4 Mathematical Modeling of WAP7 trains

The mathematical model for WAP7 locomotive developed in this research uses the equations of motion. The tractive effort $F_{trac.}$, track resistance force T_{res}, and the gradient force T_{grad} are considered as the three main operating force on the train's body. The train accelerates and decelerates as a consequence of the balance of these forces in the direction of the track.

$$F_{trac.} - F_{res} - F_{grad.} = M_{eff.} \cdot a = F_{a.} \qquad (1)$$

M_{eff} is the effective mass of the train, $F_{a.}$ Is the net resultant force accelerating the train. Each railway car features a flywheel, which is used to store energy during braking and power the vehicle during acceleration. The constrained motion of the train on tracks allows the three-dimensional parameters to be reduced to following scalar quantities, a) time, t; b) distance, x; c) velocity, v; d) acceleration, a; and e) jolt, j, all of which are related by the equations of motion. The train's movement is limited not only by physical constraints, but also by train and track features in this study.

4.1 Equations governing train dynamics

The motion of train is majorly governed by the newtons third law of motion. The effective mass of the train is dependent upon the mass of locomotives, tare mass of the carriages and the mass of the payload. On assuming that all the carriages have the same payload the effective mass of the train will be:

$$M_{eff} = M_{loco} + N_{carr.} \times (M_{tare}^{carr.} + M_{payload}) \qquad (2)$$

Here, M_{loco} is the mass of the locomotive, $N_{carr.}$ Is the total number if the carriages, M_{tare} is the tare mass of the carriage and $M_{payload}$ is payload per vehicle. By using (1) and (2) we can find the acceleration as

$$a = \frac{d^2x}{dt^2} = \frac{F_{traction} - F_{friction} - F_{gradient}}{M_{train}} \qquad (3)$$

The calculation of traction force, friction and gradient force will be done as following.

4.1.1 Tractive Effort

Tractive effort is the net force applied by the locomotive parallel to the direction of motion. The tractive effort is constrained by the motor current limits and the mechanical restriction. The values of the starting effort, continuous traction effort and braking effort is taken from the official documentation of the WAP7 locomotive.

4.1.2 Track Resistance

A train's track resistance is its velocity-dependent resistance to motion. The semi-empirical Davis equation is commonly used to calculate resistance. $F_{res} = D_A + D_B \cdot v + D_C \cdot v^2$ (4)

D_A, D_B, D_C are the coefficients for the various frictional force, the values are taken from the literature.

4.1.3 Force due to gradient

An extra force, F_{grad}, is included in the calculations for track portions with a gradient in slope. One can get the following results by using geometrical arguments.

$$F_{grad} = M_{train} \times g \sin\theta \tag{5}$$

The value of θ is track gradient.

4.1.3 Train parameters

The focus of our study is the WAP 7 locomotive, for that we have chosen Shatabdi Express through the route ADI-MMCT (Ahmedabad Jn – Mumbai Central) would be used. This convoy is chosen because it is one of the fastest convoys on one of the busiest sections. The data used is given in the table below.

Name	Description	Unit	Value
M_{loco}	Mass of the locomotive	kg	123000
$M_{carriage}$	Mass of the individual carriage	kg	45000
$M_{payload}$	Mass of payload in a carriage	kg	
$N_{carriage}$	Number of carriages		22
S	Surface area of locomotive	m^2	13.4
F_{start}	Starting force by the locomotive	kN	322.4
P_{train}	Power of the locomotive	kW	4560
V_{max}	Maximum velocity of the locomotive	m/s	50
F_{br}	Braking effort of the locomotive	kN	18600
P_{max}	Maximum power of locomotive	kW	4740

4.2 Drive Cycle and energy estimation

A driving cycle is necessary to evaluate the train's energy consumption and the potential energy savings from implementing a flywheel energy storage system. This is important because the efficiency of a hybrid power train is highly dependent on the duty cycle as well as route parameters such as stop spacing, gradient profile, and vehicle types *(Chymera M, Renfrew A, 2008) (1)*.

We used the data available on the Ahmedabad-Mumbai route of Shatabdi express for recreating the driving cycle in simulation. The recreated driving cycle's distances and time were tested and validated against figures given by erail.in (2). Table (1) below shows the input used for the driving cycle, which should be read from left to right i.e. train leaves Ahmedabad and travels for 2040 seconds to cover the distance of 46000 meters to reach next station Nadiad where it stops to wait for 120 seconds to start the journey for next station.

Sr no.	Station	waiting time (sec)	Travel Time (sec)	Distance (m)
1	Ahmedabad	0	2040	45000
2	Nadiad	120	1020	19000
3	Anand	120	1800	35000

4	Vadodara	300	2820	71000
5	Bharuch	120	2760	58000
6	Surat	300	3840	93000
7	Vapi	120	5220	140000
8	Borivali	180	3420	30000
9	MMCT	0		

Table 1 Route data for drive cycle

The track's gradient data is not given, but the model can handle gradients, allowing broad dependencies on uniform gradients to be demonstrated.

4.2.1 Drive cycle

Based on a modified version of the velocity-step approach provided in Ref [3], a driving cycle is created. The driving cycle is set up to correspond to the distance and time requirements listed in data table provided above. The approach is modified by introducing a constant velocity part in the simulation that helps in meeting the time frame for the given distance. There are three phases to each driving section: acceleration, constant velocity motion, and deceleration as depicted in the figure given below. The acceleration (section a) starts at each station and establishes a constant speed (section b) that can cover the provided distance easily. While the deceleration trajectory (section c) is dependent upon the next arriving station, based on the braking power and its restrictions. Distance covered in all the three phases (acceleration, constant velocity motion, and deceleration) are computed concurrently as the velocity is increased in each step in an iterative manner. As the net distance travelled (area under the velocity time curve) matches or exceeds the driving length of the given section we stop the acceleration. Then the train travels at a constant speed for a pre decided length, and then starts to deaccelerate as it reaches the calculated braking distance. As a result, the time criteria are met precisely, whereas the distances are near accurate.

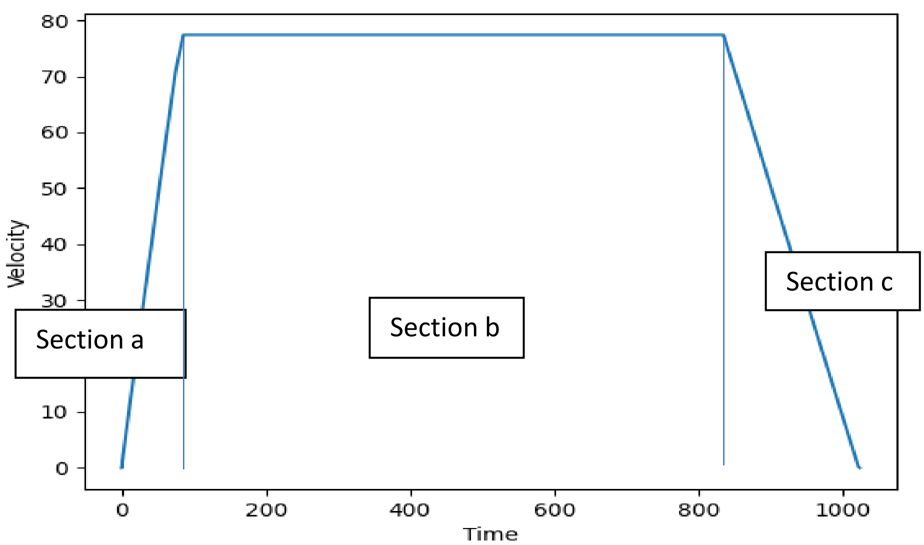


Fig 1. Process of velocity step method, showing the three sections

Equation used for determining the instantaneous acceleration is the equation (3) provided above.

$$a = \frac{F_{traction} - F_{friction} - F_{gradient}}{M_{train}}$$

After determining the acceleration, the velocity increment is calculated as

$$v = V_0 + a\Delta t. \tag{6}$$

The value of time difference or Δt is taken to be one for easier calculation and understanding. The position is updated using the equation

$$x = x_0 + v\Delta t. \quad (7)$$

Using the route data from table 1 and the process outlined above a drive cycle is obtained. Below is the velocity time and acceleration time curve for a sample train of 18 loaded LHB II AC coaches pulled by a single WAP 7 locomotive.

a. *Velocity- Time Curve*

Gradient data for this run is taken as zero, inclusion of gradient will increase or decrease the amount of traction power required. The maximum velocity is set to be 50 m/s (180 km/h). Train starts by using the maximum traction effort to counter the friction forces, continues to accelerate till achieving the constant speed at which the required distance can be covered in the prescribed time. After moving at a constant speed for a while the train starts to brake to stop at the next station. The point of braking is decided after comparing the distance left to travel with the required braking distance, if the distance remaining becomes equal to or less than the braking distance train starts to deaccelerate. The deacceleration is assumed to be done at a constant rate which is strong enough to stop the train but not uncomfortable for the passengers. The maximum velocity reached in this run is 28.72 m/s (103.39 Km/h).

