McGill University
math-315(2015-Fall): Differential Equations for Engineers

Written Assignment # 2

Due date: Tue. Oct. 21 Hand-in in class
(Total 60 point)

(1) (10 points) Perform the phase line analysis for the following autonomous equations:

$$\text{(i)} \quad \frac{\mathrm{d}y}{\mathrm{d}t} = \frac{2}{\pi}y - \sin y; \quad \text{(ii)} \quad \frac{\mathrm{d}y}{\mathrm{d}t} = y^2 - y - 6.$$

(a) Determine the equilibrium states for each of the systems.
(b) Classify these equilibrium state as stabile, unstable or semi-stable.
(c) Plot the integral curves on the (t, y) plane for these systems, respective, and show the locations of the inflection points.

(2) (10 points) One morning it began to snow very hard and continued to snow steadily through the day. A snowplow set out at 8:00 A.M. to clear a road, clearing 2 miles by 11:00 A.M. and an additional mile by 1:00 P.M. At what time did it start snowing. (You may assume that it was snowing at a constant rate and that the rate at which the snowplow could clear the road was inversely proportional to the depth of the snow.)

(3) (10 points) If two straight line in x, y plane having slopes m_1 and m_2, respectively, intersect at an angle θ, show that

$$\tan\theta(1 + m_1 m_2) = m_2 - m_1.$$

Using this fact, find the family of curves that intersects the family of curves: $x^2 + y^2 = c^2$ at an angle 45°.

(4) (10 points)
(a) Show that the identities of operators

$$\begin{aligned} L_1 &= (aD + b)^2 = a^2D^2 + 2abD + b^2; \\ L_2 &= (aD + b)^3 = a^3D^3 + 3a^2bD^2 + 3b^2aD + b^3 \\ L_3 &= (D + a) \circ (D + b) = (D + b) \circ (D + a) \end{aligned}$$

are valid, if a, b are constants. What are L_1, L_2, L_3, if $a = 2, b = 3$?
(b) Do the above identities hold, when $a = a(x), b = b(x)$ are not constants? What are L_1, L_2, L_3, if $a = x, b = x^2$?

(5) (10 points) Give a pair of functions $\{v_1(x) = x \sin x; v_2(x) = x|\sin x|\}$ on the interval $I = [-\pi, \pi]$.

(a) Determine whether these functions are linearly independent or linearly dependent.
(b) Show that the Wronskian $W(x) = W[v_1, v_2]$ exists and calculate its value on the interval (I).
(c) Prove that $\{v_1(x) = \sin x;\ v_2(x) = \sin^2 x\}$ cannot be a set of fundamental solutions of any second order linear differential equation of 2-nd order On (I).

(6) (10 points) Give a pair of linearly independent functions $\{y_1(x) = \sin x; y_2(x) = x \sin x\}$ on the interval $I = (0, \pi)$. Construct a second order linear differential equation, for which $\{y_1(x) = \sin x;\ y_2(x) = x \sin x\}$ is a set of fundamental solutions.

Marker: El Toufaili, Sam (260521406); hussein.toufaili@mail.mcgill.ca