Practice Problems

1. True, false, or neither (that is, sometimes true, sometimes false):

a) If v, w are eigenvectors of A, then so is $v + w$ and cv for c any scalar.

b) If $v \in N(A)$ is not the zero vector, then v is an eigenvector of A.

2. Describe as many eigenvalues and corresponding eigenvectors as you can (without doing any serious calculation) for

a) $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 3 & -1 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{pmatrix}$.

b) A projection matrix P onto some subspace S (pick a particular 2-dimensional subspace of $\mathbb{R}^3$ if you're confused).

c) The permutation matrix

$$M = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

d) A rank one matrix uv^T (pick particular 3-component vectors u, v if you're confused).

3. For which angles θ does the rotation matrix

$$R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix},$$

have real eigenvalues? (Hint: The roots of the quadratic equation ax^2+bx+c are $(-b\pm\sqrt{b^2-4ac})/2a$.)

4. Suppose that A, B, C are $m \times m$ matrices with eigenbases that you know. What do you know about the eigenvectors and eigenvalues of A^{2022}? A^{-1} (assuming A is invertible)? A^T? AB? $A+B$?

5. Suppose that A is the matrix

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

a) What is the pattern when you multiply A repeatedly by some vector? After _______ multiplications, you get back the same vector, so

$$A^{\_\_\_} = \_\_\_\_\_\_\_\_\_\_\_\_.$$

b) What are eigenvalues and eigenvectors of A? Is this consistent with the previous part?

c) Write the vector $x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ in the basis of the eigenvectors and give a formula for $A^n x$.

d) What are the eigenvectors and eigenvalues of $B = 2A + I$?