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A Circular-Orifice Number Describing Dependency of *Primary Pfeifenton* Frequency on Differential Pressure, Gas Density, and Orifice Geometry

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A nondimensional number $(\Delta p/\rho)^{\frac{1}{2}}/ft$ for *primary Pfeifentöne* relating the dependence of differential pressure Δp across orifice to density of gas ρ, thickness t of orifice-plate terminating pipe, and frequency f of *primary Pfeifentöne* is obtained by dimensional analysis. Correlation of experimental data presented on the basis of this number indicates that it is relatively constant in numerical value over range of variation of the parameters, pipe length, orifice diameter, differential pressure across orifice, and *Pfeifenton* frequency, studied.

INTRODUCTION

As the differential pressure across an orifice, terminating a pipe is gradually increased or decreased by varying the rate of flow of gas through the orifice, discrete ranges of self-excited eigenfrequencies, *Pfeifentöne*, are produced inside the pipe. The *primary Pfeifentöne* have been defined[1] as the frequency at the low pressure, low frequency end of the discrete ranges. A summary of results of related investigations has been made.[1] Recently it was shown[2] that the frequency of *primary Pfeifentöne* is the same as the natural longitudinal frequencies of the air column inside the pipe-orifice combination in the absence of a continuous flow of gas. The *Pfeifentöne*, however, are self-excited. It was suggested that the mechanism of their excitation is a periodic fluctuation of the effective aerodynamic orifice area produced by growth and periodic shedding of vortices from the side walls of the orifice. The fluctuation of effective orifice area creates periodic fluctuations in the discharge rate and, therefore, fluctuations in the pressure inside the pipe in the vicinity of the orifice. These periodic pulses propagate upstream in the pipe. If the frequency of the pulses is equal to, or a multiple of one of the eigenfrequencies of the pipe-orifice combination, the column of air in the pipe sings out to give a *primary Pfeifenton*.

Few studies have been reported pertinent to an understanding of the dependence of *Pfeifenton* frequency on flow velocity of gas through the orifice, differential pressure across the orifice at a given gas density, and geometry of the orifice. All studies agree, however, that jet-tone frequency is proportional to velocity of flow (for gases or water) either through rectangular slits or circular orifices. The following observations of the relation between frequency, flow velocity, and orifice geometry have been reported:

1. The *Pfeifenton* frequency produced when air discharges through a circular orifice, terminating a pipe, is proportional to the thickness of the orifice plate and to the velocity of discharge, but it is independent of orifice diameter.[3] Curiously, no experimental data to substantiate this claim were presented. The discharge velocity is proportional to the square root of the differential pressure across the orifice.[4–6]
2. "...when the orifice is circular...the general frequency of the tone f rose proportionately with the velocity of efflux; the relation between f and d (diameter of orifice), however, was not a simple one," has been stated,[7] in referring to investigations by Kohlrausch.[4] No mention was made in this reference of any dependence on orifice thickness.
3. The jet-tone frequency f, produced when air or water discharges through a rectangular orifice, is inversely proportional to slit width d and directly proportional to velocity of discharge v.[4, 8–12] For air, $fd/v=0.044$.[4] For water dischargingt hrough a rectangular orifice, $fd/v=0.046$.[8]

The first is open to question. No experimental data werepresented, and the work was carried on one hundred years ago with only the experimental techniques then available. The second is also open to question, being a misquotation from the article of Kohlrausch who confined his attention to rectangular orifices, never mentioning circular orifices. The third refers to rectangular orifices being, therefore, not entirely appropriate to the experimental results reported in this paper, which concern circular orifices.

A parallel appears to exist between *Reibungstöne* and *Drahttöne* on the one hand and jet tones and *Pfeifentöne* on the other.[15] On the one hand, as the velocity of flow of air past a wire having a circular cross section is increased from zero, a velocity is reached at which vortices are shed periodically from the wire. This gives rise to a *Reibungston*. An impulse is communicated to the wire by the shedding of each vortex. *Reibungstöne* accordingly give rise to a periodic excitation of the wire. When the *Reibungston* frequency becomes equal to one of the natural frequencies of the wire, the wire sings out in a *Drahtton*. On the other hand,

