IMAGE RESTORATION

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Image Restoration

- Recovering the an degraded image using priori knowledge of degradation phenomenon.
- Modeling the degradation and applying the inverse process in order to recover the original image
- Applying in both spatial and frequency domains

-Image degradation/restoration process

-Noise models

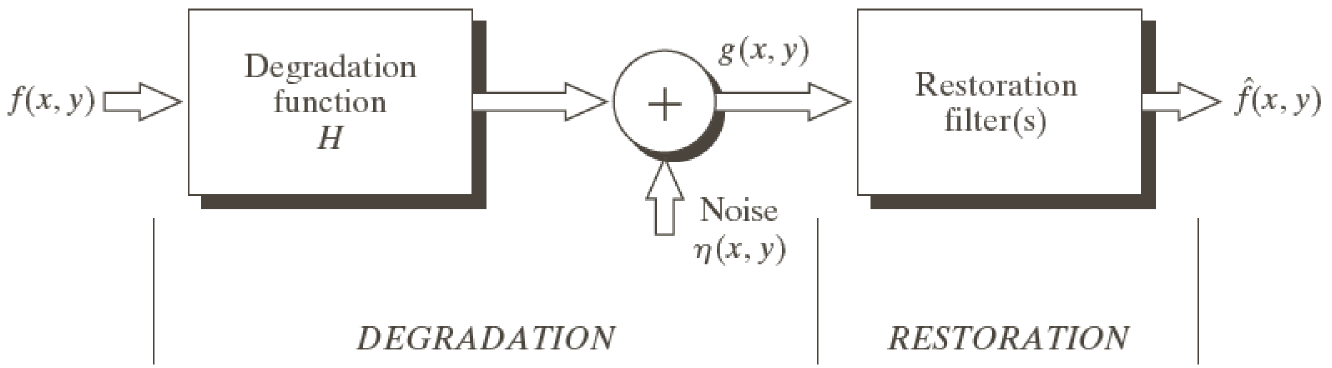
-Periodic Noise

-Estimation of Noise Parameters

DEGRADATION MODELING

Image degradation/restoration process

FIGURE 5.1 A model of the image degradation/restoration process.



$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

Noise models

- The performance of imaging sensors is affected by a variety of factors:
  - Environmental conditions: background light levels
  - Sensor temperature
  - Electricity noise
  - …
- Need a noise model capable for a specific type of imaging devices

Gaussian noise

- Gaussian noise arises in an image due to factors such as electronic circuit and sensor noise due to low illumination or high temperature

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-(z-\bar{z})^2}{2\sigma^2}}$$

Rayleigh noise

- Reyleigh density is helpful in characterizing noise in range imaging (depth sensing)

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

Impulse noise

- Known as Salt-and-pepper noise and found in situations where quick transients take place when imaging (Faulty switching)

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

Exponential and Gamma noise

- **Finds their applications in laser imaging**

Gamma noise

$$p(z)=\begin{cases}\dfrac{a^b z^{b-1}}{(b-1)!}e^{-az} & \text{for } z\geq 0\\ 0 & \text{for } z<0\end{cases}$$

Exponential noise

$$p(z)=\begin{cases}e^{-az} & \text{for } z\geq 0\\ 0 & \text{for } z<0\end{cases}$$

Uniform noise

- Used for computer simulation noise

$$p(z)=\begin{cases}\dfrac{1}{b-a} & \text{if } a\le z\le b\\ 0 & \text{otherwise}\end{cases}$$

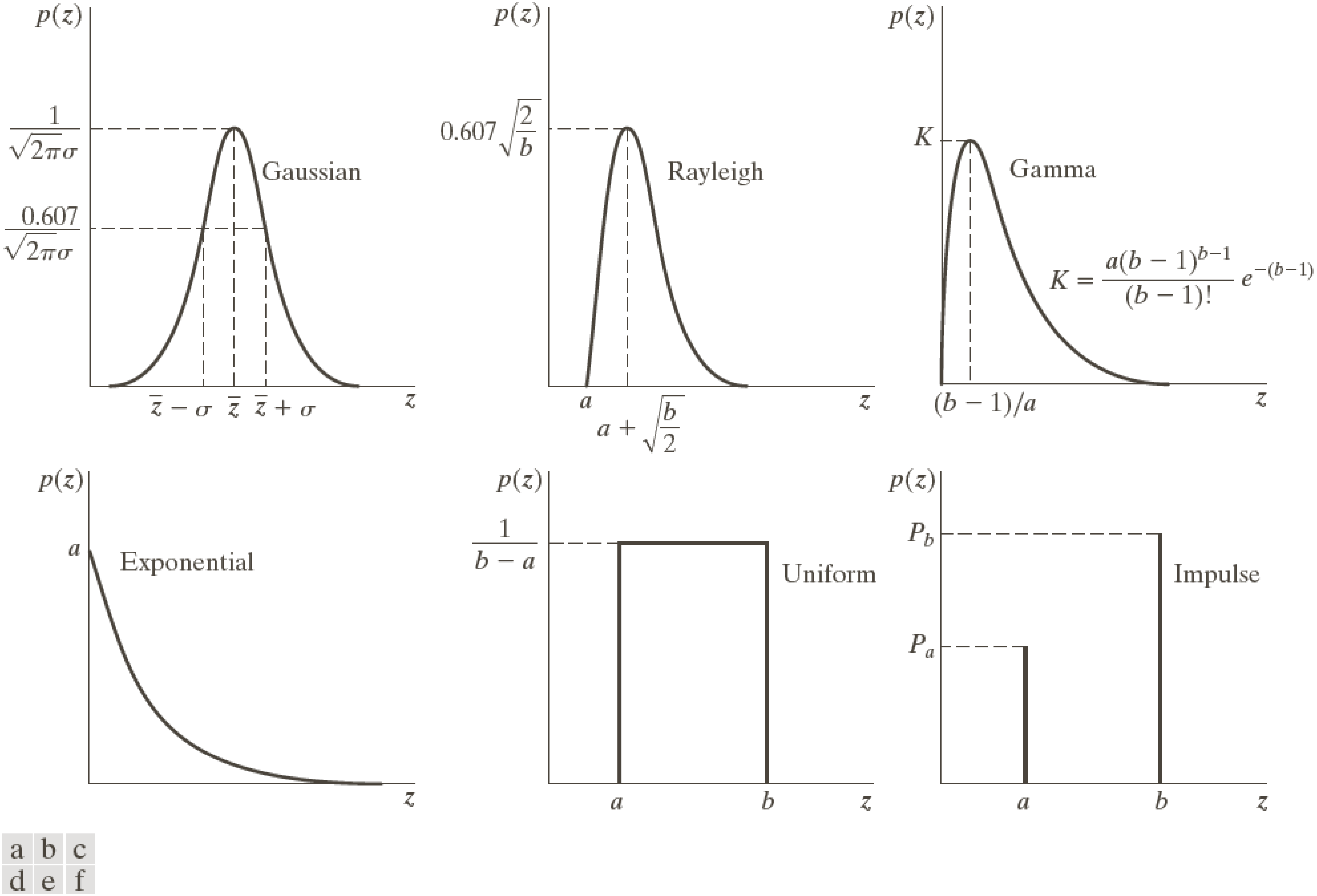


FIGURE 5.2 Some important probability density functions.

Test pattern

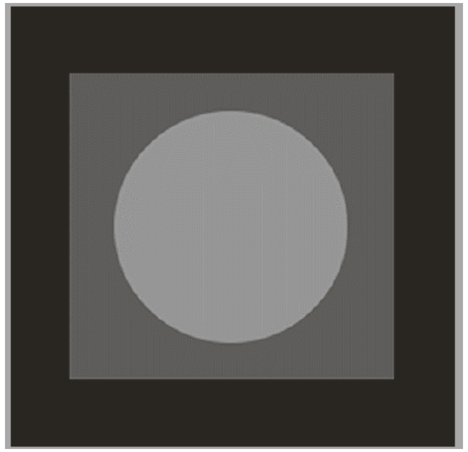


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

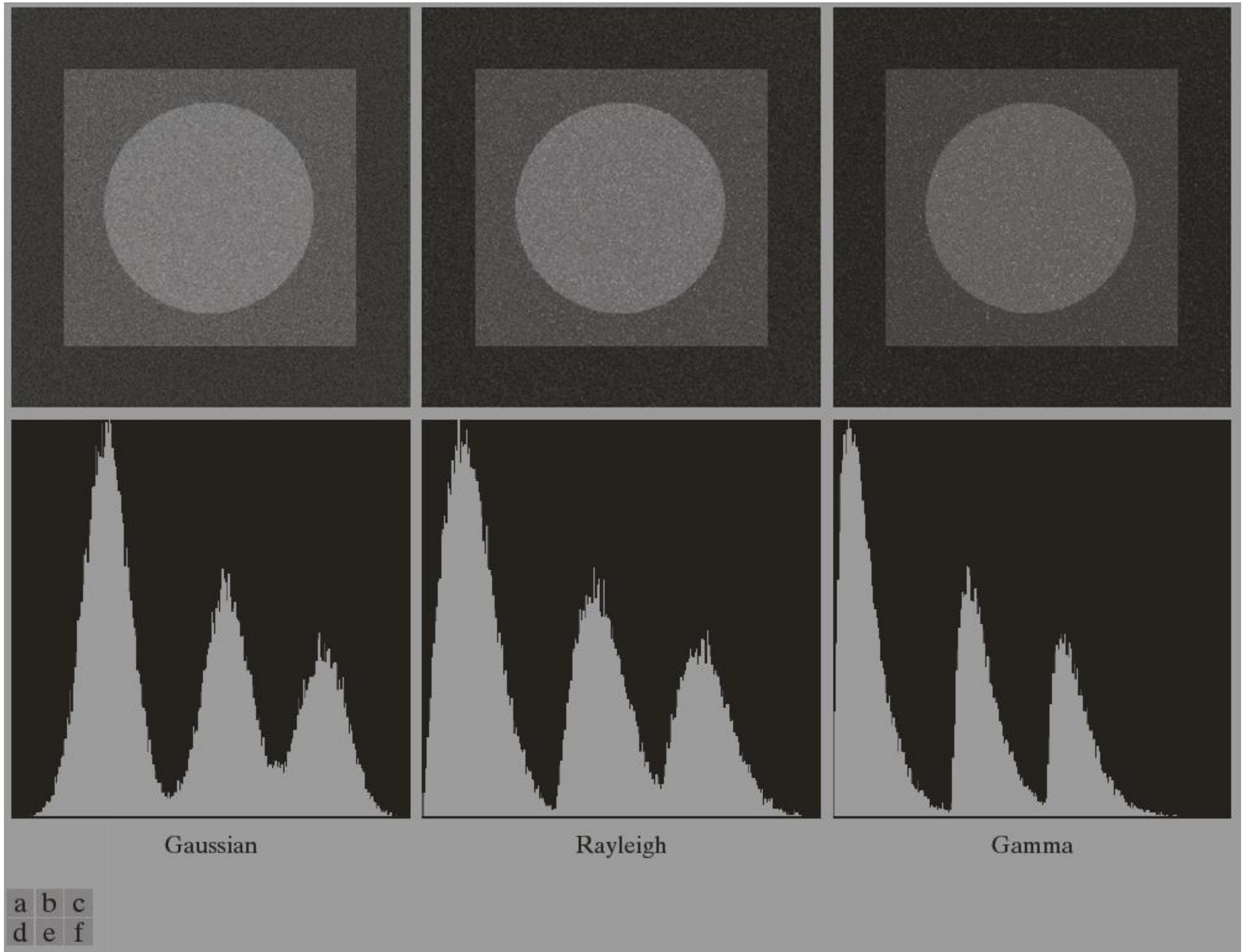


FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

HMI

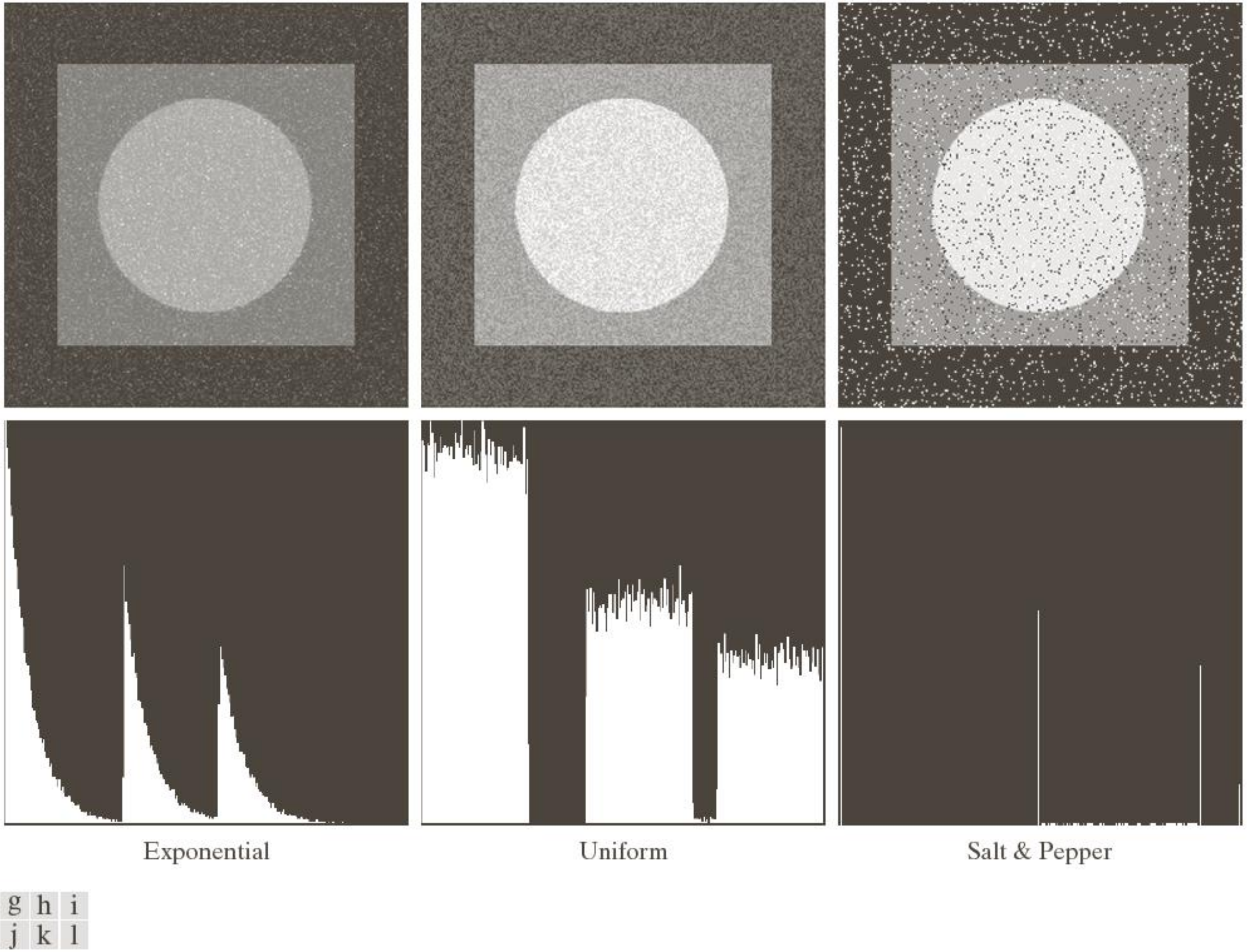
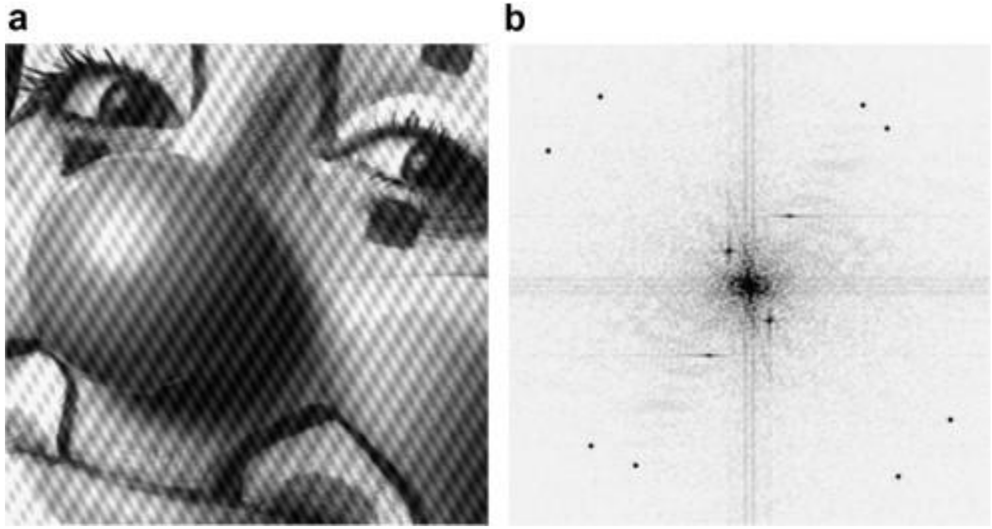
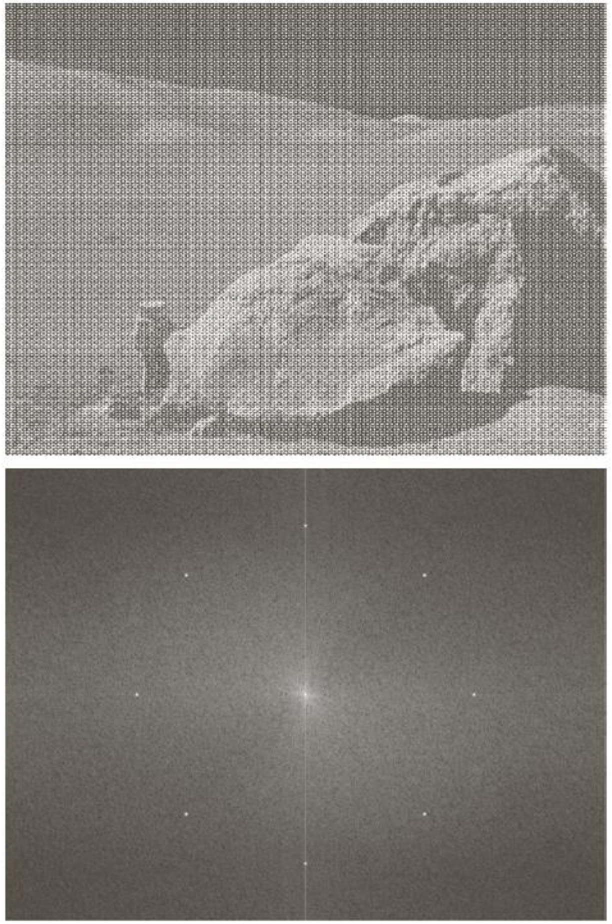


FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and salt and pepper noise to the image in Fig. 5.3.

Periodic noise

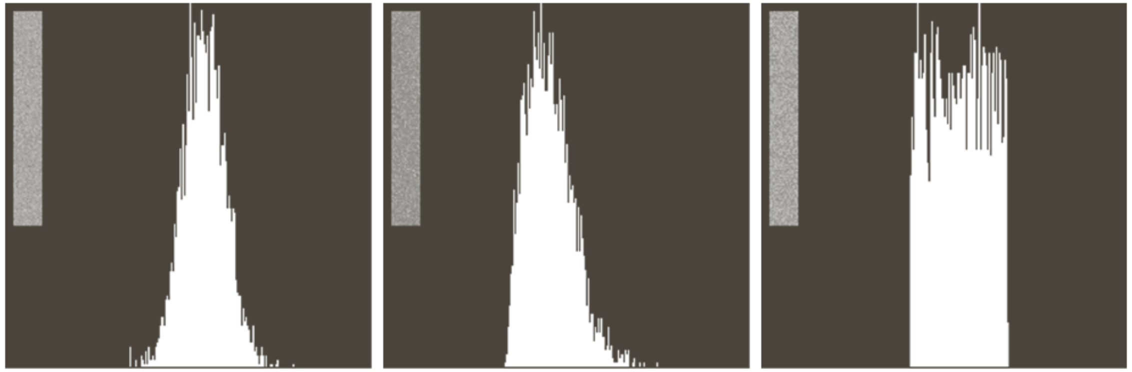
- Arises typically from electrical or electromechanical interference during image acquisition





Estimation of Noise Parameters

- Simple periodic noise parameters can be visually estimated in Fourier domain.
- Other types of noise can be estimated in spatial domain at smooth regions.



a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

-Mean filters

-Order statistic filters

-Adaptive filters

IMAGE RESTORATION

Mean filters

- Arithmetic mean filter
- Geometric mean filter
- Harmonic mean filter
- Contrahamonic mean filter

Arithmetic mean filter

- Smooths local variations in an images and noise is reduced as a result of blurring

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

Geometric mean filter

- Achieve smoothing comparable to the arithmetic mean filter but lose less image detail

