



# COMPUTER VISION

**Le Thanh Ha, Ph.D**

Assoc. Prof. at University of Engineering and Technology,  
Vietnam National University

[ltha@vnu.edu.vn](mailto:ltha@vnu.edu.vn); [lthavnu@gmail.com](mailto:lthavnu@gmail.com); 0983 692 592

# Hough Transform

- The Hough transform is a technique which can be used to isolate features of a particular shape: line, circle, ellipse
- It uses a voting mechanism in parameter space for detecting these shapes.

# Parameter function

- Object in image can be defined by a function of parameter (parameter function):

$$f(x, y, a_1, \dots, a_n) = 0$$

In which  $(x, y)$  is pixel position and  $\{a_1, \dots, a_n\}$  is a set of  $n$  parameters specifying the shape.

# Hough transform

- Transform to parameter space:
  - Every pixel  $(x,y)$  in a 2D image is transformed into a surface of  $n$ -dimension space.
- Transform back to image spatial space:
  - Every point  $(a_1, \dots, a_n)$  in the  $n$ -dimension space is transformed to a shape in the 2D image.

# Line detection

- Parameter function to be used:

$$y = ax + b$$

- It is a line in Image
- But, it is only a point (a,b) in parameter space.

# Line Detection

- A point  $(x_0, y_0) \in I$  corresponds to the line  $a = y_0/x_0 - b/x_0$  in the parameter space  $P$ .
- And, a point  $(a_0, b_0) \in P$  corresponds to the line  $y = a_0x + b_0$  in image space  $I$ .

# Line detection

- Construct the line function of following image by hough transform:

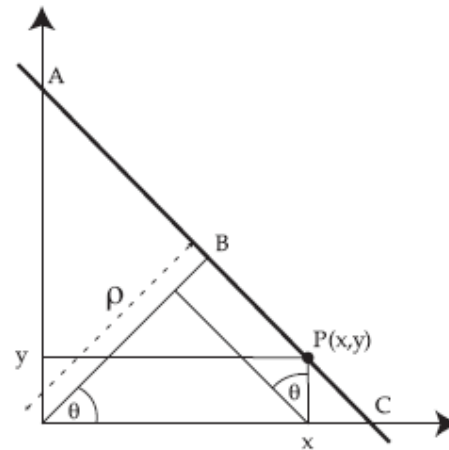
0	0	1	0
0	1	0	0
1	0	0	0
0	0	0	0

# Line detection

- Another line function can be used:

$$f(x, y, \rho_0, \theta_0) = x \cos \theta_0 + y \sin \theta_0 - \rho_0 = 0$$

in which  $\rho_0$  là is the distance between line and the origin, and  $\theta_0$  là is the angle between the distance vector and Ox axis.





# Line detection algorithm

- Create a 2D matrix  $H(\rho, \theta)$  for the parameter space. Initialize the matrix with 0 values.
- Find gradient  $G(x,y)$  of  $I(x,y)$
- For pixel  $(x,y)$  satisfies  $|G(x,y)| > T_s$ , increase  $H(\rho, \theta)$  such that:

$$\forall \theta \quad | \quad \rho = x \cos \theta + y \sin \theta$$

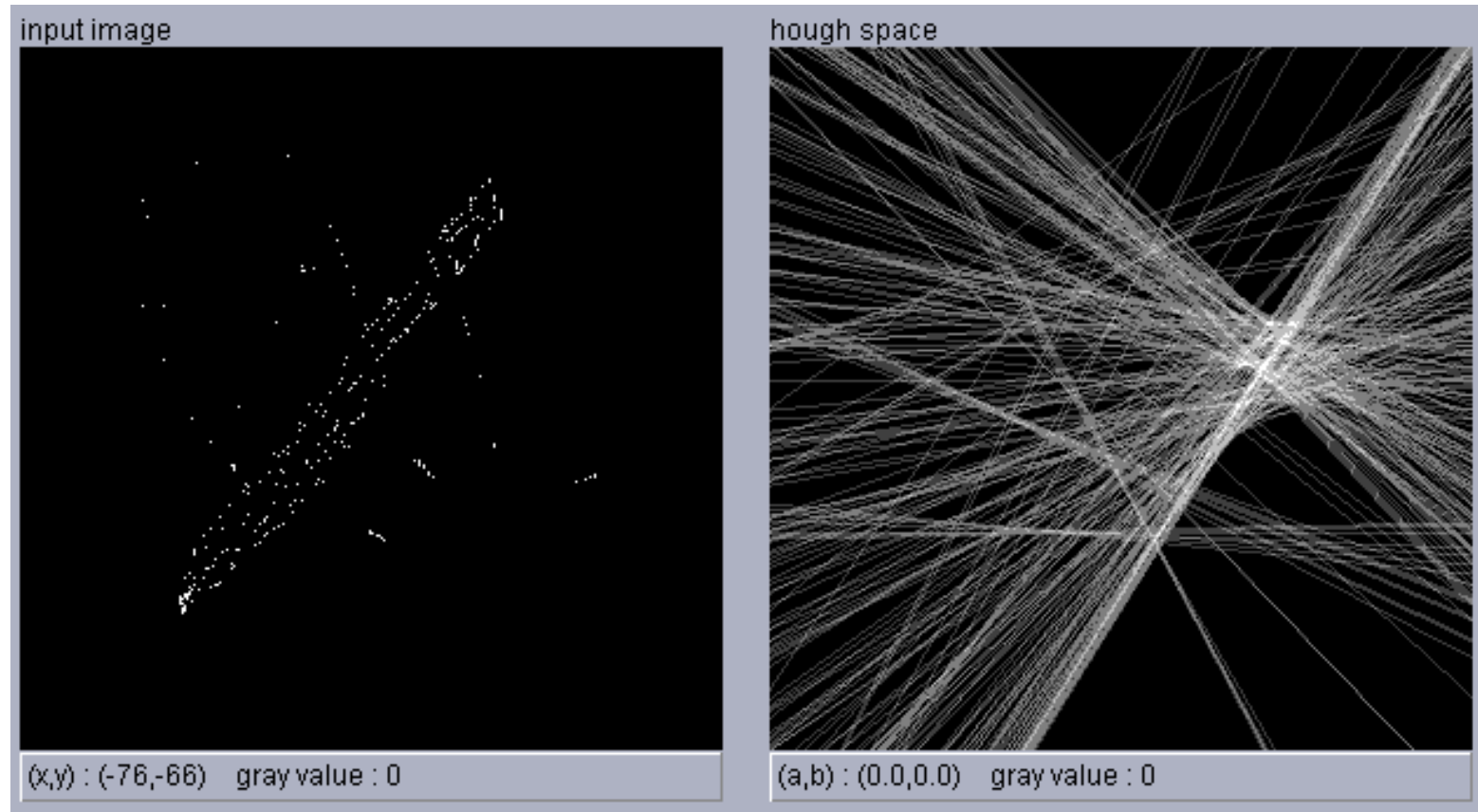
$$H(\rho, \theta) = H(\rho, \theta) + 1;$$

