Turing Machines

Our most powerful model of a computer is the Turing Machine. This is an FA with an infinite tape for storage.

A Turing Machine

A ***Turing Machine*** (TM) has three components:

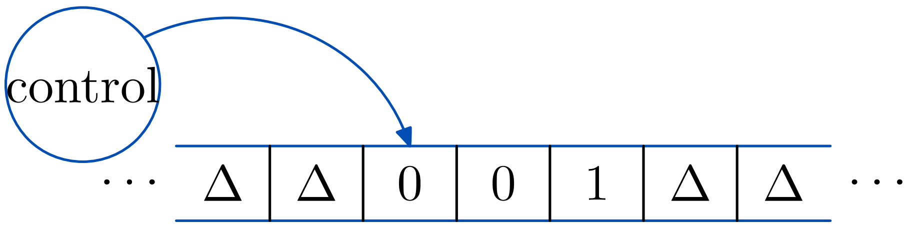
- An ***infinite tape*** divided into cells. Each ***cell*** contains one symbol.
- A ***head*** that accesses one cell at a time, and which can both read from and write on the tape, and can move both left and right.
- A ***memory*** that is in one of a fixed finite number of states.

The TM's Tape

We assume a ***two-way infinite*** tape that stretches to infinity in both directions.

Δ denotes an empty or ***blank*** cell.

The ***input*** starts on the tape surrounded by $\Delta$with the head at left-most symbol. (If input is ε, then tape is empty and head points to empty cell.)



The Program

The ***program*** of a TM is a transition function; depending on symbol under the head and state, the TM:

- writes a symbol,
- moves left or right or stays in place, and
- updates its state.

The ***language*** of TM is set of strings it accepts.

Like the PDA, once a TM enters an accept state it stops, and it terminates abnormally if there is no transition.

The Diagram

The TM is represented as a diagram, like for FA, except that each arrow is labeled with a triple:

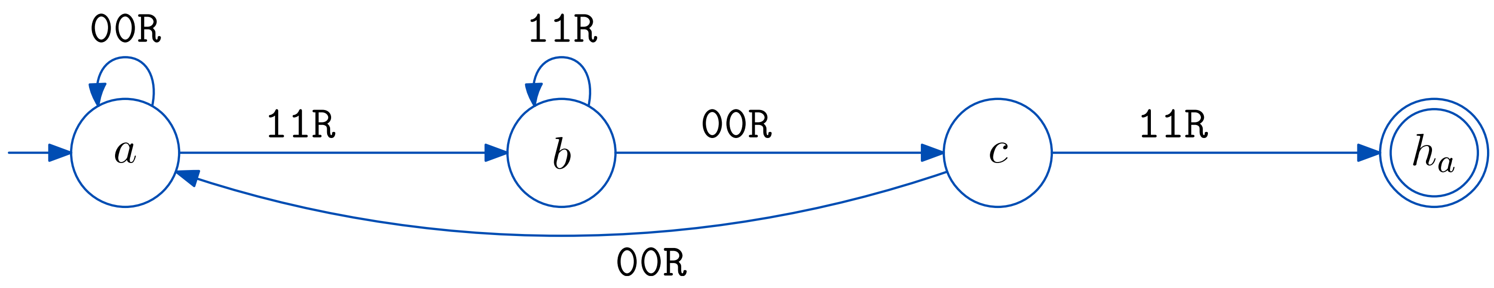
oldSymbol newSymbol moveDir ,

where "moveDir" is one of `L` (move left one cell), `R` (move right one cell), and `S` (stay in place).

For example, triple `01L` means "if reading a `0`, then write a `1` and move the head left."

Example TM: Strings Containing `101`

Here is a simple TM that mimics an FA for the language of all binary strings that contain the substring `101`.



Formal Definition

One can define a TM as a 7-tuple $(Q, \Sigma, \Gamma, q_0, h_a, h_r, \delta)$ where:

