Technische Universität München Lehrstuhl für Theoretische Informatik Ort, Datum: Garching, 05. July 2022



Paper: Correct Probabilistic Model Checking with Floating-Point Arithmetic - Arnd Hartmanns

Key points:

- Value Iteration can lead to wrong results in model checking
- -Examples given in the Paper
- Solved by using Interval Iteration and controlling the Rounding mode
- -Algorithm with rounding mode shown in the paper
- Experiments showing that the given algorithm works were done in paper

- Value Iteration can lead to wrong results in model checking
- -Examples given in the Paper
- → Verify Examples
- Solved by using Interval Iteration and controlling the Rounding mode
- -Algorithm with rounding mode shown in the paper
- Experiments showing that the given algorithm works were done in paper

- Value Iteration can lead to wrong results in model checking
- -Examples given in the Paper
- → Verify Examples
- Solved by using Interval Iteration and controlling the Rounding mode
- -Algorithm with rounding mode shown in the paper
- → Figure out how to control Rounding modes in C
- → Implement Algorithm
- Experiments showing that the given algorithm works were done in paper

- Value Iteration can lead to wrong results in model checking
- -Examples given in the Paper
- → Verify Examples
- Solved by using Interval Iteration and controlling the Rounding mode
- -Algorithm with rounding mode shown in the paper
- → Figure out how to control Rounding modes in C
- → Implement Algorithm
- Experiments showing that the given algorithm works were done in paper
- → Confirm Experiments*

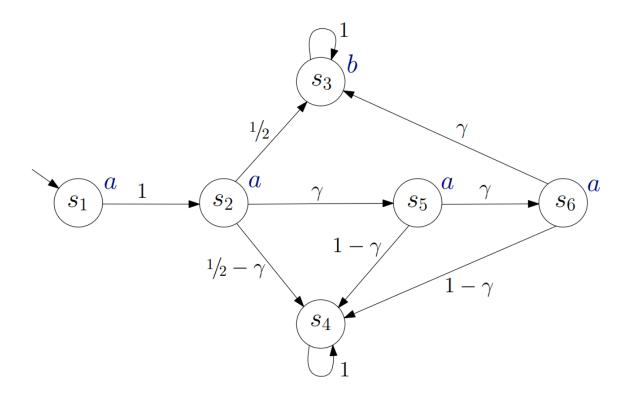
*or maybe do something else like finding counterexamples

- → Verify Examples in the paper in PRISM
- → Verify Examples in the paper in STORM
- → Figure out how to control Rounding modes in C
- → Implement Algorithm
- → Confirm Experiments*
- → Own potential ideas (more research needed first):
- When is controlled rounding is NOT needed?
- Are there cases where controlled rounding is worse than normal? (maybe higher run time in some cases)

PRISM - Example

Example from "Probabilistic Model Checking and Reliability of Results" (Wimmer et al)

