[L] Clogging in Granular Flor through a Bottleneck

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Granular clogging affects systems from hourglasses to industrial hoppers, yet the balance between geometric confinement and frictional stability remains unclear. Here we show, using molecular dynamics simulations, that clogging results from a sharp geometric transition at a critical orifice width of $D \approx 0.18$. We find that inter-particle friction stabilizes arches in intermediate regimes, yet the jamming transition persists even with particle size disorder. These results demonstrate that granular arrest is fundamentally geometry-dominated, regardless of crystalline packing artifacts.

Project Topic: Granular Matter

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I. INTRODUCTION

Granular materials exhibit a unique duality, behaving like fluids during flow but jamming like solids under stress, a transition governed by the stability of internal force networks. This complex behavior makes predicting flow arrest in confined geometries a persistent challenge in soft matter physics.

Granular flows through constrictions occur in many industrial and natural processes, such as hopper discharges, pedestrian evacuations, granular silo flows or even sand clocks [1]. A key feature of these systems is the formation of arches that block flow, known as clogging [2]. Understanding the statistical properties of clogging, including how it depends on particle and system parameters, is important for optimizing material handling and predicting jamming events. Previous work has shown that orifice size, particle size distribution, and frictional properties significantly affect clogging probability [3]. In this project, we will try to model these effects using computational methods and look at the results to see what patterns appear [4]. Some simpler models might not capture all the details, but they can still give a useful idea of how clogging behaves in different situations. The goal is to get a better sense of which parameters are most relevant for clogging and how they interact.

Here, we investigate how orifice size and friction interact using Discrete Element Method (DEM) simulations. We vary the orifice width and friction coefficient to characterize the sharp transition from steady flow to jamming. Our model is intentionally minimal, balancing physical realism with calculation speed to isolate the core causes of clogging. This method tracks individual particle contacts, allowing us to see exactly how arches form and link these small interactions to the overall flow behavior.

II. OVERVIEW

To study clogging, several computational methods are available [4, 5]. **Table I** summarizes three relevant approaches, detailing their use case, key features, and suitability for the project.
The chosen and combined application of these methods will allow the study of the clogging problem, ranging from detailed simulation of individual particles [4] to the efficient exploration of the parameter space.

Method 1: Molecular Dynamics (Particle Simulation)

The Molecular Dynamics (MD) method, or specifically the Discrete Element Method (DEM) in the granular context, is the primary approach for reproducing realistic clogging dynamics in hopper or bottleneck flows. Its main use case is simulating the motion and interaction of individual grains in these systems. This method resolves direct particle contacts, which is crucial for capturing the formation of stable arches that lead to clogging. While it is computationally intensive, which can limit system size or duration, it is the most suitable method for obtaining high-fidelity results comparable to real experiments on the mechanics of clogging, such as those discussed in literature [4]. This makes it the main tool for the project.

Method 2: Cellular Automata (Rule-based)

The Cellular Automata (CA) approach, particularly rule-based variants, provides a computationally inexpensive alternative to detailed particle simulations.[4] Its use case involves the rapid exploration of the parameter space using simplified lattice flow models. It is a grid-based model where cell states update based on local rules, allowing for very fast simulation and clear visual outputs of flow and clogging patterns.[2] Its main drawback is the lack of mechanical detail, as it does not resolve real contact forces. However, it is an excellent complementary method for performing broad parameter sweeps to quickly identify regions of interest before resorting to more costly MD simulations.

TABLE I: **Overview of Simulation Methods/Models for Granular Flow Clogging**. Summarizes the trade-offs between the three modeling approaches. While Molecular Dynamics provides the mechanical fidelity necessary to simulate clogging arches, Cellular Automata offer a computationally efficient alternative for broad parameter sweeps. Stochastic integration is reserved for scenarios requiring the analysis of noise-induced unblocking events.

Method / Model	Use Case Scenario	Key Features (Summary)	Suitable for the Project?
Molecular Dynamics / Particle Simulations	Simulating individual grains in hopper or bottleneck flows	Resolves particle contacts, friction and normal forces, realistic arch formation; computationally intensive for large systems	**Yes** — main method to reproduce realistic clogging dynamics.
Cellular Automata / Rule-based CA	Rapid exploration of parameter space using simplified grid-based dynamics	Computationally cheap, easy to visualize clog/no-clog behavior; lacks force realism and detailed contact modelling	**Possible** — less computational for wide parameter sweeps.
Brownian Dynamics / Stochastic Integration	Addition of stochastic driving or thermal-like perturbations	Captures fluctuations, simple to implement; does not model contact mechanics accurately	**Optional** — only if exploring noise-induced effects.

