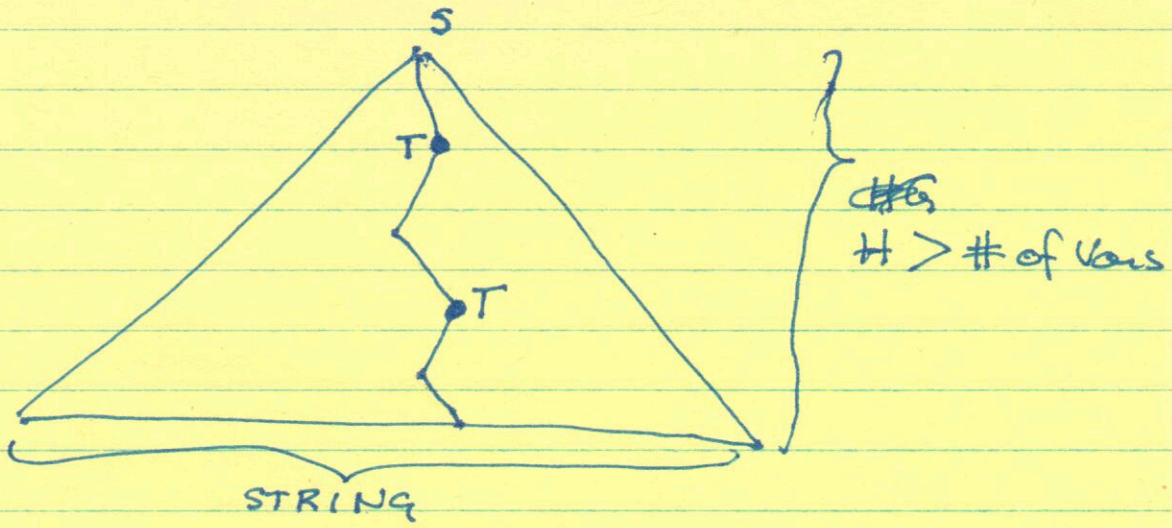
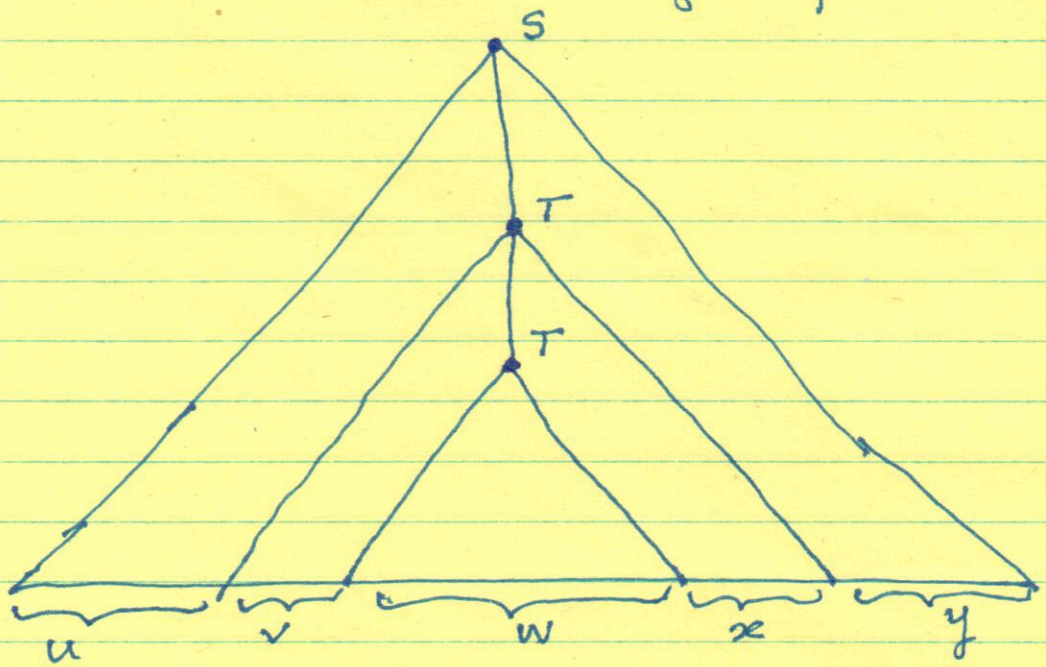
PUMPING LEMMA

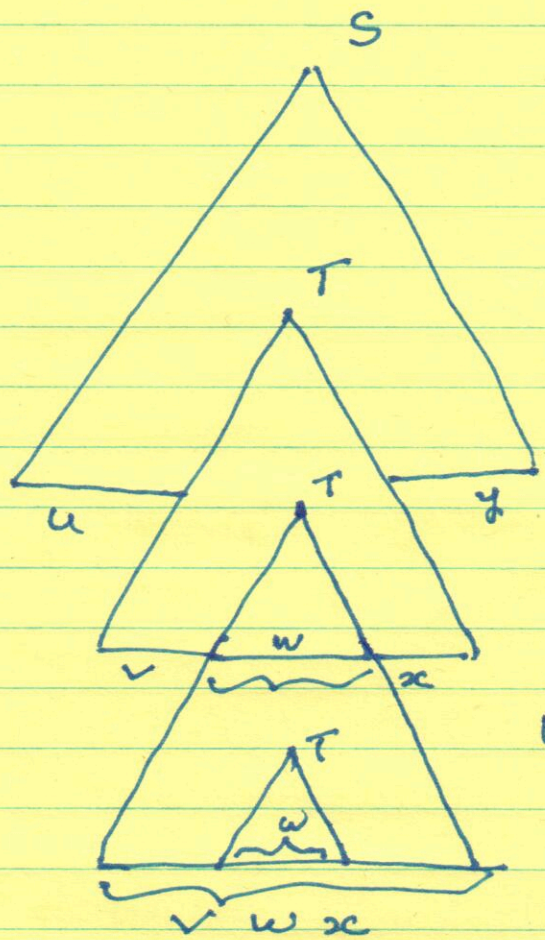


There must be a variable repeated somewhere along the path.



