머신러닝 4주차

- kernel (null space) → $\Phi(v) = 0_W$
  image (range) → $\Phi(v) = w$ $(\exists v \in V)$ ⇒ 모든 V에 대해서 linear mapping 했을때 W에 속한다.

- $A \in \mathbb{R}^{m \times n}$, $\Phi : \mathbb{R}^n \to \mathbb{R}^m$, $x \to Ax$.

  $[a_1 \cdots a_n]$의 linear combination을 모아둔 형태

  $A = [a_1 \cdots a_n]$ ⇒ $Im(\Phi) = \left\{ \sum_{i=1}^{n} x_i a_i : x_1 \cdots x_n \in \mathbb{R} \right\} = span[a_1, \cdots, a_n] \subseteq \mathbb{R}^m$

  ↳ ex) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$

  ⇒ column 들의 linear combination 형태.

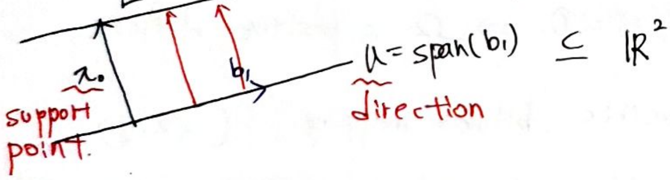
  Image : A의 ~~~~ columns의 span. = column space.

  if) column space $\subseteq \mathbb{R}^m$ ⇒ m: height of matrix. $rk(A) = dim(Im(\Phi))$

- kernel / null space → general solution to the homogeneous system $(Ax = 0)$

  if) kernel (null space) $\subseteq \mathbb{R}^n$ ⇒ n: width of matrix. $n - rk(A) = dim(Kernel(\Phi))$

- $L = x_0 + \lambda b_1$ ⇒ affine subspace, linear manifold



- Affine mapping V, W ← vector space $\phi : V \to W$ (linear mapping) $a \in W$

  $x \to a + \phi(x)$ (affine mapping from V to W)

  ↳ translation vector of ϕ

  Every affine mapping = composition of a linear mapping and translation

  $\phi = \tau \circ \phi$, uniquely determined.

  서로 다른 mapping을 합성 ⇒ 또 다른 affine mapping.

  Affine mapping ⊆ linear regression.

Chapter 3. Analytic Geometry

- Norm : vector를 하나의 실수로 mapping 하는 것. ($x \to \|x\|$)

  Absolutely homogeneous : $\|\lambda x\| = |\lambda|\|x\|$
  Triangle inequality : $\|x+y\| \le \|x\| + \|y\|$
  Positive definite : $\|x\| \ge 0$ and $\|x\| = 0 \iff x = 0$.

  Zero vector일 때만 0 이 norm이다.

  ① Manhattan Norm (1st norm) (l_1 norm)

  $\|x\|_1 = \sum_{i=1}^{n} |x_i|$

  ② Euclidean norm (2nd norm, l_2 norm)

  $\|x\|_2 = \sqrt{\sum_{i=1}^{n} x_i^2} = \sqrt{x^T x}$

- Bilinear mapping (Ω) : mapping with two arguement. (linear in each arguement)

  ↳ super position property 만족

  $\Omega(\lambda x + \psi y, z) = \lambda \Omega(x, z) + \psi \Omega(y, z)$

  If $\Omega(x, y) = \Omega(y, x) \Rightarrow \Omega$ is symmetric.

  If $\forall x \in V \setminus \{0\} : \Omega(x, x) > 0$, $\Omega(0,0) = 0 \Rightarrow \Omega$ is positive definite.

Inner product : positive definite & symmetric bilinear mapping. ($\langle x, y \rangle$)

$(V, \langle \cdot, \cdot \rangle)$: inner product space or vector space with inner product.

- $V \times V \to R$, ordered basis $B = (b_1 \cdots b_n)$ of V, $x, y \in V$

  $x = \sum_{i=1}^{n} \psi_i b_i \quad y = \sum_{j=1}^{n} \lambda_j b_j \quad \psi_i, \lambda_j \in R$.

  $\langle x, y \rangle = \left\langle \sum_{i=1}^{n} \psi_i b_i, \sum_{j=1}^{n} \lambda_j b_j \right\rangle = \sum_{i=1}^{n} \sum_{j=1}^{n} \psi_i \langle b_i, b_j \rangle \lambda_j = \hat{x}^T A \hat{y}$

  Bilinear property.

  $\hat{x}, \hat{y}$: coordinate of x, y (basis B에 관해)

  A : symmetric, positive definiteness of the inner product

  A의 null space → zero vector (consists of 0)
  Diagonal elements a_{ii} of A → positive

- Inner product induces norm. (Not every norm is induced by an Inner product)
  ↳ ex) Manhattan norm.

- Cauchy-schwarz inequality : $|\langle x, y\rangle| \le \|x\| \|y\|$. proof) $|x_1 y_1 + x_2 y_2 \cdots x_n y_n|$
  $\le \sqrt{x_1^2 + x_2^2 \cdots x_n^2} \cdot \sqrt{y_1^2 + y_2^2 \cdots y_i^2}$

- Distance : 2개의 vector간의 차이 norm.

$d(x, y) = \|x - y\| = \sqrt{\langle x-y, x-y\rangle}$ → $x-y$의 innerproduct에 induce

metric : $d : V \times V \to \mathbb{R}$. (metric = mapping)
$(x, y) \to d(x, y)$ ↳ distance = metric의 값.

properties
① d : positive definite.
② d : symmetric.
③ Triangle inequality.

- $\cos\omega = \dfrac{\langle x, y\rangle}{\|x\| \|y\|}$ ⇒ $\langle x, y\rangle = 0$: orthogonality.
$x \perp y$.

orthogonal + $\underbrace{\text{unit vector}}_{\|x\| = 1}$ = orthonormal.

- Orthogonal matrix (각각의 column 이 orthonormal)

$A^{-1} = A^T$

transformation by an orthogonal matrix preserves the length, inner product

ex) $\|Ax\|^2 = \|x\|^2$
$\cos\omega = \dfrac{x^T y}{\|x\| \|y\|}$ (Ax 와 Ay)

orthogonal matrix는 rotation 연산을 define 한다.