CALCULUS & ANALYTICAL GEOMETRY (2)
(MATH 132)-WORKSHEET#00

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1 Answers for Q1

1.1

To find the most general antiderivative of $f(x) = x^2 - 3x + 2$, we integrate term by term:

$$F(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x + C$$

Here, C is the constant of integration. To verify, differentiate $F(x)$:

$$F'(x) = x^2 - 3x + 2$$

This matches the original function $f(x)$, so the antiderivative is correct.

1.2

the antiderivative for $f(x) = 2x^3 - \frac{2}{3}x^2 + 5x$:

$$F(x) = \frac{1}{2}x^4 - \frac{2}{9}x^3 + \frac{5}{2}x^2 + C$$

To verify, differentiate $F(x)$:

$$F'(x) = 2x^3 - \frac{2}{3}x^2 + 5x$$

This matches the original function $f(x)$, so the antiderivative is correct.

1.3

the antiderivative for $f(x) = 6x^5 - 8x^4 - 9x^2$:

$$F(x) = \frac{6}{6}x^6 - \frac{8}{5}x^5 - \frac{9}{3}x^3 + C$$

$$F(x) = x^6 - \frac{8}{5}x^5 - 3x^3 + C$$

To verify, differentiate $F(x)$:

$$F'(x) = 6x^5 - 8x^4 - x^2$$

This matches the original function $f(x)$, so the antiderivative is correct.

1.4

To find the most general antiderivative of $g(t) = \frac{1+t+t^2}{\sqrt{t}}$, first rewrite the function using exponent rules:

$$g(t) = t^{-1/2} + t^{1/2} + t^{3/2}$$

Now integrate term by term:

$$G(t) = 2t^{1/2} + \frac{2}{3}t^{3/2} + \frac{2}{5}t^{5/2} + C$$

To verify, differentiate $G(t)$:

$$G'(t) = t^{-1/2} + t^{1/2} + t^{3/2}$$

This matches the original function $g(t)$, so the antiderivative is correct.

1.5

To find the most general antiderivative of $r(\theta) = \sec(\theta)\tan(\theta) - 2e^\theta$, integrate term by term:

$$R(\theta) = \sec(\theta) - 2e^\theta + C$$

To verify, differentiate $R(\theta)$:

$$R'(\theta) = \sec(\theta)\tan(\theta) - 2e^\theta$$

This matches the original function $r(\theta)$, so the antiderivative is correct.

1.6

To find the most general antiderivative of $f(x) = x(12x + 8)$, first expand the function:

$$f(x) = 12x^2 + 8x$$

Now integrate term by term:

$$F(x) = 4x^3 + 4x^2 + C$$

To verify, differentiate $F(x)$:

$$F'(x) = 12x^2 + 8x$$

This matches the original function $f(x)$, so the antiderivative is correct.

1.7

To find the most general antiderivative of $f(x) = (x-5)^2$, first expand the function:

$$f(x) = x^2 - 10x + 25$$

Now integrate term by term:

$$F(x) = \frac{1}{3}x^3 - 5x^2 + 25x + C$$

To verify, differentiate $F(x)$:

$$F'(x) = x^2 - 10x + 25$$

This matches the original function $f(x)$, so the antiderivative is correct.

1.8

To find the most general antiderivative of $h(\theta) = 2\sin(\theta) - \sec^2(\theta)$, integrate term by term:

$$H(\theta) = -2\cos(\theta) - \tan(\theta) + C$$

To verify, differentiate $H(\theta)$:

$$H'(\theta) = 2\sin(\theta) - \sec^2(\theta)$$

This matches the original function $h(\theta)$, so the antiderivative is correct.