$$\hat{f}(x, y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

Harmonic mean filter

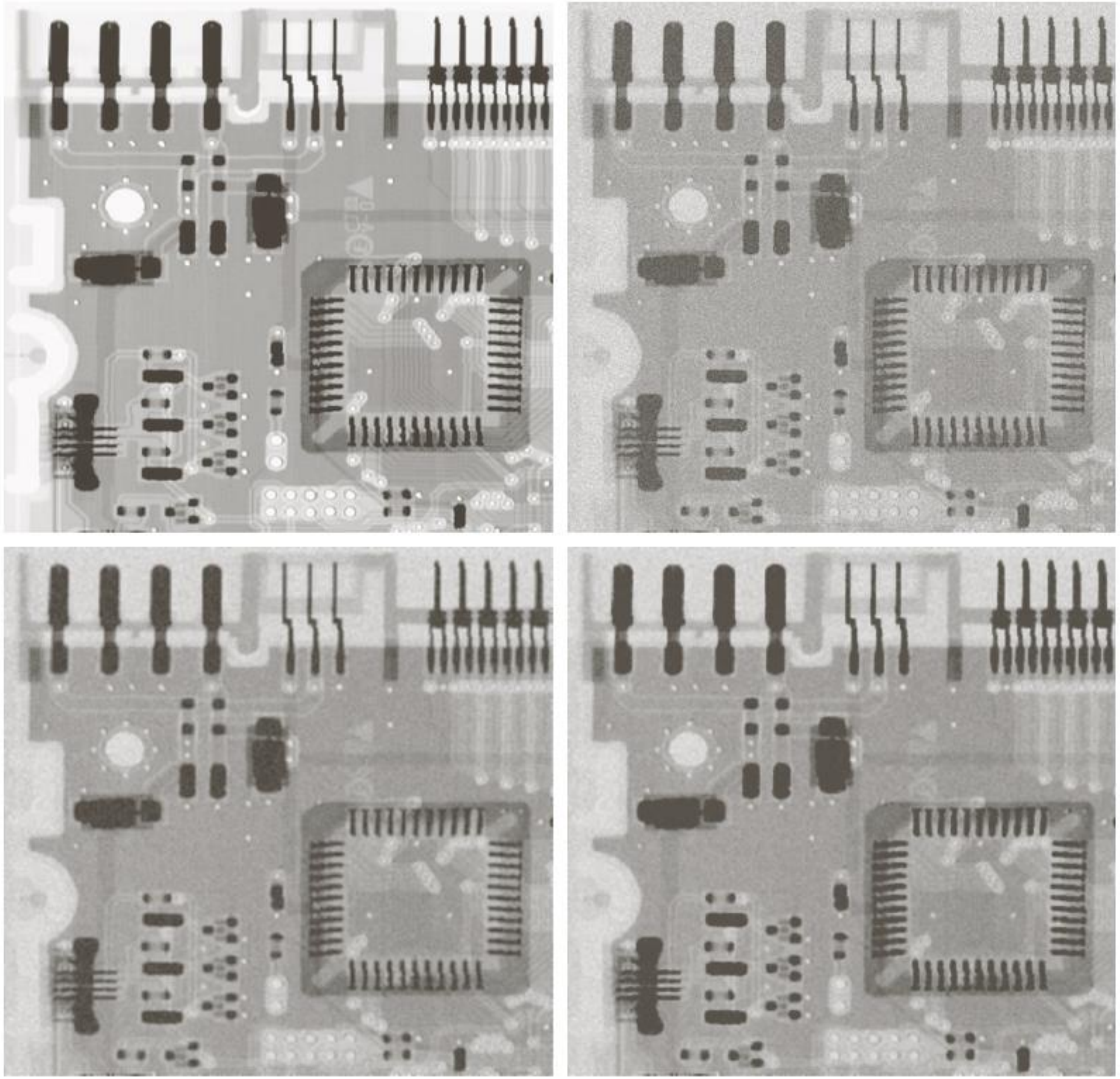
- Works well for salt noise but fails for pepper noise. It also does well with Gaussian noise.

$$\hat{f}(x, y) = \frac{mn}{\displaystyle\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

Contraharmonic mean filter

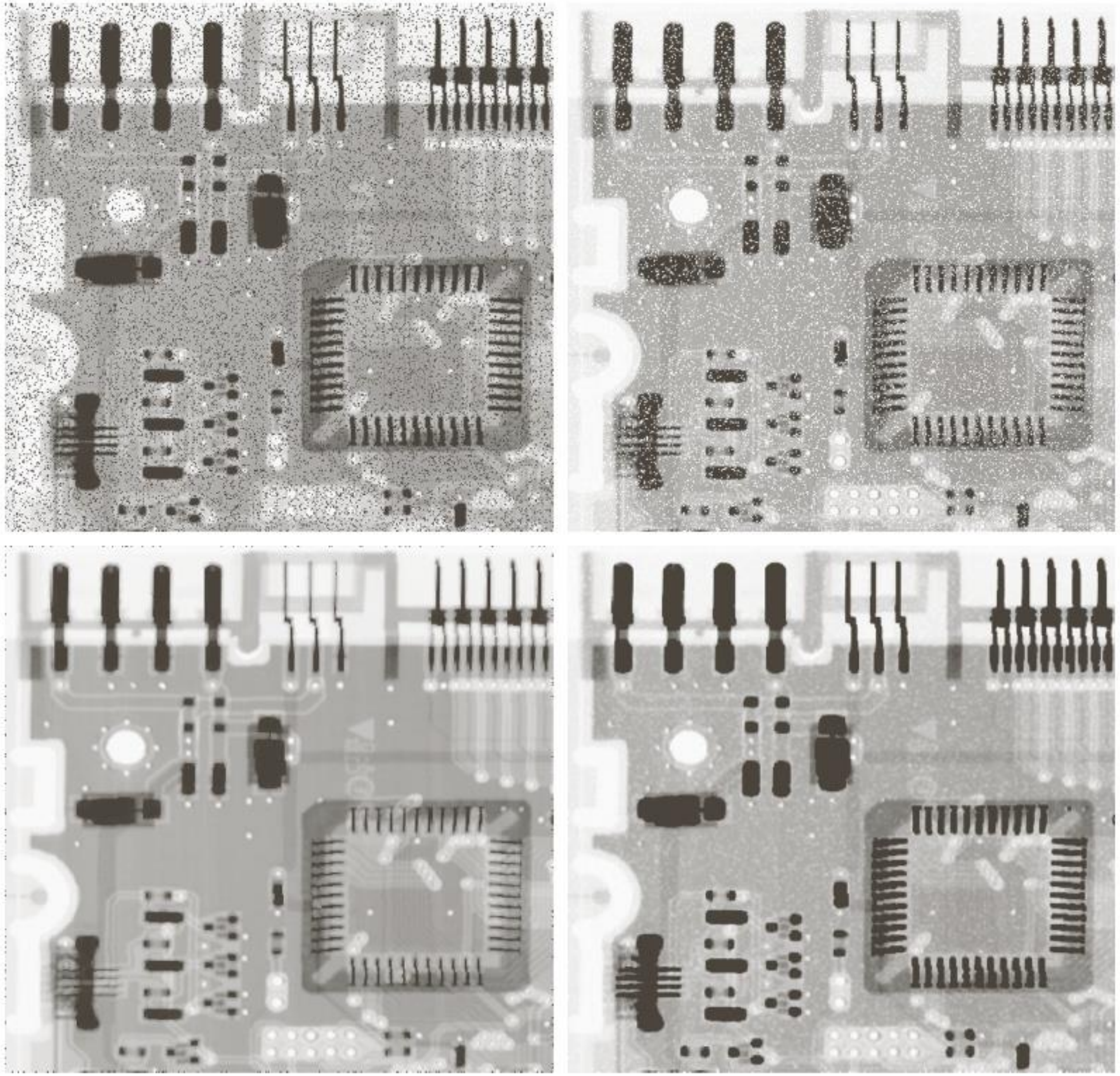
$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

- Q=0: Arithmetic mean filter
- Q=-1: Harmonic mean filter
- Q>0: the filter eliminates pepper noise
- Q<0: the filter eliminates salt noise



a	b
c	d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3×3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)



a b
c d

FIGURE 5.8
(a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contra-harmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

Order-Statistic Filters

- Median filter
- Max and min filters
- Midpoint filter
- Alpha-trimmed mean filter

