CALCULUS & ANALYTICAL GEOMETRY (2) (MATH 132)-WORKSHEET#02

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Answer for Q1

Evaluate the integral:

$$\int_1^5 (4-2x)\,dx$$

by interpreting it as a limit of a Riemann sum.

Step 1: Partition the Interval

Divide the interval $[1, 5]$ into n subintervals of equal width:

$$\Delta x = \frac{5-1}{n} = \frac{4}{n}.$$

The right endpoint of the i-th subinterval is:

$$x_i = 1 + i\Delta x = 1 + \frac{4i}{n}.$$

The Riemann sum is:

$$S_n = \sum_{i=1}^{n} \left[4 - 2\left(1 + \frac{4i}{n}\right)\right] \Delta x.$$

Step 2: Simplify the Summation

Simplifying the integrand:

$$4 - 2\left(1 + \frac{4i}{n}\right) = 2 - \frac{8i}{n}.$$

The Riemann sum becomes:

$$S_n = \sum_{i=1}^{n} \left(2 - \frac{8i}{n}\right) \cdot \frac{4}{n}.$$

Distribute $\frac{4}{n}$:

$$S_n = \sum_{i=1}^{n} \left(\frac{8}{n} - \frac{32i}{n^2}\right).$$

Separate the sums:

$$S_n = \frac{8}{n}\sum_{i=1}^{n} 1 - \frac{32}{n^2}\sum_{i=1}^{n} i.$$

Using known summations:

$$\sum_{i=1}^{n} 1 = n \quad \text{and} \quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2},$$

we get:

$$S_n = \frac{8}{n} \cdot n - \frac{32}{n^2} \cdot \frac{n(n+1)}{2}.$$

Simplify:

$$S_n = 8 - \frac{32(n+1)}{2n} = 8 - \frac{16(n+1)}{n}.$$

Step 3: Taking the Limit

Taking the limit as $n \to \infty$:

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \left[8 - \frac{16(n+1)}{n}\right].$$

Simplify the expression:

$$= \lim_{n\to\infty} \left[8 - 16\left(1 + \frac{1}{n}\right)\right] = 8 - 16 = -8.$$

Final Answer

$$\boxed{-8}$$

Answer for Q2

Evaluate the integral:

$$\int_1^5 (x^2 - 4x + 2)\, dx$$

Step 1: Partition the Interval

Divide the interval $[1, 5]$ into n equal subintervals:

$$\Delta x = \frac{5-1}{n} = \frac{4}{n}.$$

The right endpoint of the k-th subinterval is:

$$x_k = 1 + \frac{4k}{n}, \quad k = 0, 1, 2, \ldots, n.$$

Step 2: Riemann Sum Representation

The Riemann sum using right endpoints is:

$$S_n = \sum_{k=1}^{n} \left[\left(1 + \frac{4k}{n}\right)^2 - 4\left(1 + \frac{4k}{n}\right) + 2 \right] \cdot \frac{4}{n}.$$

Step 3: Expand the Function

Expand and simplify:

$$\left(1 + \frac{4k}{n}\right)^2 = 1 + \frac{8k}{n} + \frac{16k^2}{n^2}.$$

Thus:

$$\begin{aligned} x_k^2 - 4x_k + 2 &= 1 + \frac{8k}{n} + \frac{16k^2}{n^2} - 4 - \frac{16k}{n} + 2. \\ &= \frac{16k^2}{n^2} - \frac{8k}{n} - 1. \end{aligned}$$

Step 4: Full Riemann Sum

$$\begin{aligned} S_n &= \sum_{k=1}^{n} \left(\frac{16k^2}{n^2} - \frac{8k}{n} - 1 \right) \cdot \frac{4}{n}. \\ &= \sum_{k=1}^{n} \left(\frac{64k^2}{n^3} - \frac{32k}{n^2} - \frac{4}{n} \right). \end{aligned}$$

Step 5: Apply Summation Formulas

Use:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$

Substituting:

$$S_n = \frac{64}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} - \frac{32}{n^2} \cdot \frac{n(n+1)}{2} - \frac{4}{n} \cdot n.$$