[1] A. B. C. Anderson, J. Acoust. Soc. Am. **24**, 675–681 (1952).
[2] A. B. C. Anderson, J. Acoust. Soc. Am. (to be published); NOTS TM No. 903, May, 1953.
[3] M. A. Masson, Compt. rend. **36**, 257–260 (1853).
[4] W. Kohlrausch, Pogg. Ann. Phys. u. Chem. **13**, 545–569 (1881).
[5] C. Sondhauss, Pogg. Ann. Phys. u. Chem. **91**, 126–147 (1854).
[6] See reference 5, pp. 214–240.
[7] E. G. Richardson, *Sound* (Longmans, Green & Company, New York, and Edward Arnold & Company, London, 1947), p. 159.
[8] F. Krüger and E. Schmidtke, Ann. Physik **60**, 701–714 (1919).
[9] F. Krüger, Ann. Physik **62**, 673–690 (1920).
[10] F. Krüger and E. Marschner, Ann. Physik **67**, 581–611 (1922).
[11] E. Tyler and E. G. Richardson, Phil. Mag. **2**, 436–447 (1926).
[12] V. Strouhal, Weid. Ann. **5**, 216–251 (1878).

when the frequency of the periodic shedding of the vortices from an orifice, giving rise to a jet tone, is equal to one of characteristic frequencies of the pipe-orifice geometry, the pipe sings out in a *primary Pfeifenton.*

If *primary Pfeifentöne* are excited by the same aerodynamic mechanism that gives rise to jet tones, one might expect the same analytic relation holding for jet tones between discharge velocity, frequency, and some characteristic length describing the geometry of the orifice, to also hold for *primary Pfeifentöne*. For example, with air and a rectangular orifice, the relation is $fd/v=0.044$.[4] Accordingly, one might expect that the discrete *primary Pfeifenton* frequencies for a rectangular slit should be included in the continuum of jet-tone frequencies described by this relation.

The present paper is concerned with the determination of the form of this analytic relation for *primary Pfeifentöne*, when the pipe is terminated, not in a rectangular orifice, but in a circular orifice. The relation may be expressed in terms of either the velocity of flow through the orifice, or its equivalent, the square root of the differential pressure across the orifice divided by gas density. The latter will be chosen here because of the greater ease encountered in making precise differential pressure measurements.

GENERAL FORM OF RELATIONS DETERMINING *PRIMARY PFEIFENTON* FREQUENCY

The general form of the relations determining the frequency of *primary Pfeifentöne*, produced by jet tones in a pipe terminated in a circular orifice, may be obtained by dimensional analysis. Assume that the frequency f depends upon the following parameters:

$$f=F(\Delta p, \rho, t, d, \mu, L, S, c), \tag{1}$$

where the definition and dimensions of the parameters are given in Table I.

TABLE I. Definition of parameters.

Symbol	Meaning	Dimensions
Δp	Pressure difference across orifice	$ML^{-1}T^{-1}$
ρ	Gas density	ML^{-3}
t	Thickness of orifice plate	L
d	Diameter of orifice	L
μ	Coefficient of viscosity	$ML^{-1}T^{-1}$
L	Length of pipe	L
S	Internal cross-sectional area of pipe	L^2
c	Velocity of sound in gas	LT^{-1}

Equation (1) may be written

$$H(f, \Delta p, \rho, t, d, \mu, L, S, c)=0. \tag{2}$$

According to the π-theorem, Eq. (2) may be expressed in terms of the six following dimensionless π-products:

$$\pi_1=F_1(f, \Delta p, \rho, t), \quad \pi_4'=F_4(c, \Delta p, \rho, t),$$
$$\pi_2=F_2(d, \Delta p, \rho, t), \quad \pi_5=F_5(L, \Delta p, \rho, t),$$
$$\pi_3=F_3(\mu, \Delta p, \rho, t), \quad \pi_6'=F_6(S, \Delta p, \rho, t),$$

so that

$$G(\pi_1, \pi_2, \pi_3, \pi_4', \pi_5, \pi_6')=0. \tag{3}$$

The π-theorem guarantees that these six π-products can be made dimensionless and standard procedure leads to

$$\pi_1=\frac{(\Delta p/\rho)^{\frac{1}{2}}}{ft}, \quad \pi_2=\frac{d}{t}, \quad \pi_3=\frac{\rho t(\Delta p/\rho)^{\frac{1}{2}}}{\mu},$$
$$\pi_4'=\frac{(\Delta p/\rho)^{\frac{1}{2}}}{c}, \quad \pi_5=\frac{t}{L}, \quad \pi_6'=\frac{t^2}{S},$$

where π_4' has the form of Mach number. From this complete set the following complete set may be derived:

$$\pi_1=\frac{(\Delta p/\rho)^{\frac{1}{2}}}{c}, \quad \pi_2=\frac{d}{t}, \quad \pi_3=\frac{\rho t(\Delta p/\rho)^{\frac{1}{2}}}{\mu},$$
$$\pi_4=\frac{\pi_4'}{\pi_1\pi_5}=\frac{fL}{c}, \quad \pi_5=\frac{t}{L}, \quad \pi_6=\pi_2\pi_6'=\frac{td}{S}.$$

