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1 Random Signals_{Chapter 13 S713-719}

probability distribution	$F_x(\alpha) = P(x \leqslant \alpha)$
probability density	$f_x(\alpha) = \frac{d}{d\alpha} F_x(\alpha)$
mean / expected value	$E(x) = \sum_{k} \alpha_k P(x = \alpha_k) = \int_{-\infty}^{\infty} \alpha \cdot f_x(\alpha) d\alpha$
	$E(y) = E(g(x)) = \int_{-\infty}^{\infty} g(\alpha) f_x(\alpha) d\alpha$
signal power	$E(x^2) = \sum_{k} \alpha_k^2 P(x = \alpha_k) = \int_{-\infty}^{\infty} \alpha^2 f_x(\alpha) d\alpha$
average absolute value	$E(x^2) = \sum_{k} \alpha_k^2 P(x = \alpha_k) = \int_{-\infty}^{\infty} \alpha^2 f_x(\alpha) d\alpha$ $E(x) = \int_{-\infty}^{\infty} \alpha f_x(\alpha) d\alpha = \int_{0}^{\infty} \alpha [f_x(\alpha) + f_x(-\alpha)] d\alpha$
variance σ^2	$\sigma_x^2 = E\{[x - E(x)]^2\} = E(x^2) - E(x)^2 = \int_{-\infty}^{\infty} [\alpha - E(x)]^2 f_x(\alpha) d\alpha$
Joint distribution function	$F_{x(1),x(2)}(\alpha_1,\alpha_2) = Pr\{x(1) \le \alpha_1, x(2) \le \alpha_2\}$
Joint density function	$f_{x(1),x(2)}(\alpha_1,\alpha_2) = \frac{\partial^2}{\partial \alpha_1 \partial \alpha_2} F_{x(1),x(2)}(\alpha_1,\alpha_2)$
covariance	$c_{xy} = Cov(x, y) = E\{(x - E(x))(y - E(y))^*\} = E(xy^*) - E(x)E(y^*)$
correlation	$r_{xy} = E(xy^*)$ $r_{xy} = 0 \rightarrow \text{orthogonal}; \text{ if zero mean } \rightarrow \text{uncorrelated}$
correlation coefficient	$\rho_{xy} = \frac{c_{xy}}{\sigma_x \sigma_y} - 1 \leqslant \rho_{xy} \leqslant 1$ for $\rho_{xy} = 0 \to \text{uncorrelated}$
autocorrelation function	$R_{XX}(k) = E(x(n+k)x(n))$ $\hat{R}_{XX}(k) = \frac{1}{N} \sum_{n=0}^{N-1-k} x(n+k)x(n)$ (sample autocorrelation) $R_{XX}(k) = R_{XX}(-k)$
power	$\sigma_x^2 = R_{XX}(0) = E(x(n)^2) = \int_{-\pi}^{\pi} S_{XX}(\omega) \frac{d\omega}{2\pi}$
	$\sigma_y^2 = \int_{0}^{\pi} S_{YY}(\omega) \frac{d\omega}{2\pi} = \sigma_x^2 \cdot \int_{0}^{\pi} H(\omega) ^2 \frac{d\omega}{2\pi}$
power spectrum	$S_{XX}(\omega) = \sum_{k=-\infty}^{\infty} R_{XX}(k)e^{-j\omega k} = \lim_{N\to\infty} E(\hat{S}_{XX}(\omega)) \qquad \omega = \frac{2\pi f}{f_s}$ $S_{XX}(\omega) = H(\omega) ^2 S_{XX}(\omega)$
	$SYY(\omega) = II(\omega) SXX(\omega)$
periodogram spectrum	$\hat{S}_{XX}(\omega) = \frac{1}{N} X_N(\omega) ^2 \text{with } X_N(\omega) = \sum_{n=0}^{N-1} x(n)e^{-j\omega n}$
	$\hat{S}(\omega) = \frac{1}{K} [\hat{S}_1(\omega) + \dots + \hat{S}_K(\omega)] = \frac{1}{KN} [X_1(\omega) ^2 + \dots + X_K(\omega) ^2]$ (with x_i as block of length N)
Noise reduction ratio (NRR)	$NRR = \frac{\sigma_y^2}{\sigma_x^2} = \int_{-\pi}^{\pi} H(\omega) ^2 \frac{d\omega}{2\pi} = \sum_{n} h(n)^2$

2 Sampling and Reconstruction Chapter 1

2.1 Analog Signals 52

$$\Omega = 2\pi f$$
 $\left[\Omega\right] = \frac{rad}{s}$ $\omega = \Omega T = \frac{2\pi f}{f_s}$ $\left[\omega\right] = \frac{rad}{sample}$

Fourier transform	$X(\Omega) = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t}dt$
inverse Fourier transform	$x(t) = \int_{-\infty}^{\infty} X(\Omega) e^{j\Omega t} \frac{d\Omega}{2\pi}$

2.2 Digital Signals

2.2.1 Sampling Theorem 54-6

Sampling means that the analog signal is periodically measured with a sampling interval T. The discrete index n, relates to the time t as follows:

$$t = nT$$
 $n = 0, 1, 2, \dots$

The sampling frequency relates to the sampling interval as follows:

$$f_s = \frac{1}{T}$$

Nyquist interval:

Sampling Theorem (Nyquist rate):

 $f_s \geqslant 2f_{max}$ or $T \leqslant \frac{1}{2f_{max}}$

$$\left[-\frac{f_s}{2}, \frac{f_s}{2}\right]$$

2.2.2 DSP Frequency Units \$29-30

A sampled sinusoid takes the form in these units:

$$e^{2\pi jfTn} = e^{2\pi j(f/f_s)n} = e^{j\Omega Tn} = e^{j\omega n}$$

$$-f_s/2 \qquad 0 \qquad f_s/2$$

$$-f_s/2 \qquad 0 \qquad f/s/2$$

$$-1/2 \qquad 0 \qquad 1/2$$

$$-\pi \qquad 0 \qquad \pi$$

$$0 \qquad \pi$$

$$-\pi \qquad 0 \qquad \pi$$

$$-\pi \qquad 0 \qquad \pi$$

$$0 \qquad \pi \qquad \Omega = 2\pi f/f_s \qquad [radians/sample]$$

$$-\pi \qquad 0 \qquad \pi$$

2.2.3 Flat-top sampling S30

In practical sampling each sample is held for a short period of time (τ) .

