

## 第二次作业

### 2.4.4

由(25)式,

$$x(t) = \frac{x_m}{1 + ce^{-rt}}, c = \frac{x_m}{x_0} - 1$$

则

$$x''(t) = \left( \frac{2c^2 e^{-2rt} r^2}{(1 + ce^{-rt})^3} - \frac{ce^{-rt} r^2}{(1 + ce^{-rt})^2} \right) x_m$$

令  $x'' = 0$ ,

$$e^{rt_0} = c = \frac{x_m}{x_0} - 1$$

回代有

$$x(t) = \frac{x_m}{1 + e^{-r(t-t_0)}}$$

### 2.4.5

推导:

$$\begin{aligned} x(t + \Delta t) - x(t) &= k(x_m - x(t))\Delta t \\ x'(t) &= k(x_m - x(t)) \\ x(t) &= x_m - (x_m - x_0)e^{-kt} \end{aligned}$$

code:

```
x0 = 0.1;
xm = 1;
r = 1;
t = linspace(0, 6, 100)
x1 = @(t) x0 * exp(r * t);
x2 = @(t) xm ./ (1 + (xm / x0 - 1) * exp(-r * t));
x3 = @(t) xm - (xm - x0) * exp(-r * t);
plot(t, x1(t), t, x2(t), t, x3(t))
legend('指数增长模型', 'logistic模型', '新模型', 'Location', 'best')
ylim([0, 1.2])
saveas(gcf, '2.4.5.1.png');
```

图像:

