Bayesian Linear Models

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Bayesian Linear Models

- In this class we are going to revisit the linear regression model from a Bayesian point of view.
- The idea is the same as in the frequentist approach, to model the relationship of a numerical dependent variable $\mathbf{y}$ with n independent variables $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n$
- We again use a Gaussian distribution to describe our uncertainty about the response variable: $y_i \sim N(\mu_i, \sigma^2)$.
- And we also assume that each attribute has a linear relationship to the mean of the outcome.

$$\mu_i = \beta_0 + \beta_1 x_i + \ldots \beta_n x_n$$

- Instead of using least squares or maximum likelihood estimation we are going to estimate the joint posterior distribution of all the parameters of the model:

$$f(\theta|D) = f(\beta_0, \beta_1, \ldots, \beta_b, \sigma|D)$$

- This approach is more flexible as it allows incorporating prior information.
- It also allows to interpret the uncertainty of the model in a clearer way.

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- Notice the the parameters of the model are $\beta_0, \beta_1, \ldots, \beta_b$ and σ.
- The mean of the outcome μ_i is not treated as parameter because it is determined deterministically from the linear model's coefficients.
- To complete the model, we need a joint prior density:

$$f(\theta) = f(\beta_0, \beta_1, \ldots, \beta_b, \sigma)$$

- In most cases, priors are specified independently for each parameter, which amounts to assuming:

$$f(\beta_0, \beta_1, \cdots, \beta_b, \sigma) = f(\beta_0)f(\beta_1), \ldots,$$

[McElreath, 2020]

Conclusions

- Blabla

References I

McElreath, R. (2020).
Statistical rethinking: A Bayesian course with examples in R and Stan.
CRC press.