Lecture 2

An <u>alphabet</u> Σ is a finite set of symbols.
$\Sigma = \{0, 1\}$ or $\Sigma = \{a, b, c\}$, ..

A <u>word</u> over Σ is a <u>finite</u> sequence of symbols from Σ. Thus if $\Sigma = \{a, b, c\}$ words are, for example, a, ba, $abba$, $cabac$, ε : the empty string. The set of <u>all possible</u> words is Σ^*. It is <u>always</u> infinite when $\Sigma \neq \phi$. ~~If~~ We have $\phi^* = \{\varepsilon\}$. Note: $\varepsilon \neq \phi$.

A <u>language</u> is just a subset of Σ^*. If we take $\Sigma = \{a, b, c\}$ examples of languages are:

(1) $\{\varepsilon, a, aa, aaa, aaaa, \dots\} = \{a\}^*$

(2) $\{\varepsilon, ab, aa, \cancel{b}\, aba, abb, \dots$ what pattern do I have in mind

We need a way of describing languages: these ~~are~~ will be studied later.

How do we recognize patterns?

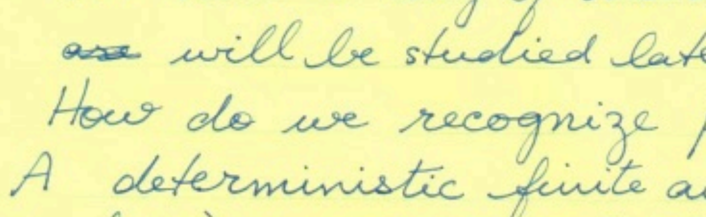
<u>Def</u> A deterministic finite automaton (or finite-state machine) is a 4-tuple $A = (\cancel{\Sigma}, S, s_0, \delta : S \times \Sigma \to S, F \subseteq S)$ where S is a finite set of <u>states</u>

$s_0 \in S$ is the <u>initial state</u>

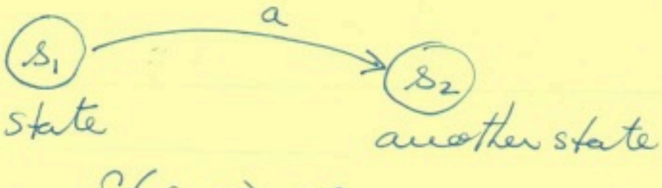
$\delta : S \times \Sigma \to S$ is the transition function

$F \subseteq S$ are the <u>final</u> states or <u>accepting</u> states.

To give examples we draw pictures:

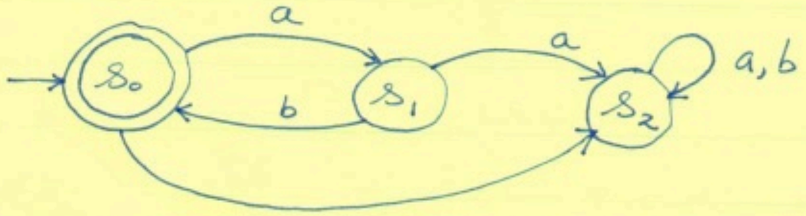


This machine accepts all strings that start with "ab". The machine reads symbols one-by-one and makes transitions. A word is accepted if the machine is in an accept state at the end of the word



$\delta(s_1, a) = s_2$

<u>Example</u>



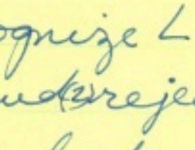
Accepts ε, ab, abab, ababab, ... $(ab)^n$, ...
and <u>nothing</u> else.

<u>Basic cycle</u> (1) read a letter (2) change state (3) read next letter (4) if there are no more symbols STOP (5) If the m/c is in a state in F then accept else reject.

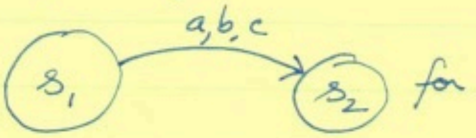
So machines or automata define <u>languages</u>.

<u>Def</u> A language that can be recognized by a DFA is called a <u>regular</u> <u>language</u>.

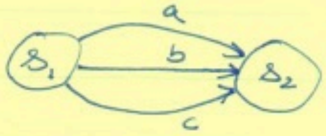
When you design a machine to recognize $L \subseteq \Sigma^*$ it must (1) accept every word in L and (2) reject every word not in L. If you do (1) but not (2) you get 0; you do not get 1/2 credit.
The "size" of a language has almost nothing to do with how hard it is to recognize it.
This mickey mouse machine recognizes Σ^* the biggest language.

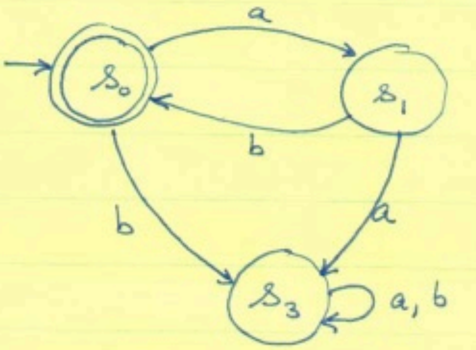


Every state has to have an arrow labelled with every symbol. Sometimes we write



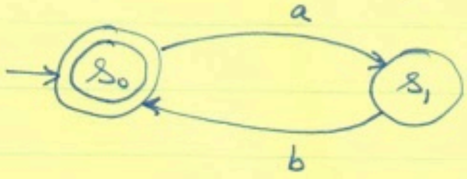
for





s_3 is a dead state: we can never get from it to an accept state ~~on a~~. We cannot even get out of s_3.

