In [1]:

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import linregress, ttest_ind
from scipy.stats.qmc import LatinHypercube, scale, Sobol
import statistics
import seaborn as sns

'''
EXERCISE 1
'''
# Define parameters
w, h = 1000, 1000 # Image size
x_min, x_max = -2.0, 0.8  # Range real part
y_min, y_max = -1.3, 1.3  # Range imaginary part
max_i = 150  # Maximum number of iterations

mandelbrot_set = np.zeros((h, w))

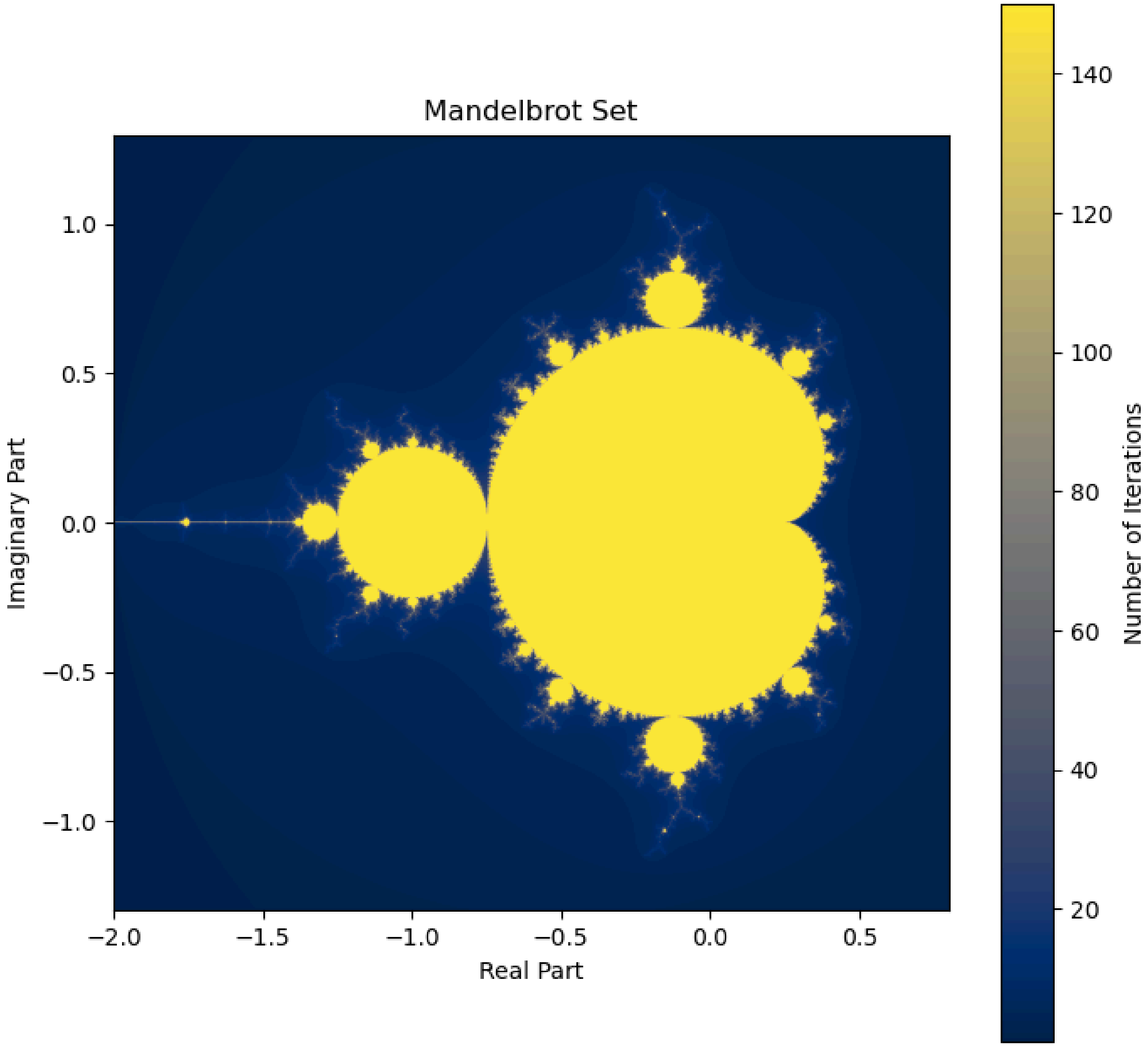
# Generate Mandelbrot set
for x in range(w):
    real = x_min + ((x / w) * (x_max - x_min))
    for y in range(h):
        # Give coordinates to complex number
        imag = y_min + ((y / h) * (y_max - y_min))
        c = complex(real, imag)

