Practice Problems

1. When does the least square solution $\hat{x}$ minimizing $||b - Ax||$ give an exact solution to Ax=b?

2. We want to project onto the plane $x - y - 2z = 0$. To do this, choose 2 vectors spanning that plane (the nullspace of what matrix?) and make them the columns of a matrix A so that $C(A)$ is the plane. Then compute the projection of the point

$$\begin{pmatrix} 0 \\ 6 \\ 12 \end{pmatrix}$$

onto this plane.

3. Say P is an $m \times m$ orthogonal-projection matrix onto an n-dimensional subspace. What is the rank of $A = (I - P)P$? What is the rank of $B = (I - P) - P$? (Hint: helpful to compute B^2, and also to draw a picture of what B does.)

4. If A is $m \times n$ with rank n, what is the complexity of finding the projection p onto $C(A)$ of a point b by

(1) forming the projection matrix P (using the formula from class) then multiplying Pb

(2) forming the normal equations, solving them for $\hat{x}$ and then computing $p = A\hat{x}$.