$$x_{flat}(t) = \sum_{n=-\infty}^{\infty} x(nT)p(t-nT)$$
 $p(t)$: flat-top pulse with duration τ

This is equivalent to filtering the perfectly sampled signal \hat{x} with a linear filter with the impulse response p(t). The spectrum of the filter looks like this

$$|P(f)| = \tau \left| \frac{\sin(\pi f \tau)}{\pi f \tau} \right|$$

2.2.4 Discrete-Time Fourier Transform (DTFT) 531

$$\hat{X}(f) = \sum_{n = -\infty}^{\infty} x(nT)e^{-2\pi jfTn}$$

$$x(nT) = \frac{1}{f_s} \int_{-f_s/2}^{f_s/2} \hat{X}(f) e^{2\pi j f T n} df = \int_{-\pi}^{\pi} \hat{X}(\omega) e^{j\omega n} \frac{d\omega}{2\pi}$$

A sampled signal has always a periodic spectrum, with its spectrum center at the multiplies of the sampling frequency.

$$\hat{X}(f) = \frac{1}{T} \sum_{m = -\infty}^{\infty} X(f - mf_s) \qquad T\hat{X}(f) = X(f) \quad \text{for } -\frac{f_s}{2} \leqslant f \leqslant \frac{f_s}{2}$$

2.2.5 Aliasing **S10**, 38

If the signal frequency f is outside the Nyquist interval, the signal will be aliased with $f \pm n \cdot f_{sampling}$.

Example: $sin(8\pi t)$ (signal frequency f=4) sampled at a rate of $f_s=5Hz$ will be aliased to $sin(2\pi(f-f_s)t)=sin(2\pi(-1)t)$

$$f_{ia} = f_i + nf_s$$

n must be selected such that f_{ia} is in the Nyquist intervall. n can also be negative.

Is the signal frequency f inside the Nyquist interval $[-f_s/2, +f_s/2]$, no aliasing will be perceived.

2.2.6 Antialiasing Prefilter S38

The stop frequency from the antialiasing prefilter is defined as:

$$f_{stop} = f_s - f_{pass}$$

The attenuation of the antialiasing prefilter is:

$$A(f) = -20 \cdot \log_{10} \left| \frac{H(f)}{H(f_0)} \right| \qquad [dB]$$

$$A_{dB} = \alpha \log_{10} \left(\frac{f}{f_{cutoff}} \right) \qquad \alpha = \frac{dB}{dek}$$

$$A_{dB} = \beta \log_2 \left(\frac{f}{f_{cutoff}} \right) \qquad \beta = \frac{dB}{okt}$$

$$A_X(f) = \underbrace{A(f)}_{Prefilter} + \underbrace{A_{X_{in}}(f)}_{Input spectrum}$$

transition region $f_{s/2}$ $f_{s/2}$ $f_{s/2}$ $f_{s/2}$ f_{pass} stopband

passband

stopband

stopband

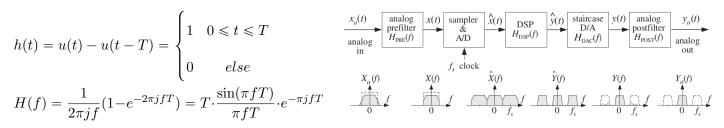
 f_0 : reference frequency (typ. DC)

2.3 Reconstructors S42

2.3.1 Ideal reconstructor S43

$$H(f) = \begin{cases} T & |f| \leq \frac{f_s}{2} \\ 0 & else \end{cases} \qquad h(t) = \frac{\sin(\pi f_s t)}{\pi f_s t} \qquad y(t) = \sum_{n = -\infty}^{\infty} y(nT)h(t - nT)$$

2.3.2 Staircase reconstructor S45



The spectral images at higher frequencies are not well suppressed, therefore an anti-image postfilter is needed.

3 Quantization_{Chapter 2}

3.1 Quantization process S62

R	full-scale range	$R = Q \cdot 2^B$
B	bits	$B = log_2\left(rac{R}{Q} ight)$
Q	quantization width	$Q = \frac{R}{2^B}$
e	quantization error (quantization noise)	$e_Q(nT) = x_Q(nT) - x(nT)$
e_{RMS}	root-mean-square error	$e_{RMS} = \frac{Q}{\sqrt{12}}$
r	dynamic range	$r = 20log_{10}\left(2^B\right) \approx 6dB \cdot B$
SNR	signal-to-noise ratio (with uniform white noise)	$SNR = 20log_{10}\left(\frac{R}{Q}\right) = 6B dB$
σ_e^2	average power / variance of quantization error	$\sigma_e^2 = E[e^2(n)] = \frac{Q^2}{12}$

3.2 Oversampling and noise shaping \$66-70

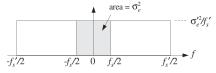
L	oversampling ratio	$L = \frac{f'_s}{f_s}$ with f'_s as higher sampling rate
ΔB	saved bits without noise shaping	$\Delta B = 0.5 \cdot log_2(L)$
	saved bits with noise shaping	$\Delta B = (p+0.5) \cdot log_2(L) - 0.5 \cdot log_2\left(\frac{\pi^{2p}}{2p+1}\right)$
		$L = \left(\frac{2^{2\Delta B} \pi^{2p}}{2p+1}\right)^{\frac{1}{2p+1}}$

p = order of the noise shaping filter

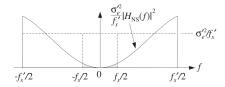
Performance of oversampling noise shaping quantizers:

p	L	4	8	16	32	64	128
0	$\Delta B \approx 0.5 \cdot log_2(L)$	1.0	1.5	2.0	2.5	3.0	3.5
1	$\Delta B \approx 1.5 \cdot log_2(L) - 0.86$	2.1	3.6	5.1	6.6	8.1	9.6
2	$\Delta B \approx 2.5 \cdot log_2(L) - 2.14$	2.9	5.4	7.9	10.4	12.9	15.4
3	$\Delta B \approx 3.5 \cdot log_2(L) - 3.55$	3.5	7.0	10.5	14.0	17.5	21.0
4	$\Delta B \approx 4.5 \cdot log_2(L) - 5.02$	4.0	8.5	13.0	17.5	22.0	26.5
5	$\Delta B \approx 5.5 \cdot log_2(L) - 6.53$	4.5	10.0	15.5	21.0	26.5	32.0

Oversampled quantization noise power, without noise shaping.