# Line detection algorithm

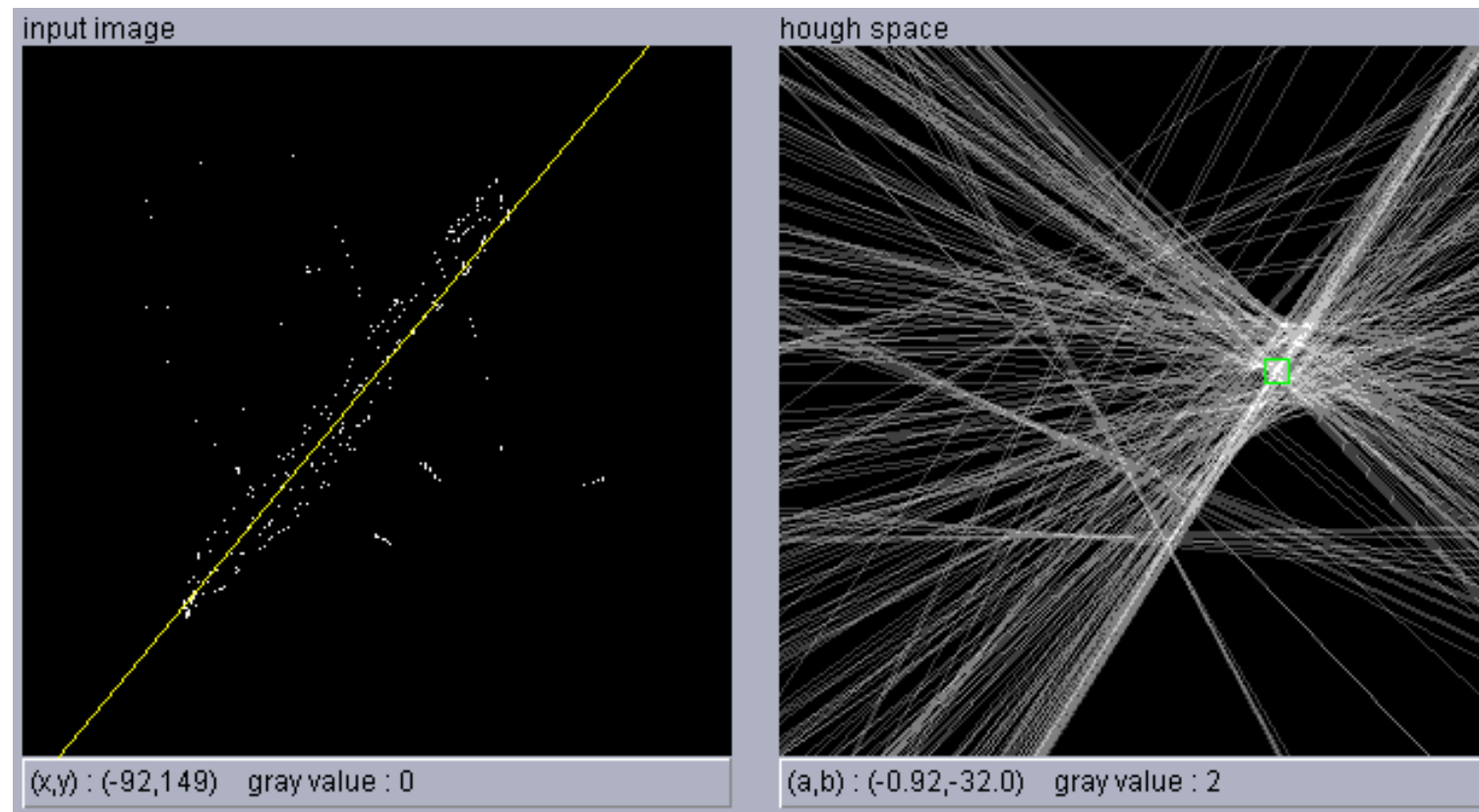
- This algorithm can be improved by using the gradient angle  $\angle G(x,y)$

$$\begin{aligned} \forall \theta \quad & | \quad \angle G(x,y) - \Delta\theta \leq \theta \leq \angle G(x,y) + \Delta\theta \\ & \rho = x \cos \theta + y \sin \theta \\ & H(\rho, \theta) = H(\rho, \theta) + 1; \end{aligned}$$

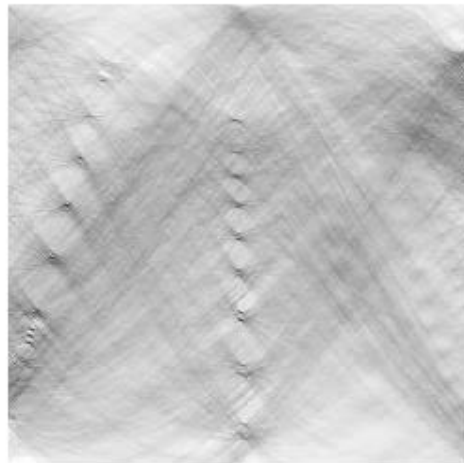
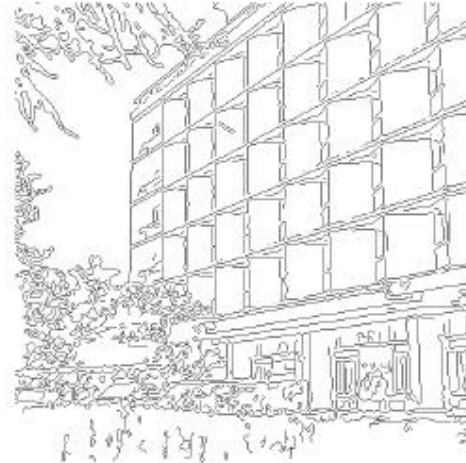
# Line detection



# Line detection



# Line detection



# Circle detection

- A circle with radius  $R$  at position  $(a,b)$  can be specified by a function:

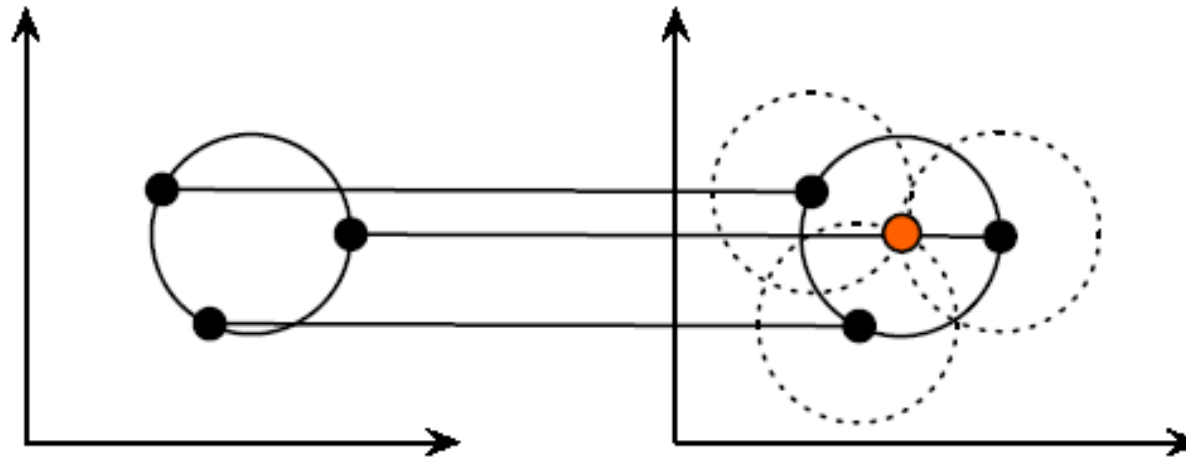
$$x = a + R \cos(\theta)$$

$$y = b + R \sin(\theta)$$

- $\theta$  scans from 0 to 360 degree
- A pixel belonging to a circle corresponds to a set of  $(a,b,\theta)$

# Circle detection

- With a fixed radius  $R$ ,  $(a,b)$  has a circle orbit centered at  $(x,y)$  and radius  $R$ .
- The coordinate of the real circle can be found by a cumulative table



# Circle Detection

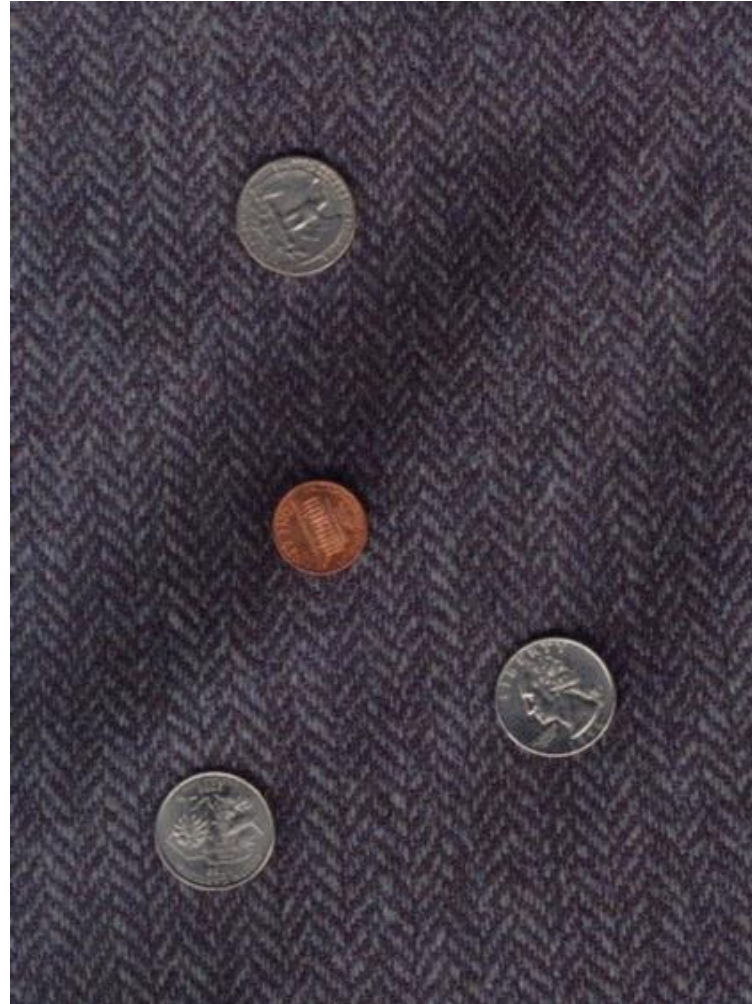
- For pixel  $(x,y)$  satisfies  $|G(x,y)| > T_s$ , increase the cumulative table in parameter space such that:

$$\forall R, \quad \begin{cases} a = x \pm R \cos \angle G \\ b = y \pm R \sin \angle G \end{cases}$$