Simplifying:

$$S_n = \frac{64(n+1)(2n+1)}{6n^2} - \frac{16(n+1)}{n} - 4.$$

Step 6: Taking the Limit

As $n \to \infty$:

$$\lim_{n\to\infty} S_n = \frac{64 \cdot 2}{6} - 16 - 4 = \frac{128}{6} - 20 = \frac{64}{3} - 20 = \frac{4}{3}.$$

Final Answer

$$\boxed{\frac{4}{3}}$$

Answer of Q3

Step 1: Set up the Riemann sum

Consider the interval $[a, b]$ divided into n equal subintervals.
The width of each subinterval is:

$$\Delta x = \frac{b-a}{n}.$$

The right endpoint of the i-th subinterval is:

$$x_i = a + i\Delta x = a + i\frac{b-a}{n}.$$

Step 2: Write the Riemann sum

The Riemann sum using right endpoints is:

$$S_n = \sum_{i=1}^{n} (x_i)^2 \Delta x.$$

Substituting x_i:

$$S_n = \sum_{i=1}^{n} \left(a + i\frac{b-a}{n}\right)^2 \cdot \frac{b-a}{n}.$$

Step 3: Expand and simplify

Expand the square:

$$\left(a + i\frac{b-a}{n}\right)^2 = a^2 + 2a\frac{b-a}{n}i + \left(\frac{b-a}{n}\right)^2 i^2.$$

Thus:

$$S_n = \frac{b-a}{n} \sum_{i=1}^{n} \left[a^2 + 2a\frac{b-a}{n}i + \left(\frac{b-a}{n}\right)^2 i^2\right].$$

Step 4: Use known summation formulas

Using the standard summation formulas:

$$\sum_{i=1}^{n} 1 = n, \quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6},$$

we get:

$$S_n = \frac{b-a}{n}\left[a^2 n + 2a\frac{b-a}{n}\cdot\frac{n(n+1)}{2} + \left(\frac{b-a}{n}\right)^2\cdot\frac{n(n+1)(2n+1)}{6}\right].$$

Step 5: Simplify the expression

Simplifying:

$$S_n = (b-a)\left[a^2 + a(b-a)\frac{n+1}{n} + \frac{(b-a)^2}{n^2}\frac{(n+1)(2n+1)}{6}\right].$$

Step 6: Take the limit as $n \to \infty$

As $n \to \infty$:

$$\frac{n+1}{n} \to 1, \quad \text{and} \quad \frac{(n+1)(2n+1)}{n^2} \to 2.$$

Hence:

$$\lim_{n\to\infty} S_n = (b-a)\left[a^2 + a(b-a) + \frac{(b-a)^2}{3}\right].$$

Expanding:

$$= (b-a)\left(a^2 + ab - a^2 + \frac{b^2 - 2ab + a^2}{3}\right),$$

$$= (b-a)\frac{b^2 + ab + a^2}{3}.$$

Step 7: Recognize the factorization

Since:

$$b^3 - a^3 = (b-a)(b^2 + ab + a^2),$$

we have:

$$\lim_{n\to\infty} S_n = \frac{b^3 - a^3}{3}.$$

Final Answer:

$$\boxed{\int_a^b x^2\,dx = \frac{b^3 - a^3}{3}}.$$

Answer of Q4

Step 1: Partition the Interval

We partition the interval $[1, 2]$ into n subintervals of equal length. The width of each subinterval is:

$$\Delta x = \frac{2-1}{n} = \frac{1}{n}.$$

Step 2: Choose Sample Points

Let x_i be the right endpoint of the i-th subinterval:

$$x_i = 1 + i\Delta x = 1 + \frac{i}{n}, \quad i = 1, 2, \ldots, n.$$

Step 3: Write the Riemann Sum

The Riemann sum for the given integral is:

$$\sum_{i=1}^{n} \sqrt{4 - \left(1 + \frac{i}{n}\right)^2} \cdot \frac{1}{n}.$$

Step 4: Take the Limit of the Riemann Sum

Taking the limit as $n \to \infty$, we get:

$$\int_1^2 \sqrt{4 - x^2}\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{4 - \left(1 + \frac{i}{n}\right)^2} \cdot \frac{1}{n}.$$