Method 3: Brownian Dynamics / Stochastic Integration
Brownian Dynamics (BD), or the inclusion of Stochastic Integration in the equations of motion, is used to incorporate the effects of randomness and fluctuations.[4]. Its use case is the optional addition of stochastic noise to particle motion, modeling scenarios where small external vibrations or inherent randomness might affect the stability of a clogging arch.[6] This feature allows it to model fluctuations and potentially smooth highly irregular flow behavior. The method is considered optional because, while it captures noise effects, the stochastic force must be carefully calibrated to avoid inaccurate representation of the granular contact mechanics. It may be used if the project requires the study of how kinetic or external noise influences clogging frequency and time.

III. METHOD

To simulate the granular discharge, we employ a numerical model that resolves the trajectories and contact interactions of individual particles under gravity.

A. Model and equations of motion

We model a two-dimensional granular silo as a collection of $N = 200$ circular grains (discs) of radius R and mass m, moving under gravity and interacting via short-range contact forces. Each particle i is characterized by its position $\mathbf{r}_i = (x_i, y_i)$ and velocity $\mathbf{v}_i = \dot{\mathbf{r}}_i$. The system evolves according to Newton's equations of motion, which are integrated numerically using a discrete-element–like molecular dynamics scheme.

The total force acting on particle i is the sum of gravitational forcing, contact forces with other grains, contact forces with the confining walls, and an optional stochastic agitation term:

$$m\,\ddot{\mathbf{r}}_i = m\,\mathbf{g} + \sum_{j\neq i} \mathbf{F}_{ij}^{\mathrm{cont}} + \mathbf{F}_i^{\mathrm{wall}} + \mathbf{F}_i^{\mathrm{noise}}. \qquad (1)$$

The gravitational acceleration is $\mathbf{g} = (0, -g)$, with g constant throughout the simulation. Time integration is performed using a fixed time step chosen small enough to resolve contact dynamics and ensure numerical stability during collisions.

B. Contact and friction law

Interactions between grains occur only when particles overlap. For two particles i and j, an overlap is defined

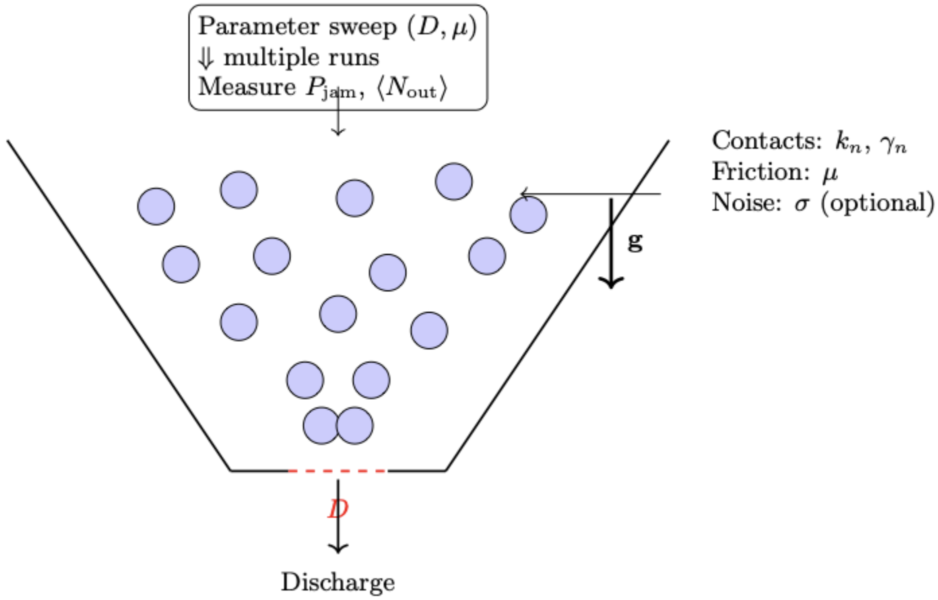


FIG. 1: **Method.** A 2D granular silo discharges discs through an orifice of width D under gravity. Particles interact via dissipative normal forces and Coulomb friction μ. We systematically sweep the parameter space (D, μ), computing jamming probabilities P_{jam} and discharge statistics over multiple independent realizations with optional geometric disorder.

