

Exercises (1)

Discrete Mathematical

(1) Which of these sentences are proposition? What are the truth values of those that are Propositions ?

- (a) London is the capital of France . (b) What time is it ? (c) Read this carefully  
(d)  $x + 1 = 2$  (e)  $2 + 2 = 3$  (f) Answer this question .

(2) Let  $P$ ,  $q$  and  $r$  be the propositions :

$P$  : you have the flu ,  $q$ : you miss the final examination ,  $r$  : you pass the course

Express each of these propositions as an English sentence.

- (a)  $(P \wedge q) \vee (\neg q \wedge r)$  (b)  $(P \rightarrow q)$  (c)  $q \rightarrow \neg r$

(3) Determine whether these biconditionals are true or false :

- (a)  $2 + 2 = 4$  if and only if  $1 + 1 = 2$   
(b)  $0 > 1$  if and only if  $2 > 1$   
(c)  $1 + 1 = 3$  if and only if monkeys can fly

(4) Determine whether each of these conditional statements is true or false :

- (a) if  $1 + 1 = 2$  then  $2 + 2 = 5$  (b) if  $1 + 1 = 3$  then  $2 + 2 = 4$   
(c) if monkeys can fly , then  $1 + 1 = 3$

(5) Construct a truth table for each of these compound propositions :

- (a)  $(p \vee q) \oplus (p \wedge q)$  (b)  $p \oplus \neg p$  (c)  $(p \rightarrow q) \vee (\neg p \rightarrow q)$

Exercise : 2

Discrete Mathematical

( 1 ) Use truth table to verify these equivalences :

$$( a ) \quad p \rightarrow q \equiv \neg q \rightarrow \neg p \qquad ( b ) \quad p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

$$( c ) \quad \neg(p \oplus q) \equiv p \leftrightarrow q \qquad ( d ) \quad \neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$$

( 2 ) Show that each of these conditional statements a tautology by using truth tables :

$$( a ) \quad (p \wedge q) \rightarrow (p \rightarrow q) \qquad ( b ) \quad \neg(p \rightarrow q) \rightarrow \neg q \qquad ( c ) \quad \neg p \rightarrow (p \rightarrow q)$$

(3) Determine whether  $(p \wedge q) \rightarrow (p \vee q)$  is a tautology by using logic laws

( 4 ) Show that the statements :

$$( a ) \quad (p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r \qquad ( b ) \quad \neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$$

by using logic laws

( 5 ) Use a direct proof to show that the sum of two odd integers is even

( 6 ) Given an indirect contrapositive proof of if  $n = ab$  then  $a \leq \sqrt{n}$  or  $b \leq \sqrt{n}$  ,  $\forall a, b \in$

( 7 ) Given an indirect contradiction proof of  $\sqrt{7}$  is irrational number