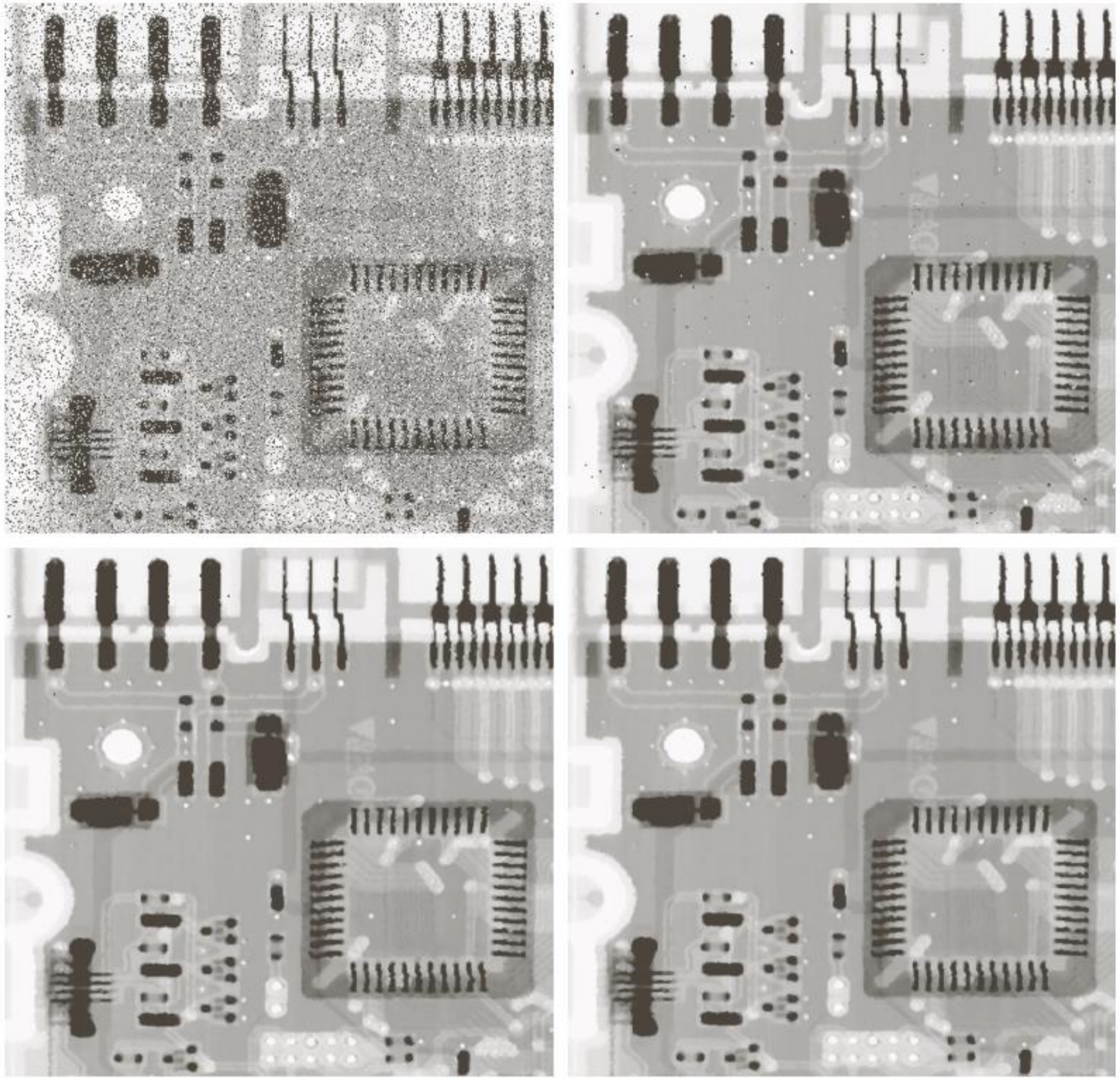
Median filter

- Excellent noise-reduction capability with considerably less blurring

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\text{median}}\{g(s,t)\}$$

a b
c d

FIGURE 5.10
(a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$.
(b) Result of one pass with a median filter of size 3×3.
(c) Result of processing (b) with this filter.
(d) Result of processing (c) with the same filter.



Max and min filters

- **Max filter is useful for finding brightest points in an image therefore for removing low values like pepper noise**

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\}$$

- Min filter is useful for finding darkest points in an image therefore for removing low values like salt noise

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$

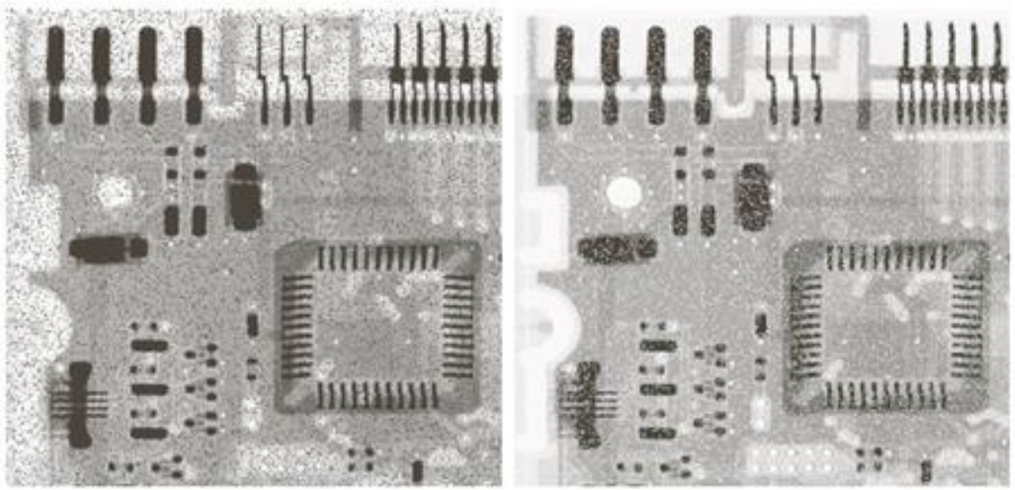
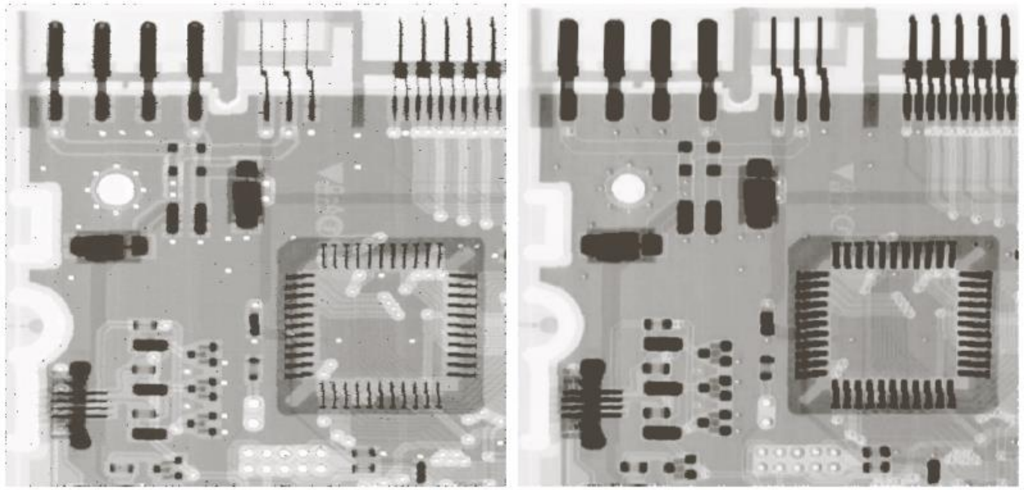


FIGURE 5.8
(a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability.



a b

FIGURE 5.11
(a) Result of filtering Fig. 5.8(a) with a max filter of size 3×3. (b) Result of filtering 5.8(b) with a min filter of the same size.

