Summarizing the Posterior

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Summarizing the Posterior

- Once our Bayesian model produces a posterior distribution, it is necessary to summarize and interpret it.
- However, a posterior distribution is (usually) a high dimensional object that is hard to visualize and work with [Murphy, 2021].
- In this class we will learn how to draw estimates (e.g., point estimates, intervals) to summarize and interpret a posterior distribution.
- Exactly how it is summarized depends upon our purpose.
- Common questions include:
  - How much posterior probability lies below some parameter value?
  - How much posterior probability lies between two parameter values?
  - Which parameter value marks the lower 5% of the posterior probability?
  - Which range of parameter values contains 90% of the posterior probability?
  - Which parameter value has highest posterior probability?

Sampling to summarize

- These questions can be usefully divided into questions about:
  - intervals of defined boundaries
  - intervals of defined probability mass
  - point estimates
- In the theoretical world (when the posterior has a closed mathematical expressions), answering these questions implies calculating complicated integrals to cancel out (or average) different variables.
- In the practical world, however, the same results can be approximated using **samples** from the posterior.
- In this class we will approach the above questions using samples from the posterior.
- Another reason to learn to work with posterior samples is that methods like MCMC produce nothing but samples from the posterior.
- This class is based on Chapter 3 of [McElreath, 2020].

Sampling from a grid-approximate posterior

- Before beginning to work with samples, we need to generate them.
- Here's a reminder for how to compute the posterior for the globe tossing model, using grid approximation:

```
p_grid <- seq( from=0 , to=1 , length.out=1000 )
prior <- rep( 1 , 1000 )
likelihood <- dbinom( 6 , size=9 , prob=p_grid )
posterior <- likelihood * prior
posterior <- posterior / sum(posterior)
```

- Now we wish to draw 10,000 samples from this posterior.
- Imagine the posterior is a bucket full of parameter values, numbers such as 0.1, 0.7, 0.5, 1, etc.
- Within the bucket, each value exists in proportion to its posterior probability, such that values near the peak are much more common than those in the tails.

Sampling from a grid-approximate posterior

- We're going to scoop out 10,000 values from the bucket.
- Provided the bucket is well mixed, the resulting samples will have the same proportions as the exact posterior density.
- Therefore the individual values of p will appear in our samples in proportion to the posterior plausibility of each value.
- Here's how you can do this in R, with one line of code:

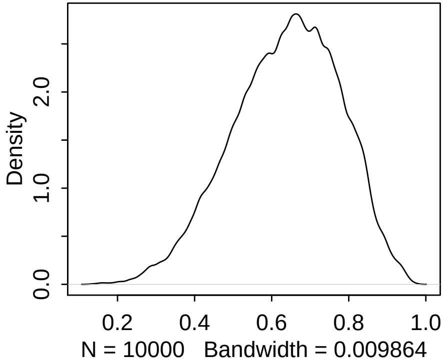
```
samples <- sample( p_grid , prob=posterior , size=1e4 ,
replace=TRUE )
```

- We are randomly pulling values from the grid of parameter values where the probability of each value is given by the posterior.

Sampling from a grid-approximate posterior

- We can visualize a density plot of our posterior sample as follows:

```
library(rethinking)
dens(samples)
```



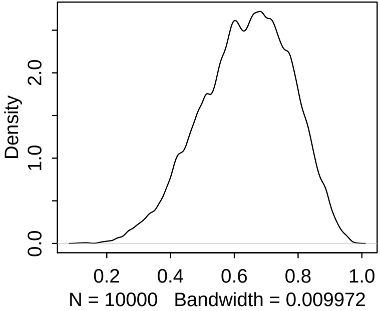
- We can see that the estimated density is very similar to to ideal posterior we computed via grid approximation in previous class.

Sampling from the theoretical posterior

- We could get the same results by sampling from the theoretical posterior using the beta distribution:

```
teo.samples<-rbeta(1e4,7,4)
dens(teo.samples)
```

- We can see that the estimated density is very similar to the theoretical posterior obtained from the beta distribution:



- However, we should keep in mind that for complex models we will not have access to the posterior closed form, so it is better to get used to working with samples.