as $\delta_{ij} = 2R - |\mathbf{r}_i - \mathbf{r}_j| > 0$. A repulsive normal force is applied along the unit normal $\mathbf{n}_{ij} = (\mathbf{r}_i - \mathbf{r}_j)/|\mathbf{r}_i - \mathbf{r}_j|$:

$$\mathbf{F}_{ij}^{n} = k_n\, \delta_{ij}\, \mathbf{n}_{ij} - \gamma_n\, (\mathbf{v}_{ij} \cdot \mathbf{n}_{ij})\, \mathbf{n}_{ij}, \tag{2}$$

where $\mathbf{v}_{ij} = \mathbf{v}_i - \mathbf{v}_j$, k_n is the normal stiffness, and γ_n introduces dissipation through velocity-dependent damping. This term effectively controls the restitution of collisions and prevents unphysical energy growth.

Tangential friction is modeled using a Coulomb-like law. During sustained contact, a tangential displacement $\boldsymbol{\xi}_{ij}$ is accumulated at the contact point and generates an elastic tangential force limited by a Coulomb threshold:

$$\mathbf{F}_{ij}^{t} = -\min\Big(k_t\, |\boldsymbol{\xi}_{ij}|,\, \mu\, |\mathbf{F}_{ij}^{n}|\Big)\, \mathbf{t}_{ij}, \tag{3}$$

where k_t is the tangential stiffness, μ is the friction coefficient, and $\mathbf{t}_{ij}$ is the local tangential unit vector. When the contact breaks, $\boldsymbol{\xi}_{ij}$ is reset to zero. The total particle–particle contact force is $\mathbf{F}_{ij}^{\text{cont}} = \mathbf{F}_{ij}^{n} + \mathbf{F}_{ij}^{t}$.
Particle–wall interactions are treated using the same normal and tangential force laws, with overlaps computed relative to the wall geometry. This ensures consistent mechanical behavior between grain–grain and grain–wall contacts.

C. Brownian agitation and geometric disorder

To probe the robustness of clogging and to mimic weak external perturbations such as vibrations, we optionally include a stochastic forcing term acting on each particle:

$$\mathbf{F}_i^{\text{noise}} = \sigma\, \boldsymbol{\eta}_i(t), \tag{4}$$

where $\boldsymbol{\eta}_i(t)$ is a Gaussian white noise with zero mean and unit variance in each Cartesian component, and σ controls the noise strength. This term introduces small, random velocity fluctuations that can destabilize marginal contact configurations and incipient arches.

In addition, a separate set of simulations includes mild radius disorder (polydispersity) by assigning particle radii from a narrow distribution around R. This breaks crystalline packing tendencies and allows us to test whether clogging is sensitive to geometric ordering or instead governed primarily by outlet geometry and frictional stabilization.

D. Silo geometry and control parameters

The confining geometry consists of two straight walls forming a symmetric funnel that terminates in a horizontal orifice of width D at the bottom (Fig. 1). Particles are initially placed above the outlet and allowed to relax under gravity before discharge begins.

The main control parameters explored in this study are: (i) the dimensionless orifice width D, reported in units of the particle diameter, and (ii) the friction coefficient μ, which applies to both particle–particle and particle–wall contacts. For each pair (D, μ) we perform multiple independent realizations, obtained by randomizing initial particle positions and, when applicable, the noise realizations. This ensemble-based approach allows us to compute statistically meaningful averages and probabilities.

E. Jamming criterion and measured observables

A simulation run is classified as *jammed* when two conditions are simultaneously satisfied: (a) no particle crosses the outlet for a sustained time window Δt_{jam}, and (b) the configuration near the orifice displays a mechanically stable arch spanning the opening, verified by the persistent presence of contacting grains bridging the outlet. This combined criterion avoids misclassifying temporary flow interruptions as true jams.

The jam probability is defined as

$$P_{\mathrm{jam}}(D, \mu) = \frac{N_{\mathrm{jam}}}{N_{\mathrm{runs}}}, \tag{5}$$

where N_{jam} is the number of jammed realizations out of N_{runs} independent simulations. In addition, we measure the mean number of discharged grains $\langle N_{\mathrm{out}} \rangle$ before jamming (or before the simulation end time in free-flow cases), as well as the number of grains remaining in the silo after a fixed simulated duration. Together, these observables provide a comprehensive characterization of discharge efficiency and flow interruption across the explored parameter space.

