Practice Problems

1. The "tribonacci numbers" are the sequence defined by $T_1 = 1$, $T_2 = 1$, $T_3 = 2$ and the recurrence

$$T_n = T_{n-1} + T_{n-2} + T_{n-3}.$$

a) Write this 3-term recurrence as a square matrix R acting on the last ____ numbers in this sequence.

b) Find a formula for T_n involving only T_1, T_2, T_3 and R.

c) The eigenvalues of this matrix are roughly $-0.42 \pm 0.61i$ and 1.84. What does this tell you about the behavior of the Tribonacci numbers for large n?

2. Consider the matrix

$$\begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}$$

from lecture, which has eigenvalues $\lambda_1 = 2$ and $\lambda_2 = 3$, and corresponding eigenvectors $x_1 = [1\ 1]$ and $x_2 = [1\ 2]$.

a) What do we get if we take the vector $x = [3\ 4] = 2x_1 + x_2$ and multiply 100 times by A^{-1}?

b) What happens if we take x and multiply many times by $(2A - 5I)^{-1}$? Does it converge to a particular vector?

c) More generally, if we have an arbitrary matrix A with all eigenvalues distinct, and we multiply a vector x repeatedly by A^{-1}, it typically approaches what eigenvector? When might this fail to happen?