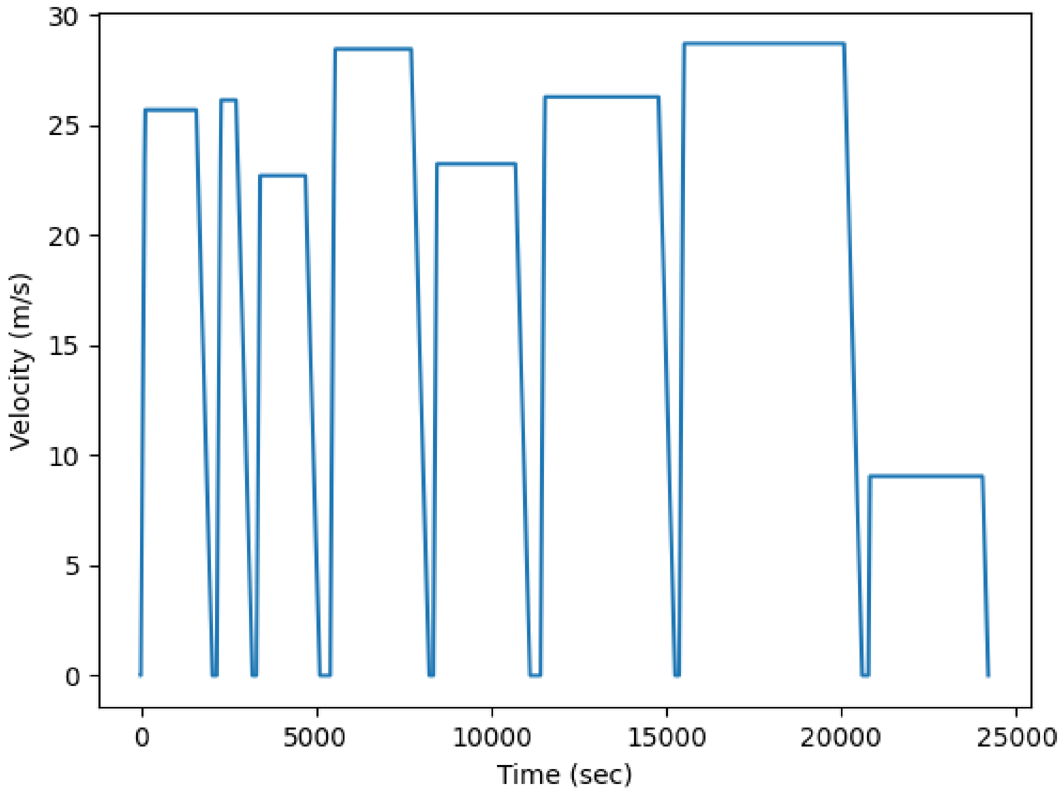


Fig 2. Velocity over time, no gradient, 18 coaches loaded train, pulled by wap7

b. *Acceleration Time Curve*

The acceleration is a direct result of the traction force that is being applied. If you look at the graph you can identify the sections of the cycle easily. In the beginning the acceleration shows a peak, this represents the accelerating segment of the route. After a while the acceleration falls to zero and stays at that value for long duration, this part represents the section of constant velocity. At last there is the dip to the negative value representing the braking of the train. The peak acceleration observed for this run is 0.31m/s^2, this provides the jolt necessary to move the train from rest. Apart from moving train from the rest, the value of acceleration never exceeds 0.21 m/s^2. Value of deacceleration value noted is 0.051 m/s^2.

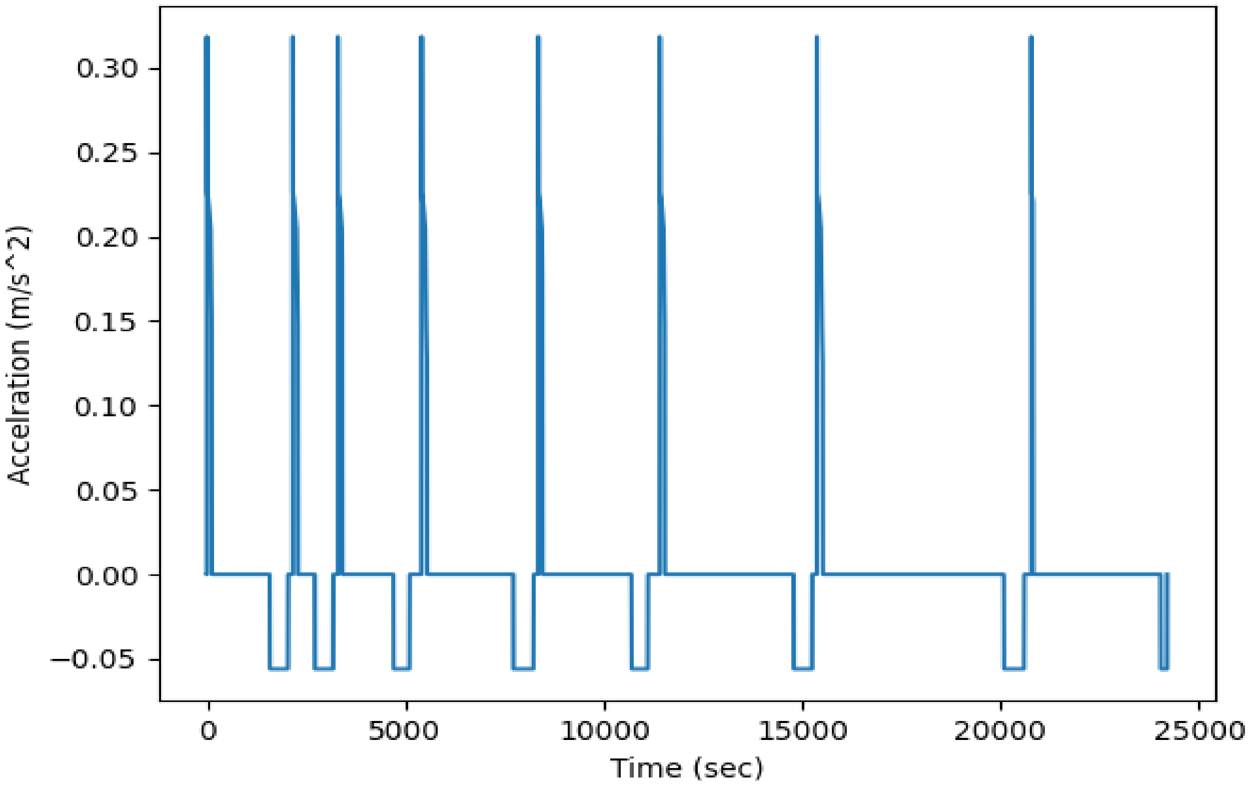


Fig 3. Acceleration time curve for 18 coach loaded train, pulled by wap7

4.2.2 Energy Estimation

The data vital to this simulation describes the power consumption throughout the journey. The instantaneous tractive power required by the train is determined by:

$$Power = F_{Traction} \times v \tag{8}$$

a. *Power Time Curve*

The figure below is the graph plotted between traction power and time for a train with loaded 18 LHB II AC coaches and WAP7 locomotive pulling it. The power time curve has peaks in both positive direction or y-axis and as well as in negative directions or y'-axis. The peaks in the upper half of the graph are from the acceleration and constant velocity phases. While the negative power represents the braking power during the deacceleration of the train. The maximum value for the power observed is 4.79 Mega watt and the lowest peak is at -1.54 Megawatts.

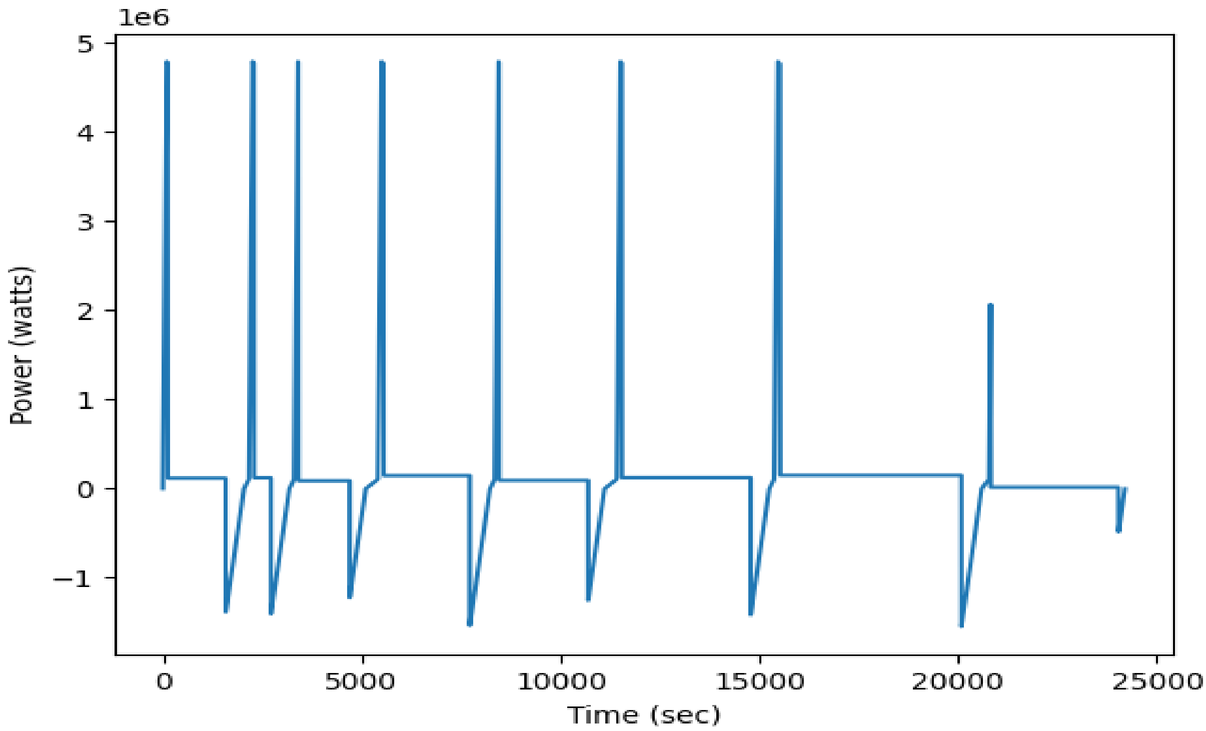


Fig 4. Power time curve, loaded 18 coach LHB II AC with WAP7, zero gradient

The energy consumption and energy recovered by the regenerative braking are what we're interested in. The following is the energy formula:

$$E = \int Pdt \tag{9}$$

The energy consumption during the traction will be calculated by using:

$$E_{traction} = \int P_{traction}dt \tag{10}$$

The energy recovered during the braking of train will be calculated using:

$$E_{braking} = \int P_{braking}dt \tag{11}$$

b. Traction energy Time Curve

The graph below shows the energy consumption during traction for the route of table 1. The train set selected is also the same, 18 LHB II AC coaches with WAP 7 locomotive. The shape of the curve is alike to steps, this is because it only represents the energy consumed only during traction. During acceleration the traction energy rises sharply, while at constant velocity the traction energy rises slowly. When the brakes are applied the energy consumed by traction remains constant, giving it the step like shape. The total consumption of energy in whole travel time is 4.65×10^{9}watt hrs.