        # Initialize z
        z = complex()
        iter = 0

        # Run iterations
        while abs(z) <= 2 and iter < max_i:
            z = (z * z) + c
            iter += 1

        # Insert corresponding number of iterations in matrix
        mandelbrot_set[y, x] = iter

# Print picture
plt.figure(figsize = (8, 8))
plt.imshow(mandelbrot_set, extent = (x_min, x_max, y_min, y_max), cmap = 'cividis')
plt.colorbar(label = 'Number of Iterations')
plt.title("Mandelbrot Set")
plt.xlabel("Real Part")
plt.ylabel("Imaginary Part")
plt.show()
```



In [2]:

```
'''
EXERCISE 2
'''
def insidetheset(c, max_i): # Define function to determine if a given c is in the set

    z = complex()

    for i in range(max_i):
        if abs(z) > 2:
            return False
        z = (z * z) + c

    return True

def slope_curve(A, area_method): # Define function that calculates the slope of the c

    slopes = []
    log_i = np.log(np.array(list_i))

    for s in range(samples):
        diff = []
        for i in list_i:
            diff.append(abs(area_method(max(list_s), i) - A) / A) # Calculate the rel
        log_diff = np.log(np.array(diff))
        slope = linregress(log_i, log_diff).slope # Convert both to log to fit a powe
```

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        slopes.append(slope)

    mean_slope = np.mean(slopes)
    variance_slope = statistics.variance(slopes)
    print(f'The average slope on a fitted power-law curve of the line corresponding t
    return slopes

def plot_diff(A, area_method): # Function that plots the relative error in area estim

    dic_diff = {s: [] for s in list_s}

    for i in list_i:
        for s in list_s:
            diff = abs(area_method(s, i) - A) / A
            dic_diff[s].append(diff)

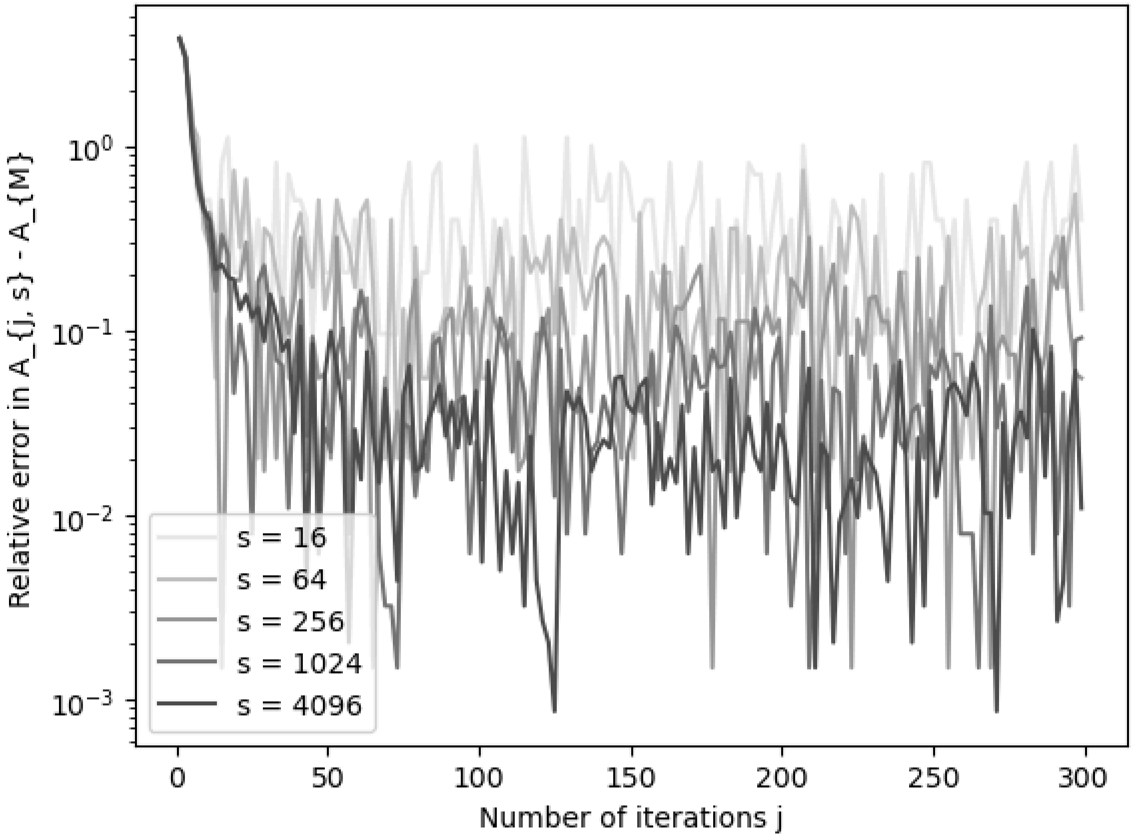
    # Plot the differences with A_{M} for every estimated area
    for index, (s, diff) in enumerate(dic_diff.items()):
        plt.plot(list_i, diff, color = plt.cm.gray(1-(0.1 + 0.6 * (index / (len(list_
    plt.xlabel('Number of iterations j')
    plt.ylabel('Relative error in A_{j, s} - A_{M}')
    plt.yscale('log')
    plt.legend()
    plt.show()

# Define general parameters
x_min, x_max = -2.0, 0.8  # Range real part
y_min, y_max = -1.3, 1.3  # Range imaginary part
# Create lists of all i and s
steps = 150
list_i = [1]
list_s = [16, 64, 256, 1024, 4096] # These number have been chosen to optimally use o
for step in range(1, steps):
    list_i.append(list_i[step - 1] + 2)
samples = 150 # Number of samples to calculate slope
```