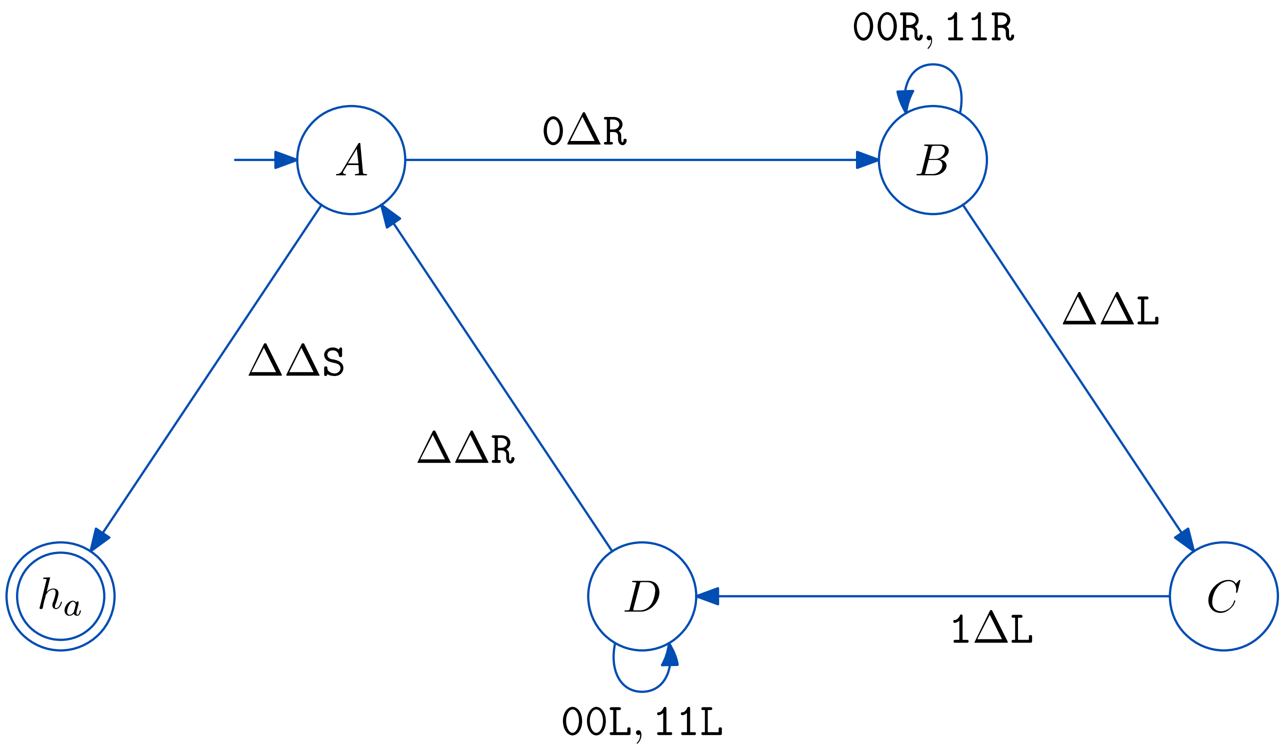
- Q is set of states.
- Σ is input alphabet.
- Γ is tape alphabet (more than Σ).
- q_0 is start state, h_a the unique halt-and-accept state, and h_r the (seldom drawn) unique halt-and-reject state.
- δ is the transition function $Q \times \Gamma \mapsto Q \times \Gamma \times \{\texttt{L}, \texttt{R}, \texttt{S}\}$.

Example TM: 0^n1^n

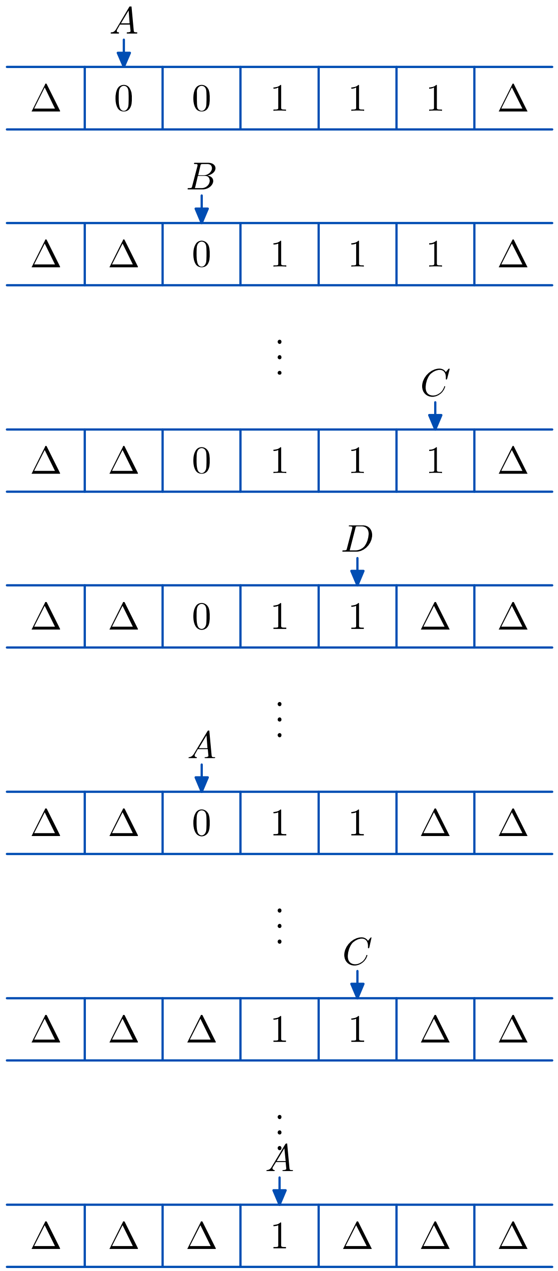
For a TM that accepts $\{ 0^n1^n \}$, pair off the 0's and 1's—repeatedly erase first 0 and last 1 until ε reached. In pseudocode:

(1) If HeadSymbol=0, then Write(Δ) else Reject.
(2) Move head right until HeadSymbol=Δ.
(3) Move head left.
(4) If HeadSymbol=1, then Write(Δ) else Reject.
(5) Move head left until HeadSymbol=Δ.
(6) Move head right.
(7) If HeadSymbol=Δ, then Accept.
(8) Goto (1).

Example Diagram: 0^n1^n



Here is what happens on input 00111...


A
Δ 0 0 1 1 1 Δ
B
Δ Δ 0 1 1 1 Δ
C
Δ Δ 0 1 1 1 Δ
D
Δ Δ 0 1 1 Δ Δ
A
Δ Δ 0 1 1 Δ Δ
C
Δ Δ Δ 1 1 Δ Δ
A
Δ Δ Δ 1 Δ Δ Δ

Reject

TMs might not halt

Here is a particularly unhelpful TM. It does not halt.