Assignment 4
COMP 330 Autumn 2015 McGill University

Due Date: 5^{th} Nov 2015

22nd October 2015

There are **6** questions for credit and one for your spiritual growth. The homework is due in class at the beginning of the class.

Question 1[20 points] Consider the grammar

$$S \to aSb|bSa|SS|\varepsilon.$$

What set is generated by this grammar? Prove your answer. You need to say "the language consists of all string of such and such type." Then you need to *prove* that every string generated by the grammar has the property that you claimed and *also* that every string with that property can be generated by the grammar. Thus there will be *two* proofs.

Question 2[15 points] a. Show that the following grammar for simple expressions containing C-style post- and pre-increment operators is ambiguous.

$$\begin{array}{lll} S & \to & V + V \\ V & \to & I|\langle Post\rangle|\langle Pre\rangle \\ I & \to & a|b|c|\ldots \\ \langle Post\rangle & \to & I{+}{+} \\ \langle Pre\rangle & \to & {+}{+}I \end{array}$$

Of course this grammar gives only a very limited fragment of the possible things that you can write in $C++$ or $Java$[1] but it is enough to illustrate the ambiguity problem.

[1]It is completely irrelevent for this question whether you know anything about either of these languages.

b. Give two unambiguous grammars that correspond to the different interpretations in (a).

Alternate Question 2[15 points] Give an example of a context-free language L such that $\mathsf{lefthalf}(L)$ is **not** context free. Note the contrast with the regular case.

Question 3[15 points] We define the language $PAREN_2$ inductively as follows:

1. $\varepsilon \in PAREN_2$,
2. if $x \in PAREN_2$ then so are (x) and $[x]$,
3. if x and y are both in $PAREN_2$ then so is xy.

Give a Chomsky Normal Form CFG for this language. Please, please, please do not come up with a CFG and then painfully convert it to Chomsky normal form (CNF). Directly design your grammar to be in CNF.

Question 4[10 points] Describe a PDA for the language of question 3 which accepts by empty stack. Give all the transitions.

Question 5[15 points] Consider the language $\{a^n b^m c^p | n \leq p$ **or** $m \leq p\}$. Show that this is context free by giving a grammar. You need not give a *formal* proof that your grammar is correct but you must explain, at least briefly, why it works.

Question 6[25 points] For *any* language L, we define $MIN(L)$ to be the language

$$MIN(L) := \{x \in L | \text{ no proper nonempty prefix of } x \in L\}.$$

Show that if L is context free then $MIN(L)$ *need not be* context free. You will find that the ideal CFG to use for this problem is the one from the previous question **slightly modified** so that we require n, m and p all to be strictly positive.

Remark: It is easy to see that if L is regular then so is $MIN(L)$; you do not have to prove this.

Question 6[0 points] Show that the language $L = \{a^n b^n | n \geq 0\} \cup \{a^n b^{2n} | n \geq 0\}$ is context free but is not accepted by any DPDA.