MATH 317: Numerical Analysis

Assignment 2 : Due 23 October, 2017

Important: Submit a complete hard copy of all your solutions either in class or to the Math Dept Office opposite the elevators on 10th floor of Burnside before 4pm on the due date; this must include Course number, name and id number (or it will get lost). Solutions should be complete and include hardcopy of all electronic output, and code, together with explanations. Work not submitted on paper will not be graded. Submit all relevant program files and documentation to explain them. Get in the habit of including comments in your code. Write your own code! Otherwise, why are you here?

1. Let's say we want to compute $\cot(0.0015)$ as accurately as possible using the following table and Lagrange interpolation by a) interpolating for $\cot(x)$, b) by interpolating for $\cos(x)$ and $\sin(x)$ and using them to calculate $\cot(x)$. Also, estimate the error in b) and explain the difference in a) and b). Here is the table:

x	$\sin x$	$\cos x$	$\cot x$
0.001	0.001000	1.000000	1000.0
0.002	0.002000	0.999998	499.999
0.003	0.003000	0.999996	333.332
0.004	0.004000	0.999992	249.999
0.005	0.005000	0.999988	199.998

2. Construct a natural cubic spline to approximate $f(x) = \sin \pi x$ by using the values at $x = 0, 0.25, 0.5, 0.75$ and 1.0. Integrate the spline over [0,1] and compare to the analytic answer. Use the derivatives of the spline to approximate $f'(0.5)$ and $f''(0.5)$. Compare to the analytic values.

3. Use Lagrange interpolation to construct an approximation to $\int_{-1}^{1} f(x)\ dx$ with nodes at $x = -1$, $-1/2$, $1/2$ and 1.

4. Find the constants x_o , x_1 and c such that the numerical integration formula

$$\int_0^2 f(x)\ dx \approx f(x_o) + cf(x_1)$$

has the highest precision possible.