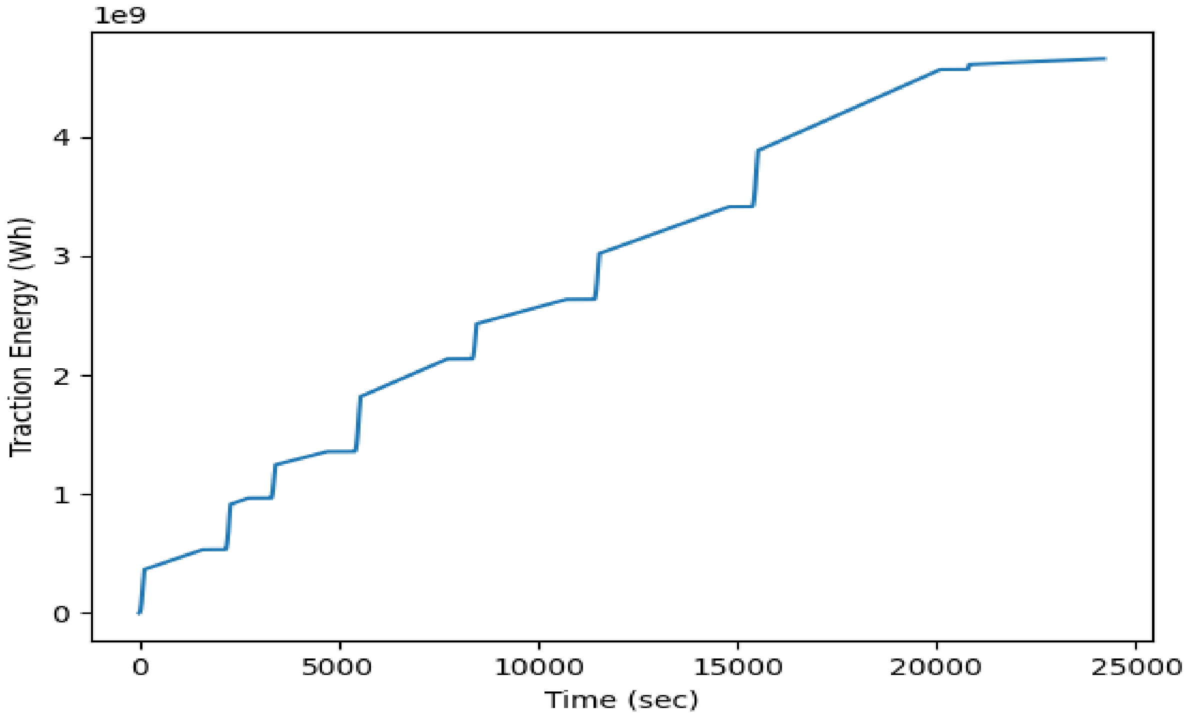


fig 5. Traction energy consumption during whole journey, zero gradient.

c. Braking energy Time Curve

The graph below shows the energy consumption during the braking for the route of table 1. The train set selected is the same, 18 LHB II AC coaches with WAP 7 locomotive. The shape of the curve similar to steps, this is because it represents the energy consumed only during braking. The time during acceleration and constant velocity the braking energy remains constant. When the brakes are applied the energy consumed by braking rises, giving it the step like shape. The total consumption of energy in whole travel time is $2.304 * 10^9$watt hrs.

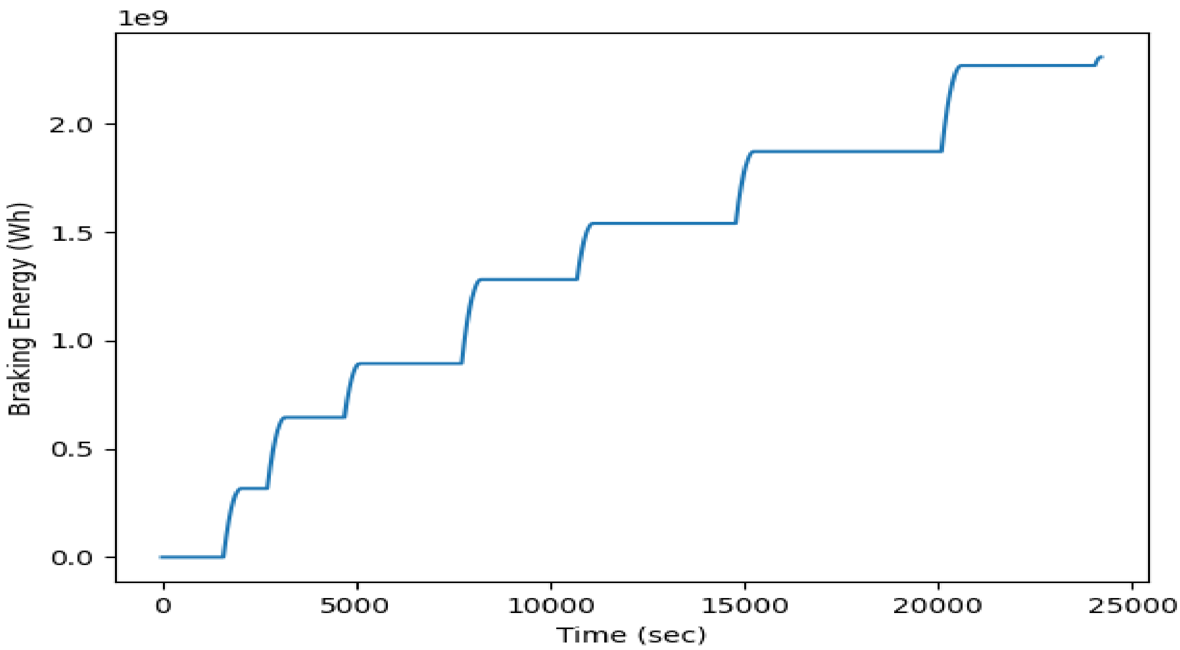


fig 6. Braking energy consumption during whole time, zero gradient.

4.3 Implementation

In Python, numerical model incorporating all the models discussed above was created. Driving cycle is computed after taking in the input of route profile, train parameters and other constraints. The velocity-step algorithm is applied at every section of track. Continuous computation is performed to calculate the distance traveled and braking distance required. For each velocity step, it starts with calculating the force required to overcome the force of friction and other resistances. Then it moves onto calculate the resultant acceleration using the equation (3) provided above. Further it updates the velocity and acceleration using equations 6 and 7. Next it checks the braking distance and braking time required to stop this train, after this the time for which the train will run at a constant speed is calculated. The distance travelled in all three (accelerating, constant speed, braking) phases is added up. This process is repeated until the sum of distances meet the prescribed distance of the section. After generating the drive cycle all time-dependent kinematic variables and forces are calculated, and the energy consumption is evaluated during postprocessing using the equations explained in section 4.2.2.

Below is the basic flow chart depicting the outline of the whole process implemented to obtain results. The detailed flow chart of the algorithm is given in the later sections of the paper.

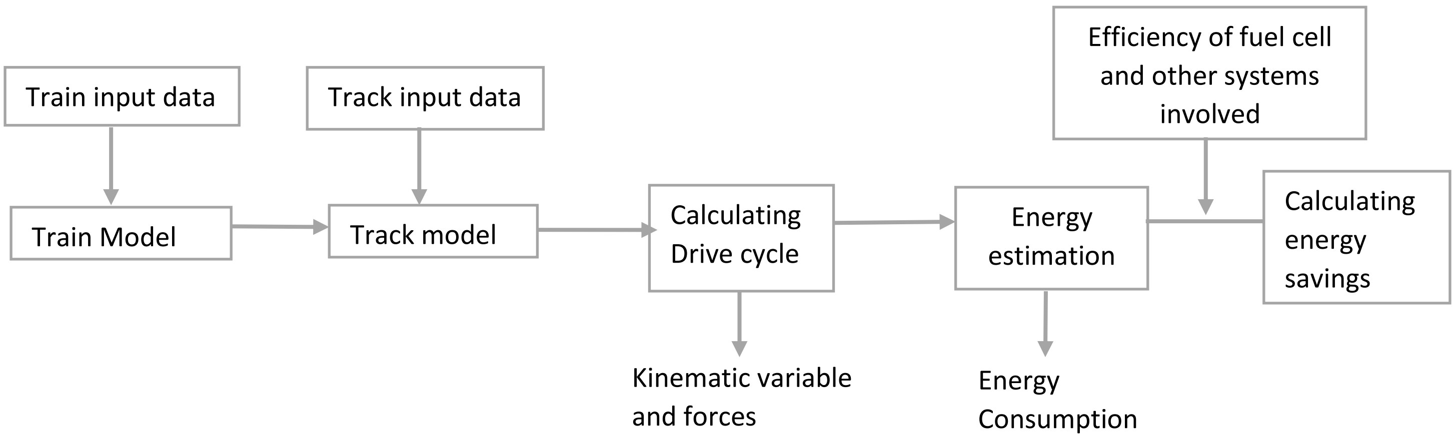


Fig 7 Basic modules and steps used in Python model.

4.4 Results

4.4.1 Without Gradient

Shown below are the results from the simulation. This simulation estimates the total energy consumed by the train in traction, we can use this to compute the energy taken from substation using equation 12.

$$E_{trac}^{Ac} = \frac{E_{trac}}{\eta_1} \tag{12}$$

Here E_{trac}^{Ac} is the total energy taken from the substation, E_{trac} is the traction energy obtained from equation 10. η_1 is the induction motor's efficiency to convert the input electrical energy to output traction energy.