1.9

To find the most general antiderivative of $g(v) = 2\cos(v) - \frac{3}{\sqrt{1-v^2}}$, integrate term by term:

$$G(v) = 2\sin(v) - 3\text{arcsin}^{1}(v) + C$$

To verify, differentiate $G(v)$:

[1] An arctrig function is the inverse of a trigonometric function. It is also denoted as trig^{-1}, e.g., $\sin^{-1}(x) = \text{arcsin}(x)$.

$$G'(v) = 2\cos(v) - \frac{3}{\sqrt{1-v^2}}$$

This matches the original function $g(v)$, so the antiderivative is correct.

1.10

To find the most general antiderivative of $f(x) = 7x^{2/5} + 8x^{-4/5}$, integrate term by term:

$$F(x) = 5x^{7/5} + 40x^{1/5} + C$$

To verify, differentiate $F(x)$:

$$F'(x) = 7x^{2/5} + 8x^{-4/5}$$

This matches the original function $f(x)$, so the antiderivative is correct.

1.11

To find the most general antiderivative of $f(x) = \sin(2x+5) + e^{-3x}$, integrate term by term:

$$F(x) = -\frac{1}{2}\cos(2x+5) - \frac{1}{3}e^{-3x} + C$$

To verify, differentiate $F(x)$:

$$F'(x) = \sin(2x+5) + e^{-3x}$$

This matches the original function $f(x)$, so the antiderivative is correct.

1.12

To find the most general antiderivative of $f(x) = \frac{1}{\sinh(x)+\cosh(x)}$, we use the identity:

$$\sinh(x) + \cosh(x) = e^x$$

Thus, the function simplifies to:

$$f(x) = \frac{1}{e^x} = e^{-x}$$

Now integrate e^{-x}:

$$F(x) = -e^{-x} + C$$

To verify, differentiate $F(x)$:

$$F'(x) = e^{-x}$$

This matches the simplified form of the original function $f(x)$, so the antiderivative is correct.

1.13

To find the most general antiderivative of $f(x) = x^{3.4} - 2x^{\sqrt{2}-1}$, integrate term by term:

$$F(x) = \frac{1}{4.4}x^{4.4} - \frac{2}{\sqrt{2}}x^{\sqrt{2}} + C$$

Simplify the constants:

$$F(x) = \frac{5}{22}x^{4.4} - \sqrt{2}x^{\sqrt{2}} + C$$

To verify, differentiate $F(x)$:

$$F'(x) = x^{3.4} - 2x^{\sqrt{2}-1}$$

This matches the original function $f(x)$, so the antiderivative is correct.

1.14

To find the most general antiderivative of $f(x) = 2^x + 4\sinh(x)$, integrate term by term:

$$F(x) = \frac{2^x}{\ln 2} + 4\cosh(x) + C$$

To verify, differentiate $F(x)$:

$$F'(x) = \frac{d}{dx}\left(\frac{2^x}{\ln 2}\right) + \frac{d}{dx}(4\cosh(x))$$

$$F'(x) = \frac{2^x \ln 2}{\ln 2} + 4\sinh(x)$$

$$F'(x) = 2^x + 4\sinh(x)$$

This matches the original function $f(x)$, so the antiderivative is correct.

1.15

To find the most general antiderivative of $f(x) = 1 + 2\sin(x) + \frac{3}{\sqrt{x}}$, integrate term by term:

$$F(x) = x - 2\cos(x) + 6x^{\frac{1}{2}} + C$$

To verify, differentiate $F(x)$:

$$F'(x) = 1 + 2\sin(x) + \frac{3}{\sqrt{x}}$$

This matches the original function $f(x)$, so the antiderivative is correct.

1.16

To find the most general antiderivative of $f(x) = \sqrt{2}$, integrate term by term:

$$F(x) = \sqrt{2}x + C$$

To verify, differentiate $F(x)$:

$$F'(x) = \sqrt{2}$$

This matches the original function $f(x)$, so the antiderivative is correct.

1.17

To find the most general antiderivative of $f(x) = e^2$, integrate term by term:

$$F(x) = e^2 x + C$$

To verify, differentiate $F(x)$:

$$F'(x) = e^2$$

This matches the original function $f(x)$, so the antiderivative is correct.

