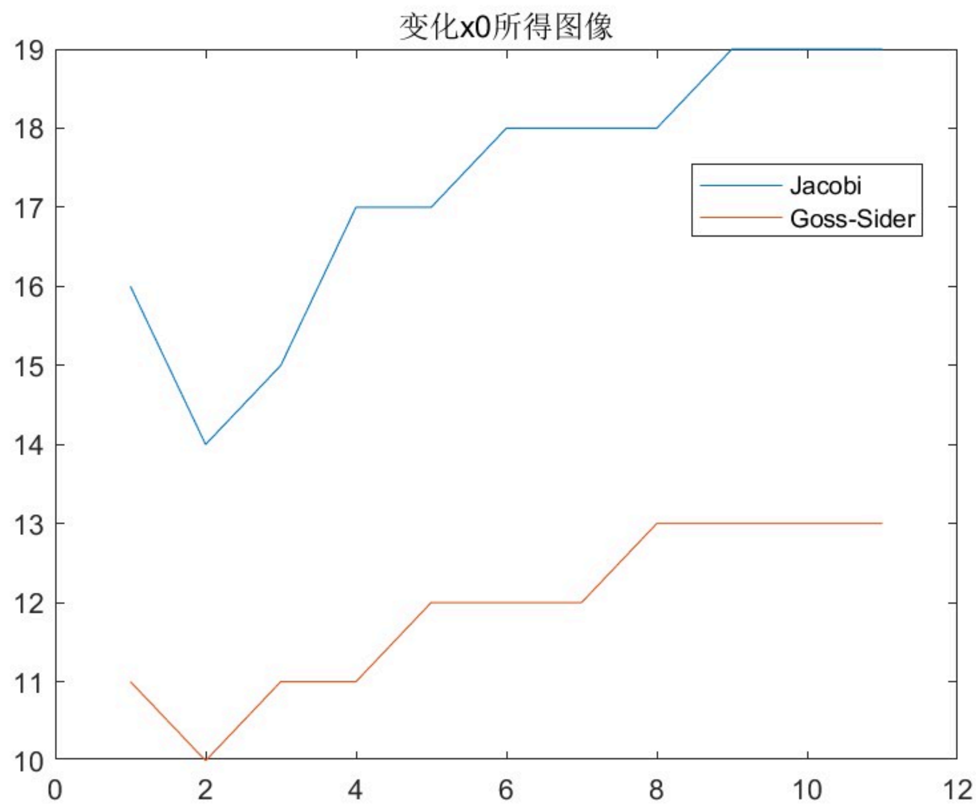
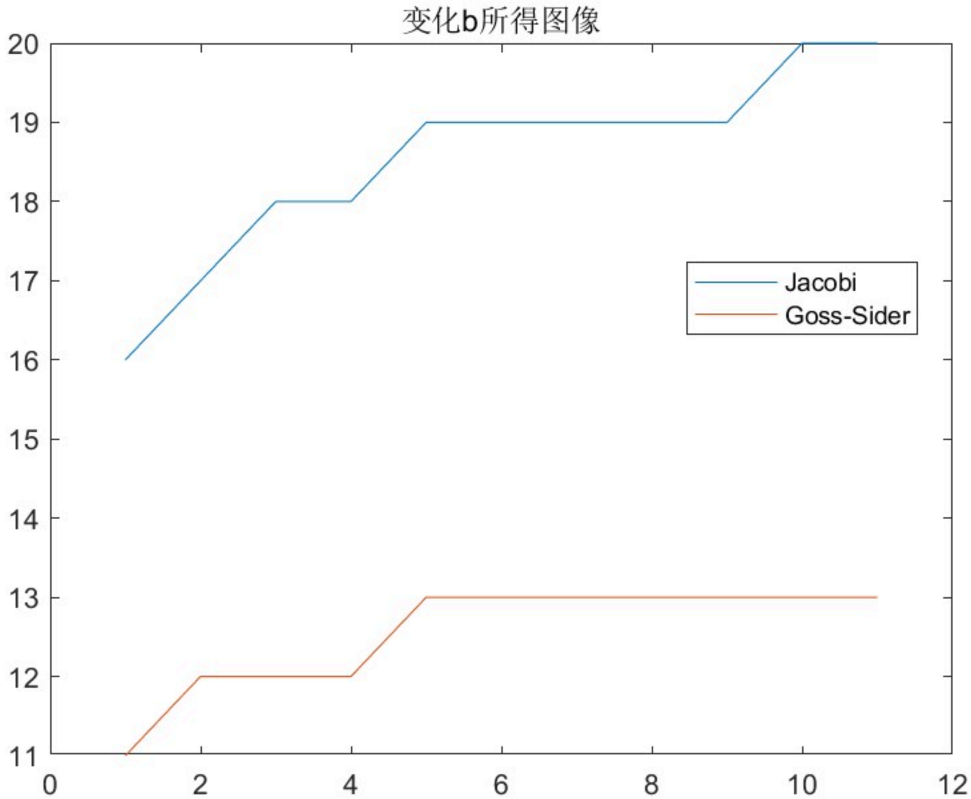
第五次作业

