An introduction to Options Pricing and the Black-Scholes Model

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What is an option?

An option is a financial contract that gives you the right, but not the obligation, to buy **or** sell an asset (like a stock) at a fixed price (which we call the strike price) on or before a certain date (the expiry).

This means there are two different options:

- **Call option**: An option that gives you the right to **buy** the asset at the strike price
- **Put option**: An option that gives you the right to **sell** the stock at the strike price

Here's a simple example of how a call option works and how it can make you money; suppose there is a stock that currently sits at £100. You now choose to purchase a call option with a strike price of £110 that expires in a month. There are two scenarios that can come from this purchase:

1. Scenario A

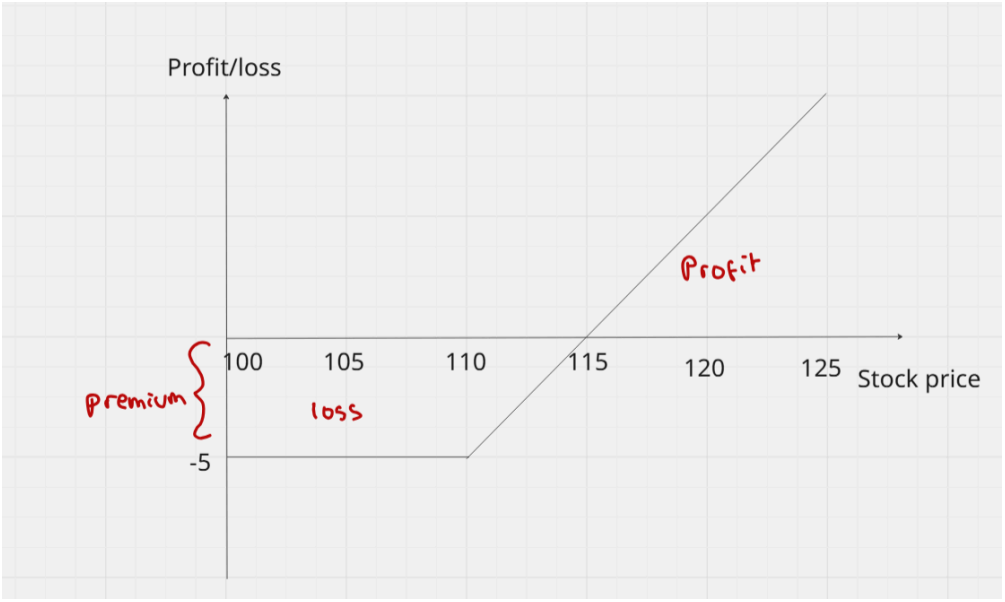
The stock goes up, say to £125. You will now buy the stock at the strike price and sell immediately to the market or sell the option contract itself and keep the profit.

Remember, buying a call option requires an upfront payment called the **premium** — the cost of the option itself. Say in this simple example, the premium was £5. Your profit is calculated based on the difference between the stock's price at expiration and the strike price, minus this premium:
Profit = Stock price at expiry - (Premium + Strike price)
So in our example it would be: Profit = 125 - (5 + 110) = £10

2. Scenario B

The stock actually goes down, say to £90, or even stays the same at £100 it wouldn't make sense for you to exercise your call option. Therefore your loss would be limited to the premium you paid for the call option (£5).

Figure 1



What we can take from this is that call options are purchased when one thinks that the price of the asset will increase as profit is only made when the stock price rises above the sum of the premium and strike price.

Let's explore a simple example of how a put option works. Suppose the current stock price is £110 and we buy a put option with a strike price of £95 that expires in one month, for a premium of £5. Simply, this just means that you have the right, but not the obligation, to sell the stock for £95 at the expiry. There are three scenarios that could occur from here:

1. Scenario A

The stock falls to say £80 at the expiry. You will now sell the stock at £95 and your profit is calculated using the strike price, stock price at expiry and the premium:

Profit = Strike price - (Premium + Stock price at expiry)
So in our example: Profit = 95 - (5 + 80) = £10

2. Scenario B

The stock stays at £110 and so selling for less than your strike price makes no sense. In this case your option expires worthless and your loss is the premium (£5).

3. Scenario C

The stock rises, say to £120. Once again there's no point selling for less, your option expires worthless and your loss is the premium.

