Recitation 3/31

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Σ as coordinate transformation

- Lecture slide 23 page 3, Summary on page 6

- A be m-by-n matrix, $u \in \text{col}(A) = \text{col}(U_1) \subset \mathbb{R}^m$, $v \in \text{row}(A) = \text{col}(V_1) \subset \mathbb{R}^n$

Since u is a vector in column space and v is a vector in row space, we have coordinates b, c(coefficients of linear combination), so that

$$u = U_1 b, \quad v = V_1 c, \quad b, c \in \mathbb{R}^r$$

If $u = Av$, we can say $b = \Sigma_r c$

Orthogonal Subspaces

If V and W are vector subspaces of $\mathbb{R}^n$, we say that V and W are orthogonal if

$$\forall v \in V, \forall w \in W \text{ we have } < v, w >= 0 \text{ (or } v \perp w \text{ or } v^T w = 0)$$

In other words, every vectors from V and W are perpendicular to each other. Denote $V \perp W$

Orthogonal Complement

Given vector subspace $V \subset \mathbb{R}^n$, the **Orthogonal complement** of V is denoted by $V^\perp$, and defined as the set of all $w \in \mathbb{R}^n$ such that w is perpendicular to all vectors in V.

It can be thought as the largest subspace orthogonal to V.

Problems

1.(a) Find any 3 orthogonal subspaces of $V \in \mathbb{R}^5$, where $V = \text{span}(\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix})$

(b) What is the orthogonal complement of V?

2. Full SVD of $A \in \mathbb{R}^{4\times5}$ is given as,

$$A = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 3 & & & & \\ & 2 & & & \\ & & 1 & & \\ & & & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & 0 & -1\sqrt{2} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

(a) What is the rank? What are U_1, V_1?

(b) $u = \begin{pmatrix} 7/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \\ 3 \end{pmatrix}$, $v = \begin{pmatrix} 5/\sqrt{2} \\ 1 \\ -1/\sqrt{2} \\ 0 \\ 0 \end{pmatrix}$ satisfies $u = Av$. Find $b, c \in \mathbb{R}^3$ such that $u = U_1 b$ and $v = V_1 c$.

(c) Find simple relationship between b and c.

3. Why are $\mathrm{col}(A)$ and $\mathrm{null}(A^T)$ orthogonal complements to each other? What about $\mathrm{row}(A)$ and $\mathrm{null}(A^T)$? Explain in terms of SVD.