5.5.3

（1）我们控制误差 $tol = 10^{-5}$,分别固定b，改变x0；固定x0，改变b，得到的迭代次数都不大，说明都是收敛的。并做出 n_J 和 n_GS 的散点图，方便比较分析。均得出结论：**GS**法收敛速度快于**Jacobi**法

代码部分是先写了函数文件，分别根据输入的向量x0和b，返回迭代次数。再在主文件中用for循环作出散点图。(为了方便直接应用到第二问，这里还增加了参数 3^t ，控制矩阵A主对角元的倍数。第一问仅需保持 $t = 0$)




变化b所得图像
Jacobi
Goss-Sider
20
19
18
17
16
15
14
13
12
11
0
2
4
6
8
10
12

主文件 code:

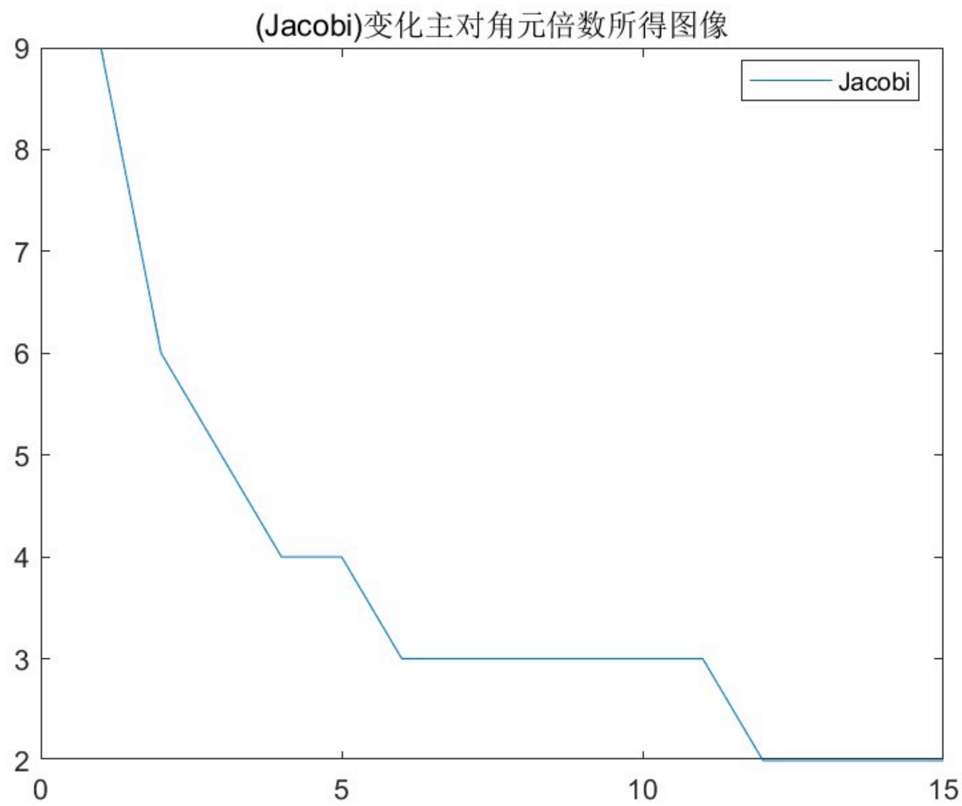
```
n = 20;
x = ones(n,1) * linspace(0,5,11);
b = ones(n,1) * (1:11);

n_Jlist = zeros(size(x,2),2);
n_GSlist = zeros(size(x,2),2);

for k = 1:size(x,2)
    x0 = x(:,k);
    b0 = b(:,1);
    [n_Jlist(k,1), n_GSlist(k,1)] = func5(x0,b0,0)
end

figure;
plot(1:size(x,2), n_Jlist(:,1)', 1:size(x,2), n_GSlist(:,1)')
legend("Jacobi", "Goss-Sider", "Location", "best")
title('变化x0所得图像')
saveas(gcf,'5_5_3_1.jpg')

for k = 1:size(b,2)
    x0 = x(:,1);
    b0 = b(:,k);
    [n_Jlist(k,2), n_GSlist(k,2)] = func5(x0,b0,0)
end

figure;
plot(1:size(b,2), n_Jlist(:,2)', 1:size(b,2), n_GSlist(:,2)')
legend("Jacobi", "Goss-Sider", "Location", "best")
title('变化b所得图像')
saveas(gcf,'5_5_3_2.jpg')
```