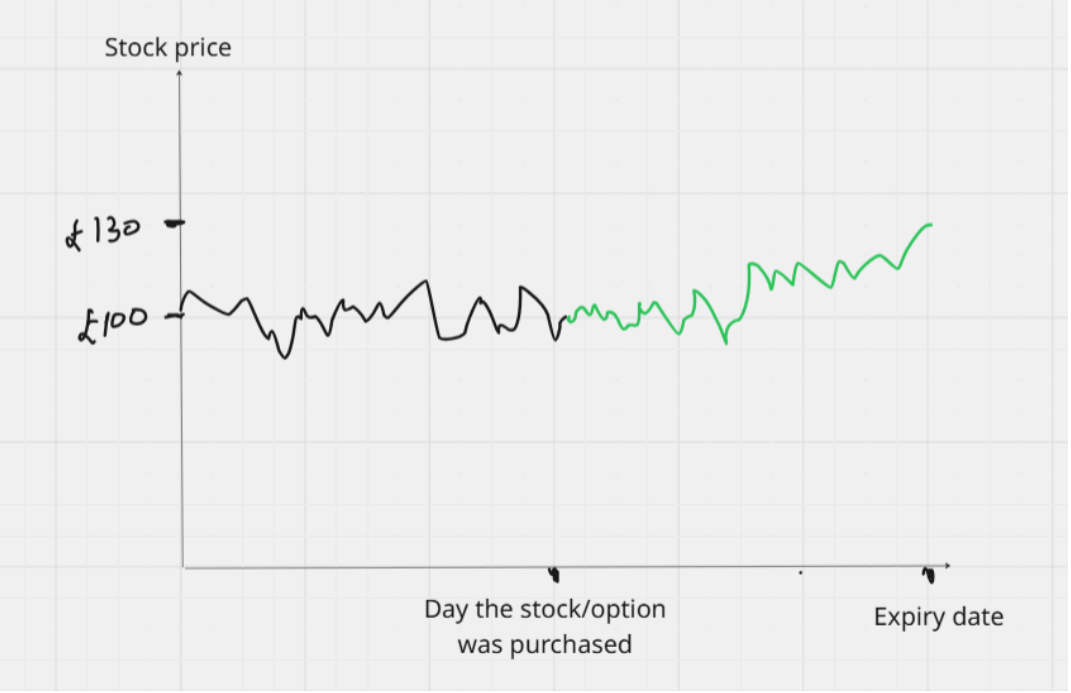
Figure 2

From this example, we can see that a put option is useful when you expect the stock price to fall. Profit is only made if the stock drops below the strike price, after accounting for the premium paid.

Options VS Stocks

- As we saw from **Figure 1** and **Figure 2** the maximum loss one can make is the price of the option (the premium). This defined risk is one of the key advantages of options. However, if you buy a stock the loss can continue until the price reaches zero. With options your downside is capped and clearly defined.
- Options also allow you to control a stock for less money in comparison to purchasing a stock outright. For example, buying 1 share of a company may cost £100, but buying a call option on that stock may only cost £5. Effectively, allowing you to potentially benefit from the stocks's movement while investing far less capital.
    - This can serve to be both positive and negative to somebody who is looking to buy an option. Take the following example:
    - Suppose the current price of a stock sits at around £100, we purchase a call option with a premium of £10 and strike price of £100. Now lets say that at the expiry, the stock price increases to £130. By now selling your stock to the market or option contract itself at the market price (£130) you would then make a profit of £20 after subtracting the £10 spent on the premium - this is a 200% return. In comparison to if we had invested £100 into buying the stock we would have only made a 30% return. The following table and graph can help visualise this example:

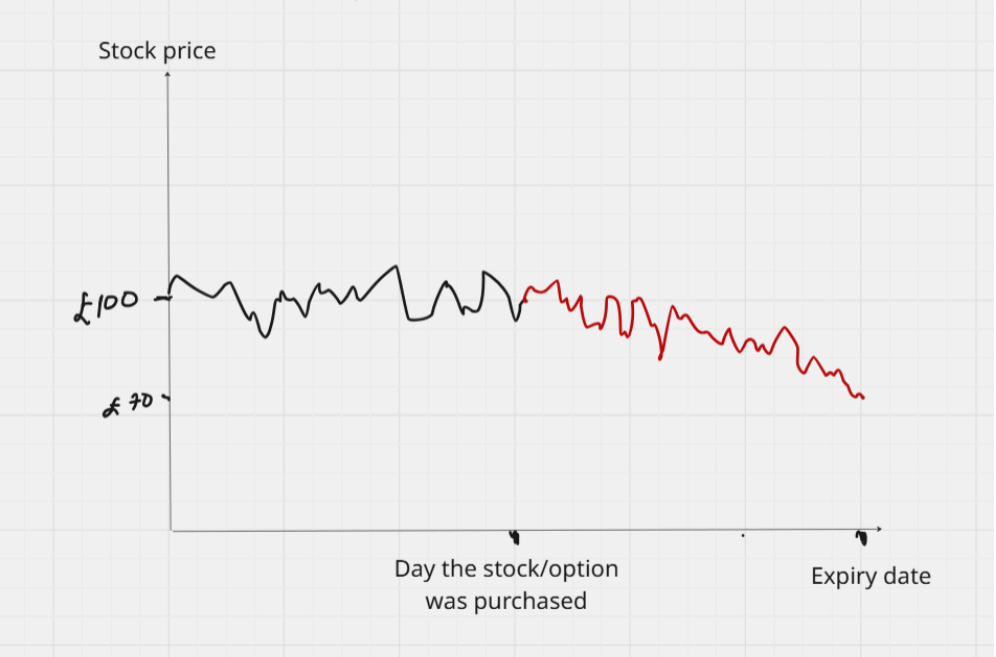
	Money invested	Profit	Return
Stock	£100	£30	30%
Option	£10	£20	200%



    - However, this can also be negative as had the price of the stock reduced, say to £70, or stayed the same we wouldn't have exercised our option and would have lost 100%

of our initial investment (the premium - £10). Whereas with stocks it would most likely be a fraction of what was initially invested. You can see this from the following table and graph that helps visualise this:

	Money invested	Loss	Return
Stock	£100	-£30	-30%
Option	£10	-£10	-100%



How can we determine the fair price of an option?

Let's quickly understand how the stock market works. Simply put, it is a continuous double auction between buyers (demand) and sellers (suppliers) of a stock, and the stock price is simply the price of the most recent trade. When more people want to buy a stock (demand increases) the price goes up, and when more people want to sell a stock (demand decreases), the price goes down. The demand and thus price of a stock is affected by many things, like politics, regulation, technology and much more.

In the early to mid 1890s, Bachelier, who is now considered the father of mathematical finance, was then only a physics student who was employed at the Paris Stock Exchange. Here he observed first hand the chaotic environment of the hand signals and shouting, noticing that the price of options were set by what the seller and buyer negotiated. After understanding how the price of a stock is determined, Bachelier reasoned that it is impossible for one to account for and quantify all these factors, in order to accurately predict where the market will move. He suggested that the best thing to do is assume that the price of a stock is as likely to go up as it is to go down, effectively proposing that the market follows a random walk.

The behaviour of the price of a stock is akin to that of a ball being dropped into a galton board. Each ball represents a possible “movement” route in the stock's price. We can see what is meant by this from the following images.

