Practice Problems

1. Say A is a 3×3 real matrix with eigenvalues $\lambda_1 = -1, \lambda_2 = -3 + 4i, \lambda_3 = -3 - 4i$, with corresponding eigenvectors x_1, x_2, x_3.

a) What are the trace and determinant of $2A$?

b) Two eigenvectors of A are $x_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $x_2 = \begin{pmatrix} 0 \\ 1 \\ i \end{pmatrix}$. What is x_3?

2. Using the same matrix A as in 1, which of the following has unbounded magnitude (i.e. magnitude blowing up) as $n \to \infty$ or $t \to \infty$? Assume y is chosen at random.

a) $A^n y$ as $n \to \infty$

b) $A^{-n} y$ as $n \to \infty$

c) The solution of $\frac{dx}{dt} = Ax$ as $t \to \infty$ for the initial condition $x(0) = y$.

d) The solution of $\frac{dx}{dt} = -Ax$ as $t \to \infty$ for the initial condition $x(0) = y$.

3. Using the same matrix A as in 1, write down the exact solution $x(t)$ to $\frac{dx}{dt} = Ax$ for the initial condition $x(0) = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$.

4. Use the series formula for e^{At} to show that

$$\frac{d}{dt} e^{At} = Ae^{At}.$$

Use this to conclude that $x(t) = e^{At} x(0)$ satisfies $\frac{dx}{dt} = Ax$.