Spectrum of oversampling **noise** shaping quantizer.



3.3 D/A converters \$71-73

Name	Output calculation	Min	Max
natural binary	$x_Q = R(b_1 2^{-1} + b_2 2^{-2} + \dots + b_B 2^{-B})$	0	R-Q
offset binary	$x_Q = R(b_1 2^{-1} + b_2 2^{-2} + \dots + b_B 2^{-B} - 0.5)$	$-\frac{R}{2}$	$\frac{R}{2}-Q$
two's complement	$x_Q = R(\overline{b_1}2^{-1} + b_22^{-2} + \dots + b_B2^{-B} - 0.5)$ $(\overline{b_1} = 1 - b_1)$	$-\frac{R}{2}$	$\frac{R}{2}-Q$

3.4 A/D converters S75

3.4.1 Successive Approximation Converters \$76

Conversion algorithms				
Natural and offset binary	Two's complement			
if mode = rounding: $y = x + \frac{Q}{2}$ else if mode = truncation: y = x for each x to be converted, do: initialize $\mathbf{b} = [0, 0, \dots, 0]$ for $\mathbf{i} = 1, 2, \dots, B$ do: $b_i = 1$ $x_Q = \operatorname{dac}(\mathbf{b}, B, R)$ $b_i = u(y - x_Q)$	if mode = rounding: $y = x + \frac{Q}{2}$ else if mode = truncation: $y = x$ for each x to be converted, do: initialize $\mathbf{b} = [0, 0, \dots, 0]$ $b_1 = 1 - u(y)$ for $\mathbf{i} = 2, 3, \dots, B$ do: $b_i = 1$ $x_Q = \operatorname{dac}(\mathbf{b}, B, R)$ $b_i = u(y - x_Q)$			

3.5 Analog and digital dither S84-86

Dither is a small white noise signal that is added to the input signal befor quantization. The variance of the noise is σ_v^2 and the variance of the quantization is σ_e^2 . The total variance is :

$$\sigma_{\epsilon}^2 = \sigma_e^2 + \sigma_v^2 = \frac{1}{12}Q^2 + \sigma_v^2$$

$$\sigma_{\epsilon}^2 = \begin{cases} \frac{Q^2}{12} & \text{undithered} \\ \frac{2Q^2}{12} & \text{rectangular dither} \\ \frac{3Q^2}{12} & \text{triangular dither} \\ \frac{4Q^2}{12} & \text{gaussian dither} \end{cases}$$

Goal of the dither is to eliminate quantization distortion and granulation and force the quantization error to look more like white noise.

4 Discrete-Time Systems_{Chapter 3}

4.1 Linearity and time invariance $_{5100-102}$

Linearity: $x(n) = ax_1(n) + bx_2(n)$ $\underline{\underline{H}}$ $y(n) = ay_1(n) + by_2(n)$

Time invariance: If x(n) \underline{H} y(n) then $x(n+\delta)$ \underline{H} $y(n+\delta)$

testing: $x_D(n) = x(n-D)$ and $y_D(n) = y(n-D)$

4.2 Impulse response \$103-105

LTI form: $y(n) = \sum_{m} x(m)h(n-m)$

direct form: $y(n) = \sum_{m} h(m)x(n-m)$

4.3 Finite and Infinite Impulse Response filters \$105,106

M	filter order	
h	filter impulse response	$\{h_0, h_1, h_2, \dots, h_M, 0, 0, \dots\}$
L_h	length of h	$L_h = M + 1$
FIR	FIR filtering equation	$y(n) = \sum_{m=0}^{M} h(m)x(n-m)$
IIR	IIR filtering equation	$y(n) = \sum_{m=0}^{\infty} h(m)x(n-m)$

4.4 Causality S112

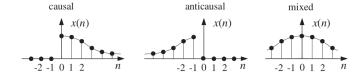
causal right sided signals, they are

non-zero for n >= 0

anticausal left sided signals, they are

non-zero for n <= -1

mixed signals double-sided signals



4.4.1 Anticausal to Causal S113

Delay the system by D to move the negative time from n=-D to 0

$$h_D(n) = h(n - D)$$

$$y(n) = \sum_m h(m)x(n - m)$$

$$y_D(n) = \sum_m h_D(m)x(n - m) = \sum_m h(m - D)x(n - m)$$

$$\xrightarrow{m := k + D} \sum_k h(k)x(n - k - D) = y(n - D)$$

4.5 Stability S115

Stability Condition $\sum\limits_{n=-\infty}^{\infty}|h(n)|<\infty$

An LTI system is stable, if a bounded input can only generate bounded outputs. Always prefer stability over causality!

5 FIR Filtering and Convolution_{Chapter 4}

5.1 Block Processing Methods

5.1.1 Convolution S121

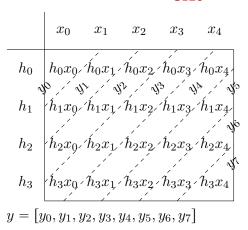
duration of data record
$$T_L = LT$$
 Signal Samples $L = T_L f_s$ $x(n)$ $n = 0, 1, ..., L-1$ sampling Time T

y(n)	direct and LTI forms of convolution	$y(n) = \sum_{m} h(m)x(n-m) = \sum_{m} x(m)h(n-m)$
y(n)	convolution table form	$y(n) = \sum_{\substack{i,j\\i+j=n}} h(i)x(j)$

5.1.2 Direct Form S123

h	$h = [h_0, h_1, \dots, h_M]$
L_h	$L_h = M + 1$
L_x	$L_x = L$
L_y	$L_y = L + M = L_x + L_h - 1$
y(n)	$y(n) = \sum_{m=max(0,n-L+1)}^{min(n,M)} h(m)x(n-m)$

