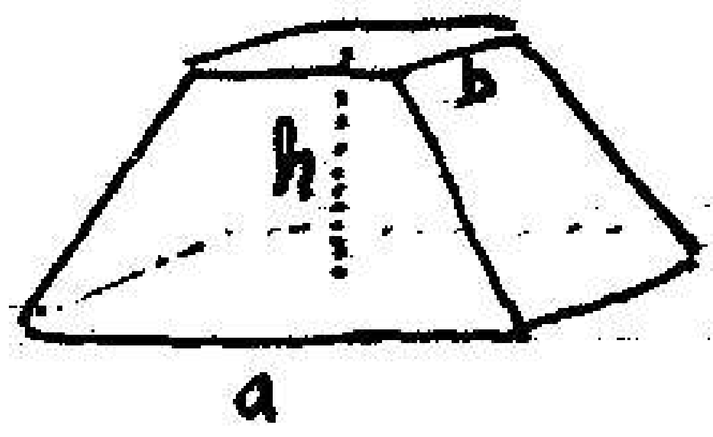
338 Homework #1 Solutions

#1 a) $\frac{1}{n+1} + \frac{1}{(n+1)(2n+1)} = \frac{2n+1}{(n+1)(2n+1)} + \frac{1}{(n+1)(2n+1)}$

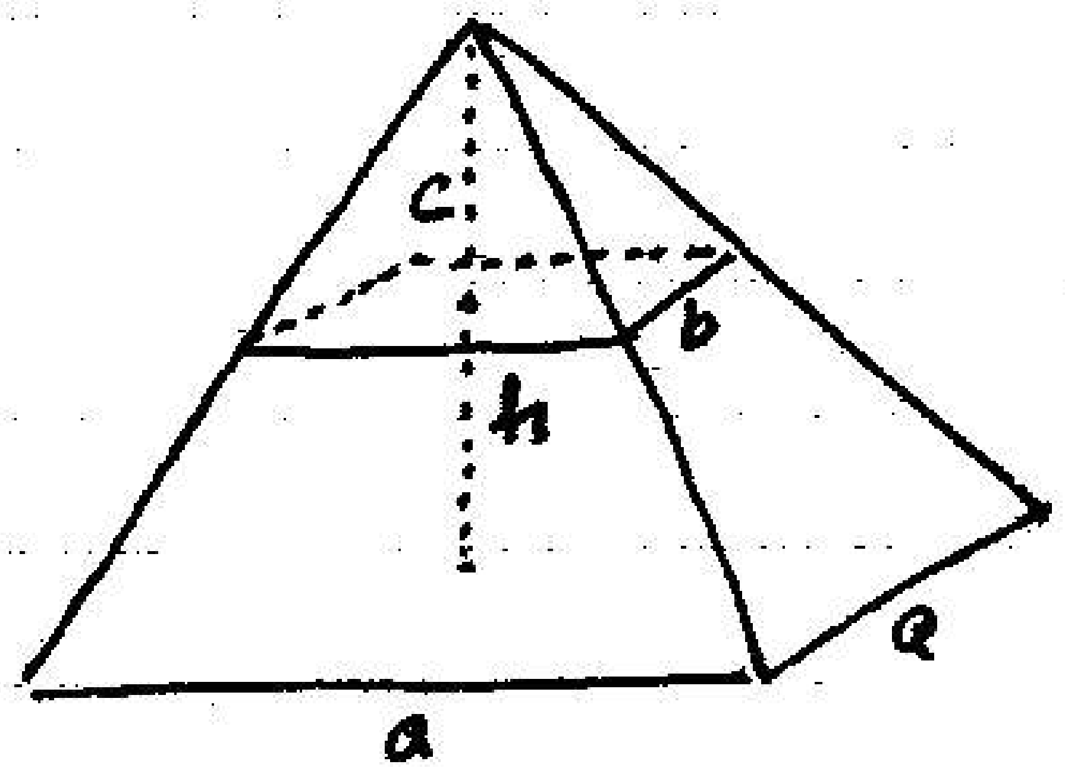
$$= \frac{2n+2}{(n+1)(2n+1)} = \frac{2(n+1)}{(n+1)(2n+1)} = \frac{2}{2n+1}$$

b) This allowed the Egyptians to write fractions as a sum of fractions with numerator 1. Where we would write $\frac{2}{13}$ they would write $\frac{1}{7} + \frac{1}{91}$

