MATH 251, homework 1, due date Monday Jan 12.

Problem 1. Let V be the set of ordered pairs of real numbers with addition defined by

$$(x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2)$$

and multiplication defined by

$$\alpha \cdot (x_1, x_2) = (\alpha x_1, x_2).$$

Is V a vector space with these operations? Justify your answer.

Problem 2. Which of the following subsets of the vector space of all functions $\mathbf{R} \to \mathbf{R}$ are its subspaces:

(i) Functions attaining a fixed value $a \neq 0$ at a specified point b;

(ii) Functions attaining the value 0 at a specified point b;

(iii) Functions with value 0 on a specified set $S \subset \mathbf{R}$;

(iv) Functions with only finitely many discontinuity points;

(v) Functions with value 0 outside of a finite set (dependent on the function).

Justify your answer.

Problem 3. Prove that the external direct sum $U \oplus V$ of vector spaces over $\mathbf{F}$ is a vector space over $\mathbf{F}$ as well.