

MATH 317: Numerical Analysis

Assignment 3 : Due 20 November, 2017

Important: Submit a complete hard copy of all your solutions either in class or to the Math Dept Office opposite the elevators on 10th floor of Burnside before 4pm on the due date; this must include Course number, name and id number (or it will get lost). Solutions should be complete and include hardcopy of all electronic output, and code, together with explanations. Work not submitted on paper will not be graded. Submit all relevant program files and documentation to explain them. Get in the habit of including comments in your code. Write your own code! Otherwise, why are you here?

1. For a point moving on a circle at constant angular velocity, ω , the x and y coordinates are given by

$$\frac{dx}{dt} = -\omega y, \quad \frac{dy}{dt} = \omega x, \quad \omega = \frac{2\pi}{T}.$$

Solve the above equations using a) the forward Euler scheme and b) the centred-difference scheme with a forward Euler first step. Use the following parameters: $T = 72\text{h}$, $\Delta t = 0.5\text{h}$, $x_o = x(t = 0) = 600\text{km}$, $y_o = 0$. Integrate for 144 h and plot the x, y coordinates on a polar plot. Comment on your results.

2. The nonlinear pendulum is governed by the equations

$$\frac{dv}{dt} = -g \sin \theta \quad \frac{d\theta}{dt} = \frac{v}{L}.$$

Solve using a) forward Euler and b) backward Euler for $L = 9\text{m}$, $\Delta t = 0.1\text{s}$, $v_o = 0.6\text{m s}^{-1}$ and $\theta_o = 0$. Integrate for 25s and plot v as a function of t . Comment on your results. Note that if a scheme is implicit, you may have to iterate to convergence at each time step. Use your own judgement on how best to do this.

3. Integrate the following set of ODE's on the computer:

$$\frac{dx}{dt} = \sigma(y - x), \quad \frac{dy}{dt} = x(\rho - z) - y, \quad \frac{dz}{dt} = xy - \beta z$$

using the Improved Euler Method. The initial conditions are $x(0) = 1$, $y(0) = -1$ and $z(0) = 30$ and the parameters take on the values $\sigma = 10$, $\beta = 8/3$ and $\rho = 28$. Use a step size of $h = 0.005$ and run it until $t = 100$, implying that you take $100/0.005 = 20,000$ steps. (It's about time we saw what computers can do!). Display your output by plotting $z(t)$ on the y -axis and $x(t)$ on the x -axis. Does the result look familiar? This is the Lorenz attractor that started the study of chaos. See <http://mathworld.wolfram.com/LorenzAttractor.html>

4. Consider the ODE $y'' + 2y' + 9y = 0$ with the initial conditions $y(0) = 0$ and $y'(0) = 1$. First, solve it analytically. Next rewrite it as a coupled set of first-order ODE's. Now, write a computer code to integrate it using the backward Euler scheme, i.e. for each equation $dA/dt = RHS$, the scheme is written $A_n - A_{n-1} = h RHS_n$, where h is the uniform step size. Write the resulting set of equations in matrix form and write a code to solve it. Finally, plot out the analytical and numerical solution over the range $[0, \pi]$. Try a few values of h and see what happens.