Midpoint filter

- Combines order statistics and averaging and works best for Gaussian or uniform noise

$$\hat{f}(x,y) = \frac{1}{2}\left[\min_{(s,t)\in S_{xy}}\{g(s,t)\} + \max_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

Alpha-trimmed mean filter

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t)$$

- $g_r(s,t)$ is obtained by trimming left and right lowest and highest values of g(s,t) in S_{xy}
- *d=0:* becomes arithmetic mean filter
- *d=mn-1:* becomes median filter
- Other value of d: useful for multiple types of noise: noise-and-pepper and Gaussian noises.

Adaptive Filters

- Filter behaviors change based on statistical characteristics of the image inside the filter region.
- Many types of adaptive filters:
  - **Adaptive local noise reduction filter**
  - **Adaptive median filter**
  - Minimum Mean Square Error (Wiener) Filter
  - Constrained Least Squares Filter
  - …

Adaptive local noise reduction filter

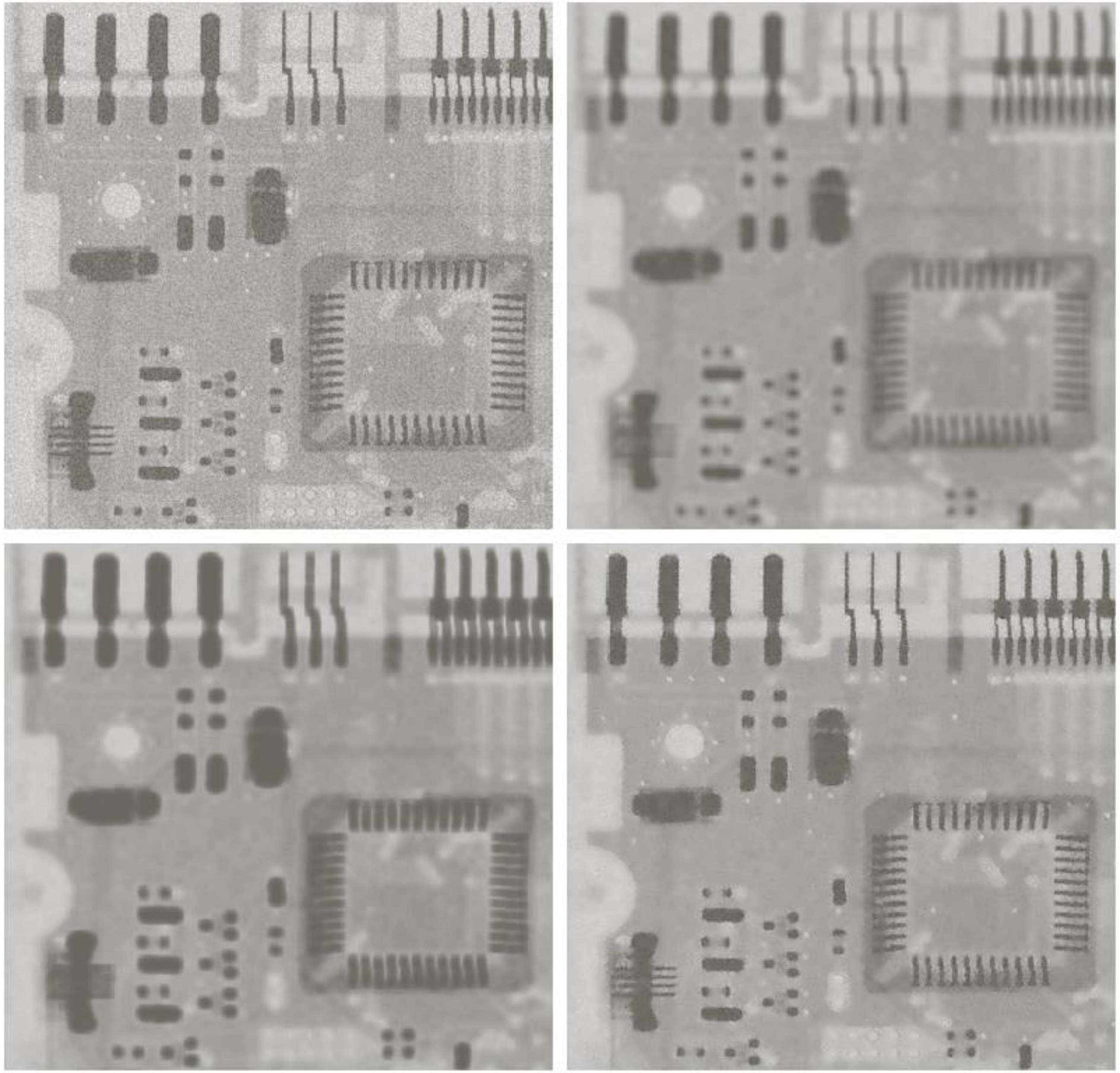
$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

σ_η^2 is the variance of the noise

σ_L^2 is the variance of pixels in S_{xy}

m_L is the local mean of pixels in S_{xy}

- If $\sigma_\eta^2 = 0$: no noise, filter returns g(x,y)
- If $\sigma_L^2 \gg \sigma_\eta^2$: edge regions, filter returns value close to g(x,y)
- if $\sigma_L^2 \approx \sigma_\eta^2$: local area has the same properties as the overall image, filter returns average value



a b
c d

FIGURE 5.13
(a) Image corrupted by additive Gaussian noise of zero mean and variance 1000.
(b) Result of arithmetic mean filtering.
(c) Result of geometric mean filtering.
(d) Result of adaptive noise reduction filtering. All filters were of size 7×7.

Adaptive median filter

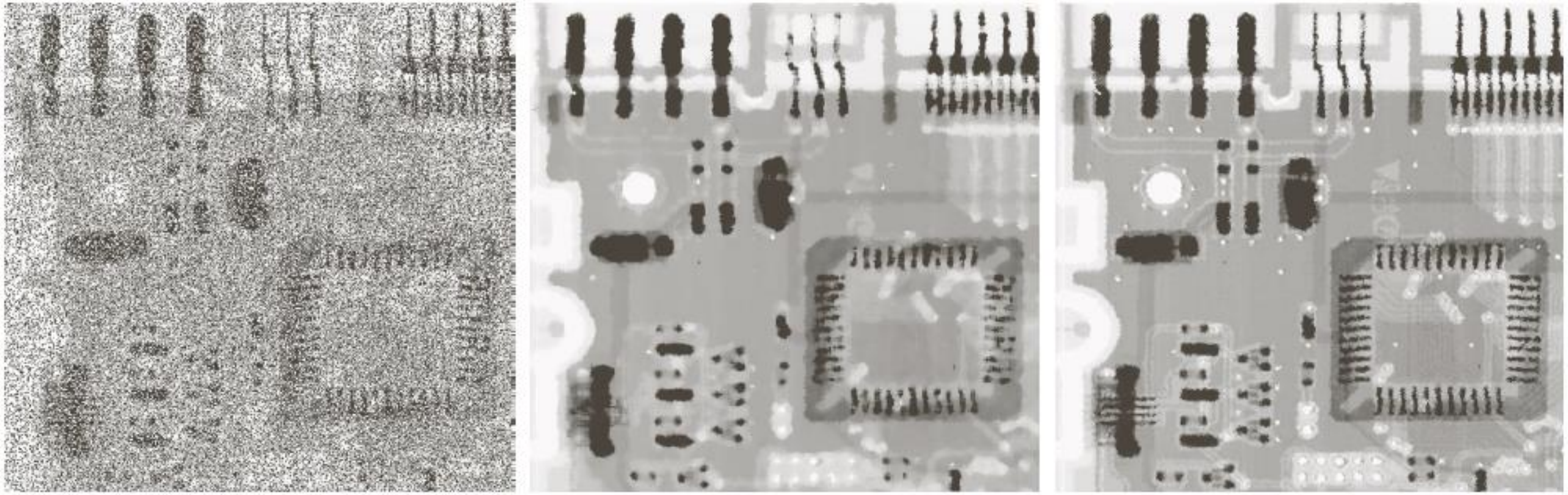
- z_{min} = minimum intensity value in S_{xy}
- z_{max} = maximum intensity value in S_{xy}
- z_{med} = median intensity value in S_{xy}
- z_{xy} = intensity value at (x,y)
- S_{max} = maximum allowed size of S_{xy}