Simulation also calculates the net braking energy generated during the drive cycles. We can use this to obtain the net amount of energy arriving at the flywheel by using equation (13).

$$E_{brake}^{Dc} = E_{brake} \times \eta_1 \quad (13)$$

Recovered energy is equal to the multiplication of energy stored in flywheel with flywheel's efficiency.

$$E_{FESS} = E_{brake}^{DC} \times \eta_2 \quad (14)$$

In the equation (13) E_{brake}^{DC} is the energy reaching the flywheel energy storage system (FESS). Multiplied with efficiency of the FESS (η_2) we obtain E_{FESS} that is the energy stored in the FESS. Due to the fact that the energy must again pass through the induction motor to reach the wheels of the train, the energy recovered is equal to the energy collected by the FESS multiplied by η_1.

$$E_{recoverd} = E_{FESS} \times \eta_1 \quad (15)$$

We also obtain the value of efficiency gain, which is defined as the ratio of energy recovered to the ratio of energy spent through-out the trip (16). As well as the value of energy recovery efficiency which is defined as the ratio of energy recovered to braking energy (17).

$$\varepsilon = \frac{E_{recovered}}{E_{trac}^{AC}} \quad (16)$$

$$\epsilon = \frac{E_{recovered}}{E_{brake}} \quad (17)$$

Table below shows the various combinations of passenger stock being pulled by the WAP 7 locomotive on the route shown in table 1. The value for the efficiency of induction motor (η_1) is taken as 0.88, and the value of η_2 as 0.90.

Train Composition	**E traction** (kWh)	**E^{AC}traction** (kWh)	**E braking** (kWh)	**E^{DC}brake** (kWh)	**E FESS** (kWh)	**E recovered** (kWh)	ε	ϵ
WAP 7 loco; 18 LHB II AC Fully load	4660000	5295454.545	2304000	2027520	1824768	1605795.84	0.30324	0.69696
WAP 7 loco; 18 LHB II AC half load	4500000	5113636.364	2199000	1935120	1741608	1532615.04	0.299711	0.69696
WAP 7 loco; 18 LHB III AC Full load	5010000	5693181.818	2536000	2231680	2008512	1767490.56	0.310457	0.69696
WAP 7 loco; 18 LHB III AC Half load	4750000	5397727.273	2348000	2066240	1859616	1636462.08	0.303176	0.69696
WAP 7 loco; 12 LHB AC CC Full Load	3570000	4056818.182	1790000	1575200	1417680	1247558.4	0.307521	0.69696
WAP 7 loco; 12 LHB AC CC Half Load	3420000	3886363.636	1706000	1501280	1351152	1189013.76	0.305945	0.69696

Table 2. Estimated recovered energy for various train configurations on route shown in table 1.

For a train consisting of 18 fully loaded LHB II AC coaches the total energy spent in traction is 5.29×10^6 kWh. The braking energy generated is 2.304×10^6 kWh. Total energy that can be recovered if on-board FESS is used 1.6×10^6 kWh, which is around 30% of total energy supplied for the traction. Similarly, for all other composition of the stock value of ε ranges from 0.29 to 0.31. Whereas, the value of ϵ is constant for all the compositions.

Fig 8 displays how energy use changes throughout the course of a driving cycle. Orange curve represents the trains without FESS or any other regenerative braking, blue curve represents train equipped FESS. The blue curve's dips correspond to the deceleration stages near the stations. The different dips' heights correspond to the amount of energy that might be saved. Total energy consumed over the trip represented by orange curve is 4.65$\times 10^6$ kWh and for the blue curve is 2.36$\times 10^6$ kWh. Estimated energy savings by using FESS in this case is 2.29$\times 10^6$ kWh.

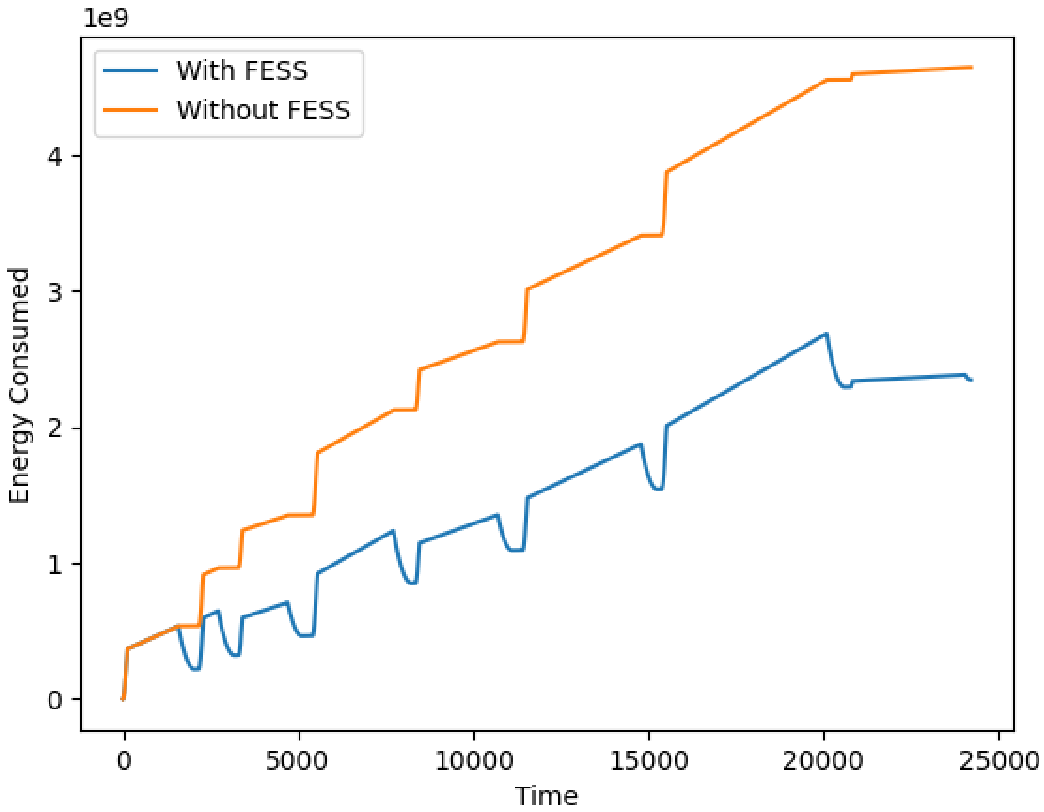


Fig 8 Energy comparison of trains running with and without FESS; WAP 7 loco, 18 LHB II AC stock

4.4.2 With Gradient

Value for gradient is taken at random from between the maximum and minimum allowed gradient. The value of gradient for the various scenarios is given in the table below:

Type of gradient	Characteristics or values
Ruling gradient for plains	0.5% to 0.67%
Ruling gradient for hilly regions	0.67% to 1.0%
Gradient in station yards	1 in 400 to 1 in 1000 is allowed

Table 3. Allowed gradients for trains in Indian railways

For the train going uphill the gravitational force due to gravity will increase the resistance and deceleration.

$$a = \frac{F_{traction} - F_{friction} - F_{gradient}}{M_{train}} \tag{18}$$

a. Velocity time curve

Graphs given below are the velocity time graphs for trainset of 18 LHB II AC coaches with WAP 7 locomotive. Value of gradient for operation is set at zero in (a) and at 1 in 150 (0.67%) in (b).