Intervals of defined boundaries

- Suppose I ask you for the posterior probability that the proportion of water is less than 0.5.
- We could calculate this from the theoretical posterior:

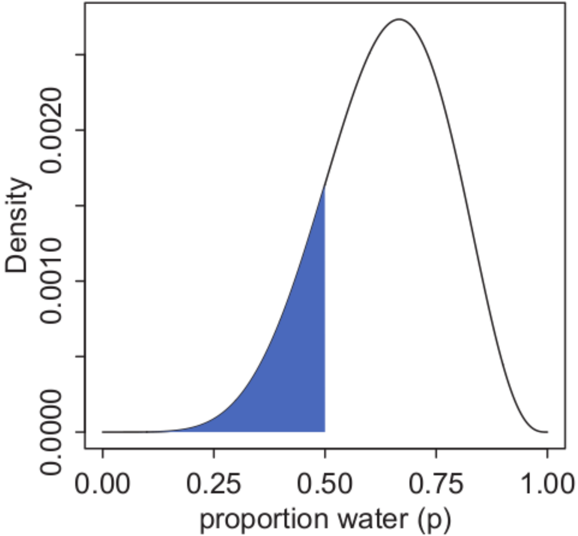
```
> pbeta(0.5,7,4)
[1] 0.171875
```

- Or alternatively we could calculate it from the grid-approximate posterior by adding up all of the probabilities where the corresponding parameter value is less than 0.5.

```
> sum( posterior[ p_grid < 0.5 ] )
[1] 0.1718746
```

- So about 17% of the posterior probability is below 0.5.

Intervals of defined boundaries



Intervals of defined boundaries

- Now, let's perform the same calculation, using samples from the posterior.
- Recall than in more complex models neither a grid-approximation nor a closed-form posterior will be available.
- All we have to do is add up all samples less than 0.5 and divide the resulting count by the total number of samples.

```
> sum( samples < 0.5 ) / 1e4
[1] 0.1752
```

- In R, the condition `samples < 0.5` returns a logical vector, so since R treats TRUE values as 1, `sum` will count all the samples satisfying the condition.

Intervals of defined boundaries

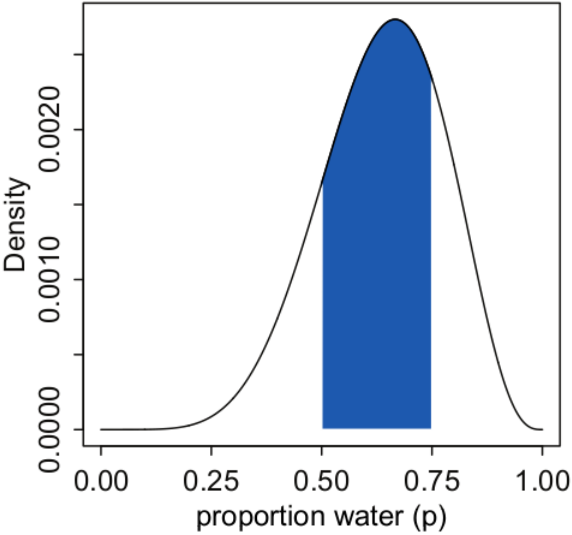
- Now, we can ask our sample how much posterior probability lies between 0.5 and 0.75.

```
> sum( samples > 0.5 & samples < 0.75 ) / 1e4
[1] 0.6043
```

- So about 61% of the posterior probability lies between 0.5 and 0.75.
- Let's validate this result using the exact posterior:

```
> pbeta(0.75,7,4)-pbeta(0.5,7,4)
[1] 0.6040001
```

Intervals of defined boundaries



Intervals of defined boundaries

- The intervals of posterior probability we are working with are called **credible interval**.
- They resemble very much the confidence intervals seen in previous lectures on frequentist inference.
- The interpretations are very different though.
- A confidence interval is a range that contains the true parameter with a certain chance after infinitely repeating the data sampling experiment.
- In contrast, a credible interval is a range of values that we believe our parameter can take with a certain probability according to both our prior beliefs and the evidence given by the data.

Conclusions

- Blablaag

References I

McElreath, R. (2020).
Statistical rethinking: A Bayesian course with examples in R and Stan.
CRC press.

Murphy, K. P. (2021).
Probabilistic Machine Learning: An introduction.
MIT Press.