F. Modeling choices, limitations, and possible extensions

We restrict the system to two-dimensional circular particles with purely repulsive frictional interactions. While real granular media are three-dimensional and may exhibit cohesion or angularity, this simplified framework allows for efficient simulation and clear visualization of the fundamental arch formation process.

The present study also focuses on global discharge observables, such as the number of grains exiting the silo and the jamming probability. A more detailed structural characterization could be obtained by explicitly analyzing contact and force networks, for example by tracking force-chain statistics or identifying load-bearing subnetworks near the outlet. Although such analyses are beyond the scope of the current work, they would provide a more direct microscopic link between contact mechanics and macroscopic flow arrest.

Finally, alternative driving protocols could be explored. In the present simulations, gravity provides a constant driving force, and stochastic agitation is modeled in a simplified Brownian-like manner. Other forms of driving, such as periodic vibration or controlled tapping, could be implemented to study unclogging dynamics and the statistics of intermittent flow in driven systems.

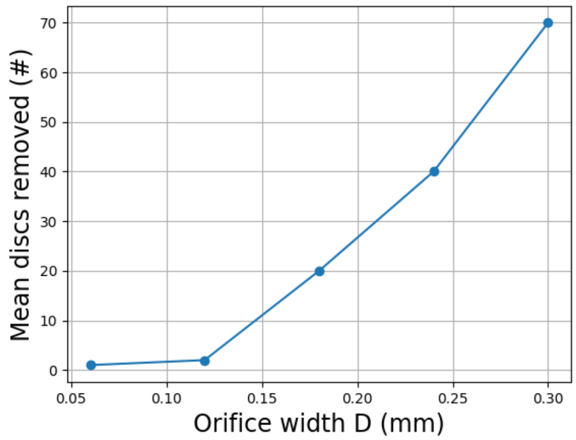
IV. RESULTS AND DISCUSSION

A. Effect of the orifice width

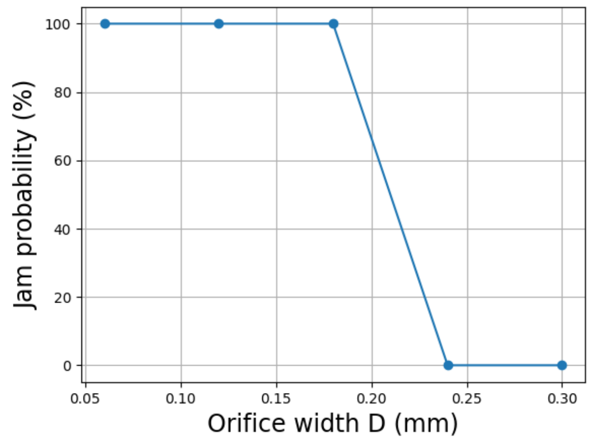
The dependence of the discharge dynamics on the outlet (orifice) width D is one of the best-established signatures of granular silo flow: small openings promote intermittent discharge and frequent flow arrest, while sufficiently large openings lead to sustained flow. In our two-dimensional simulations with $N = 200$ particles, we observe a strongly non-linear increase of the mean number of discharged particles as D increases, accompanied by a rapid drop in the probability of jamming. This qualitative picture is consistent with the broad phenomenology of granular matter, in which geometry and contact constraints combine to produce collective behavior that cannot be trivially predicted from single-particle dynamics [1, 2, 4].

For small openings (e.g. $D \leq 0.12$ in our parameter sweep), the system typically arrests shortly after discharge begins. In this regime, particles arriving at the outlet are forced to organize into a narrow "funnel", where only a small number of microstates are compatible with continued motion. The likelihood that a small set of particles simultaneously forms a mechanically stable blocking structure is therefore high. In practice, flow arrest is associated with the creation of a compact arch (or vault) spanning the orifice, whose stability is reinforced by frictional contacts and by the supporting force-chain network. This interpretation is directly supported by outlet-region snapshots and by the corresponding high jamming probabilities observed for small D (Fig. 3 and Fig. 2b).