Thus

$$G\left[\frac{(\Delta p/\rho)^{\frac{1}{2}}}{ft}, \frac{d}{t}, \frac{\rho t(\Delta p/\rho)^{\frac{1}{2}}}{\mu}, \frac{fL}{c}, \frac{t}{L}, \frac{td}{S}\right]=0. \tag{4}$$

On the other hand, if the diameter of the orifice d had been chosen as one of the three fundamental variables instead of the orifice-plate thickness t, then

$$G\left[\frac{(\Delta p/\rho)^{\frac{1}{2}}}{fd}, \frac{t}{d}, \frac{\rho d(\Delta p/\rho)^{\frac{1}{2}}}{\mu}, \frac{fL}{c}, \frac{t}{L}, \frac{td}{S}\right]=0. \tag{5}$$

It has been shown[2] that $\cot(2\pi fL/c)=(td/S)(c/2\pi fL)$. Therefore, since Eq. (4) may be rearranged into

$$G_1\left[\frac{fL}{c}, \frac{td}{S}\right]=G_2\left[\frac{(\Delta p/\rho)^{\frac{1}{2}}}{ft}, \frac{d}{t}, \frac{t}{L}, \frac{\rho t(\Delta p/\rho)^{\frac{1}{2}}}{\mu}\right],$$

one is led to the consideration that

$$G_2\left[\frac{(\Delta p/\rho)^{\frac{1}{2}}}{ft}, \frac{d}{t}, \frac{t}{L}, \frac{\rho t(\Delta p/\rho)^{\frac{1}{2}}}{\mu}\right]=0. \tag{6}$$

Likewise Eq. (5) becomes

$$G_2\left[\frac{(\Delta p/\rho)^{\frac{1}{2}}}{fd}, \frac{t}{d}, \frac{d}{L}, \frac{\rho d(\Delta p/\rho)^{\frac{1}{2}}}{\mu}\right]=0. \tag{7}$$

The first π-product in Eq. (6), and also in Eq. (7), are here referred to as orifice numbers. The last π-product in each equation has the form of a Reynolds number.

If the physical parameters in Eqs. (6) and (7) are those of importance in determining the relation between frequency of *primary Pfeifentöne* and differential pressure across the circular orifice, then the true relation describing the phenomenon must have the form of Eq. (6) or (7). Dimensional reasoning alone is not capable of determining which form, Eq. (6) or (7), is most desirable. Only with aid of experimental data may this be done. In the last analysis, the results must be tested by experiment. Later it will be shown, however, that

