A CONCISE HISTORY OF MATHEMATICS

by

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CHAPTER I

The Beginnings

1. Our first conceptions of number and form date back to times as far removed as the Old Stone Age, the Paleolithicum. Throughout the hundreds or more millennia of this period men lived in caves, under conditions differing little from those of animals, and their main energies were directed towards the elementary process of collecting food wherever they could get it. They made weapons for hunting and fishing, developed a language to communicate with each other, and in the later paleolithic ages enriched their lives with creative art forms, statuettes and paintings. The paintings in caves of France and Spain (perhaps c. 15000 years ago) may have had some ritual significance; certainly they reveal a remarkable understanding of form.

Little progress was made in understanding numerical values and space relations until the transition occurred from the mere *gathering* of food to its actual *production*, from hunting and fishing to agriculture. With this fundamental change, a revolution in which the passive attitude of man toward nature turned into an active one, we enter the New Stone Age, the Neolithicum.

This great event in the history of mankind occurred perhaps ten thousand years ago, when the ice sheet which covered Europe and Asia began to melt and made room for forests and deserts. Nomadic wandering in search of food came slowly to an end. Fishermen and hunters were in large part replaced by primitive farmers.

Such farmers, remaining in one place as long as the soil stayed fertile, began to build more permanent dwellings; villages emerged as protection against the climate and against predatory enemies. Many such neolithic settlements have been excavated. The remains show how gradually elementary crafts such as pottery, carpentry, and weaving developed. There were granaries, so that the inhabitants were able to provide against winter and hard times by establishing a surplus. Bread was baked, beer was brewed, and in late neolithic times copper and bronze were smelted and prepared. Inventions were made, notably of the potter's wheel and the wagon wheel; boats and shelters were improved. All these remarkable innovations occurred only within local areas and did not always spread to other localities. The American Indian, for example, did not learn of the existence of the wagon wheel until the coming of the white man. Nevertheless, as compared with the paleolithic times, the tempo of technical improvement was enormously accelerated.

Between the villages a considerable trade existed, which so expanded that connections can be traced between places hundreds of miles apart. The discovery of the arts of smelting and manufacturing, first copper then bronze tools and weapons, strongly stimulated this commercial activity. This again promoted the further formation of languages. The words of these languages expressed very concrete things and very few abstractions, but there was already some room for simple numerical terms and for some form relations. Many Australian, American, and African tribes were in this stage at the period of their first contact with white men; some tribes are still living in these conditions so that it is possible to study their habits and forms of expression.

2. Numerical terms—expressing some of "the most abstract ideas which the human mind is capable of forming," as Adam Smith has said—came only slowly into use. Their first occurrence was qualitative rather than quantitative, making a distinction only between one (or better "a"—"a man"—rather than "one man") and two and many. The ancient qualitative origin of numerical conceptions can still be detected in the special dual terms existing in certain languages such as Greek or Celtic. When the number concept was extended higher numbers were first formed by addition: 3 by adding 2 and 1, 4 by adding 2 and 2, 5 by adding 2 and 3.

Here is an example from some Australian tribes:

Murray River: 1 = enea, 2 = petcheval, 3 = petcheval-enea, 4 = petcheval petcheval.
Kamilaroi: 1 = mal, 2 = bulan, 3 = guliba, 4 = bulan bulan, 5 = bula guliba, 6 = guliba guliba[1].

The development of the crafts and of commerce stimulated this crystallization of the number concept. Numbers were arranged and bundled into larger units, usually by the use of the fingers of the hand or of both hands, a natural procedure in trading. This led to numeration first with five, later with ten as a base, completed by addition and sometimes by subtraction,

[1] L. Conant, *The Number Concept* (London, 1896), pp. 106-107, with many similar examples.

so that twelve was conceived as $10 + 2$, or 9 as $10 - 1$. Sometimes 20, the number of fingers and toes, was selected as a base. Of 307 number systems of primitive American peoples investigated by W. C. Eels, 146 were decimal, 106 quinary and quinary decimal, vigesimal and quinary vigesimal[1]. The vigesimal system in its most characteristic form occurred among the Mayas of Mexico and the Celts in Europe.

Numerical records were kept by means of bundling, strokes on a stick, knots on a string, pebbles or shells arranged in heaps of fives—devices very much like those of the old time inn-keeper with his tally stick. From this method to the introduction of special symbols for 5, 10, 20, etc. was only a step, and we find exactly such symbols in use at the beginning of written history, at the so-called dawn of civilization.

The oldest example of the use of a tally stick dates back to paleolithic times and was found in 1937 in Vestonice (Moravia). It is the radius of a young wolf, 7 in. long, engraved with 55 deeply encised notches, of which the first 25 are arranged in groups of 5. They are followed by a simple notch twice as long which terminates the series; then, starting from the next notch, also twice as long, a new series runs up to 30[2].

It is therefore clear that the old saying found in Jacob Grimm and often repeated, that counting started as finger counting, is incorrect. Counting by fingers, that is, counting by fives and tens, came only at a certain stage of social development. Once it was reached, num-

[1]W. C. Eels., *Number Systems of North American Indians*, Am. Math. Monthly 20 (1913), p. 293.

[2]Isis 28 (1938) pp. 462-463, from illustrated London News, Oct. 2, 1937.

bers could be expressed with reference to a base, with the aid of which large numbers could be formed; thus originated a primitive type of arithmetic. Fourteen was expressed as $10 + 4$, sometimes as $15 - 1$. Multiplication began where 20 was expressed not as $10 + 10$, but as 2×10. Such dyadic operations were used for millennia as a kind of middle road between addition and multiplication, notably in Egypt and in the pre-Aryan civilization of Mohenjo-Daro on the Indus. Divisions began where 10 was expressed as "half of a body", though conscious formation of fractions remained extremely rare. Among North American tribes, for instance, only a few instances of such formations are known, and this is in almost all cases only of 1/2, although sometimes also of 1/3 or 1/4.[1] A curious phenomenon was the love of very large numbers, a love perhaps stimulated by the all-too-human desire to exaggerate the extent of herds or of enemies slain; remnants of this tendency appear in the Bible and in other sacred writings.