As D increases into an intermediate regime (around $D \approx 0.15$–0.18 in our runs), the mean discharged count rises steeply. Two related mechanisms contribute to this behavior: (i) a broader outlet increases the number of geometrically admissible exit trajectories and reduces the probability that a small number of particles can simultaneously block the opening; and (ii) the local stress and force network near the outlet becomes more



(a) **Mean discharged grains vs. orifice width at $\mu = 0.6$.**



(b) **Jam probability vs. orifice width at $\mu = 0.6$.**

FIG. 2: (a) Larger openings allow more grains to exit before a stable arch forms, leading to a rapid increase of $\langle N_{\text{out}} \rangle$ with D. (b) Narrow bottlenecks promote stable arch formation and yield $P_{\text{jam}} \approx 1$, whereas wider openings suppress bridging and lead to $P_{\text{jam}} \approx 0$ beyond a critical D.

heterogeneous, so transient arches are more easily destabilized by rearrangements and fluctuations. This picture is compatible with force-network-based interpretations of stability in granular packings [7, 8].

Beyond a sufficiently large opening (in our study $D \gtrsim 0.18$–0.21, depending on μ), we observe essentially unjammed discharge within the simulated time window. In this regime, the jamming probability approaches zero while the number of discharged particles grows markedly (Fig. 2a and Fig. 2b). Here, the system behavior is dominated primarily by geometry rather than by fine details of contact stabilization: although transient clusters may still form, they fail to span the outlet for a sufficient time to satisfy the jamming criterion.

B. Role of disorder and stochasticity

To assess the robustness of the clogging transition, we performed additional simulations including geometric and stochastic perturbations in the form of radius noise. Figure 4 shows that, even in the presence of such disorder, the mean number of discharged particles still increases systematically with D. This indicates that the clogging transition is not an artifact of highly ordered packings.
While disorder tends to reduce the formation of perfectly symmetric local configurations, it does not eliminate clogging at small openings. Instead, flow arrest remains controlled primarily by frictional stabilization of arches at the outlet. These results support the view that clogging is a robust collective phenomenon governed mainly by geometry and contact mechanics rather than by crystalline ordering.

C. Friction dependence for multiple outlet sizes

We next analyze how friction modifies discharge statistics at fixed orifice width. Figure 6a shows the mean discharged count as a function of the friction coefficient μ for several values of D. For narrow openings, the system jams rapidly for essentially all friction values tested, indicating that geometric confinement alone is sufficient to generate stable blocking configurations.
For intermediate openings, the effect of friction becomes pronounced. Increasing μ generally promotes jamming by stabilizing tangential contacts and force chains, thereby increasing the lifetime of blocking arches. However, the dependence can be non-monotonic, since friction also modifies stress redistribution and packing structure above the outlet. Such non-linear effects are characteristic of granular constitutive behavior [3, 9].

The corresponding jamming probabilities, shown in Fig. 6b, highlight the combined role of geometry and friction. For sufficiently large D, the system remains largely unjammed across the explored range of μ, as the geometric requirement for forming a spanning arch becomes too restrictive.

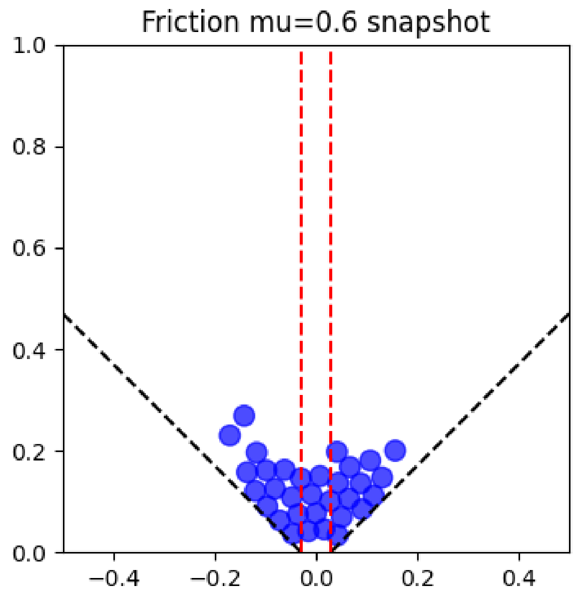


FIG. 3: **Arch formation (example).** A stable arch forms across the orifice for $\mu = 0.6$ and small D, blocking further discharge. The dashed lines indicate the funnel walls and the red markers highlight the outlet region.