In [3]:

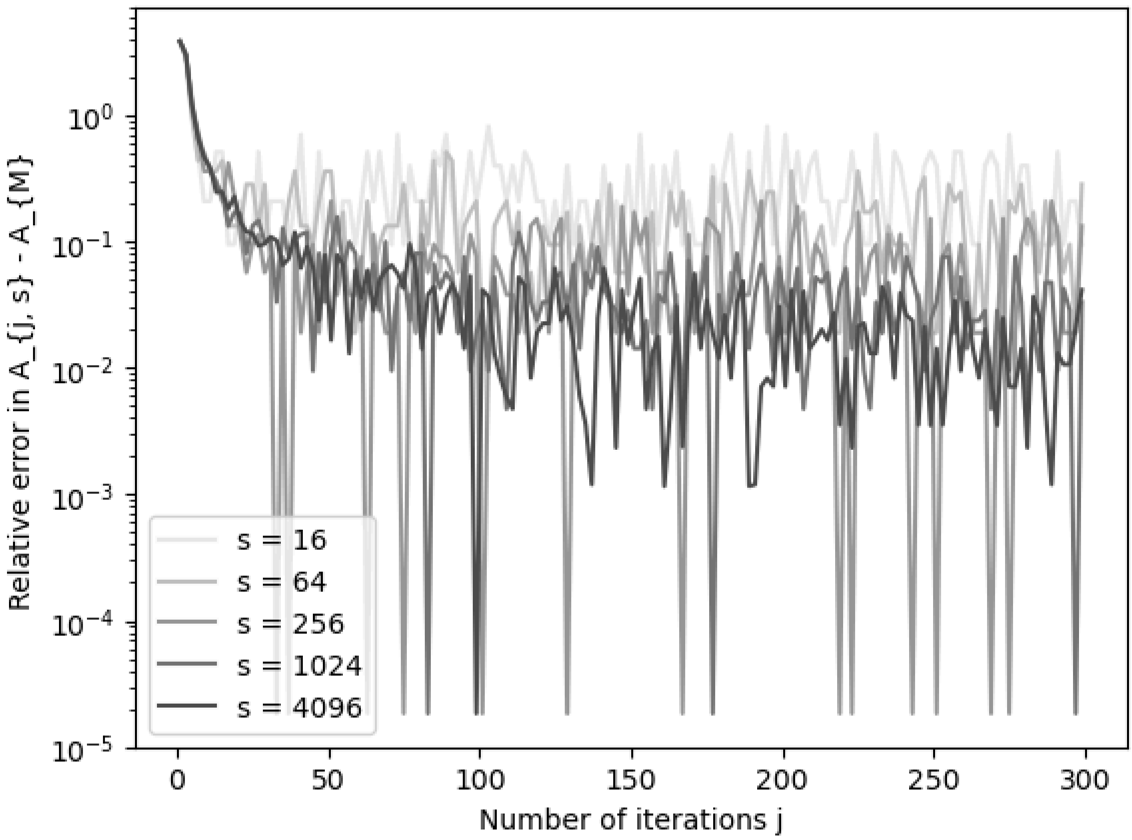
```python
def area_pure_random(s, i): # Define function to calculate area through pure random m

    total_area = (x_max - x_min) * (y_max - y_min)
    samples_in_area = 0

    for _ in range(s):
        real = np.random.uniform(x_min, x_max)
        imag = np.random.uniform(y_min, y_max)
        c = complex(real, imag)
        if insidetheset(c, i):
            samples_in_area += 1

     # Area is calculated based on percentage of samples that do not deviate
    set_area = total_area * (samples_in_area / s)
    return set_area
# Calculate an estimate of A_{PR}
A_PR = area_pure_random(262144, 10000)
print('A_{PR} ≅ ', A_PR)

slopes_PR = slope_curve(A_PR, area_pure_random)

plot_diff(A_PR, area_pure_random)
```

```
A_{PR} ≅  1.5094369506835936
The average slope on a fitted power-law curve of the line corresponding to s = 4096 is
equal to -0.9380716905204054, with a variance of 0.0031149843233349407
```