5.1.3 Convolution Table \$126



5.1.4 LTI Form S127

	h_0	$\stackrel{ }{\downarrow} h_1$	h_2	h_3	0	0	0	0
x_0	x_0h_0	x_0h_1	x_0h_2	x_0h_3	0	0	0	0
x_1	0	$x_1 h_0$	x_1h_1	x_1h_2	$x_1 h_3$	0	0	0
x_2	0	0	x_2h_0	x_2h_1	x_2h_2	x_2h_3	0	0
x_3	0	0	0	x_3h_0	$x_{3}h_{1}$	x_3h_2	x_3h_3	0
x_4	0	0	0	0	x_4h_0	x_4h_1	x_4h_2	x_4h_3
y_n	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7

$$y(n) = \sum_{m=max(0,n-M)}^{min(n,L-1)} x(m)h(n-m)$$

5.1.5 Matrix Form **5129**

The convolutional equations can also be written in the linear matrix form:

$$y = Hx$$
 or $y = Xh$

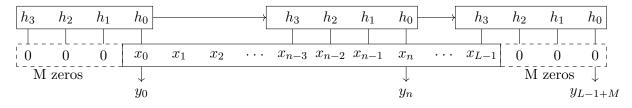
where H (Toeplitz Matrix) is built out of the filter's impulse response h or the signal matrix X is built out of the input signal. The filter matrix H, respectively the signal matrix X, must be rectangular with dimensions

$$\underbrace{L_y \times L_x = (L+M) \times L}_{\text{dimension of H}} \qquad \text{or} \qquad \underbrace{L_y \times L_h = (L+M) \times (M+1)}_{\text{dimension of X}}$$

Example:

$$y = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{bmatrix} = \begin{bmatrix} h_0 & 0 & 0 & 0 & 0 \\ h_1 & h_0 & 0 & 0 & 0 \\ h_2 & h_1 & h_0 & 0 & 0 \\ h_3 & h_2 & h_1 & h_0 & 0 \\ 0 & h_3 & h_2 & h_1 & h_0 \\ 0 & 0 & h_3 & h_2 & h_1 \\ 0 & 0 & 0 & h_3 & h_2 \\ 0 & 0 & 0 & 0 & h_3 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = Hx$$

5.1.6 Flip-and-Slide Form S131



$$y(n) = h_0 x_n + h_1 x_{n-1} + \dots + h_M x_{n-M}$$

5.1.7 Overlap-Add Form **S143.144**

- 1. Divide input x into smaller blocks x_0, x_1, \ldots of length L. If the input is not long enough for a last complete block, the last block is filled up with zeros
- 2. Calculate the output of the convolution of block x_0 with h, resulting in y_0
- 3. Repeat step 2 for all blocks, resulting in y_1, y_2, \ldots
- 4. Add y_0, y_1, \ldots up using the following table. y_{n+1} is always moved to the right with an offset of L compared to y_n .

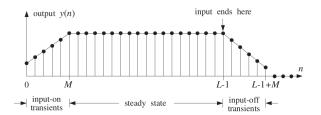
example: with L=3 and M=3

with
$$L = 3$$
 and $M = 3$
$$y_0 = h * x_0 y_1 = h * x_1 y_2 = h * x_2 x = \underbrace{n_0, n_1, n_2}_{x_0}, \underbrace{n_3, n_4, n_5}_{x_2}, \underbrace{n_6, n_7, n_8}_{x_2}$$

n	0	1	2	3	4	5	6	7	8	9	10	11
y_0	$y_{0,0}$	$y_{0,1}$	$y_{0,2}$	$y_{0,3}$	$y_{0,4}$	$y_{0,5}$						
y_1				$y_{1,0}$	$y_{1,1}$	$y_{1,2}$	$y_{1,3}$	$y_{1,4}$	$y_{1,5}$			
y_2							$y_{2,0}$	$y_{2,1}$	$y_{2,2}$	$y_{2,3}$	$y_{2,4}$	$y_{2,5}$
y	$\sum n_0$	$\sum n_1$	$\sum n_2$	Σn_3	Σn_4	Σn_5	Σn_6	Σn_7	Σn_8	Σn_9	Σn_{10}	Σn_{11}

5.1.8 Transient and Steady-State Behaviour \$132.133

input-on transient	$0 \leqslant n < M$
steady state	$M \leqslant n \leqslant L - 1$
input-off transient	$L - 1 < n \leqslant L - 1 + M$



Therefore, the direct form takes the following different forms depending on the value of the output index n:

$$y_n = \begin{cases} \sum_{m=0}^n h_m x_{n-m} & \text{if } 0 \leqslant n < M & \text{input-on} \\ \sum_{m=0}^M h_m x_{n-m} & \text{if } M \leqslant n \leqslant L-1 & \text{steady state} \\ \sum_{m=n-L+1}^M h_m x_{n-m} & \text{if } L-1 < n \leqslant L-1+M & \text{input-off} \end{cases}$$

The DC gain of a stable filter is the steady-state value to which the output converges when the input is a unit step

$$y_{dc} = \sum_{m} h(m)$$
 $y_{dc} = \sum_{m=0}^{\infty} h(m)$

5.1.9 Convolution of Infinite Sequences \$134

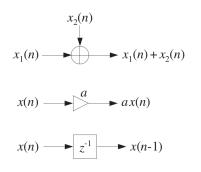
Three cases:

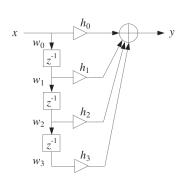
- 1. Infinite filter, finite input; i.e., $M = \infty$, $L < \infty$
- 2. Finite filter, infinite input; i.e., $M < \infty$, $L = \infty$
- 3. Infinite filter, infinite input; i.e., $M = \infty$, $L = \infty$

Therefore, the direct form takes the following different forms (See also 6.2 Region of Convergence (ROC) S186):

$$y_n = \begin{cases} \sum_{m=max(0,n-L+1)}^{n} h_m x_{n-m} & \text{if } M = \infty, L < \infty \\ \sum_{m=0}^{min(n,M)} h_m x_{n-m} & \text{if } M < \infty, L = \infty \\ \sum_{m=0}^{n} h_m x_{n-m} & \text{if } M = \infty, L = \infty \end{cases}$$

