

نماذج الاختبارات الشهرية

تجميع :

نور الجفري & فاطمة عاشور

Academic year: 2020/2021
 Day and Date: Sunday 13/6/2021
 Examiner: Mr. Awad Bin Jobah
 Time Allowed: 1:15 Hour

Test: mid exam
 Level: First
 Department: IT
 Subject: Integral Calculus

Question 1: (5+5=10 Marks)

(a) Use finite approximation to estimate the area under the graph of the function $f(x) = 4 - x^2$ between $x = 2$ and $x = -2$ using an upper sum with four rectangles of equal width

(b) Find the derivative of the following function $y = \int_{\sin x}^3 \left(\frac{1}{1+t^2}\right) dt$

Question 2: (5+5=10 Marks)

(a) Suppose that $\sum_{k=1}^{10} a_k = 15$ and $\sum_{k=1}^{10} b_k = 10$ Find $\sum_{k=1}^{10} (2a_k - 3b_k + 7)$

(b) Write $(1 \cdot 2) + (3 \cdot 4) + (5 \cdot 6) + (7 \cdot 8)$ as sigma notation

$$\sum_{k=1}^4 (2k-1) \cdot 2k$$

Question 3 (2+2+2=6 Marks)

Evaluate the following integrals

(a) $\int \sqrt{1 - \sin x} dx$ (b) $\int \frac{2 \sin^2 x + \cos 2x}{\cos^2 x} dx$ (c) $\int_0^3 \sqrt{6x - x^2} dx$ (by using the graph)

Question 4: (4 Marks)

Use the definition of definite integral to evaluate the following integral $\int_0^2 (2x^2 + 1) dx$

End of Question

End of Page



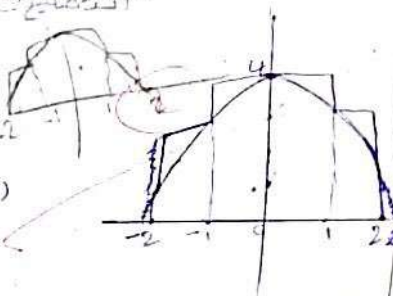
Academic year: 2019/2020
Day and Date: Sunday, 16/2/2020
Examiner: Mr. Awad Bin Jobah
Time Allowed: 1 Hour

Test: First
Level: First
Department: IT
Subject: Integral Calculus

Question 1: (5 Marks)

Use finite approximation to estimate the area under the graph of the function $f(x) = 4 - x^2$ between $x = 2$ and $x = -2$ using an upper sum with four rectangles of equal width

Solution:-
 $f(-2) = 4 - (-2)^2 = 0$
 $f(-1) = 4 - (-1)^2 = 3$
 $f(0) = 4 - 0^2 = 4$
 $f(1) = 4 - 1^2 = 3$
 $f(2) = 4 - 2^2 = 0$
 The upper sum is $A = (1 \times 3) + (1 \times 4) + (1 \times 3) + (1 \times 0) = 3 + 4 + 3 + 0 = 10$



Question 2: (3+2=5 Marks)

(a) Suppose that $\sum_{k=1}^{10} a_k = 5$ and $\sum_{k=1}^{10} b_k = 10$ Find $\sum_{k=1}^{10} (2a_k - 3b_k + 4)$

(b) Write $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots - \frac{1}{50}$ as sigma notation Solution:-
 $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots - \frac{1}{50} = \sum_{k=1}^{50} (-1)^{k+1} \cdot \frac{1}{k}$

Question 3: (2+2+2=6 Marks)

Evaluate the following integrals

(a) $\int (3x^2 - \sqrt{x} + 7) dx$
 $= \int (3x^2 - x^{\frac{1}{2}} + 7) dx$
 $= \frac{3x^3}{3} - \frac{2x^{\frac{3}{2}}}{\frac{3}{2}} + 7x + C$
 $= x^3 - \frac{2}{3}x^{\frac{3}{2}} + 7x + C$

(b) $\int (\sin 5x + \sec^2 3x) dx$
 $= -\frac{\cos 5x}{5} + \frac{\tan 3x}{3} + C$

(c) $\int (x+1)(x-1) dx$
 Solution:- $\int (x^2 - 1) dx$
 $= \frac{x^3}{3} - x + C$

Question 4: (4 Marks)

Use the definition of definite integral to evaluate the following integral $\int_0^2 (2x + 3) dx$



