

Push Down Automata (PDA)

pushdown Automata (PDA) is a way to implement a Context Free Grammar in similar way we Design Finite Automata For Regular Grammar.

- FSM has very limited memory but PDA has more memory.
- PDA = FSM + stack.

we have Discussed 2 type of grammar.

grammar type	Language accepted	Automata machine
Regular grammar	Regular Language	Finite state Automata.
Context Free grammar	Context Free grammar	pushdown Automata.

Stack is a way we arrange element one on top of another
FILO.

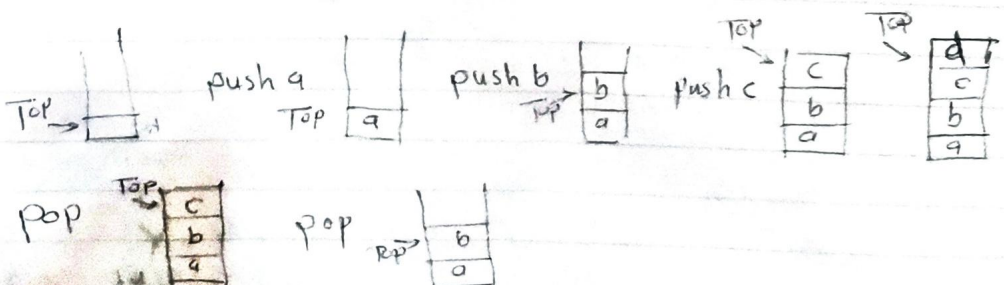
A stack has two main operation.

push: a new element is added at Top of stack

pop: The TOP element of stack is read and Removed from stack.

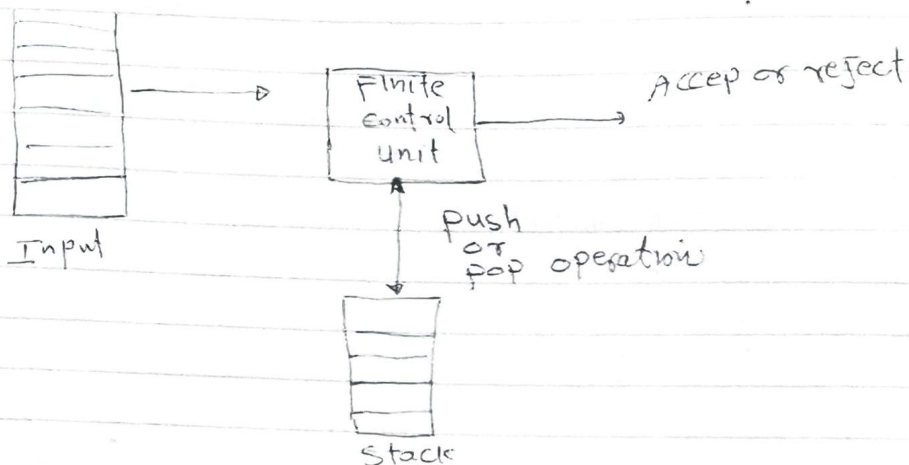
example

Insert a, b, c and d on stack



pushdown Automata has 3 component.

- 1) an input string/Tape
- 2) Finite control unit
- 3) stack with infinite size.



PDA Formal definition

PDA can be defined by 7 tuples as shown below

$$P = \{ Q, \Sigma, \Gamma, \delta, q_0, Z, F \}$$

- Q : A Finite set of states
- Σ : Finite set of input symbols
- Γ : Finite stack
- δ : Transition Function.
- q_0 : start state
- Z : start stack symbol
- F : set of final states

The Transition function δ takes Triple $\delta(q, a, x)$ where

- q : is a state in Q
- a : is input symbol in Σ or ϵ
- x : is stack symbol, That is member of Γ

The output of δ is Finite set of pairs (p, Y) where

- p : is new state
- Y : string of stack that replace x at Top of Stack.

example :

If $Y \neq \epsilon$ Then stack is popped

If $Y = x$ Then stack is unchanged

If $Y = Y^2$ Then x is replaced by 2 and Y is pushed on stack.

sample: construct PDA that accepts even palindromes of form.

$$L = \{ ww^R \mid w = (a+b)^* \}$$

palindromes: A word or sequence that read the same backwards as forwards.

example: NOON

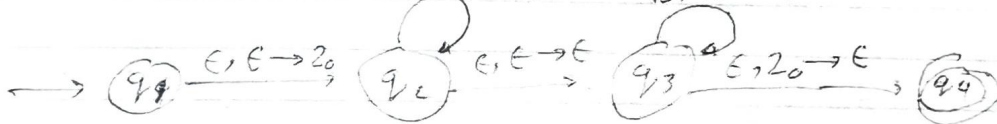
123321

abba

Race Car

$q_1, \epsilon \rightarrow q_2$
 $b, \epsilon \rightarrow b$

$q_1, a \rightarrow \epsilon$
 $b, b \rightarrow \epsilon$



ab|ba ✓

b
a
z0

X abab

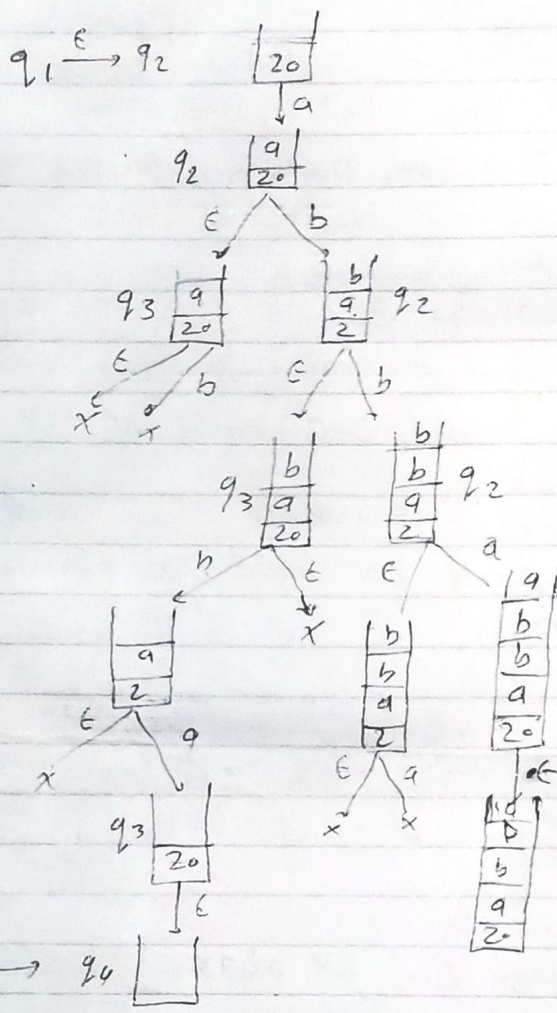
b
a
z0

How do we know that we reach the middle of string.

Example: $abb a$

→ After each alphabet we add ϵ so we get:

$\epsilon a \epsilon b \epsilon b \epsilon a \epsilon$



Final
stable

example abab

5aebbfaebbe

