MATH 317: Numerical Analysis

Assignment 4 : For practice

1. Construct an efficient routine for solving linear systems. Let

$$U = \begin{bmatrix} -10 & -7 & 0 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}, \quad A = \begin{bmatrix} 2 & 4 & -2 \\ 4 & 9 & -3 \\ -2 & -1 & 7 \end{bmatrix}, \quad b = \begin{bmatrix} 4 \\ 8 \\ -6 \end{bmatrix}$$

   (a) Create two routines for an $n \times n$ matrix, one that implements back substitution, and the other forward substituion. Each should take a matrix U and a vector b as input, and should output a vector x such that $Ux = b$. Apply your functions to solve $Ux = b$ and $U^T x = b$ and show your results.

   (b) Create a routine that, when given a matrix A, computes the LU factorization ($PA = LU$) with partial pivoting. The function should output both L and U, and should also output P. Apply your function to A above and show the result.

   (c) Create a routine that takes a matrix A, a vector b and outputs a vector x such that $Ax = b$. Your code should use the programmes from (a) and (b). Apply your routine to the pair A, b above and show the results.

2. Let's solve the first problem again, only using iterative methods. We have $A\mathbf{x} = \mathbf{b}$ as given above. Solve using the Jacobi and Gauss-Seidel methods. Which one converges faster?

3. Consider the symmetric positive definite matrix

$$A = \begin{bmatrix} 4 & 2 & 0 & 0 \\ 2 & 5 & 2 & 0 \\ 0 & 2 & 5 & 2 \\ 0 & 0 & 2 & 5 \end{bmatrix},$$

   (a) Compute the LDL^T decomposition of the matrix.

   (b) Compute its Cholesky decomposition.

4. Solve the nonlinear system

$$\begin{aligned} -\frac{1}{80}\cos x_1 + \frac{1}{9}x_2^2 + \frac{1}{3}\sin x_3 &= x_1 \\ \frac{1}{3}\sin x_1 + \frac{1}{3}\cos x_3 &= x_2 \\ -\frac{1}{9}\cos x_1 + \frac{1}{3}x_2^2 + \frac{1}{6}\sin x_3 &= x_3. \end{aligned}$$