Practice Problems

1 (Lecture recap—skip if you feel like it)**.** A function f is called "linear" if $f(x+y) = f(x) + f(y)$ and $f(cx) = cf(x)$ for any scalar c.

a) Is $f(x) = mx + b$ linear? What about $f(x) = x^2$? (In both cases, f is a function on the real numbers)

b) Show that $f(x) = Ax$ is linear for any 2×2 matrix A. (Here, x is any 2×1 vector.)

c) Show that $f(X) = AX$ is linear for any 2×2 matrix A. (Here, X is any 2×2 matrix.)

2. Say x, y, z are 4-component column vectors. The equation

$$x(y + z) = xy + xz = yx + zx$$

is nonsense (why?) but is a few symbols away from being true. Decorate with transposes to make it a true equation.

3. Say P is the 4×4 linear operation that reverses the order, i.e.

$$P \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_4 \\ x_3 \\ x_2 \\ x_1 \end{bmatrix}.$$

What does P do to the 4×4 identity matrix I? How can you use this to write down P?

More generally, if you know how a linear operation A behaves on a vector of variables, how can you write down the matrix for A?

4. Find the LU factorization of

$$A = \begin{bmatrix} a & a & a \\ a & b & b \\ a & b & c \end{bmatrix}.$$

What 3 conditions on a, b, c guarantee that $A = LU$ has 3 pivots?

5. Consider the matrices

$$U = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}, L = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 01 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

and set $A = UB^{-1}L$. Without inverting any matrices, compute the second column of A^{-1}.