Final Answer

$$\boxed{\int_1^2 \sqrt{4 - x^2}\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{4 - \left(1 + \frac{i}{n}\right)^2} \cdot \frac{1}{n}.}$$

Answer of Q5

Step 1: Divide the Interval

Divide the interval $[2, 5]$ into n subintervals of equal width:

$$\Delta x = \frac{5 - 2}{n} = \frac{3}{n}.$$

Step 2: Choose Sample Points

Choose the right-endpoint for each subinterval as the sample point:

$$x_i^* = 2 + i\Delta x = 2 + \frac{3i}{n}, \quad \text{for } i = 1, 2, \ldots, n.$$

Step 3: Write the Riemann Sum

The Riemann sum corresponding to the given function $f(x) = x^2 + \frac{1}{x}$ is:

$$\sum_{i=1}^{n} \left[(x_i^*)^2 + \frac{1}{x_i^*} \right] \Delta x.$$

Substituting x_i^* and Δx:

$$\sum_{i=1}^{n} \left[\left(2 + \frac{3i}{n}\right)^2 + \frac{1}{2 + \frac{3i}{n}} \right] \cdot \frac{3}{n}.$$

Step 4: Express as a Limit

Finally, expressing the integral as a limit of Riemann sums:

$$\int_2^5 \left(x^2 + \frac{1}{x}\right) dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(2 + \frac{3i}{n}\right)^2 + \frac{1}{2 + \frac{3i}{n}} \right] \frac{3}{n}.$$

Final Answer

$$\boxed{\int_2^5 \left(x^2 + \frac{1}{x}\right) dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(2 + \frac{3i}{n}\right)^2 + \frac{1}{2 + \frac{3i}{n}} \right] \frac{3}{n}.}$$

Answer for Q6

(a)

Step 1: Identify the Riemann Sum Structure

A Riemann sum typically has the form:

$$\sum_{k=1}^{n} f(x_k)\Delta x,$$

where:

- $\Delta x = \frac{b-a}{n}$,
- $x_k = a + k\Delta x$,
- The limit as $n \to \infty$ gives the integral $\int_a^b f(x)\,dx$.

In our case:

- $\Delta x = \frac{1}{n}$,
- Let $x_k = \frac{k}{n}$, so that as $n \to \infty$, x_k ranges from 0 to 1,
- The expression inside the sum becomes $3x_k + 2$.

Step 2: Write the Limit as a Riemann Sum

Thus, the sum can be rewritten as:

$$\frac{1}{n}\sum_{k=1}^{n} \left(3 \cdot \frac{k}{n} + 2\right) = \sum_{k=1}^{n} (3x_k + 2)\,\frac{1}{n}.$$

This is precisely the Riemann sum for the integral:

$$\int_0^1 (3x+2)\,dx.$$

Final Answer

$$\boxed{\int_0^1 (3x+2)\,dx.}$$

(b)

Step 1: Recognizing the Riemann Sum Form

The given limit can be rewritten as:

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{1+\left(\frac{k}{n}\right)^2} \cdot \frac{1}{n}.$$

This resembles a Riemann sum of the form:

$$\sum_{k=1}^{n} f(x_k)\Delta x,$$

where:

$$x_k = \frac{k}{n} \quad \text{and} \quad \Delta x = \frac{1}{n}.$$

Step 2: Converting to the Integral

As $n \to \infty$, the Riemann sum approaches the definite integral:

$$\int_0^1 \frac{1}{1+x^2}\,dx.$$

Final Answer

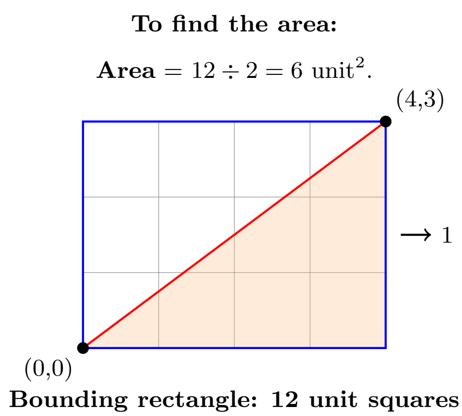
Thus, the given limit can be expressed as the integral:

$$\boxed{\int_0^1 \frac{1}{1+x^2}\,dx.}$$

Answer for Q7

- a) 4
- b) 10
- c) -3
- d) 3

It is left to the reader to verify this by counting the squares. You can find the area bounded by an inclined line by simply counting the squares in the surrounding rectangle and dividing by 2. Refer to the figure for further clarification



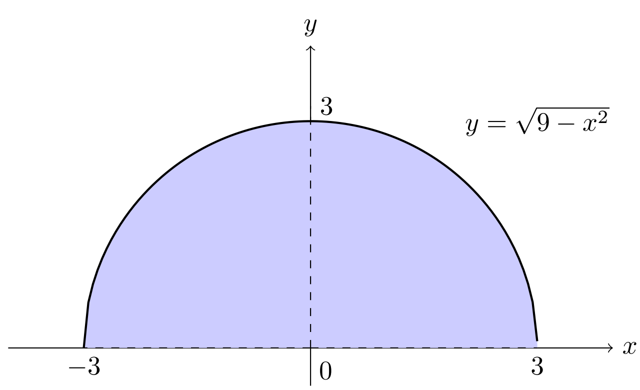
Answer for Q8

Step 1: Geometric Interpretation

The integrand $\sqrt{9-x^2}$ represents the upper half of a circle centered at the origin with radius $r = 3$. The full equation of the circle is:

$$x^2 + y^2 = 9.$$

Since the integral bounds are from -3 to 3, the integral represents the area under the upper semicircle of this circle.



Step 2: Area of the Corresponding Circle

The area of a full circle with radius r is given by:

$$\text{Area of a full circle} = \pi r^2.$$

Substituting $r = 3$:

$$\text{Area} = \pi \times 3^2 = 9\pi.$$

Step 3: Area of the Upper Semicircle

Since the integral corresponds to the area of the upper half of the circle, we take half of the total area:

$$\text{Area of upper semicircle} = \frac{1}{2} \times 9\pi = \frac{9\pi}{2}.$$

Final Answer

Thus, the value of the integral is:

$$\boxed{\frac{9\pi}{2}}.$$

Answer of Q9

Step 1: Analyze the Graph of $y = 4 - |x|$

The function $y = 4 - |x|$ forms a "V"-shaped graph symmetric about the y-axis. Key points on the graph are:

- At $x = 0$, $y = 4$.
- At $x = \pm 2$, $y = 2$.

The graph decreases linearly from $y = 4$ at $x = 0$ to $y = 2$ at $x = \pm 2$.

Step 2: Geometric Decomposition

We can interpret the region under the graph as the combination of:

1. A **rectangle** with:
   - Width $= 4$ (from -2 to 2).
   - Height $= 2$ (since $y = 2$ at the endpoints).

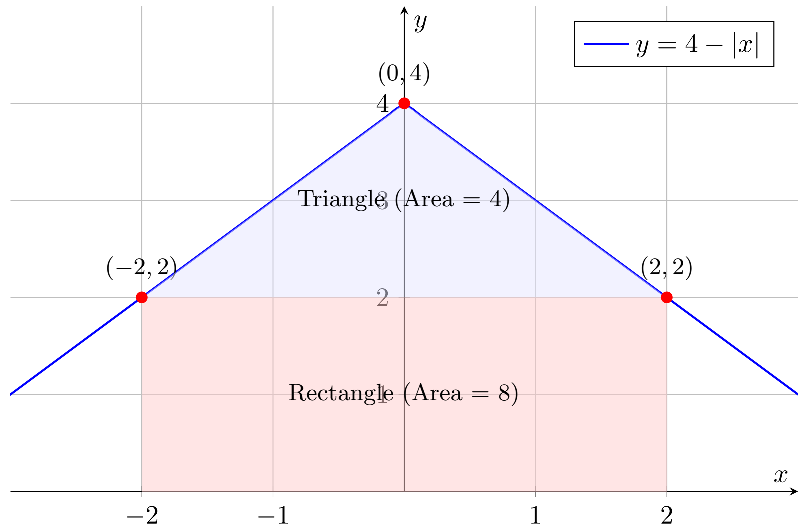
   Therefore, the area of the rectangle is:

   $$A_{\text{rectangle}} = \text{width} \times \text{height} = 4 \times 2 = 8.$$

2. A **triangle** above the rectangle with:
   - Base $= 4$ (from -2 to 2).
   - Height $= 2$ (from $y = 2$ to $y = 4$).

   The area of the triangle is:

   $$A_{\text{triangle}} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 4 \times 2 = 4.$$



Step 3: Total Area

The total area under the curve (and hence the value of the integral) is the sum of the rectangle's area and the triangle's area:

$$A_{\text{total}} = A_{\text{rectangle}} + A_{\text{triangle}} = 8 + 4 = 12.$$

Final Answer

Thus, the value of the integral is:

$$\int_{-2}^{2} (4 - |x|)\,dx = \boxed{12}.$$

Answer of Q10

We are given the following integrals:

$$\int_{2}^{8} f(x)\,dx = 7.3 \quad \text{and} \quad \int_{2}^{4} f(x)\,dx = 5.9$$

Using the property of definite integrals:

$$\int_{a}^{b} f(x)\,dx = \int_{a}^{c} f(x)\,dx + \int_{c}^{b} f(x)\,dx$$

Let $a = 2$, $b = 8$, and $c = 4$. Then:

$$\int_{2}^{8} f(x)\,dx = \int_{2}^{4} f(x)\,dx + \int_{4}^{8} f(x)\,dx$$

Substitute the given values:

$$7.3 = 5.9 + \int_{4}^{8} f(x)\,dx$$

Solving for $\int_4^8 f(x)\,dx$:

$$\int_4^8 f(x)\,dx = 7.3 - 5.9 = 1.4$$

Final Answer:

$$\boxed{1.4}$$

Anser for Q11

Step 1: Split the integral

$$\int_{-3}^{3} \left(4f(x) - 2g(x) + 5x^3\right)\,dx = 4\int_{-3}^{3} f(x)\,dx - 2\int_{-3}^{3} g(x)\,dx + \int_{-3}^{3} 5x^3\,dx$$

Step 2: Compute $\int_{-3}^{3} f(x)\,dx$

Given:

$$\int_{-3}^{2} f(x)\,dx = 10 \quad \text{and} \quad \int_{2}^{3} f(x)\,dx = -4$$

Thus:

$$\int_{-3}^{3} f(x)\,dx = \int_{-3}^{2} f(x)\,dx + \int_{2}^{3} f(x)\,dx = 10 + (-4) = 6$$

Step 3: Compute $\int_{-3}^{3} g(x)\,dx$

Given:

$$\int_{3}^{-3} g(x)\,dx = -2$$

Reversing the limits:

$$\int_{-3}^{3} g(x)\,dx = -\int_{3}^{-3} g(x)\,dx = -(-2) = 2$$

Step 4: Compute $\int_{-3}^{3} 5x^3\,dx$

Since x^3 is an odd function, $5x^3$ is also odd. The integral of an odd function over a symmetric interval $[-a, a]$ is 0:

$$\int_{-3}^{3} 5x^3\,dx = 0$$

Step 5: Combine all results

$$\int_{-3}^{3} \left(4f(x) - 2g(x) + 5x^3\right)\,dx = 4 \cdot 6 - 2 \cdot 2 + 0 = 24 - 4 = 20$$

Final Answer

$$\boxed{20}$$

Answer for Q12

Defining the Functions

Let the functions f and g on the interval $[0, 1]$ be defined as follows:

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q}, \\ 0 & \text{if } x \in \mathbb{R} \setminus \mathbb{Q}^1, \end{cases}$$

$$g(x) = \begin{cases} -1 & \text{if } x \in \mathbb{Q}, \\ 0 & \text{if } x \in \mathbb{R} \setminus \mathbb{Q}. \end{cases}$$

Why f and g are Not Integrable on $[0, 1]$?