- for small values of γ, the model checkers give wrong results



PRISM Example – big gamma, no issues (correct result 0.0)

```
Model checking: P=? [ s=8 U (P<=0.5 [ s=1|s=2|s=5|s=6 U s=3 ]) ]
Building model...
Computing reachable states...
Reachability (BFS): 4 iterations in 0.00 seconds (average 0.000000, setup 0.00)
Time for model construction: 0.021 seconds.
Type:
                                                                                                                                           wimmer_fail.pm
             6 (1 initial)
States:
Transitions: 10
Transition matrix: 31 nodes (6 terminal), 10 minterms, vars: 3r/3c
                                                                                             probabilistic
Prob0: 3 iterations in 0.00 seconds (average 0.000333, setup 0.00)
                                                                                           4 const double gamma = 0.01;
                                                                                            //default gamma = 0.000001
Prob1: 3 iterations in 0.00 seconds (average 0.000000, setup 0.00)
                                                                                           6 //for gamma = 0.01, you get 0 as result
                                                                                           8 module sys
yes = 1, no = 1, maybe = 4
Computing remaining probabilities...
Engine: Hybrid
                                                                                                     [] s=2 \rightarrow 0.5: (s'=3) + gamma: (s'=5) + (0.5-gamma): (s'=4);
Building hybrid MTBDD matrix... [levels=3, nodes=28] [1.3 KB]
                                                                                                     [] s=3 -> 1.0: (s'=3);
Adding explicit sparse matrices... [levels=3, num=1, compact] [0.1 KB]
Creating vector for diagonals... [dist=1, compact] [0.0 KB]
                                                                                                     [] s=4 -> 1.0: (s'=4);
                                                                                                     [] s=5 -> gamma: (s'=6) + (1-gamma): (s'=4);
Creating vector for RHS... [dist=2, compact] [0.0 KB]
                                                                                                     [] s=6 \rightarrow gamma: (s'=3) + (1-gamma): (s'=4);
Allocating iteration vectors... [2 x 0.0 KB]
TOTAL: [1.5 KB]
                                                                                          19 endmodule
Starting iterations...
Jacobi: 5 iterations in 0.00 seconds (average 0.000000, setup 0.00)
yes = 3, no = 3, maybe = θ
Value in the initial state: 0.0
Time for model checking: 0.01 seconds.
Result: 0.0 (exact floating point)
```

PRISM Example – small gamma, rounding issues

```
Model checking: P=? [ s=8 U (P<=0.5 [ s=1|s=2|s=5|s=6 U s=3 ]) ]
Building model...
Computing reachable states...
Reachability (BFS): 4 iterations in 0.00 seconds (average 0.000000, setup 0.00)
Time for model construction: 0.021 seconds.
                                                                                                                                             wimmer_fail.pm
             6 (1 initial)
States:
                                                                                                                        -/Modelchecking/PRISM/from_source/prism-4.7-src/prism-examples/seminar_anton
Transitions: 10
Transition matrix: 31 nodes (6 terminal), 10 minterms, vars: 3r/3c
                                                                                              probabilistic
Prob0: 3 iterations in 0.00 seconds (average 0.000000, setup 0.00)
                                                                                             4 const double gamma = 0.000001;
                                                                                             5 //default gamma = 0.000001
                                                                                            6 //for gamma = 0.01, you get 0 as result
Prob1: 3 iterations in 0.00 seconds (average 0.000000, setup 0.00)
                                                                                             8 module sys
yes = 1, no = 1, maybe = 4
Computing remaining probabilities...
Engine: Hybrid
                                                                                                       [] s=2 \rightarrow 0.5: (s'=3) + gamma: (s'=5) + (0.5-gamma): (s'=4);
Building hybrid MTBDD matrix... [levels=3, nodes=28] [1.3 KB]
Adding explicit sparse matrices... [levels=3, num=1, compact] [0.1 KB]
                                                                                                       [] S=4 -> 1.0: (S'=4);
Creating vector for diagonals... [dist=1, compact] [0.0 KB]
                                                                                                       [] s=5 -> gamma: (s'=6) + (1-gamma); (s'=4);
Creating vector for RHS... [dist=2, compact] [0.0 KB]
Allocating iteration vectors... [2 \times 0.0 \text{ KB}]
                                                                                                       [] s=6 \rightarrow gamma: (s'=3) + (1-gamma): (s'=4);
TOTAL: [1.5 KB]
                                                                                           19 endmodule
Starting iterations...
Jacobi: 3 iterations in 0.00 seconds (average 0.000000, setup 0.00)
yes = 5, no = 1, maybe = θ
Value in the initial state: 1.0
Time for model checking: 0.01 seconds.
Result: 1.0 (exact floating point)
```