For shorthand we may leave out the dead state and the arrows that go to it:

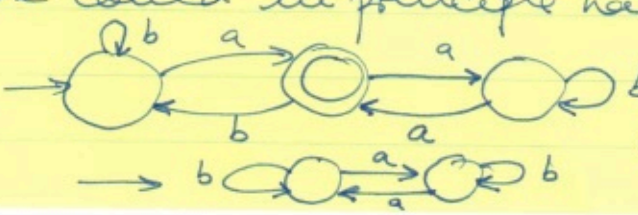


Example $\Sigma = \{1, 2, 3, 4, 5, \cancel{6}\}$ combination 45213

Drawing the dead state would introduce a whole lot of extra arrows.

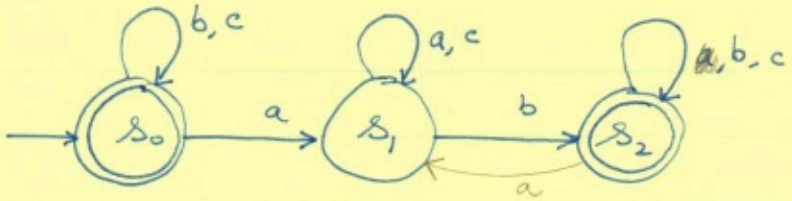
Prop Every finite language is regular.

Example We could in principle have unreachable states



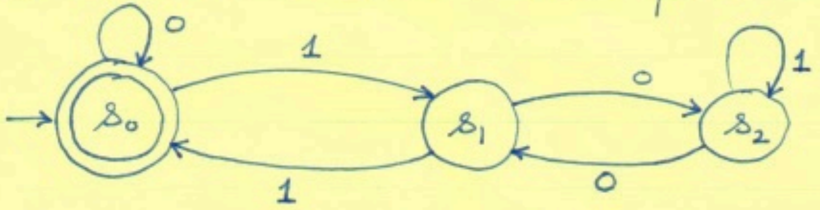
We will assume they are removed.

Useless!

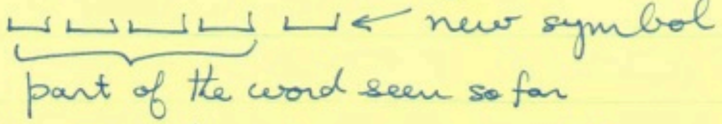


If there is an "a" there must be a "b" sometime later.
aaaccaabccb is OK ababa is <u>not</u> OK.
What must we remember? What do the states represent? They represent whatever we need to remember.

<u>Example</u> $\Sigma = \{0,1\}$ $L = \{$ strings which when interpreted as binary numbers are divisible by 3 : $L = \{11, \varepsilon, 110, 1001, 1100, \ldots\}$ but not 111 or 1 or 001. Interpret ε as 0



We read left to right



If x is the value so far then when we read another 0 we get $2x$; if we read 1 we get $2x+1$.
We only need to do the arithmetic mod 3 : s_0 means $x \equiv 0 \pmod 3$ s_1 means $x \equiv 1 \pmod 3$ & s_2 means $x \equiv 2 \pmod 3$.

$0 \times 2 = 0$ so $\delta(s_0, 0) = s_0$ $\quad 0 \times 2 + 1 = 1$ so $\delta(s_0, 1) = s_1$.

<u>Lesson</u> The states should mean something, they encode your finite memory.

<u>Prop</u> Any machine to recognize L must have at least 3 states.

<u>Proof</u> Suppose M has only 2 states one must be an accept state and the other one a non-accept state. Now consider the strings 100, 101 & 110. The string 110 must go to the accept state and 100 & 101 must both go the other state call it B. Once the machine is in B it does not matter how it got there the subsequent actions are determined. So 1001 & 1011 must go to the same state but 1001 should be accepted & 1011 must be rejected. Thus 100 & 101 cannot wind up in the same state ⊗.

General strategy: You want to prove that there must be at least n states. Find n strings such that they all have to be in different states. For each pair say u, v show that there is some string x s.t. ux is accepted & vx is rejected. This proves that u, v cannot reach the same state. Do this for all pairs.

<u>Some</u> <u>math</u> Given $M = (S, s_0, \delta, F)$ we define

$\delta^* : S \times \Sigma^* \to S$ by induction

$$\delta^*(s, \varepsilon) = s$$

$$\delta^*(s, ax) = \delta^*(\delta(s,a), x)$$

$$L_M = \{x \mid \delta^*(s_0, x) \in F\}$$

A set with a <u>binary</u> <u>associative</u> operation • and an identity ε : $(M, \bullet, \varepsilon)$ is called a monoid.