Figure 3

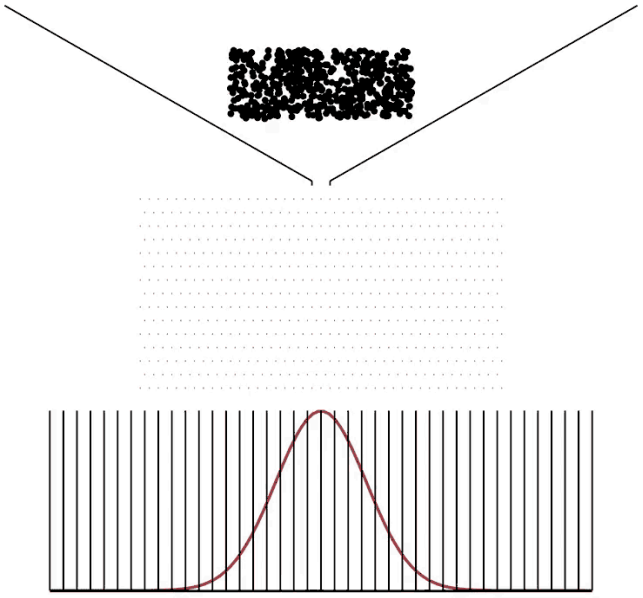


Figure 4

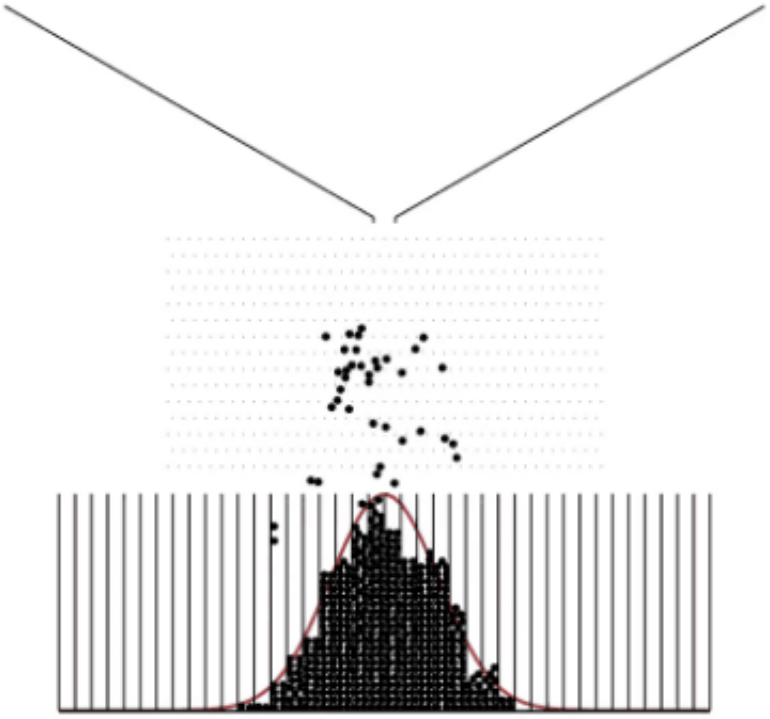
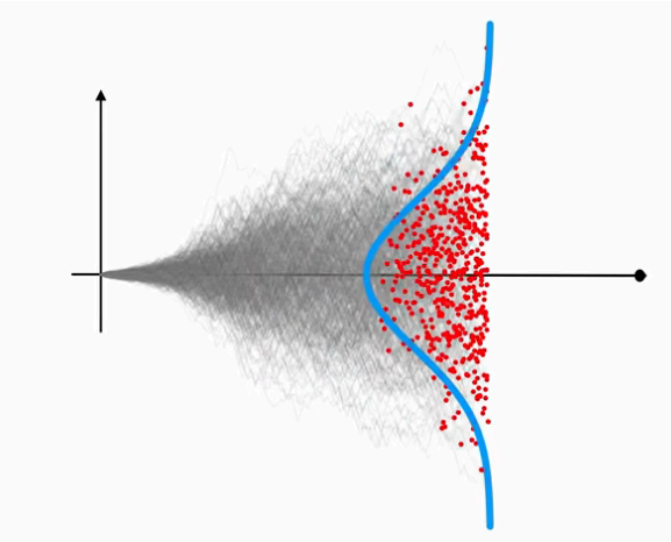


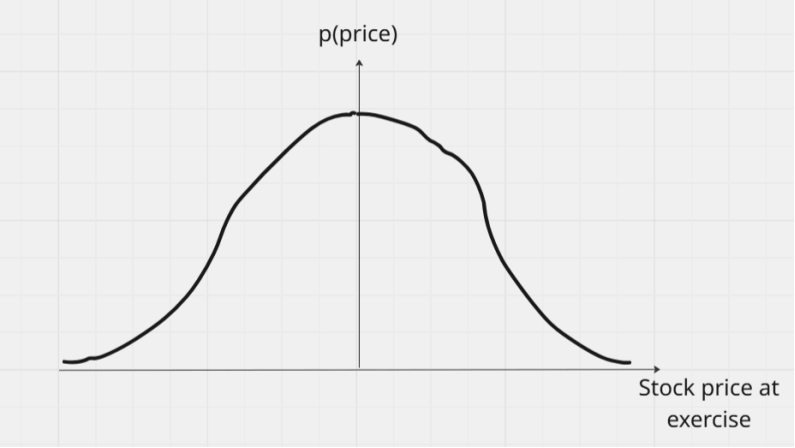
Figure 5



Bachalier believed that each additional layer of pegs represents a time step such as a day, minute or second. This means that as the time increases, the variance of the final position increases.