PRISM Example – improvement with Interval Iteration

```
Model checking: P=? [ s=8 U (P<=0.5 [ s=1|s=2|s=5|s=6 U s=3 ]) ]
Building model...
Computing reachable states...
Reachability (BFS): 4 iterations in 0.00 seconds (average 0.000000, setup 0.00)
Time for model construction: 0.019 seconds.
Type:
                                                                                                                                                  wimmer_fail.pm
             6 (1 initial)
States:
Transitions: 10
                                                                                                    probabilistic
Transition matrix: 31 nodes (6 terminal), 10 minterms, vars: 3r/3c
                                                                                                   const double gamma = 0.000001;
Prob0: 3 iterations in 0.00 seconds (average 0.000000, setup 0.00)
                                                                                                  5 //default gamma = 0.000001
                                                                                                  6 //for gamma = 0.01, you get 0 as result
Prob1: 3 iterations in 0.00 seconds (average 0.000000, setup 0.00)
                                                                                                  8 module sys
yes = 1, no = 1, maybe = 4
Computing remaining probabilities...
Engine: Hybrid
                                                                                                            [] s=2 \rightarrow 0.5: (s'=3) + gamma: (s'=5) + (0.5-gamma): (s'=4);
Building hybrid MTBDD matrix... [levels=3, nodes=28] [1.3 KB]
                                                                                                            [] S=3 -> 1.0: (S'=3);
Adding explicit sparse matrices... [levels=3, num=1, compact] [0.1 KB]
Creating vector for diagonals... [dist=1, compact] [0.0 KB]
                                                                                                            [] S=4 \rightarrow 1.0: (S'=4);
                                                                                                            [] s=5 -> gamma: (s'=6) + (1-gamma): (s'=4);
Creating vector for RHS... [dist=2, compact] [0.0 KB]
                                                                                                            [] s=6 \rightarrow gamma: (s'=3) + (1-gamma): (s'=4);
Allocating iteration vectors... [4 \times 0.0 \text{ KB}]
TOTAL: [1.6 KB]
                                                                                                 19 endmodule
Starting iterations...
Max relative diff between upper and lower bound on convergence: 1.99996E-12
Jacobi (interval iteration): 3 iterations in 0.00 seconds (average 0.000000, setup 0.00)
yes = 4, no = 2, maybe = \theta
Value in the initial state: 0.0
Time for model checking: 0.011 seconds.
Result: 0.0 (exact floating point)
```

PRISM Example – Interval Iteration correct until 10^14

```
Model checking: P=? [ s=8 U (P<=0.5 [ s=1|s=2|s=5|s=6 U s=3 ]) ]
Building model...
Computing reachable states...
Reachability (BFS): 4 iterations in 0.00 seconds (average 0.000000, setup 0.00)
Time for model construction: 0.02 seconds.
Type:
States:
            6 (1 initial)
Transitions: 10
Transition matrix: 31 nodes (6 terminal), 10 minterms, vars: 3r/3c
Prob0: 3 iterations in 0.00 seconds (average 0.000000, setup 0.00)
Prob1: 3 iterations in 0.00 seconds (average 0.001000, setup 0.00)
ves = 1, no = 1, maybe = 4
Computing remaining probabilities...
Engine: Hybrid
Building hybrid MTBDD matrix... [levels=3, nodes=28] [1.3 KB]
Adding explicit sparse matrices... [levels=3, num=1, compact] [0.1 KB]
Creating vector for diagonals... [dist=1, compact] [0.0 KB]
Creating vector for RHS... [dist=2, compact] [0.0 KB]
Allocating iteration vectors... [4 \times 0.0 \text{ KB}]
TOTAL: [1.6 KB]
Starting iterations...
Max relative diff between upper and lower bound on convergence: 1.9984E-14
Jacobi (interval iteration): 2 iterations in 0.00 seconds (average 0.000000, setup 0.00)
yes = 4, no = 2, maybe = 0
Value in the initial state: 0.0
Time for model checking: 0.01 seconds.
Result: 0.0 (exact floating point)
```

```
wimmer_fail_15gamma.pm
                     -/Modelchecking/PRISM/from_source/prism-4.7-src/prism-examples/se.
  probabilistic
 4 const double gamma = 0.0000000000001; //10^15 result: 1.0
 5 //const double gamma = 0.0000000000001; 10^-14 result: 0.0
 6 //const double gamma = 0.00000000000001; //10^15 result: 1.0
 8 //default gamma = 0.000001
 9 //for gamma = 0.01, you get 0 as result
11 module sys
           [] s=2 \rightarrow 0.5: (s'=3) + gamma: (s'=5) + (0.5-gamma): (s'=4);
           [] S=3 -> 1.0: (S'=3);
           [] S=4 \rightarrow 1.0: (S'=4);
           [] s=5 -> gamma: (s'=6) + (1-gamma): (s'=4);
           [] s=6 \rightarrow gamma: (s'=3) + (1-gamma): (s'=4);
22 endmodule
```