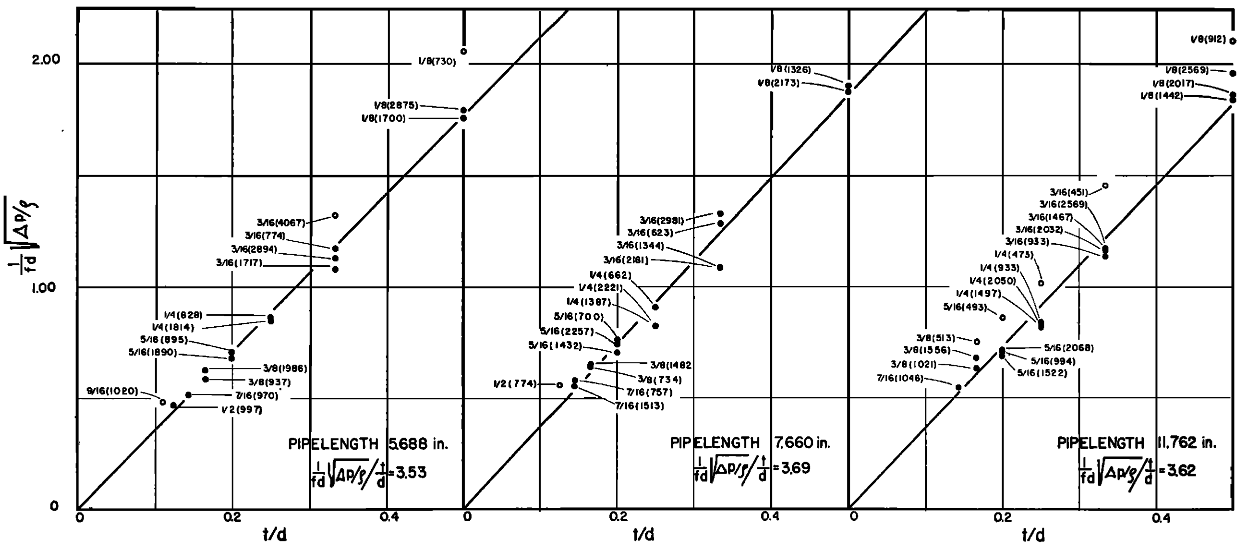

FIG. 1. Dependence of the nondimensional *primary Pfeifenton* orifice number $[(\Delta p/\rho)^{\frac{1}{2}}]/fd$ (expressed in consistent units) on t/d. Symbol f is *primary Pfeifenton* frequency; Δp, differential pressure across orifice; ρ, density of gas; t, thickness; and d, diameter of orifice terminating pipe. Dependence shown for all *primary Pfeifentöne* obtainable for pipe lengths 5.688, 7.660, and 11.762 in. Density ρ of air, 1.103 grams/cm³; inside diameter of pipe, 0.819 in.; orifice-plate thickness, $\frac{1}{16}$ in. throughout. Two numbers are attached to each point: first, diameter of orifice in inches; second, frequency of *primary Pfeifenton.* Hollow circles represent relatively doubtful experimental determinations, not considered in determining location of straight lines. See text.

the greater simplicity in form resulting by expressing the experimental data in terms of Eq. (6) favors Eq. (6).

EVALUATION OF THE NONDIMENSIONAL CIRCULAR ORIFICE NUMBER FOR *PRIMARY PFEIFENTÖNE*

Experimental values of the frequencies of the *primary Pfeifentöne* and of the differential pressure across the orifices were obtained as described elsewhere[1] except:

1. To minimize disturbances from background noise, a variable band pass filter was inserted between the sound-pressure pickup circuit and the Y axis of the oscilloscope.
2. An extremely sensitive but stable differential manometer, consisting of a U-tube filled with di-butyl phthalate and inclined at arctangent 0.10 to the horizontal, was used for the differential pressure determinations across the orifice.
3. Far more careful determinations of differential pressure across far more carefully machined circular orifices were required here, for evaluating the orifice constant, than before[1,2] for determining the relation between *Pfeifenton* frequency and geometry. This is because differential pressure is very sensitive to slight changes of *Pfeifenton* frequency and also because the differential pressure associated with a given *primary Pfeifenton* frequency appears to be greatly affected by nicks, beveling, and slight imperfections around the edge of the orifice. *Pfeifenton* frequency, however, is not. For example, reversal of an orifice plate at the end of the pipe, whose orifice contains slight imperfections, generally results in an appreciably different differential pressure, but makes no appreciable change in the frequency of the *primary Pfeifenton.*

Figures 1 and 2 show the orifice number of the *primary Pfeifenton* $(\Delta p/\rho)^{\frac{1}{2}}fd$ plotted against t/d, according to Eq. (7), for pipelengths 5.688, 7.660, 11.762, 13.718, 19.722, and 23.725 in., all having the internal pipe diameter 0.819 in. The orifice diameter ranges from $\frac{1}{8}$ to $\frac{9}{16}$ in., in steps of $\frac{1}{16}$ in., in orifice plates $\frac{1}{16}$ in. thick.

Pfeifenton frequencies and differential pressures were determined over as great range of differential pressure and orifice diameters as *primary Pfeifentöne* could be excited. Every point in the figures represents the low frequency, low pressure end of an appreciable range of *Pfeifentöne.* Only for pipe length 11.762 in., and orifice diameter $\frac{3}{16}$ in. in Fig. 3 are complete ranges presented. They are indicated by the slightly inclined almost horizontal line segments extending to the right from the corresponding *primary Pfeifenton* points.

The hollow points in the figures, deviating considerably from the others and frequently associated with the lowest differential pressures and frequencies observed, represent values that may be open to some question. For these the amplitude of the sound was often so low and the sound so prone to self-extinction that an accurate determination of *primary Pfeifenton* frequency and differential pressure may have been questionable, resulting in extinction of the *Pfeifenton* at a pressure possibly somewhat higher than that of the true *primary Pfeifenton.* Possibly this extinction is related to the observation that there is a minimum Reynolds number below which vortices are not generated.[13] The deviation of the hollow points, associated with the lowest differential pressures, from the other points may also be partly caused by an inherent rise in the value of the orifice number with decrease of Reynolds number, at the lowest Reynolds numbers.