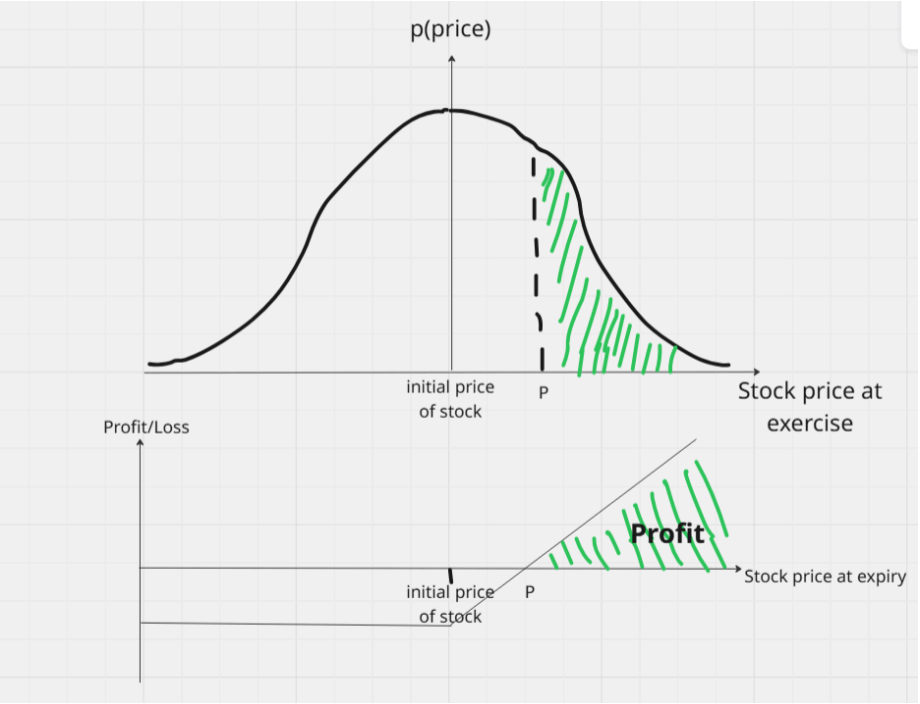
Bachelier then proposed that the expected future price of a stock can be described by a normal distribution. This idea is illustrated by the following graph, which shows a bell curve where the x-axis represents the possible prices at expiry and the y-axis indicates the probability of each outcome.

Figure 6



Now say we take a look at call options again, you only make money when the stock price increases by more than the premium you paid for it. So from **Figure 6**, the probability of the option buyer making a profit would be represented by the sum of the following probabilities.

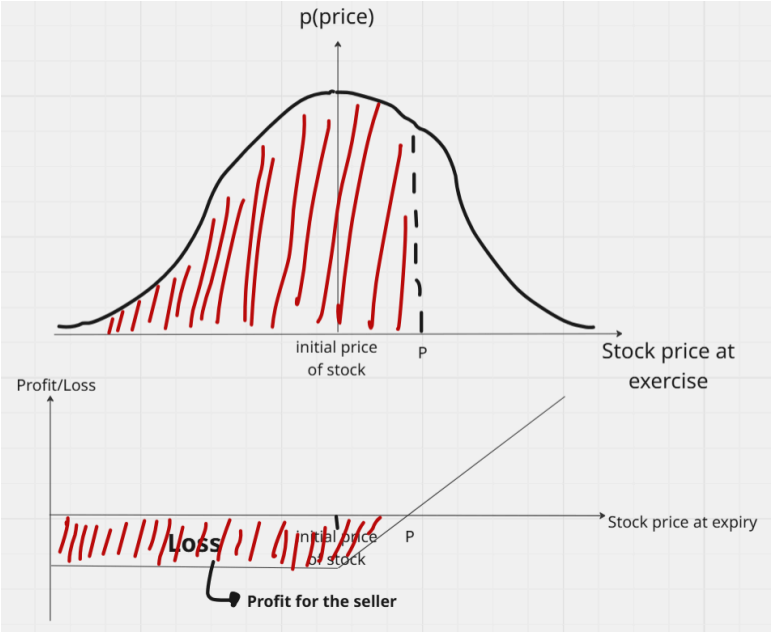
Figure 7



P is the sum of the initial price and the premium paid for it. Only once the stock price increases above this value "P" can profit be made.

The probability of the seller making a profit is: 1 - P(price>P) = P(price<P), this corresponds to the unshaded side.

