Determine whether or not these objects exist. If so, write down an example. If not, explain why not.

(1) A 3×2 matrix whose columns are linearly independent.

(2) A 2×3 matrix whose columns are linearly independent.

(3) A noninvertible 4×4 matrix whose columns span $\mathbb{R}^4$.

(4) A basis of the vector space $\operatorname{null}(\begin{pmatrix} 1 & 1 & 2 \end{pmatrix})$.

(5) An orthogonal matrix whose rows are linearly dependent.

(6) A nonidentity matrix which equals its own inverse.

(7) A matrix A such that $\operatorname{null}(A) = \operatorname{col}(A)$.

(8) A basis $\{v_1, v_2, v_3\} \in \mathbb{R}^3$ such that $\|v_i - v_j\| = 1$ for all $i \neq j$.

(9) An orthogonal matrix Q such that $\operatorname{null}(QQ^\top) \cap \operatorname{col}(QQ^\top)$ is larger than $\{0\}$.

(10) A 3×2 matrix A and a 2×3 matrix B such that $AB = \mathrm{Id}_{3\times 3}$.

(11) Two matrices A, B such that $AB = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$ and $\operatorname{null}(B)$ is larger than $\{0\}$.

(12) Two matrices A, B such that $AB = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ and $\operatorname{null}(B)$ is larger than $\{0\}$.

(13) Two linearly independent vectors in $\operatorname{null}(\begin{pmatrix} 1 & 0 & 0 \end{pmatrix})$ which are both perpendicular to $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

(14) A rank-one matrix whose columns are linearly independent.

(15) An nonzero upper-triangular matrix whose columns are linearly dependent.

(16) Two matrices A, B such that $AB = \mathrm{Id}_{4\times 4}$, the matrix A is not invertible, and the columns of B are linearly independent.

(17) A diagonal matrix Σ such that $P\Sigma P$ is not diagonal, where $P = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$.

(18) A 3×4 matrix A and a 3-vector b such that $Ax = b$ has a unique solution.

(19) A spanning set $v_1, v_2, v_3, v_4, v_5 \in \mathbb{R}^4$ with $v_i + v_j + v_k = 0$ whenever i, j, k are all different.

(20) Nonzero 2×2 projection matrices P, Q, R satisfying $P + Q + R = \mathrm{Id}_{2\times 2}$.

(21) Nonzero 2×2 projection matrices P, Q, R satisfying $P + Q + R = \frac{3}{2}\,\mathrm{Id}_{2\times 2}$.