函数文件 code:

```
function [n_J,n_GS] = func5(x0,b,t)
    n=20;
    A1=sparse(1:n,1:n,3,n,n);
    A2=sparse(1:n-1,2:n,-1/2,n,n);
    A3=sparse(1:n-2,3:n,-1/4,n,n);
    A=3^t*A1 + A2 + A2' + A3 + A3';
    D=diag(diag(A));
    U=-triu(A,1);
    L=-tril(A,-1);

    tol=1e-5;



    %Jacobi
    B_J=D\(L+U);
    f_J=D\b;
    x_J = x0 * (1:100);
    n_J=0;
    %Gauss-Seidel
    B_GS = (D-L)\U;
    f_GS = (D-L)\b;
    x_GS = x0 * (1:100);
    n_GS=0;

    for i=1:100
        x_J(:,i+1) = B_J * x_J(:,i) + f_J;
        n_J = n_J+1;
        if (norm(x_J(:,i+1) - x_J(:,i),inf) < tol)
            break;
        end
    end

    for i=1:100
        x_GS(:,i+1) = B_GS * x_GS(:,i) + f_GS;
        n_GS = n_GS+1;
        if (norm(x_GS(:,i+1) - x_GS(:,i),inf) < tol)
            break;
        end
    end

end
```

（2）利用第一问的函数文件，第二问是很容易的。注意：$|x^{(k+1)}-x^{(k)}|_\infty \leq 10^{-5}$，这已蕴含在函数文件中。结论：随着主对角占优的程度增加，收敛速度变得很快

还注意到极限情况：两次（$x^{(3)}-x^{(2)}$）即达到收敛要求，这是因为 $|x^{(3)}-x^{(2)}|_\infty \leq |B|_\infty |x^{(2)}-x^{(1)}|$, 当k足够大时，$|B|_\infty$ 非常小，于是迭代两次即达到收敛要求