(a) *(b)*

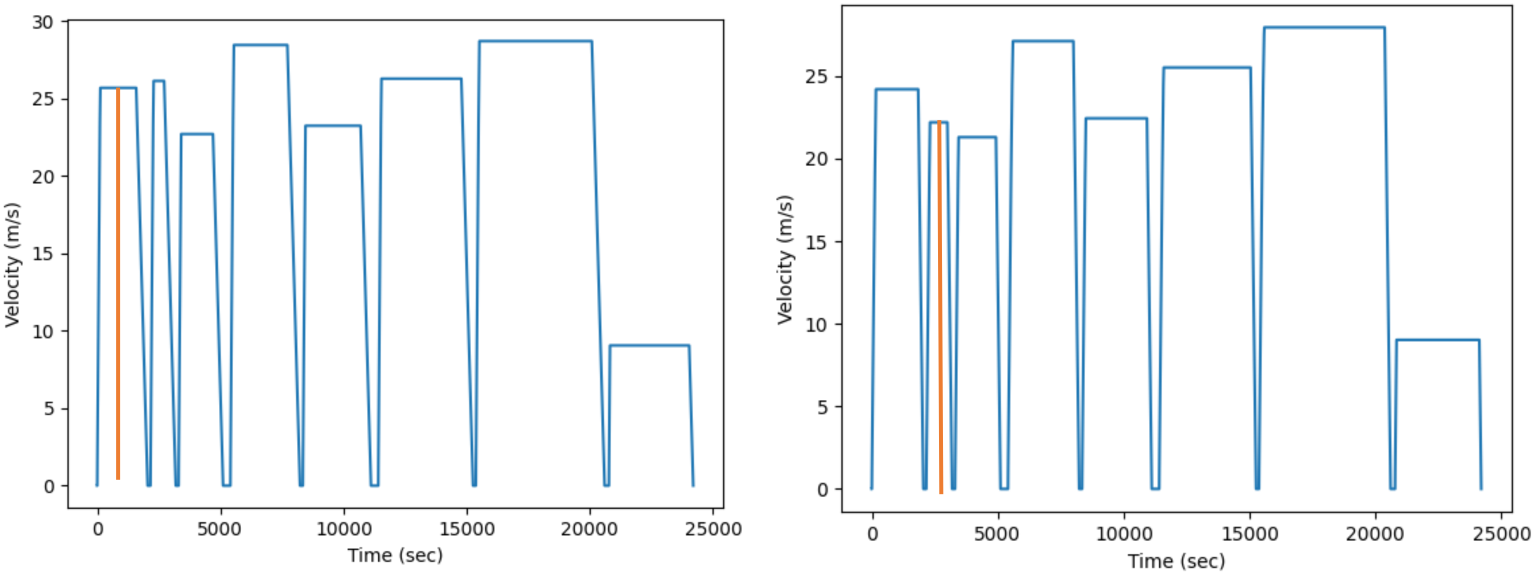
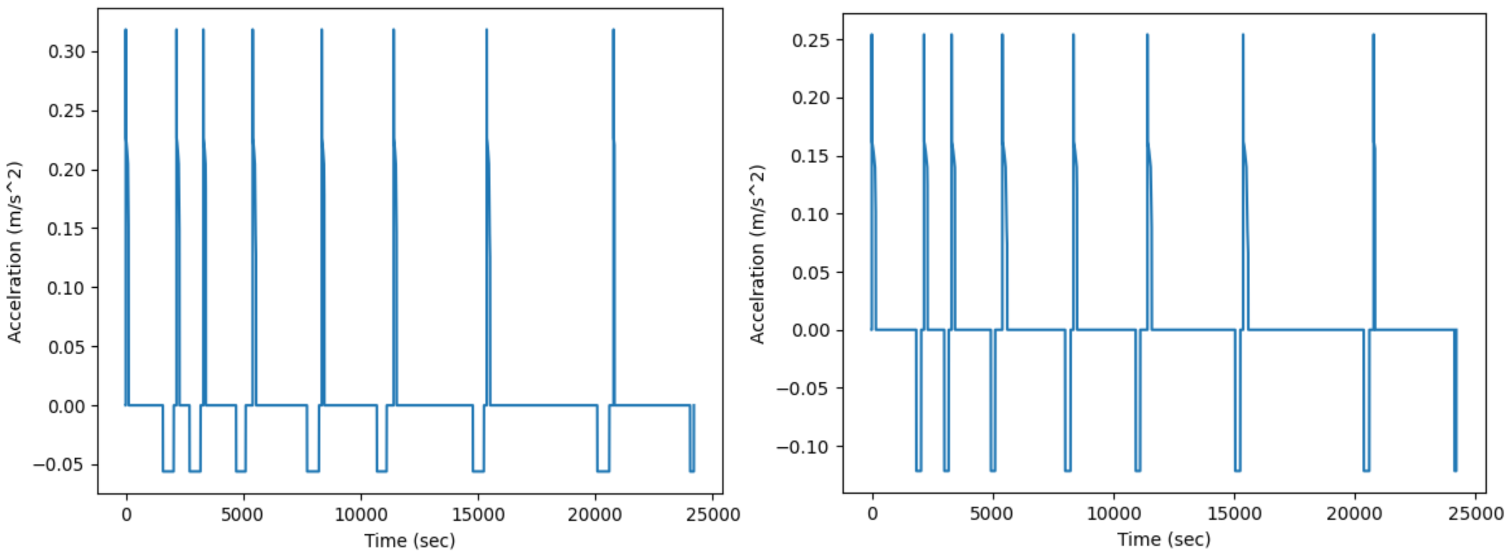


Fig 9 Velocity-Time graphs for train operating at zero gradient and 1 in 150 (0.67%) gradient.

The graph (a) is for the zero gradient and graph (b) is for the 1 in 150 (0.67%) gradient. The maximum value observed for the velocity in plot (a) is 28.72 m/s (103.3 km/h), but in plot (b) it only reaches till 28 m/s (100.8 Km/h). The decrease in maximum velocity attained for the whole route is barely noticeable. Whereas in certain sections of the route the difference is quite clear. For example, maximum velocity attained in second section of the route *(highlighted in orange),* the reduction in peak velocity is quite significant. If we examine closely, we can see that the travel time at a constant speed in the second section *(highlighted in orange)* is higher in plot b compared to plot a.

b. Acceleration time curve



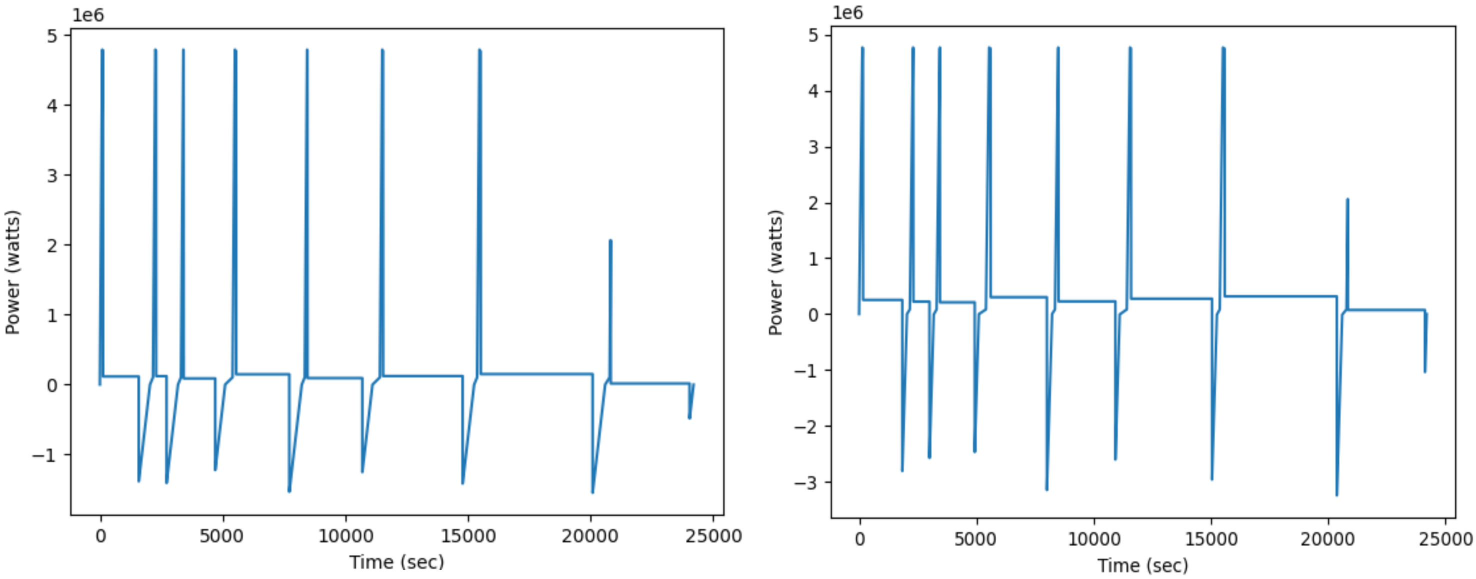
(a) *(b)*

Fig 10 Acceleration-Time graphs for train operating at zero gradient and 1 in 150 (0.67%) gradient

In fig. 10 (a) is acceleration-time plot is for zero gradient, while the (b) is for 1 in 150 (0.67%) gradient. We can clearly see the effect of uphill gradient on the peak acceleration and deacceleration values. Value for the maximum acceleration is reduced from 0.31m/s^2 to 0.25 m/s^2. While the value of deacceleration has increased from 0.05 m/s^2 to 0.12m/s^2.

c. *Power time curve*

In fig. 11 (a) is power-time plot is for zero gradient, while the (b) is for 1 in 150 (0.67%) gradient.



(a) *(b)*

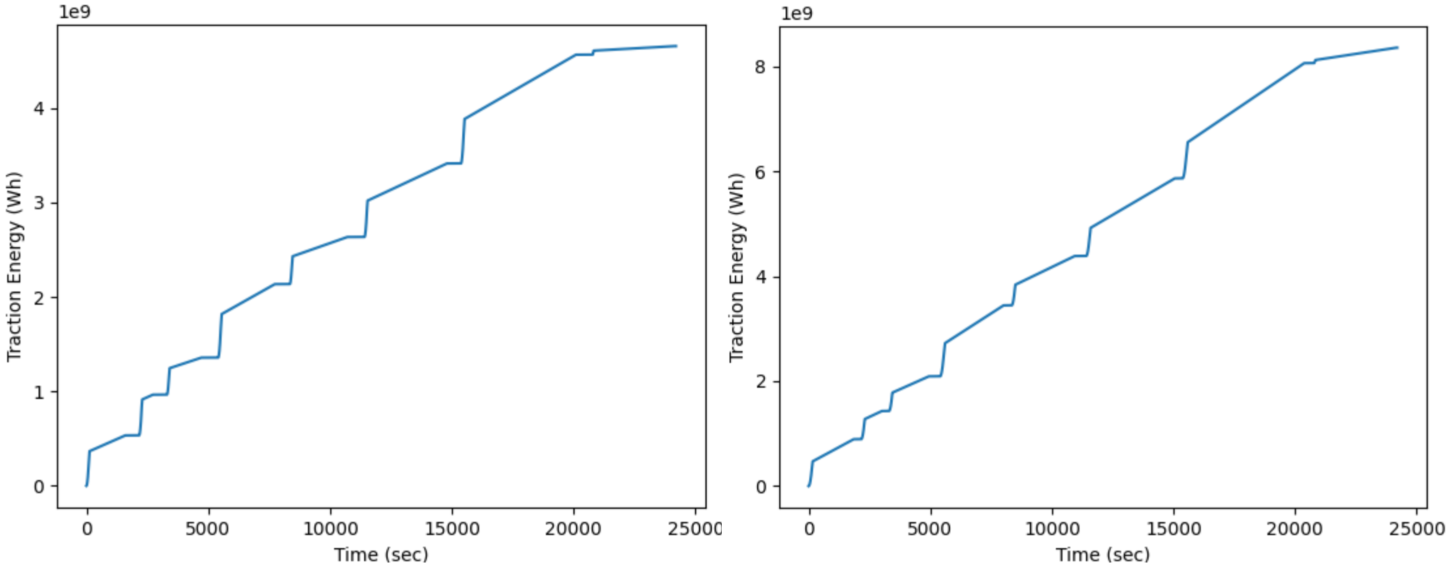
Fig 11 Power-Time graphs for train operating at zero gradient and 1 in 150 (0.67%) gradient

Peak power attained for zero gradient 4.79 Megawatts and for 1 in 150 gradient the value is 4.78 megawatts. As stated above in equation (8) $Power = F_{Traction} \times v$, both cases achieve the maximum traction force theoretical possible. Difference appears due to slight decrease in peak velocity of the system in 1 in 150 gradients. Whereas the difference in the braking power is much more noticeable as it increased from 1.54 Megawatts to 3.24 megawatts. This is also due to the increased gravitational force.

d. *Traction energy curve*

In fig. 12, plot (a) is cumulative traction energy plot is for zero gradient, while the (b) is for 1 in 150 (0.67%) gradient. Final value noted for the case of zero gradient is 4.65$\times 10^9$ watt-hrs. On other hand for 1 in 150 gradient this value is noted at 8.39$\times 10^9$ watt-hrs. This rise is because of the extra work done to counter the resistive force of gravity.

