CHAPTER 9

Linear operators

39 Definition of a linear operator. The image, the kernel, the matrix of a linear operator

3901. Which mappings are linear operators on appropriate vector spaces:

a) $x \mapsto a$ (a is a fixed vector);

b) $x \mapsto x + a$ (a is a fixed vector);

c) $x \mapsto \alpha x$ (α is a fixed scalar);

d) $x \mapsto (x, a)b$ (V is an Euclidean space, a, b are fixed vectors);

e) $x \mapsto (a, x)x$ (V is an Euclidean space, a is a fixed vector);

f) $f(x) \mapsto f(ax + b)$ ($f \in \mathbb{R}[x]_n$; a, b are fixed numbers);

g) $f(x) \mapsto f(x + 1) - f(x)$ ($f \in \mathbb{R}[x]_n$);

h) $f(x) \mapsto f^{(k)}(x)$ ($f \in \mathbb{R}[x]_n$);

i) $(x_1, x_2, x_3) \mapsto (x_1 + 2, x_2 + 5, x_3)$;

j) $(x_1, x_2, x_3) \mapsto (x_1 + 3x_3, x_2^3, x_1 + x_3)$;

k) $(x_1, x_2, x_3) \mapsto (x_1, x_2, x_1 + x_2 + x_3)$?

3902. Prove that a linear operator maps a linearly dependent system of vectors to a linearly dependent system.

3903. Prove that on n-dimensional space for any linearly independent system of vectors $a_1, \dots, a_n$ and an arbitrary system of vectors $b_1, \dots, b_n$, there exists a unique linear operator, which maps a_i to b_i ($i = 1, \dots, n$).

3904. Prove that any linear operator on one-dimensional vector space is of the form $x \mapsto \alpha x$, where α is a scalar.