

gut

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1 Modeling Bacteria, Antibiotics and Immune System

This notebook is to accompany the Report file Computation and Code section. Figures and Simulations are generated here.

1.1 Parameter Selection

Our Invariant Set \mathcal{A} implied some restriction on parameters which aligned with our biological and mathematical intuition.

```
[1]: import numpy as np
import matplotlib.pyplot as plt
```

1.2 Symbolic Calculation

Here is a Calculation of Jacobian of the system without numerical computation. Using Python Symbolic library.

```
[2]: import sympy as sp

a, s, r, p = sp.symbols('a s r p')
mu, eta_r, eta_s, alpha, beta, gamma = sp.symbols('mu eta_r eta_s alpha beta_γgamma')

vars_params = [a, s, r, p, alpha, beta, gamma, eta_s, eta_r, mu]

def f(x):
    return x - x**2 + 3/4

n = s + r

f1 = mu * (1 - a)
f2 = eta_s*(1 - n)*s - alpha * a * s - (beta * s * r)/n - gamma * s * p
f3 = eta_r*(1 - n)*r + (beta * s * r)/n - gamma * r * p
f4 = p * ( f(n) - p )

dyn = sp.Matrix([f1,f2,f3,f4])

J = dyn.jacobian([a,s,r,p])
```

```
[3]: ## for E1
li = [1 , 0, 0 , f(0) , 1,0.1, 1,1,0.3,3]

dic = dict(zip(vars_params, li))

res = J.subs(dic).evalf()
res
```

```
[3]: 
$$\begin{bmatrix} -3.0 & 0 & 0 & 0 \\ 0 & -0.75 & -0.1 & 0 \\ 0 & 0 & -0.35 & 0 \\ 0 & 0.75 & 0.75 & -0.75 \end{bmatrix}$$

```

```
[4]: from scipy.integrate import odeint

def evaluate_dyn(y , t, alpha, beta, gamma, eta_s, eta_r, mu) -> tuple:
    a, s, r, p = y
    state_vars = [a, s, r, p , alpha, beta, gamma, eta_s, eta_r, mu]
    dic = dict(zip(vars_params, state_vars))
    res = dyn.subs(dic).evalf()
    res = sp.matrix2numpy(res, dtype=np.float64)
    return res.flatten()

parset = (1, 0.1, 1, 1, 0.3, 3)

y0 = np.repeat(0.5, 4)

t = np.linspace(0, 10, 1000)

sol = odeint(evaluate_dyn, y0, t, args=parset)
```

```
[5]: fig, ax = plt.subplots(figsize=(10, 6))

variable_names = ['Antibiotic (a)', 'NARB (s)', 'ARB (r)', 'Immune System (p)']
colors = ['#1f77b4', '#ff7f0e', '#2ca02c', '#d62728'] # Professional color
↪ scheme

for i in range(4):
    ax.plot(t, sol[:, i],
            color=colors[i],
            linewidth=2,
            label=variable_names[i])

ax.set_title('Approching E1', fontsize=14)
ax.set_xlabel('Time', fontsize=12)
ax.set_ylabel('System Variables', fontsize=12)
ax.grid(True, linestyle='--', alpha=0.7)
ax.legend(loc='best')
```

```
plt.tight_layout()
plt.show()
```

