Dependence of the Primary *Pfeifenton* (Pipe Tone) Frequency on Pipe-Orifice Geometry

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A derivation is presented of an expression relating the frequency of the *primary Pfeifentöne*, produced in a column of moving gas, to the geometry of the pipe-orifice combination in which the *Pfeifenton* oscillations occur. The *Pfeifentöne* appear to be initiated in, and by the gas discharging continuously through the orifice. A comparison is made between the results obtained with this expression and experiment. Good agreement is shown between the values of the *primary Pfeifenton* frequencies predicted by the relation and the values obtained by experiment. These results apply for a thin orifice plate, and are expressed in nondimensional form.

INTRODUCTION

As the velocity of flow of a gas through a pipe terminated in an orifice is either gradually increased or decreased, a pronounced spectrum of longitudinal acoustic oscillations is produced in succession in the column of moving gas inside the pipe. These vibrations differ from ordinary organ pipe oscillations in that they appear to be initiated in, and by the gas discharging continuously from the orifice. They have been referred to as *Pfeifentöne*. A summary of information on the general nature of these, known at present, has been presented elsewhere.[1]

Figure 1 shows the dependence of the *Pfeifenton* frequency on static flow pressure, found experimentally for air, for an orifice diameter of $\frac{3}{16}$ in. The thickness of the orifice plate is $\frac{1}{16}$ in., the geometrical length of the pipe 11.762 in., and the diameter 0.819 in. The figure shows discrete ranges of *Pfeifenton* frequency. As the static pressure is raised, a sequence of harmonics appears, each made up of a continuous range of *Pfeifenton* frequencies. The sequence begins with a fundamental and is followed by overtones. In all cases an increase in static pressure inside the pipe causes an increase in *Pfeifenton* frequency. The *primary Pfeifenton* frequency[1] is the one associated with the low pressure end of each complete line segment in the figure. For the fundamental, this is the frequency corresponding to point A.

The following generalization of experimental results[1] makes it appear that, for a given gas, the *primary Pfeifenton* frequencies depend on pipe-orifice geometry alone:

1. As the diameter of the orifice approaches that of the pipe, the values of the frequencies of the *primary Pfeifentöne*, excited in the column of air moving rapidly through the pipe, tend to approach those of an open-open organ pipe.
2. As the diameter of the orifice approaches zero, the values of the frequencies of the *primary Pfeifentöne* approach roughly those of an open-closed organ pipe. The quantitative form of this dependence was not determined.

[1] A. B. C. Anderson, J. Acoust. Soc. Am. 24, 675–681 (1952).

The purpose of the present paper is to present the results of the derivation of an expression relating the frequency of the *primary Pfeifentöne* to the geometry of the pipe-orifice combination in which the *Pfeifentöne* occur, and to compare this with experiment. The derivation is based on the determination of the natural longitudinal resonant frequencies of the column of air in the pipe-orifice combination, Fig. 2, in the absence of gas flow. In practice, the velocity of flow of the gas through the system is far below sonic velocity. It is therefore not surprising that the expression, derived on the basis that the time average flow of the gas through the system is zero, applies also to the present case where an appreciable and continuous flow of gas through the system occurs.

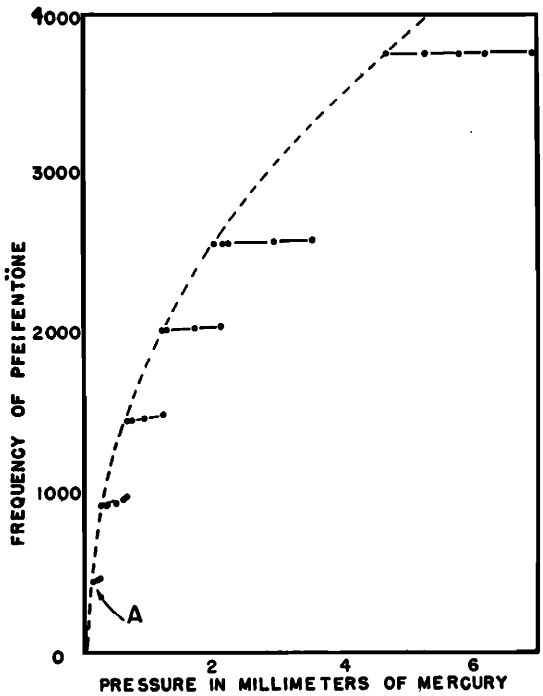

FIG. 1. Dependence of the *Pfeifenton* frequency on static flow pressure. Pipelength 11.762 in.; internal diameter of pipe, 0.819 in.; orifice diameter, $\frac{3}{16}$ in.; and orifice plate thickness, $\frac{1}{16}$ in.