State A:
$A1 = z_{med} - z_{min}$
$A2 = z_{med} - z_{max}$
if A1>0 and A2<0, goto state B
Else increase the window size
if window size $\leq S_{max}$ repeat state A
Else output z_{med}

State B:
$B1 = z_{xy} - z_{min}$
$B2 = z_{xy} - z_{max}$
If B1>0 and B2<0, output z_{xy}
Else output z_{med}

Adaptive median filter

- Statistically consider z_{min} and z_{max} are "impulse-like" components.
- State A determines if z_{med} is an impulse or not
  – If YES is an impulse increase neighbor window Sxy and repeat.
  – If NO, go to state B.
- State B determines if center pixel z_{xy} is an impulse or not
  – If YES, return z_{med}
  – If NO, return z_{xy}



a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

-Bandreject filters

-Notch filters

PERIODIC NOISE REDUCTION IN FREQUENCY DOMAIN

Bandreject filters

- Ideal bandreject filter

$$H(u,v)=\begin{cases}1 & \text{if } D(u,v)<D_0-\frac{W}{2}\\ 0 & \text{if } D_0-\frac{W}{2}\le D(u,v)\le D_0-\frac{W}{2}\\ 1 & \text{if } D(u,v)>D_0+\frac{W}{2}\end{cases}$$

- Butterworth bandreject filter

$$H(u,v)=\frac{1}{1+\left[\frac{D(u,v)W}{D^2(u,v)-D_0^2}\right]^{2n}}$$

- Gaussian bandreject filter

$$H(u,v)=1-e^{-\frac{1}{2}\left[\frac{D^2(u,v)-D_0^2}{D(u,v)W}\right]^2}$$

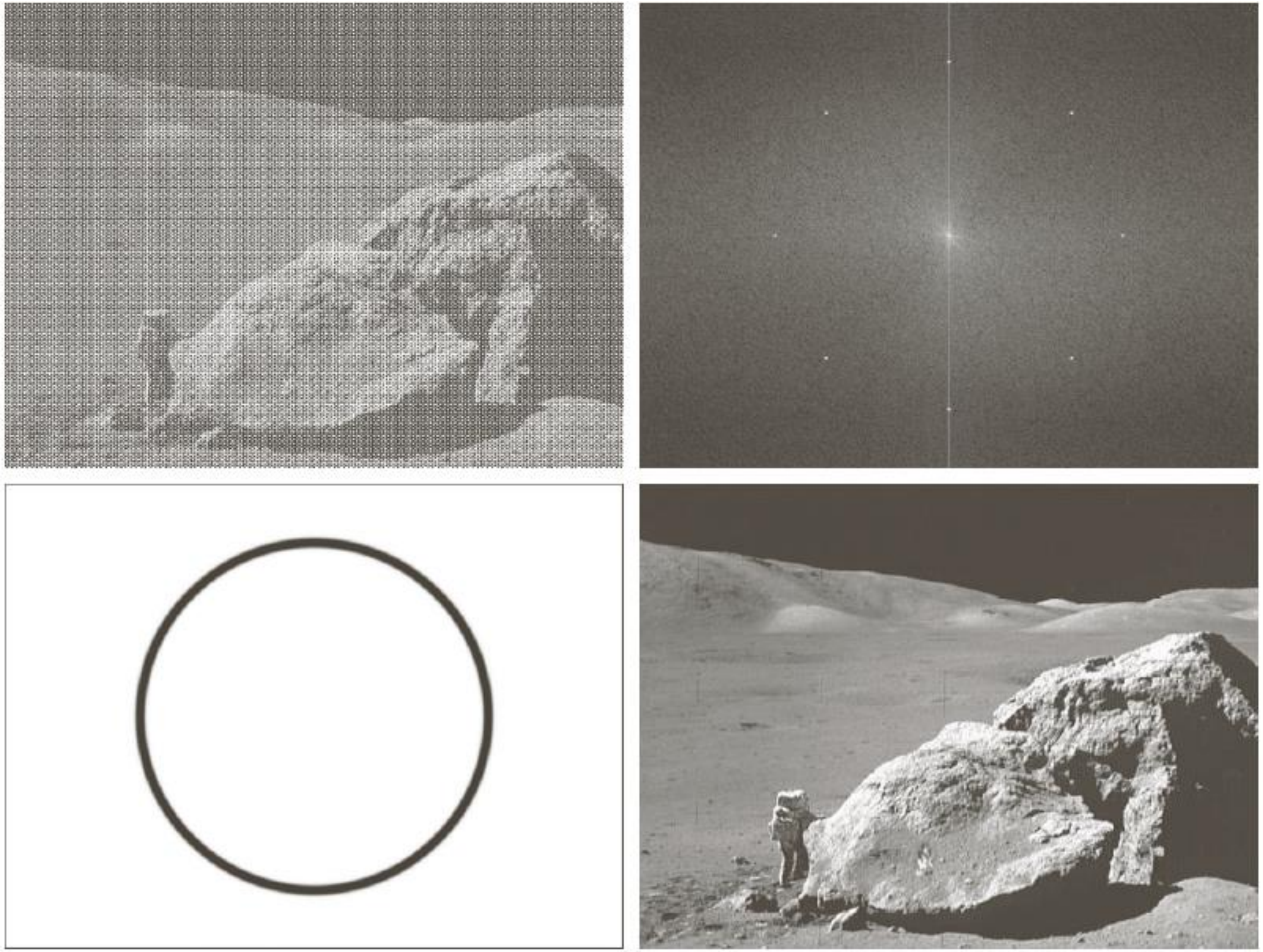
Band Reject filters

a b c

FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.

a b
c d

FIGURE 5.16
(a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)



Notch filter

- A notch filter reject frequencies in predefined neighborhoods about a center frequency.