Within experimental error an approximately constant ratio appears to exist between the orifice number $(\Delta p/\rho)^{\frac{1}{2}}/fd$ and t/d for all orifice diameters, *Pfeifenton* frequencies, and pipe lengths investigated, except for

[13] See reference 7, p. 157.

the very lowest *primary Pfeifentöne*. Little change in the numerical value of the ratio seems to occur over the range of pipe lengths 5.688 to 23.725 in.; if any, there seems only to be a very slight increase in the ratio with increase of pipe length. Accordingly, to a first approximation Eq. (7) may be placed in the following form:

$$\frac{(\Delta p/\rho)^{\frac{1}{2}}}{fd}\,\frac{t}{d}=G_3\left[\frac{d}{L},\frac{\rho d(\Delta p/\rho)^{\frac{1}{2}}}{\mu}\right], \tag{8}$$

reducing to

$$\frac{(\Delta p/\rho)^{\frac{1}{2}}}{ft}=G_3\left[\frac{d}{L},\frac{\rho d(\Delta p/\rho)^{\frac{1}{2}}}{\mu}\right]=K, \tag{9}$$

where K is constant to the order of approximation already indicated, independent of orifice diameter, differential pressure across orifice, and pipe length. Comparison of the form of Eq. (9) with (7) leads one to favor the direct adoption of Eq. (7) in the form

$$\frac{(\Delta p/\rho)^{\frac{1}{2}}}{ft}=G_4\left[\frac{d}{t},\frac{t}{L},\frac{\rho t(\Delta p/\rho)^{\frac{1}{2}}}{\mu}\right]=K, \tag{10}$$

even though the expression for Reynolds number in this equation is not the same as in Eq. (9). The relative sameness in slope of all the lines in Figs. 3 and 4 shows the degree of constancy, of the orifice-number $(\Delta p/\rho)^{\frac{1}{2}}/ft$, found experimentally for different frequencies, orifice-diameters, pipelengths, and differential pressures across the orifice. The estimated mean value of the orifice-constant for each of the pipelengths is indicated in the figures.

The seemingly smaller degree of scattering of the points in Figs. 3 and 4, derived from the same data as in Figs. 1 and 2, may be accounted for by the observation that many of the points corresponding to doubtful experimental determinations of *primary Pfeifentöne* and to the lowest *primary Pfeifenton* frequencies are concentrated at the lower left-hand corners of Figs. 3 and 4, where their effect is seemingly minimized. In Figs. 1 and 2 the doubtful points (hollow circles) are distributed uniformly throughout. In general, the relatively doubtful values, are associated with tones found to be weak, uncertain, and fluctuating. Often they occur at the lowest audible frequencies and differential pressures. The cause of this may lie in the observation that there is a minimum velocity (minimum Reynolds number) below which the relative predominance of viscous over inertial forces in the flow ensures that no vortices and, therefore, no tones are produced.[14] They also occur at the highest differential pressures and largest orifice diameters. The cause of this is still obscure but may reside in the fact that the flow becomes so turbulent that stable *Pfeifenton* vibrations cannot be maintained. It may be, therefore, that stable *primary Pfeifentöne* are maintained easily only within a definite range of Reynolds numbers. If Reynolds number of the *primary Pfeifenton* lies in the vicinity or beyond the bounds of this range, the value of the lowest frequency and differential pressure obtained for the *Pfeifenton* range will not correspond to the *primary Pfeifenton*.

DISCUSSION AND CONCLUSIONS

If it can be assumed that the mechanism involved in the creation of jet tones is also the source of excitation