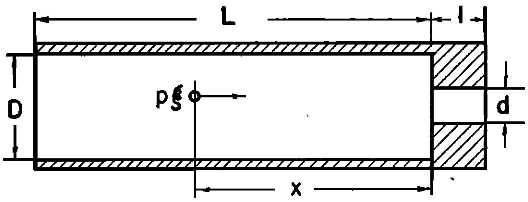

FIG. 2. Pipe terminated in an orifice.

RESONANT FREQUENCY OF A COLUMN OF GAS INSIDE A PIPE TERMINATED IN AN ORIFICE

The longitudinal resonant frequencies of a column of gas in an open pipe terminated by an orifice plate, Fig. 2, has been obtained by considering the acoustic impedance of the combination of the pipe and orifice. The concept of acoustic impedance was first introduced by Webster[2] and has been discussed in detail by others.[3–5] Assuming that the particle displacement of the gas is uniform over each cross section S in the circular pipe, that the thickness of the orifice is small with respect to the wavelength of oscillation, that the opening is circular and in a thin plate of thickness l so that the effective mass of the air moving in the orifice is that given according to Rayleigh,[6] one obtains

$$\cot\left(\frac{\omega L}{c}\right)=-\left(\frac{Ld}{S}\right)\left(\frac{1}{\omega L/c}\right),$$

relating the resonant frequency of the pipe $f(\omega=2\pi f)$ to the effective length of the pipe L, diameter of orifice d, cross-sectional area of pipe S, and velocity of sound c in the gas. It is also assumed in the derivation of this expression that the acoustic impedance of the orifice at resonance is equal but opposite in sign to the input impedance of the open-open pipe.

This equation may be solved graphically by plotting both $\cot(\omega L/c)$ and $(Ld/S)\cdot(c/\omega L)$ against $c/\omega L$ and finding where the curves intersect. This procedure was suggested by another study[7] being there used in the determination of the resonance frequencies of a cylindrical cavity terminated at one end in an orifice but

