1 Équations physiques

Dans le fluide Ω_f :

$$(\mathbf{u} \cdot \boldsymbol{\nabla})\, \mathbf{u} = -\, \boldsymbol{\nabla}\, p + \frac{2}{Re}\, \boldsymbol{\nabla} \cdot \overline{\overline{\boldsymbol{D}}}(\mathbf{u}). \tag{1}$$

Dans le disque Ω_p :

$$\left(\frac{1}{\epsilon^2} \mathbf{u} \cdot \boldsymbol{\nabla}\right) \mathbf{u} = -\, \boldsymbol{\nabla}\, p + \frac{2}{\epsilon Re}\, \boldsymbol{\nabla} \cdot \overline{\overline{\boldsymbol{D}}}(\mathbf{u}) - \frac{1}{Re} \frac{1}{Da} \left(\mathbf{u} - \boldsymbol{\Omega} \wedge \mathbf{x}\right). \tag{2}$$

Partout Ω :

$$\boldsymbol{\nabla} \cdot \mathbf{u} = \mathbf{0}. \tag{3}$$

On note

$$\mathcal{NS}(\mathbf{u}, p) = -\mathcal{S}_1 \left(\mathbf{u} \cdot \boldsymbol{\nabla}\right) \mathbf{u} - \boldsymbol{\nabla}\, p + \mathcal{S}_2 \frac{2}{Re}\, \boldsymbol{\nabla} \cdot \overline{\overline{\boldsymbol{D}}}(\mathbf{u}), \tag{4}$$

où

$$\mathcal{S}_1 = \begin{cases} 1 & \text{dans } \Omega_f \\ \frac{1}{\epsilon^2} & \text{dans } \Omega_p \end{cases} \qquad \text{et} \qquad \mathcal{S}_2 = \begin{cases} 1 & \text{dans } \Omega_f \\ \frac{1}{\epsilon} & \text{dans } \Omega_p \end{cases} \tag{5}$$

2 Formulation faible

$$\underbrace{\iint_{\Omega} q \cdot (3)\ \mathrm{d}S}_{(I)} + \underbrace{\iint_{\Omega} \mathbf{v} \cdot (4)\ \mathrm{d}S}_{(II)} + \underbrace{\iint_{\Omega_p} \mathbf{v} \cdot \left[\frac{-1}{Re\ Da} \left(\mathbf{u} - \boldsymbol{\Omega} \wedge \mathbf{x}\right)\right] \mathrm{d}S}_{(III)}. \tag{6}$$

On pose

$$\begin{bmatrix} \mathbf{u} \\ p \end{bmatrix} = \begin{bmatrix} \mathbf{u}^\star \\ p^\star \end{bmatrix} + \begin{bmatrix} \delta \mathbf{u} \\ \delta p \end{bmatrix} \tag{7}$$

On obtient tout calculs faits :

$$\begin{aligned} &\iint_{\Omega} q \cdot \operatorname{div}\left(\mathbf{u}^\star\right) + \iint_{\Omega} q \cdot \operatorname{div}\left(\delta \mathbf{u}\right) + \iint_{\Omega} -\mathcal{S}_1\ \frac{1}{2} \mathcal{C}\left(\mathbf{u}^\star, \mathbf{u}^\star\right) \cdot \mathbf{q} + \iint_{\Omega} -\mathcal{S}_1\ \mathcal{C}\left(\mathbf{u}^\star, \delta \mathbf{u}\right) \cdot \mathbf{q} \\ &\quad + \iint_{\Omega} p^\star \cdot \operatorname{div}(\mathbf{v}) + \iint_{\Omega} \delta p \cdot \operatorname{div}(\mathbf{v}) + \iint_{\Omega} -\mathcal{S}_2\ \overline{\overline{\boldsymbol{D}}}(\mathbf{v}) : \overline{\overline{\boldsymbol{D}}}(\mathbf{u}^\star) + \iint_{\Omega} -\mathcal{S}_2\ \overline{\overline{\boldsymbol{D}}}(\mathbf{v}) : \overline{\overline{\boldsymbol{D}}}(\delta \mathbf{u}) \\ &\qquad + \iint_{\Omega_p} \frac{-1}{Re\ Da} \mathbf{v} \cdot \left(\mathbf{u} - \mathbf{u_s}\right) + \iint_{\Omega_p} \frac{-1}{Re\ Da} \mathbf{v} \cdot \delta \mathbf{u} + \ \text{C.L} \ = 0 \end{aligned} \tag{8}$$