1.18

To find the most general antiderivative of $f(x) = 3\sqrt{x} - 2\sqrt[3]{x}$, rewrite the terms using exponents:

$$f(x) = 3x^{\frac{1}{2}} - 2x^{\frac{1}{3}}$$

Now integrate term by term:

$$F(x) = \frac{3}{\frac{3}{2}} x^{\frac{3}{2}} - \frac{2}{\frac{4}{3}} x^{\frac{4}{3}} + C$$

Simplify the constants:

$$F(x) = 2x^{\frac{3}{2}} - \frac{6}{4} x^{\frac{4}{3}} + C$$

$$F(x) = 2x^{\frac{3}{2}} - \frac{3}{2} x^{\frac{4}{3}} + C$$

To verify, differentiate $F(x)$:

$$F'(x) = 2 \cdot \frac{3}{2} x^{\frac{1}{2}} - \frac{3}{2} \cdot \frac{4}{3} x^{\frac{1}{3}}$$

$$F'(x) = 3x^{\frac{1}{2}} - 2x^{\frac{1}{3}}$$

This matches the original function $f(x)$, so the antiderivative is correct.

1.19

To find the most general antiderivative of $f(x) = \sqrt[3]{x^2} + x\sqrt{x}$, rewrite the terms using exponents:

$$f(x) = x^{\frac{2}{3}} + x^{\frac{3}{2}}$$

Now integrate term by term:

$$F(x) = \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C$$

Simplify the exponents and constants:

$$F(x) = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C$$

$$F(x) = \frac{3}{5}x^{\frac{5}{3}} + \frac{2}{5}x^{\frac{5}{2}} + C$$

To verify, differentiate $F(x)$:

$$F'(x) = \frac{3}{5} \cdot \frac{5}{3}x^{\frac{2}{3}} + \frac{2}{5} \cdot \frac{5}{2}x^{\frac{3}{2}}$$

$$F'(x) = x^{\frac{2}{3}} + x^{\frac{3}{2}}$$

This matches the original function $f(x)$, so the antiderivative is correct.

1.20

To find the most general antiderivative of $f(x) = \frac{2x^4+4x^3-x}{x^3}$ for $x > 0$, first simplify the expression by dividing each term by x^3:

$$f(x) = \frac{2x^4}{x^3} + \frac{4x^3}{x^3} - \frac{x}{x^3}$$

$$f(x) = 2x + 4 - \frac{1}{x^2}$$

Now, integrate term by term:

$$F(x) = x^2 + 4x + \frac{1}{x} + C$$

To verify, differentiate $F(x)$:

$$F'(x) = 2x + 4 - \frac{1}{x^2}$$

This matches the original function $f(x)$, so the antiderivative is correct.

1.21

To find the most general antiderivative of $f(t) = \frac{3t^4-t^3+6t^2}{t^4}$, first simplify the expression by dividing each term by t^4:

$$f(t) = \frac{3t^4}{t^4} - \frac{t^3}{t^4} + \frac{6t^2}{t^4}$$

$$f(t) = 3 - \frac{1}{t} + \frac{6}{t^2}$$

Now, integrate term by term:

$$F(t) = 3t - \ln|t| - \frac{6}{t} + C$$

To verify, differentiate $F(t)$:

$$F'(t) = 3 - \frac{1}{t} + \frac{6}{t^2}$$

This matches the original function $f(t)$, so the antiderivative is correct.

1.22

To find the most general antiderivative of $f(x) = \frac{1}{5} - \frac{2}{x}$, integrate term by term:

$$F(x) = \frac{1}{5}x - 2\ln|x| + C$$

To verify, differentiate $F(x)$:

$$F'(x) = \frac{1}{5} - \frac{2}{x}$$

This matches the original function $f(x)$, so the antiderivative is correct.