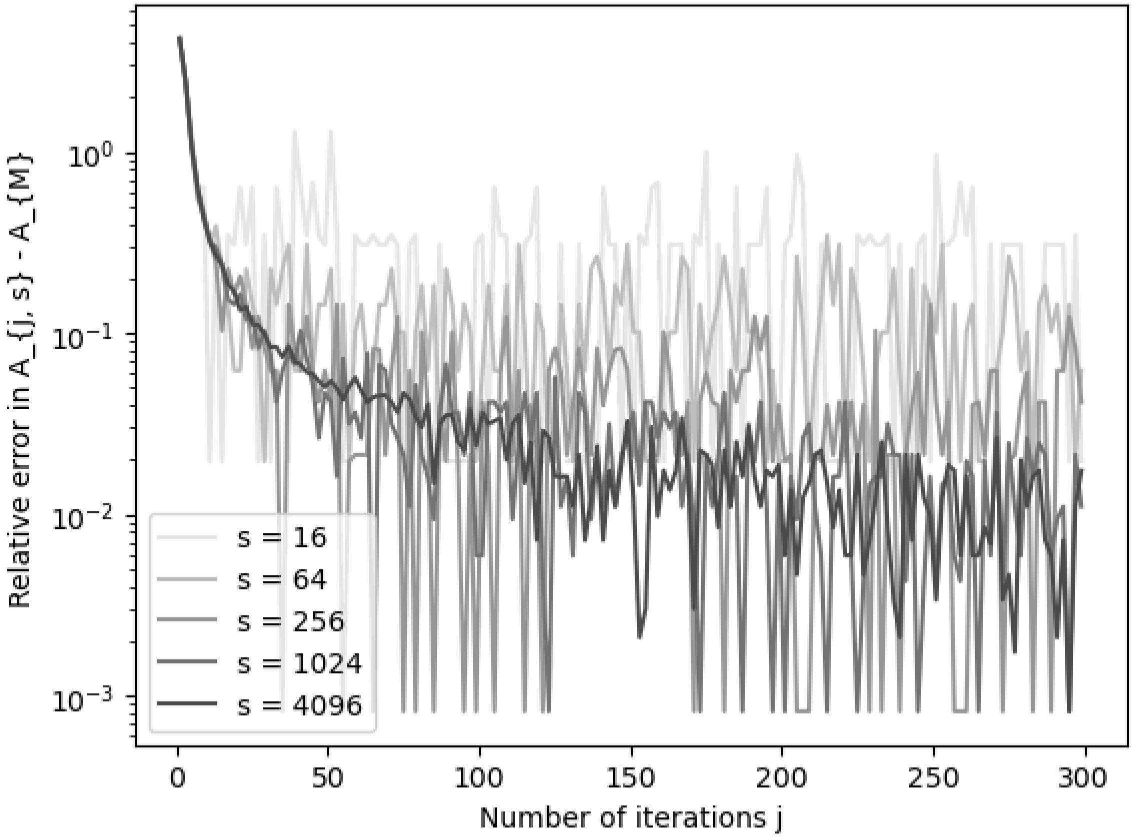
In [4]:
```
def area_latin(s, i): # Define function to calculate area through latin hypercube met

    total_area = (x_max - x_min) * (y_max - y_min)
    samples_in_area = 0

    sampler = LatinHypercube(d = 2) # 2 dimensions (real and imaginary)
    sample = scale(sampler.random(n = s), [x_min, y_min], [x_max, y_max]) # Scale the

    for real, imag in sample:
        c = complex(real, imag)
        if insidetheset(c, i):
            samples_in_area += 1

     # Area is calculated based on percentage of samples that do not deviate
    set_area = total_area * (samples_in_area / s)
    return set_area

A_LH = area_latin(262144, 10000)
print('A_{LH} ≅ ', A_LH)

slopes_LH = slope_curve(A_LH, area_latin)

plot_diff(A_LH, area_latin)
```

```
A_{LH} ≅  1.5072152709960935
The average slope on a fitted power-law curve of the line corresponding to s = 4096 is
equal to -1.0752048081939323, with a variance of 0.0034741914907378733
```



