(1) Which of these sentences are proposition ? What are the truth values of those that are Propositions ?

(a) London is the capital of France. (b) What time is it ? (c) Read this carefully

(d) $x+1=2$ (e) $2+2=3$ (f) Answer this question.

(2) Let P, q and r be the propositions:

P : you have the flu , q: you miss the final examination , r : you pass the course

Express each of these propositions as an English sentence.

(a) $(P \wedge q) \vee (\neg q \wedge r)$ (b) $(P \rightarrow q)$ (c) $q \rightarrow \neg r$

(3) Determine whether these biconditionals are true or false :

(a) $2+2=4$ *if and only if* $1+1=2$

(b) $0>1$ *if and only if* $2>1$

(c) $1+1=3$ *if and only if* monkeys can fly

(4) Determine whether each of these conditional statements is true or false :

(a) *if* $1+1=2$ *then* $2+2=5$ (b) *if* $1+1=3$ *then* $2+2=4$

(c) if monkeys can fly , then $1+1=3$

(5) Construct a truth table for each of these compound propositions :

(a) $(p \vee q) \oplus (p \wedge q)$ (b) $p \oplus \neg p$ (c) $(p \rightarrow q) \vee (\neg p \rightarrow q)$

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(1) Use truth table to verify these equivalences :

(a) $p \to q \equiv \neg q \to \neg p$ (b) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

(c) $\neg(p \oplus q) \equiv p \leftrightarrow q$ (d) $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

(2) Show that each of these conditional statements a tautology by using truth tables :

(a) $(p \wedge q) \to (p \to q)$ (b) $\neg(p \to q) \to \neg q$ (c) $\neg p \to (p \to q)$

(3) Determine whether $(p \wedge q) \to (p \vee q)$ is a tautology by using logic laws

(4) Show that the statements :

(a) $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$ (b) $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

by using logic laws

(5) Use a direct proof to show that the sum of two odd integers is even

(6) Given an indirect contrapositive proof of $if\ n = ab\ then\ a \leq \sqrt{n}\ or\ b \leq \sqrt{n}$, $\forall a, b \in \mathbb{Z}$

(7) Given an indirect contradiction proof of $\sqrt{7}$ is irrational number

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Exercise (1): Discrete Mathematical

(1) Which of these sentences are proposition? What are the truth values of those that are propositions?

(a) London is the capital of France. It's proposition (false).
(b) What time is it? It's not proposition.
(c) Read this carefully. It's not proposition.
(d) $x+1=2$. It's not proposition. (e) $2+2=3$. It's proposition (false).
(f) Answer this question. It's not proposition.

(2) Let p, q and r be the propositions:

p: you have the flu. q: you miss the final examination r: you pass the course.
- Express each of these propositions as an englishsentence.

(a) $(p \land q) \lor (\neg q \land r)$. (you have the flu and you miss the final examination) or (you don't miss the final examination and you pass the course)
(b) $(p \to q)$. (if you have the flu, then you miss the final examination).
(c) $q \to \neg r$. (if you miss the final examination, then you don't pass the course)

(3) Determine whether these biconditionals are true false:

(a) $2+2=4$ if and only if $1+1=2$. (True). (b) $0>1$ if and only if $2>1$. (false)
(c) $1+1=3$ if and only if monkeys can fly. (True)

(4) Determine whether each of these conditional statements is true or false

(a) if $1+1=2$, then $2+2=5$ (false). (b) if $1+1=3$, then $2+2=4$. (True)
(c) if monkeys can fly, then $1+1=3$. (True)

(5) Construct a truth table for each of these compound propositions:

(a) $(p \lor q) \oplus (p \land q)$ (b) $p \oplus \neg p$ (c) $(p \to q) \lor (\neg p \to q)$

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(a) $(p \vee q) \oplus (p \wedge q)$

(b) $p \oplus \neg p$

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \oplus (p \wedge q)$
T	T	T	T	F
T	F	T	F	T
F	T	T	F	T
F	F	F	F	F

p	$\neg p$	$p \oplus \neg p$
T	F	T
F	T	T

(c) $(p \rightarrow q) \vee (\neg p \rightarrow q)$

p	q	$\neg p$	$p \rightarrow q$	$\neg p \rightarrow q$	$(p \rightarrow q) \vee (\neg p \rightarrow q)$
T	T	F	T	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	F	T	T	F	T

Exercise (2):

① Use truth table to verify these equivalences:

(a) $p \rightarrow q \equiv \neg q \rightarrow \neg p$ (b) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

(c) $\neg (p \oplus q) \equiv p \leftrightarrow q$ (d) $\neg (p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

(a) $p \rightarrow q \equiv \neg q \rightarrow \neg p$

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg q \rightarrow \neg p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

$\equiv$

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(b) $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$

p	q	r	$q \lor r$	$p \land (q \lor r)$	$p \land q$	$p \land r$	$(p \land q) \lor (p \land r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

≡

(c) $\neg(p \oplus q) \equiv p \leftrightarrow q$

p	q	$p \oplus q$	$\neg(p \oplus q)$	$p \leftrightarrow q$
T	T	F	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	T	T

