**Cairo University**
**Faculty of Science**
**Department of Mathematics**

# Calculus and analytical geometry (2)
# (MATH 132)-Worksheet #01

2nd week (February 15, 2025 - February 20, 2025)

**Full Name**: ..............................................................................................

**Code number**.: ..............................................................................................

**Group number**: ..............................................................................................

## Theoretical Background

**Definition 1:** The **area** $A$ of the region $S$ that lies under the graph of the continuous function $f$ from $a$ to $b$ is the limit of the sum of the areas of approximating rectangles:

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty}[f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x] = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

Or

$$A = \lim_{n\to\infty} L_n = \lim_{n\to\infty}[f(x_0)\Delta x + f(x_1)\Delta x + \cdots + f(x_{n-1})\Delta x] = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_{i-1})\Delta x$$

Where $\Delta x = \frac{b-a}{n}$, $x_i = a + i\,\Delta x$, and $i = 0, 1, 2, \dots, n$

## Solve the following questions:

Q1) Let $S$ be the region bounded between the curves. $f(x) = 1 - x^3,\ x = 1,\ x = 5,$ and $x-axis$. Find the upper estimation for the area of $S$ using 4 rectangles.

Q2) Let $S$ be the region bounded between the curves. $f(x) = \frac{1}{(3+x^2)}$ , $x = -2$, $x = -1$, and $x - axis$. Find the lower estimation for the area of $S$ using 3 approximating rectangles.

Q3) Let $S$ be the region bounded between the curves. $f(x) = x^2 - 3x + 4$, $x = 0$, $x = 4$, and $x - axis$. Find the lower estimation for the area of $S$ using 4 rectangles.

Q4) Find Riemann sum for the function $f(x) = 4 - x^2$ over the interval $[-1,2]$ and corresponding to the partition $x_0 = -1$ , $x_1 = 0$ , $x_2 = \frac{1}{2}$ , $x_3 = \frac{5}{4}$ , $x_4 = 2$ and sample points $x_1^* = -\frac{1}{4}$ , $x_2^* = \frac{1}{4}$ , $x_3^* = 1$ , $x_4^* = \frac{5}{4}$

Q5) Find an expression for the area under the graph of $f$ as a limit. Do not evaluate the limit.

(a) $f(x) = x^3 + x,\ x \in [0,2]$ (simplify the expression)

(b) $f(x) = \sqrt{\sin x}, 0 \leq x \leq \pi$

(c) $f(x) = x^2 - 3x,\ x \in [-2,2]$ (simplify the expression)

(d) $f(x) = \sin\left(\frac{\pi}{2}x\right),\ x \in [0,1]$

Q6) Determine the region whose area is expressed by the following limit:

$$\lim_{n\to\infty} \frac{2}{n}\left(\sum_{k=1}^{n} \left(5+\frac{2k}{n}\right)^{10}\right)$$

Q7) (True or False) and Justify your answer for the following

a. If the area $A$ of region $S$ bounded by $y = x^4 + 1,\ y = 0,\ x = 0,$ and $x = 1$ , then $1 < A < 1.5$

b. $\sum_{i=1}^{n} i + \sin\left(\frac{i\pi}{2}\right) = \frac{n(n+1)}{2} + \sin\left(\frac{n(n+1)\pi}{4}\right)$

## Check your knowledge

Q8) Let $A$ be the area under the graph of an increasing continuous function $f$ from $a$ to $b$, and let $L_n$ and $R_n$ be the approximations to $A$ with n subintervals using left and right endpoints, respectively.

(a) How are $A$, $L_n$, and $R_n$ related?

(b) Show that $R_n - L_n = \frac{b-a}{n}[f(b) - f(a)]$