1.23

To find the most general antiderivative of $f(x) = 2\sin(x)\cos(x)$, use the trigonometric identity:

$$2\sin(x)\cos(x) = \sin(2x)$$

Now, integrate $\sin(2x)$:

$$F(x) = -\frac{1}{2}\cos(2x) + C$$

To verify, differentiate $F(x)$:

$$F'(x) = \sin(2x)$$

This matches the original function $f(x)$, so the antiderivative is correct.

1.24

Sure! To find the most general antiderivative of $f(x) = \tan^2(x)$, we'll start by deriving the identity for $\tan^2(x)$ from the Pythagorean identity $\sin^2(x) + \cos^2(x) = 1$.

We know that:

$$\tan^2(x) = \frac{\sin^2(x)}{\cos^2(x)}$$

Now, we can use the identity $\sin^2(x) + \cos^2(x) = 1$, which leads to:

$$\sin^2(x) = 1 - \cos^2(x)$$

So,

$$\tan^2(x) = \frac{1-\cos^2(x)}{\cos^2(x)} = \sec^2(x) - 1$$

Now, we can proceed with the antiderivative of $f(x) = \sec^2(x) - 1$:

$$F(x) = \tan(x) - x + C$$

To verify, differentiate $F(x)$:

$$F'(x) = \sec^2(x) - 1$$

This matches the original function $f(x)$, so the antiderivative is correct.

2 Answer for Q2

Given the values:

$$x_1 = 0.5, \quad x_2 = -1, \quad x_3 = 2, \quad x_4 = 1.5$$

We need to compute the sum:

$$\sum_{k=1}^{4} (x_k - 2)^2$$

Step 1: Calculate each term $(x_k - 2)^2$ individually:

$$\begin{aligned}
(x_1 - 2)^2 &= (0.5 - 2)^2 = (-1.5)^2 = 2.25 \\
(x_2 - 2)^2 &= (-1 - 2)^2 = (-3)^2 = 9 \\
(x_3 - 2)^2 &= (2 - 2)^2 = 0^2 = 0 \\
(x_4 - 2)^2 &= (1.5 - 2)^2 = (-0.5)^2 = 0.25
\end{aligned}$$

Step 2: Sum all the calculated terms:

$$\sum_{k=1}^{4} (x_k - 2)^2 = 2.25 + 9 + 0 + 0.25 = 11.5$$

Final Answer:

$$\boxed{11.5}$$

3 Answers for Q3

(a)

We are asked to find the value of the following sum:

$$S = \sum_{k=0}^{3} (k^2 + 7)$$

We can split this into two separate sums:

$$S = \sum_{k=0}^{3} k^2 + \sum_{k=0}^{3} 7$$

Step 1: Sum of Squares $\sum_{k=0}^{3} k^2$

The formula for the sum of squares of the first n integers is:

$$\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Substituting $n = 3$:

$$\sum_{k=0}^{3} k^2 = \frac{3(3+1)(2(3)+1)}{6} = \frac{3(4)(7)}{6} = \frac{84}{6} = 14$$

Thus, the sum of squares is 14.

Step 2: Sum of Constants $\sum_{k=0}^{3} 7$

The formula for the sum of a constant c over $n+1$ terms is:

$$\sum_{k=0}^{n} c = c \cdot (n+1)$$

Here, $c = 7$ and the sum is from $k = 0$ to $k = 3$, so there are $3 + 1 = 4$ terms. Using the formula:

$$\sum_{k=0}^{3} 7 = 7 \cdot (3+1) = 7 \cdot 4 = 28$$

Thus, the sum of constants is 28.

