Determine whether or not these objects exist. If so, write down an example. If not, explain why not.

(1) A 2×4 matrix A such that $\operatorname{null}(A) = \left\{ \begin{pmatrix} x \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ for all } x \in \mathbb{R} \right\}$.

(2) An invertible matrix of the following form:

$$\begin{pmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & 0 & 0 & 0 \\ ? & 0 & 0 & 0 \end{pmatrix}$$

(3) Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$. A matrix B such that $\operatorname{col}(B) = \operatorname{row}(A)$ and $\operatorname{null}(B) = \operatorname{null}(A^\top)$.

(4) A matrix $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ such that the system

$$\begin{aligned} a_{11}x + a_{12}y &= 1 \\ a_{21}x + a_{22}y &= 2 \end{aligned}$$

has no solution, the system

$$\begin{aligned} a_{11}x + a_{12}y &= 1 \\ a_{21}x + a_{22}y &= 1 \end{aligned}$$

has exactly one solution, and the system

$$\begin{aligned} a_{11}x + a_{12}y &= 1 \\ a_{21}x + a_{22}y &= 0 \end{aligned}$$

has infinitely many solutions.

(5) Two subspaces $V_1, V_2 \subseteq \mathbb{R}^3$ such that $\dim(V_1) = \dim(V_2) = 2$ and $V_1 \cap V_2 = \{0\}$.

(6) Let $A = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$. A basis $\{v_1, v_2, v_3\} \subset \mathbb{R}^3$ such that $v_1 \in \operatorname{null}(A)$ and $v_2, v_3 \in \operatorname{row}(A)$.

(7) Let $A = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$. A basis $\{v_1, v_2, v_3\} \subset \mathbb{R}^3$ such that $v_1, v_2 \in \operatorname{null}(A)$ and $v_3 \in \operatorname{row}(A)$.

(8) An $n \times n$ matrix P such that $P^2 = P$, $\operatorname{rank}(P) = n$, and $P \neq \mathrm{Id}_{n\times n}$.

(9) An $n \times n$ matrix A such that all singular values of A are equal to 1, and $A \neq \mathrm{Id}_{n\times n}$.

(10) Two subspaces $V_1, V_2 \subseteq \mathbb{R}^3$ such that $V_1 \cap V_2 = \{0\}$ and $V_1^\perp \cap V_2^\perp = \{0\}$.

(11) A 3×5 matrix A such that $\dim(\operatorname{null}(A)) + \dim(\operatorname{null}(A^\top)) = 5$.

(12) Matrices A and B such that $\operatorname{pinv}(A) = \operatorname{pinv}(B)$ and $A \neq B$.

(13) Matrices A and B such that $A^\top A = B^\top B$ and $A \neq B$.

(14) A square matrix A such that $A^\top A + AA^\top$ is noninvertible.

(15) An $m \times n$ matrix A and a nonzero vector $v \in \operatorname{row}(A)$ such that $Av \in \operatorname{null}(A^\top)$.