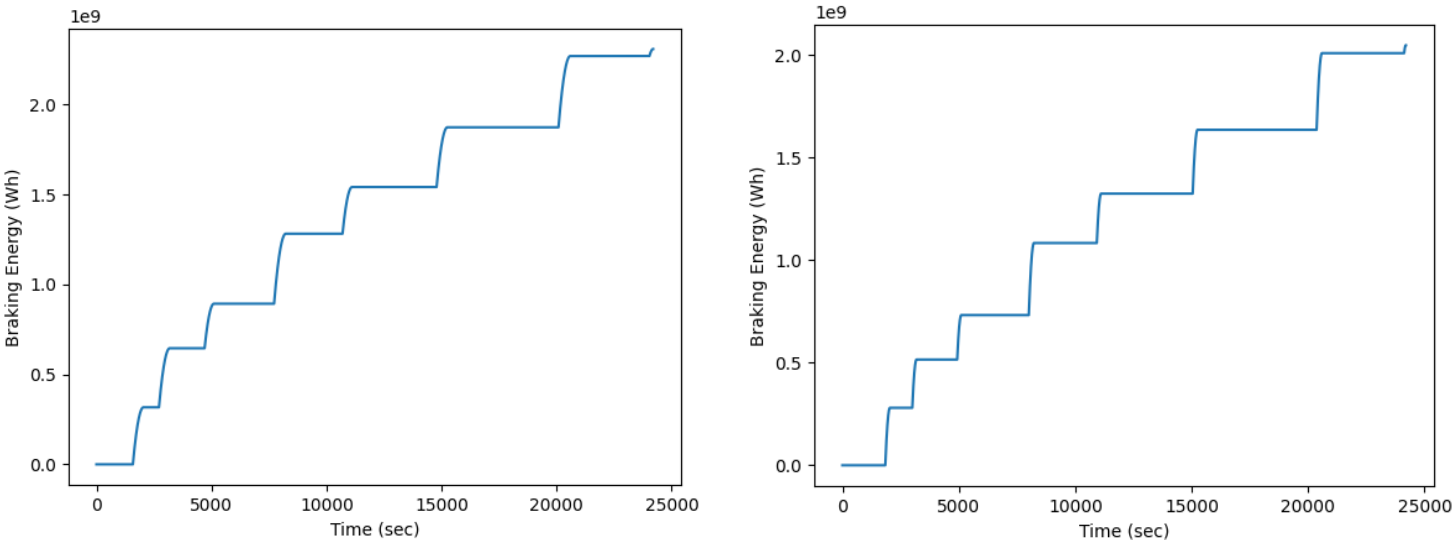
The pattern for traction energy through the journey depicted by the both curves is the same. The increased consumption of traction energy for the gradient is observed from the start. In plot a energy consumed in first 5000 seconds is noted to be 1.35$\times 10^9$ watt-hrs, while in plot b the traction energy consumption for the first 5000 seconds is 2.12$\times 10^9$ watt-hrs. Similarly, the consumption of energy for the whole trip is higher in case of the gradient.



(a) *(b)*

Fig 12 Traction energy graphs for train operating at zero gradient and 1 in 150 (0.67%) gradient

e. Braking energy curve



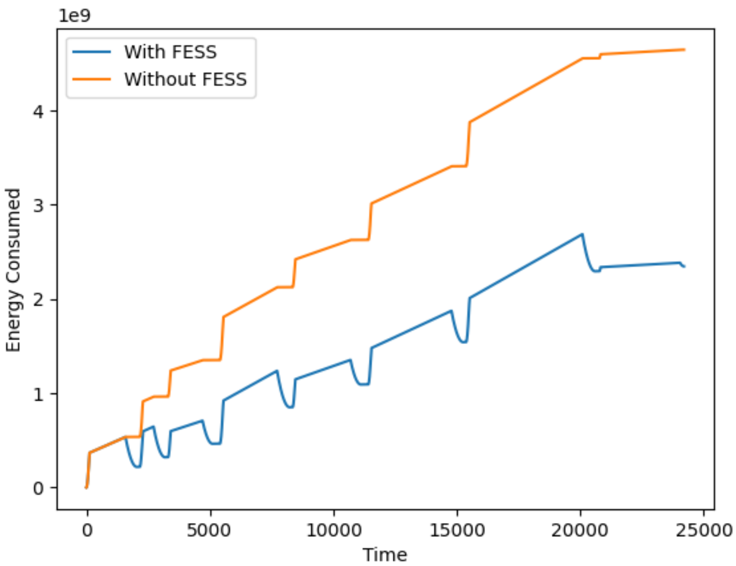
(a) *(b)*

Fig 13 Braking energy graphs for train operating at zero gradient and 1 in 150 (0.67%) gradient

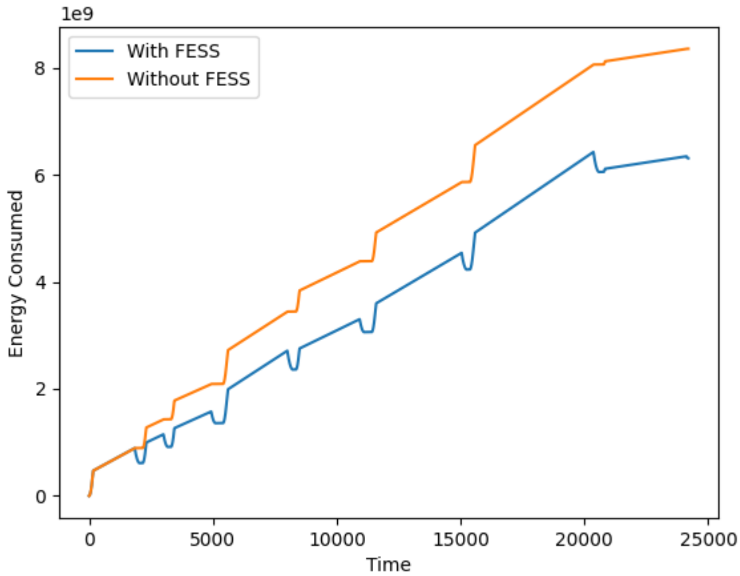
In fig. 13, part (a) is cumulative braking energy plot is for zero gradient, while the (b) is for 1 in 150 (0.67%) gradient. Final value noted for the case of zero gradient is 2.304×10^{9} watt-hrs. On other hand for 1 in 150 gradient this value is noted at 2.049×10^{9} watt-hrs. A slight reduction in final braking energy is noticed. Although, the pattern for braking energy through the journey depicted by the both curves are the same.

f. Energy comparison of trains running with and without FESS

In fig. 14, part (a) is comparative plot of train with fess to without fess at zero gradient, while the (b) is for 1 in 150 (0.67%) gradient.



(a)



(b)

Fig 14 Comparative plot of train with fess to without fess operating at zero gradient and 1 in 150 (0.67%) gradient

Pattern visible in both the cases a & b are very similar to one another. Although the gap between the orange and blue curve in a appears to be larger than in b.

In case of zero gradient the energy consumption without FESS (orange curve) reached value of 4.65×10^6 kWh. If FESS was used in the same scenario (blue curve) the energy consumed falls to 2.36×10^6 kWh giving the savings of 2.29×10^6 kWh. For gradient of 0.67% the orange curve reaches the value of 8.39×10^6 kWh whereas the blue curve provides the value of 6.30×10^6 kWh. So, using FESS in with 0.67% gradient will provide us with 2.09×10^6 kWh of savings.

g. *Results for various gradients*

This section will discuss the results obtained for the various values of gradient for a train set of 18 LHB II AC coaches. The value for the efficiency of induction motor (η_1) is taken as 0.88, and the value of flywheel efficiency (η_2) as 0.90.

Gradient	Train Composition	$E_{traction}$ (kWh)	$E^{AC}_{traction}$ (kWh)	$E_{braking}$ (kWh)	E^{DC}_{brake} (kWh)	E_{FESS} (kWh)	$E_{recovered}$ (kWh)	ε	ϵ
0%	WAP 7 loco; 18 LHB II AC Full load	4660000	5295454.545	2304000	2027520	1824768	1605795.84	0.303240416	0.69696
	WAP 7 loco; 18 LHB II AC half load	4500000	5113636.364	2199000	1935120	1741608	1532615.04	0.299711386	0.69696
1 in 400 (0.25%)	WAP 7 loco; 18 LHB II AC Full load	5840000	6636363.636	2137000	1880560	1692504	1489403.52	0.224430667	0.69696
	WAP 7 loco; 18 LHB II AC half load	5640000	6409090.909	2042000	1796960	1617264	1423192.32	0.222058376	0.69696
1 in 300 (0.33%)	WAP 7 loco; 18 LHB II AC Full load	6320000	7181818.182	2111000	1857680	1671912	1471282.56	0.204862129	0.69696
	WAP 7 loco; 18 LHB II AC half load	6080000	6909090.909	2016000	1774080	1596672	1405071.36	0.203365592	0.69696
1 in 200 (0.5%)	WAP 7 loco; 18 LHB II AC Full load	7250000	8238636.364	2058000	1811040	1629936	1434343.68	0.174099647	0.69696
	WAP 7 loco; 18 LHB II AC half load	6990000	7943181.818	1978000	1740640	1566576	1378586.88	0.173556002	0.69696
1 in 150 (0.67%)	WAP 7 loco; 18 LHB II AC Full load	8390000	9534090.909	2049000	1803120	1622808	1428071.04	0.149785759	0.69696
	WAP 7 loco; 18 LHB II AC half load	8020000	9113636.364	1959000	1723920	1551528	1365344.64	0.149813377	0.69696
1 in 100 (1%)	WAP 7 loco; 18 LHB II AC Full load	10990000	12488636.36	2049000	1803120	1622808	1428071.04	0.114349637	0.69696
	WAP 7 loco; 18 LHB II AC half load	10430000	11852272.73	1951000	1716880	1545192	1359768.96	0.114726432	0.69696

Table 4. Results obtained for the various gradients for rack of 18 LHB II AC coaches.

