MATH 251, homework 9, due date Monday Mar 23.

Problem 1. Let V be the space of infinitely differentiable functions on $[0,1]$. Prove that the family of functions $\{e^{\lambda x}\}_{\lambda \in \mathbf{R}}$ is linearly independent. Hint: eigenvectors.

Problem 2. Let $a_0 = a_1 = 1$ and for $n \geq 1$ let $a_{n+1} = 2a_n + a_{n-1}$. Find a closed formula for a_n. Justify your answer.

Problem 3. For a permutation σ of the set $\{1, \ldots, n\}$ consider the n by n matrix A_σ whose ith column is the standard vector $e_{\sigma(i)}$. For which σ is A_σ diagonalizable

(i) over $\mathbf{C}$,

(ii) over $\mathbf{R}$?

Justify your answers.