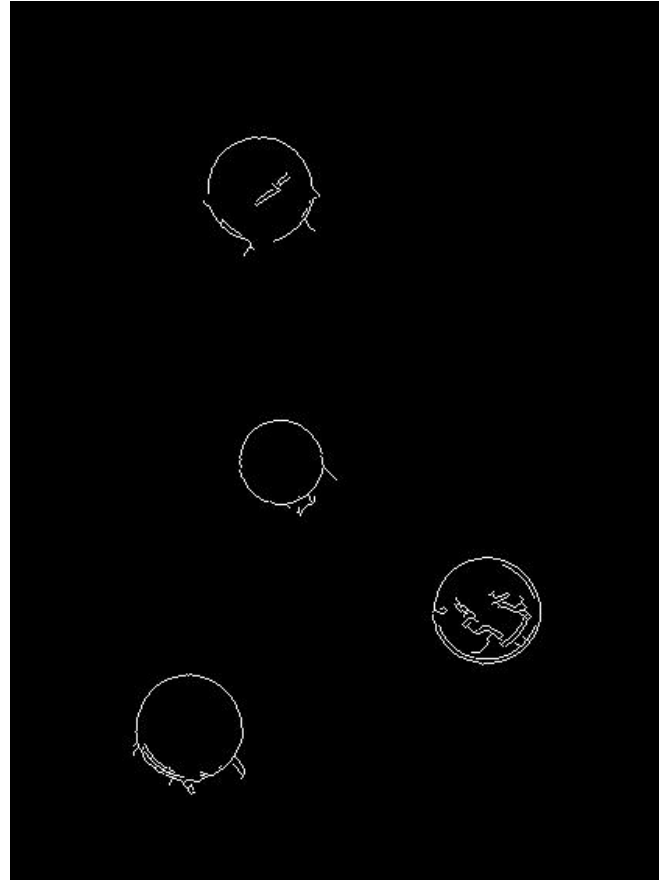
$$H(a,b,R) = H(a,b,R) + 1$$



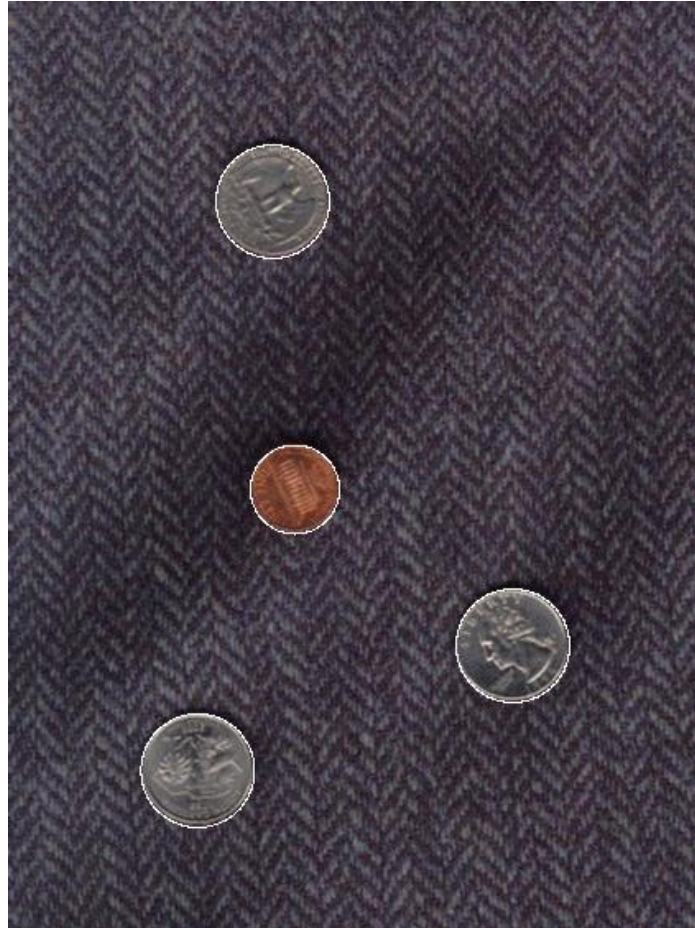
# Circle Detection



# Circle Detection



# Circle Detection



# Summary

- Hough transform
- It can be used to detect some image features having a particular shape properties: line, circle, ...