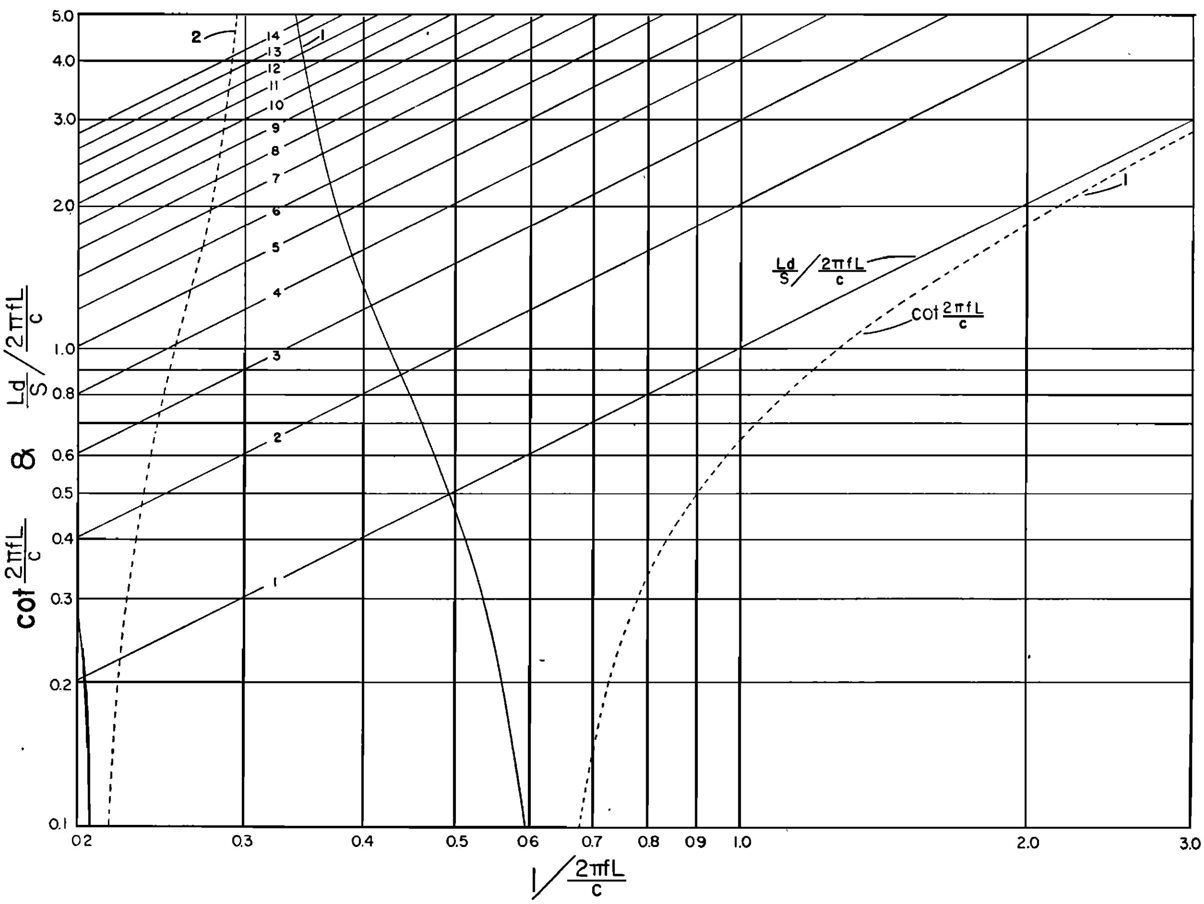

FIG. 3. Graph for solving the equation $\cot(2\pi fL/c)=-(Ld/S)(c/2\pi fL)$ for the fundamental frequency.

[2] A. G. Webster, Nat. Acad. Sci. Proc. **6**, 316–320 (1920).
[3] G. W. Stewart and R. B. Lindsay, *Acoustics* (D. Van Nostrand Company, Inc., New York, 1930).
[4] E. G. Richardson, *Sound* (Longmans, Green and Company, New York, and Edward Arnold and Company, London, 1947).
[5] Alexander Wood, *Acoustics* (Interscience Publishers, New York, 1947).
[6] Lord Rayleigh, *Theory of Sound* (Reprint, New York, Dover Publications 1945), Vol. II, pp. 173 and 178.
[7] E. G. Richardson, Proc. Phys. Soc. (London) **40**, 206–219 (1928).

