MATH 251, homework 4, due date Monday Feb 2.

Problem 1. Let $T\colon V \to W$ be a linear map and let $U \subset V$ be a subspace such that $U \cap \ker T = \{0_V\}$. Prove that if B is a linearly independent family of vectors in U, then $T(B)$ is also linearly independent.

Problem 2. Consider the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 1 & 2 & 4 \end{pmatrix}.$$

(i) Compute $\operatorname{sgn}(\sigma)$.

(ii) Let π be an arbitrary other permutation of $\{1, \ldots, 5\}$. What is the sign of the composition $\pi^{-1}\sigma\pi$?

Problem 3. Prove that $\det A = \det A^t$.