

MATH 251, homework 7, due date Monday Mar 9.

Problem 1. For $a_1, \dots, a_n > 0$ prove

$$\left(\sum_{i=1}^n a_i\right)\left(\sum_{i=1}^n \frac{1}{a_i}\right) \geq n^2.$$

When do we have equality? Justify your answer.

Problem 2. Find in $\mathcal{C}[0, 1]$ with inner product $\langle f, g \rangle = \int_0^1 f(x)g(x)dx$ the projection of the function e^x onto the subspace of polynomials of degree ≤ 1 .

Problem 3. Let $V = \mathcal{C}[0, 1]$ with the inner product as in Problem 2. Find a functional $f \in V^*$ for which there does not exist a vector $v \in V$ satisfying $f(w) = \langle v, w \rangle$. Justify your answer.