#2 Start with



. Extend it up to form



a completed pyramid with height $h+c$

The big pyramid has volume $\frac{1}{3}a^2(c+h)$

and the little pyramid on top has volume $\frac{1}{3}b^2c$

so the truncated pyramid has volume $\frac{1}{3}a^2(c+h) - \frac{1}{3}b^2c = V$

Now similar triangles says $\frac{c+h}{a} = \frac{c}{b} \Rightarrow c = \frac{-bh}{b-a}$

Put this into formula for V and simplify.

#3 a) $(a^2-b^2)^2+(2ab)^2 = a^4-2a^2b^2+b^4+4a^2b^2$

$= a^4+2a^2b^2+b^4 = (a^2+b^2)^2$

b) By choosing values for a and b we get Pythagorean Triples

#4. When you write a decimal number like 647.352 you're saying

$$647.352 = \underline{6}\times 100 + \underline{4}\times 10 + \underline{7}\times 1 + \underline{3}\times\frac{1}{10} + \underline{5}\times\frac{1}{100} + \underline{2}\times\frac{1}{1000}$$

To write $13942\frac{3}{16}$ in sexagesimal we want numbers $a, b, c, d, e, \ldots$ so that

☆ $$13942\frac{3}{16} = a\times 3600 + b\times 60 + c\times 1 + d\times\frac{1}{60} + e\times\frac{1}{3600} + \ldots$$

dividing 13942 by 3600 we get

$13942\frac{3}{16} = 3\times 3600 + 3142 + \frac{3}{16}$. Now divide 3142 by 60 to get

$$13942\frac{3}{16} = \underline{\underline{3}}\times 3600 + \underline{\underline{52}}\times 60 + \underline{\underline{22}}\times 1 + \frac{3}{16}$$

Now we want $\frac{3}{16} = \frac{d}{60} + \frac{e}{3600} + \ldots$ Multiply by 60

$\frac{180}{16} = d + \frac{e}{60} +$ Divide 16 into 180 to get

$$\frac{180}{16} = 11 + \frac{1}{4} = d + \frac{e}{60} + \ldots \quad \text{so } d = 11$$

and now we want $\frac{1}{4} = \frac{e}{60} + \ldots$ This gives $e = 15$

We found

$$13942\tfrac{3}{16} = 3 \times 3600 + 52 \times 60 + 22 \times 1 + 11 \times \frac{1}{60} + 15 \times \frac{1}{3600}$$

$$= 3,52,22.11,15 \text{ in sexagesimal.}$$

Note if you follow closely this is the same as how you turn a fraction into a decimal using long division, except instead of "bring down the zero" you "multiply by 60", which is the same process. Here is an example:

$$\begin{array}{l} \quad 0.8,34,17,8 \\ 7 \overline{)\,1} \\ \quad 60 \\ \quad \underline{56} \\ \quad \cancel{4} \\ \quad 240 \\ \quad \underline{21} \\ \quad\quad 30 \\ \quad\quad \underline{28} \\ \quad\quad\quad \cancel{2} \\ \quad\quad\quad 120 \\ \quad\quad\quad \underline{119} \\ \quad\quad\quad\quad \cancel{1} \\ \quad\quad\quad\quad 60 \end{array}$$

so $\frac{1}{7} = .8,34,17,\overline{8,34,17,}$

$$\begin{array}{l} \quad 0.11,15 \\ 16 \overline{)\,\cancel{3}} \\ \quad 180 \\ \quad \underline{176} \\ \quad \cancel{4} \\ \quad 240 \\ \quad \underline{240} \\ \quad\quad 0 \end{array}$$

as we saw before