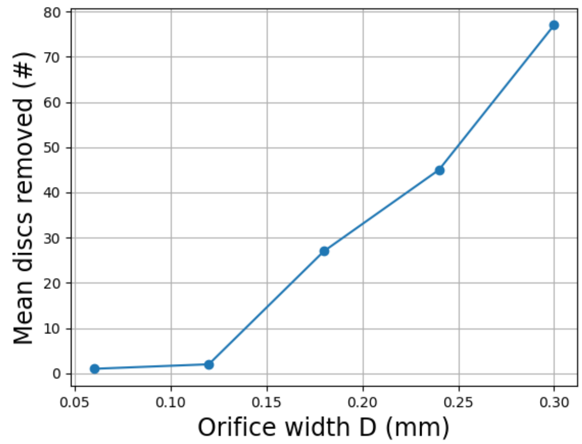


FIG. 4: **Mean discharged grains vs. D with radius noise.** Mild size disorder preserves the qualitative dependence of discharge on D, indicating that clogging is controlled primarily by outlet geometry and frictional stabilization.

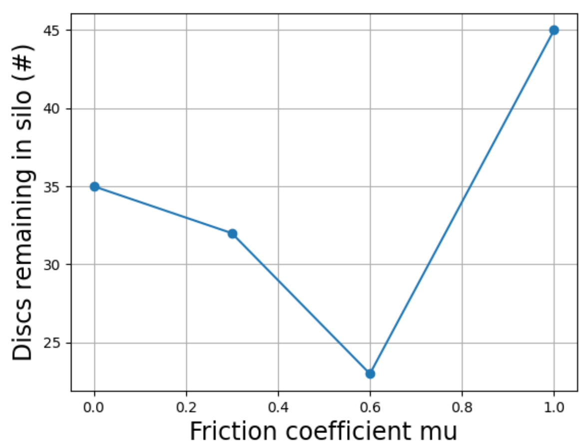


FIG. 5: **Grains remaining in the silo vs. friction.** The remaining population reflects the competition between rearrangement at low friction and arch stabilization at higher friction.

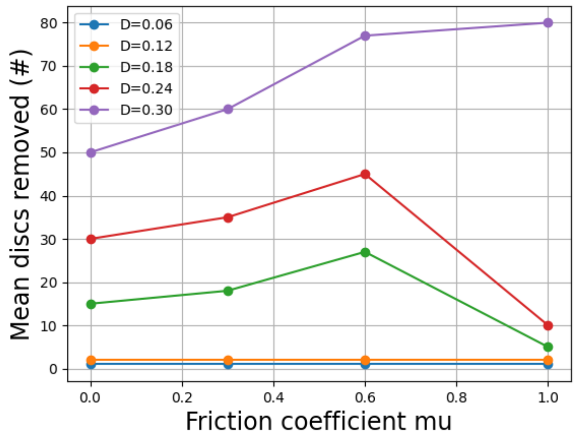
D. Remaining particles and long-time flow interruption

A complementary observable is the number of grains remaining in the silo after a fixed simulation time, which captures both early and late clogging events. As shown in Fig. 5, this quantity exhibits a non-trivial dependence on friction. At very low μ, particles rearrange efficiently and escape more easily, whereas at intermediate friction stable arches form more readily, leaving a larger fraction of grains trapped inside the silo. At high friction, the remaining count saturates, consistent with an early transition to a strongly jammed mechanical state.

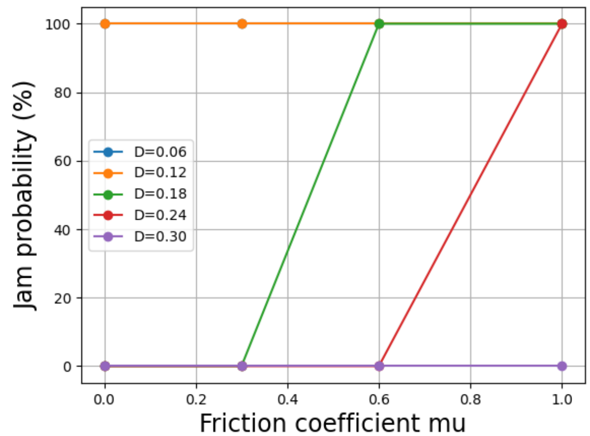
E. Microscopic picture: arch formation at the outlet

A microscopic interpretation of the macroscopic trends can be obtained by inspecting particle configurations near the outlet. Figure 3 shows a representative snapshot for $\mu = 0.6$ and a small opening, illustrating how a small number of grains can form a load-bearing arch spanning the orifice. Such structures redirect stresses toward the silo walls and prevent further discharge.