Academic year: 2019/2020

Day and Date: Sunday 15/3/2020
Examiner: Mr. Awad Bin Jobah
Time Allowed: 1 Hour

Test: Second
Level: First
Department: IT
Subject: Integral Calculus

Question 1: (3+2=5 Marks)

- (a) Find an upper bound and lower bound of the following integral: $\int_0^{\frac{\pi}{2}} (\sin x - \cos x) dx$
- Solutions: $f(x) = \sin x - \cos x \rightarrow f'(x) = \cos x + \sin x > 0, \forall x \in [0, \frac{\pi}{2}]$ (increasing)
- $\max(f) = \max(\sin x - \cos x) = (1 - 0) = 1$
 $\min(f) = \min(\sin x - \cos x) = (0 - 1) = -1$
- Since $\min(b-a) \leq f(x) \leq \max(b-a)$
 Thus $-1 \leq \int_0^{\frac{\pi}{2}} (\sin x - \cos x) dx \leq 1$
- $\rightarrow -1 \leq \int_0^{\frac{\pi}{2}} (\sin x - \cos x) dx \leq 1$
- The upper bound is 1
 The lower bound is -1

- (b) Verify the inequalities without evaluate the integral: $\int_1^3 \sqrt{2x^2 + 7} dx \geq 0$
- Solutions: $f(x) = \sqrt{2x^2 + 7} \geq 0, \forall x \in [1, 3]$
- Thus $\int_1^3 \sqrt{2x^2 + 7} dx \geq 0, \forall x \in [1, 3]$

Question 2: (3+2=5 Marks)

- (a) Graph the integrands and use area to evaluate the following integral: $\int_{-2}^4 (\frac{x}{2} + 3) dx$

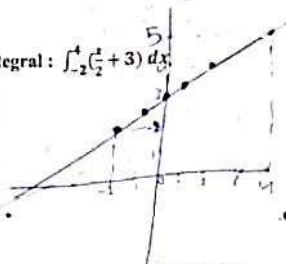
Solutions: $\frac{x}{2} + 3$

-2	-1	0	1	2
2	2.5	3	3.5	4

$y = (\frac{x}{2} + 3) = 5$

$A = (3+2) \cdot 2 + (5+3) \cdot 4$

$= \frac{5}{2} \cdot 2 + \frac{3}{2} \cdot 4 = 5 + 6 = 11$



- (b) Evaluate the following integrals: (1) $\int \frac{e^{\tan x}}{1 - \sin^2 x} dx$ (2) $\int_0^{\frac{\pi}{2}} \frac{\cos \sqrt{x}}{\sqrt{x}} dx$

(1) $\int \frac{e^{\tan x}}{1 - \sin^2 x} dx = \int \frac{e^{\tan x}}{\cos^2 x} dx = \int e^{\tan x} \sec^2 x dx$

Put $u = \tan x \Rightarrow du = \sec^2 x dx$

$\int e^u du = e^u + C = e^{\tan x} + C$

(2) $\int_0^{\frac{\pi}{2}} \frac{\cos \sqrt{x}}{\sqrt{x}} dx$

Let $u = \sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}} dx \Rightarrow dx = 2\sqrt{x} du$

$\int_0^{\frac{\pi}{2}} \cos u \cdot 2 du = 2 \sin u \Big|_0^{\frac{\pi}{2}} = 2(\sin \frac{\pi}{2} - \sin 0) = 2(1 - 0) = 2$

Question 3: (2+3=5 Marks)

(a) Find the derivative of the following function: $y = \int_0^{\tan x} \sqrt{1+t^2} dt$

Solution: $y = \sqrt{1+\tan^2 x} \cdot \sec^2 x$

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(b) Find the number C that satisfies the mean value theorem of the following integral: $\int_0^9 \sqrt{x} dx$

Solution: $AV(f) = \frac{1}{9-0} \int_0^9 \sqrt{x} dx = \frac{1}{9} \cdot \left[\frac{2}{3} x^{3/2} \right]_0^9 = \frac{1}{9} \cdot \frac{2}{3} \cdot 27 = \frac{2}{3} \cdot 9 = 2$

Thus $AV(f) = 2$

Since $f(c) = AV(f)$

Thus $\sqrt{c} = 2 \Rightarrow c = 4 \in [0, 9]$

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Question 4: (5 Marks)

Find the area of the region bounded by the following curves $x = y^2$ and $x = y+2$