≡

(d) $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

p	q	$\neg q$	$p \leftrightarrow q$	$\neg(p \leftrightarrow q)$	$p \leftrightarrow \neg q$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	F

≡

2. Show that each of these conditional statement's a tautology by using truth tables:

(a) $(p \land q) \to (p \to q)$ (b) $\neg(p \to q) \to \neg q$ (c) $\neg p \to (p \to q)$

(a) $(p \land q) \to (p \to q)$

p	q	$p \land q$	$p \to q$	$(p \land q) \to (p \to q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

p	q	$\neg q$	$p \to q$	$\neg(p \to q)$	$\neg(p \to q) \to \neg q$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	T	F	T
F	F	T	T	F	T

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(c) $\neg p \to (p \to q)$

p	q	$\neg p$	$p \to q$	$\neg p \to (p \to q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

③ Determine whether $(p \wedge q) \to (p \vee q)$ is a tautolog by using logic laws.

$(p \wedge q) \to (p \vee q) \equiv \neg(p \wedge q) \vee (p \vee q) \equiv \neg p \vee \neg q \vee p \vee q$

$(\neg p \vee p) \vee (\neg q \vee q) \equiv T \vee T \equiv T$

④ Show that the statements:

(a) $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$ (b) $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

by using logic laws

(a) $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

$(p \to r) \vee (q \to r) \equiv \neg p \vee r \vee \neg q \vee r \equiv (\neg p \vee \neg q) \vee (r \vee r) \equiv \neg(p \wedge q) \vee r$

$\equiv p \wedge q \to r$

(b) $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg(\neg p \wedge q) \equiv \neg p \wedge (p \vee \neg q) \equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$

$\equiv F \vee (\neg p \wedge \neg q) \equiv \neg p \wedge \neg q$

⑤ Use a direct proof to show that the sum of two odd integers is even.

Proof: Let u, v are odd, then $u = 2k+1$, $v = 2J+1$ (for some $k, J \in \mathbb{Z}$)

$u+v = 2k+1+2J+1 = 2k+2J+2 = 2(k+J+1) = 2m$ (such that $m = (k+J+1)$)

Thus $u+v$ is even.

⑥ Given an indirect contrapositive proof of if $n = ab$, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$, $\forall a, b \in \mathbb{Z}$

Proof: $n = ab$, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$

$\neg(a \leq \sqrt{n}$ or $b \leq \sqrt{n}) = \neg(a \leq \sqrt{n})$ and $\neg(b \leq \sqrt{n})$

$= a > \sqrt{n}$ and $b > \sqrt{n}$ $= ab > \sqrt{n} \cdot \sqrt{n} = n \Rightarrow ab\ n$

$\Rightarrow ab \neq n$

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7 Given an indirect contradiction proof of $\sqrt{7}$ is irrational number.

Proof : Let $\sqrt{7}$ is rational number.

$(\sqrt{7} = \frac{a}{b}$ where $a, b \in \mathbb{Z}$, $b \neq 0$, have no common factor$)$

$a = \sqrt{7}\, b \Rightarrow a^2 = 7b^2$ this means a is prime

$\boxed{a = 7k}$

$(7k)^2 = 7b^2 \Rightarrow 49k^2 = 7b^2 \Rightarrow b^2 = 7k^2$

this means b is prime $\boxed{b = 7J}$

Thus a, b have a common factor

this contradiction

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Exercise : (3) Discrete math

(1) List the members of this set : { x / x is a real number such that $x^2 = 1$ }

(3) Let $A = \{a, b, c\}$, $B = \{f, g\}$ and $C = \{d, e\}$ find (a) AXB (b) $AXBXC$

(4) Suppose that the universal set is $U = \{1,2,3,4,5,6,7,8,9,10\}$express each of these sets with bit string (a) $\{1,4,5\}$ $(b)\{1,3,6,10\}$ $(c)\{2,3,4,7,8,9\}$

(5) Using the same universal set in the last problem , find set specified by each of these bit strings : $(a)1111001111$ $(b)0101111000$ $(c)1000000001$

(6) Let f be the function from $\{a, b, c, d\}$ to $\{5,6,7,8\}$ $f(a) = 5$, $f(b) = 6$, $f(d) = 7$

$f(C) = 5$ $IS\ f\ onto\ ?\ Why?$

(7) Determine whether each of these functions is a bijection from $\mathcal{R}$ to $\mathcal{R}$ (i) $f(x) = x^3+1$ (ii) $g(x) = 3x-2$

(8) Find fOg and gOf , where $f(x) = 3x + 1$ and $g(x) = x + 2$ are functions from $\mathcal{R} to \mathcal{R}$

(9) Let A and B be two sets , show that $(A \cup B)^c = A^c \cap B^c$

(10) Use the set identity $X - Y = X \cap Y^c$ to prove $(A - B) \cap (C - B) = (A \cap C) - B$

