Cairo University
Faculty of Science
Department of Mathematics

Calculus and analytical geometry (2) (MATH 132)-Worksheet #00

1st week (February 8, 2025 - February 13, 2025) Spring 2025

Full Name: ....

Code number.: ....

Group number: ....

Theoretical Background

Theorem: If $F(x)$ is an antiderivative of $f(x)$ on an interval I, then the most general antiderivative of $f(x)$ on I is

$$F(x) + C$$

where C is an arbitrary constant.

Table of some Antidifferentiation Formulae

Function	Particular antiderivative	Function	Particular antiderivative
$cf(x)$	$cF(x)$	$\sin x$	$-\cos x$
$f(x)+g(x)$	$F(x)+G(x)$	$\sec^2 x$	$\tan x$
$x^n (n \neq -1)$	$\frac{x^{n+1}}{n+1}$	$\sec x \tan x$	$\sec x$
$\frac{1}{x}$	$\ln\|x\|$	$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x$
e^x	e^x	$\frac{1}{1+x^2}$	$\tan^{-1} x$
b^x, $(b > 0, b \neq 1)$	$\frac{b^x}{\ln b}$	$\cosh x$	$\sinh x$
$\cos x$	$\sin x$	$\sinh x$	$\cosh x$

Definition: If $a_m, a_{m+1}, \ldots, a_n$ are real numbers and m and n are integers such that $m \leqslant n$, then

$$\sum_{i=m}^{n} a_i = a_m + a_{m+1} + a_{m+2} + \cdots + a_{n-1} + a_n$$

Theorem: If c is any constant (that is, it does not depend on i), then

(a) $\sum_{i=m}^{n} ca_i = c\sum_{i=m}^{n} a_i$

(b) $\sum_{i=m}^{n} (a_i + b_i) = \sum_{i=m}^{n} a_i + \sum_{i=m}^{n} b_i$

(c) $\sum_{i=m}^{n} (a_i - b_i) = \sum_{i=m}^{n} a_i - \sum_{i=m}^{n} b_i$

Theorem: Let c be a constant and n a positive integer. Then

(a) $\sum_{i=1}^{n} 1 = n$

(b) $\sum_{i=1}^{n} c = nc$

(c) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

(d) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

(e) $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$

Solve the following questions:

Q1) Find the most general antiderivative of the function. (Check your answer by differentiation.)

1. $f(x) = x^2 - 3x + 2$

13. $f(x) = x^{3.4} - 2x^{\sqrt{2}-1}$

2. $f(x) = 2x^3 - \frac{2}{3}x^2 + 5x$

14. $f(x) = 2^x + 4\sinh x$

3. $f(x) = 6x^5 - 8x^4 - 9x^2$

15. $f(x) = 1 + 2\sin x + 3/\sqrt{x}$

4. $g(t) = \frac{1+t+t^2}{\sqrt{t}}$

16. $f(x) = \sqrt{2}$

5. $r(\theta) = \sec\theta\tan\theta - 2e^{\theta}$

17. $f(x) = e^2$

6. $f(x) = x(12x + 8)$

18. $f(x) = 3\sqrt{x} - 2\sqrt[3]{x}$

7. $f(x) = (x - 5)^2$

19. $f(x) = \sqrt[3]{x^2} + x\sqrt{x}$

8. $h(\theta) = 2\sin\theta - \sec^2\theta$

20. $f(x) = \frac{2x^4+4x^3-x}{x^3}, \; x > 0$

9. $g(v) = 2\cos v - \frac{3}{\sqrt{1-v^2}}$

21. $f(t) = \frac{3t^4-t^3+6t^2}{t^4}$

10. $f(x) = 7x^{2/5} + 8x^{-4/5}$

22. $f(x) = \frac{1}{5} - \frac{2}{x}$

11. $f(x) = \sin(2x+5) + e^{-3x}$

23. $f(x) = 2\sin x\cos x$

12. $f(x) = \frac{1}{\sinh x + \cosh x}$

24. $f(x) = \tan^2 x$

Q2) If

$$x_1 = 0.5,\ x_2 = -1,\ x_3 = 2,\ x_4 = 1.5$$

Then find $\sum_{k=1}^{4}(x_k - 2)^2$

Q3) Find the value of each of the following sums:

(a) $\sum_{k=0}^{3}(k^2 + 7)$

(b) $\sum_{k=1}^{5}(k^2 - 1)(k - 2)$

(c) $\sum_{r=1}^{20}(r^3 + 1)$

Q4) Simplify the following sums in terms of n only:

(a) $\sum_{k=1}^{n}(3k - 2)$

(b) $\sum_{j=2}^{n}(j^2+j)$

(c) $\sum_{k=1}^{n}(k^2-1)(k+1)$

Q5) Given that

$$\sum_{j=1}^{10} x_j^2 = 15, \ \sum_{j=1}^{10} y_j^2 = 26, \ \sum_{j=1}^{10} \left(x_j + y_j\right)^2 = 73$$

find $\sum_{r=1}^{10} x_r y_r$.