Solution: $x = y+2$

$0 = y+2 \Rightarrow y = -2$

$x = 0+2 \Rightarrow x = 2$

$y^2 = y+2 \Rightarrow y^2 - y - 2 = 0$

$\Rightarrow (y+1)(y-2) = 0$

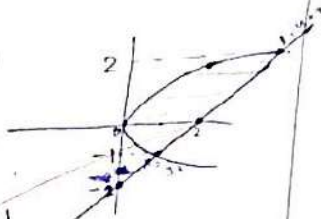
$\Rightarrow y = -1$ and $y = 2$

$A = \int_{-1}^2 [(y+2) - y^2] dy = \int_{-1}^2 (y^2 + 2 - y^2) dy$

$= \left[\frac{y^3}{3} + 2y - \frac{y^3}{3} \right]_{-1}^2 = (2+4-\frac{8}{3}) - (-\frac{1}{3}-2+\frac{1}{3})$

$= \frac{y^3}{3} + 2y - \frac{y^3}{3} = \frac{10}{3} + \frac{7}{3} = \frac{20+7}{6} = \frac{27}{6} = \frac{9}{2}$

$= (6 - \frac{8}{3}) - (-\frac{1}{3} + \frac{1}{3}) = \frac{10}{3} + \frac{7}{3} = \frac{20+7}{6} = \frac{27}{6} = \frac{9}{2}$



$u = \sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}} dx \Rightarrow 2 du = \frac{1}{\sqrt{x}} dx$

(22) b) 12

at $x = \frac{11}{2} \Rightarrow y = \sqrt{\frac{11}{2}}$

at $x = 0 \Rightarrow y = 0$

$\frac{1}{2} \int_0^{\sqrt{11/2}} \cos u du = 2 \sin u \Big|_0^{\sqrt{11/2}} = 2 [\sin \sqrt{11/2} - \sin 0] = 2 \sin \sqrt{11/2}$

End of Questions

Rest of book

نماذج الامتحانات النهائية

تجميع :

نور الجفري & فاطمة عاشور



HADRAMOUT UNIVERSITY
COLLEGE OF COMPUTERS & INFORMATION TECHNOLOGY
FINAL EXAMINATION



Academic year: 2019/2020
Day and Date: Tuesday, 22/9/2020
Examiner: Mr. Awad Bin Jobah
Time Allowed: 130 Hour

Level: First
Department: IT موزي
Subject: Integral Calculus

Question 1: (8+8+9 = 25 Marks)

- (a) Verify the formula by differentiation: $\int (7x+2)^3 dx = \frac{(7x+2)^4}{28} + c$ $\frac{d}{dx} \int = \int \frac{d}{dx}$
- (b) Solve the initial value problem: $\frac{d^2y}{dx^2} = 2 - 6x$; $y'(0) = 4$; $y(0) = 1$
- (c) Evaluate the following integrals: (1) $\int \sin x \sec^2 x dx$ (2) $\int \frac{1}{\tan x \sin x} dx$

Question 2: (9+8+8 = 25 Marks)

- (a) Use finite approximation to estimate the area under the graph of the function $f(x) = x^3$ between $x=0$ and $x=1$ using an upper sum with four rectangles of equal width.
- (b) Suppose that $\sum_{k=1}^{10} a_k = 5$ and $\sum_{k=1}^{10} b_k = 10$ find $\sum_{k=1}^{10} (2a_k + 3b_k + 4)$
- (c) Evaluate the following integral: (1) $\int_0^{\frac{\pi}{4}} \tan^3 x \sec^2 x dx$

Question 3: (8+8+9 = 25 Marks)

- (a) Find the definition of definite integral to evaluate the following integral. $\int_0^1 (2x+1) dx$
- (b) Find an upper bound and lower bound of the following integral. $\int_{-2}^{-1} (x^2 - 4x) dx$
- (c) Find the number C that satisfies the mean value theorem $\int_0^3 (x^2 - 2x + 1) dx$

Question 4: (8+8+9 = 25 Marks)

- (a) Find the derivative of the following function $y = \int_{\sec x}^2 (\sqrt{t^2 - 1}) dt$
- (b) Find the area of the region in closed by the following curves $y = x^2$ and $y = -x^2 + 4x$
- (c) Find the volume of the solid generated by revolving the region bounded by $y = x^3$, $x = 2$ and $y = 0$ about the x -axis

End of Questions

God of luck