Cairo University
Faculty of Science
Department of Mathematics

Calculus and analytical geometry (2)
(MATH 132)-Worksheet #01

2nd week (February 15, 2025 - February 20, 2025)

Full Name: .....

Code number.: .....

Group number: .....

Theoretical Background

Definition 1: The **area** A of the region S that lies under the graph of the continuous function f from a to b is the limit of the sum of the areas of approximating rectangles:

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} [f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x] = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

Or

$$A = \lim_{n\to\infty} L_n = \lim_{n\to\infty} [f(x_0)\Delta x + f(x_1)\Delta x + \cdots + f(x_{n-1})\Delta x] = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_{i-1})\Delta x$$

Where $\Delta x = \frac{b-a}{n}$, $x_i = a + i\,\Delta x$, and $i = 0, 1, 2, \ldots, n$

Solve the following questions:

Q1) Let S be the region bounded between the curves. $f(x) = 1 - x^3$, $x = -5$, $x = -1$, and $x - axis$. Find the upper estimation for the area of S using 4 rectangles.

Q2) Let S be the region bounded between the curves. $f(x) = \frac{1}{(3+x^2)}$, $x = -2$, $x = -1$, and $x - axis$. Find the lower estimation for the area of S using 3 approximating rectangles.

Q3) Let S be the region bounded between the curves. $f(x) = x^2 - 3x + 4$, $x = 2$, $x = 6$, and $x - axis$. Find the lower estimation for the area of S using 4 rectangles.

Q4) Find Riemann sum for the function $f(x) = 4 - x^2$ over the interval $[-1,2]$ and corresponding to the partition $x_0 = -1$, $x_1 = 0$, $x_2 = \frac{1}{2}$, $x_3 = \frac{5}{4}$, $x_4 = 2$ and sample points $x_1^* = -\frac{1}{4}$, $x_2^* = \frac{1}{4}$, $x_3^* = 1$, $x_4^* = \frac{5}{4}$

Q5) Find an expression for the area under the graph of f as a limit. Do not evaluate the limit.

(a) $f(x) = x^3 + x,\ x \in [0,2]$ (simplify the expression)

(b) $f(x) = \sqrt{\sin x}\,, 0 \le x \le \pi$

(c) $f(x) = x^2 - 3x,\ x \in [-2, 0]$ (simplify the expression)

(d) $f(x) = \sin\left(\frac{\pi}{2}x\right),\ x \in [0,1]$

Q6) Determine the region whose area is expressed by the following limit:

$$\lim_{n\to\infty} \frac{2}{n}\left(\sum_{k=1}^{n} \left(5+\frac{2k}{n}\right)^{10}\right)$$

Q7) (True or False) and Justify your answer for the following

a. If the area A of region S bounded by $y = x^4 + 1,\ y = 0,\ x = 0,$ and $x = 1$, then $1 < A < 1.5$

b. $\sum_{i=1}^{n} i + \sin\left(\frac{i\pi}{2}\right) = \frac{n(n+1)}{2} + \sin\left(\frac{n(n+1)\pi}{4}\right)$

Check your knowledge

Q8) Let A be the area under the graph of an increasing continuous function f from a to b, and let L_n and R_n be the approximations to A with n subintervals using left and right endpoints, respectively.

(a) How are A, L_n, and R_n related?

(b) Show that $R_n - L_n = \frac{b-a}{n}[f(b) - f(a)]$