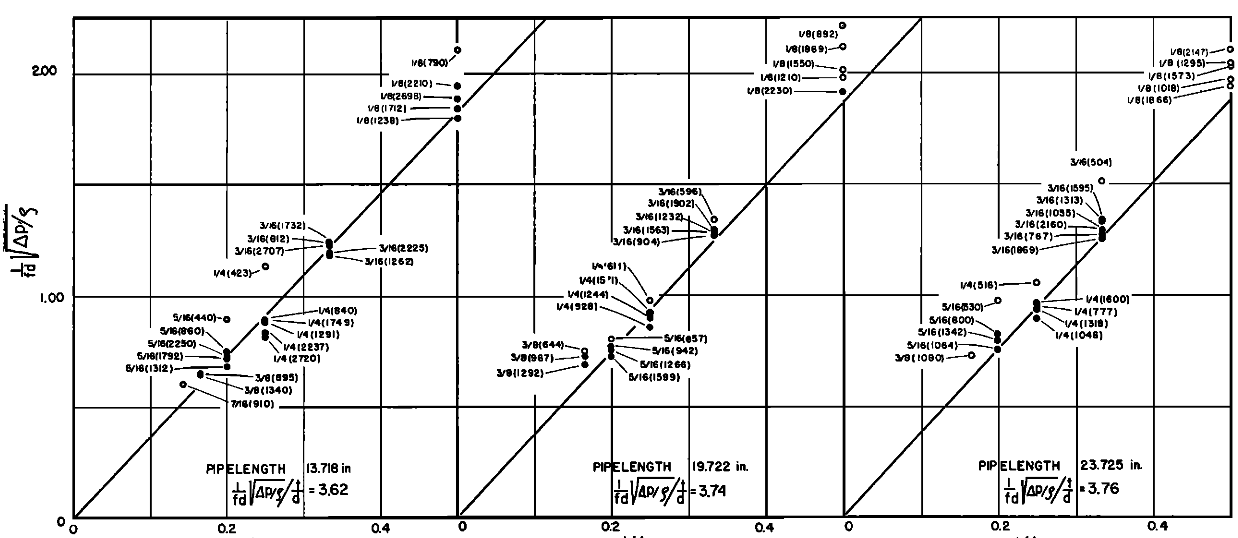

FIG. 2. Dependence of the nondimensional *primary Pfeifenton* orifice number $[(\Delta p/\rho)^{\frac{1}{2}}]/fd$ (expressed in consistent units) on t/d. Symbol f is *primary Pfeifenton* frequency; Δp, differential pressure across orifice; ρ, density of gas; t, thickness; and d, diameter of orifice terminating pipe. Dependence shown for all *primary Pfeifentöne* obtainable for pipe lengths 13.718, and 19.722, 23.725 in. Density ρ of air 1.103 grams/cm³; inside diameter of pipe, 0.819 in.; orifice-plate thickness, $\frac{1}{16}$ in. throughout. Two numbers are attached to each point: first, diameter of orifice in inches; second, frequency of *primary Pfeifenton*. Hollow circles represent relatively doubtful experimental determinations, not considered in determining location of straight lines. See text.

[14] See reference 7, p. 155.

of *Pfeifentöne*,[1,2,12] it might be presumed that the same relation (between differential pressure across the orifice and jet-tone frequency) found for jet tones should also hold for *Pfeifentöne*. A cursory survey of recent literature can easily lead one to the conclusion that v/fd equals a constant for jet tones produced by circular orifices, that is, $(\Delta p/\rho)^{\frac{1}{2}}/fd$ equals a constant. For example, it has been stated[15] that "Kohlrausch and also Krüger have shown (Annalen der Physik, 1881 and 1920) that in a jet issuing from a circular nozzle, vortices are produced periodically with frequency f, given by $v/fd=$ a constant, d being a linear dimension dependent on the bore of the nozzle." A similar statement found elsewhere[7] has already been referred to in the Introduction. It might be expected, therefore, that the analytic relation found in this paper for *primary Pfeifentöne* should have the same form as the foregoing, reported for jet tones produced by circular orifices. Accordingly, the data presented in this paper for *primary Pfeifentöne* were first plotted, according to Eq. (7), in Figs. 1 and 2.

No constancy of the expression $(\Delta p/\rho)^{\frac{1}{2}}/fd$ whatever is found, since it depends linearly on t/d. An examination of the original work[4,9] referred to in later articles,[7,15] in which it is claimed that v/fd was found constant for circular orifices, discloses that the original work was carried out with rectangular slits of width d and not circular orifices as claimed.

On the other hand, the expression $(\Delta p/\rho)^{\frac{1}{2}}/ft$ of Eq. (6), involving the orifice plate thickness t, is *relatively* constant over the range of variation of parameters considered, Figs. 3 and 4. This is illustrated by the *relative* constancy and sameness in slope of the straight lines describing the trend of the points. Why then has v/fd been found constant for rectangular orifices elsewhere, whereas the present study of *primary Pfeifentöne* with circular orifices appears to indicate $(\Delta p/\rho)^{\frac{1}{2}}/ft$ (that is, v/ft, where v is the flow velocity) *relatively* constant over the range of variation of parameters considered in this paper? Sufficient evidence for an explanation does not appear obvious.

Could it be the explanation of the discrepancy lies in a difference in nature of the aerodynamics of vortex development from rectangular and from circular orifices? For example,[16] "...the resulting vortex formation (by a column of air issuing from a slit whose width is small compared with its length) is not identical with that found in a jet of circular cross section. In the first case the vortices are cylindrical and have their axes parallel to the longest length of the slit: as they grow in size they occupy the whole of the triangular space into which streams of this kind spread....In the case of the