5.2 Sample Processing Methods \$146





- 5.2.1 Pure Delays S147-151
- 5.2.2 FIR Filtering in Direct Form \$152-156
- 5.2.3 Hardware realizations and circular buffers \$162

6 z-Transform_{Chapter 5}

6.1 Basic Properties \$183

$$X(z) = \sum_{n = -\infty}^{\infty} x(n)z^{-n}$$

linearity	$a_1x_1(n) + a_2x_2(n)$	\xrightarrow{Z}	$a_1 X_1(z) + a_2 X_2(z)$
delay	x(n-D)	\xrightarrow{Z}	$z^{-D}X(z)$
convolution	y(n) = h(n) * x(n)	\xrightarrow{Z}	Y(z) = H(z)X(z)
modulation	$a^ng(n)$	\xrightarrow{Z}	$G(\frac{z}{a})$
time inversion	g(-n)	\xrightarrow{Z}	$G(z^{-1})$

6.2 Region of Convergence (ROC) \$186

$$\{z \in \mathbb{C} \mid X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \neq \pm \infty\}$$

infinite geometric series 1	$1 + x + x^2 + x^3 + \dots = \sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$	for $ x < 1$
infinite geometric series 2	$x + x^2 + x^3 + \dots = \sum_{m=1}^{\infty} x^m = \frac{x}{1-x}$	for $ x < 1$

If there is no ROC specified, we assume that the system is causal.

6.3 Causality and Stability \$193

causal signals	$ROC z > \max_{i} p_i $
mixed signals	$ROC \min_{i} p_i < z < \max_{i} p_i $
anticausal signals	$ROC z < \min_{i} p_i $
stable signals	$\{z (z = 1)\} \in ROC$

For a signal or system to be **simultaneously stable and causal**, it is necessary that all its poles lie strictly **inside** the unit circle in the z-plane.

$$1 > \max_{i} |p_i|$$

A signal or system can also be **simultaneously stable and anticausal**, but in this case all its poles must lie strictly **outside** the unit circle.

$$1 < \min_{i} |p_i|$$

Marginally stable signals have poles, that fall exactly onto the unit circle!

6.4 Frequency Spectrum \$196-210

$$z=e^{j\omega} \qquad \omega=\frac{2\pi f}{f_s}$$

$$X(\omega)=\sum_{n=-\infty}^{\infty}x(n)e^{-j\omega n} \qquad \text{(DTFT)}$$

$$H(\omega)=\sum_{n=-\infty}^{\infty}h(n)e^{-j\omega n} \qquad \text{(frequency response)}$$

$$H(\omega)=H(z)|_{z=e^{j\omega}} \qquad -\pi\leqslant\omega\leqslant\pi \qquad \text{nyquist interval}$$

$$x(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi}X(\omega)e^{j\omega n}d\omega \qquad \text{(inverse DTFT)}$$

Another useful relationship is Parseval's equation, which relates the total energy of a sequence to its spectrum:

$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$$
 (Parseval)

For real valued discrete time sequences:

$$X(\omega)^* = X(-\omega)$$
$$|X(\omega)| = |X(-\omega)|$$
$$argX(\omega) = -argX(-\omega)$$

Some DTFT-Transforms:

$\delta[n]$	$X_{2\pi}(\omega) = 1$
$\delta[n-M]$	$X_{2\pi}(\omega) = e^{-i\omega M}$
u[n]	$X(\omega) = \frac{1}{1 - e^{-i\omega}} + \pi \cdot \delta(\omega)$
$e^{-i\omega_0 n}$	$X(\omega) = 2\pi \cdot \delta(\omega + \omega_0), -\pi \leqslant \omega_0 < \pi$
$\cos(\omega_0 n)$	$X(\omega) = \pi [\delta(\omega + \omega_0) + \delta(\omega - \omega_0)], -\pi < \omega_0 < \pi$
$\sin(\omega_0 n)$	$X(\omega) = -j\pi\delta(\omega - \omega_0) + j\pi\delta(\omega + \omega_0)$

Some z-Transforms:

x(n)	$\mathbf{X}(\mathbf{z})$	ROC
u(n)	$\frac{1}{1-z^{-1}}$	z > 1
-u(-n-1)	$\frac{1}{1-z^{-1}}$	z < 1
$(-1)^n u(n)$	$\frac{1}{1+z^{-1}}$	z > 1
$-(-1)^n u(-n-1)$	$\frac{1}{1+z^{-1}}$	z < 1
$a^n u(n)$	$\frac{1}{1-az^{-1}}$	z > a (causal)
$-a^n u(-n-1)$	$\frac{1}{1-az^{-1}}$	z < a (anticausal)
$u(n)e^{\alpha n}$	$\frac{1}{1 - e^{\alpha} z^{-1}}$	$ z > e^{\alpha} $
$u(n)\cos(\omega n)$	$\frac{1 - \cos(\omega)z^{-1}}{1 - 2\cos(\omega)z^{-1} + z^{-2}} = \frac{1}{2} \left[\frac{1}{1 - e^{j\omega}z^{-1}} + \frac{1}{1 - e^{-j\omega}z^{-1}} \right]$	z > 1
$u(n)\sin(\omega n)$	$\frac{\sin(\omega)z^{-1}}{1 - 2\cos(\omega)z^{-1} + z^{-2}} = \frac{1}{2j} \left[\frac{1}{1 - e^{j\omega}z^{-1}} - \frac{1}{1 - e^{-j\omega}z^{-1}} \right]$	z > 1
$A\delta(n)$	A	all z
$A\delta(n-D)$	Az^{-D}	$z \neq 0$

6 z-Transform_{Chapter 5}

6.5 Inverse z-Transform S202-204

$$X(z) = \frac{N(z)}{D(z)} = \frac{N(z)}{(1 - p_1 z^{-1})(1 - p_2 z^{-1})\dots(1 - p_M z^{-1})} = A_0 + \frac{A_1}{1 - p_1 z^{-1}} + \dots + \frac{A_M}{1 - p_M z^{-1}}$$
 with $A_0 = X(z)|_{z=0}$ otherwise $z = p_i$