Step 3: Combine the Two Sums

Now we combine the two sums:

$$S = 14 + 28 = 42$$

Thus, the value of the sum is:

$$\boxed{42}$$

(b)

Evaluate the sum:

$$\sum_{k=1}^{5} (k^2 - 1)(k - 2).$$

Step 1: Expand $(k^2 - 1)(k - 2)$

First, expand the expression:

$$(k^2 - 1)(k - 2) = k^2(k - 2) - 1(k - 2) = k^3 - 2k^2 - k + 2.$$

Thus, the sum becomes:

$$\sum_{k=1}^{5}(k^2-1)(k-2)=\sum_{k=1}^{5}(k^3-2k^2-k+2).$$

Step 2: Break into Individual Sums

Split the sum into four separate sums:

$$\sum_{k=1}^{5}(k^3-2k^2-k+2)=\sum_{k=1}^{5}k^3-2\sum_{k=1}^{5}k^2-\sum_{k=1}^{5}k+\sum_{k=1}^{5}2.$$

Step 3: Compute Each Sum

Compute each of the four sums individually:

1. $\sum_{k=1}^{5} k^3$:

$$1^3+2^3+3^3+4^3+5^3=1+8+27+64+125=225.$$

2. $\sum_{k=1}^{5} k^2$:

$$1^2+2^2+3^2+4^2+5^2=1+4+9+16+25=55.$$

Thus, $-2\sum_{k=1}^{5} k^2=-2\cdot 55=-110$.

3. $\sum_{k=1}^{5} k$:

$$1+2+3+4+5=15.$$

Thus, $-\sum_{k=1}^{5} k=-15$.

4. $\sum_{k=1}^{5} 2$:

$$2+2+2+2+2=10.$$

Step 4: Combine the Results

Add the results of the four sums:

$$225-110-15+10=110.$$

Final Answer

$$\boxed{110}$$

(c)

Evaluate the sum:

$$\sum_{r=1}^{20}(r^3+1).$$

Step 1: Break into Individual Sums

The sum can be split into two separate sums:

$$\sum_{r=1}^{20}(r^3+1) = \sum_{r=1}^{20} r^3 + \sum_{r=1}^{20} 1.$$

Step 2: Compute $\sum_{r=1}^{20} r^3$

The formula for the sum of cubes is:

$$\sum_{r=1}^{n} r^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

For $n = 20$:

$$\sum_{r=1}^{20} r^3 = \left(\frac{20 \cdot 21}{2}\right)^2 = (210)^2 = 44100.$$

Step 3: Compute $\sum_{r=1}^{20} 1$

The sum of 1 repeated 20 times is:

$$\sum_{r=1}^{20} 1 = 20.$$

Step 4: Combine the Results

Add the two sums together:

$$\sum_{r=1}^{20}(r^3+1) = 44100 + 20 = 44120.$$

Final Answer

$$\boxed{44120}$$

4 Answers for Q4

(a)

Evaluate the sum:

$$\sum_{k=1}^{n}(3k-2).$$

Step 1: Break into Individual Sums

The sum can be split into two separate sums:

$$\sum_{k=1}^{n}(3k-2) = 3\sum_{k=1}^{n} k - 2\sum_{k=1}^{n} 1.$$

Step 2: Compute $\sum_{k=1}^{n} k$

The formula for the sum of the first n natural numbers is:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

Step 3: Compute $\sum_{k=1}^{n} 1$

The sum of 1 repeated n times is:

$$\sum_{k=1}^{n} 1 = n.$$

Step 4: Substitute and Simplify

Substitute the results from Steps 2 and 3 into the expression:

$$\sum_{k=1}^{n} (3k-2) = 3 \cdot \frac{n(n+1)}{2} - 2 \cdot n.$$

Simplify the expression:

$$\sum_{k=1}^{n} (3k-2) = \frac{3n(n+1)}{2} - 2n.$$

Combine the terms into a single fraction:

$$\sum_{k=1}^{n} (3k-2) = \frac{3n(n+1) - 4n}{2}.$$

Expand and simplify the numerator:

$$3n(n+1) - 4n = 3n^2 + 3n - 4n = 3n^2 - n.$$

Thus, the final simplified form is:

$$\sum_{k=1}^{n} (3k-2) = \frac{3n^2 - n}{2}.$$

Final Answer

$$\boxed{\frac{3n^2 - n}{2}}$$

(b)