- Both f and g are examples of Dirichlet-type functions.
- For any partition of $[0, 1]$, the upper and lower Riemann sums for f and g differ, implying that neither f nor g is Riemann integrable on $[0, 1]$.
- More precise explanation that the set of rational numbers ($\mathbb{Q}$) is dense in $[0, 1]$ and has measure zero.

Consider the Sum $f + g$

Now, consider:

$$(f + g)(x) = f(x) + g(x) = \begin{cases} 1 + (-1) = 0 & \text{if } x \in \mathbb{Q}, \\ 0 + 0 = 0 & \text{if } x \in \mathbb{R} \setminus \mathbb{Q}. \end{cases}$$

Hence, the sum simplifies to:

$$(f + g)(x) = 0 \quad \text{for all } x \in [0, 1].$$

Since $(f + g)(x) = 0$ for all x in $[0, 1]$, it is the zero function, which is continuous and hence Riemann integrable.

Answer for Q13

Consider the function:

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q}, \\ -1 & \text{if } x \in \mathbb{R} \setminus \mathbb{Q}. \end{cases}$$

[1] The notation $\setminus$ denotes the set difference. Specifically, $\mathbb{R} \setminus \mathbb{Q}$ represents the set of real numbers excluding the rational numbers, i.e., the set of irrational numbers.

Why $f(x)$ is not Riemann integrable:

- Both rational and irrational numbers are dense in $[0, 1]$.
- In every subinterval, $f(x)$ takes values 1 (rationals) and -1 (irrationals).
- The function keeps oscillating between 1 and -1 without settling, so the Riemann integral does not exist.

Why $|f(x)|$ is Riemann integrable:

The absolute value is:

$$|f(x)| = \begin{cases} 1 & \text{if } x \in \mathbb{Q}, \\ 1 & \text{if } x \in \mathbb{R} \setminus \mathbb{Q}. \end{cases}$$

This simplifies to:

$$|f(x)| = 1 \quad \text{for all } x \in [0, 1].$$

Since $|f(x)|$ is the constant function 1, it is Riemann integrable.

Answer for Q14

Step 1: Prove the inequality $1 \leq \sqrt{1 + x^2} \leq \sqrt{2}$

Given:

$$-1 \leq x \leq 1$$

Squaring all sides gives:

$$0 \leq x^2 \leq 1$$

Adding 1 to all sides:

$$1 \leq 1 + x^2 \leq 2$$

Taking the square root of all sides (since all terms are non-negative, we keep the positive root):

$$1 \leq \sqrt{1 + x^2} \leq \sqrt{2}$$

Step 2: Use properties of integrals to establish the given inequality

Since for all x in $[-1, 1]$:

$$1 \leq \sqrt{1 + x^2} \leq \sqrt{2},$$

we can use the **comparison property of integrals**, which states:

If $f(x)$ is continuous on $[a, b]$ and $m \leq f(x) \leq M$ for all $x \in [a, b]$, then:

$$m(b - a) \leq \int_a^b f(x)\, dx \leq M(b - a)$$

In our case:

$$f(x) = \sqrt{1 + x^2}, \quad m = 1, \quad M = \sqrt{2}, \quad a = -1, \quad b = 1.$$

The length of the interval is:

$$b - a = 1 - (-1) = 2.$$

Thus:

$$1 \times 2 \leq \int_{-1}^{1} \sqrt{1 + x^2}\, dx \leq \sqrt{2} \times 2$$

$$2 \leq \int_{-1}^{1} \sqrt{1 + x^2}\, dx \leq 2\sqrt{2}$$

Final Answer

$$\boxed{2 \leq \int_{-1}^{1} \sqrt{1 + x^2}\, dx \leq 2\sqrt{2}}$$

Answer for Q15

Proposition 0.1. If f is continuous on $[a, b]$, then

$$\left| \int_a^b f(x)\, dx \right| \leq \int_a^b |f(x)|\, dx.$$

Proof.

