1102. What will happen with a determinant of size n if

a) the first column is moved to the last column and the other columns are moved to the left while retaining their order;

b) its rows are written down in inverse order?

1103. What will happen to a determinant if

a) we add to each column, starting with the second, the previous column;

b) we add to each column, starting with the second, all previous columns;

c) we subtract from each row, except from the last, the next row and from the last row we subtract the former first row;

d) we add to each column, starting with the second, the previous column, and to the first column we add the former last one?

1104. Prove that the determinant of a skew-symmetric matrix of odd size is equal to 0.

1105. Integers 20604, 53227, 25755, 20927 and 289 are divisible by 17. Prove that the determinant

$$\begin{vmatrix} 2 & 0 & 6 & 0 & 4 \\ 5 & 3 & 2 & 2 & 7 \\ 2 & 5 & 7 & 5 & 5 \\ 2 & 0 & 9 & 2 & 7 \\ 0 & 0 & 2 & 8 & 9 \end{vmatrix}$$

is divisible by 17.

1106. Calculate the determinant without using its expansion:

$$\begin{vmatrix} x & y & z & 1 \\ y & z & x & 1 \\ z & x & y & 1 \\ \frac{x+z}{2} & \frac{x+y}{2} & \frac{y+z}{2} & 1 \end{vmatrix}.$$

1107. What is the value of the determinant in which the sum of rows with even indices is equal to the sum of all rows with odd indices?

1108. Prove that any determinant is equal to the half-sum of two determinants, one of which is obtained from the given one by addition of a number b to all entries of the ith row, while the other one is obtained by similar addition of the number $-b$.

1109. Prove that if all entries of a determinant of size n are differentiable functions in one variable, then the derivative of this determinant is equal to the sum of n determinants D_i, where all rows of D_i, except the ith one, are the same as in D, and the ith row consists of derivatives of entries of the ith row of D.

1110. Calculate the determinants:

a) $$\begin{vmatrix} a_1+x & x & \dots & x \\ a_1 & a_2+x & \dots & x \\ \dots & \dots & \dots & \dots \\ x & x & \dots & a_n+x \end{vmatrix};$$

b) $$\begin{vmatrix} a_1+x & a_2 & \dots & a_n \\ a_1 & a_2+x & \dots & a_n \\ \dots & \dots & \dots & \dots \\ a_1 & a_2 & \dots & a_n+x \end{vmatrix};$$

c) $$\begin{vmatrix} 1+x_1y_1 & 1+x_1y_2 & \dots & 1+x_1y_n \\ 1+x_2y_1 & 1+x_2y_2 & \dots & 1+x_2y_n \\ \dots & \dots & \dots & \dots \\ 1+x_ny_1 & 1+x_ny_2 & \dots & 1+x_ny_n \end{vmatrix};$$

d) $$\begin{vmatrix} f_1(a_1) & f_1(a_2) & \dots & f_1(a_n) \\ f_2(a_1) & f_2(a_2) & \dots & f_2(a_n) \\ \dots & \dots & \dots & \dots \\ f_n(a_1) & f_n(a_2) & \dots & f_n(a_n) \end{vmatrix},$$

where $f_i(x)$ are polynomials of degree, at most, $n-2$ $(i = 1, 2, \dots, n)$;

e) $$\begin{vmatrix} 1+a_1+b_1 & a_1+b_1 & \dots & a_1+b_n \\ a_2+b_1 & 1+a_2+b_2 & \dots & a_2+b_n \\ \dots & \dots & \dots & \dots \\ a_n+b_1 & a_n+b_1 & \dots & 1+a_n+b_n \end{vmatrix}.$$

12 Expanding a determinant according to the elements of a row or a column

1201. Calculate the determinant by expanding it according to the elements of the third row

$$\begin{vmatrix} 2 & -3 & 4 & 1 \\ 4 & -2 & 3 & 2 \\ a & b & c & d \\ 3 & -1 & 4 & 3 \end{vmatrix}.$$