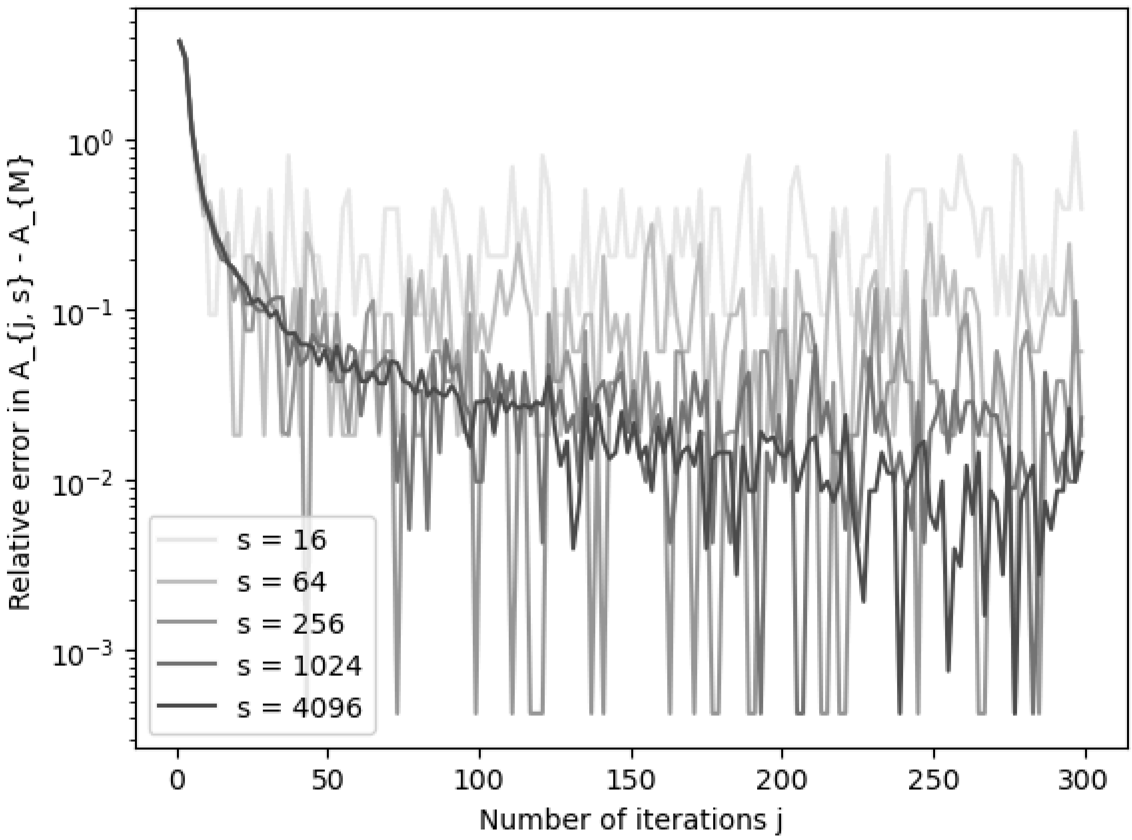
In [5]:
```
def area_orthogonal(s, i): # Define function to calculate area through orthogonal met

    total_area = (x_max - x_min) * (y_max - y_min)
    samples_in_area = 0

    n = int(np.sqrt(s)) # This is done to obtain s samples
    if n * n != s:
        raise ValueError('s should be perfect square for orthogonal sampling')

    x_intervals = np.linspace(x_min, x_max, n + 1)
    y_intervals = np.linspace(y_min, y_max, n + 1)

    for j in range(n):
        for k in range(n):
            real = np.random.uniform(x_intervals[j], x_intervals[j+1])
            imag = np.random.uniform(x_intervals[k], x_intervals[k+1])
            c = complex(real, imag)
            if insidetheset(c, i):
                samples_in_area += 1

     # Area is calculated based on percentage of samples that do not deviate
    set_area = total_area * (samples_in_area / s)
    return set_area

A_0 = area_orthogonal(262144, 10000)
print('A_{0} ≅ ', A_0)

slopes_0 = slope_curve(A_0, area_orthogonal)

plot_diff(A_0, area_orthogonal)
```

```
A_{0} ≅  1.3922988891601562
The average slope on a fitted power-law curve of the line corresponding to s = 4096 is
equal to -1.1638958383022908, with a variance of 0.0012162312871785749
```