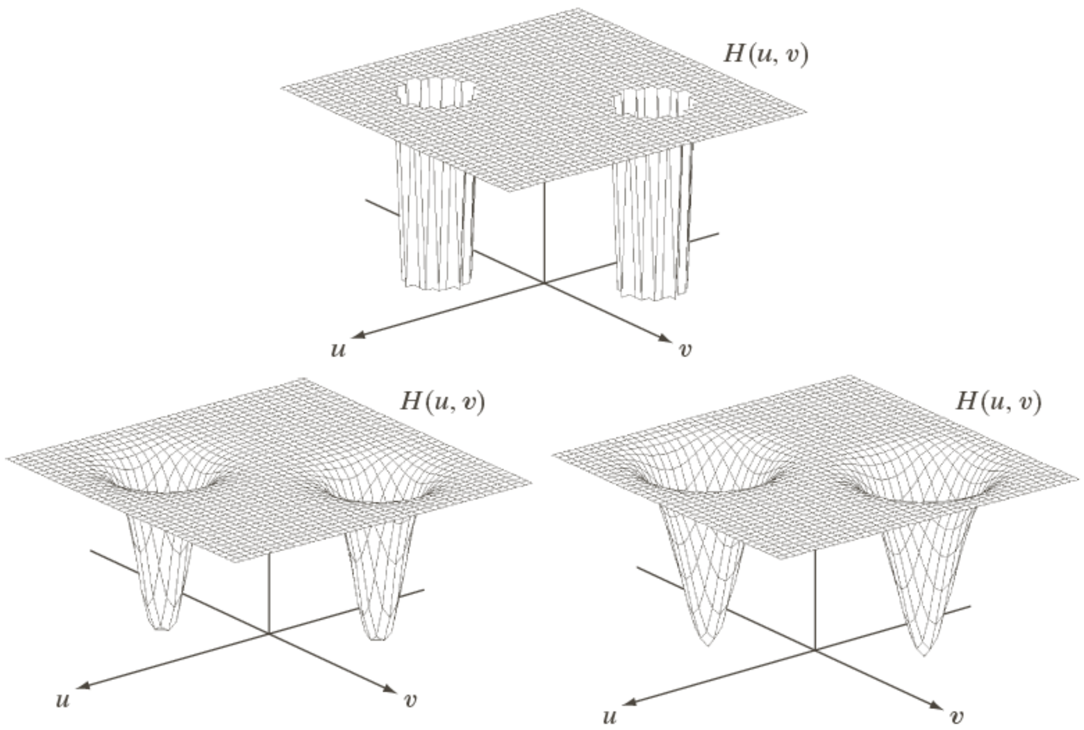


FIGURE 5.18 Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.

- Linear, Position-invariant Degradations
- Estimating the degradation function
- Inverse Filtering
- Different approaches

DEGRADATION ESTIMATION

Linear, position-invariant Degradation

- The linear, position-invariant Degradation can be written in convolution term:

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

where $\eta(x,y)$ is position-invariant noise, h(x,y) is a degradation model. Many types of degradations can be approximated by this process.

Restoration seeks to find filters that apply the process in reverse (Deconvolution filters)

Estimation by image observation

- Estimation by image observation
  - Degradation system H is completely characterized by its impulse response
  - Select a small section from the degraded image

$$g_s(x, y)$$

  - Reconstruct an unblurred image of the same size

$$\hat{f}_s(x, y)$$

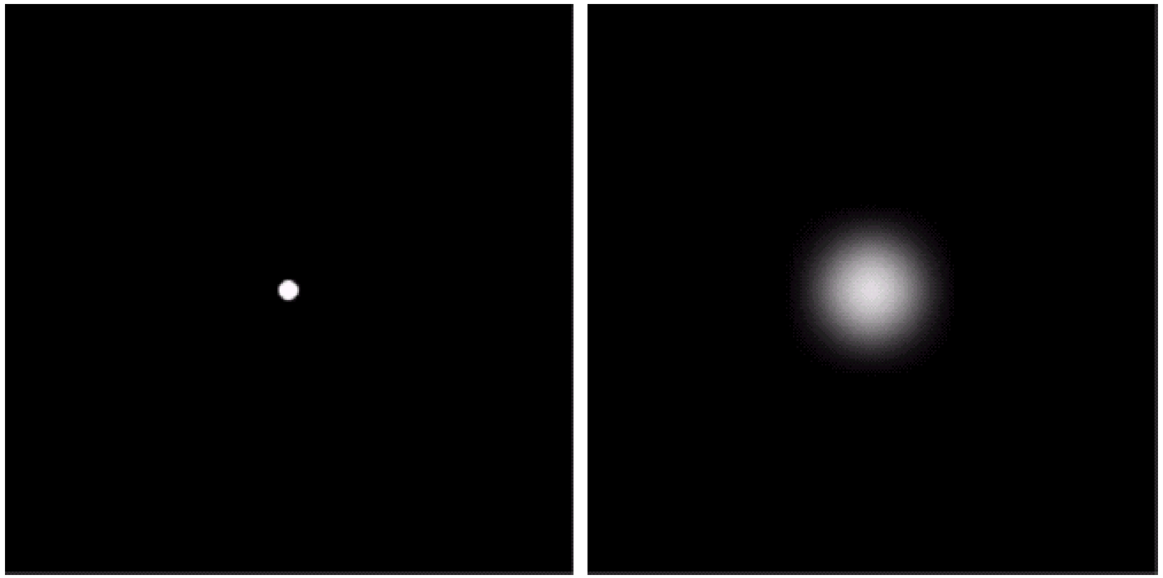
  - The degradation function can be estimated by

$$H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$$

Estimation by experimentation

Point spread function (PSF)

- Used in optics
- The impulse becomes a point of light
- The impulse response is commonly referred to as the PSF



a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

$$H(u, v) = \frac{G(u,v)}{A},$$

Estimation by modeling

- Atmospheric turbulence

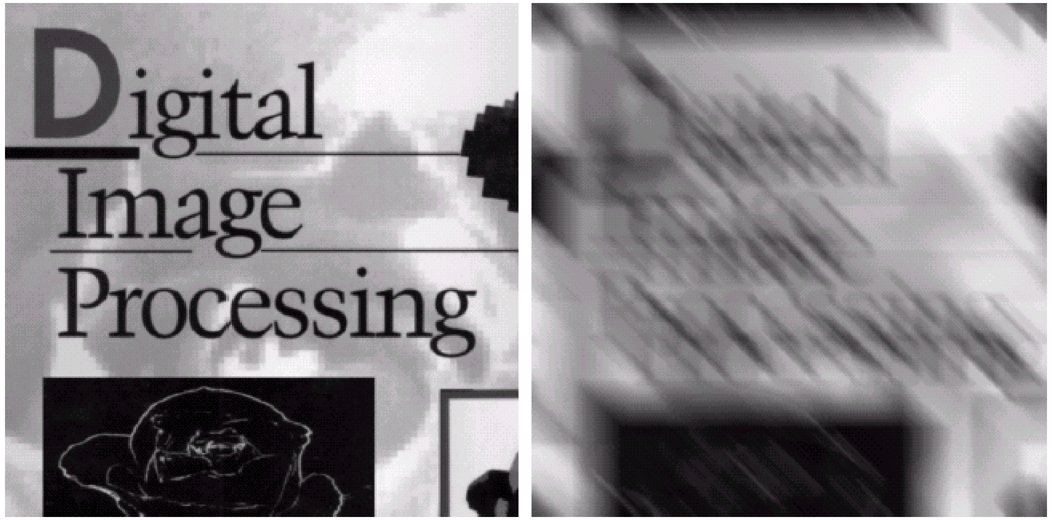
a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

Estimation by modeling

- Linear motion blurring



a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

$$H(u,v) = \frac{T}{\pi(ua+vb)} \cdot \sin[\pi(ua+vb)]e^{-j\pi(ua+vb)}$$

Inverse filtering

- Degradation model

$$g(x,y) = f(x,y) \otimes h(x,y) + \eta(x,y)$$
$$G(u,v) = F(u,v)H(u,v) + N(u,v)$$

- Inverse filter

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} + \frac{N(u,v)}{H(u,v)}$$
$$= F(u,v) + \frac{N(u,v)}{H(u,v)}$$