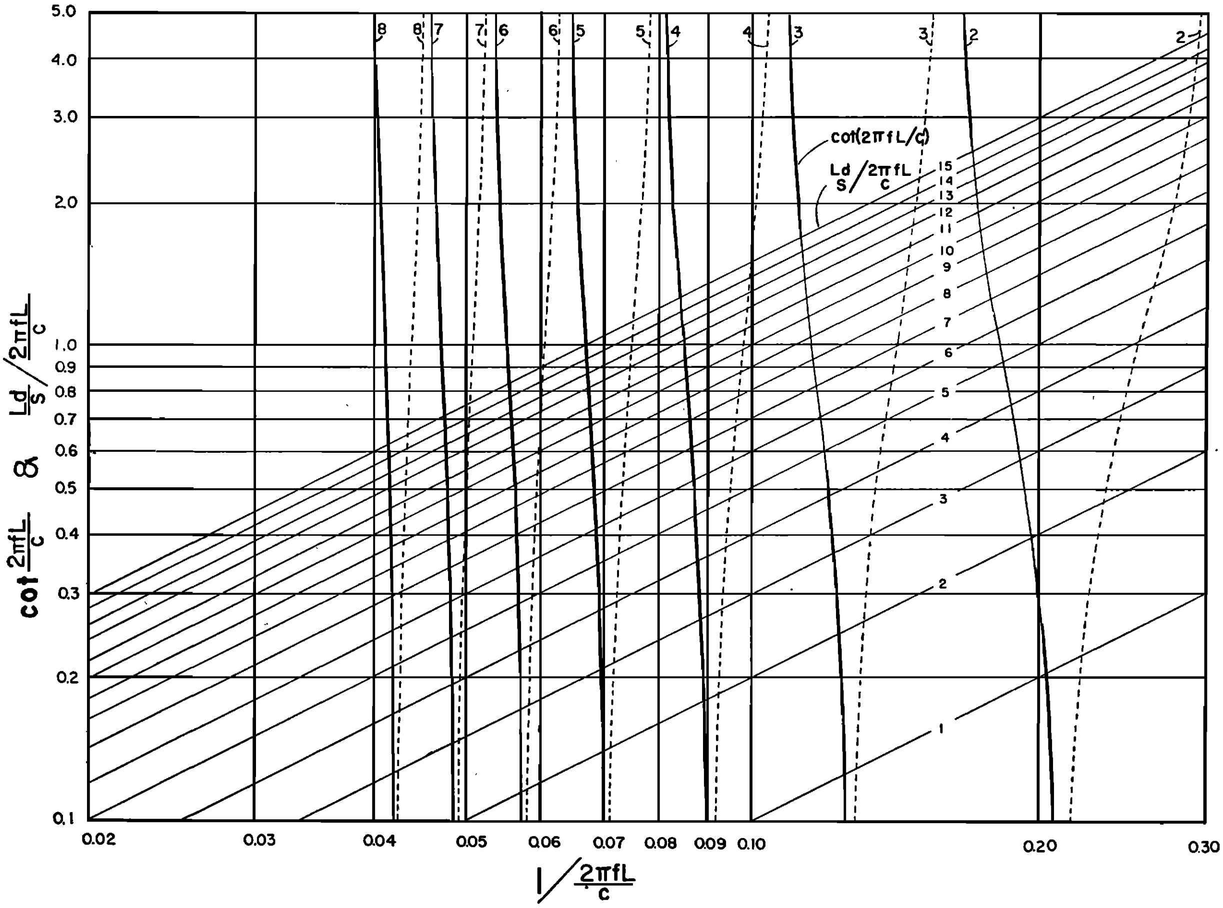

FIG. 4. Graph for solving the equation $\cot(2\pi fL/c) = -(Ld/S)(c/2\pi fL)$ for the frequencies above the fundamental.

closed at the other. The present results are shown in Figs. 3 and 4. The family of straight lines represent $(Ld/S)(c/\omega L)$ for integral values of Ld/S beginning with unity. Only the intersections of the solid part of the plot of $\cot(\omega L/c)$ with the straight lines are applicable to the solution in the present case. The frequency of the fundamental starts with the branch of $\cot(\omega L/c)$ at the extreme right of Fig. 3. As one proceeds to the left, from one branch to the next, the order of the harmonic increases by one.

The frequencies of the *primary Pfeifentöne* may be determined with this equation by first determing the value of (Ld/S) from the pipe orifice geometry. This value determines a straight line, parallel to the family shown in Figs. 3 and 4, whose position in the figure is obtained by interpolation. The intersection of this line with the curved segments determines a sequence of values of $(c/2\pi fL)$. Since the velocity of sound and the effective length of the pipe are known, one can calculate the sequency of *primary Pfeifentöne.*

COMPARISON OF EXPERIMENTAL WITH THEORETICAL VALUES OF PRIMARY *PFEIFENTÖNE*

The experimental values of the *primary Pfeifentöne,* using air, were obtained according to the procedure described elsewhere.[1] The transitions in frequency between adjacent ranges of *Pfeifenton* frequency were, in general, discontinuous as shown by the step-like nature of the ranges shown in Fig. 1. Nevertheless, extreme care was required in determining the value of the lowest frequency (the *primary Pfeifentöne*) in each range.

The first column, Table I, shows the experimental values of the *primary Pfeifenton* frequencies obtained for a range of orifice diameters varying from $\frac{1}{8}$ to $\frac{1}{2}$, in steps of $\frac{1}{16}$ in. The length of the pipe was 11.762 in.; the internal cross-sectional area, 0.527 in. sq; and, the orifice plate thickness $\frac{1}{16}$ in. throughout. In general, several experimental determinations were made of the same *primary Pfeifenton,* not all on the same day. Since all of them almost always agreed with each other well within one percent, only their average is presented in the table.

The first column of Table I (b) and Table II, on the other hand, shows the experimental values of the *primary Pfeifentöne* obtained for the pipe lengths, 13.719, 11.762, 7.660, and 5.688 in. The orifice diameter was $\frac{3}{16}$ in. throughout, but the orifice plate thickness and the internal cross-sectional area of the pipe were the same as before.