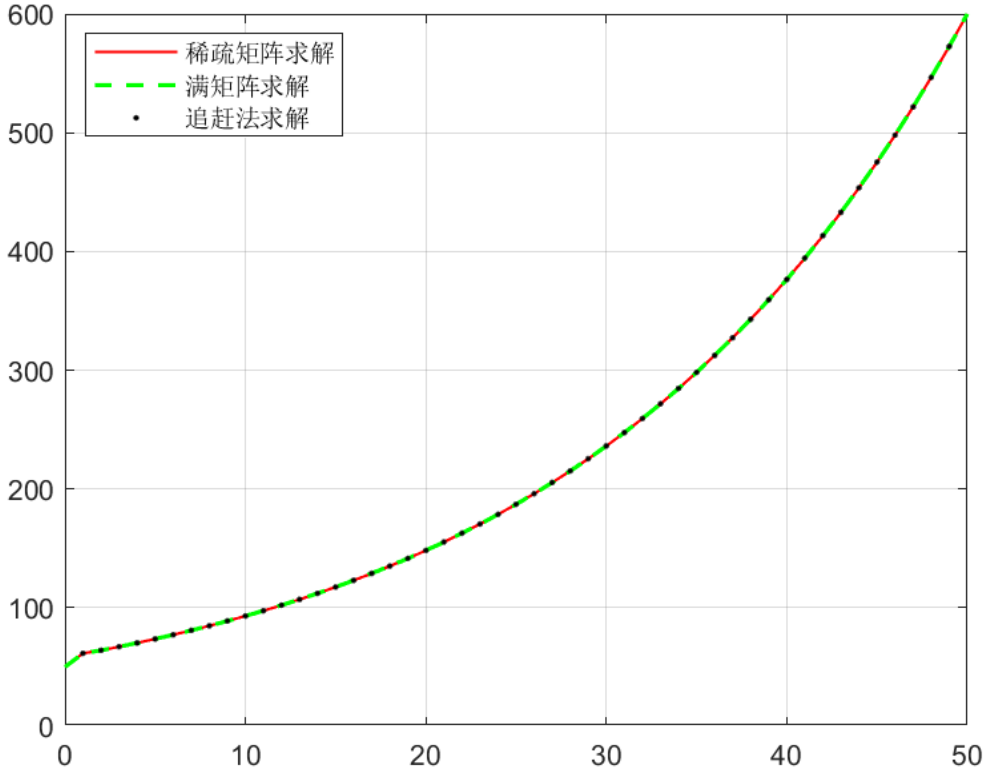


code:

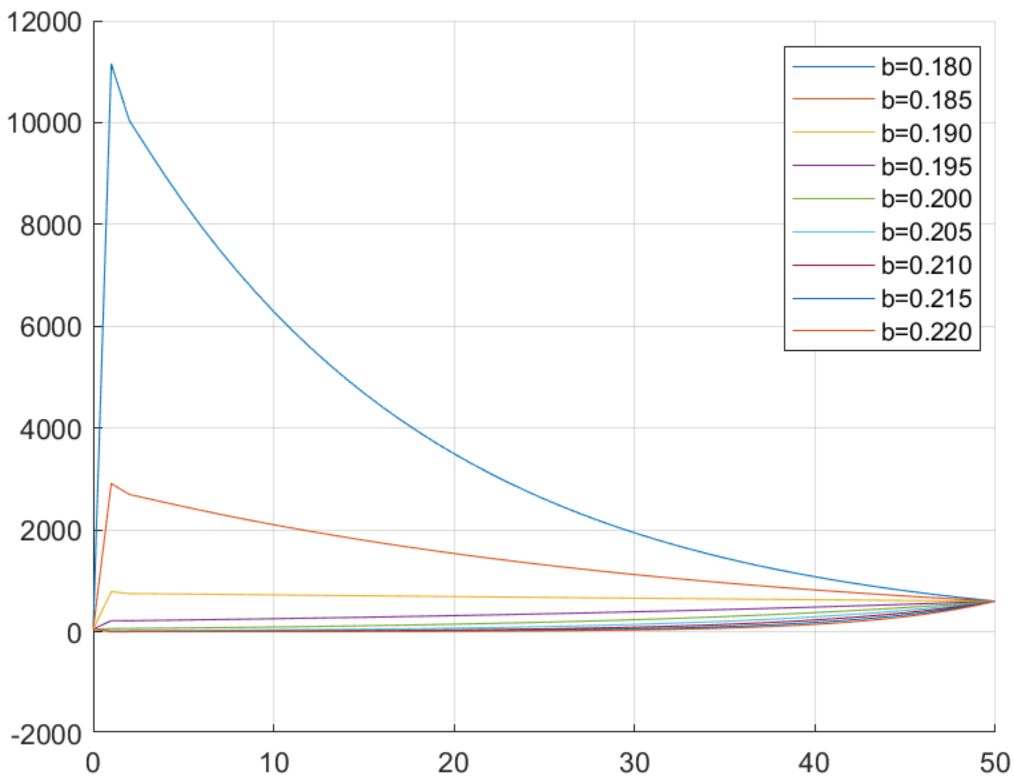
```
n = 20;
m=15;
x = ones(n,1) * linspace(0,5,11);
b = ones(n,1) * (1:11);

n_Jlist = zeros(m,1);
n_GSlist = zeros(m,1);

for k = 1:m
    x0 = x(:,1);
    b0 = b(:,1);
    [n_Jlist(k,1), n_GSlist(k,1)] = func5(x0,b0,k);
end

figure;
plot(1:m, n_Jlist(:,1)')
legend("Jacobi", "Location", "best")
title('(Jacobi)变化主对角元倍数所得图像')
saveas(gcf,'5_5_3_3.jpg')
```