Partial fraction expansion S203: for Order of $N(z) \leq$ Order of D(z)

$$A_i = [(1 - p_i z^{-1}) X(z)]_{z=p_i} = \left[\frac{N(z)}{\prod_{j \neq i} (1 - p_j z^{-1})} \right]_{z=p_i}$$

for $A_0 \to z = 0$

Euler:

$$\cos(\alpha) = \frac{e^{j\alpha} + e^{-j\alpha}}{2} \qquad \sin(\alpha) \qquad = \frac{e^{j\alpha} - e^{-j\alpha}}{2j} \qquad e^{\pm jn\omega} = \cos n\omega \pm j \sin n\omega$$

$$e^{j\frac{\pi}{2}n} = j^n \qquad e^{-j\frac{\pi}{2}n} \qquad = (-j)^n \qquad e^{jn\pi} = e^{jn\pi} = (-1)^n$$

$$\sqrt[n]{1} = e^{j \cdot 2\pi \frac{k}{n}} \quad k \in [1, n]$$

Complex valued poles: $(1 - ae^{j\omega}z^{-1})(1 - ae^{-j\omega}z^{-1}) = 1 - 2a\cos(\omega)z^{-1} + a^2z^{-2}$

7 Transfer Functions_{Chapter 6}

7.1 Equation description \$\sigma_{215,216}\$

 $H(z) = \frac{5+2z^{-1}}{1-0.8z^{-1}}$ transfer function:

 $h(n) = -2.5\delta(n) + 7.5(0.8)^n u(n)$ impulse response:

impulse response coefficient: $h(n) = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}$

difference equation: $h(n) = 0.8h(n-1) + 5\delta(n) + 2\delta(n-1)$

y(n) = 0.8y(n-1) + 5x(n) + 2x(n-1)I/O difference equation:

 $H(\omega) = \frac{5+2e^{-j\omega}}{1-0.8e^{-j\omega}}$ frequency response:

 $|H(\omega)| = \sqrt{H(\omega) \cdot H(\omega)^*}$ magnitude response:

7.2 IIR-Form: **S223,224**

$$H(z) = \frac{N(z)}{D(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_N z^{-N}}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_M z^{-M}}$$
 $a_0 = 1$ normalize to 1

if D(z) = 1, the IIR Form can be reduced to a FIR Filter:

$$H(z) = N(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_N z^{-N}$$

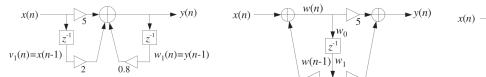
Example: find H(z) for h(n) = [1, 3, 4, 5]

$$H(z) = 1 + 3z^{-1} + 4z^{-2} + 5z^{-3}$$

Example: $y(n) = 0.25 \cdot y(n-2) + x(n)$ (I/O difference equation)

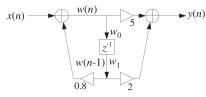
$$Y(z) = 0.25z^{-2}Y(z) + X(z)$$

direct form S217



$$\begin{array}{lll} H(z) = \frac{Y(z)}{X(z)} = \frac{5 + 2z^{-1}}{1 - 0.8z^{-1}} \\ Y(z)(1 - 0.8z^{-1}) &= X(z)(5 + & H(z) = \frac{Y(z)}{X(z)} = \frac{5 + 2z^{-1}}{1 - 0.8z^{-1}} \\ 2z^{-1}) & W(z) = \frac{1}{1 - 0.8z^{-1}} X(z) \\ Y(z) &= & W(z) = X(z) + 0.8z^{-1} W(z) \\ 5X(z) + 2z^{-1} X(z) + 0.8z^{-1} Y(z) & Y(z) = (5 + 2z^{-1}) W(z) \end{array}$$

canonical form $_{\mathbf{S220}}$



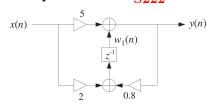
$$H(z) = \frac{Y(z)}{X(z)} = \frac{5+2z^{-1}}{1-0.8z^{-1}}$$

$$W(z) = \frac{1}{1-0.8z^{-1}}X(z)$$

$$W(z) = X(z) + 0.8z^{-1}W(z)$$

$$Y(z) = (5+2z^{-1})W(z)$$

transposed form S222



Transposition rules: replace adders by nodes, nodes by adders, reversing all flows and exchanging input with output

7.3 Steady state response \$229-232

$$\cos(\omega_0 n) \xrightarrow{H} |H(\omega_0)| \cos(\omega_0 n + \arg(H(\omega_0))) \qquad \sin(\omega_0 n) \xrightarrow{H} |H(\omega_0)| \sin(\omega_0 n + \arg(H(\omega_0)))$$

$$e^{j\omega_0 n} \xrightarrow{H} H(\omega_0) e^{j\omega_0 n} = |H(\omega_0)| e^{j\omega_0 n + j\arg(H(\omega_0))}$$

phase delay
$$d(\omega) = -\frac{\arg(H(\omega))}{\omega}$$
 $\arg H(\omega) = -\omega d(\omega)$
group delay $d_g(\omega) = -\frac{d}{d\omega}(\arg(H(\omega)))$

7.4 Transient Response S232

Input sine:
$$x(n) = e^{j\omega_0 n} \cdot u(n) \implies X(z) = \frac{1}{1 - e^{j\omega_0} z^{-1}}$$
 ROC $|z| > |e^{j\omega_0}| = 1$

time until the output is stable: $n_{eff} = \frac{\ln \epsilon}{\ln \rho}$ [sample] typ. $\epsilon = 1\%$

 $\rho = \max_{i} |p_i|$ $|p_i|$ is the magnitude of the pole

 $\tau = n_{eff} \cdot T$ time constant:

time constant:
$$H(\omega) = \frac{b}{1 - ae^{-j\omega}} \qquad \Rightarrow |H(\omega)| = \frac{b}{\sqrt{1 - 2a\cos(\omega) + a^2}}$$
$$\left|1 - ae^{-j\omega}\right| = \sqrt{1 - 2a\cos(\omega) + a^2}$$

