COMP 360 - Fall 2015 - Assignment 3

Due: 6:00 pm Nov 10th.

General rules: In solving these questions you may collaborate with other students but each student has to write his/her own solution. There are in total 105 points, but your grade will be considered out of 100. You should drop your solutions in the assignment drop-off box located in the Trottier Building.

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1. (10 Points) Show that if we strengthen linear programming by also allowing constraints of the form $\sum_{i,j=1}^{n} a_{ij}x_ix_j = b$ (for integers b and a_{ij}), then the problem becomes NP-complete.

2. (10 Points) Show that if we strengthen linear programming by also allowing constraints of the form $|\sum_{i=1}^{n} a_ix_i| \geq b$ (for integers b and a_i), then the problem becomes NP-complete.

3. For each one of the following problems either prove that they are NP-complete or prove that they belong to P.

   (a) (10 Points)

   - Input: A CNF ϕ and a positive integer M.
   - Question: Is there a truth assignment that satisfies ϕ and assigns True to exactly M variables.

   (b) (15 Points)

   - Input: Positive integers $a_1, \ldots, a_n$ and a positive integer M.
   - Question: Is there a subset $S \subseteq \{1, \ldots, n\}$ such that $\sum_{i \in S} a_i \in \{M-1, M, M+1\}$?

   (c) (15 Points)

   - Input: A CNF ϕ.
   - Question: Is there a truth assignment that satisfies none of the clauses in ϕ.

   (d) (15 Points)

   - Input: A graph G and a positive integer M.
   - Question: Does G have a proper 2-coloring with colors R, G such that exactly M vertices receive the color R?

   (e) (15 Points)

   - Input: A graph G.
   - Question: Does G have a proper 3-coloring with colors R, G, B such that at most 100 vertices receive the color R?

   (f) (15 Points) Recall that a graph is called Hamiltonian if it contains a cycle that visits all the vertices.

   - Input: A graph G.
   - Question: Is G a Hamiltonian *bipartite* graph?