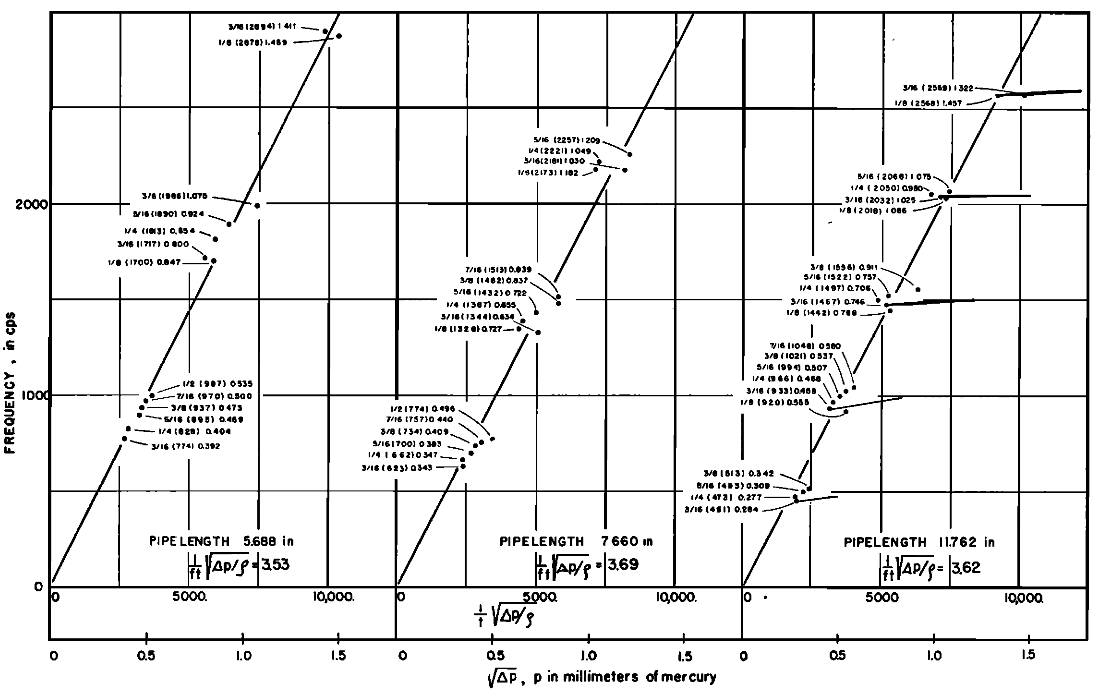

FIG. 3. Constancy in numerical value of nondimensional *primary Pfeifenton* orifice number $[(\Delta p/\rho)^{\frac{1}{2}}]/ft$ (expressed in consistent units). Constancy is indicated by sameness in slope of three lines, representing results for all *primary Pfeifentöne* obtainable for pipe lengths 5.688, 7.660, and 11.762 in. Symbol f is *primary Pfeifenton* frequency; Δp, differential pressure across orifice; ρ, density of gas; t, thickness of orifice terminating pipe. Density ρ of air, 1.103 grams/cm^3; inside diameter of pipe, 0.819 in.; orifice-plate thickness, $\frac{1}{16}$ in. throughout. Three numbers are attached to each point: first, diameter of orifice in inches; second, frequency of *primary Pfeifenton*; third, square root of differential pressure expressed in millimeters of mercury. Lower abscissa indicated in graph by $(\Delta p)^{\frac{1}{2}}$ represents square root of differential pressure expressed in millimeters of mercury.

[15] E. G. Richardson, Nature **116**, 171–172 (1925).

[16] G. Burniston Brown, Proc. Phys. Soc. (London) **47**, 703–732 (1935).

