

Practice Problems

1. Consider the matrix

$$A = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}.$$

One eigenvalue of A is -1 . The other eigenvalue is a double root of $\det(A - \lambda I)$. What is the other eigenvalue? (Hint: there is a way to do this without computing the characteristic polynomial. What are some equations you know involving eigenvalues?)

2. Suppose M is a positive Markov matrix (so one eigenvalue equals 1, all other eigenvalues have $|\lambda| < 1$). Why is M^∞ a rank-1 matrix?
3. $x^T A y = \text{tr}(AB)$ where B is what matrix? (Hint: recall that the trace of a 1x1 matrix a is a).
4. Suppose A is an $m \times n$ full column-rank matrix with thin SVD $U\Sigma V^T$. By inspection of $A^T A$ in comparison with the diagonalization formula, the eigenvectors of $A^T A$ are ___ and its eigenvalues are ___.
5. Suppose A is $m \times m$, full rank, and we compute its QR factorization $A = QR$, e.g. by Gram-Schmidt. Claim: the matrix $B = RQ$ has the same eigenvalues as A . Why?