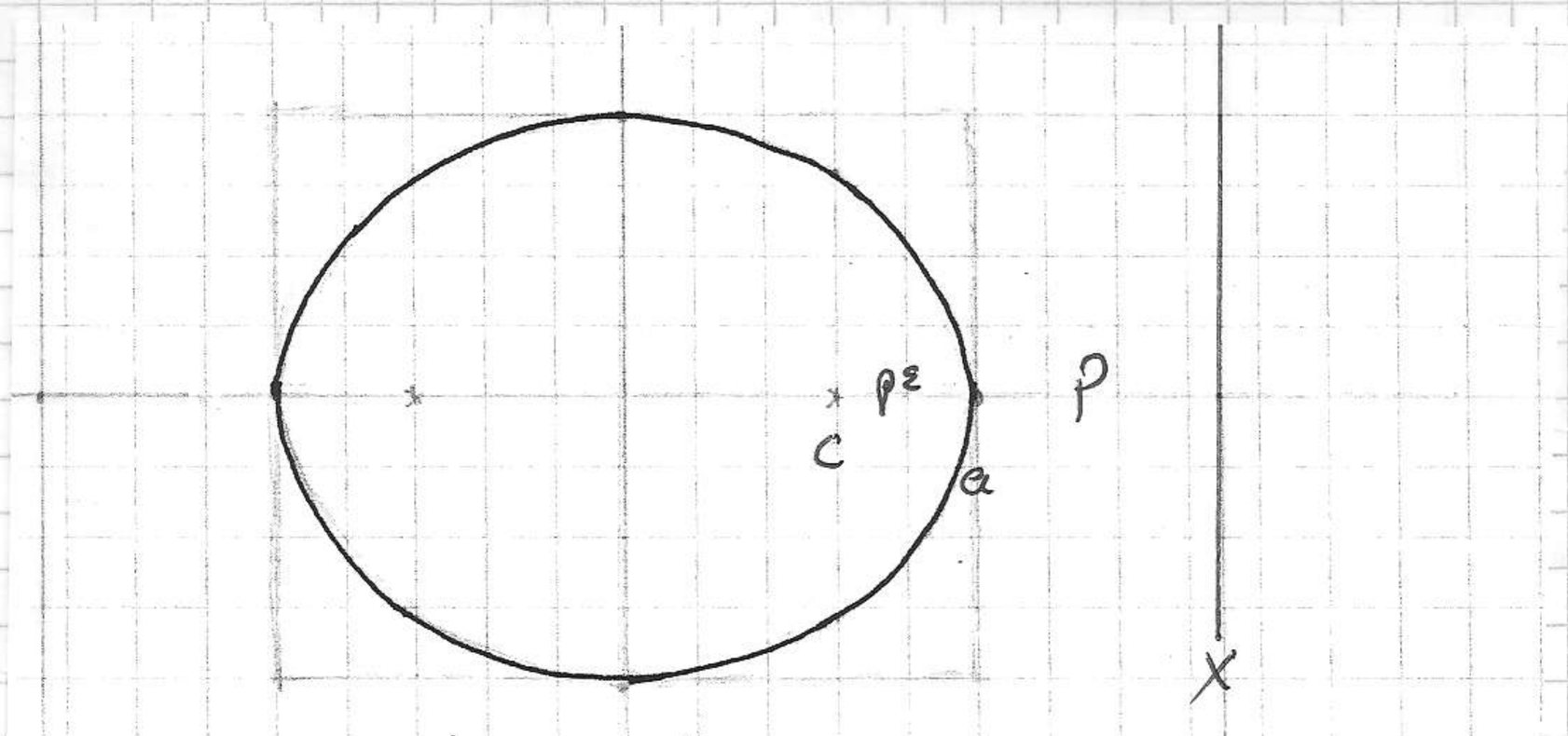
Homework 5: Answers

#3



we know $\varepsilon = \frac{c}{a}$

we see $c + \rho\varepsilon = a$, so $c + \rho\frac{c}{a} = a$. Solve for ρ

$\rho = \frac{a^2 - ca}{c}$, so $X = a + \frac{a^2 - ca}{c} = \frac{a^2}{c}$

#4. Two ways to do this

a) The function is $y = 10x$ is a one to one function from $0 \le x \le 1$ onto $0 \le y \le 10$ and it carries rational numbers to rational numbers.

b) $\#\mathbb{Q} = \aleph_0$ is the smallest transfinite number, so an infinite subset of $\mathbb{Q}$ cannot have fewer than $\aleph_0$ elements, nor more, so every infinite subset of $\mathbb{Q}$ has $\aleph_0$ elements.

#5 a) This is almost the definition of "countably infinite".

$\mathbb{N}$ is countably infinite if there is a one to one function of $\mathbb{N}$ onto $\mathbb{N}$.

There is one - the identity $f(x) = x$.

b) The angles in any triangle <u>in the Euclidean universe</u> add up to 180°, but not in any non-Euclidean geometry. You must specify where you are before answering the question.

c) The criteria for "truth" are completely different. The laws of physics depend on empirical evedence and can neves have the same truth value as the theorems of mathematics, which will remain true when the unviese collapses into a black hole where all the "laws" change. On the other hand the various mathematical worlds, where our theorems are true forever, are just worlds of the imaginatios which may or may not exist in any real sense.

d) This is the "Continuum Hypothesis" which is true in some mathematical models and false in others. If you start with the usual axioms for mathematics you must say the Continuum Hypothesis is neither true nor false. It is independent.
So the answer to "True or False?" is "NO".