Practice Problems

1. Say P is a permutation. Why is $P^{-1} = P^T$? (You're allowed to use this fact in the pset without justification, but it's good to know why it's true! If you get stuck, answer this question for a particular permutation.)

Bonus question: Say $P[x_1, \ldots, x_n] = [x_{p_1}, \ldots, x_{p_n}]$ and $P^{-1}[x_1, \ldots, x_n] = [x_{q_1}, \ldots, x_{q_n}]$. What's the relationship between the lists $(p_1, \ldots, p_n)$ and $(q_1, \ldots, q_n)$?

2. Say

$$A = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 3 \\ 1 & 1 & 0 & 1 \end{bmatrix}.$$

Solve

$$Ax = \begin{bmatrix} 12 \\ 6 \\ 32 \\ 2 \end{bmatrix}$$

for x *without* using Gaussian elimination.

3. Consider the following *tridiagonal* matrix

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 2 & -1 & 1 & 0 & 0 \\ 0 & 3 & 4 & -2 & 0 \\ 0 & 0 & -2 & 5 & -2 \\ 0 & 0 & 0 & -1 & 3 \end{bmatrix}.$$

a) Compute its LU factorization using Gaussian elimination. What do you notice about the pattern of nonzero entries?

b) Compute the 3rd column of A^{-1}. What do you notice about the pattern of nonzero entries?

c) Suppose you carried out arithmetic at the same rate, but A was a 250×250 tridiagonal matrix instead of 5×5. How much longer would a) have taken?

4. Are the following subsets of $\mathbb{R}^2$ vector spaces? Why or why not?

a) $[x, y]$ satisfying $mx + b = y$, where m, b are fixed scalars.

b) $[x, y]$ satisfying $x^2 + y^2 = 1$ (the unit circle).

c) $[x, y]$ satisfying

$$A \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

where A is a fixed 2×2 matrix.