HOMEWORK 4

Due Tuesday, November 7

- The same rules as for previous assignments apply.
- Working through the assignment on your own will help you to learn the material and identify those areas which you need to study more.
- If you have questions, make sure to clear them up during *office hours* or by asking on the myCourses *discussion board*.

Readings:

- Read the textbook Chapters 8 and 9 carefully.
- Read Nagel and Newman's "Gödel's Proof" (1956).

Problems:

- You should not need to consult other sources for answering these questions.
  If you do so nevertheless, intellectual honesty requires you to state the sources!

1. *Typographical and arithmetical rules*.

   Explain in a paragraph or two what the statement in the middle of p. 264 means: "Typographical rules for manipulating *numerals* are actually arithmetical rules for operating on *numbers*." In your explanation, discuss a typographical rule and its corresponding arithmetical rule as an example.

2. *Gödel numbering*.

   a) What is the Gödel number of the formula '0 = 0' according to the numbering scheme presented in GEB?

   b) What is the Gödel number of the formula '0 = 0' according to the numbering scheme presented in the article "Gödel's proof"?

   c) State one more sentence in first-order arithmetic (TNT) with their Gödel numbers in the GEB scheme and the "Gödel's proof" scheme.

3. *TNT numbers*.

   a) Is the mapping from TNT-sentences to natural numbers a function? And, if so, what kind of function; if not, why not?

   b) Is the set of all numbers that are Gödel numbers of *wff*s in first-order arithmetic decidable? Justify your answer.

   c) Explain why TNT-numbers (i. e., the Gödel numbers of theorems of first-order arithmetic) are recursively enumerable (see GEB, p. 269).

d) Is the set of all numbers that are Gödel numbers of theorems of first-order arithmetic decidable? Justify your answer.

4. *History*. Explain in one or two sentences what is

   a) *Principia Mathematica*, and

   b) Hilbert's Program.

   (If you use other sources that the textbook, don't forget to cite them!)

⋆ *Optional question.*

The MU-Puzzle. Present the solution to the MU-puzzle from GEB, pp. 260–261 as rigorously as you can, as a *proof by induction* on the length of MIU-derivations.