MATH 251, homework 8, due date Monday Mar 16.

Problem 1. Find the eigenspaces and eigenvalues

(i) of the map $T(y) = t\frac{dy}{dt}$ on $\mathbf{R}[t]_n$;

(ii) of the map $T(A) = A^t$ on $\mathcal{M}_{n\times n}$.

Problem 2. Let V be a finite dimensional vector space over $\mathbf{C}$ and let $T_1\colon V \to V, \dots, T_k\colon V \to V$ be pairwise commuting linear maps (that is, $T_i \circ T_j = T_j \circ T_i$ for all i, j). Show that there exists $v \in V$ that is an eigenvector of all T_i.

Hint: Prove that the eigenspaces of T_i are preserved by T_j.

Problem 3. Let $T\colon V \to V$ be a linear map. Prove that if λ^2 is an eigenvalue of T^2, then λ or $-\lambda$ is an eigenvalue of T.