Hadhramout Univ.
IT department

Test In Linear Algebra

Date : 28-12-20
Time : 90mints

Name : [redacted]

Answer all the following questions

(Q_1) Let $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 & -1 \\ 1 & 0 & 2 \end{bmatrix}$ then :

(a)What the size of A . 3×2

size of A 3×2 (2)

(b)Find B^T

$B^T = \begin{bmatrix} 3 & 1 \\ -2 & 0 \\ -1 & 2 \end{bmatrix}$ (3)

(c) Compute $A - 3B^T$ $\quad 3B^T = 3\begin{bmatrix} 3 & 1 \\ -2 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 9 & 3 \\ -6 & 0 \\ -3 & 6 \end{bmatrix}$

$A - 3B^T = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 9 & 3 \\ -6 & 0 \\ -3 & 6 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} -9 & -3 \\ 6 & 0 \\ 3 & -6 \end{bmatrix} = \begin{bmatrix} -8 & -4 \\ 6 & 1 \\ 5 & -3 \end{bmatrix}$ (4)

(d) Compute the (3,2)-entry of AB : (row$_3$ A, col$_2$ of B)

$\begin{bmatrix} 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \times -2 + 3 \times 0 \end{bmatrix}$

$\begin{bmatrix} -4 + 0 \end{bmatrix}$ (3)

$\begin{bmatrix} -4 \end{bmatrix}$

$29\frac{1}{2}$ / 30

( ) Express the second column of AB as linear combination of columns of A .

second col AB = A col B

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 3 \end{bmatrix} \quad \text{col}(B) = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

$$-2\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + 0\begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} =$$

(Q_2)Show that If A and B are symmetric matrix and AB=BA , then AB is symmetric .

when A and B is symmetric Then $A = A^T \quad B^T = B$

$$(AB)^T = (BA)^T$$

$$B^T A^T = AB$$

$$BA = AB .$$

Then AB Is symmetric .

(Q_3) Let $A = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 1 \\ 1 & -2 & -3 \end{bmatrix}$ find a symmetric matrix S and K askew symmetric matrix K ,such that A=S+K .

while :- $S = \frac{1}{2}(A^T + A)$ / $A^T = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 4 & -2 \\ 3 & 1 & -3 \end{bmatrix}$

$K = \frac{1}{2}(A - A^T)$

Solution

Good Luck

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 1 \\ 1 & -2 & -3 \end{bmatrix} \qquad A^T = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 4 & -2 \\ 3 & 1 & -3 \end{bmatrix}$$

$$S = \frac{1}{2}(A^T + A)$$

$$\frac{1}{2}\left(\begin{bmatrix} 2 & 0 & 1 \\ -1 & 4 & -2 \\ 3 & 1 & -3 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 1 \\ 1 & -2 & -3 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 4 & -1 & 4 \\ -1 & 8 & -1 \\ 4 & -1 & -6 \end{bmatrix}$$

$$\begin{bmatrix} 2+2 & 0-1 & 1+4 \\ -1+0 & 4+4 & -2+1 \\ 3+1 & 1+(-2) & -3+3 \end{bmatrix}$$

$$S = \begin{bmatrix} 2 & -\frac{1}{2} & 2 \\ -\frac{1}{2} & 4 & -\frac{1}{2} \\ 2 & -\frac{1}{2} & -3 \end{bmatrix}$$

(7)

$$K = \frac{1}{2}(A - A^T) =$$

$$\frac{1}{2}\left(\begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 1 \\ 1 & -2 & -3 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 1 \\ -1 & 4 & -2 \\ 3 & 1 & -3 \end{bmatrix}\right) \Rightarrow \begin{bmatrix} 2-2 & -1-0 & 3-1 \\ 0-(-1) & 4-4 & 1-(-2) \\ 1-3 & -2-1 & -3-(-3) \end{bmatrix}$$

$$\frac{1}{2}\left(\begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix}\right) =$$

$$K = \begin{bmatrix} 0 & -\frac{1}{2} & 1 \\ \frac{1}{2} & 0 & \frac{3}{2} \\ -1 & -\frac{3}{2} & 0 \end{bmatrix}$$

$$S + K = \begin{bmatrix} 2 & -\frac{1}{2} & 2 \\ -\frac{1}{2} & 4 & -\frac{1}{2} \\ 2 & -\frac{1}{2} & -3 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{1}{2} & 1 \\ \frac{1}{2} & 0 & \frac{3}{2} \\ -1 & -\frac{3}{2} & 0 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 1 \\ 1 & -2 & -3 \end{bmatrix}$$

A

is skew

HADHRAMOUT UNIVERSITY
COLLEGE OF COMPUTERS & INFORMATION TECHNOLOGY
FINAL EXAMINATION

Academic year: *2020- 2021*
Day and Date: Wednesday 3/2/2021
Examiner: *Ahlam Omer BinBadr*
Time allowed:2hours

Exam Semester:*1*
Level:2
Department: *IT*
Subject: Linear Algebra

Solve the following questions

(Q1) (mark 45)

(a) *Let* $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 5 & 1 \\ 4 & 1 & -1 \end{bmatrix}$ *and* $C = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ *then*

(1) Find $A^T + B$

(2)Find if possible AB and CB^T

(3) Express the second column of AB as a linear combination of the column of A .

(4) Find C^{-1}

A(col$_2$(B)

(b)*Let* $A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 1 & 6 \\ k & 3 & 2 \end{bmatrix}$

(1) Find $|A|$

(2) Find all values of (k) so that A is singular .

(Q2) (30 mark)

(a) Solve the following system using Guass-Jordan method

$x + 2y + 3z = 9$

$2x - y + z = 8$

$3x \quad - z = 3$

(b) Find all values of (a) for which the system

$x + \quad z = 4$

$2x + y + \quad 3z = 5$

$-3x - 3y + (a^2 - 5a)z = a - 8$

has (a) on solution , (b)unique solution ,and(c) infinite many solution .

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 2 & 1 & 3 & 5 \\ -3 & -3 & (a^2-5a) & 8 \end{array}\right]$$

$a^2 - 5a + 6$

Follow →

(Q3) (mark25)

(a) Prove of the following

(1) If A and B are symmetric matrix, then $AB - BA$ is a skew- symmetric matrix .

(2) If u and v be two solution of the non-homogenous system $Ax = B$, *then* u-v is a solution to the homogenous Ax=0 .

(b) Let $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$=3 find $\begin{vmatrix} a_1 & a_2 & a_3 \\ -2b_1 & -2b_2 & -2b_3 \\ 3a_1 - c_1 & 3a_2 - c_2 & 3a_3 - c_3 \end{vmatrix}$

Good Luck