The second column shows the theoretical values of the *primary Pfeifenton* frequencies determined, in the man-

TABLE I. Comparison of experimental and calculated values of the *Pfeifenton* frequency f expressed in nondimensional form. In this table the geometrical length of the pipe is 11.762 in.; the effective length of the pipe L is 12.013 in.; the cross-sectional area S, 0.527 sq in.; and the orifice plate thickness, $\frac{1}{16}$ in.

Frequency f, experimental value, cps	Frequency f, calculated value, cps	Velocity of sound, ft/sec	$2\pi fL/c$ experimental	$2\pi fL/c$ calculated
(a) Orifice diameter 0.125 in.; Ld/S, 2.850				
900	934	1129	5.01	5.20
1440	1469	1128	8.03	8.18
2006	2027	1131	11.16	11.26
(b) Orifice diameter 0.1875 in.; Ld/S, 4.275				
454	464	1127	2.48	2.59
926	966	1129	5.15	5.38
1458	1500	1130	8.12	8.34
2018	2042	1129	11.25	11.39
2564	2593	1130	14.26	14.43
3125	3138	1132	17.37	17.42
3756	3710	1130	20.91	20.65
(c) Orifice diameter 0.250 in.; Ld/S, 5.70				
473	485	1131	2.63	2.70
961	988	1129	5.35	5.50
1485	1520	1130	8.26	8.46
2038	2069	1130	11.35	11.51
2574	2610	1129	14.34	14.51
3750	3710	1127	20.92	20.72
(d) Orifice diameter 0.3125 in.; Ld/S, 7.12				
492	496	1131	2.73	2.76
993	1012	1130	5.52	5.63
1515	1536	1131	8.42	8.54
2060	2077	1131	11.46	11.54
(e) Orifice diameter 0.375 in.; Ld/S, 8.54				
509	509	1130	2.84	2.83
1019	1025	1129	5.68	5.70
1537	1550	1128	8.57	8.64
2072	2090	1129	11.54	11.62
(f) Orifice diameter 0.4375 in., Ld/S, 9.97				
518	523	1129	2.88	2.91
1036	1034	1128	5.78	5.76
1558	1564	1128	8.68	8.71
2087	2105	1129	11.63	11.72
(g) Orifice diameter 0.500 in., Ld/S, 11.40				
525	523	1129	2.94	2.91
1051	1041	1127	5.87	5.81
1575	1575	1128	8.76	8.77

ner already described, according to the equation by means of Figs. 3 and 4. Column 3 gives the velocity of sound in the pipe. The equation indicates it should be possible, by means of only the two nondimensional variables $(2\pi fL/c)$ and (Ld/S), to express the dependence of the frequency of the *Pfeifentöne* on the nature of the gas and the pipe-orifice geometry. Both the experimental and the theoretical values of the frequency have therefore been expressed in the form $(2\pi fL/c)$ and are shown in columns four and five.

The effective length of the pipe is used for L in calculating Ld/S. The manner in which this is determined from geometrical length has been described. The need for this correction in length[8–10] is based upon the fact that even for a cylindrical pipe with a clean-cut end, the antinode at the end of the pipe does not coincide with the end of the pipe.

Figure 5 shows both the experimental, as well as the theoretical dependence of $(2\pi fL/c)$ on (Ld/S), a variable based purely upon the geometry. The order of the *Pfeifenton* harmonic increases as one proceeds to higher and higher lines in the figure, the lowest corresponding to the fundamental. The geometrical length of the pipe in inches is placed alongside those points for which the length of the pipe differs from 11.762 in.

Consideration of Fig. 5 leads to the following generalizations:

1. Good agreement is shown between the values calculated from experimental data and those predicted by the equation proposed, especially for the larger orifice diameters.
2. Several factors may help to account for the slightly increasing divergence between theory and experiment as the orifice diameter decreases. The equation, for example, was derived on the assumption that the orifice plate thickness is small compared to the orifice diameter. This assumption becomes less true as one proceeds to the left in Fig. 5 to smaller values of the orifice diameter. Also, there may be some question as to the true value of the end correction that should be added to the geometrical length of the pipe to obtain the effective length. The correction used[10] was the end correction for an unflanged open circular pipe $0.6133R$, where R is the

TABLE II. Comparison of experimental and calculated values of the *Pfeifenton* frequency f expressed in nondimensional form. In this table the orifice diameter is 0.1875 in., but the cross-sectional area of the pipe and the thickness of the orifice plate is the same as for Table I.

