McGill University
math-315(2015-Fall): Differential Equations

Written Assignment # 1: Due on Tuesday, Oct. 6, 2015, hand-in in class
(Total 50 point)

Solve the following problems with first order equations:

(1) (10 points) Find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = y + \mathrm{e}^{2x}y^3.$$

(2) (10 points) Solve the initial value problem

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{y} + \frac{y}{x}, \quad y(1) = -4.$$

(3) (10 points) Find, in implicit form, the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x + 3y + 8}{5x + 4y + 1}.$$

(Hint: By use the new variables: $X = x - x_0$, $Y = y - y_0$ with some proper constants x_0, y_0).

(4) (10 points) Show that the differential equation $M + Ny' = 0$ has an integrating factor which is a function of $z = x + y$ only if and only if

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M - N}$$

is a function of z only. Use this result to solve the differential equation

$$x^2 + 2xy - y^2 + (y^2 + 2xy - x^2)y' = 0.$$

(5) (10 points) Check that the equation below is not exact but becomes exact when multiplied by the integrating factor $\mu(x, y) = P(x)Q(y)$.

$$x^2y^3 + x(1 + y^2)y' = 0.$$

Solve the differential equation.

(Hint: integrating factor: $\mu(x, y) = 1/(xy^3)$).

Marker: El Toufaili, Sam (260521406); hussein.toufaili@mail.mcgill.ca