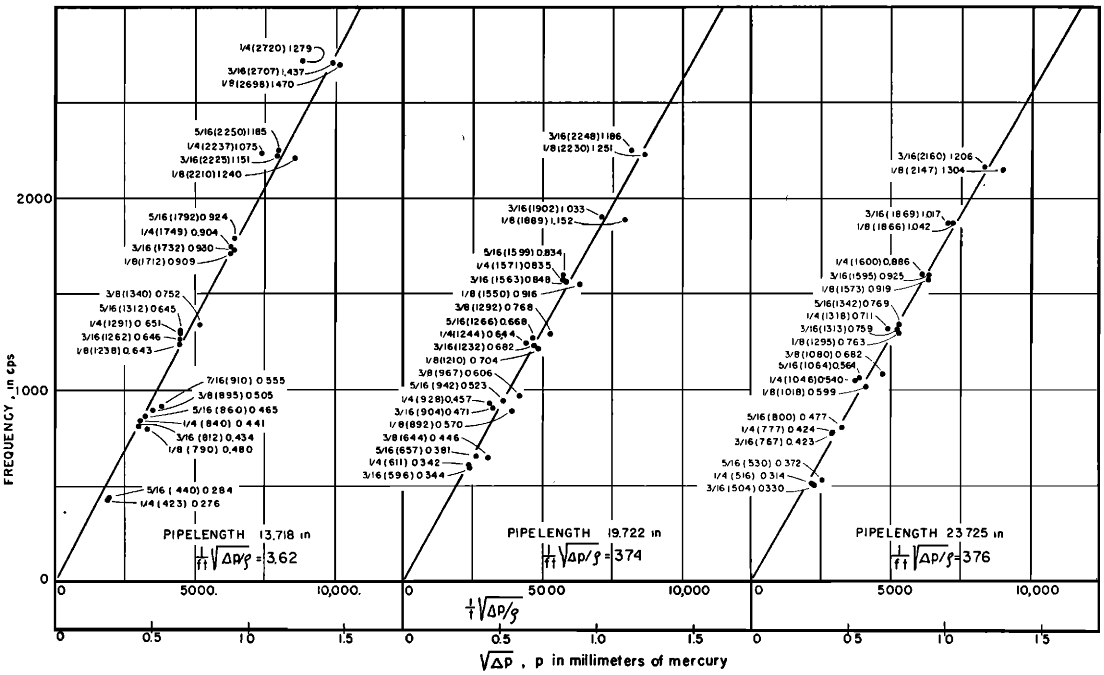

FIG. 4. Constancy in numerical value of nondimensional *primary Pfeifenton* orifice number $[(\Delta p/\rho)^{\frac{1}{2}}]/ft$ (expressed in consistent units). Constancy is indicated by sameness in slope of three lines, representing results for all *primary Pfeifentöne* obtainable for pipe lengths 13.718, 19.722, 23.725 in. Symbol f is *primary Pfeifenton* frequency; Δp, differential pressure across orifice; ρ, density of gas; t, thickness of orifice terminating pipe. Density ρ of air, 1.103 grams/cm^3; inside diameter of pipe, 0.819 in.; orifice plate thickness, $\frac{1}{16}$ in. throughout. Three numbers are attached to each point: first, diameter of orifice in inches; second, frequency of *primary Pfeifenton*; third, square root of differential pressure expressed in millimeters of mercury. Lower abscissa indicated in graph by $(\Delta p)^{\frac{1}{2}}$ represents square root of differential pressure expressed in millimeters of mercury.

jet of circular cross section the axes of the vortices are more or less semicircular and these vortices may interact, in favorable circumstances, in such a way as to cause the stream to bifurcate, one vortex being thrown to the right and the next to the left and so on...."

The numerical value of $[(\Delta p/\rho)^{\frac{1}{2}}/fd]/[t/d]$ given by the slope of the lines in Figs. 1 and 2 should equal the value $(\Delta p/\rho)^{\frac{1}{2}}/ft$ given by the slope of the lines in Figs. 3 and 4. The values represented by hollow circles in Figs. 1 and 2, considered somewhat doubtful, were not considered in determining the lines in either Figs. 1 and 2 or 3 and 4.

Although the numerical value of the *primary Pfeifenton* orifice number $(\Delta p/\rho)^{\frac{1}{2}}/ft$ appears to be relatively constant for the range of variation of parameters (orifice diameter, pipe length, *Pfeifenton* frequency, and differential pressure across orifice) considered in this paper, the present studies, however, have not demonstrated it to be a universal constant. For example, on the one hand, Eq. (6) implies that the numerical value of the orifice constant might depend on Reynolds number. The results of the present studies do not permit consideration of the effect of varying viscosity, gas density, and orifice-plate thickness. On the other hand, it has been stated that Strouhal's number describing the shedding frequency of vortices from a circular cylinder, a phenomenon similar in nature to the foregoing, is independent of Reynolds number once vortices are formed.[17] Most agree, however, that Strouhal's number varies to a small extent with Reynolds number,[12,18–20] especially at low Reynolds numbers. Within the range of variation of the parameters considered in the present study, the numerical value of the *primary Pfeifenton* orifice constant $(\Delta p/\rho)^{\frac{1}{2}}/ft$ does not appear to show an appreciable dependence on the differential pressure Δp involved in Reynolds number $[\rho t(\Delta p/\rho)^{\frac{1}{2}}]/\mu$ in Eq. (6), except possibly at the lowest Reynolds numbers.

Two relations have now been found,[2] $\cot(2\pi fL/c) = (Ld/S)(c/2\pi fL)$ and $(\Delta p/\rho)^{\frac{1}{2}}/ft =$ a constant, describing the dependence of *primary Pfeifentöne* on pertinent parameters investigated up to the present. Both together may be used in estimating the *primary Pfeifenton* frequency and the differential pressure, across an orifice, resulting from a given pipe-orifice geometry. For example, if the length and cross-sectional area of the pipe, the diameter of the orifice, and the thickness of the "thin" orifice plate are given, the *primary Pfeifenton* frequency may be estimated from the first relation. The second then gives the differential pressure across the orifice.

[17] See reference 7, p. 158.

[18] Lord Rayleigh, *Theory of Sound, Vol. II* (Dover Publications, New York, 1945) p. 413.

[19] L. S. G. Kovasznay, Proc. Roy. Soc. (London) **A198**, 174–190 (1949).

[20] Alexander Wood, *Acoustics* (Interscience Publishers, New York, 1947), p. 390.