Practice Problems

1. True or False (give a good reason if true/counterexample or reason if false)

1. If the zero vector is in the column space of a matrix A, then the columns of A are linearly dependent.

2. If the columns of a matrix are dependent, so are the rows.

3. The column space of a 2×2 matrix is the same as its row space.

4. The column space of a 2×2 matrix has the same dimension as its row space.

5. The columns of a matrix are a basis for the column space.

6. A and A^T have the same number of pivots.

7. A and A^T have the same left nullspace.

8. If the row space equals the column space then $A^T = A$.

9. If $A^T = -A$, then the row space of A equals the column space.

2. If w_1, w_2, w_3 are independent vectors in $\mathbb{R}^3$, show that the differences

$$v_1 = w_2 - w_3$$
$$v_2 = w_1 - w_3$$
$$v_3 = w_1 - w_2.$$

are *dependent.* Find the matrix A so that

$$\begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix} A = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}.$$

Which matrices above are singular?

3. Construct $A = uv^T + wz^T$ whose column space has basis $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$ and whose row space has basis $(1, 0), (1, 1)$. Write A as a 3×2 matrix times a 2×2 matrix.

4. If a subspace S is contained in a subspace V, prove that $S^\perp$ contains $V^\perp$.