LINEAR ALGEBRA. VASILY KRYLOV. RECITATION 10: EXERCISES.

1. Problem 1

a) Find $A^T A$ and AA^T and the singular vectors v_1, v_2, u_1, u_2 for A:

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}.$$

Check the equations $Av_1 = \sigma_1 u_1$, $Av_2 = \sigma_2 u_2$

b) Find (and check) the SVD decomposition:

$$A = U\Sigma V^T.$$

Recall that matrices U, V should be orthogonal and the matrix Σ is diagonal.

2. Problem 2

Find $A^T A$ and AA^T and the singular vectors v_1, v_2, u_1, u_2 for A:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 8 \\ 0 & 0 & 0 \end{bmatrix} \text{ has rank } r = 2. \text{ The eigenvalues are } 0, 0, 0.$$

Check the equations $Av_1 = \sigma_1 u_1$, $Av_2 = \sigma_2 u_2$ and $A = \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T$. If you remove row 3 of A, show that σ_1 and σ_2 do not change.

(This is a problem from PSet 8)

3. Problem 3

Find the SVD factors U and Σ and V^T for

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

4. Problem 4

(a) For this rectangular matrix find v_1, v_2, v_3 and u_1, u_2 and σ_1, σ_2:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

(b) Write the SVD for A as $U\Sigma V^T = (2 \times 2)(2 \times 3)(3 \times 3)$.

5. Problem 5

(a) Why is the trace of $A^T A$ equal to the sum of all a_{ij}^2?

(b) For every rank-one matrix, why is $\sigma_1^2 =$ sum of all a_{ij}^2?

(This is a problem from PSet 8)