In [6]:
```
def area_sobol(s, i): # Define function to calculate area through sobol method

    total_area = (x_max - x_min) * (y_max - y_min)
    samples_in_area = 0

    sampler = Sobol(d = 2)
    sample = scale(sampler.random(n = s), [x_min, y_min], [x_max, y_max])

    for real, imag in sample:
        c = complex(real, imag)
        if insidetheset(c, i):
            samples_in_area += 1

     # Area is calculated based on percentage of samples that do not deviate
    set_area = total_area * (samples_in_area / s)
    return set_area

A_S = area_sobol(262144, 10000)
print('A_{S} ≅ ', A_S)

slopes_S = slope_curve(A_S, area_sobol)

plot_diff(A_S, area_sobol)
```

```
A_{S} ≅  1.5065487670898436
The average slope on a fitted power-law curve of the line corresponding to s = 4096 is
equal to -1.1771522018615943, with a variance of 0.001110324797681524
```



In [7]:
```
slopes_list = [slopes_PR, slopes_LH, slopes_O, slopes_S]
names_list = ['Pure Random', 'Latin Hypercube', 'Orthogonal', 'Sobol']

alpha = 0.05 # Set alpha for t test on the average slopes

# Test for the difference in convergence for every method against the others through 
for index, sl1 in enumerate(slopes_list[:-1]):
    for sl2 in slopes_list[index+1:]:
        t_stat, p_value = ttest_ind(sl1, sl2, equal_var = False)
        print(f'H0: there is no significant difference in convergence between {names_
        if p_value < alpha:
            print(f'H0 is rejected with P-value = {p_value}')

            higher_conv = sl1 if max(abs(np.mean(sl1)), abs(np.mean(sl2))) == abs(np.
            lower_conv = sl1 if min(abs(np.mean(sl1)), abs(np.mean(sl2))) == abs(np.m
            print(f'{names_list[slopes_list.index(higher_conv)]} method has a higher 
        else:
            print(f'H0 is not rejected with P-value = {p_value}')
            print(f'{names_list[slopes_list.index(sl1)]} does not significantly diffe
        print('---------')

for index, slope in enumerate(slopes_list):
    sns.kdeplot(slope, label = names_list[index], alpha = 0.6)

plt.legend()
plt.xlabel("Slope")
plt.ylabel('Density')
plt.show()
```

```
H0: there is no significant difference in convergence between Pure Random and Latin Hy
percube methods
H0 is rejected with P-value = 1.607570696245069e-59
Latin Hypercube method has a higher convergence (the slope is 0.1371 units higher in m
agnitude) compared to Pure Random
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H0: there is no significant difference in convergence between Pure Random and Orthogon
al methods
H0 is rejected with P-value = 2.623961887791756e-115
Orthogonal method has a higher convergence (the slope is 0.2258 units higher in magnit
ude) compared to Pure Random
---------
H0: there is no significant difference in convergence between Pure Random and Sobol me
thods
H0 is rejected with P-value = 5.061639647831175e-120
Sobol method has a higher convergence (the slope is 0.2391 units higher in magnitude)
compared to Pure Random
---------
H0: there is no significant difference in convergence between Latin Hypercube and Orth
ogonal methods
H0 is rejected with P-value = 2.5404479512576268e-39
Orthogonal method has a higher convergence (the slope is 0.0887 units higher in magnit
ude) compared to Latin Hypercube
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H0: there is no significant difference in convergence between Latin Hypercube and Sobo
l methods
H0 is rejected with P-value = 1.3675744350075095e-47
Sobol method has a higher convergence (the slope is 0.1019 units higher in magnitude)
compared to Latin Hypercube
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H0: there is no significant difference in convergence between Orthogonal and Sobol met
hods
H0 is rejected with P-value = 0.0008629078582886535
Sobol method has a higher convergence (the slope is 0.0133 units higher in magnitude)
compared to Orthogonal
---------
```