1. **Start with the given inequality:**
   From the given hint, we have:

   $$-|f(x)| \leq f(x) \leq |f(x)|.$$

2. **Integrate over $[a, b]$:**
   Since the integral preserves inequalities, we obtain:

   $$\int_a^b -|f(x)|\, dx \leq \int_a^b f(x)\, dx \leq \int_a^b |f(x)|\, dx.$$

3. **Simplify the inequality:**
   The integral of $-|f(x)|$ is simply the negative of the integral of $|f(x)|$:

   $$-\int_a^b |f(x)|\, dx \leq \int_a^b f(x)\, dx \leq \int_a^b |f(x)|\, dx.$$

4. **Apply the equivalence $-b \leq a \leq b \iff |a| \leq b$:**
   Since we have:

   $$-\int_a^b |f(x)|\, dx \leq \int_a^b f(x)\, dx \leq \int_a^b |f(x)|\, dx,$$

   it follows by the equivalence $-b \leq a \leq b \iff |a| \leq b$ that:

   $$\left| \int_a^b f(x)\, dx \right| \leq \int_a^b |f(x)|\, dx.$$

$\square$

Theorem 0.1. If f is continuous on $[a, b]$, then

$$\left| \int_a^b f(x)\, dx \right| \leq \int_a^b |f(x)|\, dx.$$

We want to show that

$$\left| \int_0^{2\pi} f(x) \sin(2x)\, dx \right| \leq \int_0^{2\pi} |f(x)|\, dx.$$

Proof. **Step 1: Define a Suitable Function**

Let

$$g(x) = f(x) \sin(2x).$$

Since $f(x)$ is continuous on $[0, 2\pi]$ and $\sin(2x)$ is also continuous on $[0, 2\pi]$, their product $g(x)$ is continuous on $[0, 2\pi]$.

Step 2: Apply Theorem 0.1

By applying Theorem 0.1 to $g(x)$, we have:

$$\left| \int_0^{2\pi} g(x)\, dx \right| \leq \int_0^{2\pi} |g(x)|\, dx.$$

Substituting $g(x) = f(x) \sin(2x)$:

$$\left| \int_0^{2\pi} f(x) \sin(2x)\, dx \right| \leq \int_0^{2\pi} |f(x) \sin(2x)|\, dx.$$

Step 3: Simplify the Absolute Value

Since $|\sin(2x)| \leq 1$ for all x, we have:

$$|f(x) \sin(2x)| = |f(x)| \cdot |\sin(2x)| \leq |f(x)|.$$

Thus:

$$\int_0^{2\pi} |f(x) \sin(2x)|\, dx \leq \int_0^{2\pi} |f(x)|\, dx.$$

Step 4: Combine the Inequalities

Combining the inequalities from the previous steps:

$$\left| \int_0^{2\pi} f(x) \sin(2x)\, dx \right| \leq \int_0^{2\pi} |f(x)|\, dx.$$

Final Answer

Hence, we have shown that:

$$\boxed{\left|\int_0^{2\pi} f(x)\sin(2x)\,dx\right| \leq \int_0^{2\pi} |f(x)|\,dx.}$$

$\square$

About The Repository

This document is part of the `github.com/3ndlyb/Math132Answers` repository, which contains solutions to Math132 problem sheets. Each sheet is organized in its own folder, including:

- The original problem sheet in PDF format (`Math - 132 - sheetX.pdf`).
- A LaTeX source file (`main.tex`) containing the answers.
- The compiled PDF of the answers (`SheetXAns.pdf`).

To view the answers, simply open the corresponding `SheetXAns.pdf` file. If you wish to edit the answers, you can modify the `main.tex` file and recompile it using:

```
$ pdflatex main.tex
```

Make sure you have a suitable LaTeX distribution installed, such as TeX Live or MiKTeX.

Contributions and Feedback

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2. Fork the repository, make your changes, and submit a pull request.

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Contact

For further questions or feedback, feel free to reach out via Discord at `3ndlybalabyd` .

Acknowledgments

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