$T \xrightarrow{*} v\,w\,x$

$T \xrightarrow{*} w$



$u v^2 w x^2 y$

$\forall i \geq 0 \quad u v^i w x^i y \in L$

$\forall$ CFLs L $\exists p \geq 0 > 0$

$\forall s \in L \quad |s| \geq p$

$\exists u, v, w, x, y \in \Sigma^*$ s.t.

- $s = uvwxy$
- $|vx| \neq 0$
- $|vwx| \leq p$

$\forall i \geq 0 \quad uv^i w x^i y \in L$

Fix L some language

$\forall p > 0$

$\exists s \in L \quad |s| \geq p$

$\forall u, v, w, x, y \in \Sigma^*$

$s = uvwxy$

$|vx| > 0$

$|vwx| \leq p$

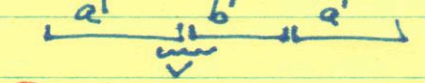
$\exists i \geq 0 \quad uv^i w x^i y \notin L$

Then L is <u>not</u> context-free.

$L = \{ a^n b^n a^n \mid n \geq 0 \}$

Demon chooses $p > 0$

I choose $a^p b^p a^p$



Possible choices for u, v, w, x, y

- v (or x) straddles a block boundary.
  Demon loses because pumping gets a's & b's out of order
- v, x contain only a's then they must be in the same block. so pumping with $i = 2$ will put the string out of L.
- same argument if v, x contain only b's.

- v contains only a's
  x contains only b's
  so one of the blocks is <u>untouched</u> by the pumping & $i = 2$ will put the string out of L.

$\Sigma = \{a, b, c\}$

$L = \{ a^{i+j} b^{j+k} c^{i+k} \mid i, j, k > 0 \}$

Σ : arbitrary $|\Sigma| \geq 2$

$L = \{ww \mid w \in \Sigma^*\}$ $\quad L' = \{w w^{REV} \mid w \in \Sigma^*\}$

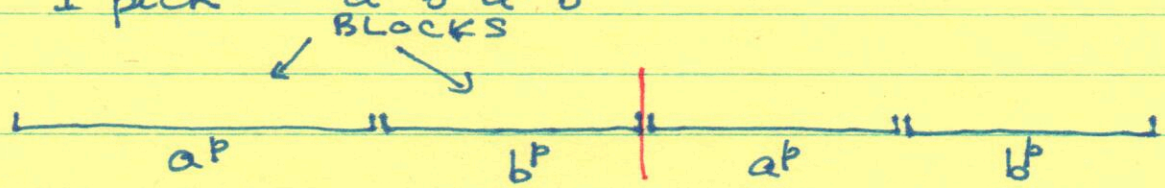
FACT: If L is a CFL, R regular then
$L \cap R$ is a CFL.

so if R is regular & $L \cap R$ is not a CFL then L cannot be a CFL.

$\hat{L} \stackrel{def}{=} L \cap a^*b^*a^*b^*$ $\quad [\Sigma = \{a,b\}]$

Demon picks $p > 0$

I pick $a^p b^p a^p b^p$



- If v,x straddles a block boundary then - - - -
- If v,x are in the same block demon has no hope . . . [please write more on your HW]
- If v's are in the first block & x's are in the second block
  choose $i = 2$
  The first ~~word~~ w starts with "a" so the second copy cannot start with "b".
  with $i=2$ the first "word" is longer than the second.

$\Sigma = \{a, b\}$ $L = \{ww \mid w \in \Sigma^*\}$ is not a CFL but $\overline{L}$ is a CFL.

$S \to AB \mid BA \mid A \mid B$

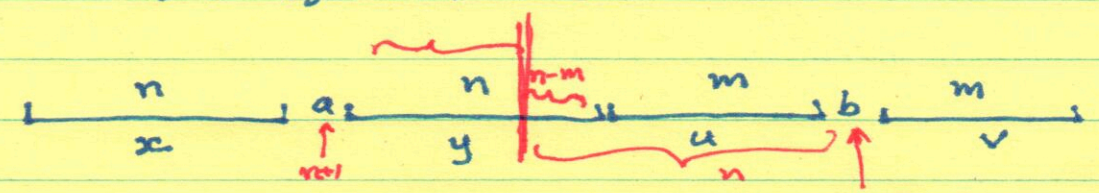
$A \to CAC \mid a \qquad B \to CBC \mid b$

$C \to a \mid b$

A generates strings of the form $x\,a\,y$ $\quad |x| = |y|$ $\quad x, y \in \Sigma^*$

B " $x\,b\,y$ - - - -

$S \to AB$ or BA will generate $x\,a\,y\,u\,b\,v$ where $|x| = |y|$ & $|u| = |v|$



TOTAL $= 2n + 2m + 2$

$\frac{1}{2}$ LENGTH $= n + m + 1$

$n+1$ in its 2nd word.

ASSUME $n > m$

$L_1 = \{a^n b^n c^m \mid n, m \geq 0\}$ $\quad L_2 = \{a^m b^n c^n \mid n, m \geq 0\}$

$L_1 \cap L_2 = \{a^n b^n c^n \mid n \geq 0\}$ not a CFL.