chapter 2.

- Algebra: objects의 set들을 다루고, 다루는 규칙의 set.
  - linear Algebra: Vector들의 연구, vector들을 다루는 규칙들의 연구
    - → 이것들 끼리 더하거나, scalar를 곱해도 같은 종류의 다른 것이 나오는것.
    - → 종류: Geometric vector, Polynomials, Audio signal ...

- linear equation 의 solution 종류: ① No solution ② Unique solution ③ infinitely many solution

- Matrices ① $R^{m\times n}$: real valued (m,n) matrices의 집합.
  - ② Matrix addition (Element-wise sum)
  - ③ Matrix multiplication

    $C = AB \to C_{ij}$ = inner product A의 i번째 row B의 j번째 column.

    $AB \neq BA$.
  - ④ ②,③의 basic properties
    - Associativity
    - Distributivity
    - Multiplication with the Indentity matrix.
  - ⑤ Matrix inverse. (square matrix에서만 존재)

    inverse exist = regular, invertible, nonsingular

    " not " = noninvertible, singular
  - ⑥ Transpose : $A^{m\times n}$, $B^{n\times m}$ $\Rightarrow A_{ij} = B_{ji}$
  - ⑦ Symmetric matrix : $A^T = A$ (A와 B가 symmetric matrix $\Rightarrow A+B$: ")

- Solving systems of linear system.

  Elementary transformations (elementary row operation) $\xRightarrow{\text{이용해서}}$ Simpler form으로 만든다.

  ⇒ row-echelon form ⇒ reduced row-echelon form
  → (Every pivot = 1)

  Augmented matrix $\xrightarrow{\text{Gaussian elimination}}$ row-echelon form. ((variables corresponding pivot) basic variable / free variable) 분류

  No free variable ⇒ unique solution.

- A : square, invertible ⇒ $Ax=b \to x = A^{-1}b$
- A : linearly independent columns (⇒ inverse를 가진다) (square X) ⇒ $Ax=b \longleftrightarrow A^TAx = A^Tb$

  $x = (A^TA)^{-1}A^Tb$.

Vector space.

- Group : $G \times G \to G$ 조건 ① closure ⇒ +조건 ⑤ commutative : Abelian Group.
  ② Associativity
  ③ Neural element
  ④ Inverse element.

- Vector space : vector addition (~~Vector addition~~ inner operation), multiplication by scalars (outer operation)

  위 2가지 operation에 대한 Group.

- Vector subspace : V = Vector space, $U \subseteq V$, $U \neq 0$

  inner operation, outer operation 에 대해 closure ⇒ 다른 Group의 조건 만족.

  * 모든 Vector space의 subspace는 최소 자기자신과 zero vector 포함.

- Linear combination → $v = \sum_{i=1}^{k} \lambda_i x_i \in V$

  ↳ vector space V의 원소들 linear combination → 그것도 V의 sub space.

- Linear independence.

  $v = \sum_{i=1}^{k} \lambda_i x_i \in V = 0$ ⇒ λ_i set이 0이 아닌 값들로 구성 : linearly dependent

  ⇏ 〃 〃 〃인 값들로 구성 : linearly independent.

  How check?

  Gaussian elimination ⇒ row-echelon form ⇒ (pivot column들 끼리 linearly independent.

  ↳ Row operation ⇒ column들 간의 관계는 유지.

Basis and rank.

- span → $x_1 \cdots x_k$를 linear combination을 통해 만든 set(vector)

- Basis = minimal generating set of V = maximal linearly independent subset of V.

  $x = \sum_{i=1}^{k} \lambda_i b_i = \sum_{i=1}^{k} \psi_i b_i$ (basis) $\lambda_i = \psi_i$ (Unique 하게 계수가 정해짐)

  한 Vector space에 여러가지 basis가 있을 수 있다.

- Dimension = basis vector의 개수.

∘ Rank : linearly independent 인 columns (rows) 의 개수.