5.5.4

b=0.2 ,三者图像基本重合，只有计算时间差异，图像对比如下



若考虑b的误差，$b \in [0.18, 0.22]$ 我们取步长 0.005。计算每个b值下的cond（A），发现前四个比较大，非常病态，后四个在二三十左右，病态没那么严重。



code:

```
c=10;
a1=0.5;
a2=0.25;
b=0.2;
p=-a1*b*c;
q=-a2*b*(1-a1)*b*c;

% 稀疏矩阵法
x0 = 50; xn = 600; n = 49;
A1 = sparse(1:n, 1:n, p, n, n); % 输入A的对角元素
A2 = sparse(1:n-1, 2:n, 1, n, n); % 输入A的(上)次对角元素
A3 = sparse(2:n, 1:n-1, q, n, n); % 输入A的(下)次对角元素
A = A1 + A2 + A3;
i = [1,n], j = [1,1];
s = [-q * x0, -xn];
bb = sparse(i, j, s, n, 1);
x = A \ bb;
k = 0:n+1;
xxxs = [x0, x', xn];
% 满矩阵
AA=full(A);
x = AA \ bb;
xxman = [x0, x', xn];

% 追赶法
f=zeros(n, 1); f(1)=-q*x0;f(n) = -xn;
u(1)=p;
y(1)=f(1);
for i=2:n
    l(i)=q/u(i-1);
    u(i) = p-l(i);
    y(i) = f(i) -l(i) *y(i-1);
end
xzg= zeros(1,n);
xzg(n) = y(n)/u(n);
for i=n-1:-1:1
    xzg(i) = (y(i)-xzg(i+1))/u(i);
end

figure(1)
plot(k, xxxs, 'r', 'LineWidth', 1); hold on;
plot(k, xxman, '--g', 'LineWidth', 1.5);
plot(1:n, xzg, '.k', 'LineWidth', 1)
grid on;
legend('稀疏矩阵求解', '满矩阵求解','追赶法求解' ,'Location', 'best');
saveas(gcf, '5_5_4_1.png')



figure(2)
hold on;
grid on;
for r = 0:8
```

```
    b = 0.18 + (0.005 * r);
    p=-a1*b*c;
    q=-a2*b*(1-a1)*b*c;
    A1 = sparse(1:n, 1:n, p, n, n); % 输入A的对角元素
    A2 = sparse(1:n-1, 2:n, 1, n, n); % 输入A的(上)次对角元素
    A3 = sparse(2:n, 1:n-1, q, n, n); % 输入A的(下)次对角元素
    A = A1 + A2 + A3;
    condest(A)
    i = [1,n];
    j = [1,1];
    s = [-q * x0, -xn];
    bb = sparse(i, j, s, n, 1);
    x = A \ bb;
    xxxs = [x0, x', xn];
    plot(k, xxxs)
    legends{r+1} = sprintf('b=%.3f', b);

end
legend(legends, 'Location', 'best');
saveas(gcf, '5_5_4_2.png')
```

5.5.9

（1）

$$\begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \vdots \\ \tilde{x}_n \end{bmatrix} = \begin{bmatrix} b_1 & b_2 & \cdots & b_n \\ s_1 & 0 & \cdots & 0 \\ 0 & s_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & s_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} - \begin{bmatrix} 0 \\ h_1 \\ \vdots \\ h_{n-1} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

即

$$(A - I_n)X = H$$

这里

$$A = \begin{bmatrix} b_1 & b_2 & \cdots & b_n \\ s_1 & 0 & \cdots & 0 \\ 0 & s_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & s_{n-1} \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad H = \begin{bmatrix} 0 \\ h_1 \\ \vdots \\ h_{n-1} \end{bmatrix}$$

（2）x_i 依次如下

```
ans =

   1.0e+03 *

    8.4810    2.8924    1.3354    0.6013    0.1405
```

code:

```
A1 = sparse(1, [3, 4], [5, 3], 5, 5);
A2 = sparse(2:5, 1:4, [0.4, 0.6, 0.6, 0.4], 5, 5);
A = A1 + A2;
M = A - eye(5);
h = [0 500 400 200 100]';
x = M \ h;
x'
```

（3）h_i无法均为500，这是种群数量出现负值

```
ans =

   1.0e+04 *

    1.1772    0.4209    0.2025    0.0715   -0.0214
```