7.5 Unit Step Response \$239

DC-Gain:
$$H(0) = H(z)|_{z=1} = \sum_{n=0}^{\infty} h(n)$$

AC-Gain:
$$H(\pi) = H(z)|_{z=-1} = \sum_{n=0}^{\infty} (-1)^n h(n)$$

7.6 Pole/Zero Design S242-258

7.6.1 First-Order Filters S242

Transfer function:
$$H(z) = \frac{G(1+bz^{-1})}{1-az^{-1}}$$
 $a = \epsilon^{1/n_{eff}}$

$$b \text{ can be calculated from:} \quad \frac{H(\pi)}{H(0)} = \frac{\text{AC Gain}}{\text{DC Gain}} \begin{cases} > 1 & HP \\ < 1 & LP \end{cases}$$
 $a,b \leqslant 1 \text{ and } G = \text{gain}$

7.6.2 2 pole conjugate filter S244-246

poles:

$$p = Re^{j\omega_0}$$

$$p = Re^{j\omega_0}$$
 and $p^* = Re^{-j\omega_0}$

Transfer function:

$$(1-Re^{-j\omega_0}z)$$

$$H(z) = \frac{G}{(1 - Re^{-j\omega_0}z^{-1})(1 - Re^{j\omega_0}z^{-1})} = \frac{G}{1 + a_1z^{-1} + a_2z^{-2}}$$

Parameter:

$$a_1 = -2R\cos(\omega_0) \qquad ; \qquad a_2 = R^2$$

$$a_2 = R^2$$

filter impulse Response

$$h(n) = \frac{G}{\sin(\omega_0)} R^n \sin(\omega_0 n + \omega_0)$$

$$G = (1 - R)\sqrt{1 - 2R\cos(2\omega_0) + R^2}$$
 only for $|H(\omega_0)| = 1$

3dB width

$$\Delta\omega \simeq 2(1-R)$$

 $\Delta\omega \simeq 2(1-R)$ =: R is the magnitude of the pole

full width at half maximum of the magnitude $|H(\omega)|^2 = \frac{1}{2}|H(\omega_0)|^2 = \frac{1}{2}$ squared response

7.6.3 2 pole 2 zero filter 5228.249

poles:

$$p = Re^{j\omega_0}$$

$$p = Re^{j\omega_0}$$
 and $p^* = Re^{-j\omega_0}$

zeros:

$$z_1 = re^{j\omega_0}$$

$$z_1 = re^{j\omega_0}$$
 and $z_1^* = re^{-j\omega_0}$

Transfer function:
$$H(z) = \frac{(1-re^{j\omega_0}z^{-1})(1-re^{-j\omega_0}z^{-1})}{(1-Re^{j\omega_0}z^{-1})(1-Re^{-j\omega_0}z^{-1})} = \frac{1+b_1z^{-1}+b_2z^{-2}}{1+a_1z^{-1}+a_2z^{-2}}$$

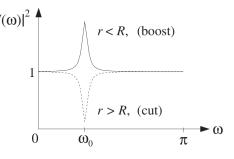
Parameter:

$$a_1 = -2R\cos(\omega_0) \qquad ; \qquad a_2 = R^2$$

$$a_2 = R$$

$$b_1 = -2r\cos(\omega_0) \qquad ; \qquad b_2 = r^2$$

$$b_2 =$$



7.6.4 Notch and Comb Filter S249-251

The zeros of the filters located on the unit circle and the poles are in the unit circle.

Transfer function: $H(z) = \frac{N(z)}{D(z)}$

$$H(z) = \frac{N(z)}{D(z)}$$

$$D(z) = N(\rho^{-1}z) = \prod_{i=1}^{M} (1 - e^{j\omega_i}\rho z^{-1})$$
 $\rho = \text{Radius}$

Notch filter:

$$N(z) = \prod_{i=1}^{M} (1 - e^{j\omega_i} z^{-1})$$

$$i=1$$

$$H(z) = \frac{N(z)}{(N\rho^{-1}z)} = \frac{1 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M}}{1 + \rho b_1 z^{-1} + \rho^2 b_2 z^{-2} + \dots + \rho^M b_M z^{-M}} \qquad \text{with } 0 < |\rho| < 1$$

with
$$0 < |\rho| < 1$$

$$_{
m mit}$$

$$a_i = \rho^i b_i$$
 mit $i = 1, 2, \dots, M$

Comb filter:

$$N(z) = \prod_{i=1}^{M} (1 - e^{j\omega_i} r z^{-1})$$

$$H(z) = \frac{N(r^{-1}z)}{N(\rho^{-1}z)} = \frac{1 + rb_1z^{-1} + \dots + r^Mb_Mz^{-M}}{1 - \rho b_1z^{-1} + \dots + \rho^Mb_Mz^{-M}} \quad \text{with } |r| < |\rho| < 1$$

with
$$|r| < |\rho| < 1$$

7.7 Deconvolution, Inverse Filters and Stability S254-259

$$H_{inv}(z) = \frac{1}{H(z)} = \frac{D(z)}{N(z)}$$

Deconvolution: $x(n) = h_{inv}(n) * y(n)$

$$\hat{x}(n) = h_{inv}(n) * y(n) = x(n) + \hat{\nu}(n)$$

Filtered noise: $\hat{\nu}(n) = h_{inv}(n) * \nu(n)$

$$\tilde{h}_{inv}(n) = \begin{cases} h_{inv}(n) & \text{if } n \ge -D \\ 0 & \text{if } n < -D \end{cases}$$

Because $H_{inv}(z)$ can have poles outside the unit circle, the stable inverse z-transform $h_{inv}(n)$ will necessarily be anticausal. To make a causal system, by clipping of the anticausal tail of the impulse response by a time n = -D and delayed by D time units.