From a force-network perspective, jamming corresponds



(a) **Mean discharged grains vs. friction for multiple D.**



(b) **Jam probability vs. friction for multiple D.**

FIG. 6: (a) Small openings jam rapidly regardless of μ, while larger openings sustain discharge at low-to-moderate friction before stable arches dominate at higher μ. (b) Friction stabilizes arches in intermediate outlets, whereas the largest opening remains largely unclogged within the explored friction range.

to the emergence of a self-supporting contact subnetwork that transfers load away from the outlet. Although contact forces are not explicitly analyzed here, the observed dependence on D and μ is consistent with theoretical and experimental studies emphasizing the role of force chains and microscopic heterogeneity in granular arrest [1, 2, 7, 10]. The inclusion of stochastic forcing further destabilizes marginal arches, leading to intermittent bursts of flow in the vicinity of the clogging transition, in line with broader discussions of dynamical heterogeneities in jammed systems [6].

Overall, our results support the standard picture of clogging in bottleneck driven granular flows: increasing the outlet size rapidly suppresses jamming, while increasing friction promotes the stability of arches and force chains, thereby enhancing the probability of flow interruption. Despite its two-dimensional and idealized nature, the present model captures the key qualitative features reported in experimental and numerical studies of granular discharge through constrictions [4, 5, 11].

V. CONCLUSIONS AND OUTLOOK

In this work, we investigated the discharge and clogging behavior of a granular assembly in a two-dimensional silo using numerical simulations. By systematically varying the orifice width D and friction coefficient μ, we characterized the interplay between geometric confinement and contact mechanics.

We identified the outlet size as the dominant parameter driving the sharp transition from jamming to continuous flow. However, our findings demonstrate that interparticle friction is critical in intermediate regimes, where it stabilizes blocking arches and significantly expands the jamming phase space. Despite its simplicity, the 2D model captures the essential physical mechanisms of flow arrest1111, validating the dominance of geometric constraints over packing details.

Limitations of this study include the neglect of particle shape, cohesion, and 3D effects, which are known to influence arch stability in real systems. Future work could extend this framework by incorporating particle polydispersity, detailed force-network analysis, or external driving mechanisms such as vibration. These extensions would allow for a more direct comparison with experimental silo flows and a deeper understanding of jamming as an emergent phenomenon.

VI. CONTRIBUTIONS

César Mejía and Marnick Huisman jointly conceived the project, designed the numerical approach, and defined the research objectives. Both authors contributed equally to the development of the simulation, the execution of the experiments, and the analysis of the results. The figures were produced collaboratively, and both authors participated in the interpretation of the data. The manuscript was written jointly by César Mejía and Marnick Huisman, and both authors reviewed and approved the final version of the paper.

VII. CONFLICT OF INTEREST

The authors declare that there are no competing or conflicting interests that could have influenced the work reported in this manuscript.

VIII. DATA AND CODE AVAILABILITY

All simulation data and analysis scripts used in this study are available upon reasonable request. The source code implementing the numerical model and producing the results presented in this paper is publicly available on GitHub at:

`https://github.com/3lsesar/FFR120`

The repository contains the version of the code used for the simulations reported here, along with example scripts to reproduce the main figures.

[1] P.-G. de Gennes, "Granular matter: a tentative view," *Reviews of Modern Physics*, vol. 71, no. 2, p. S374, 1999, relevant because it provides a foundational conceptual introduction to granular matter and clarifies why granular systems defy classical continuum descriptions. [Online]. Available: https://doi.org/10.1103/RevModPhys.71.S374

[2] I. S. Aranson and L. S. Tsimring, "Patterns and collective behavior in granular media: Theoretical concepts," *Reviews of Modern Physics*, vol. 78, no. 3, p. 641, 2006, relevant because it develops theoretical frameworks for understanding emergent structures such as vortices, waves, and segregation patterns. [Online]. Available: https://doi.org/10.1103/RevModPhys.78.641

[3] M. Massoudi, "Remarks on constitutive modeling of granular materials," *Eng*, vol. 4, no. 4, pp. 2856–2878, 2023, relevant because it surveys constitutive principles such as yield behavior, dilatancy, and frictional laws. [Online]. Available: https://doi.org/10.3390/eng4040161