We are asked to find the value of the following sum:

$$S = \sum_{j=2}^{n} (j^2 + j)$$

We can split this into two separate sums:

$$S = \sum_{j=2}^{n} j^2 + \sum_{j=2}^{n} j$$

Step 1: Sum of Squares $\sum_{j=2}^{n} j^2$
The formula for the sum of squares from 1 to n is:

$$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$$

To get the sum from $j = 2$ to $j = n$, we subtract the first term 1^2 from the sum:

$$\sum_{j=2}^{n} j^2 = \sum_{j=1}^{n} j^2 - 1^2$$

Substituting the formula for $\sum_{j=1}^{n} j^2$:

$$\sum_{j=2}^{n} j^2 = \frac{n(n+1)(2n+1)}{6} - 1$$

Step 2: Sum of Integers $\sum_{j=2}^{n} j$
The formula for the sum of integers from 1 to n is:

$$\sum_{j=1}^{n} j = \frac{n(n+1)}{2}$$

To get the sum from $j = 2$ to $j = n$, we subtract the first term 1 from the sum:

$$\sum_{j=2}^{n} j = \sum_{j=1}^{n} j - 1$$

Substituting the formula for $\sum_{j=1}^{n} j$:

$$\sum_{j=2}^{n} j = \frac{n(n+1)}{2} - 1$$

Step 3: Combine the Two Sums
Now, we combine the two sums:

$$S = \left(\frac{n(n+1)(2n+1)}{6} - 1 \right) + \left(\frac{n(n+1)}{2} - 1 \right)$$

We first combine the constants:

$$S = \left(\frac{n(n+1)(2n+1)}{6} \right) + \left(\frac{n(n+1)}{2} \right) - 2$$

Now, we find a common denominator. The least common denominator between 6 and 2 is 6. We rewrite the second term:

$$\frac{n(n+1)}{2} = \frac{3n(n+1)}{6}$$

Substituting this into the expression:

$$S = \frac{n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{6} - 2$$

Now, combine the fractions:

$$S = \frac{n(n+1)(2n+1) + 3n(n+1)}{6} - 2$$

Factor out $n(n+1)$ from the numerator:

$$S = \frac{n(n+1)\left((2n+1)+3\right)}{6} - 2$$

Simplify inside the parentheses:

$$S = \frac{n(n+1)(2n+4)}{6} - 2$$

Factor out 2 from $(2n+4)$:

$$S = \frac{n(n+1)\cdot 2(n+2)}{6} - 2$$

Simplify the factor of 2:

$$S = \frac{2n(n+1)(n+2)}{6} - 2$$

Now, simplify the fraction by dividing 2 by 6:

$$S = \frac{n(n+1)(n+2)}{3} - 2$$

Final Answer:

Thus, the simplified expression for the sum is:

$$S = \frac{n(n+1)(n+2)}{3} - 2$$

(c)

We are tasked with evaluating the sum:

$$\sum_{k=1}^{n}(k^2-1)(k+1).$$

Step 1: Simplify the Expression

First, expand $(k^2-1)(k+1)$:

$$(k^2-1)(k+1) = k^2(k+1) - 1(k+1) = k^3 + k^2 - k - 1.$$

Thus, the sum becomes:

$$\sum_{k=1}^{n}(k^2-1)(k+1) = \sum_{k=1}^{n}(k^3+k^2-k-1).$$

Step 2: Break into Individual Sums

Split the sum into four separate sums:

$$\sum_{k=1}^{n}(k^3 + k^2 - k - 1) = \sum_{k=1}^{n} k^3 + \sum_{k=1}^{n} k^2 - \sum_{k=1}^{n} k - \sum_{k=1}^{n} 1.$$

Step 3: Use Known Summation Formulas

Using the following formulas:

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} 1 = n,$$

we substitute into the expression:

$$\sum_{k=1}^{n}(k^3 + k^2 - k - 1) = \left(\frac{n(n+1)}{2}\right)^2 + \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} - n.$$