y(n) is bounded by some maximal value $|y(n)| \leq A$, so the deconvolution error can be calculated by

$$|x(n) - \tilde{x}(n)| \le A \sum_{m=-\infty}^{-D-1} |h_{inv}(m)|$$

8 Idiotenseite

8.1 Funktionswerte für Winkelargumente

deg	rad	sin	cos	tan	deg	rad	sin	cos	deg	rad	sin	cos	deg	rad	sin	cos
0 °	0	0	1	0	90 °	$\frac{\pi}{2}$	1	0	180°	π	0	-1	270 °	$\frac{3\pi}{2}$	-1	0
30 °	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	120 °	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	210 °	$\frac{7\pi}{6}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	300 °	$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
45 °	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	135 °	$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	225 °	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	315 °	$\frac{7\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
60 °	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	150 °	$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	240°	$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	330 °	$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$

8.2 Periodizität

$$cos(a + k \cdot 2\pi) = cos(a)$$
 $sin(a + k \cdot 2\pi) = sin(a)$ $(k \in \mathbb{Z})$

8.3 Additionstheoreme

$$\sin(a \pm b) = \sin(a) \cdot \cos(b) \pm \cos(a) \cdot \sin(b)$$

$$\cos(a \pm b) = \cos(a) \cdot \cos(b) \mp \sin(a) \cdot \sin(b)$$

$$\tan(a \pm b) = \frac{\tan(a) \pm \tan(b)}{1 \mp \tan(a) \cdot \tan(b)}$$

8.5 Summe und Differenz

$$\begin{aligned} \sin(a) + \sin(b) &= 2 \cdot \sin\left(\frac{a+b}{2}\right) \cdot \cos\left(\frac{a-b}{2}\right) \\ \sin(a) - \sin(b) &= 2 \cdot \sin\left(\frac{a-b}{2}\right) \cdot \cos\left(\frac{a+b}{2}\right) \\ \cos(a) + \cos(b) &= 2 \cdot \cos\left(\frac{a+b}{2}\right) \cdot \cos\left(\frac{a-b}{2}\right) \\ \cos(a) - \cos(b) &= -2 \cdot \sin\left(\frac{a+b}{2}\right) \cdot \sin\left(\frac{a-b}{2}\right) \\ \tan(a) &\pm \tan(b) &= \frac{\sin(a \pm b)}{\cos(a)\cos(b)} \end{aligned}$$

8.4 Doppel- und Halbwinkel

$$\begin{aligned} &\sin(2a) = 2\sin(a)\cos(a) \\ &\cos(2a) = \cos^2(a) - \sin^2(a) = 2\cos^2(a) - 1 = 1 - 2\sin^2(a) \\ &\cos^2\left(\frac{a}{2}\right) = \frac{1 + \cos(a)}{2} &\sin^2\left(\frac{a}{2}\right) = \frac{1 - \cos(a)}{2} \end{aligned}$$

8.6 Produkte

$$\sin(a)\sin(b) = \frac{1}{2}(\cos(a-b) - \cos(a+b))$$

$$\cos(a)\cos(b) = \frac{1}{2}(\cos(a-b) + \cos(a+b))$$

$$\sin(a)\cos(b) = \frac{1}{2}(\sin(a-b) + \sin(a+b))$$

8.7 Reihenentwicklungen

$$\begin{array}{lll} \textbf{Geometrische Reihe} & \sum\limits_{n=0}^{\infty} x^n & = \frac{1}{1-x} & |x| < 1 \\ & \sum\limits_{k=0}^{\infty} k \, x^k & = x \sum\limits_{k=1}^{\infty} k \, x^{k-1} = \frac{x}{(1-x)^2} & x \neq 1 \\ \\ \textbf{Binominalreihe} & \sum\limits_{n=0}^{\infty} \binom{\alpha}{n} x^n & = (1+x)^{\alpha} & x \in (-1,1) \\ \textbf{E-Funktion} & \sum\limits_{k=0}^{\infty} \frac{x^k}{k!} & = e^x \\ \end{array}$$

8.8 Eigenschaften unterschiedlicher Schwingungsformen

Form	Funktion	Gleichrichtwert	Formfaktor	Effektivwert	Scheitelfaktor	X_0	X^2	var(X)
Formel		$\overline{ x } = \frac{1}{T} \int_0^T x(t) dt$	X X	$X = \sqrt{X^2} = \sqrt{\frac{1}{T}} \int_{t_0}^{t_0 + T} x^2(t) dt$	$k_s = rac{X_{ m max}}{X_{ m eff}}$			
	$A \cdot \sin(t)$	$\frac{2}{\pi} \approx 0.637$	$\frac{\pi}{2\sqrt{2}} \approx 1.11$	$\frac{1}{\sqrt{2}} \approx 0.707$	$\sqrt{2} \approx 1.414$	0	$\frac{A^2}{2}$	$\frac{A^2}{2}$
	$A\cdot \sin(t) $	$\frac{2}{\pi} \approx 0.637$	$\frac{\pi}{2\sqrt{2}} \approx 1.11$	$\frac{1}{\sqrt{2}} \approx 0.707$	$\sqrt{2} \approx 1.414$	$\frac{2A}{\pi}$	$\frac{A^2}{2}$	$\frac{A^2}{2} - \frac{4A^2}{\pi^2}$
	$\begin{cases} A \cdot \sin(t) & 0 < t < \pi \\ 0 & \text{True} \end{cases}$	$\frac{1}{\pi} \approx 0.318$	$\frac{\pi}{2} \approx 1.571$	$\frac{1}{2} = 0.5$	2	A #	$\frac{A^2}{4}$	$\frac{A^2}{4} - \frac{A^2}{\pi^2}$
	$A\cdot \Lambda(t)$	$\frac{1}{2} = 0.5$	$\frac{2}{\sqrt{3}} \approx 1.155$	$\frac{1}{\sqrt{3}} \approx 0.557$	$\sqrt{3} \approx 1.732$	0	$\frac{A^2}{3}$	$\frac{A^2}{3}$
	$\begin{cases} A & 0 < x < t \\ 0 & \text{True} \end{cases}$	1	1	1	1	0	A^2	A^2
	1	1	1	1	1	ı	1	ı
		$T_1^{t_1}$	$\sqrt{rac{T}{t_1}}$	$\sqrt{rac{t_1}{T}}$	$\sqrt{rac{T}{t_1}}$	$A rac{t}{T}$	$A^2 rac{t}{T}$	$\frac{A^2t}{T} - \frac{A^2t^2}{T^2}$