Introduction to Bayesian Inference

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Some Critics to the Frequentist Approach

- The statistical methods that we have discussed so far are known as frequentist (or classical) methods.
- The frequentist approach requires that all probabilities be defined by connection to the frequencies of events in very large samples.
- This leads to frequentist uncertainty being premised on imaginary resampling of data.
- If we were to repeat the measurement many many times, we would end up collecting a list of values that will have some pattern to it.
- It means also that parameters and models cannot have probability distributions, only measurements can.
- The distribution of these measurements is called a sampling distribution.
- This resampling is never done, and in general it doesn't even make sense.

Bayesian Inference

There is another approach to inference called Bayesian inference [**?**], which is based on the following postulates:

- Probability describes **degree of belief**, not limiting frequency.
  - We can make probability statements about lots of things, not just data which are subject to random variation.
  - For example, I might say that ”the probability that Albert Einstein drank a cup of tea on August 1, 1948” is .35.
  - This does not refer to any limiting frequency.
  - It reflects my strength of belief that the proposition is true.
- We can make probability statements about parameters, even though they are fixed constants.
- We make inferences about a parameter θ by producing a probability distribution for θ. Inferences, such as point estimates and interval estimates, may then be extracted from this distribution.

Bayesian Inference

- In modest terms, Bayesian data analysis is no more than counting the numbers of ways the data could happen, according to our assumptions [**?**].
- In Bayesian analysis all alternative sequences of events that could have generated our data are evaluated.
- As we learn about what did happen, some of these alternative sequences are pruned.
- In the end, what remains is only what is logically consistent with our knowledge [**?**].
- Warning: understanding the essence of Bayesian inference can be hard.
- The following toy example tries to explain it in a gentle way.

Counting Possibilities

- Suppose there's a bag, and it contains four marbles.
- These marbles come in two colors: blue and white.
- We know there are four marbles in the bag, but we don't know how many are of each color.
- We do know that there are five possibilities:
  (1) [○○○○], (2) [●○○○], (3) [●●○○], (4) [●●●○], (5) [●●●●]
- These are the only possibilities consistent with what we know about the contents of the bag. Call these five possibilities the conjectures.
- Our goal is to figure out which of these conjectures is most plausible, given some evidence about the contents of the bag.
- We do have some evidence: A sequence of three marbles is pulled from the bag, one at a time, replacing the marble each time and shaking the bag, in that order. These before drawing another marble.
- The sequence that emerges is: ●○●, in that order. These are the data.

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