Frequency f, experimental value, cps	Frequency f, calculated value, cps	Velocity of sound, ft/sec	$2\pi fL/c$ experimental	$2\pi fL/c$ calculated
(a) Geometrical length of pipe, 13.719 in.; effective length L, 13.970 in.; and Ld/S, 4.970				
807	842	1131	5.22	5.44
1257	1299	1131	8.12	8.39
1722	1775	1131	11.13	11.46
2210	2238	1131	14.30	14.46
2685	2715	1131	17.36	17.55
3684	3684	1131	23.82	23.80
(b) Geometrical length of pipe, 7.660 in.; effective length L, 7.911 in.; and Ld/S, 2.813				
626	663	1130	2.29	2.43
1336	1420	1130	4.90	5.20
2178	2232	1130	7.98	8.18
2950	3070	1130	10.82	11.25
3856	3910	1130	14.13	14.32
(c) Geometrical length of pipe, 5.688 in.; effective length L, 5.939 in.; and Ld/S, 2.11				
768	842	1132	2.11	2.31
1717	1840	1132	4.71	5.05
2873	2994	1132	7.89	8.22
4067	4050	1132	11.17	11.11

[8] See reference 4, p. 174.

[9] A. E. Bate, Phil. Mag. 24, 453–458 (1937).

[10] Harold Levine and Julian Schwinger, Phys. Rev. **73**, 383–406 (1948).

internal radius of the pipe. Although the pipe actually had no flange, its thick walls (0.45 inch thick) probably did contribute some slight flanging effect in the mathematical sense.

3. The *primary Pfeifenton* frequency, expressed in terms of the nondimensional variable $(2\pi fL/c)$, appears fairly closely to be a multiple valued function of only the nondimensional variable (Ld/S).

4. For any given *Pfeifenton* harmonic the dependence of $2\pi fL/c$ on (Ld/S) is relatively small. It is still, however, seen to be appreciable.

5. The frequencies of the different *Pfeifenton* harmonics are not harmonically related, as in the case of an organ pipe, because the frequencies of the primary harmonics are not exact multiples of the fundamental.

The foregoing results seem to add some confirmation to the hypothesis proposed elsewhere[1] as to the mechanism of excitation of the *Pfeifentöne*. It was suggested that the *Pfeifentöne* are excited by periodic fluctuations of the orifice area; these fluctuations having the same frequency, or a multiple of the frequency of the *Pfeifentöne*, and being produced by the development and periodic shedding of vortices from the orifice.

The fluctuation of effective orifice diameter, or orifice area, creates periodic fluctuations in the discharge rate and therefore fluctuations in the pressure inside the pipe in the vicinity of the orifice. These periodic pulses propagate upstream in the pipe. If the frequency of these pulses is just equal to *one of the natural resonant frequencies of the* pipe-orifice combination, the column of air in the pipe will sing out to give a *primary Pfeifenton.*

The present paper indicates that the frequency of the *primary Pfeifentöne* are very closely the same as the natural resonant frequencies produced in the system if one excited the air column into longitudinal standing waves.

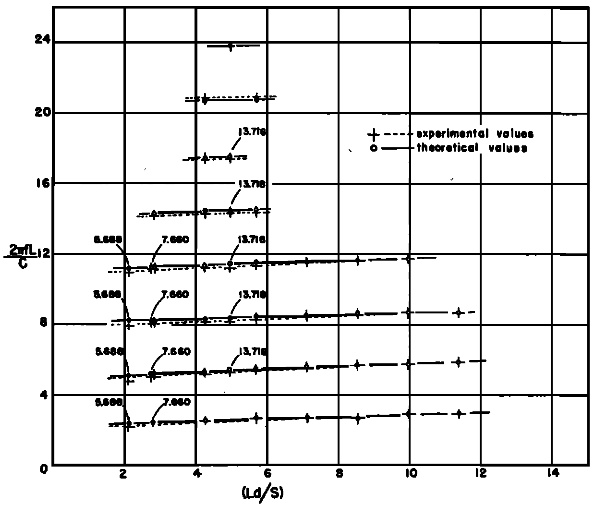

FIG. 5. Comparison of the experimental and theoretical dependence of $2\pi fL/c$ on Ld/S. The geometrical length of the pipe in inches is placed alongside those points for which the length of the pipe differs from 11.762 in.