X=\begin{bmatrix}-2\\ 0\\ 2\end{bmatrix}\ ,X=\begin{bmatrix}-1\\ 1\\ -1\end{bmatrix},X=\begin{bmatrix}3\\ 6\\ 3\end{bmatrix}$$

4) Find the Eigenvalues of $A^{-1}\ and\ A^{T}$ if

$$a)\ A=\begin{bmatrix}3 & 0 & 0\\ 7 & \frac{5}{4} & 0\\ -5 & 1 & 2\end{bmatrix}\qquad Ans:\ \text{Eigenvalues of } A^{-1}\ are\ \ 3\,,\frac{5}{4}\,,2\ \text{Eigenvalues of } A^{T}\ are\ \frac{1}{3}\,,\frac{4}{5}\,,\frac{1}{2}$$

$$b)\ A=\begin{bmatrix}2 & -2 & 2\\ 1 & 1 & 1\\ 1 & 3 & -1\end{bmatrix}\qquad Ans:\ \text{Eigenvalues of } A^{-1}\ are\ \ 2\ \,,-2\ \text{Eigenvalues of } A^{T}\ are\ \tfrac{1}{2}\ \,,-\tfrac{1}{2}$$

5) Find the sum and product of the Eigenvalues of the following matrices

$a)\ A=\begin{bmatrix}-6 & 0 & 0\\ 2 & 2 & 0\\ 4 & 5 & 1\end{bmatrix}$ $Ans:$ The sum of the Eigenvalues -3. The product of the Eigenvalues -12

$b)\ A=\begin{bmatrix}3 & 1 & 2\\ 1 & 7 & 4\\ 0 & -2 & -3\end{bmatrix}$ $Ans:$ The sum of the Eigenvalues 1. The product of the Eigenvalues -40

6) Show that the matrix $A=\begin{bmatrix}1 & -1 & 4\\ 3 & 2 & -1\\ 2 & 1 & -1\end{bmatrix}$ satisfies its own characteristic equation

7) Find A^{-1} and A^{5}, by using **Cayley − Hamilton Theorem**, where

$$a)\ A=\begin{bmatrix}3 & -2\\ 1 & 2\end{bmatrix}\ \ Ans{:}\ A^{-1}=-\frac{1}{8}\begin{bmatrix}-2 & -2\\ 1 & -3\end{bmatrix}\qquad b)\ A=\begin{bmatrix}1 & 0 & 1\\ -1 & 1 & -3\\ 2 & 2 & 4\end{bmatrix}\quad Ans{:}\ A^{-1}=\begin{bmatrix}\frac{5}{3} & \frac{1}{3} & -\frac{1}{6}\\ -\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ -\frac{2}{3} & -\frac{1}{3} & \frac{1}{6}\end{bmatrix}$$