The value of $E_{traction}$ or energy consumed increases as we increase the gradient but $E_{braking}$ or braking energy reduces. This is because the train is climbing uphill, this requires higher energy to move against the gravity but takes less energy to stop. The ratio of energy consumed to energy recovered ε falls from 0.30 at zero gradient to 0.11 at 1% gradient.

As given in table 3, range of allowed gradient for ruling plain is 0.5-0.67%. For the maximum gradient allowed on a ruling plain that is 0.67% value of energy recovered is 1.428×10^9 watthours. If we increase the gradient even more, which is to 1%, the maximum allowed gradient for the hilly regions. The recovered energy is 1.359×10^9 watthours. If we compare the magnitude of these recovered energy, it's not much less than that of a 0% gradient. But the percentage recovered on the amount spent for traction has reduced significantly. For

0% gradient the energy recovered was 30.32 % for full loaded and 29.97% for half loaded, whereas for 1% gradient it reduced to mere 11.43% for full loaded and 11.47% for half loaded.

4.5 Discussions

A comprehensive research *(Perez-Martinez,2010) (4)* examined state-of-the-art technology for energy recovery systems in railway systems and revealed real-world energy savings of 20 to 30 percent. Savings of 24 percent *(Domniguez, M, Cuala, A, et.al 2011) (5)* and 18.6–35.6 percent *(Barrero,R, e.al, 2008) (6)* have been recorded for on-board energy storage. According to one research *(Richardson, 2002) [7]*, FESS applications in trains have been used in New York since 2000. This highlights how energy storage may lower peak power demands at the rectifier and reinforce the supply infrastructure, as well as deliver extra energy savings using inbuilt regenerative motors. The literature on FESS storage in railway applications is fairly sparse, as Rupp et al. *(Rupp, A.; Baier, H.; et.al) (8)* have already pointed out. According to their research, savings ranged from 9.83 percent to 31.21 percent. Subject of their research was a light transit convoy with on-board FESS was the. The track, according to their theory, had no gradient. Because of fundamental variations in the convoy Rupp et al. analyzed and our own convoy model, our data for no-gradient settings indicates efficiency between 29.9% and 31.6 percent depending on the convoy.

Based on the analysis performed using the simulation following conclusions are derived:

1. Using FESS on board we can avoid power losses in transmission, converter stations and in transformers.
2. With current technology WAP-7, WAG-9 class of locomotives are able to save up to 20% of total energy through regenerative braking. Our results are much higher, 29-31%.
3. If we increase the gradient the value of traction energy spent to climb uphill increases, whereas the braking energy decreases.
4. The efficiency of the system reduces as the gradients increase.
5. The value of energy recovery efficiency remained the same throughout all the configurations, this is because the model and efficiency of induction motor as well as the flywheel were kept the same.
6. Efficiency gain (ratio of energy recovered to the ratio of energy spent through-out the trip) for the fully loaded stock has been higher than the energy gain for half loaded, because higher mass implies more powerful accelerations and decelerations, hence greater amount of energy can be saved.
7. The energy saving during the whole journey is very high, to avoid the capacity limitations of the storage it is recommended to use up all the energy stored before reaching next station.
8. Tractive effort and acceleration that a locomotive can achieve is limited by the power capability of the induction motor.
9. It is not necessary for a more powerful FESS to be more cost effective. Cheaper, less powerful systems can save money in the long run by being less sensitive to operational settings and so minimizing the risk of unpredictability.

Several general issues should be taken into account while implementing a FESS in trains. While the electrical coupling between a FESS and trains looks to be relatively easy when compared to other direct current energy storage options like as capacitors and batteries, safety and space considerations for a FESS must be considered *(Rail Safety and Standards Board 2009.) (9)*. To ensure operational safety, the FESS housing must survive any rotor failures, and the influence of the FESS on train stability and maneuverability must be studied. Due to the train's velocity, the rotor produces gyroscopic effects. According to Herbst et al., the curvature causes the highest rotation rate, hence the rotor axis should be oriented vertically *(Herbst et al 2003) (10)*. A FESS's space needs might be significant, making integration onto current trains difficult *(Steiner et.al) (11)*. The mounting of the FESS console on the roof is a common solution for low-floor cars *(Seiemns) (12)*.

4.6 Conclusion

A mathematical model for a passenger train was created and used to assess a particular route's train (Ahmedabad -Mumbai Central). A technique was proposed for predicting the train driving cycle based on inter-station distance and time. Train energy consumption was then estimated using the train and drive cycle models. The train and driving cycle mathematical models were validated when energy consumption predictions without a FESS were compared to values reported by railway operators, and similar results were obtained, validating the train and driving cycle mathematical models.

To calculate the possible energy savings on using the FESS data of its efficiency was taken from the previously done work by *(Rupp, A. et.al) (13)*. The efficiency of the flywheel was multiplied to the braking energy generated to obtain the total energy stored. The stored energy will travel again through the whole system to be supplied as traction energy to the wheels hence efficiency of induction motor was again multiplied to obtain the net recovered energy.

The results show that utilizing the FESS can save a significant amount of energy. The estimated energy savings varied depending on the track, payload, number of cars per train, and kind of FESS. The net recovered energy ranges from 29-31% of the total energy consumed. The feasibility of a FESS for decreasing energy usage in passenger rail transportation systems is confirmed in this study. The technique and results can be applicable to various rail routes, even if the results were achieved for a specific route.

Appendix A. Velocity-step method for determining the driving cycle

The velocity step method uses the distance and time spans given in table 1 as input. The acceleration is determined by the user defined parameters like maximum power capacity of the locomotive. The deacceleration is taken as a constant value to reduce complex calculations, it is set to a value satisfying all the safety and comfort parameters. The time is gradually increased in a step like manner to obtain the change in velocity ΔV. Then final velocity, distance travelled and time spent are updated as shown in equations (18), (19) & (20).

$$V_{i+1} = V_i + \Delta V \tag{18}$$

$$t_{i+1} = t_i + \Delta t \tag{19}$$

$$X_{i+1} = X_i + (V_i + 0.5\Delta V)\Delta t \tag{20}$$

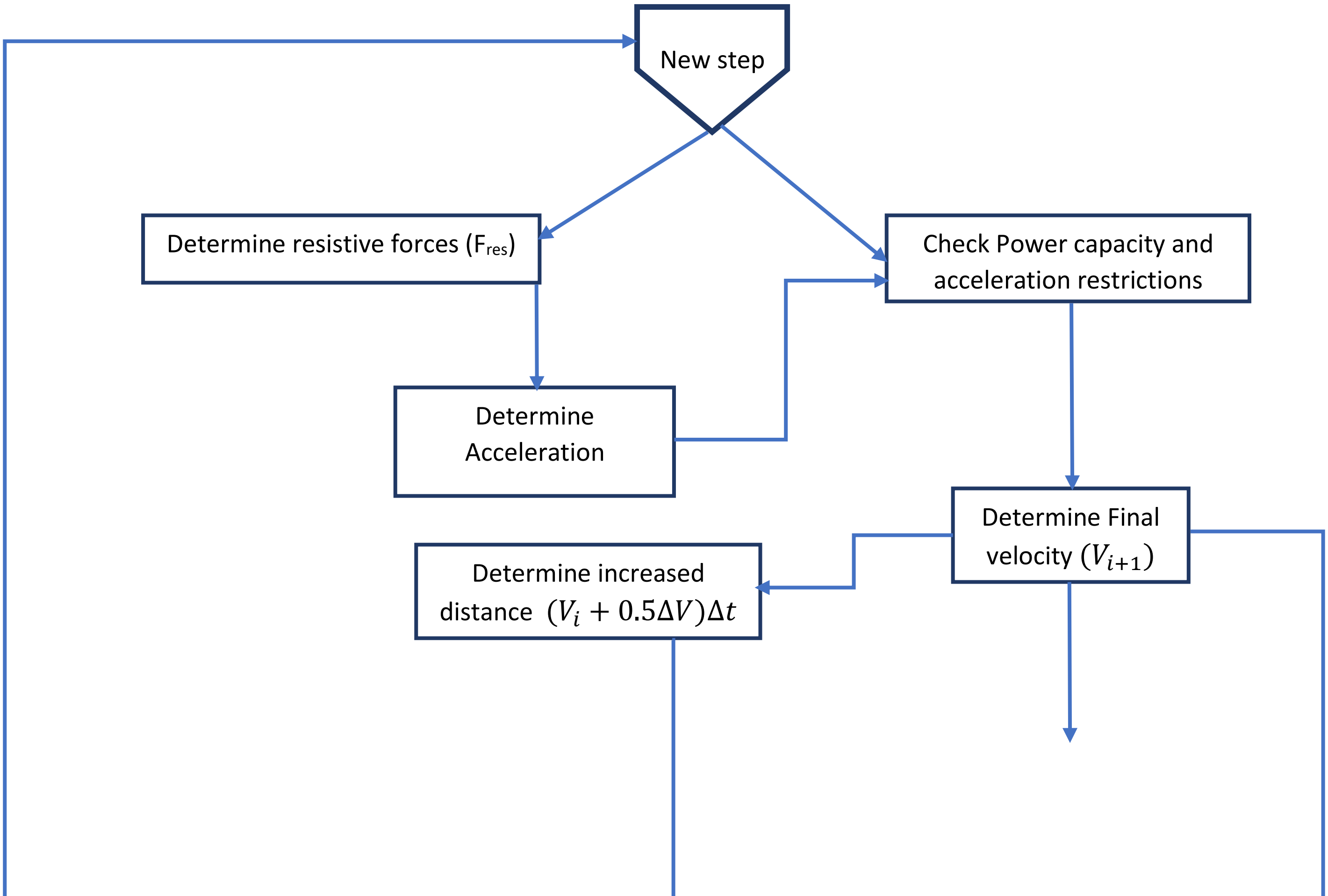
Here $V_i, t_{i,} X_i$ are the velocity, time and distance at the current step respectively, while V_{i+1} t_{i+1}, X_{i+1} are associated with the next steps. Since time is increased in constant step the value of acceleration is needed to be determined for every next step. Following are the parameters on which the acceleration is determined.

