Determine whether or not these objects exist. If so, write down an example. If not, explain why not.

(1) A 2×2 matrix P satisfying $P^2 = P$, $\begin{pmatrix} 1 \\ 1 \end{pmatrix} \in \operatorname{col}(P)$, and $\begin{pmatrix} 1 \\ 2 \end{pmatrix} \in \operatorname{null}(P)$.

(2) An invertible matrix V such that

$$V \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} V^{-1} = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}.$$

(3) Two vectors $v, w \in \mathbb{R}^2$ such that $v \cdot w = 1$ and $vv^\top + ww^\top = \mathrm{Id}_{2\times 2}$.

(4) A square orthogonal matrix Q such that $Q^3 = \mathrm{Id}$, but neither Q nor Q^2 equal the identity.

(5) An upper triangular[1] matrix U such that $U^3 = \mathrm{Id}$, but but neither U nor U^2 equal the identity.

(6) A square orthogonal matrix Q such that $\det(Q) < 0$.

(7) An orthogonal matrix such that $\det(QQ^\top) < 0$.

(8) A symmetric matrix A such that $\det(A) < 0$.

(9) A matrix A such that $\det(A^\top A) < 0$.

(10) A real number a such that the matrix

$$A = \begin{pmatrix} 3 & a \\ a & 1 \end{pmatrix}$$

transforms a shape with area 1 into a shape with area 4.

(11) A 2×2 matrix which transforms the parallelogram with vertices $(1,1), (2,-1), (-2,1), (-1,-1)$ into a square of area 4.

(12) A 2×2 matrix A such that $\det(A) = 1$ and A transforms the square with vertices $(1,1), (1,-1), (-1,1), (-1,-1)$ into a shape which lies inside the unit disk $D := \{v \in \mathbb{R}^2 \text{ such that } \|v\| \leq 1\}$.

[1] This means that the entries of U lying *strictly* below the main diagonal are zero.