[4] F. Radjai, J.-N. Roux, and A. Daouadji, "Modeling granular materials," *Journal of Engineering Mechanics*, vol. 143, no. 4, p. 04017002, 2017, relevant because it synthesizes micromechanical insights with continuum approaches, showing how force networks translate into macroscopic behavior. [Online]. Available: https://doi.org/10.1061/(ASCE)EM.1943-7889.0001196

[5] M. E. Sivritas, K. T. Kularathna, and E. O. Aydin, "A comparative study of sph and dem for granular flow," *Powder Technology*, vol. 415, p. 118163, 2023, relevant because it compares numerical approaches (DEM vs SPH) for granular flow, highlighting strengths and limitations. [Online]. Available: https://doi.org/10.1016/j.powtec.2022.118163

[6] O. Dauchot, G. Marty, and G. Biroli, "Dynamical heterogeneities in grains and foams," in *Dynamical Heterogeneities in Glasses, Colloids, and Granular Media*. Oxford University Press, 2011, relevant because it discusses intermittency and cooperative rearrangements that appear near jamming and clogging thresholds. [Online]. Available: https://doi.org/10.1093/acprof:oso/9780199691470.003.0006

[7] J. H. Snoeijer, T. J. H. Vlugt, M. van Hecke, and W. van Saarloos, "Force network ensemble: A new approach to static granular matter," *Physical Review Letters*, vol. 92, no. 5, p. 054302, 2004, relevant because it formalizes a statistical ensemble description of force networks, offering a predictive framework to analyze stability. [Online]. Available: https://doi.org/10.1103/PhysRevLett.92.054302

[8] S. Ostojic, E. Somfai, and B. Nienhuis, "Scale invariance and universality of force networks in static granular matter," *Nature*, vol. 439, no. 7078, pp. 828–830, 2006, relevant because it shows that granular force-chain networks exhibit universal scaling, suggesting deep structural regularities even in disordered packings. [Online]. Available: https://doi.org/10.1038/nature04549

[9] G. Buscarnera, R. C. Hurley, and G. M. B. Viggiani, "The mechanics of brittle granular materials with coevolving grain size and shape," *Proceedings of the Royal Society A*, vol. 477, no. 2249, p. 20201005, 2021, relevant because it investigates how grain breakage and evolving particle morphology modify bulk mechanical response. [Online]. Available: https://doi.org/10.1098/rspa.2020.1005

[10] N. Brodu, J. A. Dijksman, and R. P. Behringer, "Spanning the scales of granular materials through microscopic force imaging," *Nature Communications*, vol. 6, no. 1, p. 6361, 2015, relevant because it experimentally resolves grain-scale forces, validating DEM-like simulations at the microscopic level. [Online]. Available: https://doi.org/10.1038/ncomms7361

[11] Q. Zheng, Q. Luo, and A. Yu, "A unified theory for granular matter," *Powder Technology*, vol. 434, p. 119370, 2024, relevant because it presents a theoretical model that captures both solid-like and fluid-like regimes,

which is central for describing flow–clog transitions. [Online]. Available: https://doi.org/10.1016/j.powtec.2023.119370

[12] L. Papadopoulos, M. A. Porter, K. E. Daniels, and D. S. Bassett, "Network analysis of particles and grains," *Journal of Complex Networks*, vol. 6, no. 4, pp. 485–565, 2018, relevant because it introduces graph-theoretic tools to quantify connectivity and mesoscale organization in granular assemblies. [Online]. Available: https://doi.org/10.1093/comnet/cny005

[13] A. Escobar, J. Baker, F. Guillard, T. Faug, and I. Einav, "Experimental confirmation of secondary flows within granular media," *Nature Communications*, vol. 16, p. 7446, 2025, relevant because it reveals hidden internal circulation patterns that affect transport and mixing in granular flows. [Online]. Available: https://doi.org/10.1038/s41467-025-62669-y

[14] R. S. Sharma and A. Sauret, "Experimental models for cohesive granular materials: a review," *Soft Matter*, vol. 21, pp. 2193–2208, 2025, relevant because it describes modern laboratory techniques for studying cohesive grains and their effect on clog stability. [Online]. Available: https://doi.org/10.1039/D4SM01324G