Step 4: Simplify the Expression

To simplify, we first rewrite all terms with a common denominator. The least common, multiple, denominator (LCD) for 4, 6, 2, and 1 is 12. Rewrite each term with denominator 12:

1. $\left(\frac{n(n+1)}{2}\right)^2 = \frac{n^2(n+1)^2}{4}$ becomes:

$$\frac{n^2(n+1)^2}{4} = \frac{3n^2(n+1)^2}{12}.$$

2. $\frac{n(n+1)(2n+1)}{6}$ becomes:

$$\frac{n(n+1)(2n+1)}{6} = \frac{2n(n+1)(2n+1)}{12}.$$

3. $\frac{n(n+1)}{2}$ becomes:

$$\frac{n(n+1)}{2} = \frac{6n(n+1)}{12}.$$

4. $-n$ becomes:

$$-n = \frac{-12n}{12}.$$

Now, combine all terms into a single fraction:

$$\frac{3n^2(n+1)^2 + 2n(n+1)(2n+1) - 6n(n+1) - 12n}{12}.$$

Step 5: Expand and Simplify the Numerator

Expand each term in the numerator:

1. Expand $3n^2(n+1)^2$:

$$3n^2(n+1)^2 = 3n^2(n^2 + 2n + 1) = 3n^4 + 6n^3 + 3n^2.$$

2. Expand $2n(n+1)(2n+1)$:

$$2n(n+1)(2n+1) = 2n(2n^2+3n+1) = 4n^3+6n^2+2n.$$

3. Expand $-6n(n+1)$:

$$-6n(n+1) = -6n^2-6n.$$

4. The last term is $-12n$.

Now, combine all the expanded terms:

$$3n^4+6n^3+3n^2+4n^3+6n^2+2n-6n^2-6n-12n.$$

Simplify by combining like terms:

1. $3n^4$ (no other n^4 terms),
2. $6n^3+4n^3 = 10n^3$,
3. $3n^2+6n^2-6n^2 = 3n^2$,
4. $2n-6n-12n = -16n$.

Thus, the numerator simplifies to:

$$3n^4+10n^3+3n^2-16n.$$

Step 6: Write the Final Simplified Form

The final simplified form of the sum is:

$$\boxed{\frac{3n^4+10n^3+3n^2-16n}{12}}.$$

5 Answer for Q5

Given the following information:

$$\sum_{j=1}^{10} x_j^2 = 15, \quad \sum_{j=1}^{10} y_j^2 = 26, \quad \sum_{j=1}^{10} (x_j+y_j)^2 = 73,$$

find the value of $\sum_{j=1}^{10} x_j y_j$.

Step 1: Expand $\sum_{j=1}^{10} (x_j+y_j)^2$

Expand the square inside the summation:

$$(x_j+y_j)^2 = x_j^2 + 2x_j y_j + y_j^2.$$

Thus, the sum becomes:

$$\sum_{j=1}^{10}(x_j + y_j)^2 = \sum_{j=1}^{10} x_j^2 + 2\sum_{j=1}^{10} x_j y_j + \sum_{j=1}^{10} y_j^2.$$

Step 2: Substitute the Given Values

Substitute the known values into the equation:

$$73 = 15 + 2\sum_{j=1}^{10} x_j y_j + 26.$$

Step 3: Simplify the Equation

Combine the constants on the right-hand side:

$$73 = 41 + 2\sum_{j=1}^{10} x_j y_j.$$

Subtract 41 from both sides:

$$73 - 41 = 2\sum_{j=1}^{10} x_j y_j.$$

Simplify:

$$32 = 2\sum_{j=1}^{10} x_j y_j.$$

Step 4: Solve for $\sum_{j=1}^{10} x_j y_j$

Divide both sides by 2:

$$\sum_{j=1}^{10} x_j y_j = \frac{32}{2} = 16.$$

Final Answer

$$\boxed{16}$$