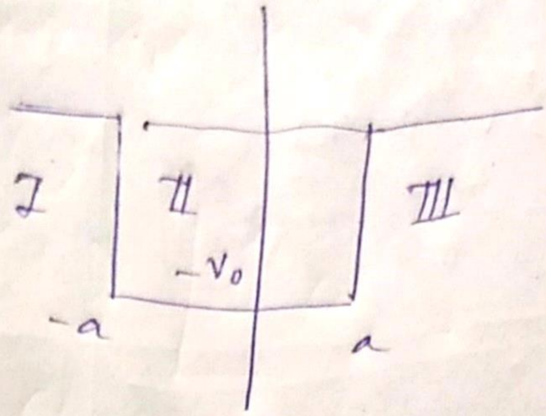
Region I

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$$



$$\Rightarrow \frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi$$

$$= k^2\psi$$

$$k = \sqrt{\frac{-2mE}{\hbar^2}} = \sqrt{\frac{2m|E|}{\hbar^2}}$$

$$\psi_1 = A_1 e^{kx} + B_1 e^{-kx}.$$

when $x \to -\infty$, $\psi_1 \to 0$

$\Rightarrow B_1 = 0$

$\therefore \psi_1 = A_1 e^{kx}$

Region II

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} - V_0\psi = E\psi$$

$$\Rightarrow \frac{d^2\psi}{dx^2} = -\frac{2m(E+V_0)}{\hbar^2}\psi$$

$$= -\frac{2m(V_0-|E|)}{\hbar^2}\psi$$

$$= -l^2\psi$$

$$l = \frac{\sqrt{2m(V_0-|E|)}}{\hbar}$$

$$\psi_{II} = C_1 e^{-ilx} + C_2 e^{ilx}.$$

$$= C_1\cos(lx) + C_2\sin(lx)$$

Considering even solutions:

$$\psi_{II} = C_1\cos(lx)$$

Since ψ and $\frac{d\psi}{dx}$ are continuous at $-a$ (and a)

$\psi_I(-a) = \psi_{II}(-a)$

$\Rightarrow A_1 e^{-ka} = C_1 \cos(la)$ —(i)

$\psi_I'(-a) = \psi_{II}'(-a)$

$\Rightarrow k A_1 e^{ka} = +C_1 l \sin(la)$ —(ii)

from (i) and (ii)

$l \tan(la) = k$

$\Rightarrow la \tan(la) = ka$

let ~~y=la~~ $y = la$, $x = ka$

$\therefore$ ~~y tan(la~~ $y \tan y = x$ —(iii)

$$l^2 = \frac{2mV_0}{\hbar^2} - \frac{2m|E|}{\hbar^2}$$

multiply by a^2

$$\Rightarrow \cancel{a^2}\, a^2 l^2 = \frac{2mV_0}{\hbar^2}a^2 - \frac{2m|E|}{\hbar^2}a^2$$

$$\Rightarrow$$

$$= \frac{2mV_0}{\hbar^2}a^2 - k^2a^2$$

$$\Rightarrow y^2 + x^2 = \frac{2mV_0}{\hbar^2}a^2 = \lambda \text{ (say)} \quad \text{—(iv)}$$

putting (iii) in (iv)

$$y^2 + y^2\tan^2 y = \lambda$$

$$\Rightarrow \boxed{y^2 \sec^2 y = \lambda}$$ solved this using root finding method. (line 16)

let solution be y_0

$$y_0^2 = l^2a^2 = \frac{2mV_0a^2}{\hbar^2} - \frac{2m|E|}{\hbar^2}$$

set $m=1$ and $\hbar = 1$

$$y_0^2 = 2V_0a^2 - 2|E|$$

$$\Rightarrow \boxed{|E| = \frac{1}{2}(2V_0a^2 - y_0^2)}$$ (line 163)

from (i)

$A_1 e^{-\kappa a} = C_1 \cos(la)$

$\Rightarrow \frac{A_1}{C_1} = e^{\kappa a}\cos(la)$

$= e^{x_0}\cos(y_0)$ [$x_0 = y_0 \tan y_0$ from (iii)]

$\hookrightarrow$ line (164)

$\psi_1 = A_1 e^{\kappa x}$

$\psi_{11} = C_1 \cos(lx)$

$\psi_{111} = A_1 e^{-\kappa x}$

$$\int_{-\infty}^{-a} \psi_1^2\,dx + \int_{-a}^{a} \psi_{11}^2\,dx + \int_{+a}^{\infty} \psi_{111}^2\,dx = 1$$

$$\Rightarrow \int A_1^2 e^{2\kappa x}\,dx + \int C_1^2 \cos^2 lx\,dx + \int A_1^2 e^{-2\kappa x} = 1$$

$$\Rightarrow C_1^2\left[\frac{A_1^2}{C_1^2}\left(\underbrace{\int_{-\infty}^{-a} e^{2\kappa x}\,dx}_{I_1} + \underbrace{\int_{a}^{\infty} e^{-2\kappa x}\,dx}_{I_2}\right) + \underbrace{\int_{-a}^{a} \cos^2 lx\,dx}_{I_3}\right] = 1$$

find I_1, I_2, I_3 using numerical integration.

$\hookrightarrow$ line 170

C_1 is found, $\frac{A_1}{C_1}$ is known

$\Rightarrow A_1$ is found.