(11) Find two finite sets A and B such that $A \in B$ and $A \subset B$

(12) Given a proof of or a counterexample to the following statement :

$$A \cap (B \cup C) = (A \cup B) \cap (A \cup C)$$

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Exercise (3) Discrete Mathematical

① List the members of this set: $\{x \mid x \text{ is a real number s.t } x^2=1\}$

Set $= \{-1, 1\}$

② Let $A=\{a,b,c\}$, $B=\{f,g\}$ and $C=\{d,e\}$ find:

a) $A \times B = \{(a,f),(a,g),(b,f),(b,g),(c,f),(c,g)\}$

b) $A \times B \times C = \{(a,f,d),(a,f,e),(a,g,d),(a,g,e),(b,f,d),(b,f,e),(b,g,d),$
$(b,g,e),(c,f,d),(c,f,e),(c,g,d),(c,g,e)\}$

③ Suppose that the universal set is $U=\{1,2,3,4,5,6,7,8,9,10\}$
express each of these sets with bit string:

a) $\{1,4,5\} = 1001100000$, b) $\{1,3,6,10\} = 1010010001$
c) $\{2,3,4,7,8,9\} = 0111001110$

④ Using the same universalset in the last problem, find set specified by each of these bit string:

a) $1111001111 = \{1,2,3,4,7,8,9,10\}$, b) $0101111000 = \{2,4,5,6,7\}$
c) $1000000001 = \{1,10\}$

⑤ Let f be the function from $\{a,b,c,d\}$ to $\{5,6,7,8\}$ $f(a)=5, f(b)=6$
$f(d)=7, f(c)=5$ IS f onto? Why?

It's not onto because Domain $=\{a,b,c,d\} \neq$ Codomain $=\{5,6,7,8\}$

⑥ Determine whether each of these function is a bijection from R to R:

a) $f(x)=x^3+1$ ⓘ Let x_1, x_2 such that $x_1 \neq x_2$
$x_1^3 \neq x_2^3 \Rightarrow x_1^3+1 = x_2^3+1 \Rightarrow f(x_1) \neq f(x_2)$, $x_1 \neq x_2$ $f(x_1) \neq f(x_2)$
f is injective (ii) Let $y=f(x) \Rightarrow y=x^3+1 \Rightarrow y-1=x^3$
$x=\sqrt[3]{y-1} \Rightarrow$ surjective from (i) and (ii) $\Rightarrow f$ is a bijection.

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b) $g(x) = 3x - 2$

(i) Let x_1, x_2 such that $x_1 \neq x_2 \xrightarrow{\times 3} x_1 \neq x_2 \Rightarrow 3x_1 \neq 3x_2 \xrightarrow{-2}$

$3x_1 - 2 \neq 3x_2 - 2 \Rightarrow g(x_1) \neq g(x_2)$, $x_1 \neq x_2 \Rightarrow g(x_1) \neq g(x_2)$

f is injective

(ii)

Let $y = g(x) \Rightarrow y = 3x - 2 \xrightarrow{+2} y + 2 = 3x \xrightarrow{\div 3} x = \frac{y+2}{3}$

f is surjective.

from (i) and (ii) $\Rightarrow$ f is a bijection

⑦ Find fog and gof, where $f(x) = 3x + 1$ and $g(x) = x + 2$ are functions from R to R.

(i) $f \circ g = f(g(x)) = f(x+2) = 3(x+2) + 1 = 3x + 6 + 1 = 3x + 7$

(ii) $g \circ f = g(f(x)) = g(3x+1) = 3x + 1 + 2 = 3x + 3$

⑧ Let A and B two sets, show that $(A \cup B)^c = A^c \cap B^c$

$x \in (A \cup B)^c = \{x / x \notin A \cup B\} = \{x \mid \neg(x \in A \cup B)\} = \{x \mid \neg(x \in A \vee x \in B)\}$

$= \{x \mid \neg(x \in A) \wedge \neg(x \in B)\} = \{x \notin A \wedge x \notin B\} = \{x \in A^c \wedge x \in B^c\}$

$= A^c \cap B^c$

⑨ Use the set identity $X - Y = X \cap Y^c$ to prove $(A-B) \cap (C-B) = (A \cap C) - B$

$(A - B) \cap (C - B) = A \cap B^c \cap C \cap B^c = (A \cap C) \cap (B^c \cap B^c)$

$= (A \cap C) \cap B^c = (A \cap C) - B$

⑩ Find two finite sets A and B such that $A \in B$ and $A \subset B$.

$A = \{1\}$, $B = \{1, \{1\}\}$ $A \in B$ and $A \subset B$

⑪ Given a proof of or a counterexample to the following statement:

$A \cap (B \cup C) = (A \cup B) \cap (A \cup C)$

Let $A = \{a\}$, $B = \{a, b\}$, $C = \{b\} \Rightarrow A \cap (B \cup C) = \{a\} \cap \{a, b\} = \{a\}$

$(A \cup B) \cap (A \cup C) = \{a, b\} \cap \{a, b\} = \{a, b\}$

$A \cap (B \cup C) \neq (A \cup B) \cap (A \cup C)$

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