1. Maximum allowed acceleration ($\hat{a}$): The acceleration (a) must be less than the maximum allowed acceleration on the particular section.
2. The acceleration is dependent on the traction effort which is limited by the power capacity of the motor.
3. Safety and comfort parameters are also kept in mind. Rate of change of acceleration or jolt must be restricted for the safety and comfort of the passengers.

After increasing time by a definite step, power consumption on the current velocity is determined by multiplying the constant traction force, and then compared to the maximum power limit. The minimum of both powers is selected and the corresponding force is taken as the final tractive effort. After obtaining tractive force acceleration is obtained and is made sure to be in the limits. The velocity for the next step is determined, the time spent & distance travelled during this step is updated.

Parallelly the braking distance along with the braking time for the updated velocity is computed. Subtracting the sum of time travelled and the braking time from the total journey time we obtain the time travelled at constant speed. On obtaining the value of constant speed journey time we also get the distance travelled at a constant speed. The sum of all the three distances (travelled, braking and at constant speed) is compared with the journey distance. If the sum of distances is equal to or more than journey distance than further acceleration is not required, else further acceleration is needed.

The flow chart below explains the drive cycle computation:



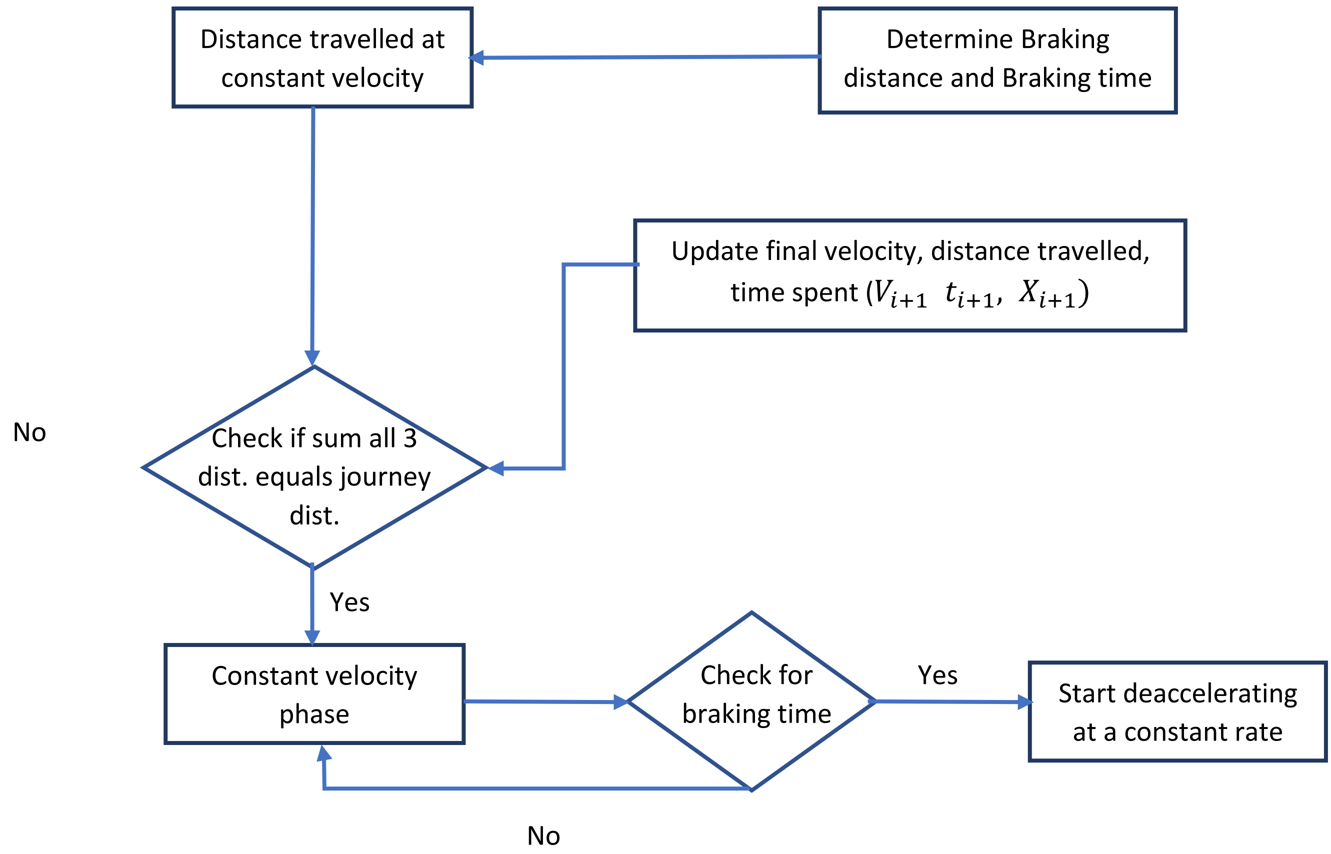


Fig. 9. Flow chart of the driving cycle computation

Appendix B Code snippets from python simulation

1. Libraries used for making the simulation: pandas to import the track profile, numpy for using mathematical functions, matplotlib.pyplot for plotting the result graphs

```
from cProfile import label
from cmath import sin
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import math
```

2. Taking route profile as an input from the user.

```
route_file=input("Input the name of excel file with route details, ex- 'test.xlsx'")
route_profile=pd.read_excel(route_file)
```

3. Determining traction force and calculating acceleration.

```
ftrac=min(pmax/u,fconti)
if stop==False:
    acc=(ftrac-fres)/net_mass
else: acc=0
v=u+acc
```

4. Line 82,85,93 and 104 checks for different conditions of increasing velocity and determine the traction force and acceleration according to it.
5. Determining the braking distance and braking time.

```
deacc=(brake)/net_mass
dist+=u+(acc/2)
brtime=v/deacc
brakdist=(v*v)/(2*deacc)
midtime=i-(brtime+t1)
middist=v*midtime
```

```
83            v=u+acc
84            p=fstart*v
if (j)<=(dist+brakdist+middist):
86            fres=((0.0069*net_mass)+130*20)+(504*10**-6)*net_mass*u+(0.046+0.0065*(carriages-1))*13.4*3.6*3.6*u*u
87            ftrac=min(pmax/u,fconti)
88            if stop==False:
89                acc=(ftrac-fres)/net_mass
90            else: acc=0
91            v=u+acc
92            p=ftrac*v
93        elif u>=cspeed and u<umax:
94            fres=((0.0069*net_mass)+130*20)+(504*10**-6)*net_mass*u+(0.046+0.0065*(carriages-1))*13.4*3.6*3.6*u*u
95            ftrac=min(pmax/u,fconti)
96            #ftrac=fconti
97            if stop==False:
98                acc=(ftrac-fres)/net_mass
99                v=u+acc
100           else: acc=0
101           if v>umax:
102               v=umax
103           p=ftrac*v
104       elif u==umax:
105           acc=0
106           v=u
107           fres=((0.0069*net_mass)+130*20)
108           p=fres*v
```

6. Checking the condition to stop the acceleration.
7. Checking if the braking time has reached.
8. Deaccelrating the train at a constant rate.

```
elif braking==True:
 deacc=-1*(brake)/net_mass
 v=u+deacc
 acc=deacc
 dist+=u+0.5*acc
 u=v
```

9. Plotting various curves. (Velocity- time, accelration-time, power-time, traction energy-time,etc.).

```
x_val=[x[1] for x in Vplot]
y_val=[x[0] for x in Vplot]
plt.plot(x_val,y_val)
plt.xlabel('Time (sec)')
plt.ylabel('Velocity (m/s)')
plt.show()
x_val1=[x[1] for x in Aplot]
y_val1=[x[0] for x in Aplot]
plt.plot(x_val1,y_val1)
plt.xlabel('Time (sec)')
plt.ylabel('Accelration (m/s^2)')
plt.show()
x_val2=[x[1] for x in powerplot]
y_val2=[x[0] for x in powerplot]
plt.plot(x_val2,y_val2)
plt.xlabel('Time (sec)')
plt.ylabel('Power (watts)')
plt.show()
x_val3=[x[1]for x in etr]
y_val3=[x[0] for x in etr]
plt.plot(x_val3,y_val3)
plt.xlabel('Time (sec)')
plt.ylabel('Traction Energy (Wh)')
plt.show()
x_val4=[x[1]for x in ebr]
y_val4=[x[0] for x in ebr]
plt.plot(x_val4,y_val4)
plt.xlabel('Time (sec)')
plt.ylabel('Braking Energy (Wh)')
plt.show()
```

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