HOMEWORK 3

Due Tuesday, October 24

(Note that this assignment is due Tuesday!)

- The same rules as for previous assignments apply.
- Working through the assignment on your own will help you to learn the material and identify those areas which you need to study more.
- If you have questions, make sure to clear them up during *office hours* or by asking on the myCourses *discussion board*.

Readings:

- Read the textbook Chapters 7 and 8 (up to p. 215) carefully.
- Carefully study Handouts 2, 2a, and 3 on propositional and first-order logic.

  Note, that while you are asked to answer only some of the exercises of the handouts in this assignment, you should nevertheless be able to answer all of them for the midterm and final exams.
- Read Sections I–II of 'Investigations into Logical Deduction' (1935) by Gerhard Gentzen (pp. 68–81). The text can be found in "Readings" on myCourses.

Problems:

- You should not need to consult other sources for answering these questions.
  If you do so nevertheless, intellectual honesty requires you to state the sources!

1. *Propositional logic*.

   Consider the formula $\mathsf{B} \lor (\mathsf{A} \supset \mathsf{A})$.

   (a) Show that it is a tautology.
   (b) Prove it using the Natural Deduction calculus.
   (c) Prove it using the axiomatic calculus.
   (d) Prove it using the GEB calculus.

2. *Propositional logic.*.

   (a) From Handout 2 'A short introduction to propositional logic' do Exercises 8, 10, 11, 13, 15.
   (b) From Handout 2a 'Formula trees and other notations for propositional logic' do Exercise 2.

3. *Soundness and completeness*.

   Explain in two or three paragraphs what the difference between *semantic entailment* and *provability* are, and what it means for propositional logic to be *sound* and *complete*. Try to write in such a way that a friend of yours who isn't taking this course could understand.

   Name a friend who has read your explanations. (Have they understood them?)

4. *Predicate logic*.

   On Handout 3, 'A short introduction to first-order predicate logic', do Exercises 4, 5, and 7

5. *Free variables and substitution*.

   (a) What is a *free variable* in the language of first-order logic?

   (b) Describe when it is allowed to substitute a *term* for a *free variable* without changing the meaning of the formula in question.

6. *Formalization in first-order logic*. Consider the two first-order arithmetic formulas on the bottom of p. 212 of GEB.

   State their meaning in English (try to be as informal as possible) and whether they are true or false in the standard interpretation.

7. *A translation exercise*. Pick two of the five statements listed on p. 215 of GEB and state it in the language of first-order arithmetic.

8. *Predicate logic*.

   Say in English what the conclusion of the deduction below means.

   Say which inference rules appear to have been used in the following Natural Deduction proof. If some inference rule was used incorrectly, explain why.

$$\dfrac{\dfrac{\dfrac{\dfrac{[\mathsf{x} = 0]}{\forall \mathsf{y} : (\mathsf{y} = 0)}}{\mathsf{x} = 0 \supset \forall \mathsf{y} : (\mathsf{y} = 0)}}{\forall \mathsf{z} : (\mathsf{z} = 0 \supset \forall \mathsf{y} : (\mathsf{y} = 0))}}{0 = 0 \supset \forall \mathsf{y} : (\mathsf{y} = 0)}$$

9. *ω-incomplete*.

   Explain to a friend in a paragraph or two what it means for a theory to be ω-incomplete.

⋆ *Optional questions.*

   Answer the following two questions on Gentzen in a brief paragraph:

   a) What is the difference between the two calculi *NJ* and *NK*?

   b) Which calculus is presented in Handout 3?

   Read the excerpt from Frege's *Begriffsschrift* (1879).

   c) On p. 17, Frege speaks of restricting his calculus to 'a single mode of inference'. Explain what this mode of inference is.

   d) How are $\mathsf{A} \wedge \mathsf{B}$, $\mathsf{A} \vee \mathsf{B}$, and $\mathsf{A} \supset \sim(\sim \mathsf{B} \supset \sim \mathsf{A})$ expressed in Frege's notation?