Figure 8

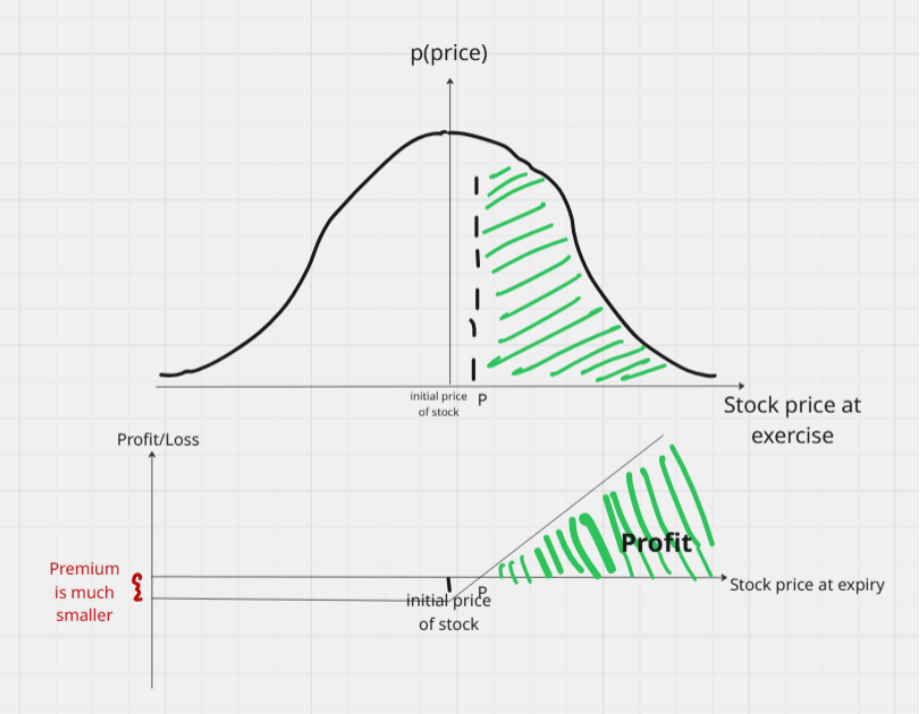


When it falls below the P price it means the seller makes a profit because the buyer won't exercise the option and they get to keep the premium

For this example, the expected profit/loss for this stock sold at this premium is in favour of the seller, meaning the option is overpriced and thus nobody will want to buy it. Hence Bachalier proposed that

the fair price of an option is when the buyer is as likely to make a profit as the seller is - both parties should stand to gain or lose the same amount.

Figure 9



However in this example above where the premium is much smaller the probability of the buyer profiting is much higher and hence not fair for the seller. This is because the seller will **most likely** be forced to sell the stock at a reduced cost (the strike price), below its market value, rather than keeping the stock and profiting. If the premium is too low, the trade disproportionately benefits the buyer and isn't an attractive deal for the seller.

In 1973, Myron Scholes, Fischer Black and Robert Merton developed the most famous equation in finance - relating the price of any contract to any asset:

Figure 10

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^s \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

When solving this partial differential equation, you get an explicit formula for the 'fair' price of the option as a function of a bunch of parameters, such as the strike price, volatility etc.

Figure 11

$$C = N(d_1)S_t - N(d_2)Ke^{-rt}$$

$$\text{where } d_1 = \frac{\ln\frac{S_t}{K} + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

$$\text{and } d_2 = d_1 - \sigma\sqrt{t}$$

C = call option price
N = cumulative distribution function of the normal distribution
S_t = spot price of an asset
K = strike price
r = risk-free interest rate
t = time to maturity
σ = volatility of the asset

How does the Black–Scholes model guide a traders' decision?

The Black-Scholes Model (BSM) gives a theoretical "fair" price of an option based on factors like the strike price, volatility etc. Traders will compare this fair price to the market price of the option to make the decision of whether they should buy it or not. If the option is trading below its BSM value then according to the model it is undervalued and incentivises the buyer to purchase it. However if it is trading above, it is over valued - so you shouldn't buy and should consider selling an option in order to collect the premium.

Further Reading & Sources

1. Investopedia – Black-Scholes Model
2. Khan Academy – Introduction to the Black-Scholes Formula
3. Corporate Finance Institute – Black-Scholes-Merton Model
4. Macroption – Black-Scholes Formulas & Greeks
5. MoontowerMeta – Intuition Behind Black-Scholes
6. Veritasium – The Trillion Dollar Equation (YouTube)