3. It also became necessary to measure the length and contents of objects. The standards were rough and often taken from parts of the human body, and in this way units originated like fingers, feet, or hands. Names like ell, fathom, cubit also remind us of this custom. When houses were built, as among the agricultural Indians or the pole house dwellers of Central Europe,

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[1]G. A. Miller has remarked that the words *one-half*, *semis*, *moitié* have no direct connection with the words *two*, *duo*, *deux* (contrary to *one-third*, *one-fourth*, etc.), which seems to show that the conception of ½ originated independent of that of integer. Nat. Math. Magazine 13 (1939) p. 272.

rules were laid down for building along straight lines and at right angles. The word "straight" is related to "stretch," indicating operations with a rope[1]; the work "line" to "linen," showing the connection between the craft of weaving and the beginnings of geometry. This was one way in which interest in mensuration evolved.

Neolithic man also developed a keen feeling for geometrical patterns. The baking and coloring of pottery, the plaiting of rushes, the weaving of baskets and textiles, and later the working of metals led to the cultivation of plane and spatial relationships. Dance patterns must also have played a role. Neolithic ornamentation rejoiced in the revelation of congruence, symmetry, and similarity. Numerical relationships might enter into these figures, as in certain prehistoric patterns which represent triangular numbers; others display "sacred" numbers.

Here follow some interesting geometrical patterns occurring in pottery, weaving or basketry.

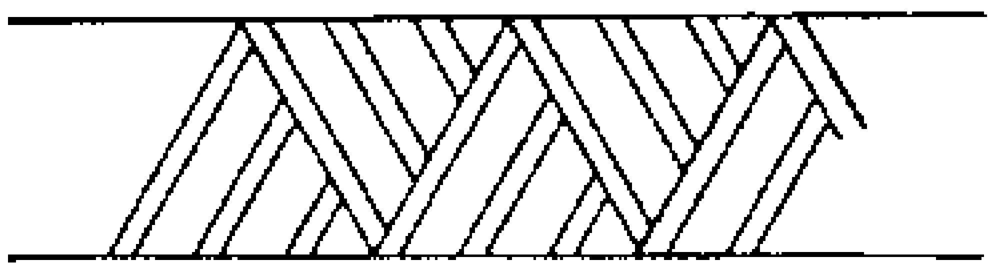

Fig. 1.

This can be found on neolithic pottery in Bosnia and on objects of art in the Mesopotamian Ur-period[2].

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[1]The name "rope-stretchers" (Greek: "harpedonaptai," Arabic: "massah," Assyrian: "masiḫānu") was attached in many countries to men engaged in surveying—see S. Gandz, *Quellen und Studien zur Geschichte der Mathematik* I (1930) pp. 255-277.

[2]W. Lietzmann, *Geometrie und Praehistorie,* Isis 20(1933) pp. 436-439.

Patterns of this kind have remained popular throughout historical times. Beautiful examples can be found on dipylon vases of the Minoan and early Greek periods, in the later Byzantine and Arabian mosaics, on Persian and Chinese tapestry. Originally there may have been a religious or magic meaning to the early patterns, but their esthetic appeal gradually became dominant.

In the religion of the Stone Age we can discern a primitive attempt to contend with the forces of nature. Religious ceremonies were deeply permeated with magic, and this magical element was incorporated into existing conceptions of number and form as well as in sculpture, music, and drawing. There were magical numbers, such as 3, 4, 7 and magical figures such as the Pentalpha and the Swastica. Some authors have even considered this aspect of mathematics the determining factor in its growth[1], but though the social roots of mathematics may have become obscured in modern times, they are fairly obvious during the early section of man's history. "Modern" numerology is a leftover from magical rites dating back to neolithic, and perhaps even to paleolithic, times.

4. Even among very primitive tribes we find some reckoning of time and, consequently, some knowledge of the motion of sun, moon, and stars. This knowledge attained its first more scientific character when farming and trade expanded. The use of a lunar calendar goes very far back into the history of mankind, the changing aspects of vegetation being connected with the changes

[1]W. J. McGee, *Primitive Numbers*, Nineteenth Annual Report, Bureau Amer. Ethnology 1897-98 (1900) pp. 825-851.

of the moon. Primitive people also pay attention to the solstices or rising of the Pleiades at dawn. The earliest civilized people attributed a knowledge of astronomy to their most remote, prehistoric periods. Other primitive peoples used the constellations as guides in navigation. From this astronomy resulted some knowledge of the properties of the sphere, of angular directions, and of circles.

5. These few illustrations of the beginnings of mathematics show that the historical growth of a science does not necessarily pass through the stages in which we now develop it in our instruction. Some of the oldest geometrical forms known to mankind, such as knots and patterns, only received full scientific attention in recent years. On the other hand some of our more elementary branches of mathematics, such as the graphical representation or elementary statistics, date back to comparatively modern times. As A. Speiser has remarked with some asperity: "Already the pronounced tendency toward tediousness, which seems to be inherent in elementary mathematics, might plead for its late origin, since the creative mathematician would prefer to pay his attention to the interesting and beautiful problems."[1]

Literature.

Apart from the texts by Conant, Eels, Smith, Lietzmann, McGee, and Speiser already quoted, see:

[1]A. Speiser, *Theorie der Gruppen von endlicher Ordnung* (Leipzig 1925, reprint New York 1945) p. 3.