PRISM Example – rounding issues with <=10^15

```
Model checking: P=? [ s=8 U (P<=0.5 [ s=1|s=2|s=5|s=6 U s=3 ]) ]
Building model...
Computing reachable states...
Reachability (BFS): 3 iterations in 0.00 seconds (average 0.000000, setup 0.00)
Time for model construction: 0.019 seconds.
                                                                                                                                           wimmer_fail_15gamma.pm
Type:
                                                                                                                              -/Modelchecking/PRISM/from_source/prism-4.7-src/prism-examples/se.
States:
             4 (1 initial)
Transitions: 5
                                                                                                             probabilistic
Transition matrix: 13 nodes (3 terminal), 5 minterms, vars: 3r/3c
                                                                                                               const double gamma = 0.000000000000001; //10^15 result: 1.0
                                                                                                             //const double gamma = 0.0000000000001; 10^-14 result: 0.0
Prob0: 3 iterations in 0.00 seconds (average 0.000000, setup 0.00)
                                                                                                             //const double gamma = 0.00000000000001; //10015 result: 1.0
Probl: 3 iterations in 0.00 seconds (average 0.000000, setup 0.00)
                                                                                                           # //not 10^11 and 10^12 like in paper
yes = 1, no = 1, maybe = 2
                                                                                                           //default gamma = 0.000001
                                                                                                           1/for gamma = 0.01, you get 0 as result
Computing remaining probabilities...
Engine: Hybrid
                                                                                                            module sys
Building hybrid MTBOD matrix... [levels=3, nodes=11] [0.5 KB]
Adding explicit sparse matrices... [levels=3, num=1, compact] [0.0 KB]
Creating vector for diagonals... [dist=1, compact] [0.0 KB]
Creating vector for RHS... [dist=2, compact] [0.0 KB]
                                                                                                                     [] s=2 \rightarrow 0.5: (s'=3) + gamma: (s'=5) + (0.5-gamma): (s'=4);
Allocating iteration vectors... [4 x 0.0 KB]
                                                                                                                     [] S=3 -> 1.0: (S'=3);
TOTAL: [0.7 KB]
                                                                                                                     [] s=4 -> 1.0: (s'=4);
                                                                                                                     [] s=5 → gamma: (s'=6) + (1-gamma): (s'=4);
Starting iterations...
                                                                                                                     [] s=6 -> gamma: (s'=3) + (1-gamma): (s'=4);
Max relative diff between upper and lower bound on convergence: 0
Jacobi (interval iteration): 2 iterations in 0.00 seconds (average 0.000000, setup 0.00)
                                                                                                             endmodule
yes = 3, no = 1, maybe = \theta
Value in the initial state: 1.0
Time for model checking: 0.01 seconds.
Result: 1.0 (exact floating point)
```

Open Tasks and Timeline

- → Verify Examples in the paper in PRISM Done
- → Verify Examples in the paper in STORM
- → Figure out how to control Rounding modes in C
- → Implement Algorithm until 15.07

- → Confirm Experiments until 01.08
- → Own potential ideas (more research needed first):
 - When is controlled rounding is NOT needed?
 - Are there cases where controlled rounding is worse than normal? (maybe higher run time in some cases)