특성 ① Transpose 해도 변화X

② A의 col $\rightarrow \mathbb{R}^{m\times 1} \Rightarrow U = span(A) \quad U \subseteq \mathbb{R}^m \quad \therefore \dim(U) = rk(A)$

row $\rightarrow \mathbb{R}^{1\times n} \Rightarrow W = span(A) \quad W = \mathbb{R}^n \quad \therefore \dim(W) = rk(A)$

③ full rank $\longleftrightarrow$ inverse 존재

∘ Linear mapping. $\Phi : V \rightarrow W$. (V, W : vector space)

$\forall x, y \in V \quad \forall \lambda, \psi \in \mathbb{R} : \phi(\lambda x + \psi y) = \lambda\phi(x) + \psi\phi(y) \Rightarrow$ superposition property.

∘ Coordinate vector.

B: ordered basis $= \{b_1 \cdots b_n)$

$x = \alpha_1 b_1 + \alpha_2 b_2 \cdots \alpha_n b_n \Rightarrow \alpha = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix} \in \mathbb{R}^n$: coordinate vector.

Transformation matrix.

- ordered basis : $B = \{b_1 \cdots b_n\}$, $C = \{c_1 \cdots c_m\}$
- $\phi(b_j) = \alpha_{1j} c_1 + \cdots + \alpha_{mj} c_m = \sum_{i=1}^{m} \alpha_{ij} c_i$

$\hookrightarrow A_\phi(i,j) = \alpha_{ij} \quad i = 1 \sim m, \; j = 1 \sim n.$

- $\hat{y} = A_\phi \hat{x}$

∘ Equivalence and similarity.

Equivalent : $\tilde{A} = T^{-1} A S$ (regular matrix $S \in \mathbb{R}^{n\times n}$, $T \in \mathbb{R}^{m\times m}$)

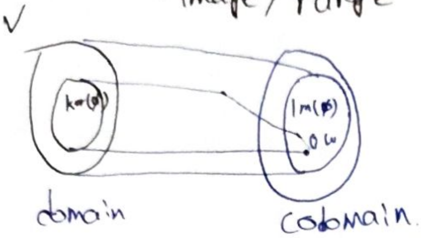
Similar : $\tilde{A} = S^{-1} A S$ (〃 〃)

$A \rightarrow P^{-1}AP \Rightarrow$ similarity transformation or conjugation of the matrix A.

Linear mapping.

∘ $\phi : V \to W$.

- kernel/null space $\quad \ker(\phi) := \phi^{-1}(0_W) = \{v \in V : \phi(v) = 0_W\}$
- image/range $\quad \mathrm{Im}(\phi) := \phi(V) = \{w \in W \mid \exists v \in V : \phi(v) = w\}$



∘ null space and column space.

$\phi : \mathbb{R}^n \to \mathbb{R}^m, \quad x \to Ax. \quad (A = [a_1 \cdots a_n]$

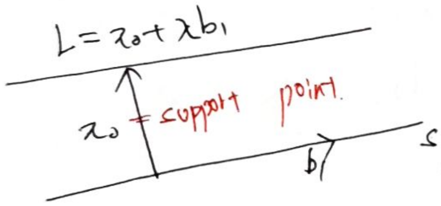
$\mathrm{Im}(\phi) = \mathrm{span}[a_1 \cdots a_n] \subseteq \mathbb{R}^m$ ⇒ span of the columns of A = column space

A의 rank = column space의 dimension.

kernel / null space : $\ker(\phi) \Rightarrow \dim(\ker(\Phi)) = n - rk(A)$

∘ Affine subspace

V: vector space, $x_0 \in V$, $U \subseteq V$



$\mathrm{span}(b_1) = U \subseteq V$ → direction

$\quad \| $

$\quad \lambda b_1$

∘ Affine mapping.

$\phi : V \to W \qquad a \in W \qquad\qquad \phi : V \to W$

$\qquad\qquad\qquad\qquad\qquad\qquad x \to a + \phi(w)$

translation vector of ϕ

→ linear mapping과 translation으로 구성.

→ dimension과 parallelism은 보존시킴

기울기